Sweet `Cython` code for planet search finding

@`dfm` & @`mirca`

February 16, 2018

1 Why period search algorithms are important?

1.1 Statistical assumptions

We assume that $y_i \sim \mathcal{N}(m(t_i), \sigma_i)$, $i = 1, 2, ..., n$, where

$$m(t_i) = \begin{cases} l, & \text{if } t_0 \leq t_i \leq t_0 + w, \\ h, & \text{otherwise.} \end{cases} \tag{1}$$

such that $h > l$, in which t_0 is the transit time, w is the transit duration, h is the mean (not quite) flux level out-of-transit, l is the mean (not quite) flux level in-transit.

Then the log-likelihood function of $\boldsymbol{y} \triangleq (y_1, y_2, ..., y_n)$, assuming y_i's are pair-wise idependent, can be written as

$$\log p(\boldsymbol{y}) = -\frac{1}{2}\sum_{i=1}^{n}\left(\frac{y_i - m(t_i)}{\sigma_i}\right)^2, \tag{2}$$

up to an additive constant.

Note further that, the log-likelihood function can be expressed as

$$\log p(\boldsymbol{y}) = -\frac{1}{2}\left[\sum_{i \in I}\left(\frac{y_i - l}{\sigma_i}\right)^2 + \sum_{i \in I^c}\left(\frac{y_i - h}{\sigma_i}\right)^2\right], \tag{3}$$

in which $I \triangleq \{i | t_0 \leq t_i \leq t_0 + w\}$ and I^c denotes the complement of I.

And the maximum likelihood estimator for h and l, denoted as $h^\star$ and $l^\star$, respectively, can be written as

$$l^\star(\boldsymbol{y}) = \frac{\displaystyle\sum_{i \in I}\frac{y_i}{\sigma_i^2}}{\displaystyle\sum_{i \in I}\frac{1}{\sigma_i^2}}, \quad h^\star(\boldsymbol{y}) = \frac{\displaystyle\sum_{i \in I^c}\frac{y_i}{\sigma_i^2}}{\displaystyle\sum_{i \in I^c}\frac{1}{\sigma_i^2}}. \tag{4}$$

Note that

$$\operatorname{var}\left(l^\star(\boldsymbol{y})\right) = \frac{\displaystyle\sum_{i\in I} \frac{\operatorname{var}(y_i)}{\sigma_i^4}}{\left(\displaystyle\sum_{i\in I} \frac{1}{\sigma_i^2}\right)^2} = \left(\sum_{i\in I} \frac{1}{\sigma_i^2}\right)^{-1} \tag{5}$$

$$\operatorname{var}\left(h^\star(\boldsymbol{y})\right) = \frac{\displaystyle\sum_{i\in I^c} \frac{\operatorname{var}(y_i)}{\sigma_i^4}}{\left(\displaystyle\sum_{i\in I^c} \frac{1}{\sigma_i^2}\right)^2} = \left(\sum_{i\in I^c} \frac{1}{\sigma_i^2}\right)^{-1} \tag{6}$$

The maximum likelihood for the transit depth, denoted as $d^\star$, can then be written as

$$d^\star(\boldsymbol{y}) = h^\star(\boldsymbol{y}) - l^\star(\boldsymbol{y}). \tag{7}$$

Then the variance on $d^\star$ can be expressed as

$$\begin{aligned} \operatorname{var}\left(d^\star\right) &= \operatorname{var}\left(h^\star\right) + \operatorname{var}\left(l^\star\right) &(8)\\ &= \left(\sum_{i\in I} \frac{1}{\sigma_i^2}\right)^{-1} + \left(\sum_{i\in I^c} \frac{1}{\sigma_i^2}\right)^{-1} &(9)\end{aligned}$$