Simulating the Aurora

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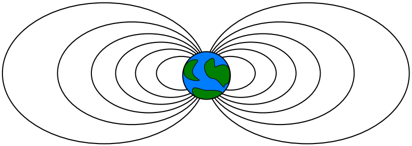
What is the Aurora?

- Solar wind sends charged particles (electrons) toward Earth
- Earth's magnetic field funnels the electrons to the poles and accelerates them downward toward Earth
- Electrons travel at high speeds (10 keV)



Colours

- electrons excite atomic oxygen and nitrogen
- excited state decays, emits photon
- wavelength depends on atom type:
  - O: red & green
  - N: blue (rare)

Colours

- Excited states take varying times to decay:
  - Red: 100+ seconds
  - Green: 0.5 seconds
  - Blue: 0.01 seconds
- Between excitation and emission, if atom collides with another atom, energy will be dissipated without photon emission

- Long-lived photons (red) produced only in low density (high altitude)

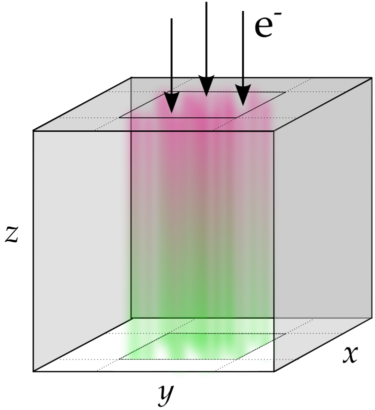
Structure

- **Swirls:** ionospheric currents, density variations
- **Vertical banding:** Collective electric field of the electrons

Aside from accelerating the particles, magnetic field has very little effect.

The simulation box

- $200 \times 200 \times 200$ simulation box (each voxel ~ 1 km^3)
- Periodic boundary conditions in x and y.
- Electron flux through top
- track the electrons as they progress through time



Algorithm: Motion

- Each (non-interacting) electron has a
  - position vector $\mathbf{x}$,
  - velocity vector $\mathbf{v}$,
  - force vector $\mathbf{F}$
- External influences (e.g. $\mathbf{B}$, $\mathbf{E}$ field) can interact through $\mathbf{F}$.

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- Basic Newton integration used to time-evolve electrons:

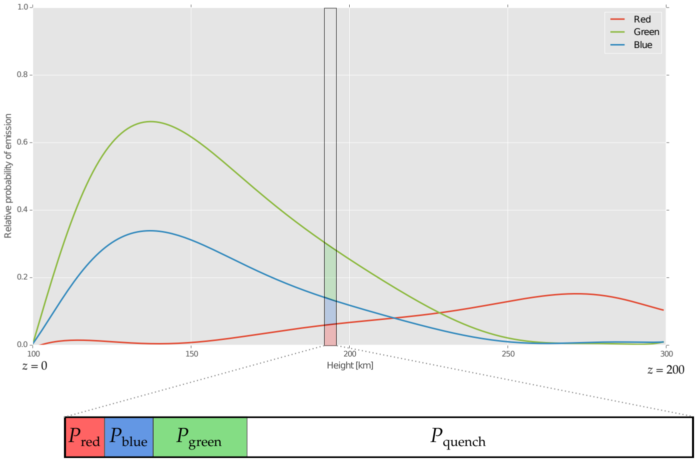
$$\mathbf{v}(t+dt) = \mathbf{v}(t) + \frac{\mathbf{F}}{m}dt; \qquad \mathbf{x}(t+dt) = \mathbf{x}(t) + \mathbf{v}(t+dt)dt.$$

Algorithm: “Interaction” with atoms

Stochastic approach to emission

- **Monte Carlo:** probability of a collision depends on height (ie: $P_{\text{emit}} = P_{\text{emit}}(z)$)
- If there is a collision, use **Kinetic Monte Carlo** to determine if a photon is emitted, and the colour. Rates dependent on height.

The colour dependence on height can be dealt with by using probability thresholds derived from observed colours.



data source: Baranoski [2003]

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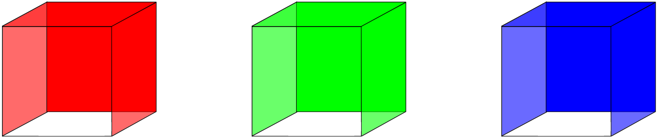
Algorithm: “Interaction” with atoms

- If an emission occurs:
  - increment the corresponding “photon density” voxel.
  - Transfer the photon energy ($\Delta E = hc/\lambda$) from the electron to an array keeping track of energy deposition.
- One photon intensity array per colour channel (red, green blue)



- Can later map this 3D data to a 2D image for visualization.

