Lecture Presentation

Advanced Formation Evaluation

PGE385 (M,K)

Petrophysics and Acoustic Logging

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The University of Texas at Austin

Objectives:

1. To understand the physical principles behind the operation of sonic logging tools,

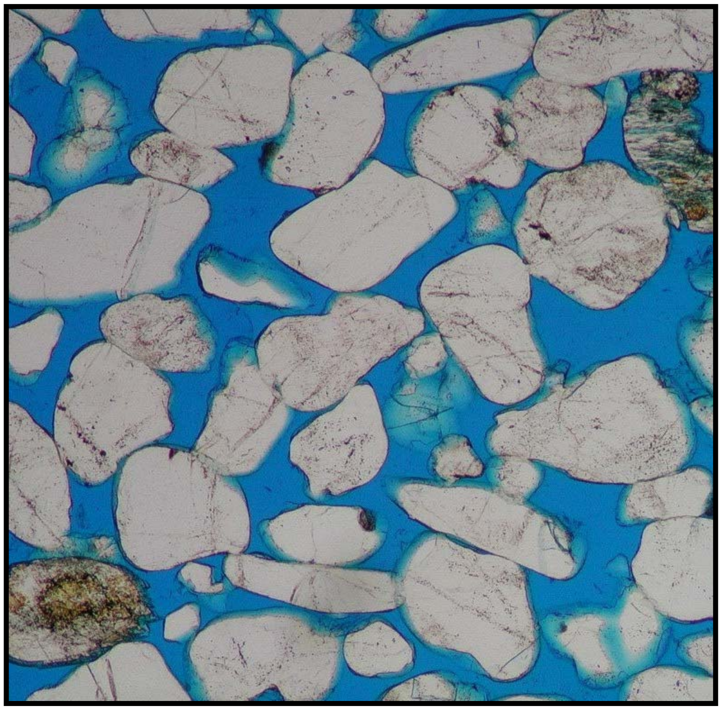
2. To learn how to interpret sonic logs in terms of lithology, types of fluids, and porosity,

3. To learn how to combine gamma-ray, resistivity, neutron, bulk density, and sonic logs to estimate lithology, types of fluids, and porosity, and

4. To understand the importance of environmental and interpretation corrections applied to sonic logs.

5. To understand and learn how to diagnose the limitations of sonic logs.

What do acoustic and shear logs respond to?



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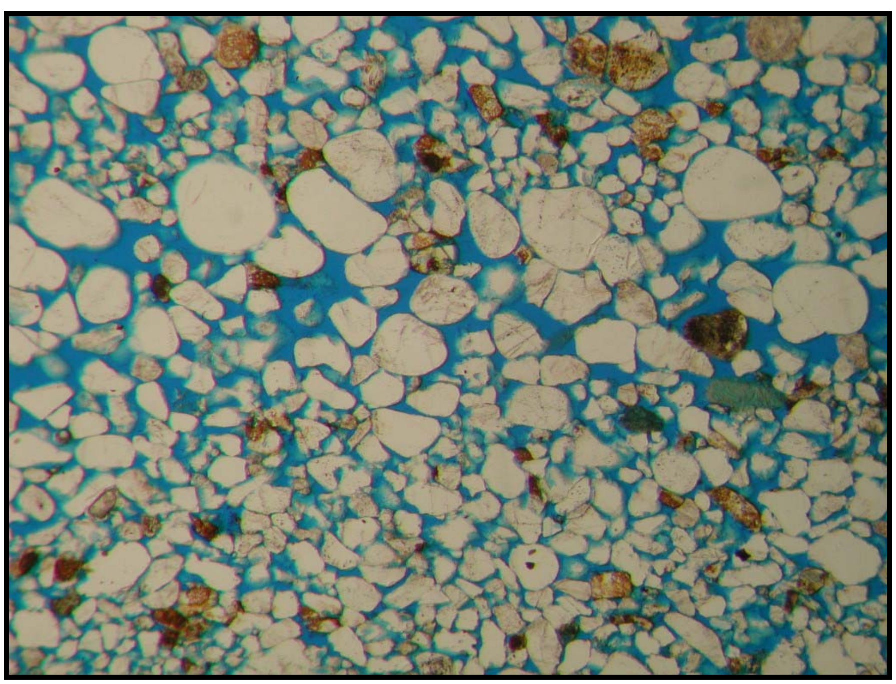

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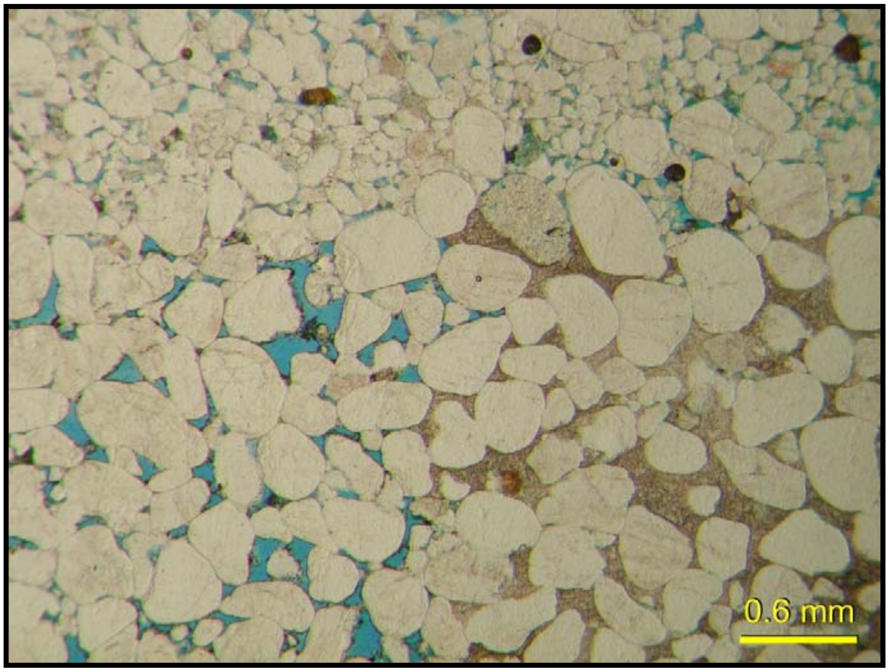

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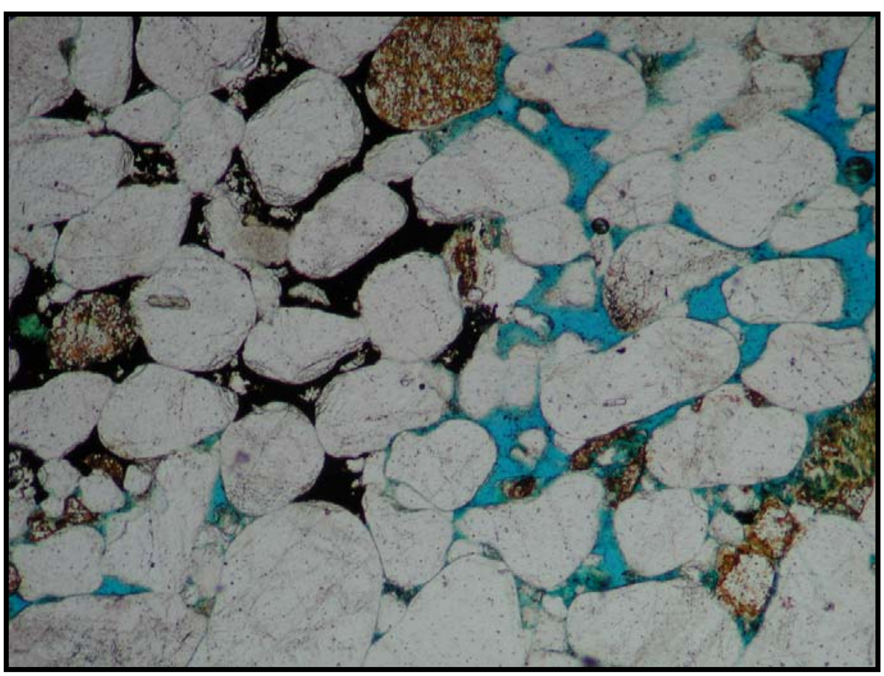
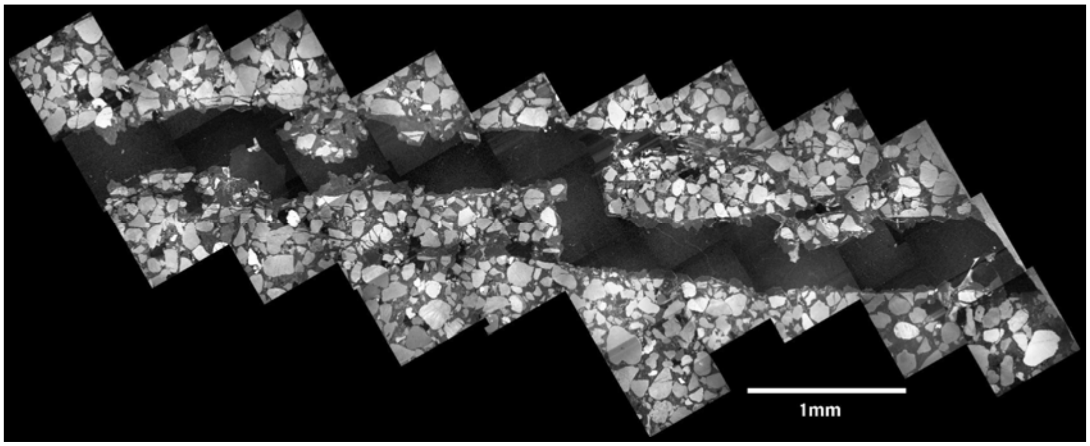


What do acoustic and shear logs respond to?

Example of Microfracturing



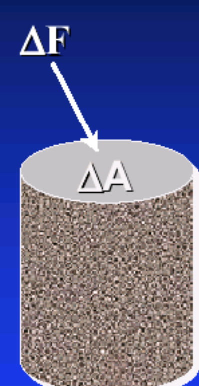
Photograph courtesy of Prof. Jon Olson

Stress and Strain

Stress (σ)

- Defines the mechanical force field related to a material
- Ratio of force over area
- is a Tensor.

$$\sigma = \frac{\Delta F}{\Delta A}$$

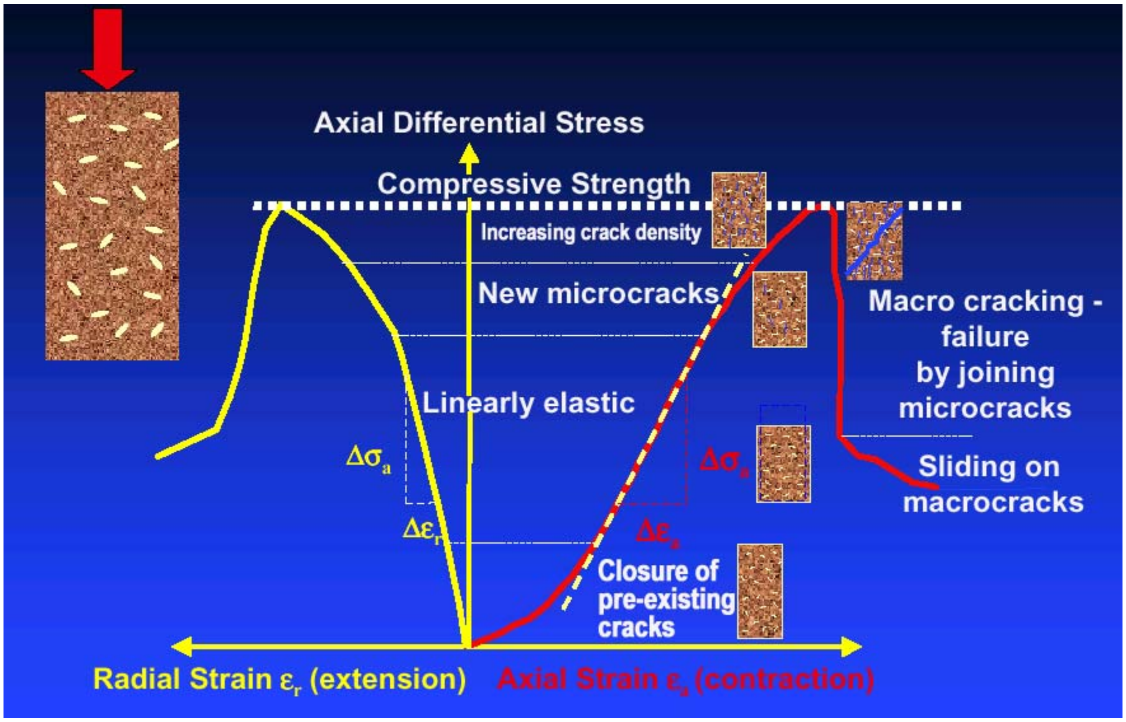


Strain (ε)

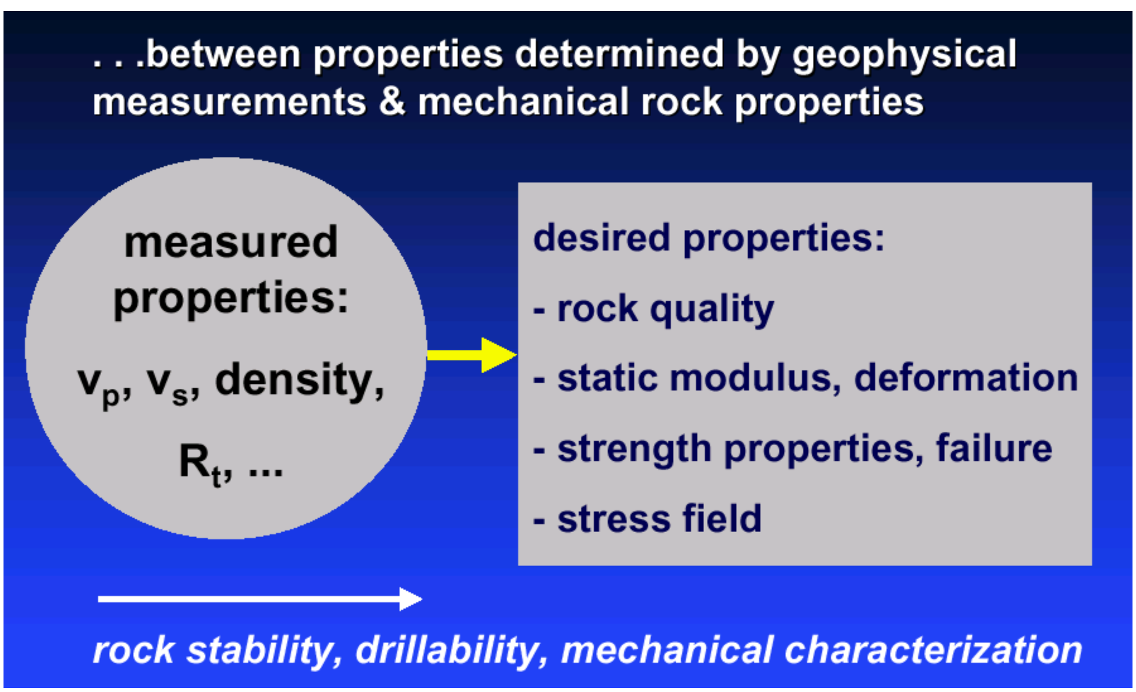
- Defines the deformation of a material in a stress field.
- Relative change in material dimension
- is a Tensor.

$$\varepsilon = \frac{\Delta L}{L_1} = \frac{\text{change in dimension}}{\text{initial dimension}}$$

Stress and Strain



Mechanical Properties and Petrophysics



Units and Conversions

- **Elastic moduli (*E*, *μ*, …): SI-unit Pa (Pascal)**

  1 Gpa = 10^9 Pa　　1 MPa = 10^6 Pa

  1 Pa = 1.0197 · 10^{-5} kp cm^{-2} = 1.4504 · 10^{-4} psi

  1 MPa = 10.2 kp cm^{-2} = 145 psi

- **Poissons ratio : dimensionless**
- **Velocity: SI-unit m s^{-1}**

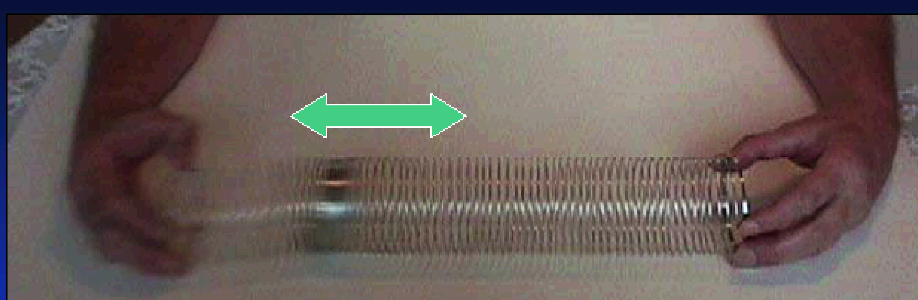
  1 m s^{-1} = 3.2808 ft s^{-1}　　1 ft s^{-1} = 0.3048 m s^{-1}

- **Interval transit time: SI-unit μs m^{-1}**

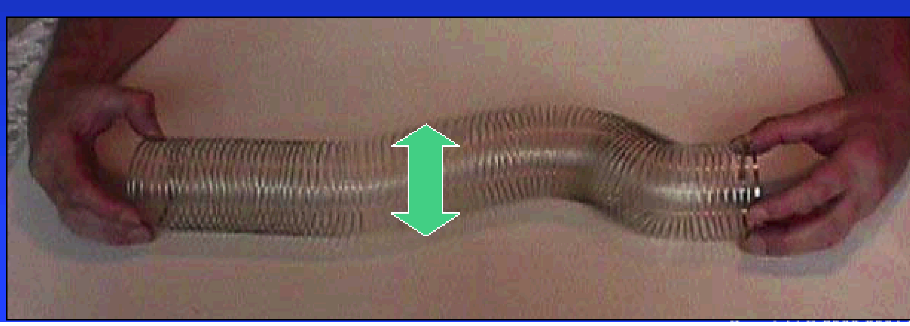
  1 μs m^{-1} = 0.3048 μs ft^{-1}　　1 μs ft^{-1} =3.2808 μs m^{-1}

P and S WAVES

P-Waves (Compressional), vibrate in direction of travel



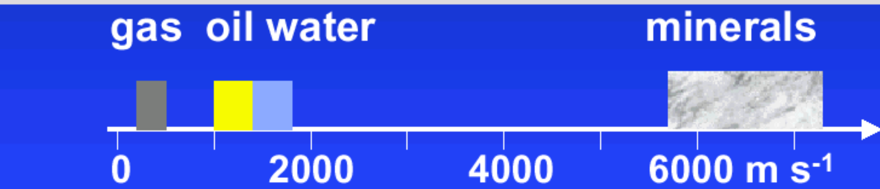
S-Waves (Shear), vibrate perpendicular to direction of travel



Typical Ranges of Velocities

Mean values & overview

Minerals:	v_P m/s	Δt_p µs/ft	v_s m/s	Δt_s µs/ft
Quartz	6000	51	4100	74
Calcite	6600	46	3400	90
Dolomite	7300	42	4000	76
Pore fluids				
Water	1450 … 1700	210…180		
Oil	1000 … 1400	305…220		
Gas	300 … 400	1010…760		



→ Velocity decreases with increasing porosity

P-WAVE VELOCITIES OF GASES

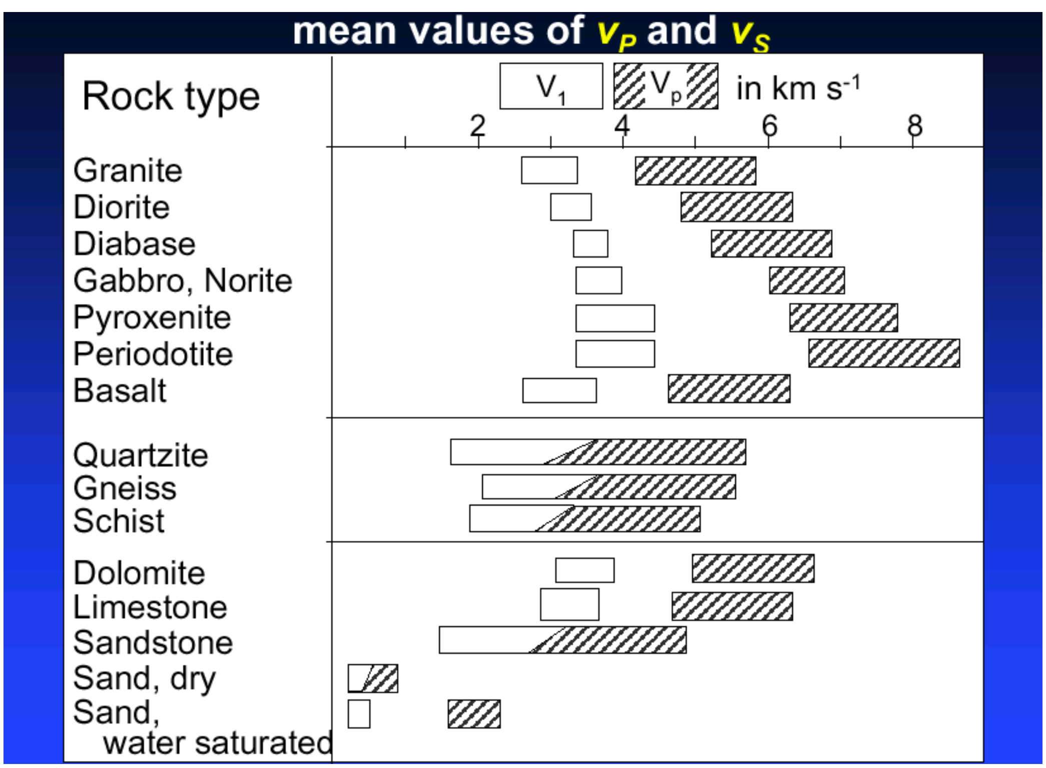
Gas	v_P m/s	Δt µs/ft	Remarks
Air	263	1160	T = 173 °K
	306.5	993	T = 223° K
	331.8	919	T = 273 °K
	355.3	858	T = 313 °K
	387.2	788	T = 373 °K
Methane	487.7	874	p = 15 psi (= 0.103 MPa)
Ethane	307.8	990	T = 10 °C = 283 °K, d = 1.25 kg m^{-3}
Carbondioxide	259.1	1177	ρ = 1.9776 kg m-3

P- and S-WAVE VELOCITIES OF SEDIMENTARY ROCKS

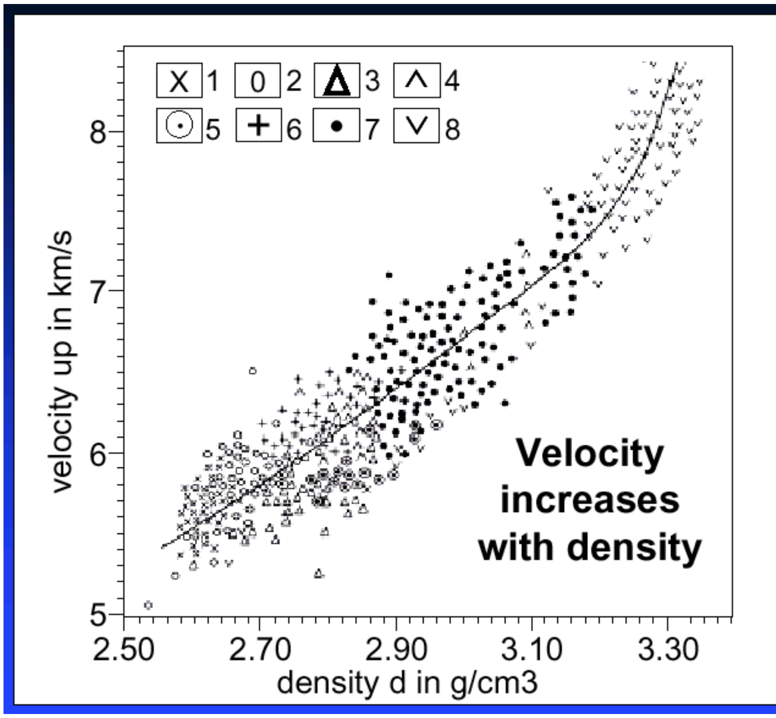
(v in km/s, Δt in µs/ft; values are partly converted & rounded)
n - number data; F - porosity, numbers in brackets are densities (kg/m³).

