

**Lecture Presentation**  
PGE385(K,M)  
Advanced Multi-Well Formation Evaluation

Lecture Presentation

**Estimation of Volumetric  
Concentrations of Mineral  
Constituents from Logs**

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**Mineralogy Determination, Why?**

- Determines grain density → Porosity
- Mechanical properties and borehole stability
- Propensity to fracture
- Clay type - formation damage (effect of fresh water, acid, ...)
- Limestone vs. Dolomite - Dolomite is generally more porous in low  $\phi$  carbonates.
- Used to infer other petrophysical properties (e.g. velocity, ...)

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# Minerals for Petroleum Geoscientists

- Limestone, Dolomite
- Quartz, Chert, Opal (hydrous silica)
- Syderite, Ankerite
- Anhydrite, Gypsum (hydrous anhydrite)
- Halite
- Clay minerals (illite, smectite, kaolinite, chlorite)
- Feldspars
- Coals
- Glauconite

Important in the determination of depositional environment and reservoir quality

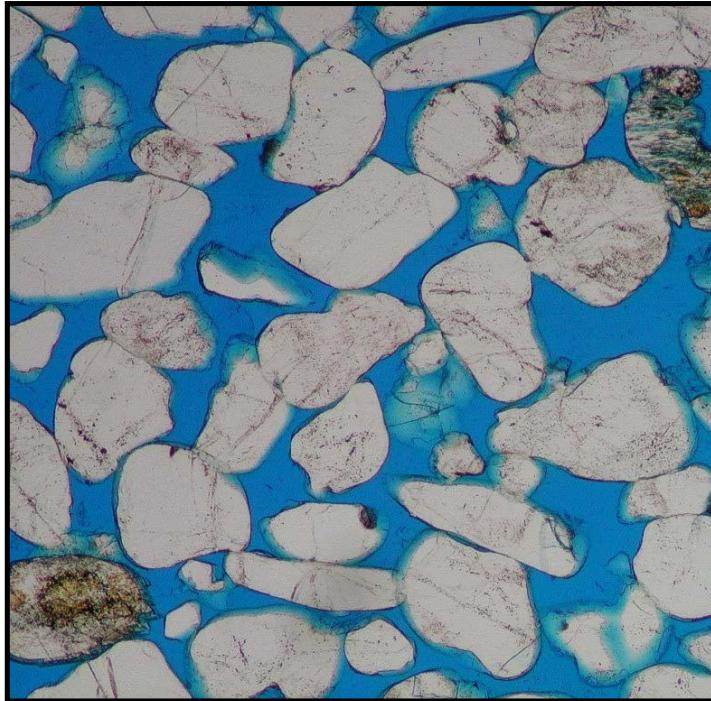
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## Objectives:

1. To introduce the methodology used to estimate apparent matrix density in a rock whose solid composition is a mixture of elemental constituents,
2. To introduce basic procedure to identify the elemental constituents included in the solid component of a rock,
3. To assess porosity, fluid saturation, and permeability in a rock with mixed solid composition, and
4. To exercise the estimation of volumetric concentrations using basic linear algebra and linear minimization with constraints.

