ELEC207 Coursework: Design of a Stable Martian Segway ("Experiment 81")

Prof Simon Maskell 1 February 2022

Module	ELEC207
Coursework name	Experiment 81
Component Weight	25% = 3.75 Credits
Semester	2
HE Level	5
Lab location	Online (for 2020-21 academic year)
Work	Individually
Timetabled time	Check Canvas announcements for lab module
Suggested Private Study time	16h
Assessment method	Individual report
Submission Format	On line submission - Canvas
Late Submission	Standard University Penalties
Resit Opportunity	By arrangement
Marking Policy	Numerical mark
Anonymous marking	Yes
Feedback	Canvas
Subject of relevance	Control Engineering
AHEP Learning Outcomes	LO1

This coursework component[1] of ELEC207 relates to Part B, "Control", and focuses on the content up to and including lecture 10 of Part B of the module. The mark you will receive for the coursework constitutes 25% of your mark for ELEC207 and is intended to enable you to demonstrate your understanding of how to:

- use the position of poles to demonstrate whether systems are stable;
- use your knowledge of control to define a controller that ensures that a system is stable;
- use root locus to ensure the closed-loop time-response has specific properties;
- use Simulink to validate that the closed-loop time-response is as expected;
- explain your work in a clear and concise fashion.

You are expected to make use of the lecture notes and explicit references to numbered lectures are therefore included in this document. The demonstrators associated with the lab are available to support you in undertaking this coursework. Queries can also be submitted via the discussion board for ELEC207 on Canvas.

The mark you will receive (out of a total of 40 marks) will quantify the following aspects of your write-up:

- Demonstration of your understanding of ELEC207 (75% and out of 30 marks);
- Clarity of exposition (25% and out of 10 marks).

The marking descriptors are provided in the appendix.

[1] Assessment of ELEC207 has previously included Experiment 81, which may result in some legacy references to experiment 81 in documentation that has not yet been updated to reflect the change. This coursework takes the place of experiment 81.

Any emboldened text in a box herein implies that a specific response should be included in your write-up with the number in brackets indicating the number of marks associated with that component of the write-up. Failure to include such a response is liable to result in you obtaining fewer marks than would have been the case otherwise.

The assessment of the assignment is intended to be sufficiently straightforward that a diligent student should be able to achieve a pass mark of 40% but sufficiently challenging that achieving a first (ie 70% or above) requires deep understanding of the subject matter. To aid you in understanding how challenging each mark is to obtain, marks are annotated with E for Easy, M for Moderate and H for Hard: 8 of the marks are deemed to be easy; 14 are deemed moderate; 8 are deemed hard.

You should submit your coursework on or before the deadline announced by the lab coordinator (check Canvas announcements).

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https://www.liverpool.ac.uk/media/livacuk/tqsd/code-of-practice-on-assessment/appendix_L_cop_assess.pdf

1. Mathematical Modelling

A Segway (as shown in figure 1) is a physical system that can be modelled as an inverted pendulum.

Figure 1: A Segway

More specifically, we will assume that the pendulum can be approximated as a point mass, of mass *m*, at a distance, *l*, and at a (small) angle, $\theta(t)$, defined clockwise from vertical. Gravity is assumed to act downwards and exert an acceleration of *g*. A motor provides a torque, $T(t)$. The actuator that converts the Torque control signal to the physical Torque can be assumed to have a gain of unity. The angular acceleration of the pendulum can then be approximated as being defined by:

$$\frac{g}{l}\theta(t) + T(t) = m\frac{d^2\theta(t)}{dt^2}$$

Our task as an engineer designing the controller for the Martian Segway is to optimise the time-response to changes to an input, $X(t)$, which defines the desired values for $\theta(t)$. We will consider this design criterion to be achieved if the settling time is equal to t_s.

To ensure your Martian Segway is as unique as your coursework, please assume that:

- *l* is the day of the month when you were born (where *l* is in metres);
- *m* is the month of the year when you were born (where *m* is in kg);
- t_s is the year when you were born divided by 250 (where t_s is in seconds).