Rock type	range v_p	mean v_p	Δt_p	range v_s	mean v_s	Δt_s
Sandstone	2.80…4.46	3.63	84			
				1.59…2.93	2.26	135
	3.05…5.94	5.34	57			
- semi-consolidated		5.49	56			
- consolidated		5.79	53			
Shale	2.13…5.18					
Silt- and (parallel)	3.91…5.55	4.73	64			
claystone (perpend.)	2.83…5.37	4.10	74			
Limestone dense		6.40	48		3.38	90
Φ = 0.20…0.05	3.96…5.64			2.13…2.90		
Dolomite	3.50…6.50		42	1.90…3.60		84
Φ = 0.20…0.05	4.57…6.10			2.29…3.35		
Anhydrite		6.09	50		3.47	88
Gypsum	5.75…5.81	5.80	52			
Salt, Halite		4.57	67			
Lignite	1.00…2.18					
Uncons. sediments	0.55…1.95			0.15…0.4		

Typical Ranges of Velocities



Relationship with Depth



1 - granite

2 - gneiss (biotitic amphibolitic)

3 - gneiss (garnet biotitic)

4 - amphibole

5 - granulite

6 - diorite

7 - gabbro-norite

8 - ultrabasite

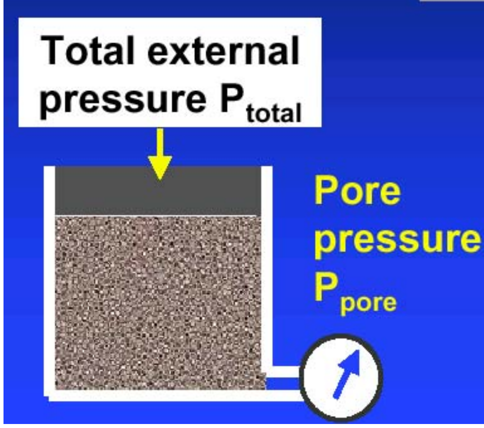
(Dortman 1976)

JS96

Relationship with Pressure

Pressure

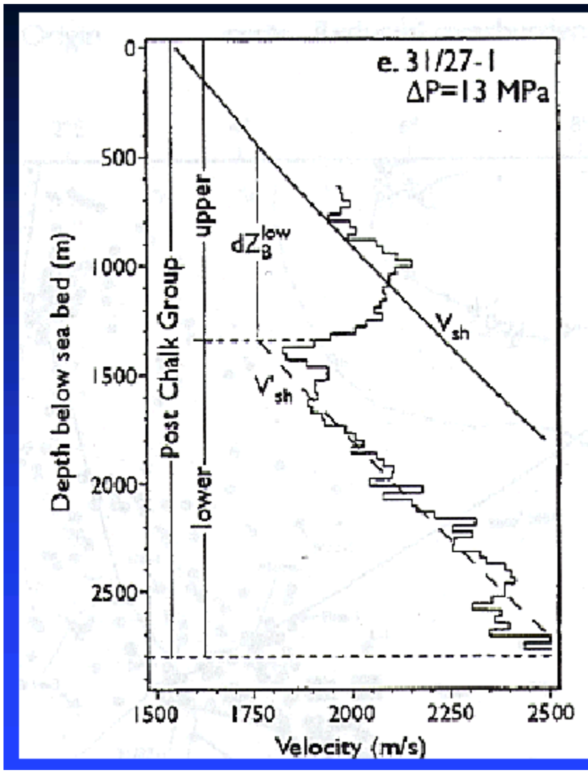
- reduces porosity
- changes grain to grain contact elasticity



Change of elasticity (porosity and grain to grain elasticity) is pressure effect acting on rock skeleton (effective pressure):

$$P_{eff} = \Delta P = P_{total} - P_{pore}$$

Formation Over-Pressure

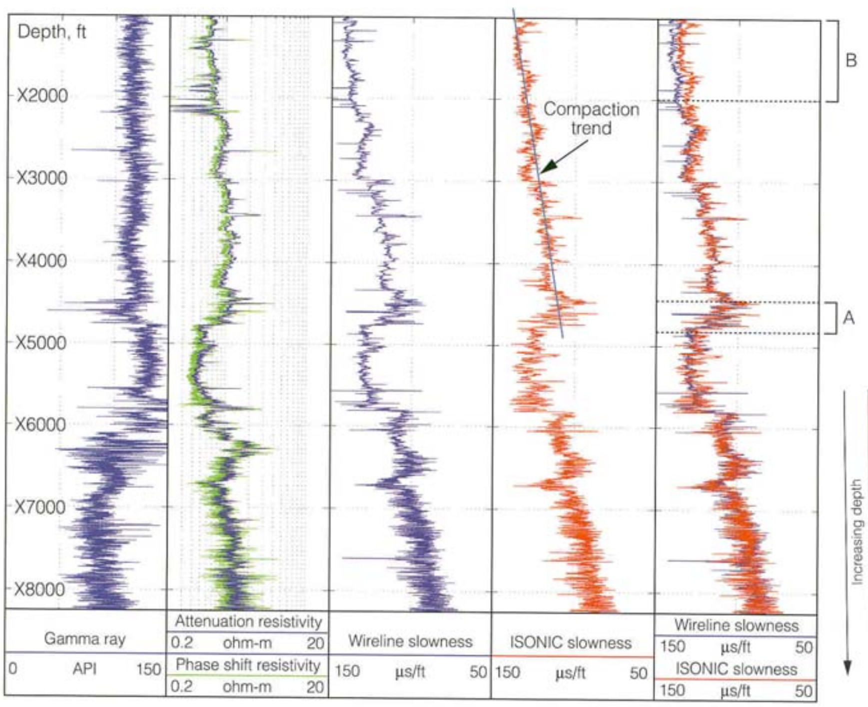


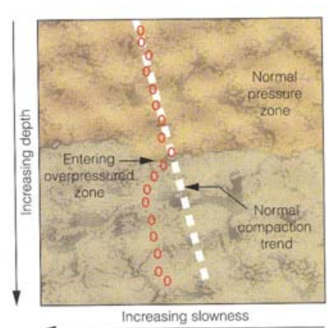
Sonic logs are averaged over intervals of 40 m.

Overpressured Cenozoic shale (North Sea); the burial anomaly of the lower Post Chalk Group (893 m) is proportional to the overpressure in the wells

(Japsen, 1999)

Compaction and Over Pressure





Influence of Saturating Fluids

v_p increases from air --> oil --> water

v_s decreases from air --> oil --> water

why ?

$$v_p = \sqrt{\frac{E}{\rho} \cdot \frac{(1-\nu)}{(1+\nu)(1-2\nu)}}$$

E strong increases

ρ moderate increases

$$v_s = \sqrt{\frac{\mu}{\rho}}$$

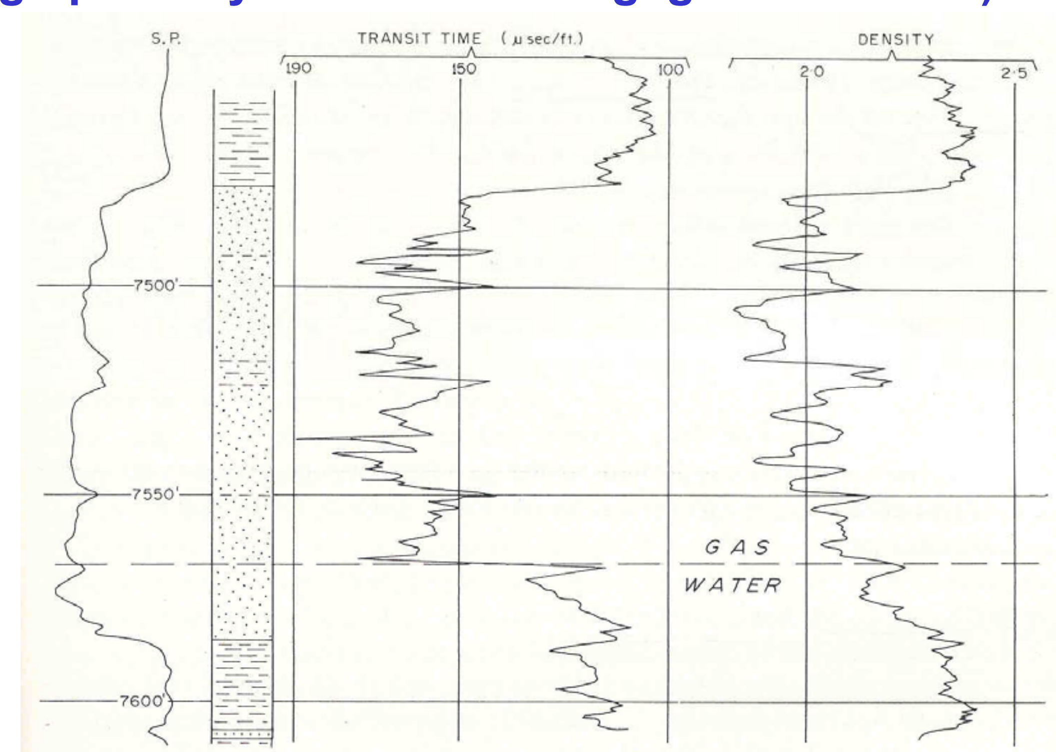
μ non influenced

ρ moderate increases

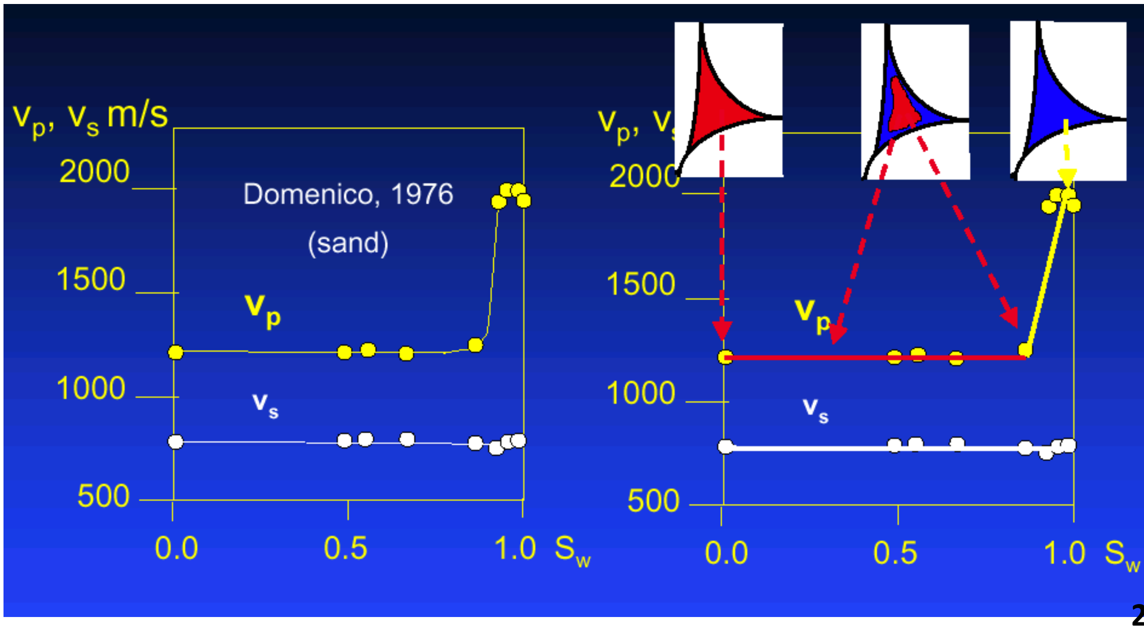
→ v_p/v_s ratio as a pore fluid indicator

Example of Gas Effect on a P-wave Sonic Log

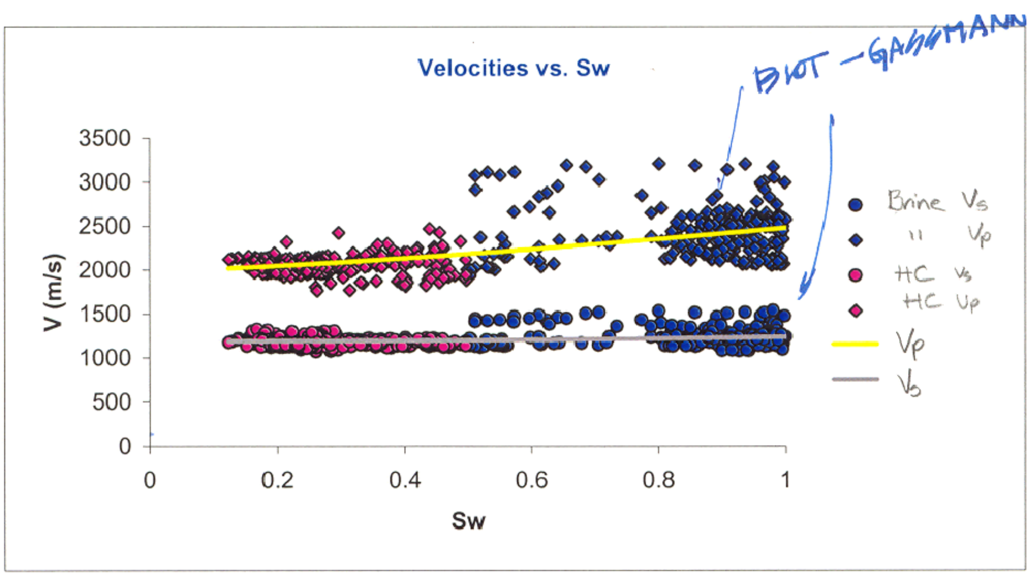
(high porosity formation → negligible invasion)



