

$\mathbb{R}[x]$

$$f(x) = a_i x^i$$

$\mathbb{R}[x, y]$

$$a_0 + a_1 x + a_2 y + a_3 x^2 + a_4 xy + a_5 y^2$$

$$+ a_6 x^3 + a_7 x^2 y + a_8 xy^2 + a_9 y^3$$

$$a_0 + (a_1 + a_4 y + a_5 y^2)x$$

$$+ (a_2 + a_7 y)x^2$$

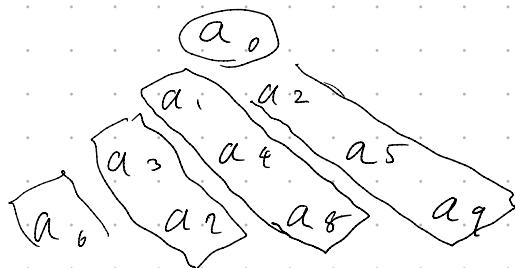
$$+ a_6 x^3$$

$$a_0 + (a_1 + a_4 y + a_5 y^2)x$$

$$+ (a_2 + a_7 y)x^2$$

$$a_6 x^3 + a_2 y + a_5 y^2$$

$$+ a_9 y^3$$



a_0

$a_1 \ a_4$

$a_2 \ a_5 \ a_7$

$a_3 \ a_6 \ a_8 \ a_9$

$$a_0 + a_1 x + a_4 y + a_2 x^2 + a_5 xy + a_7 y^2$$

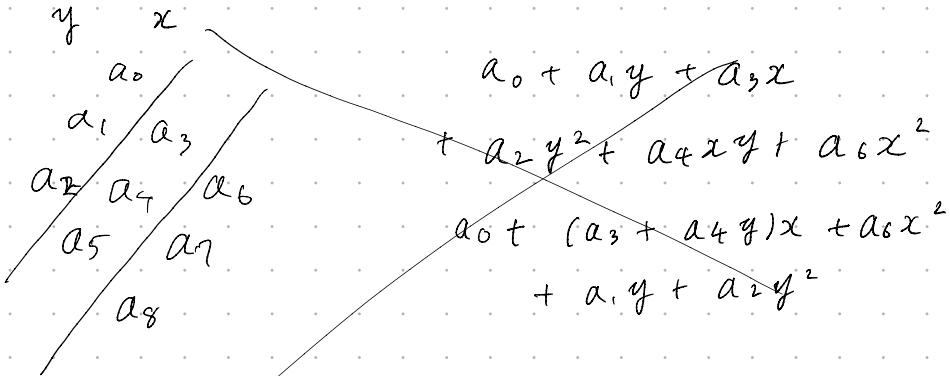
$$+ a_3 x^3 + a_6 x^2 y + a_8 xy^2 + a_9 y^3$$

$$\begin{aligned}
 & a_0 + (a_1 + a_5 y + a_8 y^2)x \\
 & + (a_2 + a_6 y)x^2 \\
 & + a_3 x^3 \\
 & a_4 y + a_7 y^2 + a_9 y^3
 \end{aligned}$$

$$\begin{aligned}
 & (a_1 + y(a_5 + a_8 y))x \\
 & + (a_2 + a_6 y)x^2 \\
 & + a_3 x^3 \\
 & a_0 + y(a_4 + y(a_7 + a_9 y))
 \end{aligned}$$

$$\begin{array}{ccccccccc}
 & & & a_0 & & & & & \\
 & a_4 & a_1 & & & & & & \\
 & a_8 & a_5 & a_2 & & & & & \\
 a_{12} & a_9 & a_6 & a_3 & & & & & \\
 \hline
 a_{13} & a_{10} & a_7 & & & & & & \\
 & a_{14} & a_{11} & & & & & & \\
 & & a_{15} & & & & & &
 \end{array}$$

$$a_0 + y(a_1 + a_2 y(a_3$$



$\boxed{\deg = 2}$

$x \quad y$

$$\begin{array}{c} x \quad y \\ a_0 \\ a_3 \quad a_1 \\ \hline a_6 \quad a_4 \quad a_2 \\ a_7 \quad a_5 \\ a_8 \end{array}$$

$$\begin{aligned}
 & a_0 + a_3x + a_1y \\
 & + a_6x^2 + a_4xy + a_2y^2 \\
 = & a_0 + y(a_1 + a_2y) \\
 & + (a_3 + a_4y)x \\
 & + a_6x^2
 \end{aligned}$$