Please define the values for *l, m* and t_s that you will use for your coursework. [1E]

Now derive the transfer function, $H(s) = \theta(s)/T(s)$, of the Segway in terms of *l*, *m* and *g*. [1E]

Using your values for *l* and *m* along with *g*=3.711 ms^{-2}, write the transfer function with the denominator and numerator of your transfer function in polynomial form. [1E]

Calculate the position of the poles for your Segway and plot the poles on the complex plane. [1E]

$l: 13\,m$

$m: 12\,kg$

$t_s: \frac{2000}{250}\,s = 8s$

$$\frac{g}{l}\theta(t) + T(t) = m\frac{d^2}{dt^2}\theta(t)$$

$$\frac{g}{l}\Theta(s) + T(s) = m\left(s^2\Theta(s) - s\,\theta(0^-) - \dot{\theta}(0^-)\right)$$

Assume $\theta(0^-)$ and $\dot{\theta}(0^-)$ equals zero

$$\frac{g}{l}\Theta(s) + T(s) = ms^2\Theta(s)$$

$$H(s) = \frac{\Theta(s)}{T(s)} = \frac{1}{ms^2 - \frac{g}{l}}$$

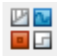
$$\frac{1}{12s^2 - \frac{3.711}{13}}$$

$$12s^2 - \frac{3.711}{13} = 0$$

$$s = \pm\sqrt{\frac{3.711}{13\times12}}$$

2. Validating that the Open-loop System is Unstable

With no controller, the system is believed to be unstable. We can use Simulink to validate this. Open MATLAB R2020a or later (as available using Apps Anywhere or the Remote Teaching Centre Service[2]) and then click Simulink. Information is available online (at: https://uk.mathworks.com/help/simulink) on how to use Simulink (please also check the supporting materials available in your Canvas lab module page). Here we provide a brief description of what is needed for our purposes:

1. Press “Blank Model”;
2. Click the “Library Browser” button, ;
3. Search for a step function by entering “Step” into the box where it says “enter search term”;
4. Drag and drop the “Step” block onto the Simulink Editor;
5. Click the “Library Browser” button again;
6. Search for “Transfer Fcn” and drag and drop the “Transfer Fcn” block onto the Simulink Editor. Be careful to select the continuous-time transfer function block (parameterised by *s*) and not the discrete-time transfer function block (parameterised by *z*);
7. Double click the “Transfer Fcn” block on the Simulink Editor and enter the polynomials that you calculated above. Be careful that any coefficients of zero need to be explicitly entered (ie coefficients of [1 0 2] describe s^2+2 but coefficients of [1 2] describe $s+2$);
8. Click on the triangle on the right of the “Step” block such that an arrow appears;
9. Connect the output signal from the “Step” block to the input signal for the “Transfer Fcn” block;
10. Add an “Outport” block;
11. Connect the output signal from the “Transfer Fcn” block to the input signal for the “Outport” block;
12. Selecting the signal that connects the “Transfer Fcn” block to the input for the “Outport” block such that it turns blue;
13. Click “Add viewer” (under “Simulation” and “Prepare”) and select “Scope”;
14. Press “Run” (under “Simulation” and “Simulate”). You should now see a graph showing the time-response of your Segway to a unit-step.
15. Press “File” -> “Print” -> “Microsoft Print to PDF” -> “OK” and save the time-response to a convenient location.

Insert a picture of the time-response of your Segway to the unit-step. [2E]

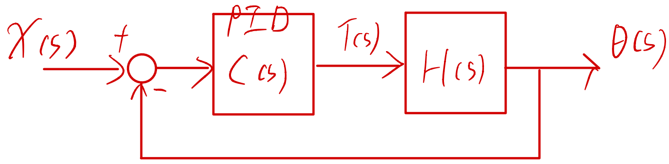
Comment on whether this time-response indicates that the open-loop system is stable. [1M]

3. Ensuring that the Closed-loop System is Stable Using PID Control

We will now add a cascade controller, as explained in lecture 7. We will use a combination of Proportional, Integral and Derivative control, ie a PID controller. The transfer function of a PID controller is:

[2] You can also download a copy of Matlab/Simulink from the University’s Computer Services Department website at https://www.liverpool.ac.uk/csd/software/software-downloads/#matlab

The open-loop system is not stable because the response which is y-axis grows faster over time.