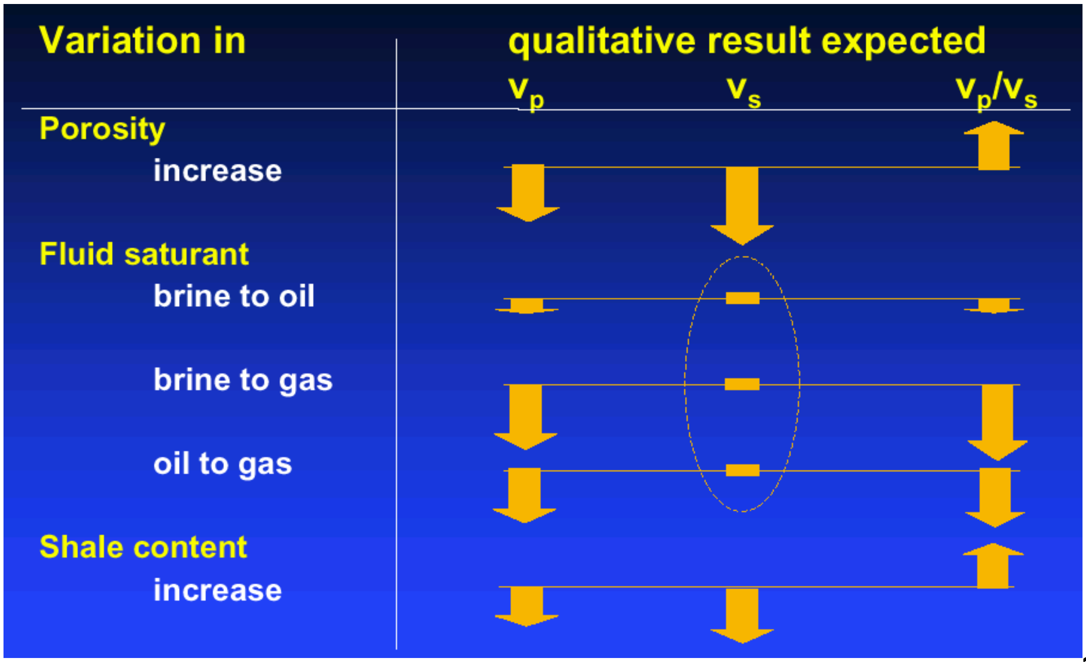
Influence of Gas and Water Saturation



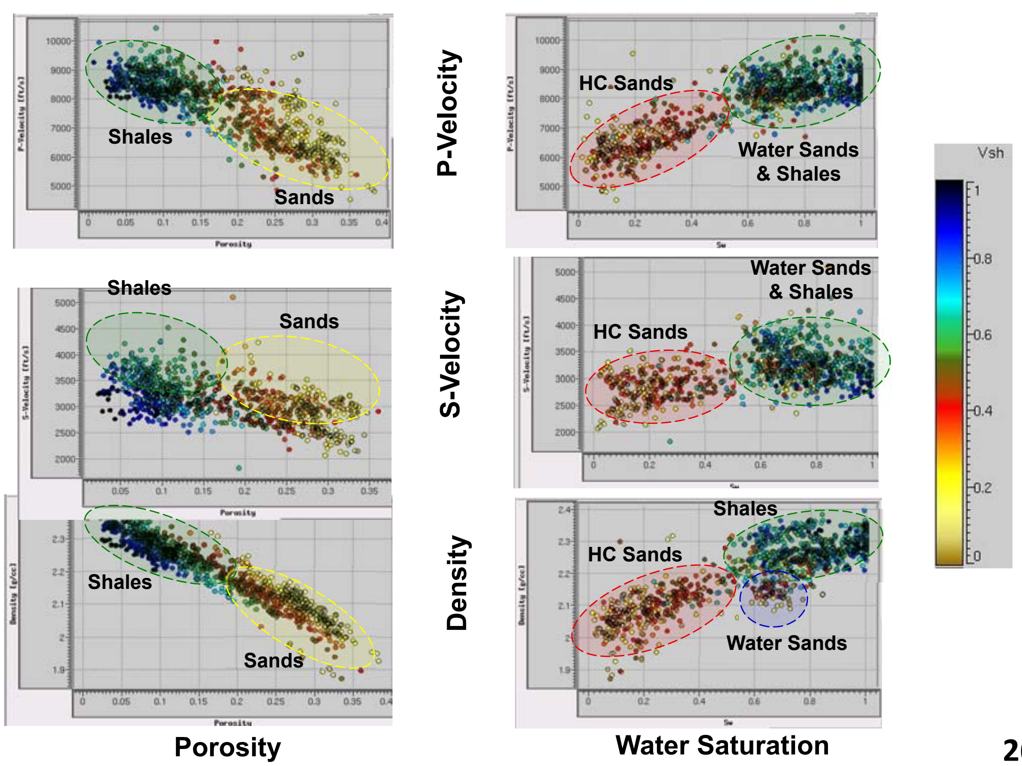
Example of Saturation Effect



Qualitative Summary

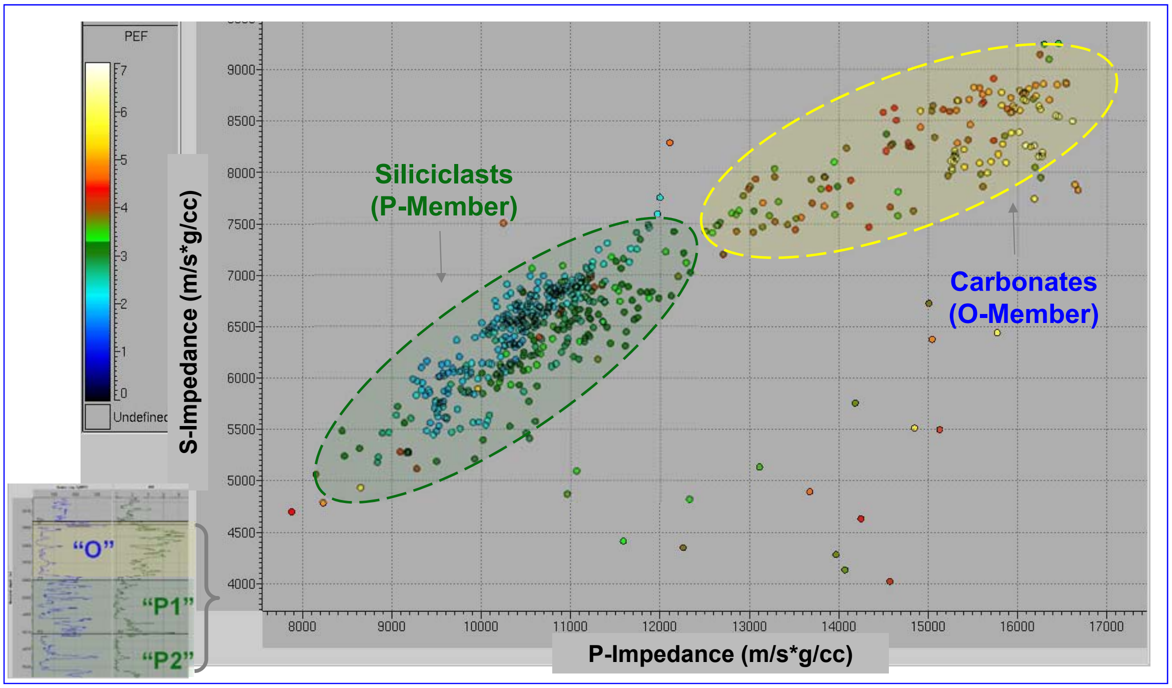


Example:
Deep Water Gulf of Mexico

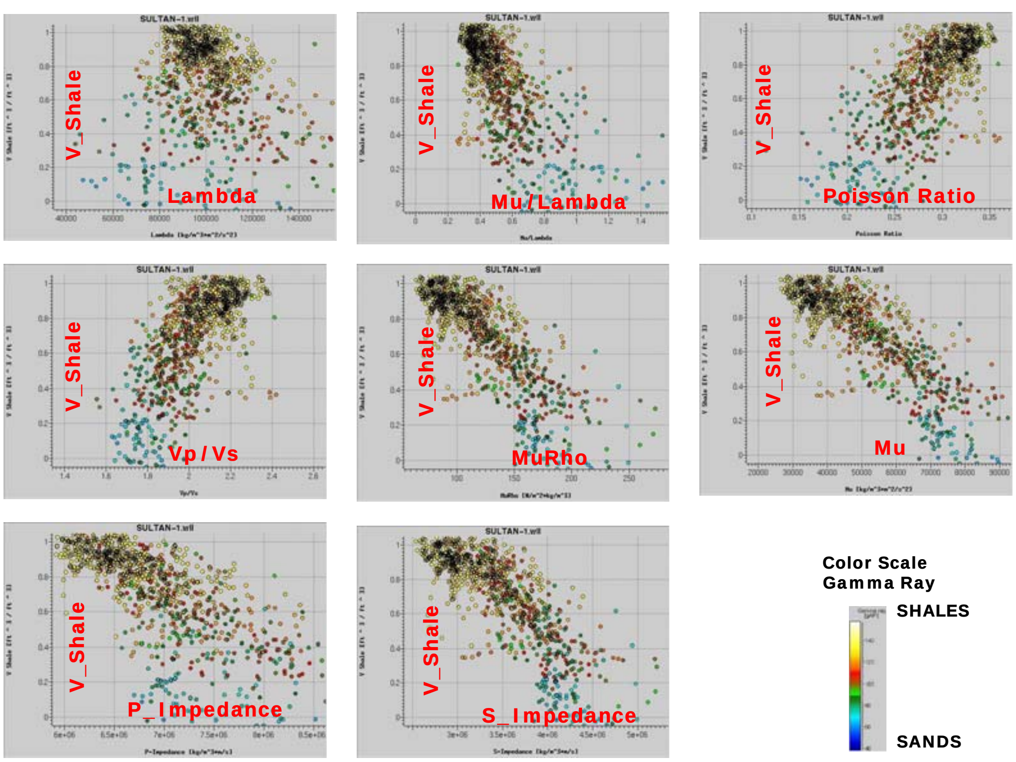


Example: CrossPlot Analysis

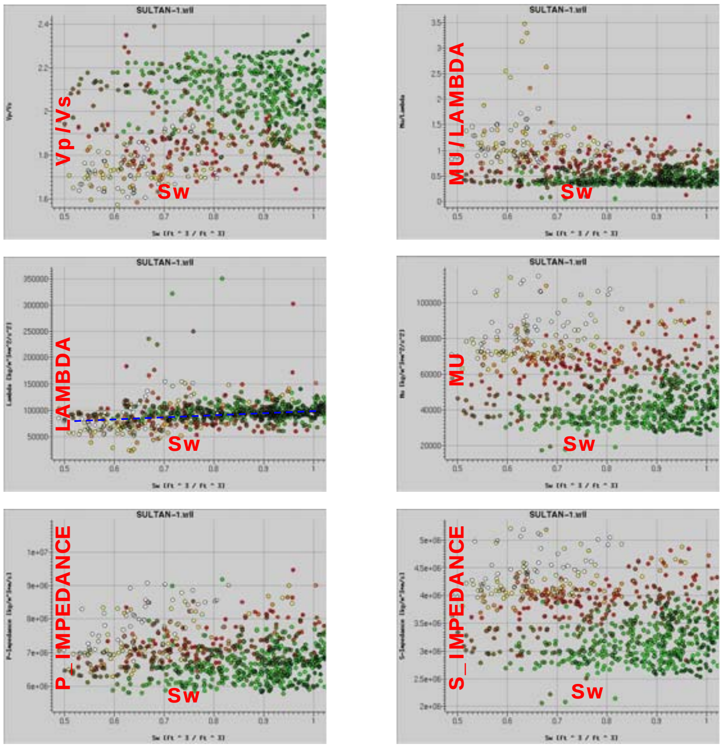
P-Impedance vs. S-Impedance



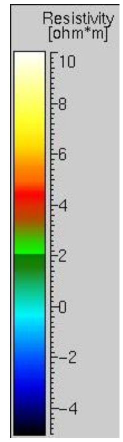
EXAMPLE: BURGOS BASIN, MEXICO



Example from Burgos Basin, Mexico



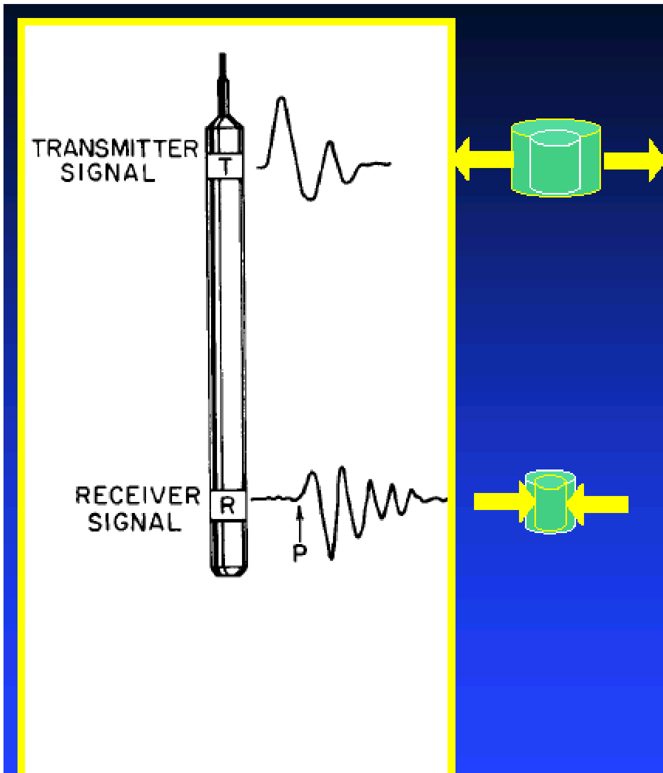
Color Scale
Resistivity



SANDS

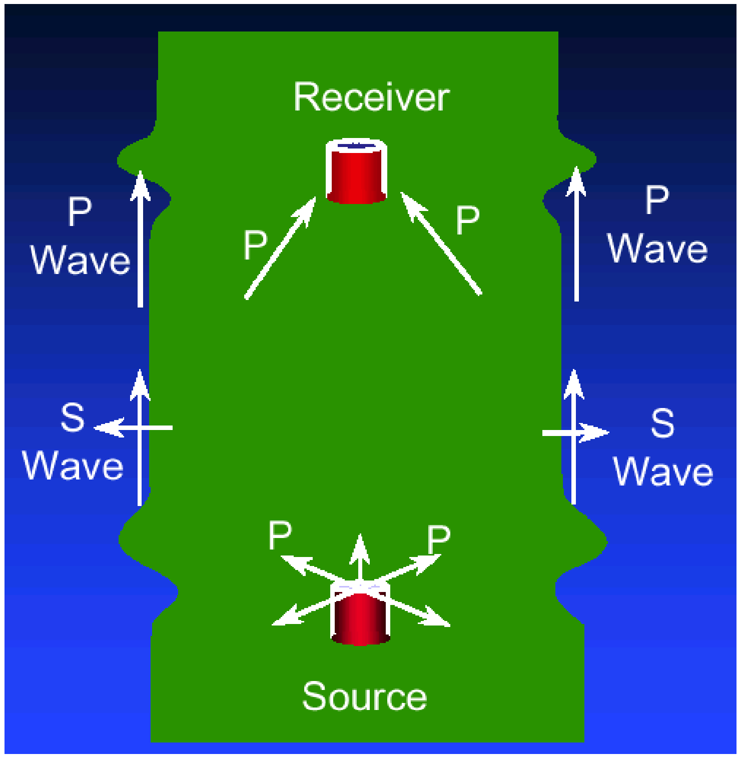
SHALES

BASIC PRINCIPLES



- Transmitter
  - Physical displacement when voltage is applied
  - Creates compressional wave in borehole fluid
  - Monopole is omni-directional
- Receiver
  - Outputs voltage when physically displaced by

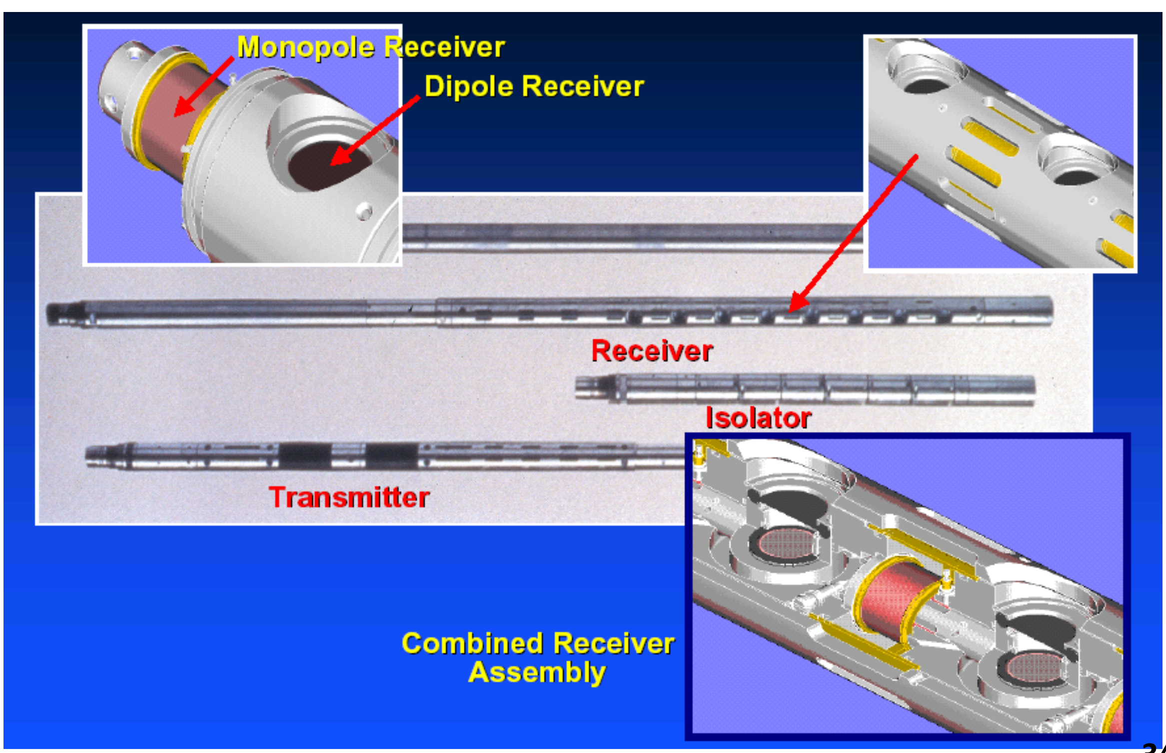
Traditional Monopole Tool

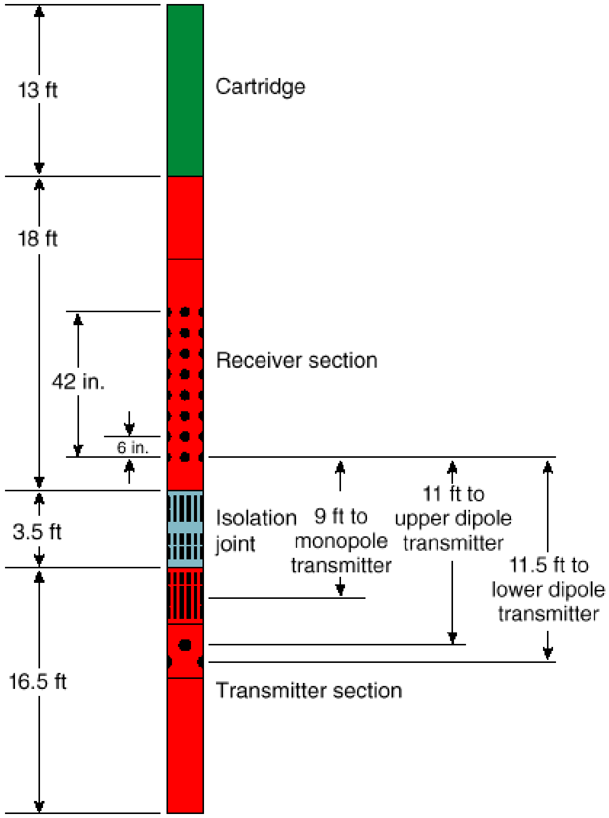


STANDARD BAKER ATLAS SONIC TOOL (FOCUS)

STANDARD BAKER ATLAS SONIC TOOL (FOCUS)