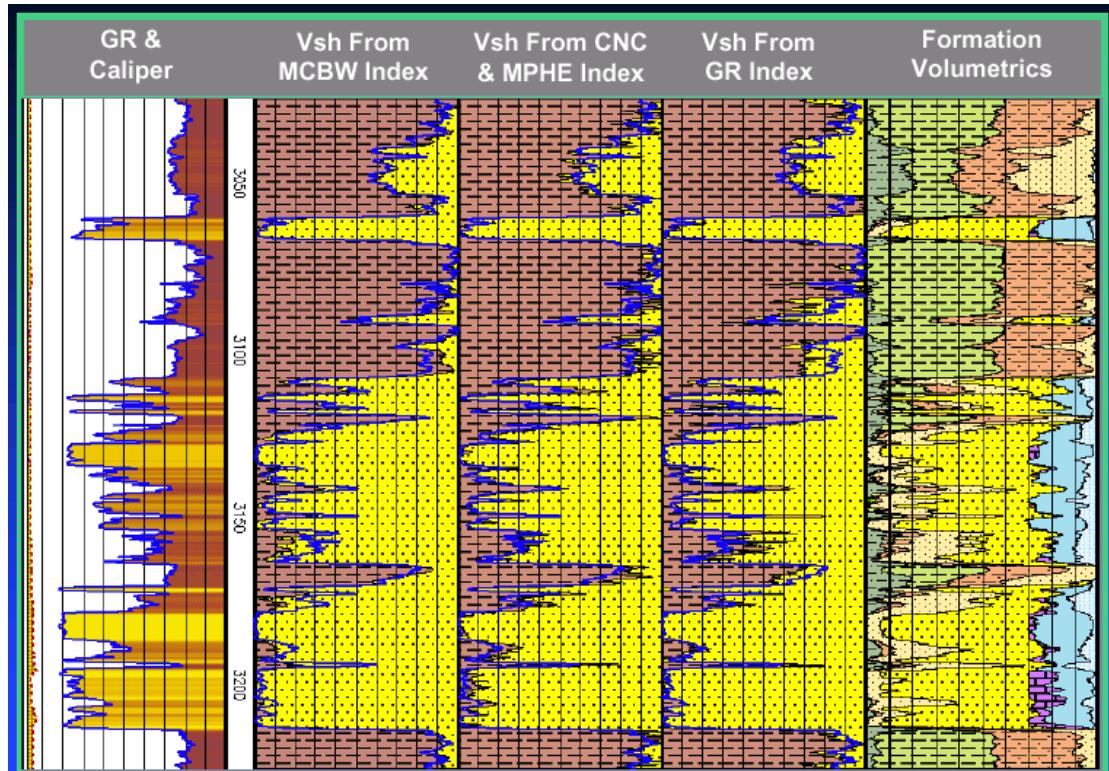
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# Mineral Composition of Grains: Pure or Mixed Constituents?



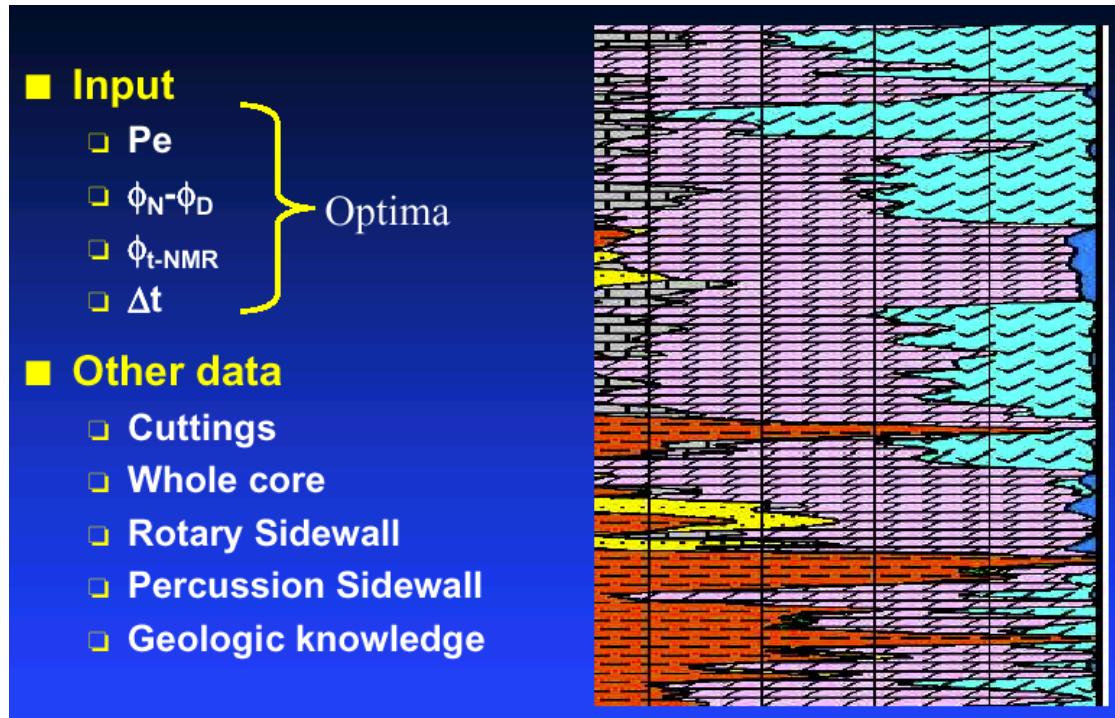
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## FORMATION VOLUMETRICS: Example