$$C(s) = K_p + \frac{K_I}{s} + K_D s = \frac{K_p s + K_I + K_D s^2}{s}$$

where K_p, K_I and K_D are respectively the proportional, integral and derivative control constants. We know that the closed-loop transfer function for a negative feedback system with open-loop transfer function of $C(s)H(s)$ and that relates the input, $X(s)$, to the output, $\theta(s)$, is:

?
不是T(s)吗

$$\frac{\theta(s)}{X(s)} = \frac{C(s)H(s)}{1 + C(s)H(s)}$$

Write the closed-loop transfer function for your Segway in terms of K_p, K_I and K_D as a ratio of polynomials in *s*. Ensure that the highest order term in s in the denominator has a coefficient of unity. [3M]

To ensure that the system is stable, we start by placing the poles at *s*=-1, *s*=-2 and *s*=-3.

What is the characteristic polynomial that would result in these pole positions? [1M]

By equating the coefficients in the closed-loop transfer function's denominator and this characteristic function, deduce values for K_p, K_I and K_D which will ensure that the closed-loop system is stable. [3M]

4. Validating That the Closed-loop System is Stable

Simulink includes a "Subtract" block and a "PID" block. Use these blocks (with the parameters of the PID controller that you have defined) to simulate the time-response from a stable closed-loop system.

Insert a picture of the time-response of your closed-loop system to the unit-step. [2M]

5. Optimising the Time-Response Using Root Locus

The open-loop system has poles and zeros that are the union of those associated with the plant and those associated with the PID controller.

Calculate the positions of the open-loop zeros (ie the zeros of $C(s)H(s)$) for the values of *l*, *m*, K_p, K_I and K_D that you have used. [1M]

Recall that the PID controller has a pole at the origin and the Segway has poles that you have calculated above.

State the positions of the open-loop poles (ie the poles of $C(s)H(s)$) for the values of *l* and *m* that you have used. [2E]

We are going to use the root locus to choose the open-loop gain to achieve the time-response we want. More specifically, we are going to move the closed-loop poles along the root locus such that we can achieve the settling time that you have defined at the start of this coursework assignment.

As explained in lecture 6, the settling time for a generalised second-order system with natural frequency, ω, and damping coefficient, β, is $t_s \approx 4/\beta\omega$. As explained in lecture 5, the real part of the location of the poles of a generalised second-order system is $Re(s) = -\beta\omega$. We therefore want to ensure that the dominant poles (the poles that are closest to the

$$C(s) = \frac{K_P s + K_I + K_D s^2}{s}$$

$$H(s) = \frac{\theta(s)}{T(s)} = \frac{1}{ms^2 - \frac{g}{l}}$$

$$\frac{\theta(s)}{x(s)} = \frac{C(s)H(s)}{1 + C(s)H(s)}$$

$$= \frac{\frac{K_P s + K_I + K_D s^2}{s} \frac{1}{ms^2 - \frac{g}{l}}}{1 + \frac{K_P s + K_I + K_D s^2}{s} \frac{1}{ms^2 - \frac{g}{l}}}$$

$$= \frac{\frac{K_P s + K_I + K_D s^2}{ms^3 - \frac{g}{l} s}}{\frac{K_P s + K_I + K_D s^2 + ms^3 - \frac{g}{l} s}{ms^3 - \frac{g}{l} s}}$$

$$= \frac{K_P s + K_I + K_D s^2}{K_P s + K_I + K_D s^2 + ms^3 - \frac{g}{l} s}$$

$$= \frac{\frac{K_D}{m} s^2 + \frac{K_P}{m} s + \frac{K_I}{m}}{s^3 + \frac{K_D}{m} s^2 + \frac{1}{m}\left(K_P - \frac{g}{l}\right) s + \frac{K_I}{m}}$$

$$(s+1)(s+2)(s+3) = s^3 + 6s^2 + 11s + 6$$

characteristic polynomial

$$s^3 + \frac{K_D}{m}s^2 + \frac{1}{m}\left(K_P - \frac{g}{\ell}\right)s + \frac{K_I}{m} = s^3 + 6s^2 + 11s + 6$$