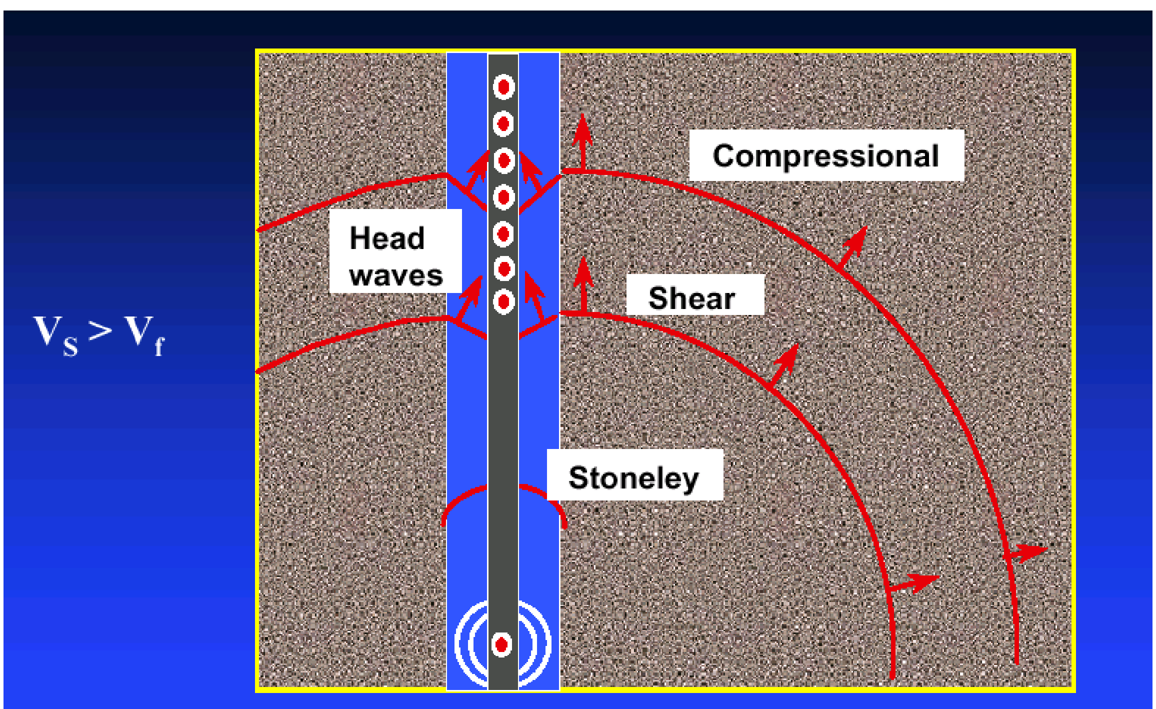
Baker-Atlas MAC Instrument



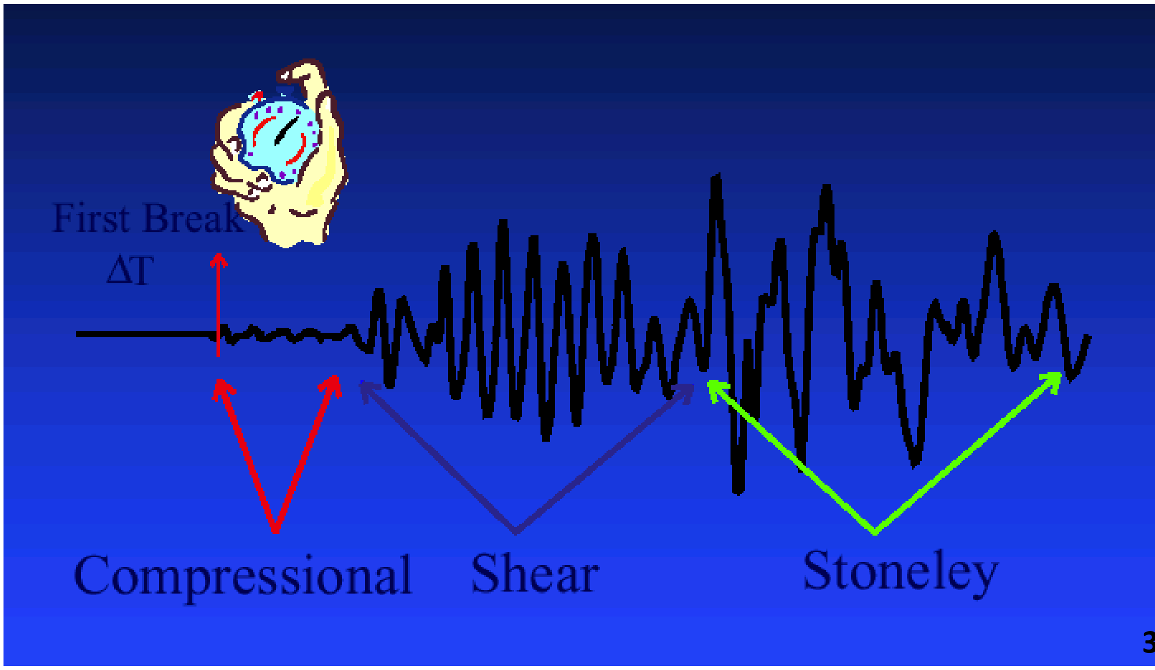


DIPOLE SONIC ARRAY TOOL

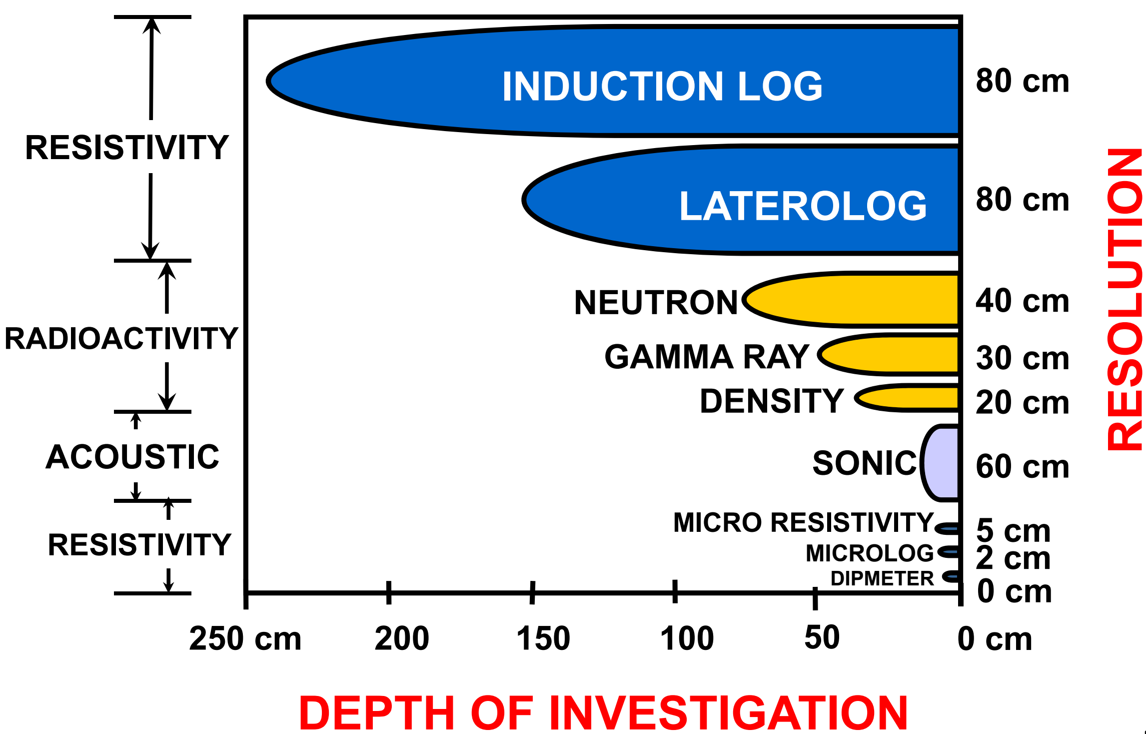
Monopole in a Fast Formation



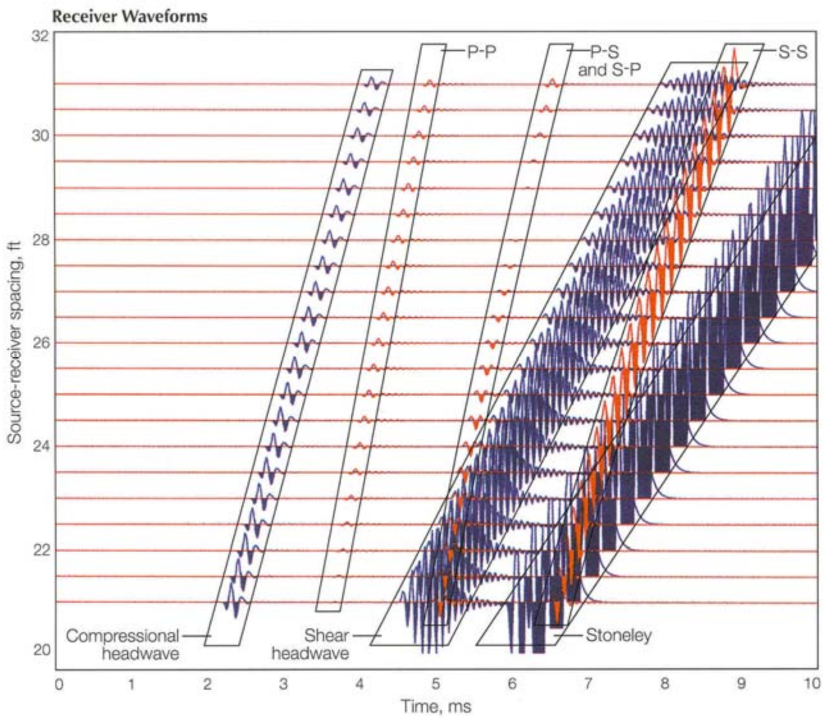
MONOPOLE WAVEFORM

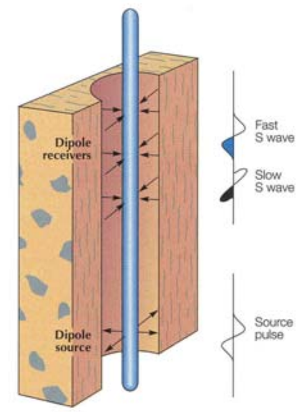


Logging Tools

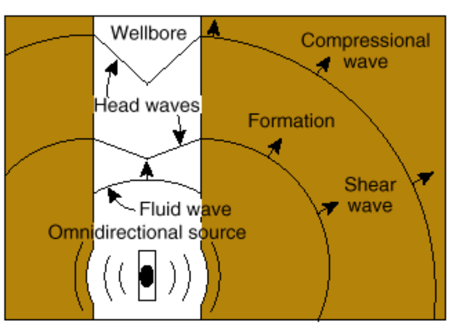


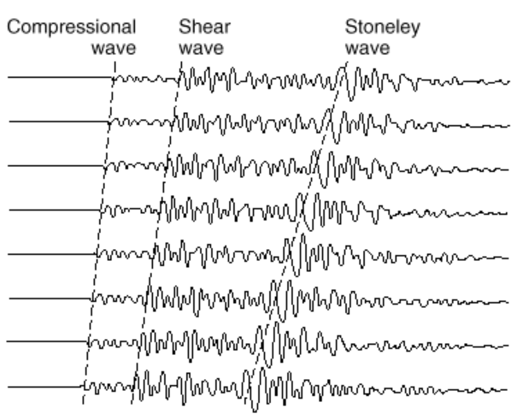
BOREHOLE WAVES

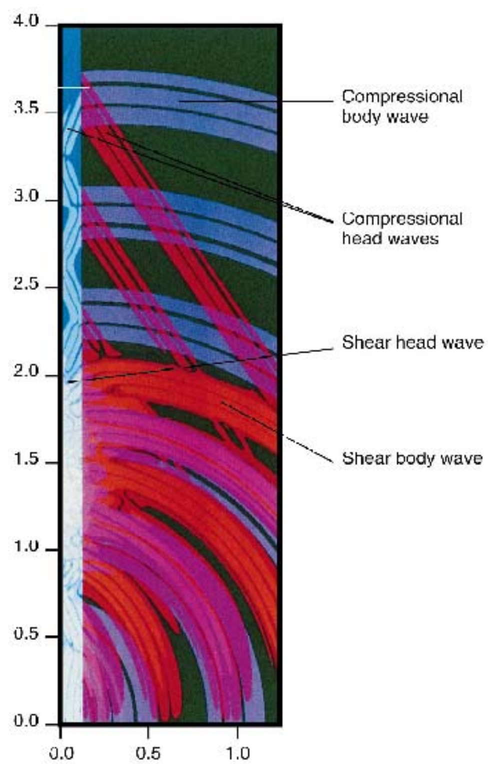




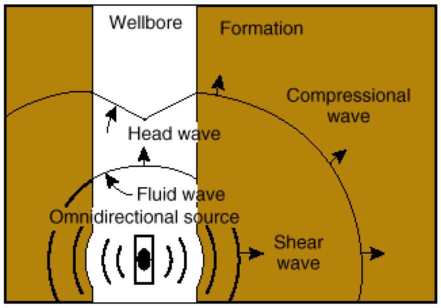
HARD FORMATION, MONOPOLE EXCITATION

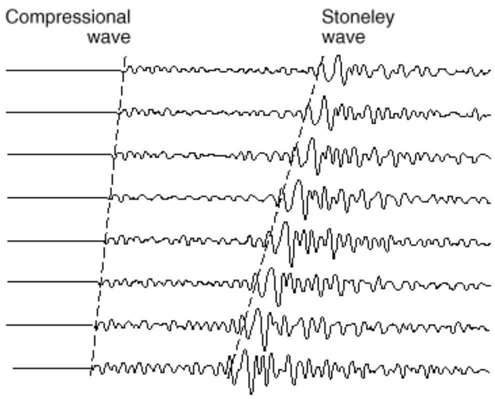


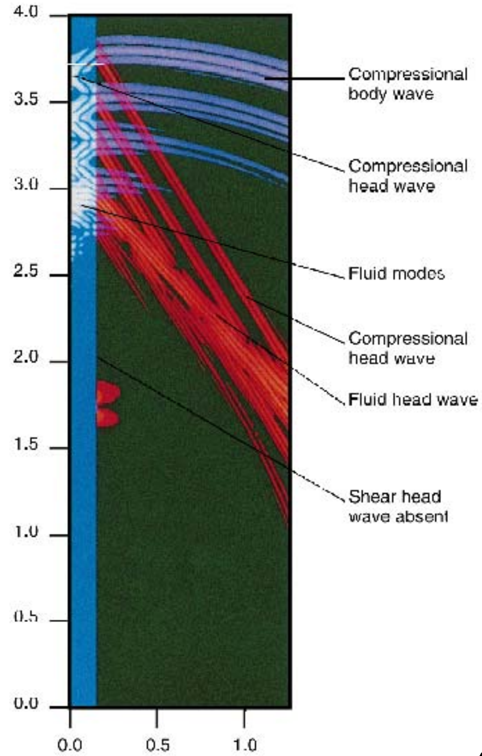




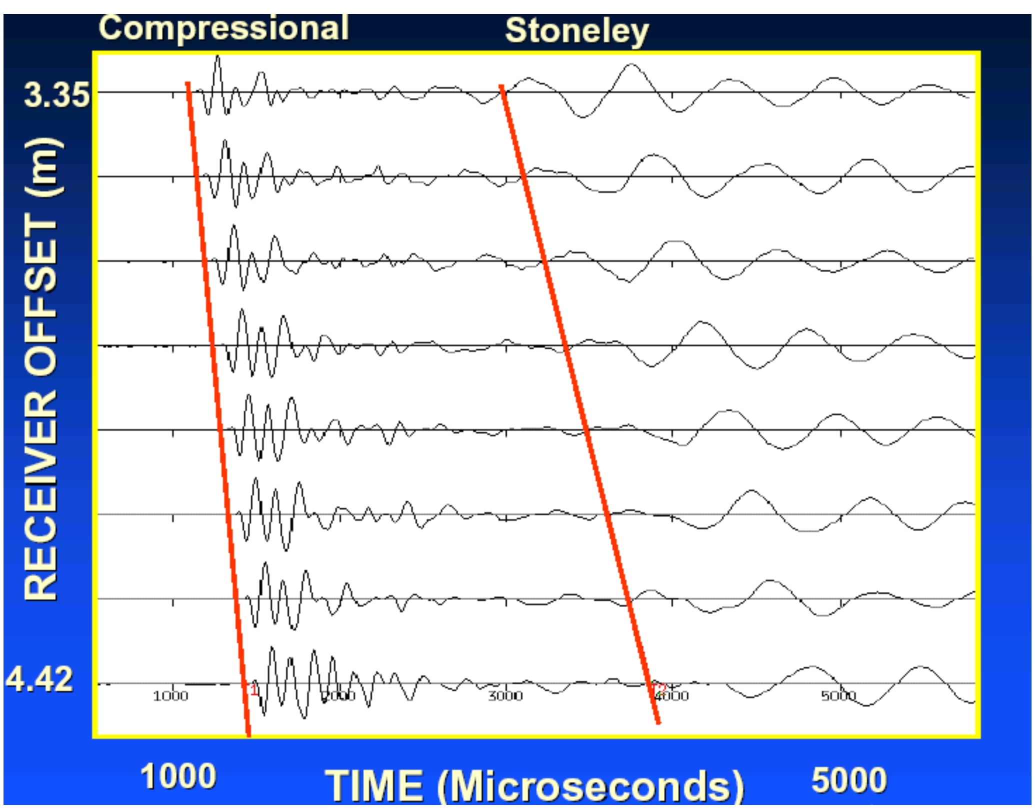
SOFT FORMATION, MONOPOLE EXCITATION







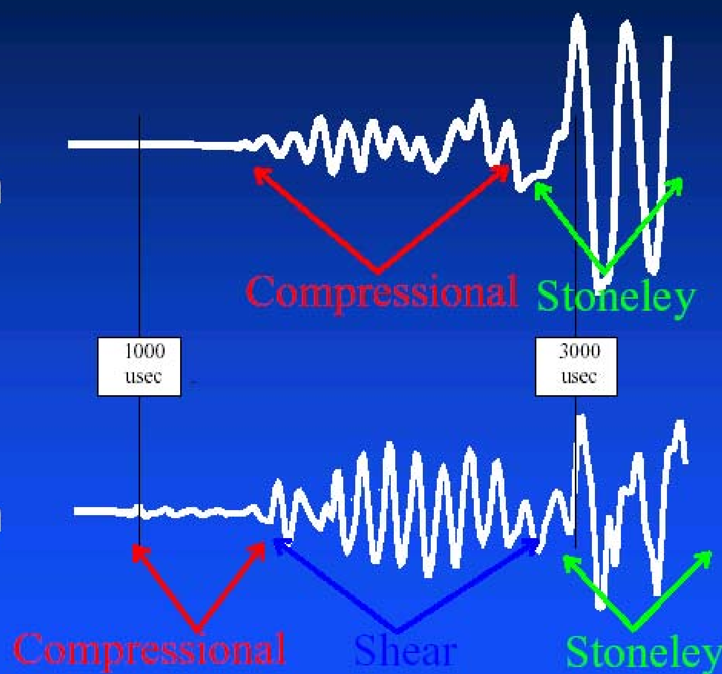
Monopole Data in Slow Formations



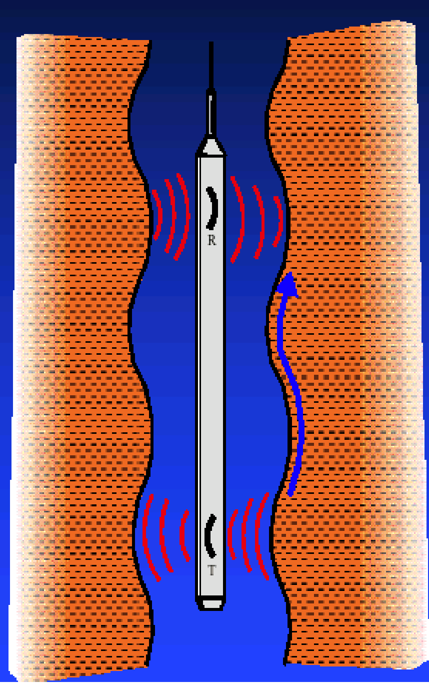
Monopole Limitation

- Slow Formation

- Fast Formation



DIPOLE FLEXURAL WAVE



Dipole

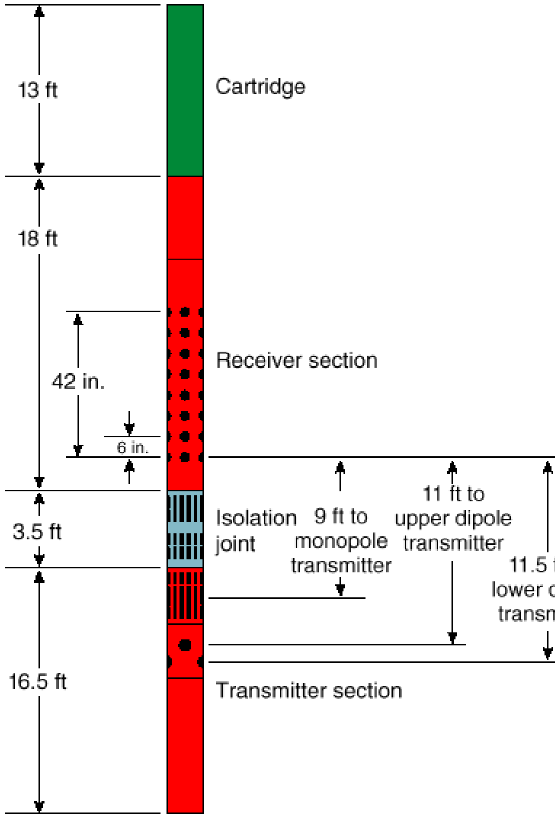
Flexural Shear

=

Monopole

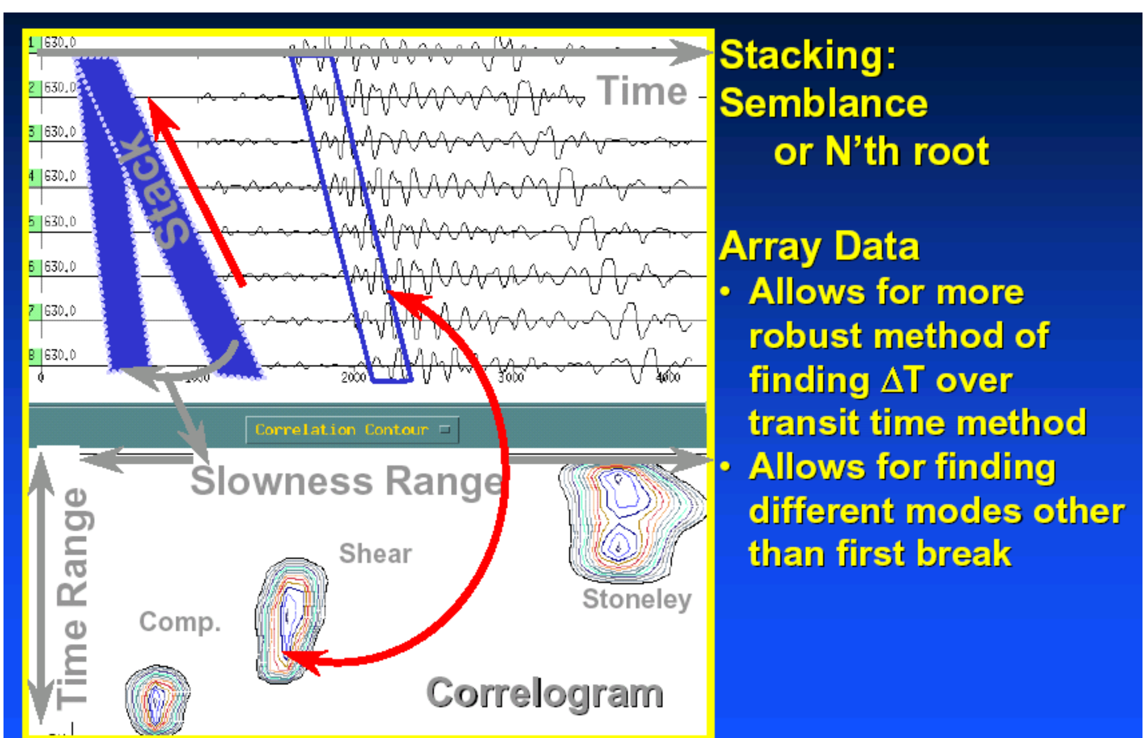
Refracted Shear

@ low frequency < 2kHz

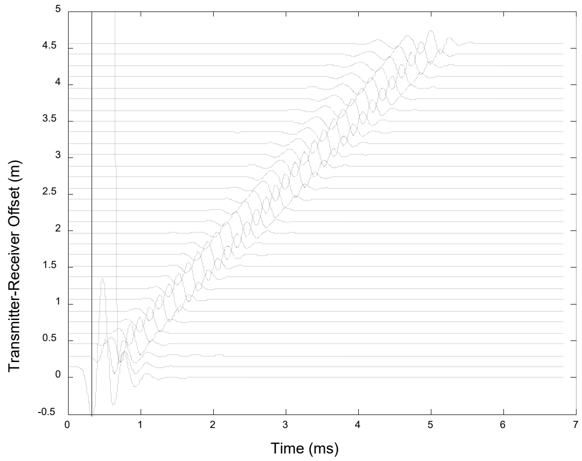


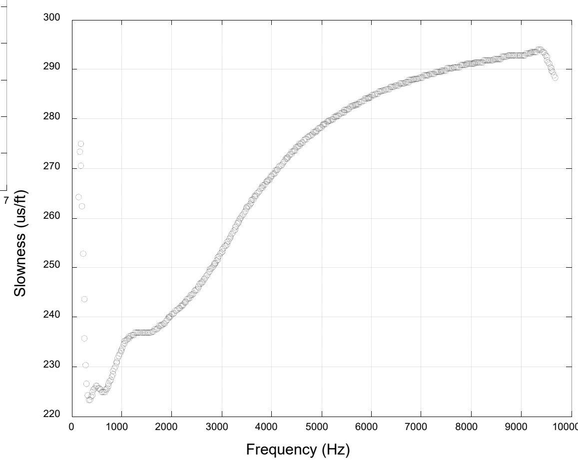
DIPOLE SONIC ARRAY TOOL

Waveform Processing: Array Correlation



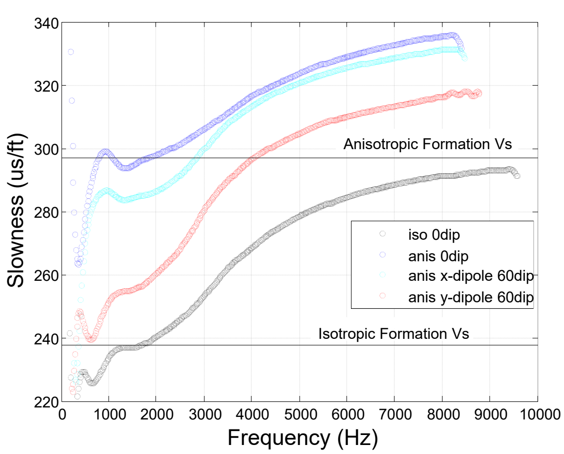
Time-Domain Waveforms and Frequency Dispersion



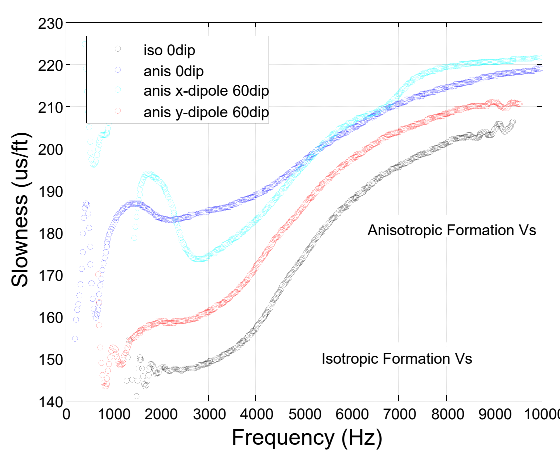


Dispersion Behavior of Flexural Waves

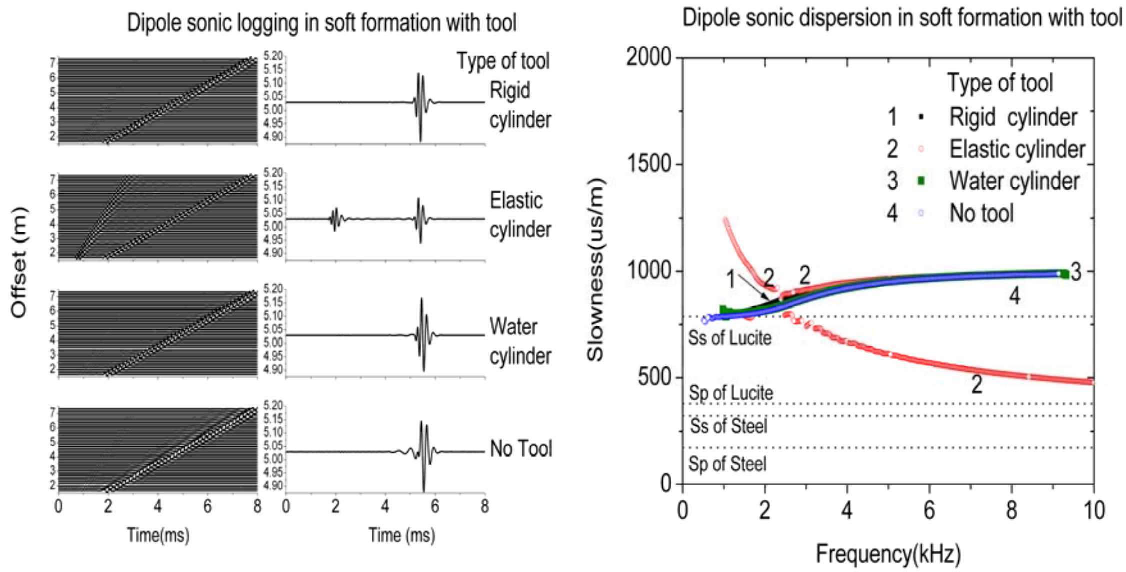
Slow Formation



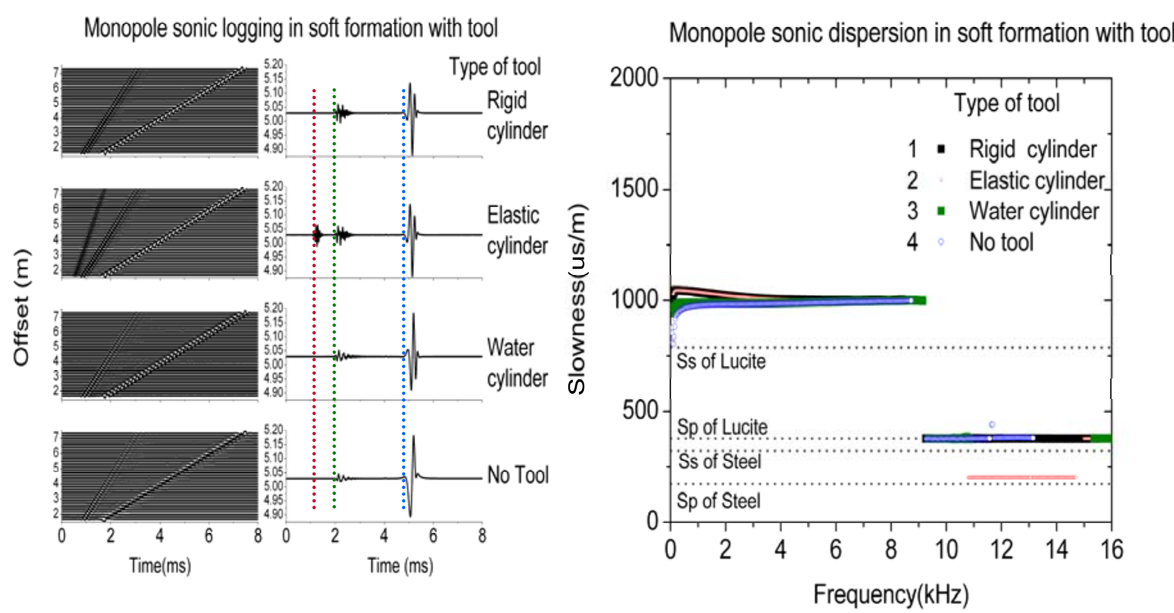
Hard Formation



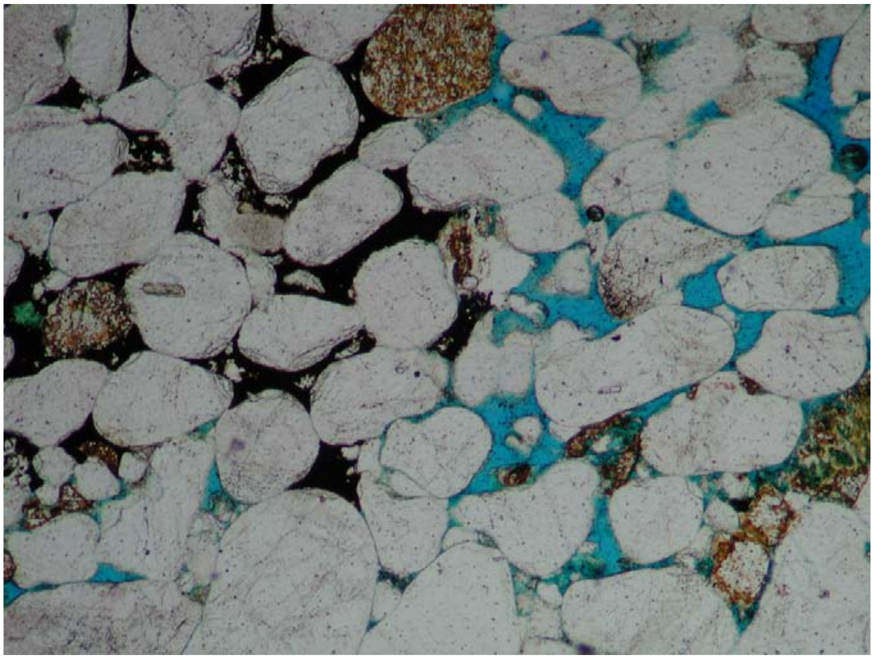
Dispersion Processing of Flexural Waves



Dispersion Processing of Monopole Waves

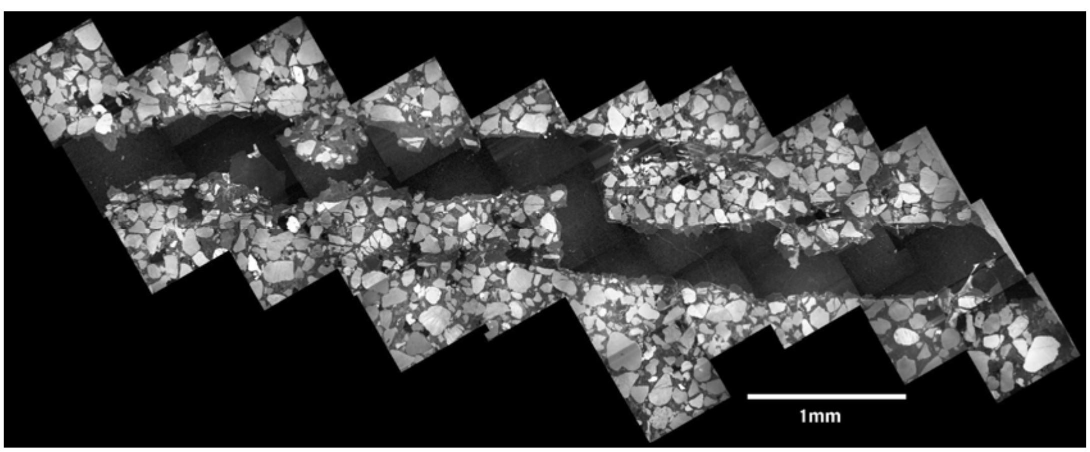


SONIC RESPONSE OF POROUS MEDIA:
Parameters that have a primary influence on the fastest arrival time



SONIC RESPONSE OF POROUS MEDIA:
Parameters that have a primary influence on the fastest arrival time

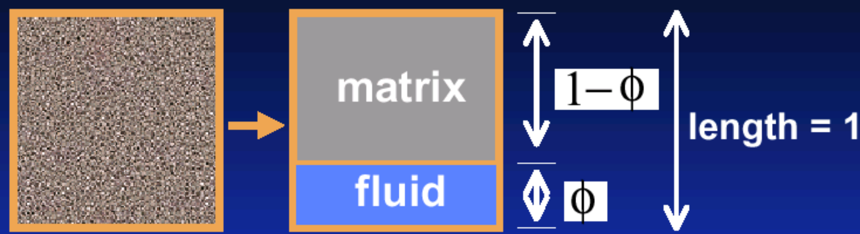
Example of Microfracturing



Photograph courtesy of Prof. Jon Olson

Sonic Porosity: Intuitive Model

(Wyllie et al. 1956)



medium	distance	velocity	travel time
matrix	1 - φ	$v_{matrix} = \Delta t_{matrix}^{-1}$	$(1-\phi)\cdot\Delta t_{matrix}$
fluid	φ	$v_{fluid} = \Delta t_{fluid}^{-1}$	$\phi\cdot\Delta t_{fluid}$
rock, model	1	$v = \Delta t^{-1}$	Δt

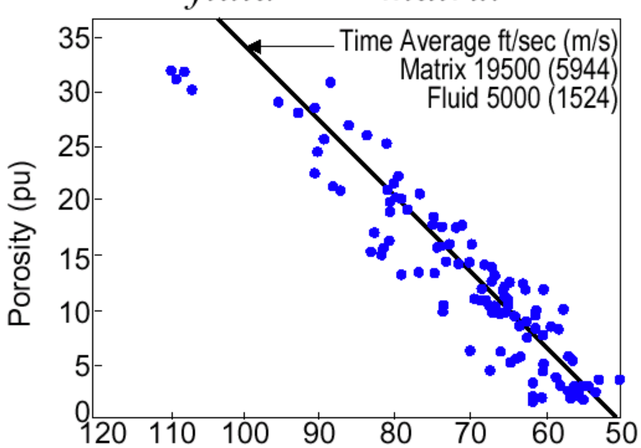
Addition of the partial travel times results in:

$$\Delta t = (1-\phi)\cdot\Delta t_{matrix} + \phi\cdot\Delta t_{fluid}$$

Theory and Measurements

$$\Delta t = (1-\phi)\cdot\Delta t_{matrix} + \phi\cdot\Delta t_{fluid}$$

$$\phi = \frac{\Delta t - \Delta t_{matrix}}{\Delta t_{fluid} - \Delta t_{matrix}}$$



from Wyllie's original publication, reprinted by Ellis 1987

Compaction Effects

- Gives good average porosity value for consolidated sediments;
- Does not consider influence of pressure differential;
- Needs corrections for unconsolidated sands : “compaction correction”;

$$\phi = \frac{\Delta t - \Delta t_{matrix}}{\Delta t_{fluid} - \Delta t_{matrix}} \times \frac{1}{C_p} \qquad C_p \; compaction\ coeff.$$

- Needs correction for shale influence (laminated or dispersed shale correction).