$\boxed{\deg = 3}$

$x \quad y$

$$\begin{array}{c} x \quad y \\ a_0 \\ a_4 \quad a_1 \\ a_8 \quad a_5 \quad a_2 \\ \hline a_2 \quad a_9 \quad a_6 \quad a_3 \\ a_{13} \quad a_{10} \quad a_7 \\ a_{14} \quad a_{11} \\ a_{15} \end{array}$$

$$\begin{aligned}
 & a_0 + a_4x + a_1y \\
 & + a_8x^2 + a_5xy + a_2y^2 \\
 & + a_{12}x^3 + a_9x^2y + a_6xy^2 + a_3y^3 \\
 = & a_0 + a_1y + a_2y^2 + a_3y^3 \\
 & + (a_4 + a_5y + a_6y^2)x \\
 & + (a_8 + a_9y)x^2 \\
 & + a_{12}x^3 \\
 = & a_0 + y(a_1 + y(a_2 + y(a_3))) \\
 & + (a_4 + y(a_5 + y(a_6)))x \\
 & + (a_8 + y(a_9))x^2 \\
 & + a_{12}x^3
 \end{aligned}$$

$$\begin{array}{ccccc}
 0 & 0 & \longrightarrow & [a_0, a_1, a_2, a_3]_y^{\nwarrow A_0} \\
 4 & n+1 & & + [a_4, a_5, a_6]_y^{A_4}x \\
 8 & 2n+2 & & + [a_8, a_9]_y^{A_8}x^2 \\
 12 & 3n+3 & & + [a_{12}]_y^{A_{12}}x^3
 \end{array}$$

$$\longrightarrow A_0 + x(A_4 + x(A_8 + x(A_{12})))$$

$\boxed{\deg = n}$

$$[a_0, a_1, \dots, a_n]_y^{n+1}$$

$$+ [a_{(n+1)}, a_{(n+1)+1}, \dots, a_{(n+1)+(n-1)}]_y x^1$$

$$+ [a_{2(n+1)}, a_{2(n+1)+1}, \dots, a_{2(n+1)+(n-2)}]_y x^2$$

\cdots

$$+ [a_{m(n+1)}, a_{m(n+1)+1}, \dots, a_{m(n+1)+(n-m)}]_y x^m$$

\cdots

$$+ \underbrace{[a_{n(n+1)}, a_{n(n+1)+1}, \dots, a_{n(n+1)+(n-n)}]}_{n(n+1)} \underbrace{[a_{n(n+1)}, a_{n(n+1)+1}, \dots, a_{n(n+1)+(n-n)}]}_{n(n+1)} x^n$$

$$= A_0(y) + A_{n+1}(y)x + \cdots + A_{m(n+1)}x^m + \cdots + A_{nm}x^n$$

$$= A_0(y) + x(A_{n+1}(y)) + x(A_{2(n+1)}) + x(\cdots + x A_{nm})$$

$$= [A_0(y), A_{n+1}(y), A_{2(n+1)}(y), \dots, A_{m(n+1)}(y)]_x$$

□

$$A_{m(n+1)}(y) \stackrel{\text{def}}{=} \underbrace{[a_{m(n+1)}, \dots, a_{m(n+1)+(n-m)}]}_{n-m+1 \text{ つ}}$$

$$\stackrel{\text{def}}{=} \sum_{i=m(n+1)}^{m(n+1)-(n-m)} a_i y^i$$