$$\frac{K_D}{m} = 6 \qquad K_D = 6 \times 12 = 72$$

$$\frac{1}{m}\left(K_P - \frac{g}{\ell}\right) = 11 \qquad K_P = 11 \times 12 + \frac{3.711}{13} = 132.285$$

$$\frac{K_I}{m} = 6 \qquad K_I = 6 \times 12 = 72$$

$$G(s)H(s) = \frac{K_P s + K_I + K_D s^2}{ms^3 - \frac{g}{\ell}s}$$

$$K_D s^2 + K_P s + K_I = 0$$

Zeros $s = \dfrac{-K_P \pm \sqrt{K_P^2 - 4K_D K_I}}{2K_D} = -0.9186 \pm 0.3951j$

$$ms^3 - \frac{g}{\ell}s = 0$$

$$s\left(ms^2 - \frac{g}{\ell}\right) = 0$$

Poles $s = 0, \quad s = \pm\sqrt{\dfrac{g}{\ell m}} = \pm 0.1542$

imaginary axis) are located such that $Re(s) = -4/t_s$. We can understand how to achieve this by sketching the root locus for the open-loop system with the compensator.

Sketch the root locus for $C(s)H(s)$ and identify the points on the root locus that are such that $Re(s) = -4/t_s$. [3M]

We now want to calculate the value of the open-loop gain that will ensure that the closed-loop poles are such that we achieve the desired time-response.

Write the open-loop transfer function, $C(s)H(s)$, as a ratio of polynomials in s. [1H]

If the numerator of the open-loop transfer function is $Z(s)$ and the denominator of the open-loop transfer function is $P(s)$, ie $C(s)H(s) = Z(s)/P(s)$, then the closed-loop poles occur when $P(s) + KZ(s) = 0$.

Write $P(s) + KZ(s) = 0$ as a polynomial in s involving K. [1H]

To identify the value of K, we need to re-express this polynomial as a polynomial in $\tilde{s} = s + 4/t_s$. We can achieve this by substituting $s = \tilde{s} - 4/t_s$ (using your value for t_s) and simplifying.

Write $P(\tilde{s}) + KZ(\tilde{s}) = 0$ as a polynomial in $\tilde{s}$ involving K. [1H]

As explained in lecture 9, we can then use Routh-Hurwitz to deduce the value of K that is such that $\tilde{s}$ is on the imaginary axis (ie when $Re(s) = -4/t_s$ and so we achieve our desired time-response). We achieve this by choosing K to be such that there is a row of zeros.

Complete a Routh table for $P(\tilde{s}) + KZ(\tilde{s})$. Deduce the value of K that is such that $Re(s) = -4/t_s$ [3H]

6. Validating the Response of Optimised System

You can now use Simulink to simulate the time-response from the closed-loop system with the gain you have chosen, K. Note that there is a "Gain" block that you may find useful.

Insert a picture of the time-response of your improved closed-loop system to the unit-step. [2H]

7. Further Directions for Private Study (Not Assessed)

Should you find this coursework assignment interesting and wish to continue to work on designing a Martian Segway, it might be interesting to consider:

1) Using a compensator (eg a PD controller) to ensure that the overshoot adheres to some design criterion;
2) Using a further compensator (eg a PI controller) to ensure that the steady-state error is reduced.

For the avoidance of doubt, your mark for the coursework will not be affected by whether you design these compensators.