Electrostatics

- Structure comes from electric field **E**
- Need to compute **E** from the charge density ρ.
    - At each timestep, compute ρ
    - Solve Poisson's equation:

$$\nabla^2\phi = -\frac{\rho}{\epsilon_0}$$

    - Compute **E** through

$$\mathbf{E} = -\nabla\phi$$

Electrostatics

- Electron reset if
  - it exits box through the bottom
  - its energy falls below the energy of a photon

Electrostatics

- Implemented using Fourier transforms (FFTW3).
- Spectral methods used to directly compute $\mathbf{E}$ from $\mathcal{F}_\rho$ (the Fourier transform of ρ).

$$\phi(\mathbf{r}) = \frac{1}{\epsilon_0}\int \frac{\rho(\mathbf{r}')}{\mathbf{r}^2}\mathrm{d}^3\mathbf{r} = -\frac{1}{2\pi\epsilon_0}\int \frac{\mathcal{F}_\rho(\mathbf{k})}{\mathbf{k}^2}e^{i\mathbf{k}\cdot\mathbf{r}}\mathrm{d}^3\mathbf{k}$$

$$\mathbf{E} = -\nabla\phi = \frac{i}{2\pi\epsilon_0}\int \frac{\mathbf{k}\mathcal{F}_\rho(\mathbf{k})}{\mathbf{k}\cdot\mathbf{k}}e^{i\mathbf{k}\cdot\mathbf{r}}\mathrm{d}^3\mathbf{k}$$

Electrostatics

- Electrons interact with **E** through the Lorentz force

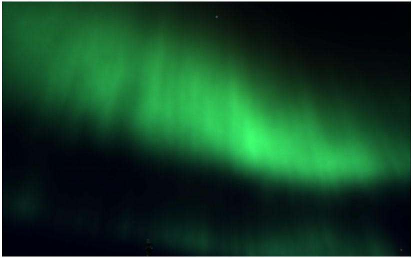
$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

- **B** taken as constant field in $-y$ direction (**B** points toward North Pole)

Results

Baranoski uses a complicated visual rendering procedure:

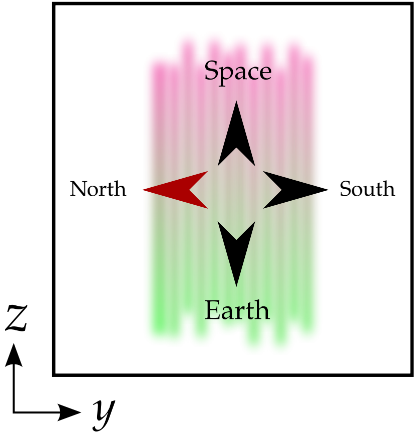
- perspective projection to trace photons to Earth-bound observer
- Gaussian kernel convolution to blur the images to smooth it out



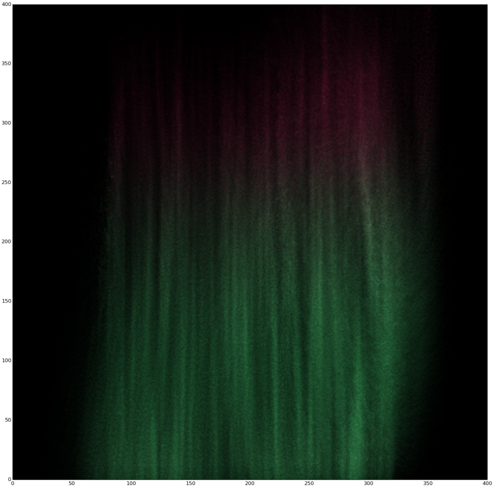
Baranoski [2003]