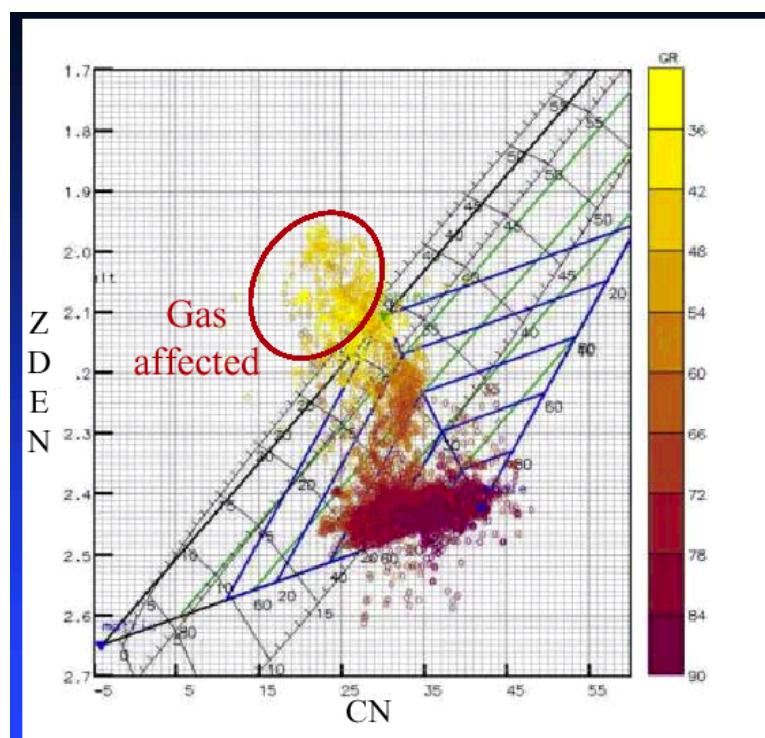
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# Mineralogy Determination



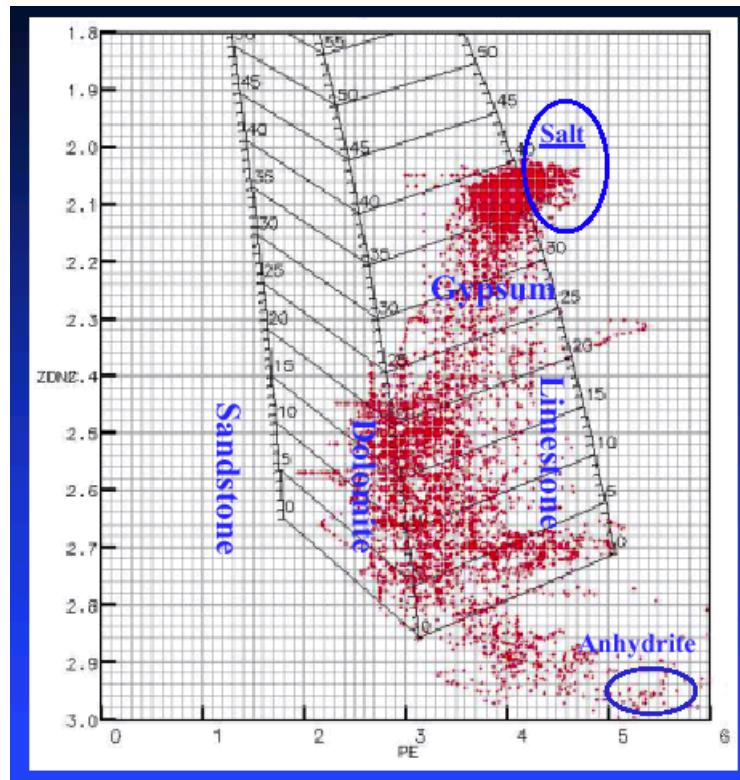
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## Cross-Plot Analysis: Example