Biot- Gassmann

$$K_b = \overline{K} + \frac{\left(1 - \frac{\overline{K}}{K_{ma}}\right)^2}{\frac{\phi}{K_f} + \frac{1-\phi}{K_{ma}} - \frac{\overline{K}}{K_{ma}^2}}$$

$$G_b = \overline{G}$$

K and G are the dry frame bulk moduli

$$\Delta t_c = \frac{304.8}{\sqrt{\frac{K + \frac{4}{3}G}{\rho}}} \qquad \Delta t_s = \frac{304.8}{\sqrt{\frac{G}{\rho}}}$$

Intuitive Understanding

Elastic properties (compressional modulus *k*, shear modulus *μ*) of saturated rocks are described by properties:
- solid material (k_s, μ_s)
- pore fluid (k_f, μ_f)
- (dry) rock skeleton ($\bar{k}$, $\bar{\mu}$)

Equations for saturated porous rock are derived by coupling deformation of rock-skeleton (static) & fluid in pores:

$$k = \bar{k} + \frac{\left(1 - k/k_S\right)^2}{\frac{1-\phi}{k_s} + \frac{\phi}{k_f} - \frac{k}{k_S^2}}$$

$$\mu = \mu$$

$$v_p = \sqrt{\frac{M}{\rho}} = \sqrt{\frac{3k + 4\mu}{3\rho}}$$

$$v_S = \sqrt{\frac{\mu}{\rho}}$$

Example of Computation

measured:

$v_{p,dry}$ = 3500 m/s $v_{s,dry}$ = 1750 m/s ϕ = 0.2

assumed:

ρ_{water} = 1.0 g cm^{-3}	ρ_{gas} = 0 g cm^{-3}	ρ_{solid} = 2.65 g cm^{-3}
k_{water} =2.2 Gpa	k_{gas} = 0 GPa	k_{solid} = 37 GPa

Calculation:

$$M_{dry} = \bar{M} = v_{p,dry}^2 \cdot \rho solid \cdot (1-\phi) = \_\_ = 2.597 \cdot 10^{10}\,\mathrm{P}a$$

$$\bar{\mu} = \mu = v_{S,dry}^2 \cdot \rho solid \cdot (1-\phi) = \_\_ = 0.649 \cdot 10^{10}\,\mathrm{P}a$$

$$\bar{k} = \frac{3 \cdot \bar{M} - 4 \cdot \mu}{3} = 1.73 \cdot 10^{10}\,\mathrm{P}a$$

Example of Computation

measured: $v_{p,dry} = 3500$ m/s $\quad v_{s,dry} = 1750$ m/s $\quad \phi = 0.2$

Insertion into equation

$$M = k + \frac{4}{3}\mu = k + \frac{4}{3}\bar{\mu} = \bar{M} + \frac{\left(1 - \bar{k}/k_S\right)^2}{(1-\phi)/k_S + \phi/k_f - \bar{k}/k_S^2}$$

results in

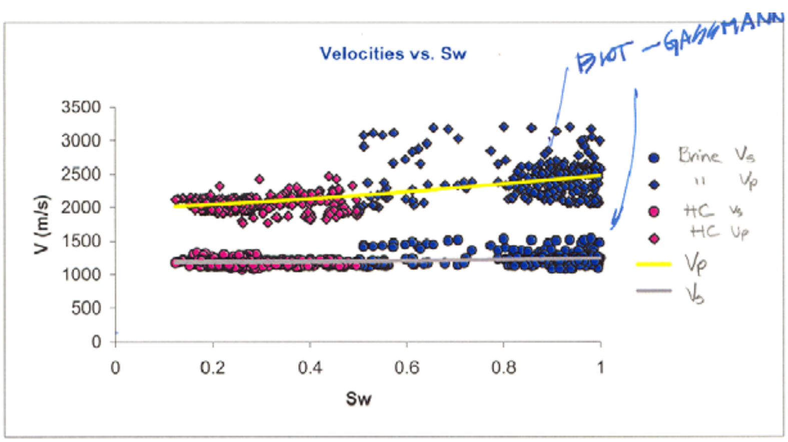
$M = 3.38\ 10^{10}$ Pa for the water saturated rock

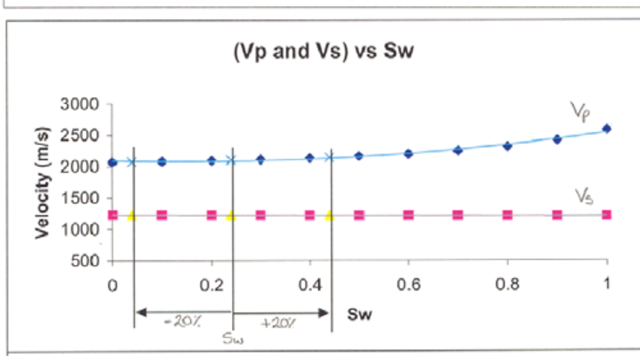
and the compressional wave velocity is

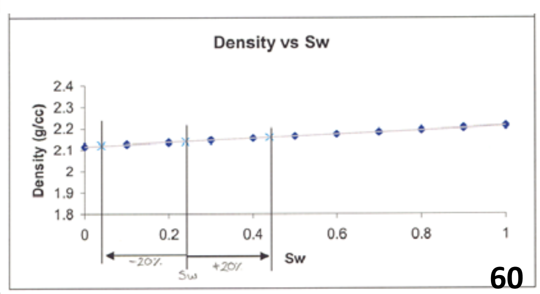
$v_p = 3816$ m/s for the water saturated rock.

for the shear wave results with $\mu = 0.65\ 10^{10}$ Pa a value $v_s = 1672$ m/s

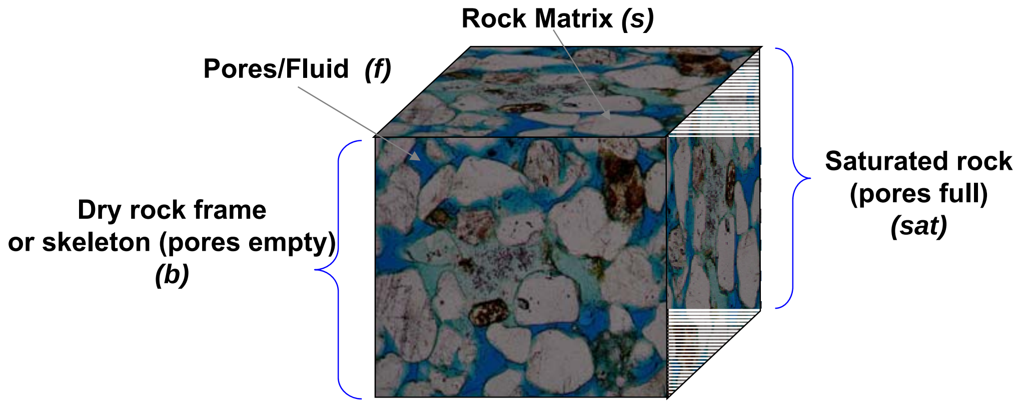
Comparison: Well-Log Data vs. Theory







Biot-Gassmann Fluid Substitution Analysis



Dry Skeleton Term **Mixed Fluid/Rock Term**

$$V_P = \sqrt{\frac{K_b + \frac{4}{3}\mu_b + \dfrac{\left(1 - \frac{K_b}{K_s}\right)^2}{\left(1 - \frac{K_b}{K_s}\right)\frac{1}{K_s} + \frac{\phi}{K_f}}}{\rho_{sat}}}$$

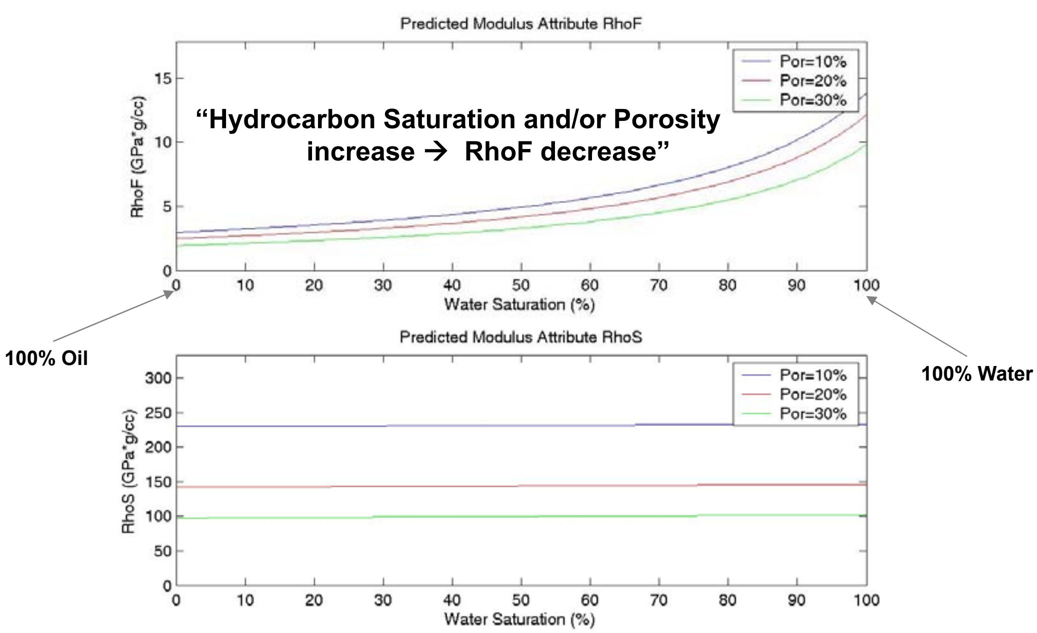
$$V_S = \sqrt{\frac{\mu_b}{\rho_{sat}}}$$

K = Bulk Modulus
μ = Shear Rigidity
ϕ = Porosity
ρ = Density

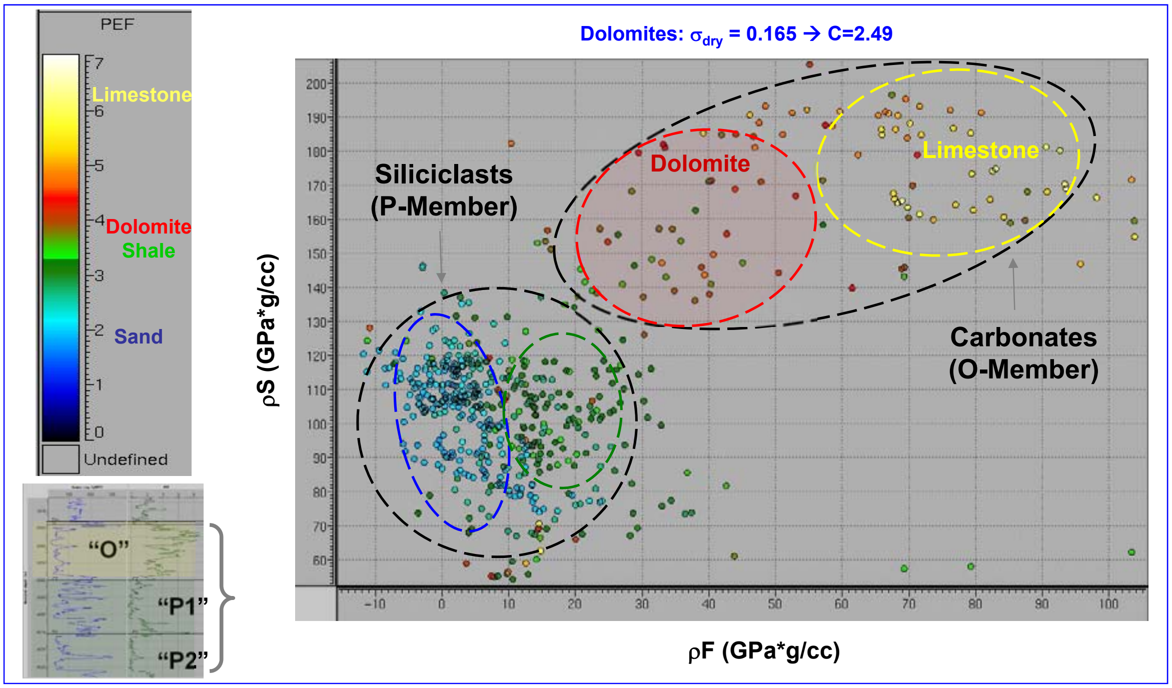
(Modified from Russell et al., 2003)

Biot-Gassmann Fluid Substitution Analysis

Dolomites: σ_{dry} = 0.165 → C=2.49

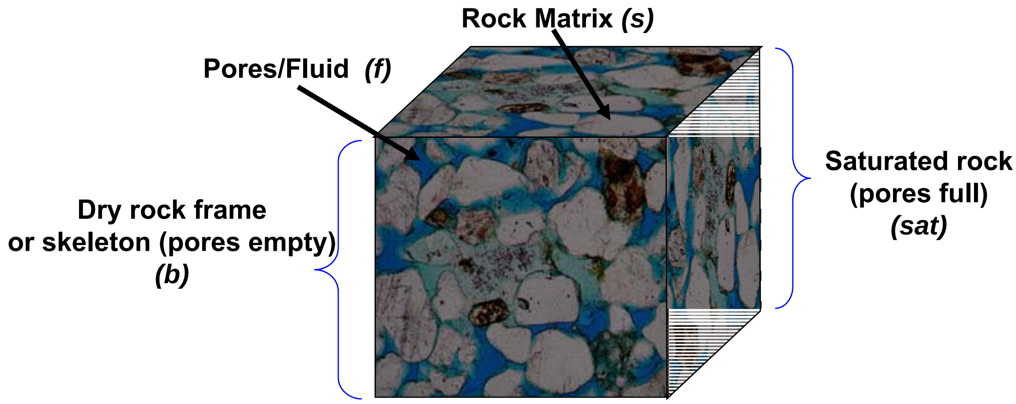


Biot-Gassmann Fluid Substitution Analysis



EXAMPLE
OF
METHOD

Biot-Gassmann Fluid Substitution



Dry Skeleton Term — **Mixed Fluid/Rock Term**

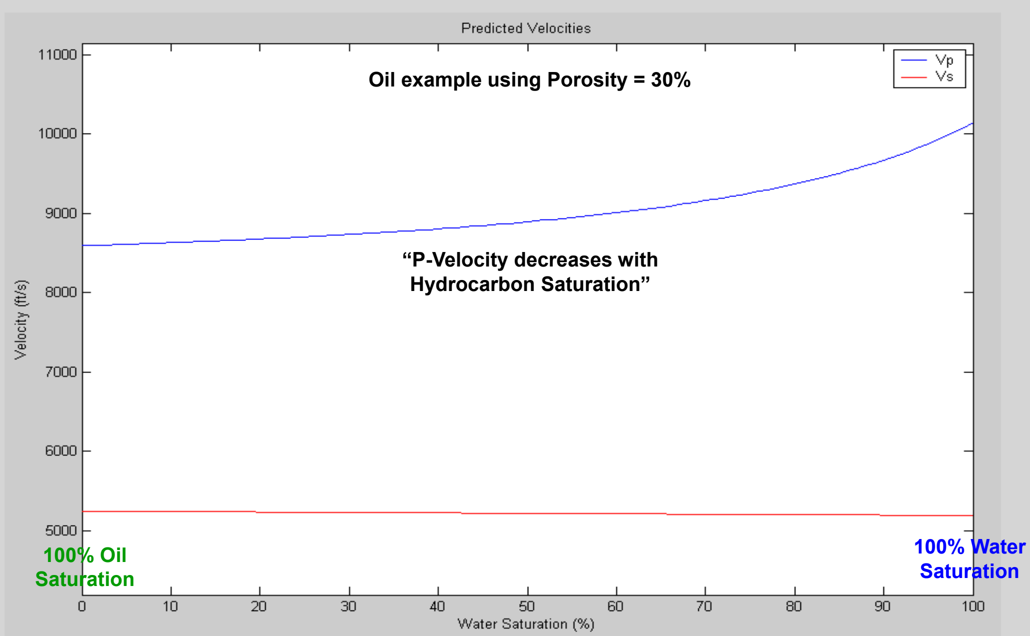
$$V_P = \sqrt{\frac{K_b + \frac{4}{3}\mu_b + \dfrac{\left(1-\frac{K_b}{K_s}\right)^2}{\left(1-\frac{K_b}{K_s}\right)\frac{1}{K_s}+\frac{\phi}{K_f}}}{\rho_{sat}}}$$

$$V_S = \sqrt{\frac{\mu_b}{\rho_{sat}}}$$

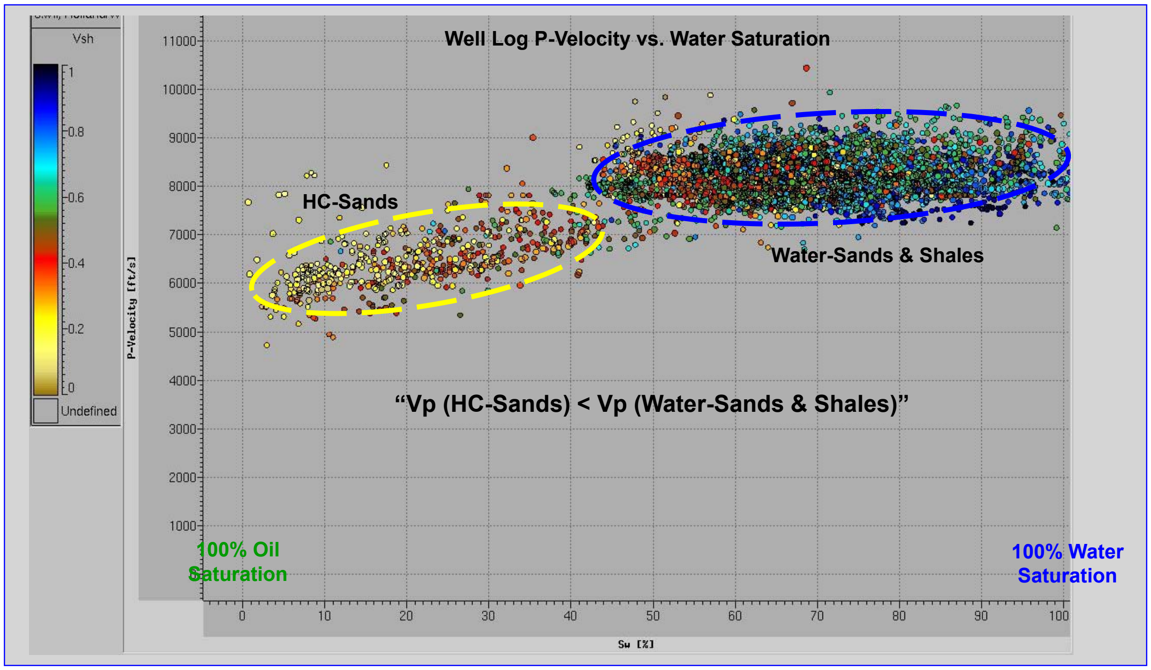
K = Bulk Modulus
μ = Shear Rigidity
ϕ = Porosity
ρ = Density

(Modified from Russell et al., 2003)

Biot-Gassmann Fluid Substitution Predictions

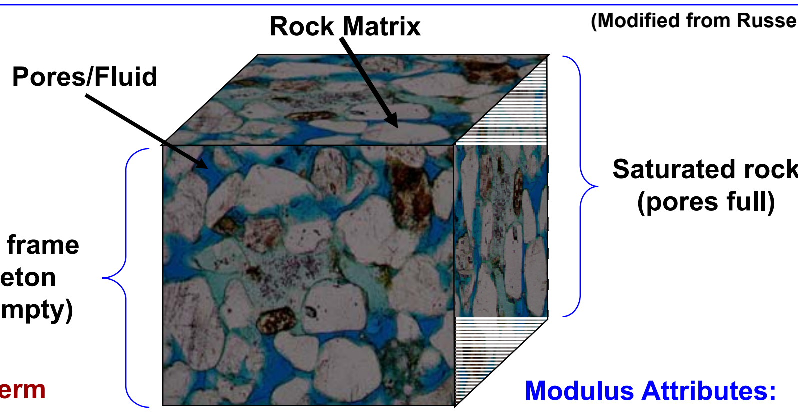


Biot-Gassmann Fluid Substitution



Biot-Gassmann Fluid Substitution

(Modified from Russell et al., 2003)



Dry Skeleton Term

Mixed Fluid/Rock Term

$$V_P = \sqrt{\frac{S + \boxed{F}}{\rho_{sat}}} \rightarrow I_P^2 = \rho * (S + F) \rightarrow$$

$$V_S = \sqrt{\frac{\mu_b}{\rho_{sat}}} \qquad I_S^2 = \rho * \mu$$

Modulus Attributes:

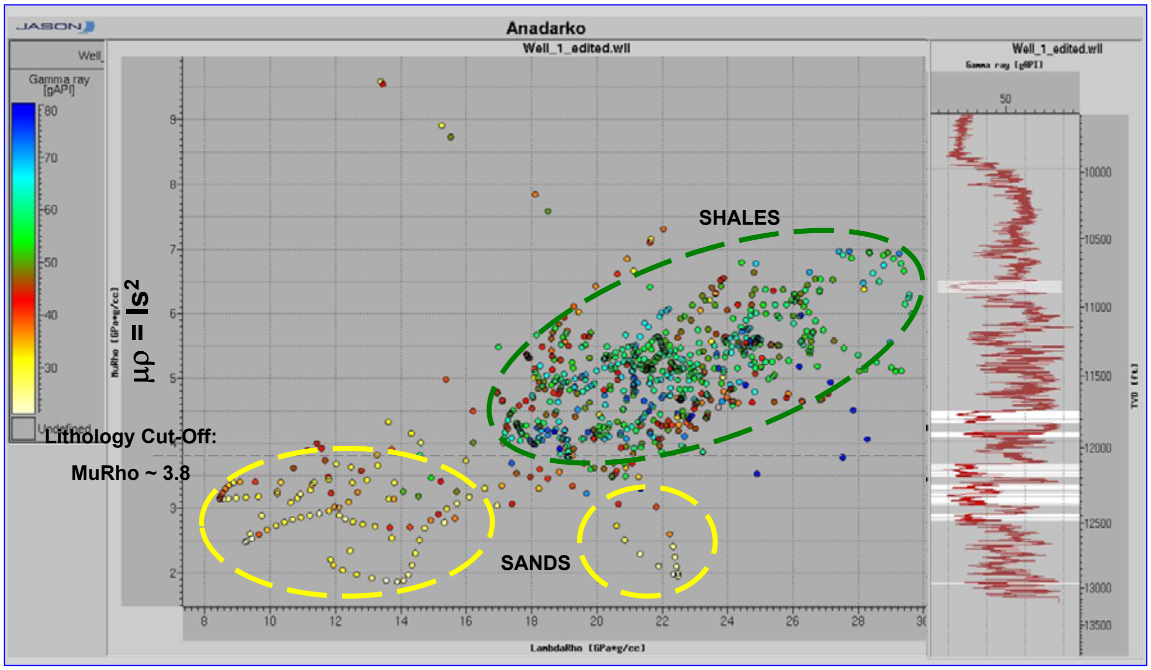
$$\boxed{\rho F = I_P^2 - c I_S^2}$$ **Fluid Sensitive**

$$\boxed{\rho S = c I_S^2}$$ **Rock Sensitive**

$$c = (\sigma_{dry} - 1) / (\sigma_{dry} - 0.5)$$

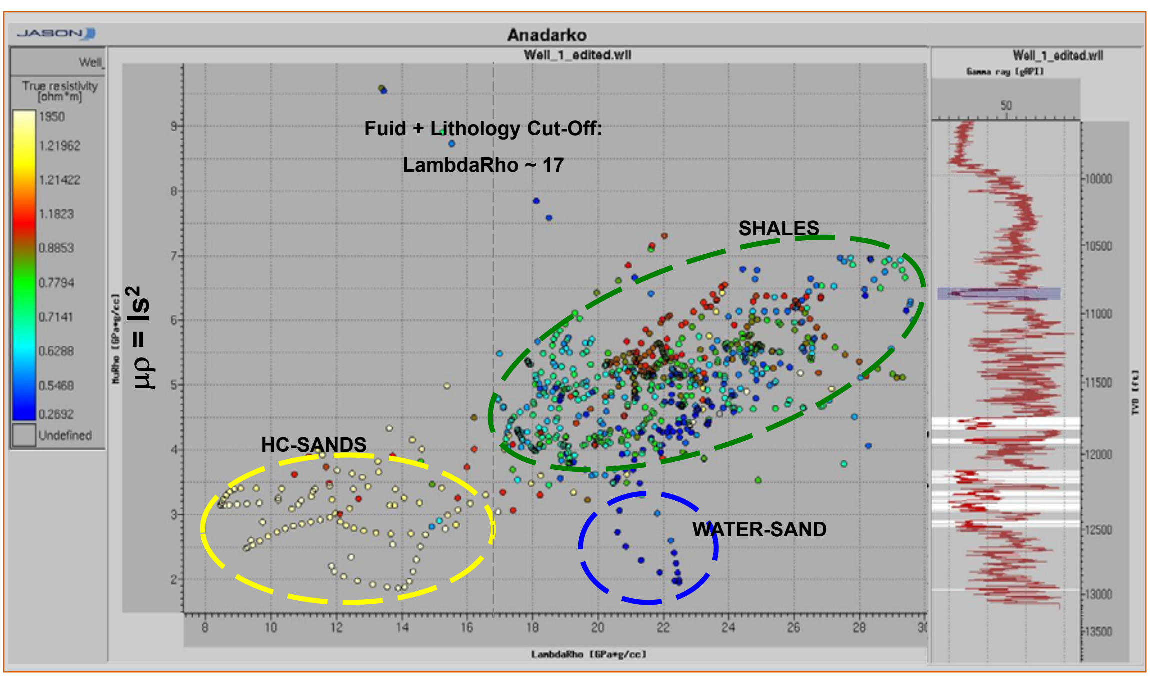
σ_{dry} = Poisson's Ratio of the dry rock
Sands: σ_{dry} = 0.12 → C=2.33

