$\mathbb{R}[x]$

$f(x) = a_i x^i$

$\mathbb{R}[x, y]$

$$\overset{0}{a} + \overset{1}{b}x + \overset{2}{c}y + \overset{3}{d}x^2 + \overset{4}{e}xy + \overset{5}{f}y^2$$

$$\underset{6}{g}x^3 + \underset{7}{h}x^2y + \underset{8}{i}xy^2 + \underset{9}{j}y^3$$

$$a + (b + ey + iy^2)x$$
$$+ (d + hy)x^2$$
$$+ gx^3$$

$$a_0 + (a_1 + a_4 y + a_8 y^2)x$$
$$+ (a_3 + a_7 y)x^2$$
$$a_6 x^3 + a_2 y + a_5 y^2$$
$$+ a_9 y^3$$

a_0

$a_1 \quad a_2$

$a_3 \quad a_4 \quad a_5$

$a_6 \quad a_7 \quad a_8 \quad a_9$

a_0

$a_1 \quad a_4$

$a_2 \quad a_5 \quad a_7$

$a_3 \quad a_6 \quad a_8 \quad a_9$

$$a_0 + a_1 x + a_4 y + a_2 x^2 + a_5 xy + a_7 y^2$$
$$+ a_3 x^3 + a_6 x^2 y + a_8 xy^2 + a_9 y^3$$

$$a_0 + (a_1 + a_5 y + a_8 y^2)x$$
$$+ (a_2 + a_6 y)x^2$$
$$+ a_3 x^3$$
$$a_4 y + a_7 y^2 + a_9 y^3$$

$$(a_1 + y(a_5 + a_8 y))x$$
$$+ (a_2 + a_6 y)x^2$$
$$+ a_3 x^3$$
$$a_0 + y(a_4 + y(a_7 + a_9 y))$$

a_0

$a_4 \quad a_1$

$a_8 \quad a_5 \quad a_2$

$a_{12} \quad a_9 \quad a_6 \quad a_3$

$a_{13} \quad a_{10} \quad a_7$

$a_{14} \quad a_{11}$

a_{15}

$$a_0 + y(a_1 + a_2 y(a_3$$

$y \quad x$

a_0

$a_1 \quad a_3$

$a_2 \quad a_4 \quad a_6$

$a_5 \quad a_7$

a_8

$$a_0 + a_1 y + a_3 x$$
$$+ a_2 y^2 + a_4 xy + a_6 x^2$$

$$a_0 + (a_3 + a_4 y)x + a_6 x^2$$
$$+ a_1 y + a_2 y^2$$

$\boxed{deg = 2}$

$$\begin{array}{ccccc} x & & & & y \\ & & a_0 & & \\ & a_3 & & a_1 & \\ a_6 & & a_4 & & a_2 \\ \hline & a_7 & & a_5 & \\ & & a_8 & & \end{array}$$

$$a_0 + a_3 x + a_1 y + a_6 x^2 + a_4 xy + a_2 y^2$$

$$= a_0 + y(a_1 + a_2 y) + (a_3 + a_4 y)x + a_6 x^2$$

$\boxed{deg = 3}$

$$\begin{array}{ccccccc} x & & & & & & y \\ & & & a_0 & & & \\ & & a_4 & & a_1 & & \\ & a_8 & & a_5 & & a_2 & \\ a_{12} & & a_9 & & a_6 & & a_3 \\ \hline & a_{13} & & a_{10} & & a_7 & \\ & & a_{14} & & a_{11} & & \\ & & & a_{15} & & & \end{array}$$

$$a_0 + a_4 x + a_1 y + a_8 x^2 + a_5 xy + a_2 y^2 + a_{12} x^3 + a_9 x^2 y + a_6 x y^2 + a_3 y^3$$

$$= a_0 + a_1 y + a_2 y^2 + a_3 y^3 + (a_4 + a_5 y + a_6 y^2) x + (a_8 + a_9 y) x^2 + a_{12} x^3$$

$$= a_0 + y(a_1 + y(a_2 + y(a_3))) + \left(a_4 + y(a_5 + y(a_6))\right) x + (a_8 + y(a_9)) x^2 + a_{12} x^3$$

0	0
4	n+1
8	2n+2
12	3n+3

$$\longrightarrow \underbrace{[a_0, a_1, a_2, a_3]_y}_{= A_0} + \underbrace{[a_4, a_5, a_6]_y}_{= A_4} x + \underbrace{[a_8, a_9]_y}_{= A_5} x^2 + \underbrace{[a_{12}]_y}_{= A_{12}} x^3$$

$$\longrightarrow A_0 + x(A_4 + x(A_8 + x(A_{12})))$$

$\boxed{\deg = n}$

$$
\begin{aligned}
& [a_0, a_1, \overset{n+1}{\cdots} a_n]_y \\
& + [a_{(n+1)}, a_{(n+1)+1}, \overset{n}{\cdots}, a_{(n+1)+(n-1)}]_y x^1 \\
& + [a_{2(n+1)}, a_{2(n+1)+1}, \cdots, a_{2(n+1)+(n-2)}]_y x^2 \\
& \cdots\cdots
\end{aligned}
$$

$x : m$次 $\rightarrow$ $+ [a_{m(n+1)}, a_{m(n+1)+1}, \cdots, a_{m(n+1)+(n-m)}]_y x^m$

$$
\begin{aligned}
& \cdots\cdots \\
& + [\underbrace{a_{n(n+1)}, \cancel{a_{n(n+1)+1}, \cdots, a_{n(n+1)+(n-n)}}}_{n(n+1)}]_y x^n \\
&= A_0(y) + A_{n+1}(y)x + \cdots A_{m(n+1)}x^m + \cdots + A_{n(n+1)}x^n \\
&= A_0(y) + x(A_{n+1}(y) + x(A_{2(n+1)}(y) + x(\cdots + xA_{n(n+1)}) \\
&= [A_0(y), A_{n+1}(y), A_{2(n+1)}(y), \cdots, A_{n(n+1)}(y)]_x
\end{aligned}
$$

$\square$

$$
A_{m(n+1)}(y) \overset{\text{def}}{=} [\underbrace{a_{m(n+1)}, \cdots, a_{m(n+1)-(n-m)}}_{n-m+1\text{個}}]_y
$$

$$
\overset{\text{def}}{=} \sum_{i=m(n+1)}^{m(n+1)-(n-m)} a_i y^i
$$