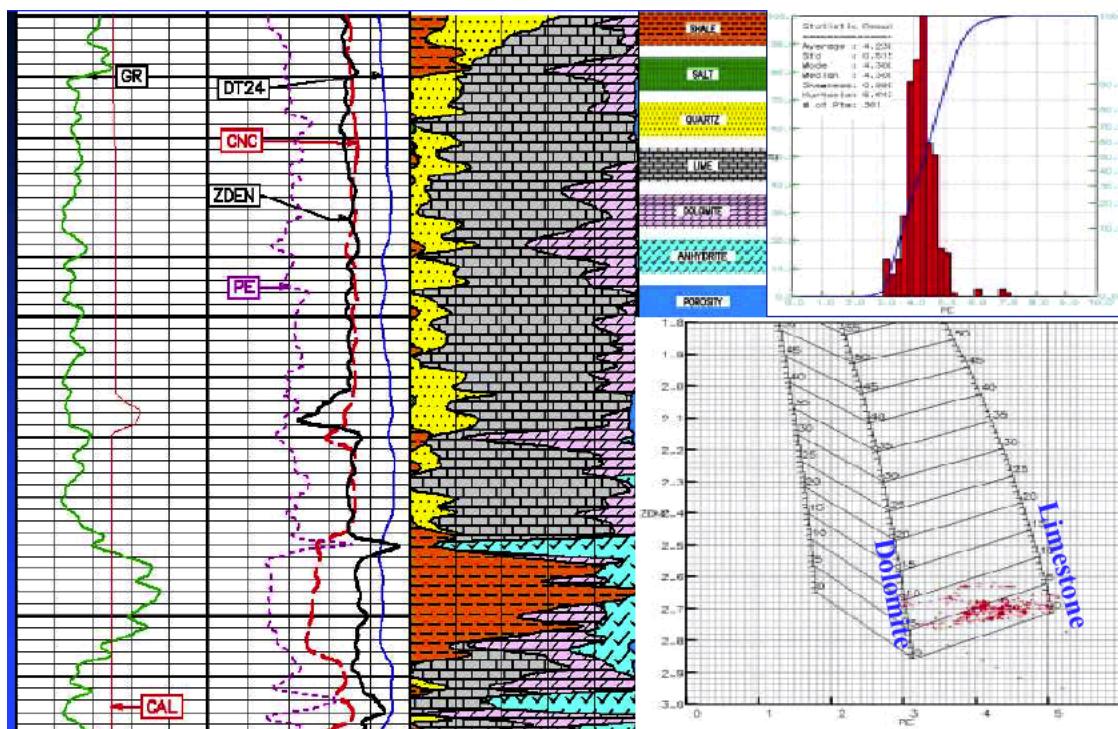
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## PEF-Density Cross-Plot Analysis: Example



