

# Sweet **Cython** code for planet search finding

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## 1 Why period search algorithms are important?

### 1.1 Statistical assumptions

We assume that  $y_i \sim \mathcal{N}(m(t_i), \sigma_i)$ ,  $i = 1, 2, \dots, n$ , where

$$m(t_i) = \begin{cases} l, & \text{if } t_0 \leq t_i \leq t_0 + w, \\ h, & \text{otherwise.} \end{cases} \quad (1)$$

such that  $h > l$ , in which  $t_0$  is the transit time,  $w$  is the transit duration,  $h$  is the mean (not quite) flux level out-of-transit,  $l$  is the mean (not quite) flux level in-transit.

Then the log-likelihood function of  $\mathbf{y} \triangleq (y_1, y_2, \dots, y_n)$ , assuming  $y_i$ 's are pair-wise independent, can be written as

$$\log p(\mathbf{y}) = -\frac{1}{2} \sum_{i=1}^n \left( \frac{y_i - m(t_i)}{\sigma_i} \right)^2, \quad (2)$$

up to an additive constant.

Note further that, the log-likelihood function can be expressed as

$$\log p(\mathbf{y}) = -\frac{1}{2} \left[ \sum_{i \in I} \left( \frac{y_i - l}{\sigma_i} \right)^2 + \sum_{i \in I^c} \left( \frac{y_i - h}{\sigma_i} \right)^2 \right], \quad (3)$$

in which  $I \triangleq \{i | t_0 \leq t_i \leq t_0 + w\}$  and  $I^c$  denotes the complement of  $I$ .

And the maximum likelihood estimator for  $h$  and  $l$ , denoted as  $h^*$  and  $l^*$ , respectively, can be written as

$$l^*(\mathbf{y}) = \frac{\sum_{i \in I} \frac{y_i}{\sigma_i^2}}{\sum_{i \in I} \frac{1}{\sigma_i^2}}, \quad h^*(\mathbf{y}) = \frac{\sum_{i \in I^c} \frac{y_i}{\sigma_i^2}}{\sum_{i \in I^c} \frac{1}{\sigma_i^2}}. \quad (4)$$

Note that

$$\text{var}(l^*(\mathbf{y})) = \frac{\sum_{i \in I} \frac{\text{var}(y_i)}{\sigma_i^4}}{\left(\sum_{i \in I} \frac{1}{\sigma_i^2}\right)^2} = \left(\sum_{i \in I} \frac{1}{\sigma_i^2}\right)^{-1} \quad (5)$$

$$\text{var}(h^*(\mathbf{y})) = \frac{\sum_{i \in I^c} \frac{\text{var}(y_i)}{\sigma_i^4}}{\left(\sum_{i \in I^c} \frac{1}{\sigma_i^2}\right)^2} = \left(\sum_{i \in I^c} \frac{1}{\sigma_i^2}\right)^{-1} \quad (6)$$

The maximum likelihood for the transit depth, denoted as  $d^*$ , can then be written as

$$d^*(\mathbf{y}) = h^*(\mathbf{y}) - l^*(\mathbf{y}). \quad (7)$$

Then the variance on  $d^*$  can be expressed as

$$\text{var}(d^*) = \text{var}(h^*) + \text{var}(l^*) \quad (8)$$

$$= \left(\sum_{i \in I} \frac{1}{\sigma_i^2}\right)^{-1} + \left(\sum_{i \in I^c} \frac{1}{\sigma_i^2}\right)^{-1}. \quad (9)$$

Then the signal to noise ratio of the transit depth can be computed as

$$\text{SNR}_{d^*} = \frac{d^*}{\sqrt{\left(\sum_{i \in I} \frac{1}{\sigma_i^2}\right)^{-1} + \left(\sum_{i \in I^c} \frac{1}{\sigma_i^2}\right)^{-1}}}. \quad (10)$$