Modulus Attributes ($\lambda\rho$ vs. $\mu\rho$)



$\lambda\rho = Ip^2 - 2Is^2$ (for C=2, $\lambda\rho = \rho F$)

Modulus Attributes ($\lambda\rho$ vs. $\mu\rho$)



$\lambda\rho = Ip^2 - 2Is^2$

QUALITY

CONTROL

Assessment of Volumetric Fractions
(including porosity)

$\delta - (C_{sh} \cdot \delta_{sh}) = C_q \cdot \delta_q + C_l \cdot \delta_l + \phi \cdot \delta_f + C_d \cdot \delta_d$

$\phi_n - (C_{sh} \cdot (\phi_n)_{sh}) = C_q \cdot (\phi_n)_q + C_l \cdot (\phi_n)_l + \phi \cdot (\phi_n)_f + C_d \cdot (\phi_n)_d$

$\Delta t - (C_{sh} \cdot \Delta t_{sh}) = C_q \cdot \Delta t_q + C_l \cdot \Delta t_l + \phi \cdot \Delta t_f + C_d \cdot \Delta t_d$

$\upsilon - (C_{sh} \cdot \upsilon_{sh}) = C_q \cdot \upsilon_q + C_l \cdot \upsilon_l + \phi \cdot \upsilon_f + C_d \cdot \upsilon_d$

$1 - C_{sh} = C_q + C_l + \phi + C_d$

$$\underbrace{\begin{pmatrix} \delta_{quartz} & \delta_{limestone} & \delta_{fluid} & \delta_{dolomite} \\ (\phi_n)_{quartz} & (\phi_n)_{limestone} & (\phi_n)_{fluid} & (\phi_n)_{dolomite} \\ \Delta t_{quartz} & \Delta t_{limestone} & \Delta t_{fluid} & \Delta t_{dolomite} \\ \upsilon_{quartz} & \upsilon_{limestone} & \upsilon_{fluid} & \upsilon_{dolomite} \\ 1 & 1 & 1 & 1 \end{pmatrix}}_{C} * \underbrace{\begin{pmatrix} C_q \\ C_l \\ \phi \\ C_d \end{pmatrix}}_{x} = \underbrace{\begin{pmatrix} \delta - (C_{sh} \cdot \delta_{sh}) \\ \phi_n - (C_{sh} \cdot (\phi_n)_{sh}) \\ \Delta t - (C_{sh} \cdot \Delta t_{sh}) \\ \upsilon - (C_{sh} \cdot \upsilon_{sh}) \\ 1 - C_{sh} \end{pmatrix}}_{d}$$

A= [1 1 1 1]

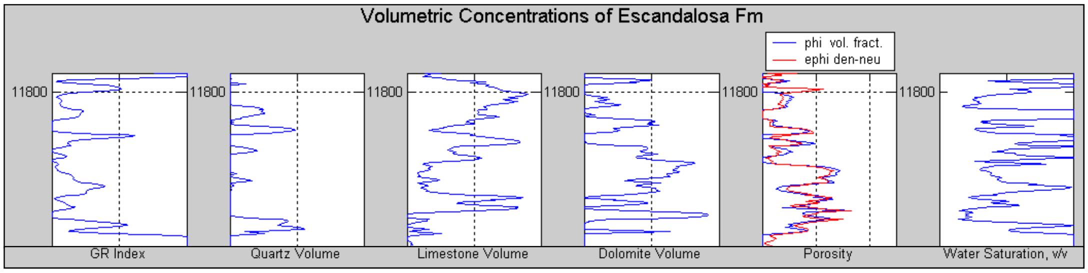
b= 1

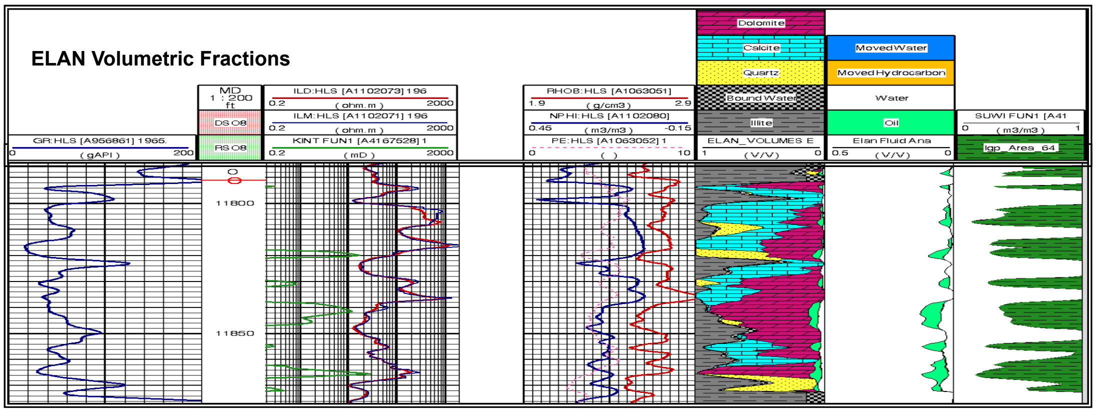
Aeq= [ ]

Beq= [ ]

$\boxed{\text{x= lsqlin (C, d, A, b, Aeq, Beq, zeros(4,1), ones(4,1))}}$

Quality Control of Solid Component Volumes and Porosity





Assessment Solid Component Volumes

(exluding porosity)

$$
\begin{aligned}
\delta - (C_{sh} \cdot \delta_{sh}) &= C_q \cdot \delta_q + C_l \cdot \delta_l + C_d \cdot \delta_d \\
\phi_n - (C_{sh} \cdot (\phi_n)_{sh}) &= C_q \cdot (\phi_n)_q + C_l \cdot (\phi_n)_l + C_d \cdot (\phi_n)_d \\
\Delta t - (C_{sh} \cdot \Delta t_{sh}) &= C_q \cdot \Delta t_q + C_l \cdot \Delta t_l + C_d \cdot \Delta t_d \\
\upsilon - (C_{sh} \cdot \upsilon_{sh}) &= C_q \cdot \upsilon_q + C_l \cdot \upsilon_l + C_d \cdot \upsilon_d \\
1 - C_{sh} &= C_q + C_l + C_d
\end{aligned}
$$

$$
\underbrace{\begin{pmatrix}
\delta_{quartz} & \delta_{limestone} & \delta_{dolomite} \\
(\phi_n)_{quartz} & (\phi_n)_{limestone} & (\phi_n)_{dolomite} \\
\Delta t_{quartz} & \Delta t_{limestone} & \Delta t_{dolomite} \\
\upsilon_{quartz} & \upsilon_{limestone} & \upsilon_{dolomite} \\
1 & 1 & 1
\end{pmatrix}}_{C}
*
\underbrace{\begin{pmatrix}
C_q \\ C_l \\ C_d
\end{pmatrix}}_{x}
=
\underbrace{\begin{pmatrix}
\delta - (C_{sh} \cdot \delta_{sh}) \\
\phi_n - (C_{sh} \cdot (\phi_n)_{sh}) \\
\Delta t - (C_{sh} \cdot \Delta t_{sh}) \\
\upsilon - (C_{sh} \cdot \upsilon_{sh}) \\
1 - C_{sh}
\end{pmatrix}}_{d}
$$

A= [1 1 1]

b= 1

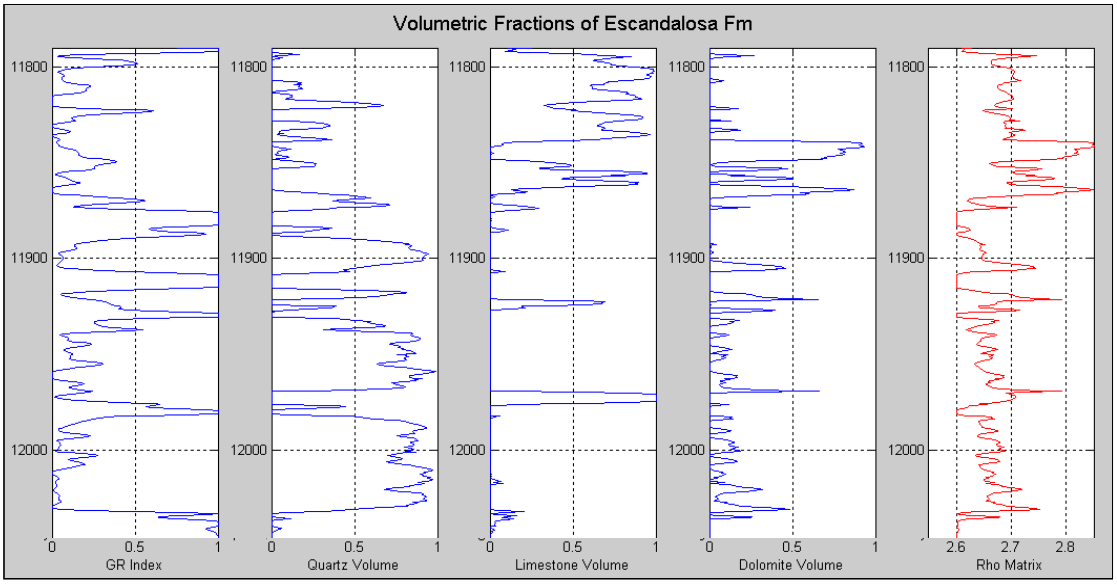
Aeq= [ ]

Beq= [ ]

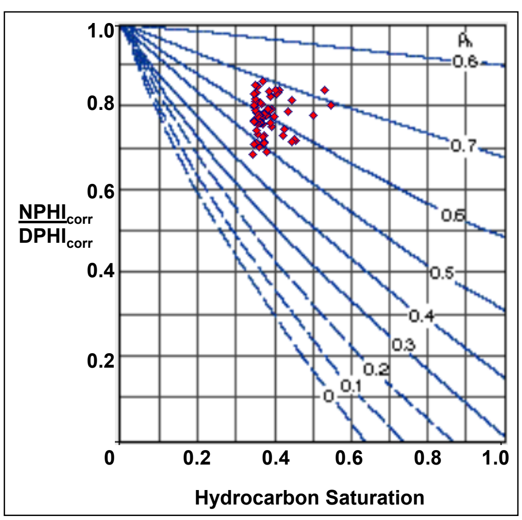
$\boxed{\textbf{x= lsqlin (C, d, A, b, Aeq, Beq, zeros(3,1), ones(3,1))}}$

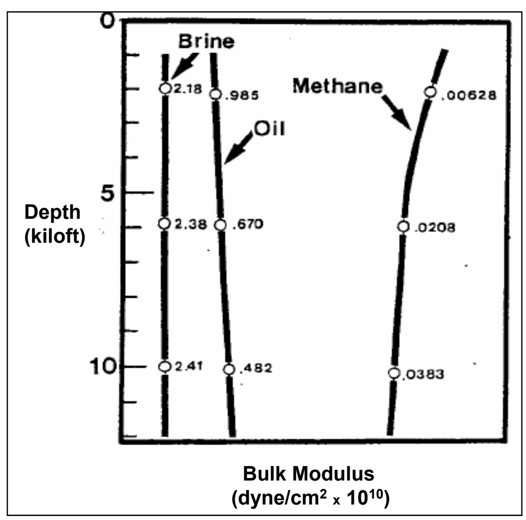
Quality Control of Solid Component Volumes and Estimation of Rho Matrix

Rho Matrix= Vq * 2.65 + Vl * 2.71 + Vd * 2.87 + Vsh * 2.6

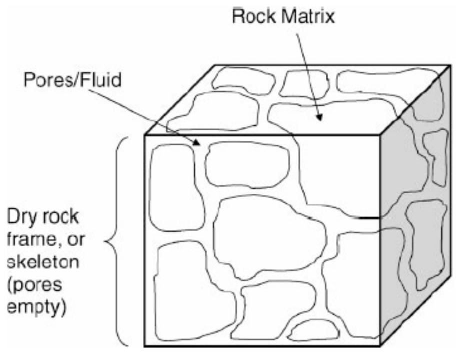


Estimation of Hydrocarbon Density and Fluid Bulk Modulus





GASSMAN-BIOT-GEERTSMA EQUATIONS (I)



(1) $$\rho_f = \left[(S_w)(\rho_{H_2O})\right] + \left[(1 - S_w)(\rho_{gas,oil})\right]$$

(2) $$\rho_b = \left[(\phi)(\rho_f)\right] + \left[(1 - \phi)(\rho_s)\right]$$

(3) $$\boxed{K_f = \frac{1}{\left(S_w / K_{H_2O}\right) + \left((1 - S_w) / K_{gas,oil}\right)}}$$

(4) $$S = \frac{3(1 - \sigma)}{(1 + \sigma)}$$

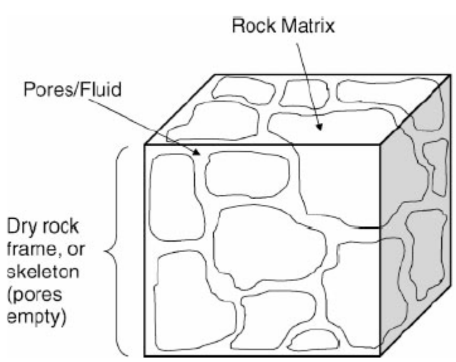
(5) $$M = (V_p^2)(\rho_b)(929 * 10^{-10})$$

(6) $$A = S - 1$$

(7) $$B = \left[(\phi)(S)\left(\frac{K_s}{K_f} - 1\right)\right] - [S] + \left[\frac{M}{K_s}\right]$$

where Ks = $Ks_{qz} * V_{qz}$ + $Ks_{sh} * V_{sh}$ + $Ks_{li} * V_{li}$ + $Ks_{do} * V_{do}$

GASSMAN-BIOT-GEERTSMA EQUATIONS (II)



(8) $$C = (-\phi)\left(S - \frac{M}{K_s}\right)\left(\frac{K_s}{K_f} - 1\right)$$

(9) $$Y = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$$

(10) $$\boxed{K_b = (1 - Y)K_s}$$

$$V_p^2 = \left[K_b + \frac{4}{3}\mu_b + \frac{\left(1 - K_b / K_s\right)^2}{\left(1 - K_b / K_s\right) 1 / K_s + \phi / K_f}\right] \frac{1}{\rho_b};$$

$$V_s^2 = \frac{\mu_b}{\rho_b}$$

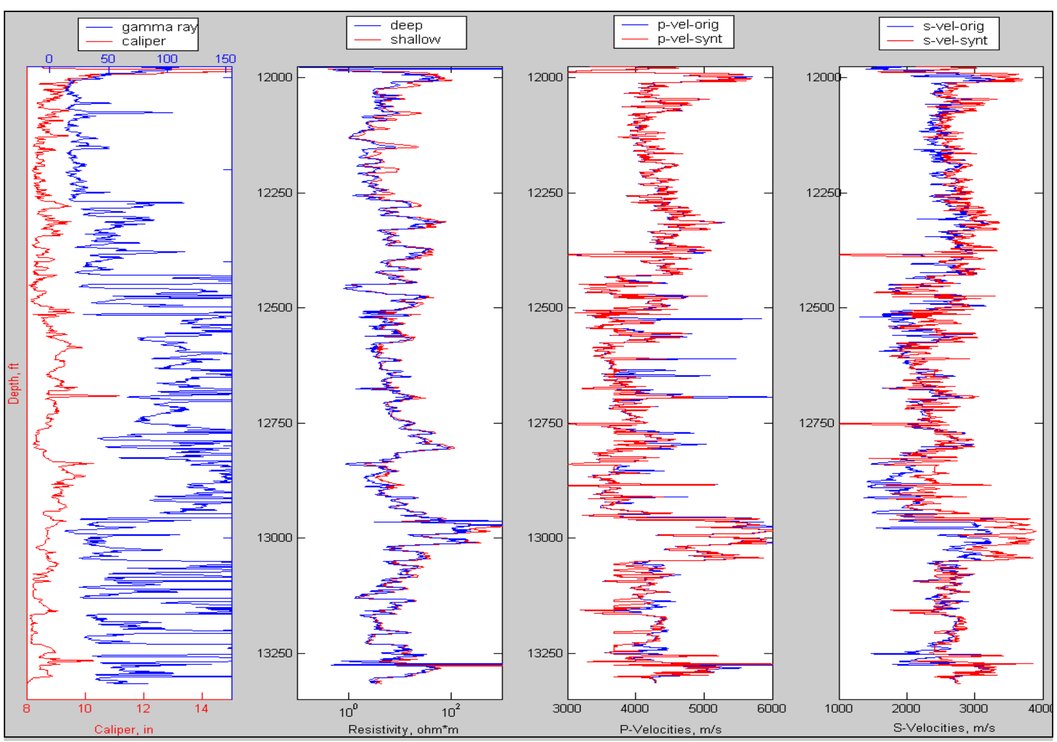
Applying Biot-Gassman's formula in MatLab

```
for i=1:length(depth)
    rho_f(i)=(Sw(i)*rho_w)+((1-Sw(i))*rho_hc);           %rho_f is density of fluid;
    rho_b(i)=(ephi(i)*rho_f(i))+((1-ephi(i))*rho_s);     %rho_b is density of skeleton;
    Kf(i)=1/((Sw(i)/Kw)+((1-Sw(i))/Khc));                %kf is bulk modulus of fluid;
    S=(3*(1-Sg))/(1+Sg);                                 %S is a single variable;
    M(i)=((Vp_ft_s(i))^2)*rho_b(i)*(929e-10);            %M is another variable;
    J=S-1;                                               %J is the A variable (no petroph. signif.);
    G(i)=(ephi(i)*S*((Ks(i)/Kf(i))-1))-S+(M(i)/Ks(i));   %G is the B variable (no petroph. signif.);
    W(i)=(-1*ephi(i))*(S-(M(i)/Ks(i)))*((Ks(i)/Kf(i))-1); %W is the C variable;
    Y(i)=(-G(i)+sqrt(G(i)^2-4*J*W(i)))/(2*J);            %Y is another variable;
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    Kb(i)=(1-Y(i))*Ks(i);                                %Kb is the BULK MODULUS OF DRY ROCK;
    Kp(i)=ephi(i)*((1/Kb(i))-(1/Ks(i)))^(-1);            %Kp is the Pore Bulk Modulus;
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    Mu_b(i)=0.75*Kb(i)*(S-1);                           %Mu_b is Shear Rigidity of Skeleton;
    Vp_syn2(i)=(Kb(i)+(1.333*Mu_b(i))+((1-(Kb(i)/Ks(i)))^2/(((1-(Kb(i)/Ks(i)))*(1/Ks(i)))+ (ephi(i)/Kf(i)))))*(1/rho_b(i));
    Vp_syn(i)=sqrt(Vp_syn2(i))*0.0328e5*0.3048;       Synthetic P-Velocity in m/s
    Vs_syn(i)=0.0328e5*0.3048*sqrt(Mu_b(i)/rho_b(i)); Synthetic S-Velocity in m/s
end
```

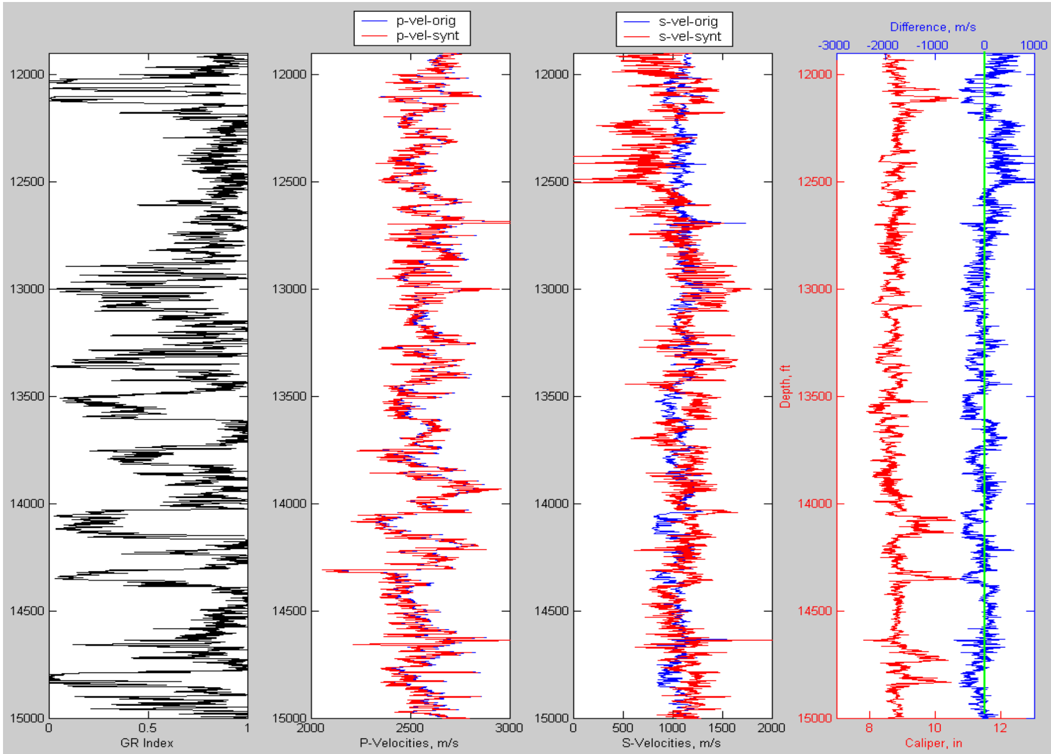
Where:

`Ks_qz=36;`	**Bulk Modulus of Quartz**
`Ks_sh=15;`	**Bulk Modulus of Shale**
`rho_w=1;`	**Density of Fresh Water**
`rho_hc=0.75;`	**Density of Hydrocarbon**
`rho_s=2.65;`	**Density of the Solid Component**
`Kw=2.38;`	**Bulk Modulus of Water**
`Khc=0.67;`	**Bulk Modulus of Hydrocarbon**
`Sg=0.12;`	**Poisson's Ratio of Dry Rock**