I use a

- direct orthographic projection
- photon brightness decays as $1/x^2$



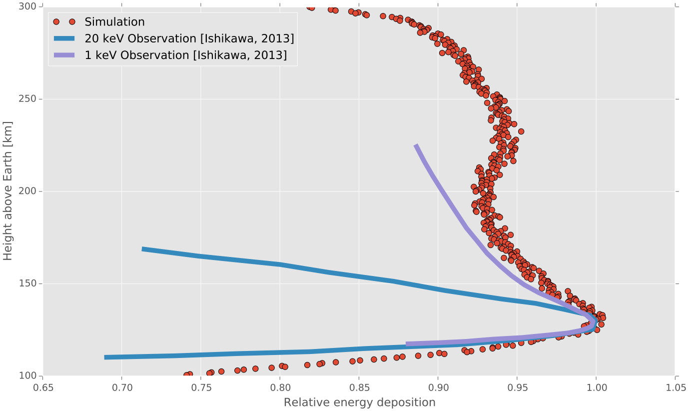
My projection


0
50
100
150
200
250
300
350
400

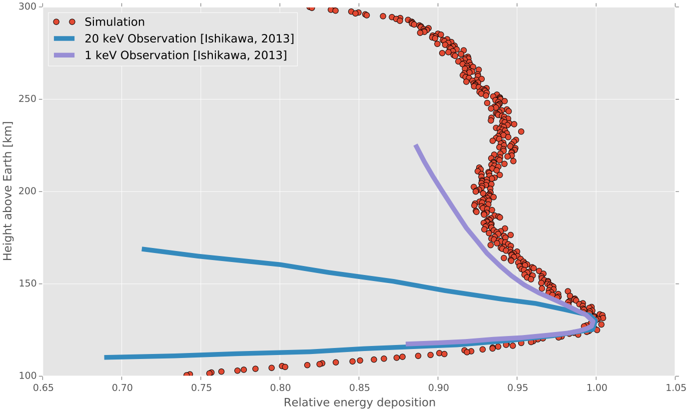
Extension

- Baranoski's goal was to produce photorealistic *images*, not necessarily give a realistic physical representation of the Aurora.
- I want to see how close my model matches the real Aurora, in terms of energy

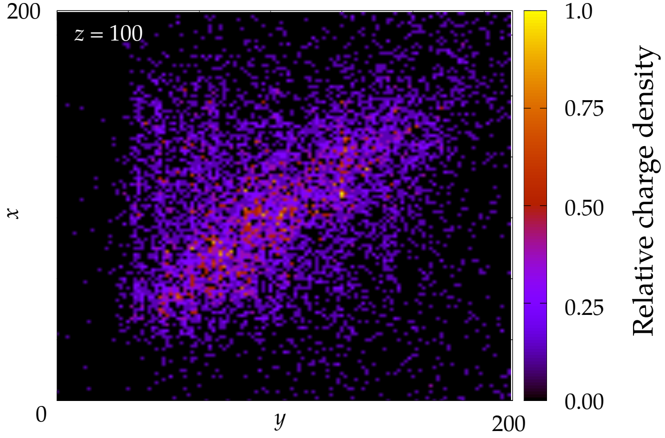
- Every photon emission causes energy to be deposited into the system ($\Delta E = hc/\lambda$). We can compare with atmospheric measurements:



- This could be improved by adding in more features that I neglected such as scattering, ionosphere fluid dynamics.

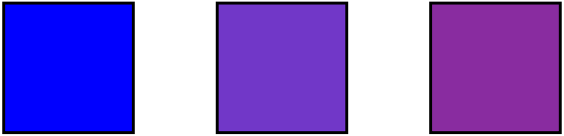


Charge density

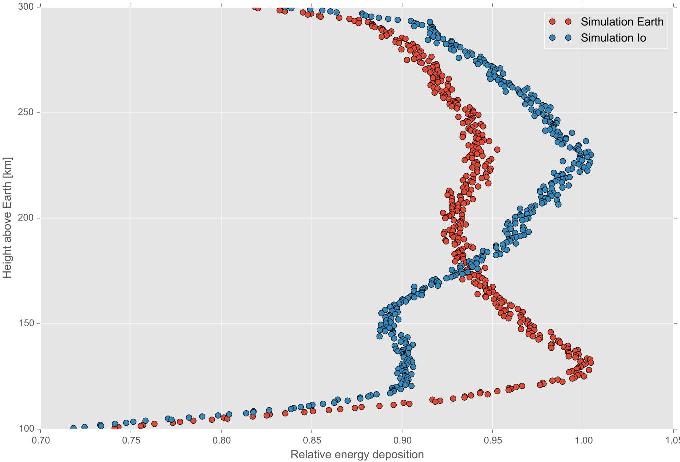


Aurora on Io

- Jupiter's moon Io has intense Aurora, thousands of times more energetic than Earth's
- Atmosphere composed of SO_2 from volcanic activity
- 3 main emission wavelengths: 450 nm, 335 nm & 300 nm



- Magnetic field of Io dominated by magnetic field of Jupiter
    - Larger magnetic field in z direction
- Spreads electrons more, hides the banding structure



Simulation Earth
Simulation Io
Height above Earth [km]
Relative energy deposition
300
250
200
150
100
0.70
0.75
0.80
0.85
0.90
0.95
1.00
1.05


0
50
100
150
200
250
300
350
400
0
50
100
150
200
250
300
350
400