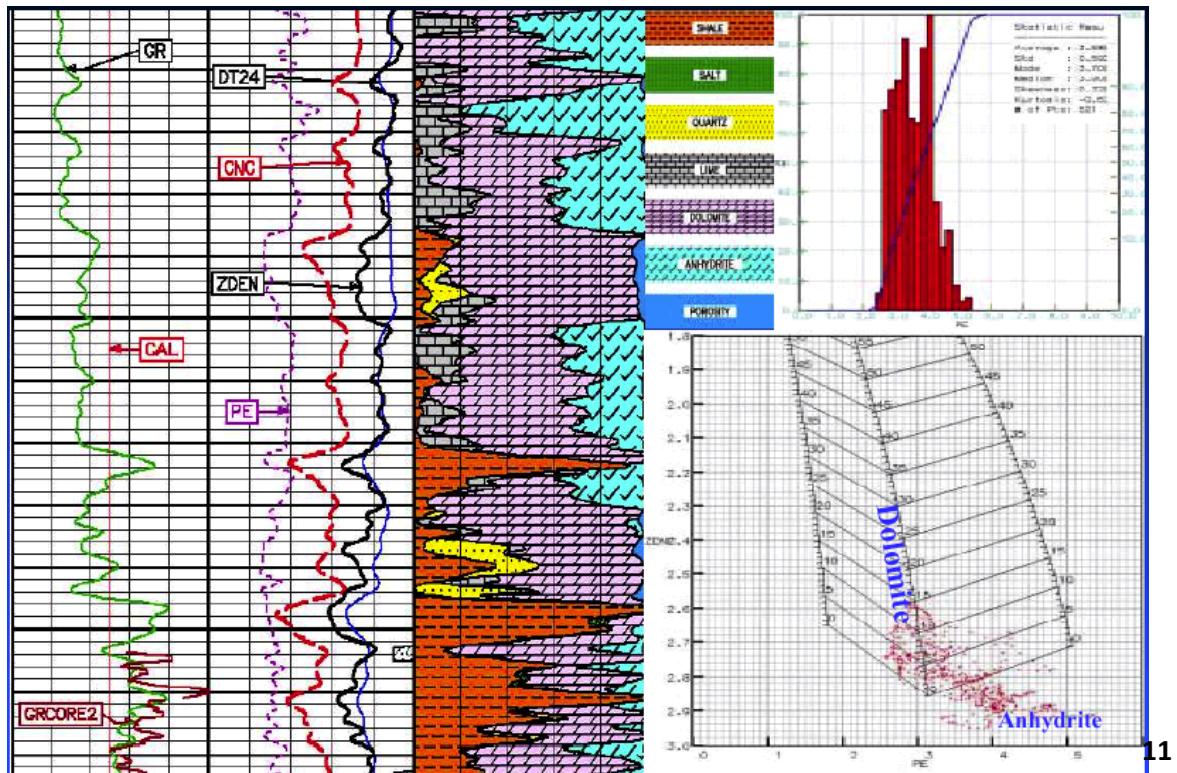
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## Porosity and Lithology From PEF and Density: Example