Evaluation of Results in Consolidated Fms



Evaluation of Results in Unconsolidated Fms



Castagna-Greenberg's Shear Wave Estimation

$$V_S = \frac{1}{2}\left\{\left[\sum_{i=1}^{L} X_i \sum_{j=0}^{N_i} a_{ij} V_P^j\right] + \left[\sum_{i=1}^{L} X_i \left(\sum_{j=0}^{N_i} a_{ij} V_P^j\right)^{-1}\right]^{-1}\right\}$$

$$\sum_{i=1}^{L} X_i = 1$$

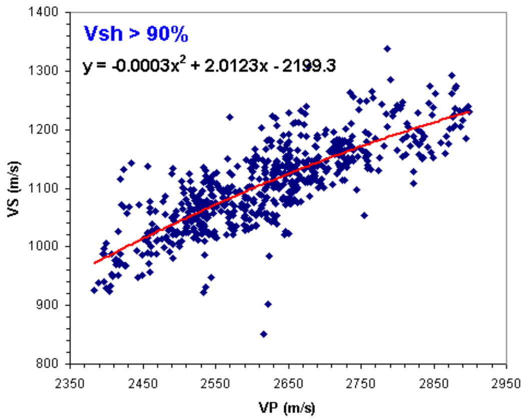
where

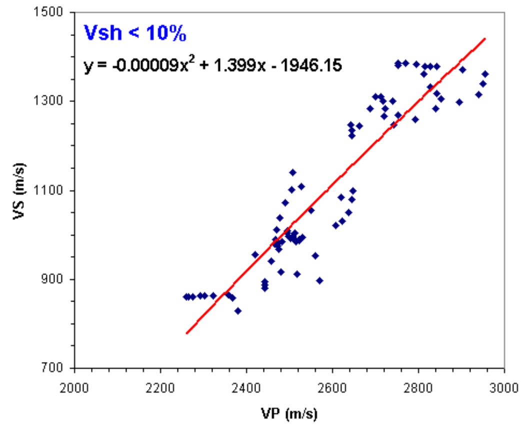
L = number of pure monomineralic lithologic constituents
X_i = volume fractions of lithological constituents
a_{ij} = empirical regression coefficients
N_i = order of polynomial for constituent i
V_P, V_S = P- and S-wave velocities (km/s) in composite brine-saturated, multimineralic rock

Empirical regression coefficients (a_{ij}) with:

Lithology	a_{i2}	a_{i1}	a_{i0}
Sandstone	0	0.80416	-0.85588
Limestone	-0.05508	1.01677	-1.03049
Dolomite	0	0.58321	-0.07775
Shale	0	0.76969	-0.86735

Regression Coefficients for Unconsolidated Formations



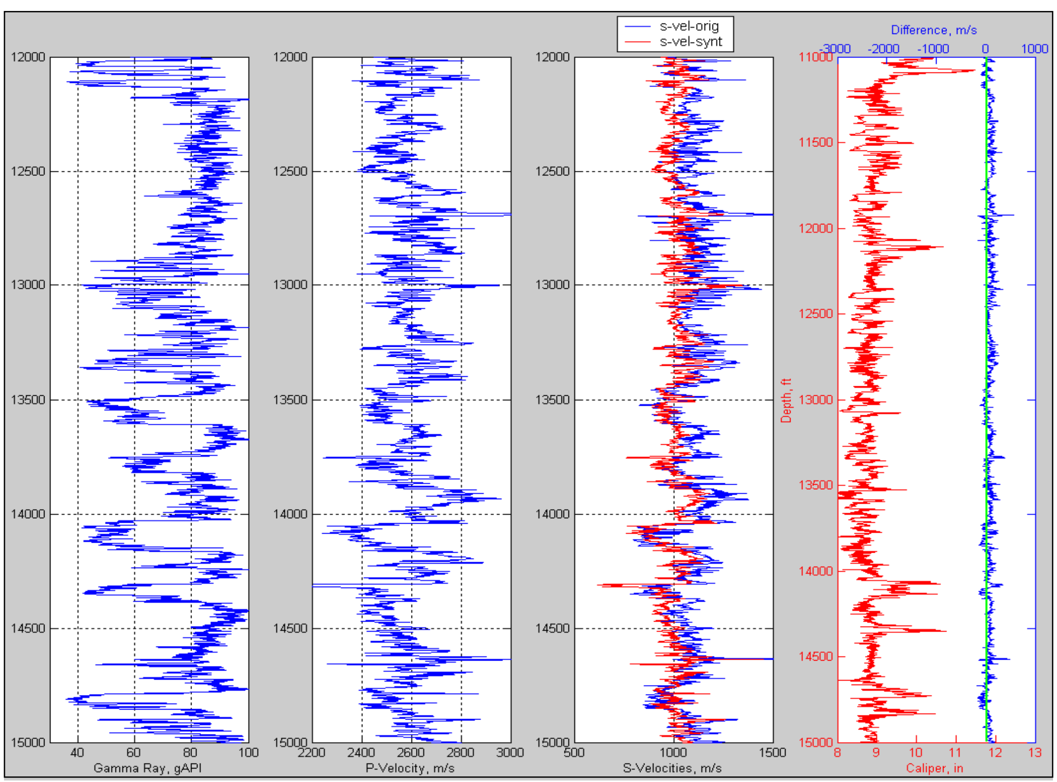


So, regression coefficients* (a_{ij}) are:

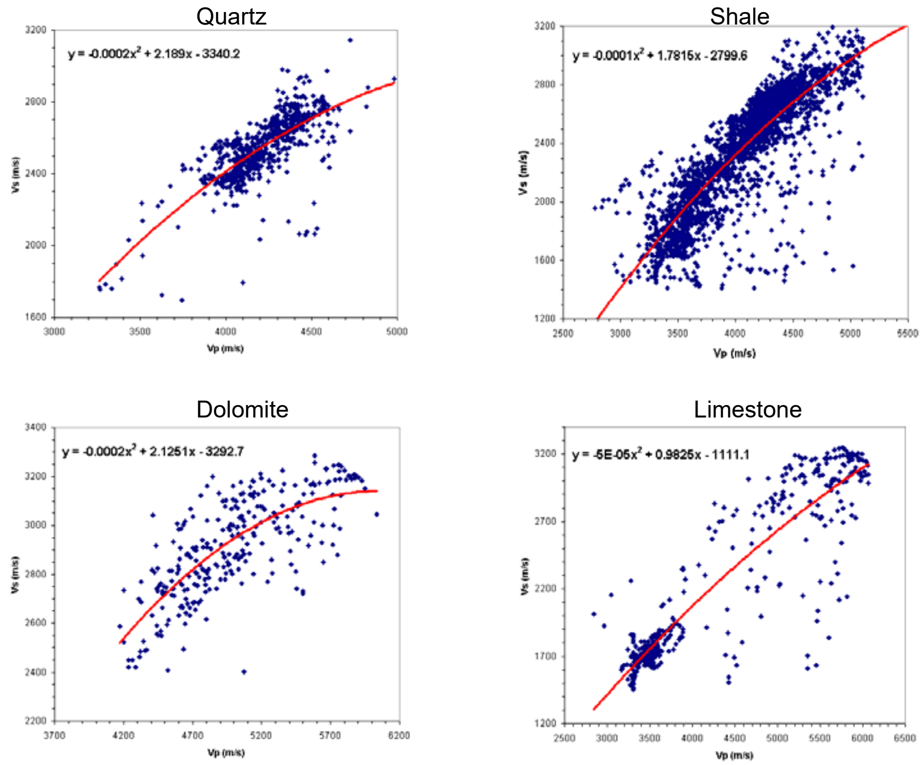
Lithology	a_{i2}	a_{i1}	a_{i0}
Sandstone	**-0.00009**	**1.399**	**-1946.15**
Shale	**-0.00030**	**2.012**	**-2199.30**

*** coefficients for velocities in meters/seconds**

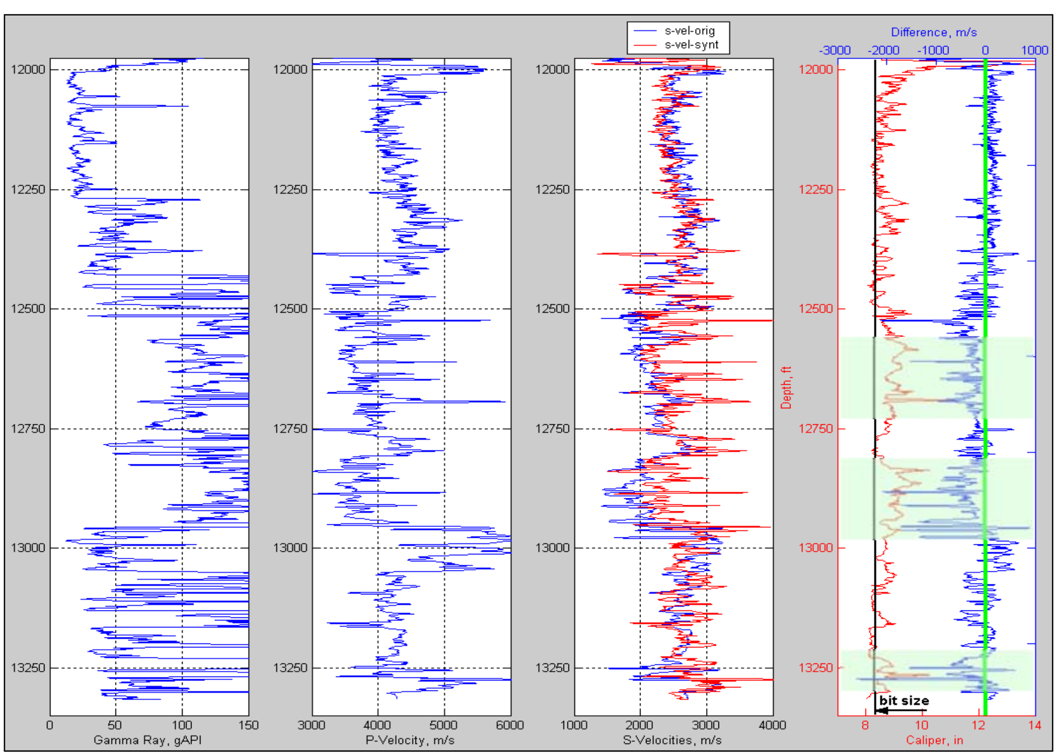
Evaluation in Poorly Consolidated Formations



Regression Coefficients for Consolidated Formations

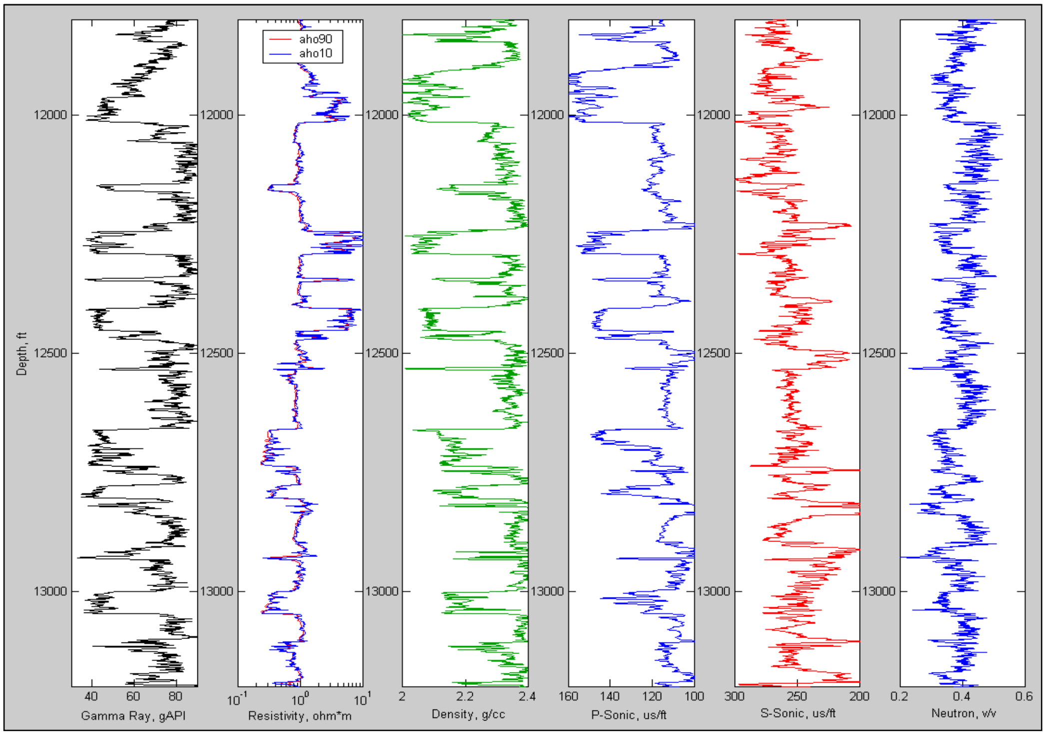


Evaluation in Consolidated Formations



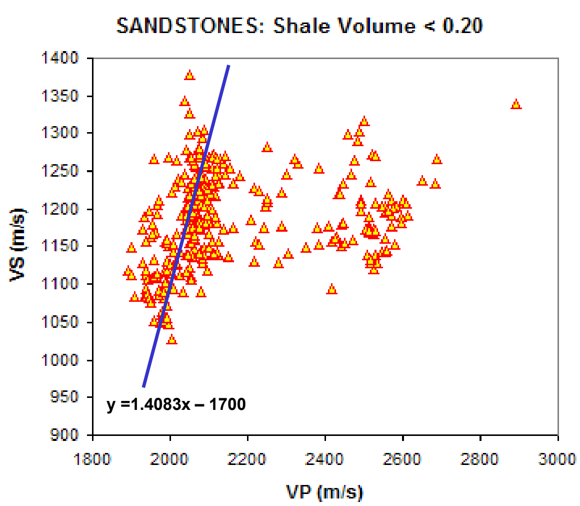
Deep-Water Gulf of Mexico Well

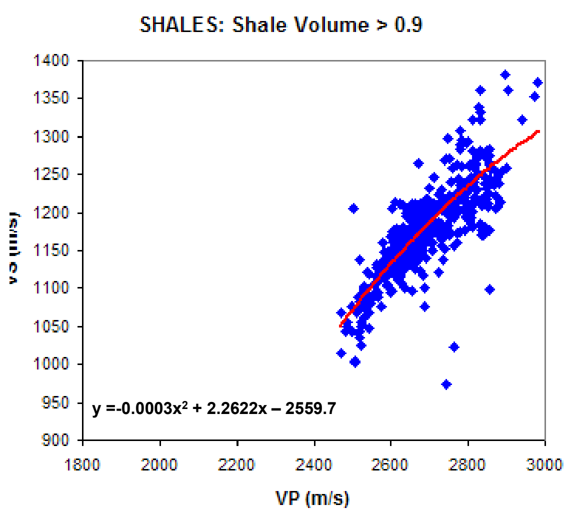
Basic Well Logs



Regression Coefficients BEFORE Fluid Substitution

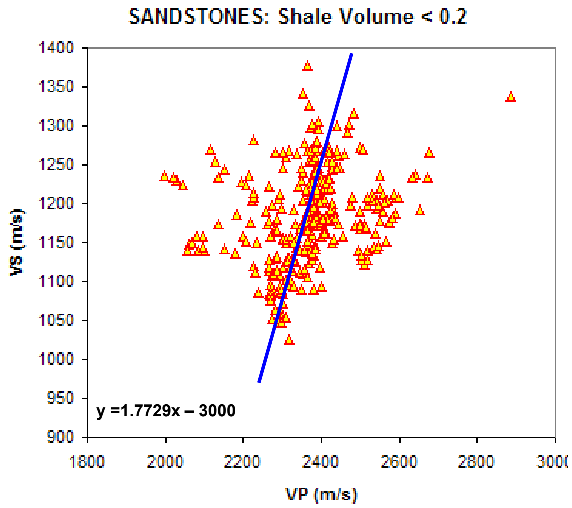
(for the Castagna-Greenberg formula)

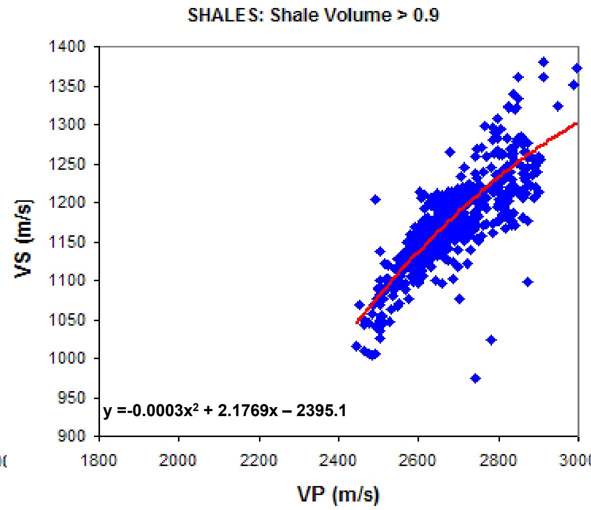




Regression Coefficients AFTER Fluid Substitution

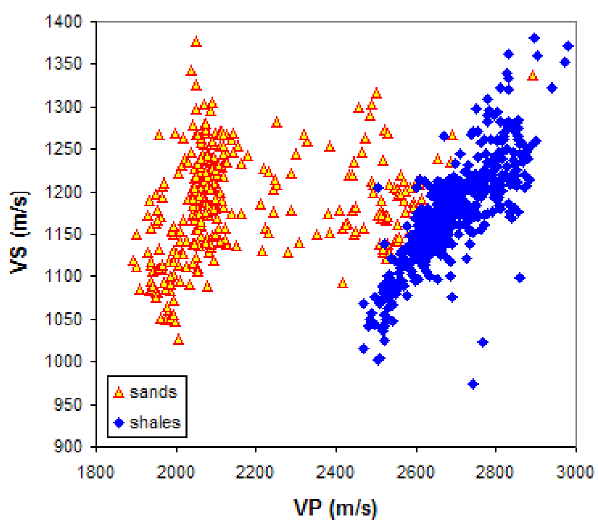
(for the Castagna-Greenberg formula)

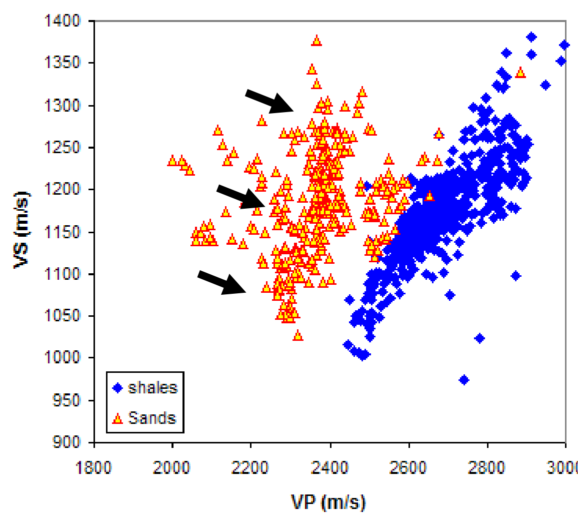




Result of Biot's Fluid Substitution (Sw=1)

(for the Castagna-Greenberg formula)

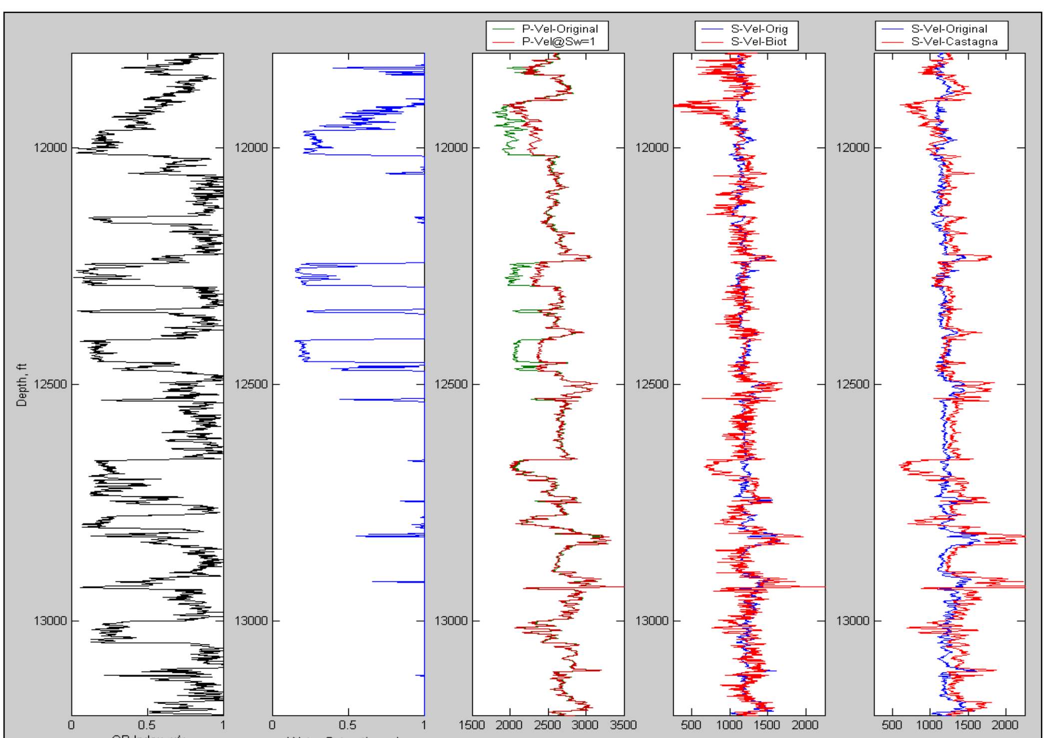




Movement of Cluster After Fluid Substitution

EXAMPLE
No. 2

Evaluation of Synthetic S-Velocities
(using Biot-Gassman and Castagna-Greenberg formulas)



METHOD

EDITION OF WELL LOGS IN WASHOUT INTERVALS

$$
\begin{aligned}
\delta - (C_{sh} . \delta_{sh}) &= C_{qz} . \delta_{qz} + C_{li} . \delta_{li} + \phi . \delta_{fl} + C_{do} . \delta_{do} \\
\phi_n - (C_{sh} . (\phi_n)_{sh}) &= C_{qz} . (\phi_n)_{qz} + C_{li} . (\phi_n)_{li} + \phi . (\phi_n)_{fl} + C_{do} . (\phi_n)_{do} \\
\Delta t - (C_{sh} . \Delta t_{sh}) &= C_{qz} . \Delta t_{qz} + C_{li} . \Delta t_{li} + \phi . \Delta t_{fl} + C_{do} . \Delta t_{do} \\
\upsilon - (C_{sh} . \upsilon_{sh}) &= C_{qz} . \upsilon_{qz} + C_{li} . \upsilon_{li} + \phi . \upsilon_{fl} + C_{do} . \upsilon_{do} \\
1 - C_{sh} &= C_{qz} + C_{li} + \phi + C_{do}
\end{aligned}
$$

$$
\underbrace{\begin{pmatrix} \delta_{quartz} & \delta_{limestone} & \delta_{fluid} & \delta_{dolomite} \\ (\phi_n)_{quartz} & (\phi_n)_{limestone} & (\phi_n)_{fluid} & (\phi_n)_{dolomite} \\ \Delta t_{quartz} & \Delta t_{limestone} & \Delta t_{fluid} & \Delta t_{dolomite} \\ \upsilon_{quartz} & \upsilon_{limestone} & \upsilon_{fluid} & \upsilon_{dolomite} \\ 1 & 1 & 1 & 1 \end{pmatrix}}_{C} * \underbrace{\begin{pmatrix} C_{qz} \\ C_{li} \\ \phi \\ C_{do} \end{pmatrix}}_{x} = \underbrace{\begin{pmatrix} \delta - (C_{sh} . \delta_{sh}) \\ \phi_n - (C_{sh} . (\phi_n)_{sh}) \\ \Delta t - (C_{sh} . \Delta t_{sh}) \\ \upsilon - (C_{sh} . \upsilon_{sh}) \\ 1 - C_{sh} \end{pmatrix}}_{d}
$$

$$C . x = d$$

$$[W . C] . x = [d . W]$$

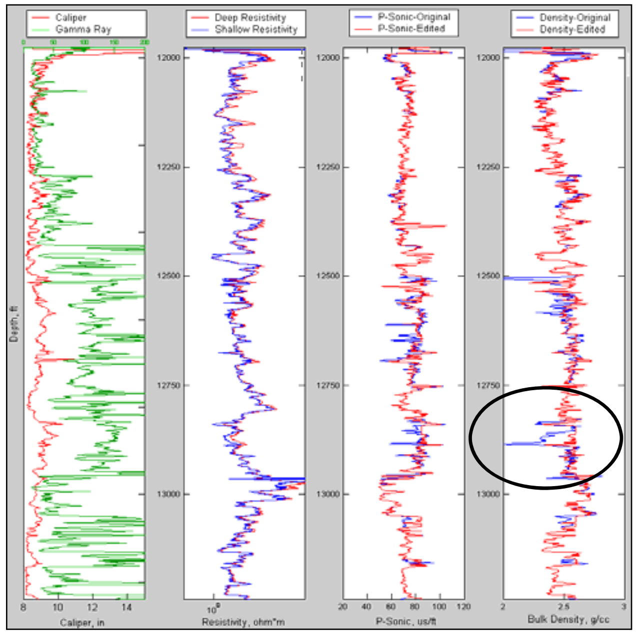
where :

$$
\underbrace{\begin{pmatrix} 1/(1+dcal^2) & 0 & 0 & 0 \\ 0 & 1/(1+dcal^2) & 0 & 0 \\ 0 & 0 & 1/(1+dcal^2) & 0 \\ 0 & 0 & 0 & 1/(1+dcal^2) \end{pmatrix}}_{W}
$$

then, 1- re-compute "x"

