Sweet **Cython** code for planet search finding

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February 16, 2018

1 Why period search algorithms are important?

1.1 Statistical assumptions

We assume that $y_i \sim \mathcal{N}(m(t_i), \sigma_i)$, $i = 1, 2, ..., n$, where

$$m(t_i) = \begin{cases} l, & \text{if } t_0 \leq t_i \leq t_0 + w, \\ h, & \text{otherwise.} \end{cases} \tag{1}$$

such that $h > l$, in which t_0 is the transit time, w is the transit duration, h is the mean (not quite) flux level out-of-transit, l is the mean (not quite) flux level in-transit.

Then the log-likelihood function of $\boldsymbol{y} \triangleq (y_1, y_2, ..., y_n)$, assuming y_i's are pair-wise idependent, can be written as

$$\log p(\boldsymbol{y}) = -\frac{1}{2} \sum_{i=1}^{n} \left(\frac{y_i - m(t_i)}{\sigma_i} \right)^2, \tag{2}$$

up to an additive constant.

Note further that, the log-likelihood function can be expressed as

$$\log p(\boldsymbol{y}) = -\frac{1}{2} \left[\sum_{i \in I} \left(\frac{y_i - l}{\sigma_i} \right)^2 + \sum_{i \in I^c} \left(\frac{y_i - h}{\sigma_i} \right)^2 \right], \tag{3}$$

in which $I \triangleq \{i | t_0 \leq t_i \leq t_0 + w\}$ and I^c denotes the complement of I.

And the maximum likelihood estimator for h and l, denoted as $h^\star$ and $l^\star$, respectively, can be written as

$$l^\star(\boldsymbol{y}) = \frac{\sum\limits_{i \in I} \dfrac{y_i}{\sigma_i}}{\sum\limits_{i \in I} \dfrac{1}{\sigma_i}}, \quad h^\star(\boldsymbol{y}) = \frac{\sum\limits_{i \in I^c} \dfrac{y_i}{\sigma_i}}{\sum\limits_{i \in I^c} \dfrac{1}{\sigma_i}}. \tag{4}$$

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Note that

$$\text{var}\left(l^{\star}(\boldsymbol{y})\right) = \frac{\sum\limits_{i\in I}\dfrac{\text{var}(y_i)}{\sigma_i^2}}{\left(\sum\limits_{i\in I}\dfrac{1}{\sigma_i}\right)^2} = \frac{\sum\limits_{i\in I}1}{\left(\sum\limits_{i\in I}\dfrac{1}{\sigma_i}\right)^2} \tag{5}$$

$$\text{var}\left(h^{\star}(\boldsymbol{y})\right) = \frac{\sum\limits_{i\in I^c}\dfrac{\text{var}(y_i)}{\sigma_i^2}}{\left(\sum\limits_{i\in I^c}\dfrac{1}{\sigma_i}\right)^2} = \frac{\sum\limits_{i\in I^c}1}{\left(\sum\limits_{i\in I^c}\dfrac{1}{\sigma_i}\right)^2} \tag{6}$$

The maximum likelihood for the transit depth, denoted as $d^{\star}$, can then be written as

$$d^{\star}(\boldsymbol{y}) = h^{\star}(\boldsymbol{y}) - l^{\star}(\boldsymbol{y}). \tag{7}$$

Then the variance on $d^{\star}$ can be expressed as

$$\text{var}\left(d^{\star}\right) = \text{var}\left(h^{\star}\right) + \text{var}\left(l^{\star}\right) \tag{8}$$

$$= \frac{\sum\limits_{i\in I^c}1}{\left(\sum\limits_{i\in I^c}\dfrac{1}{\sigma_i}\right)^2} + \frac{\sum\limits_{i\in I}1}{\left(\sum\limits_{i\in I}\dfrac{1}{\sigma_i}\right)^2} \tag{9}$$