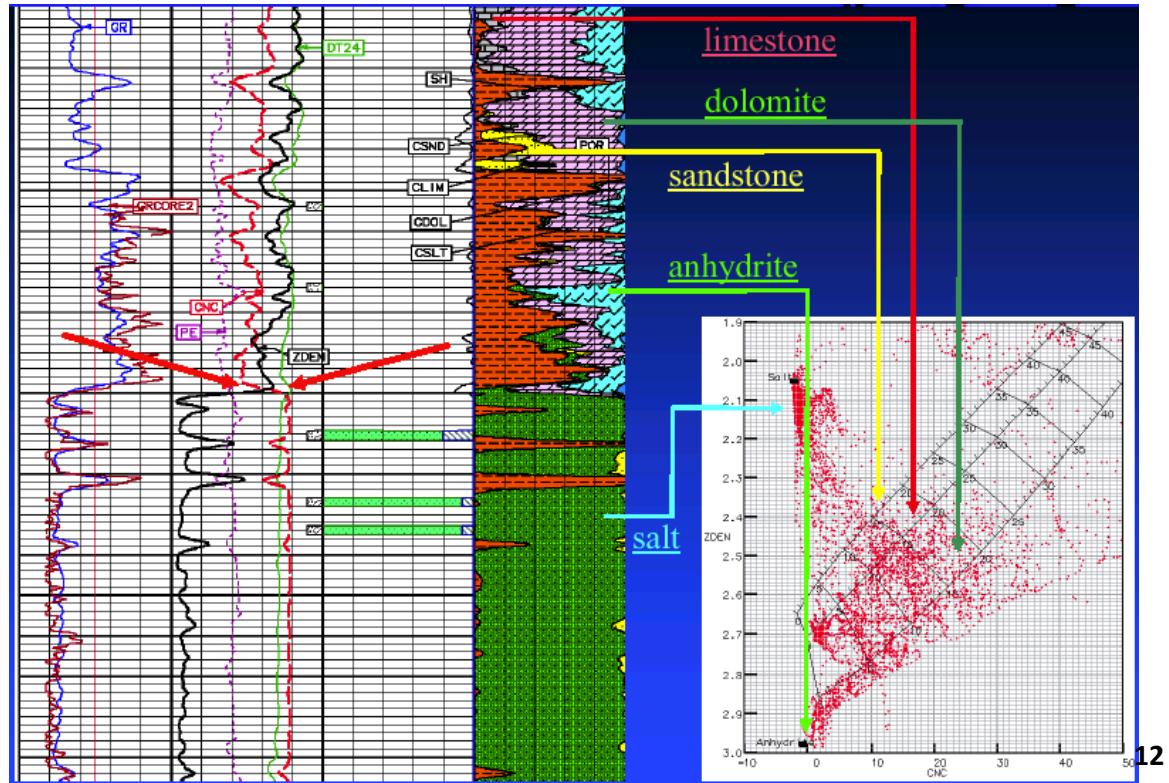


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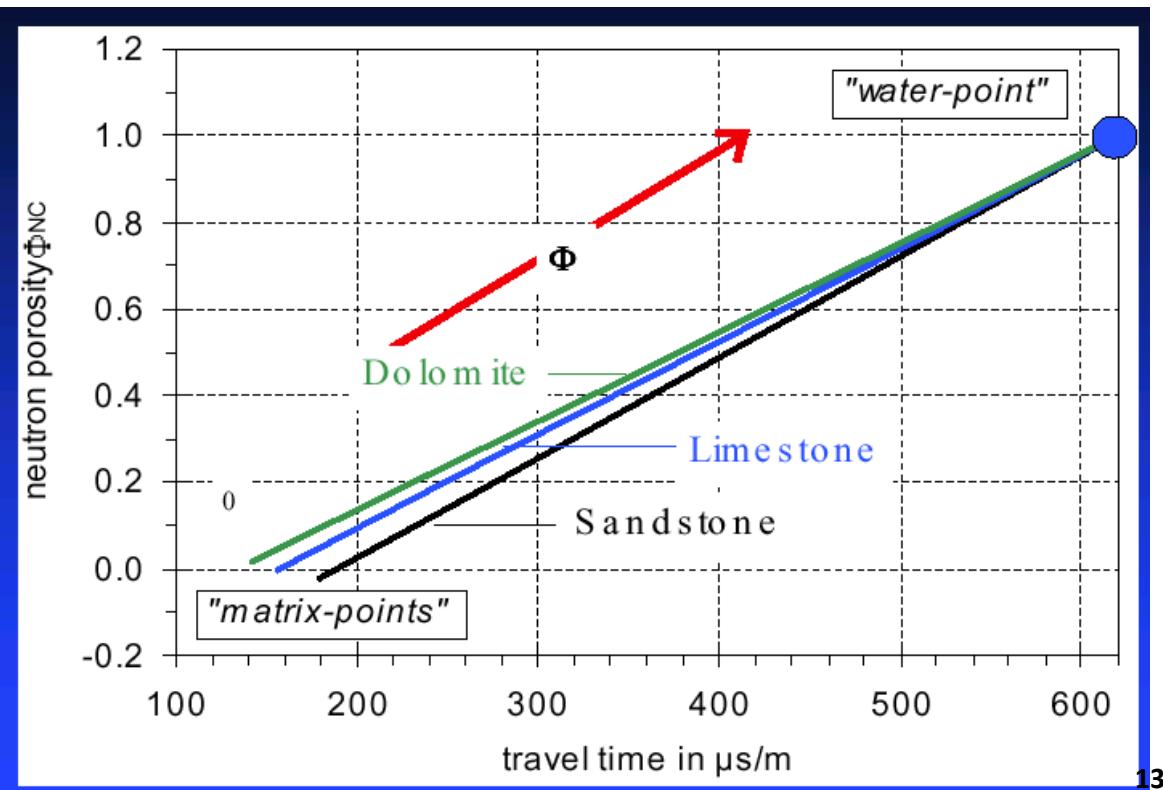
## Porosity and Lithology From PEF and Density: Example