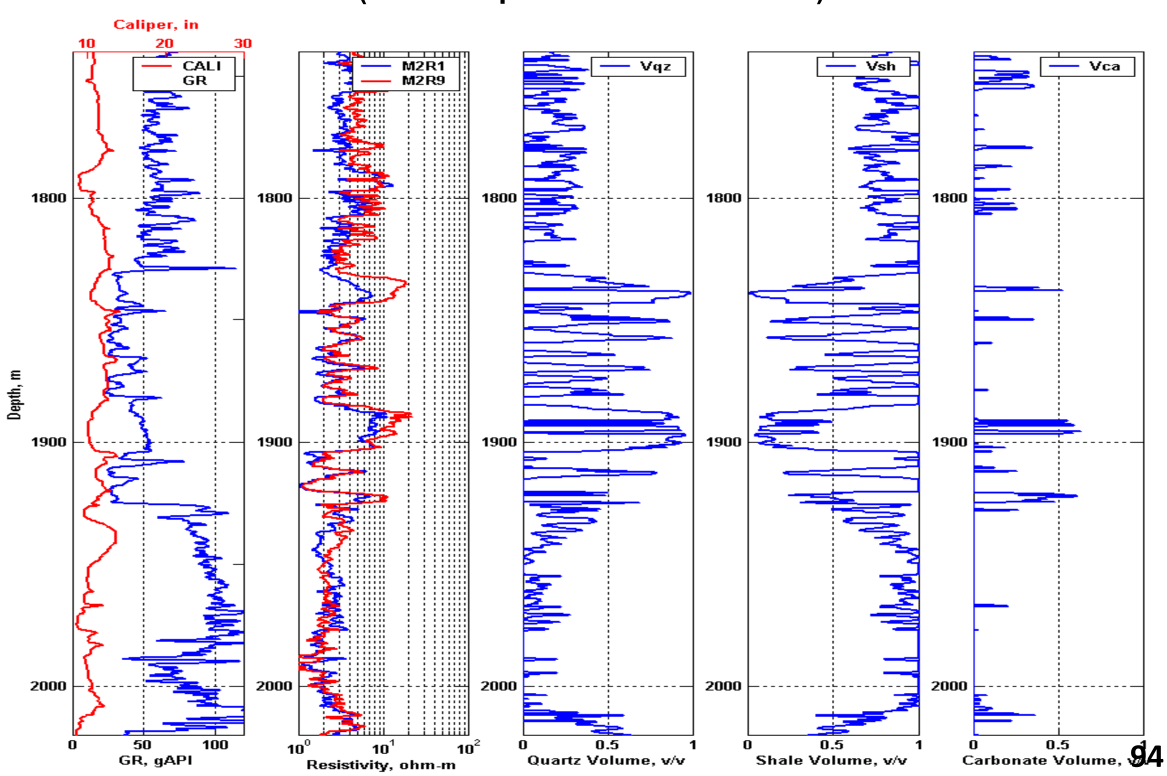
2- estimate synthetic logs (for instance: $\delta_{syn} = (C_{qz} . \delta_{qz} + C_{li} . \delta_{li}$ ....]

3- replace original logs by synthetic logs in washout zones

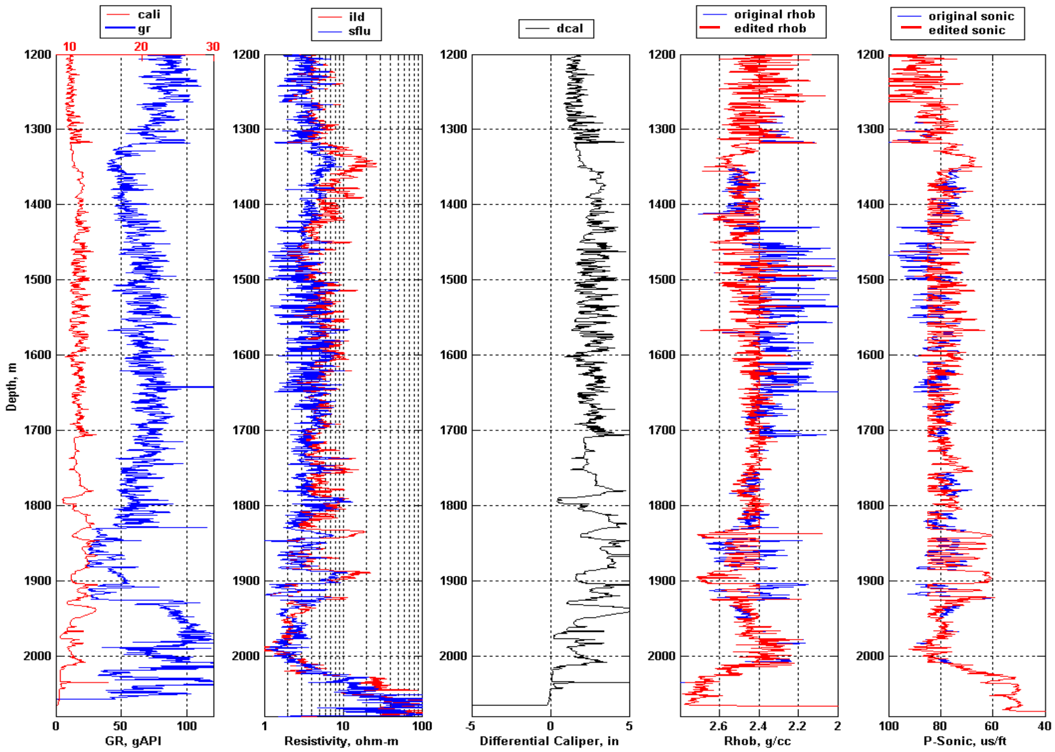


Volumetric Fractions

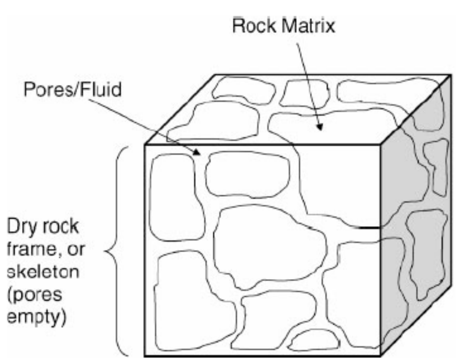
(within depth interval of interest)



RESULTS
(complete depth interval)



GASSMAN-BIOT-GEERTSMA EQUATIONS (I)



(1) $$\rho_f = \left[(S_w)(\rho_{H_2O})\right] + \left[(1-S_w)(\rho_{gas,oil})\right]$$

(2) $$\rho_b = \left[(\phi)(\rho_f)\right] + \left[(1-\phi)(\rho_s)\right]$$

(3) $$\boxed{K_f = \frac{1}{\left(S_w / K_{H_2O}\right) + \left((1-S_w) / K_{gas,oil}\right)}}$$

(4) $$S = \frac{3(1-\sigma)}{(1+\sigma)}$$

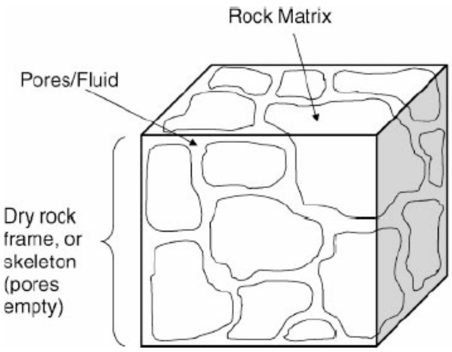
(5) $$M = \left(V_p^2\right)(\rho_b)\left(929 * 10^{-10}\right)$$

(6) $$A = S - 1$$

(7) $$B = \left[(\phi)(S)\left(\frac{K_s}{K_f} - 1\right)\right] - [S] + \left[\frac{M}{K_s}\right]$$

where $Ks = Ks_{qz} * V_{qz} + Ks_{sh} * V_{sh} + Ks_{ca} * V_{lca}$

GASSMAN-BIOT-GEERTSMA EQUATIONS (II)



$$(8) \qquad C = (-\phi)\left(S - \frac{M}{K_s}\right)\left(\frac{K_s}{K_f} - 1\right)$$

$$(9) \qquad Y = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$$

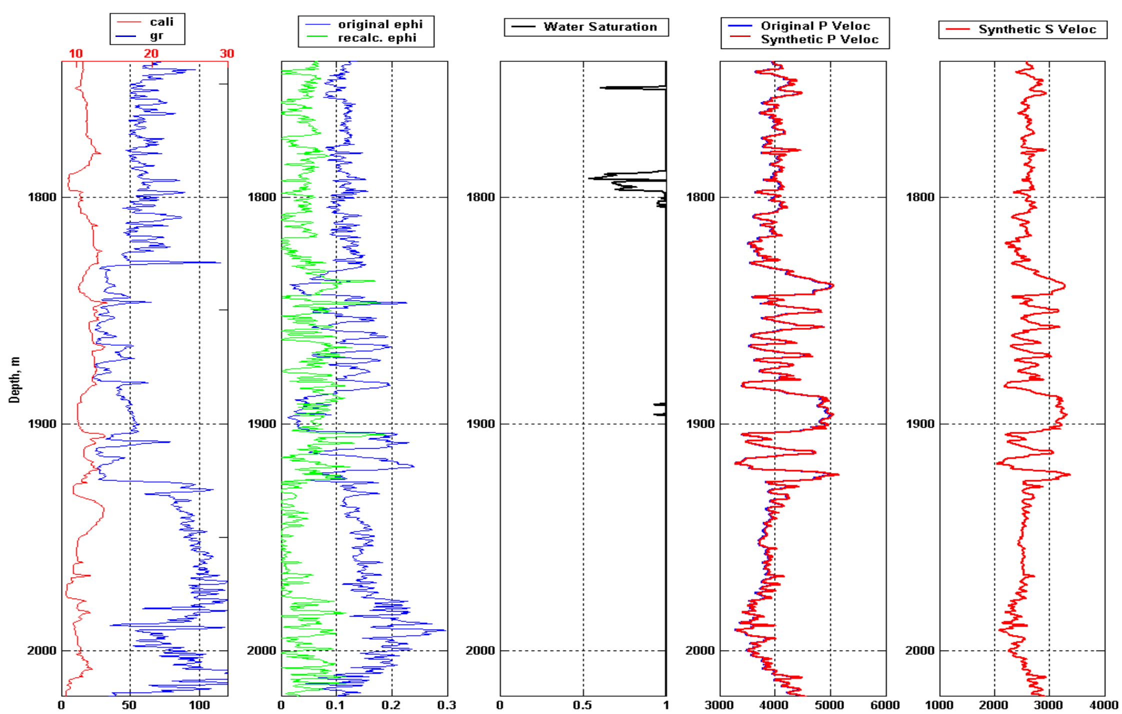
$$\boxed{\underline{(10)} \qquad K_b = (1 - Y)K_s}$$

$$V_p^2 = \left[K_b + \frac{4}{3}\mu_b + \frac{\left(1 - K_b/K_s\right)^2}{\left(1 - K_b/K_s\right)1/K_s + \phi/K_f}\right]\frac{1}{\rho_b};$$

$$V_s^2 = \frac{\mu_b}{\rho_b}$$

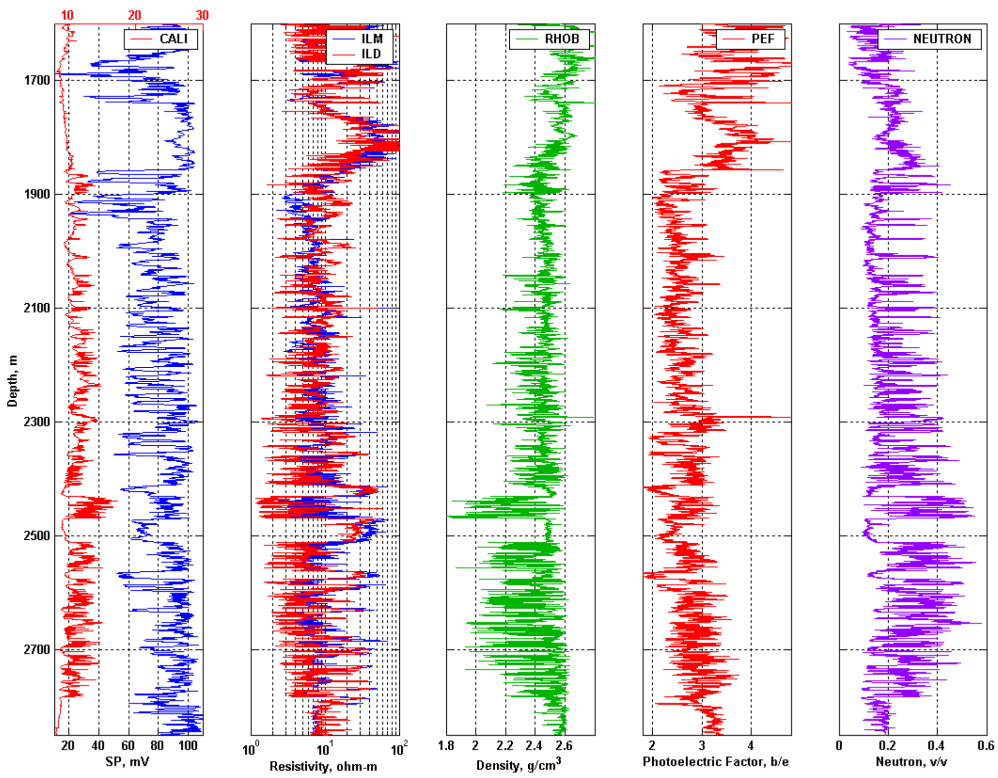
SYNTHETIC "S" VELOCITY

(within the depth zone of interest)

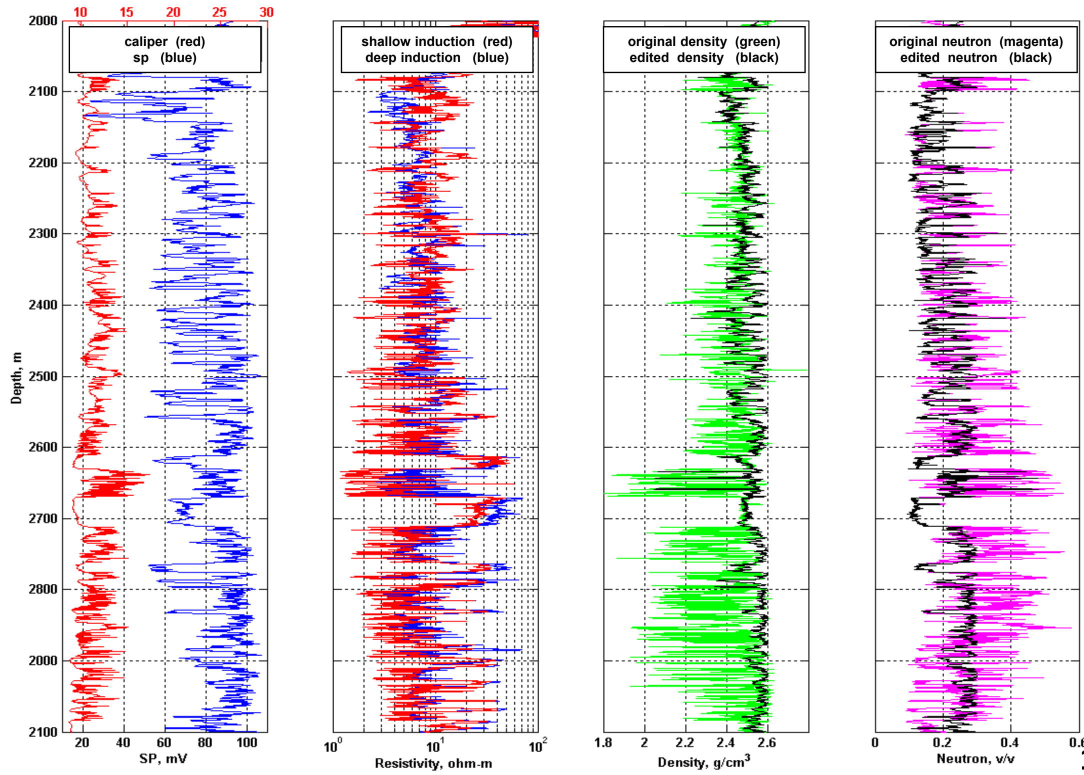


EXAMPLE
No. 3

WELL LOGS



CALCULATION OF SYNTHETIC SONIC AND NEUTRON LOGS



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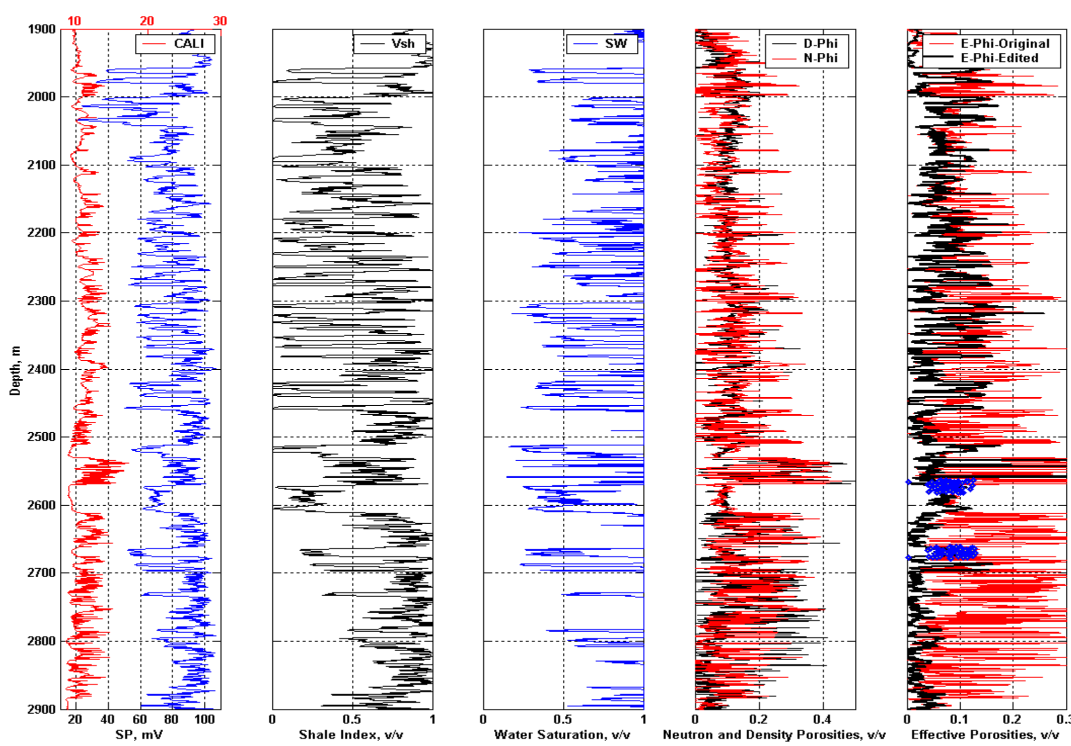
CALCULATION OF PETROPHYSICAL PROPERTIES



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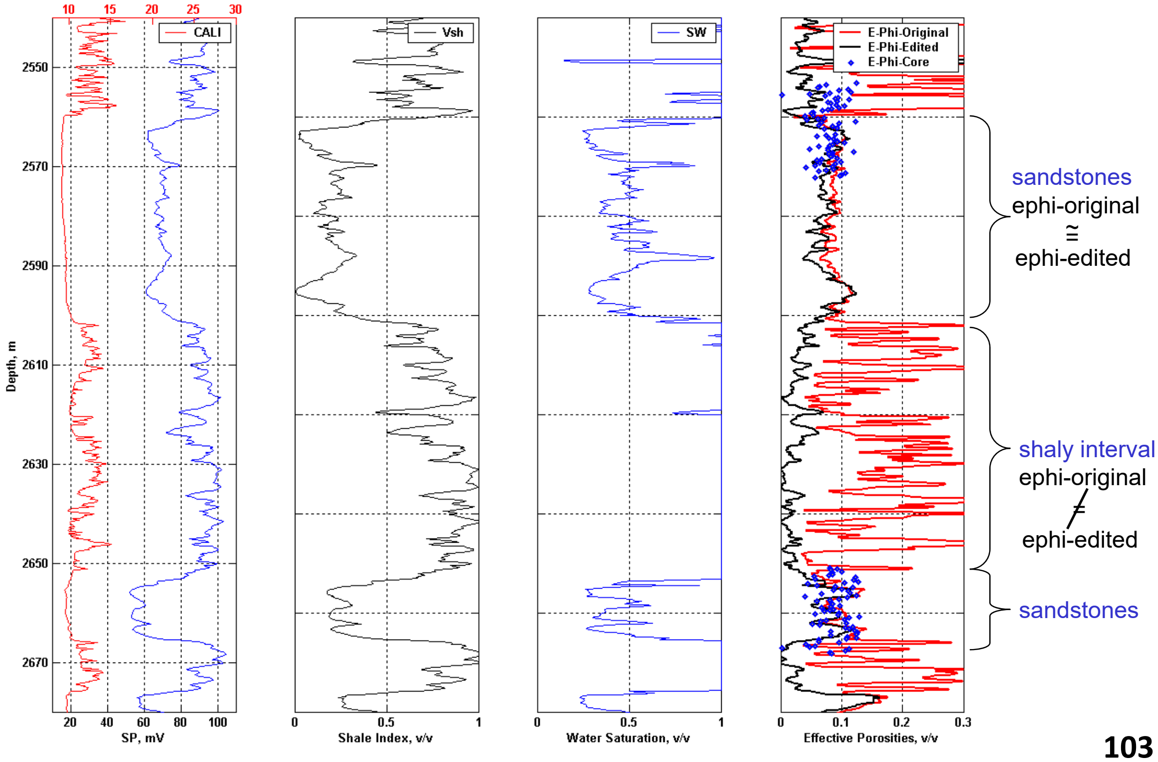
MATCH BETWEEN CORE AND LOG POROSITIES



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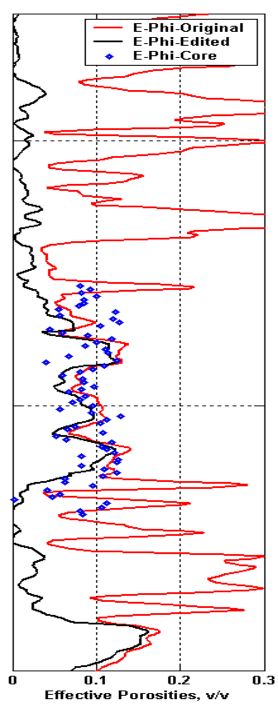
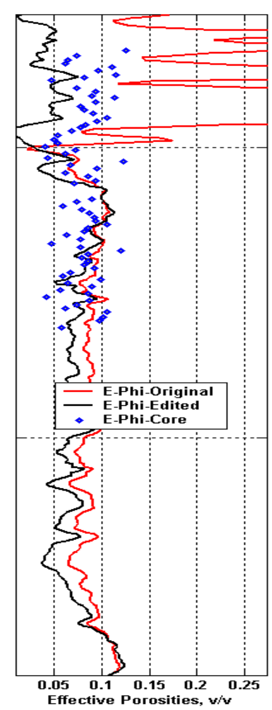
DETAIL ALONG DEPTH INTERVAL OF INTEREST





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Acknowledgements:

- Baker Atlas
- Schlumberger