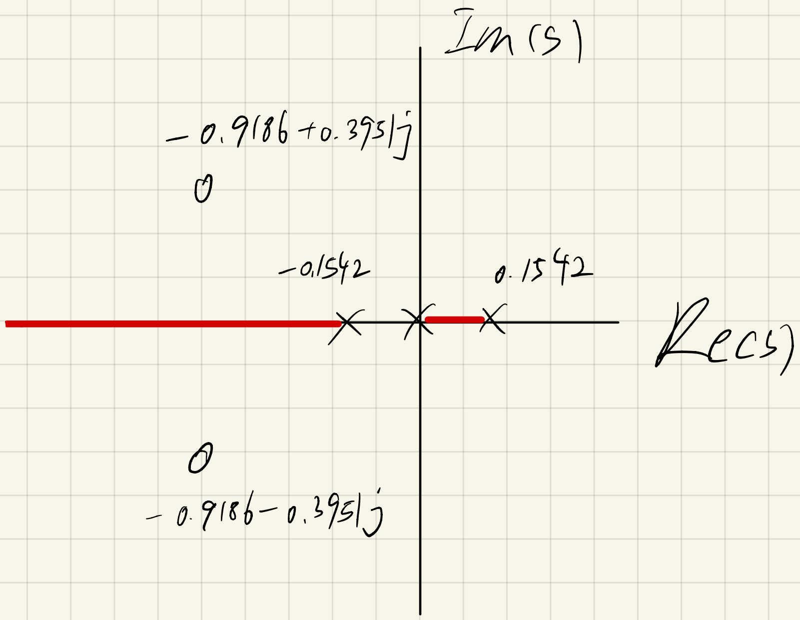
(1) Open poles and zeros

They have been calculated in the previous questions

zeros @ $-0.9186 \pm 0.3951j$

poles @ $0, \pm 0.1542$

(2) Real axis line segment



(3) Infinite poles and zeros

poles = 3, # zeros = 2

Need 1 infinite zero such that the number of poles and zeros are balanced

(4) Asymptotes

where the asymptotes touch the real axis

$$\sigma_a = \frac{\sum_{i=1}^{m} p_i - \sum_{i=1}^{n} z_i}{m-n}$$

$$= \frac{0 + 0.1542 - 0.1542 - (-0.9186 + 0.395j + -0.9186 - 0.395j)}{3-2}$$

$$= 1.8372$$

$$\theta_a = \frac{(2k+1)\pi}{m-n}$$

$$= \frac{(2k+1)\pi}{3-2}$$

$$= (2k+1)\pi$$

(5) Real-Axis Breakaway Points

$$1 + KG(s)H(s) = 0$$

$$KG(s)H(s) = -1$$

$\because K > 0$

$\therefore K = \frac{1}{|G(s)H(s)|}$

K is a local max or min when root locus breaks away from the real axis.

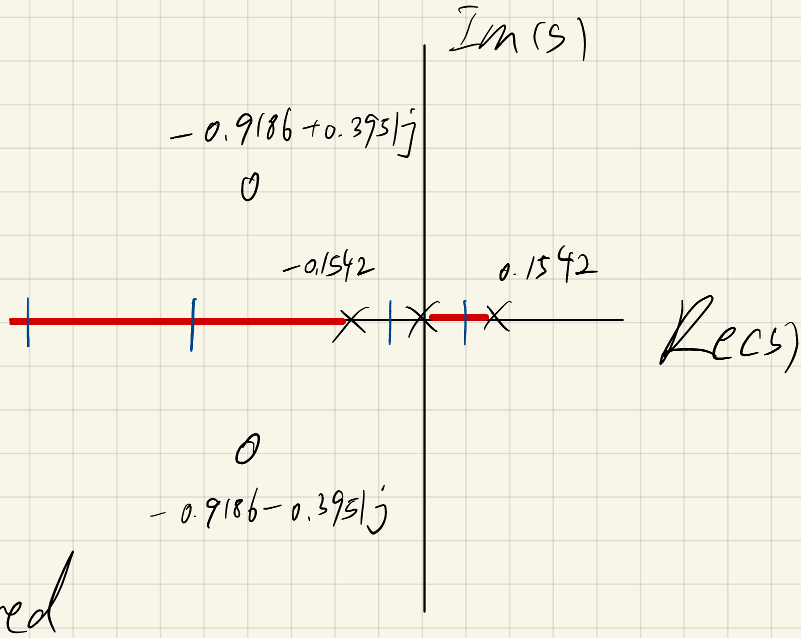
$$\frac{d}{ds}K = \frac{d}{ds}\frac{1}{|C(s)H(s)|} = 0$$