## Mineral Identification From Neutron and Density: Example



## EXAMPLE: Sonic-Neutron Cross-Plot

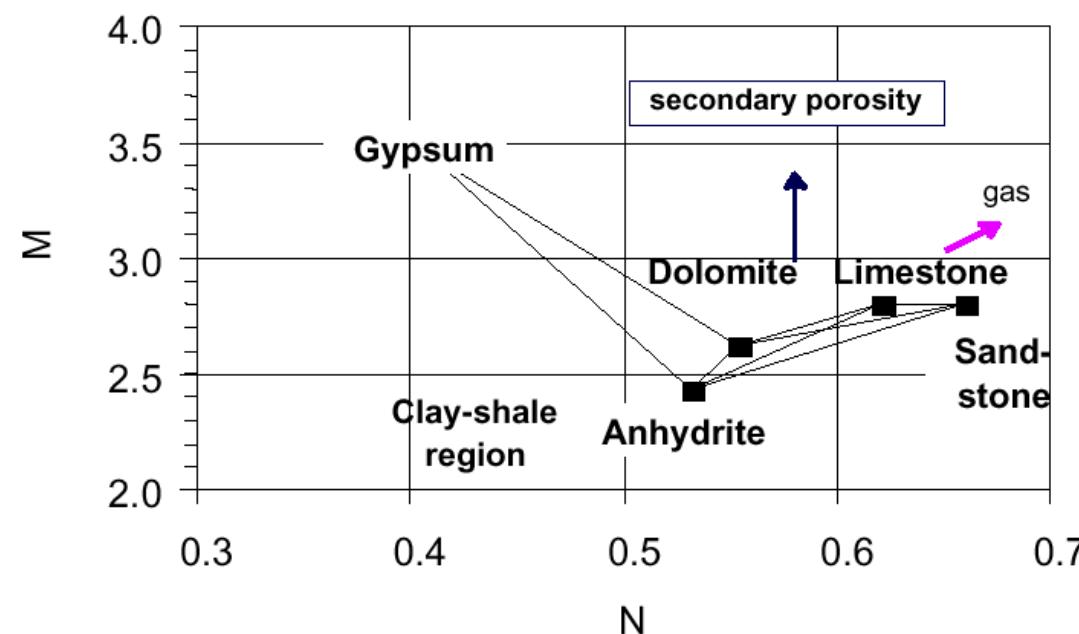


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## EXAMPLE: M-N Cross-Plots

$$M = \frac{\Delta t_{fl} - \Delta t}{\rho - \rho_{fl}} \cdot 100$$

$$N = \frac{\Phi_{CN, fl} - \Phi_{CN}}{\rho - \rho_{fl}}$$



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## Linear Equations and Volumetric Models

resulting rock property is sum of contributions of components

examples are density, Pe & neutron-porosity.

Density of a porous rock

$$\rho = (1 - \Phi) \cdot \rho_m + \Phi \cdot \rho_{fl}$$

Density of n-component rock

$$\rho = \sum_{i=1}^n \rho_i \cdot V_i \quad \sum_{i=1}^n V_i = 1$$

$V_i$  volume fraction component  $i$

Model is exactly valid for scalar rock properties, that don't depend on structure or distribution of components.