$$\frac{d}{ds}\frac{1}{C(s)H(s)} = 0$$

$$\frac{d}{ds}\frac{1}{\frac{K_P s + K_I + K_D s^2}{ms^3 - \frac{g}{l}s}} = 0$$

$$\frac{d}{ds}\frac{ms^3 - \frac{g}{l}s}{K_P s + K_I + K_D s^2} = 0$$

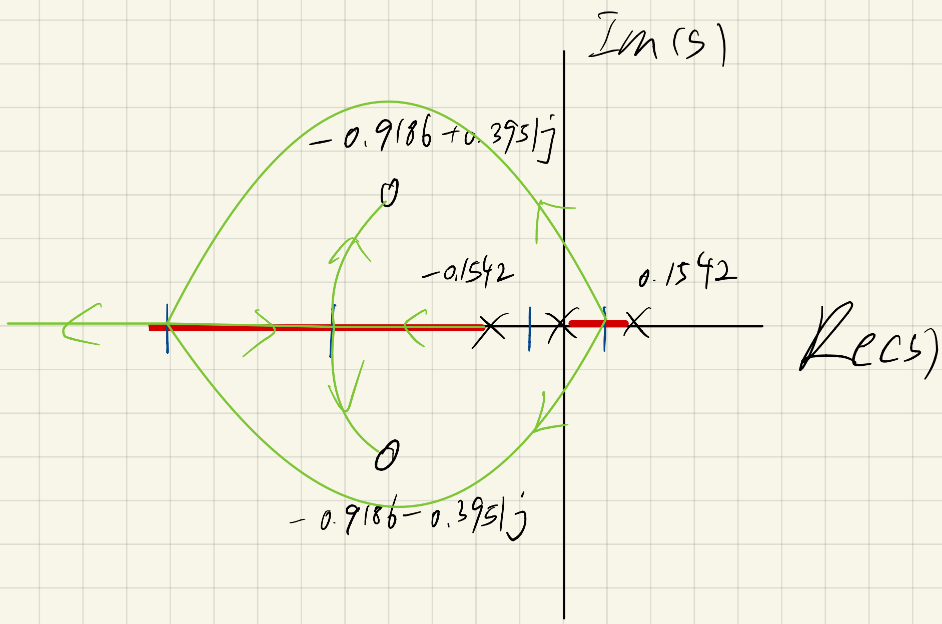
$$s = \begin{bmatrix} 0.0844 \\ -0.0941 \\ -1.2310 \\ -2.4338 \end{bmatrix}$$



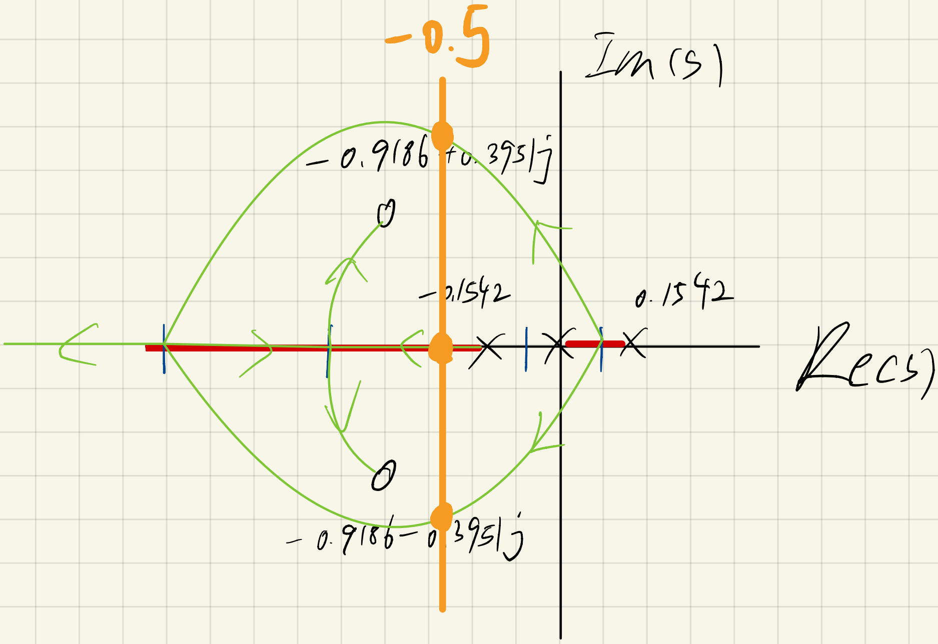
However, -0.0941 does not sit on the route locus, so it should not be considered as a breakaway point; hence there are three breakaway point.

In summary, we have one infinite zero, one asymptote which touches 1.8372 at the real-axis and has π rad points to the infinite zero, there breaking points which are $\begin{bmatrix} 0.0844 \\ -1.2310 \\ -2.4338 \end{bmatrix}$

One possible route locus that can meet the above requirements is



$$Re(s) = -\frac{4}{t_s} = -\frac{4}{8} = -0.5$$



$$G(s)H(s) = \frac{72s^2+132.3s+72}{12s^3-0.2855s} \quad (1)$$

$P(s) = 12s^3 - 0.2855s$ is the den of (1)

$Z(s) = 72s^2+132.3s+72$ num (1)

$$P(s) + KZ(s) = 0$$

$$12s^3 - 0.2855s + K(72s^2+132.3s+72) = 0$$

$$12s^3 + 72Ks^2 + (132.3K - 0.2855)s + 72K = 0 \quad (2)$$

let $\hat{s} = s + \frac{4}{t_s}$

$$= s + \frac{4}{8}$$

$$= s + 0.5$$

Replace s with $\hat{s}$ in ②

$$12\hat{s}^3 + (72K - 18)\hat{s}^2 + (60.3K + 8.7145)\hat{s} + 23.85K - 1.35725 = 0$$

$$\begin{array}{llll}
\tilde{s}^3 & 12 & 60.3K + 8.7145 & 0 \\
\tilde{s}^2 & 72K - 18 & 23.85K - 1.357 & 0 \\
\tilde{s}^1 & -\frac{1}{72K-18}\begin{vmatrix} 12 & 60.3K + 8.7145 \\ 72K - 18 & 23.85K - 1.357 \end{vmatrix} & 0 & \\
\tilde{s}^0 & & &
\end{array}$$

$$> 0$$

Marking descriptors

	Demonstration of your understanding of ELEC207 (75%)	Exposition and structure of the report (25%)
90-100% 'Outstanding'	Total coverage of the experiment aims, objectives and task set. An exceptional demonstration of knowledge and understanding, appropriately grounded in theory and relevant literature. Outstanding research and academic content.	Extremely clear exposition. Excellently logical structure. Excellent presentation, only the most insignificant errors. Scientific dissemination.
80-89% 'Excellent'	As 'Outstanding' but with some minor weaknesses in knowledge. Original and novel aspects presented but not fully developed.	As 'Outstanding' but with some minor weaknesses in structure, logic and/or presentation.
70-79% 'Very Good'	Full coverage of the task set. A very good demonstration of knowledge and understanding but with some modest gaps. A very good grounding in theory.	Very clear exposition. Satisfactory structure. Very good presentation, largely free of grammatical and other errors. All compulsory sections present.
60-69% 'Comprehensive'	As 'Very Good' but with some gaps in knowledge and understanding and/or gaps in theoretical grounding.	As 'Very Good' but with some weaknesses in exposition and/or structure, a few more grammatical and other errors.
50-59% 'Competent'	Covers most of the task set. Patchy knowledge and understanding with a limited grounding in literature.	Competent exposition and structure. Competent presentation but some significant grammatical and other errors. Minor errors in labelling graphs and figures.
40-49% 'Adequate'	As 'Competent' but patchy coverage of the task set and more weaknesses and/or omissions in knowledge and understanding. Just meets the threshold level.	As 'Competent' but with more weaknesses in exposition, structure, presentation and/or errors. Just meets the threshold level.
35-39% 'Compensatable fail'	Some parts of the set task are likely to have been omitted. Major gaps in knowledge and understanding. Some significant confusion. Very limited grounding. Falls just short of the threshold level.	Somewhat confused and limited exposition. Confused structure. Some weaknesses in presentation and some serious grammatical and other errors. Falls just short of the threshold level.
20-34% 'Deficient'	As 'Compensatable Fail' but with more serious weaknesses in presentation and/or grammar. Falls substantially below the threshold level.	As 'Compensatable Fail' but with more serious weaknesses in presentation and/or grammar. Falls substantially below the threshold level.
0-20% 'Extremely weak'	Largely confusing exposition and structure. Many serious grammatical and other errors.	Largely confusing exposition and structure. Many serious grammatical and other errors.