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## Linear Equations and Volumetric Models

$n$  components

$$g = \sum_{i=1}^n g_i \cdot V_i$$

Measured property  $g$

Volume fraction  $V_i$

$$\sum_{i=1}^n V_i = 1$$

Component  $i$

system of linear equations:



$m$  independent equations (logs)

$n = m + 1$  unknown components

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# Applications

Foundation for various techniques for determining:

- rock composition
- porosity

Example:

- sonic-neutron crossplot, sonic-density crossplot
- mineral-identification (MID) plots
- M-N-plots

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# Applications

a. Determination of

rock composition (lithology) and porosity  
crossplot, M-N-plot etc.

Matrix-inversion techniques

b. Shaly sand analysis based on nuclear  
measurements

Thomas-Stieber and Juhasz Model

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# Linear Equations: Four Components

Substance	$\rho$ (g/cm <sup>3</sup> )	$\Phi_{CN}$	$\Delta t$ (μs/m)
quartz	2.65	-0.02	180
calcite	2.71	0	156
dolomite	2.87	0.02	143
water	1.00	1.00	620

$$\rho = 2.71 V_{\text{calcite}} + 2.87 V_{\text{dolomite}} + 2.65 V_{\text{quartz}} + 1.00 \Phi$$

$$\Phi_{CN} = 0.02 V_{\text{dolomite}} - 0.02 V_{\text{quartz}} + 1.00 \Phi$$

$$\Delta t = 156 V_{\text{calcite}} + 143 V_{\text{dolomite}} + 180 V_{\text{quartz}} + 620 \Phi$$

$$1 = V_{\text{calcite}} + V_{\text{dolomite}} + V_{\text{quartz}} + \Phi$$

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# Linear Equations: Four Components

Example:

Measured values  $\rho = 2.485$  g/cm<sup>3</sup>       $\Phi_{CN} = 0.158$        $\Delta t = 225$  μs/m

1. Choose the model: C + D +Q +W

Insert

$$2.485 = 2.71 V_{\text{calcite}} + 2.87 V_{\text{dolomite}} + 2.65 V_{\text{quartz}} + 1.00 \Phi$$

$$0.158 = 0.02 V_{\text{dolomite}} - 0.02 V_{\text{quartz}} + 1.00 \Phi$$

$$225 = 156 V_{\text{calcite}} + 143 V_{\text{dolomite}} + 180 V_{\text{quartz}} + 620 \Phi$$

$$1 = V_{\text{calcite}} + V_{\text{dolomite}} + V_{\text{quartz}} + \Phi$$

3. Solve for volume fractions:

$$V_{\text{calcite}} = 0.52 \quad V_{\text{dolomite}} = 0.23 \quad V_{\text{quartz}} = 0.10 \quad \Phi = 0.15$$

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## Matrix Representation: Four Components

$$\rho = 2.71 V_{\text{calcite}} + 2.87 V_{\text{dolomite}} + 2.65 V_{\text{quartz}} + 1.00 \Phi$$

$$\Phi_{\text{CN}} = 0.02 V_{\text{dolomite}} - 0.02 V_{\text{quartz}} + 1.00 \Phi$$

$$\Delta t = 156 V_{\text{calcite}} + 143 V_{\text{dolomite}} + 180 V_{\text{quartz}} + 620 \Phi$$

$$1 = V_{\text{calcite}} + V_{\text{dolomite}} + V_{\text{quartz}} + \Phi$$

$$\begin{bmatrix} \rho \\ \phi_{\text{NC}} \\ \Delta t \\ 1 \end{bmatrix} = \begin{bmatrix} 2.71 & 2.87 & 2.65 & 1.00 \\ 0.00 & 0.02 & -0.02 & 1.00 \\ 156 & 143 & 180 & 620 \\ 1.00 & 1.00 & 1.00 & 1.00 \end{bmatrix} \cdot \begin{bmatrix} V_{\text{calcite}} \\ V_{\text{dolomite}} \\ V_{\text{quartz}} \\ \phi \end{bmatrix}$$

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## Matrix Inversion: Four Components

$$\mathbf{M} = \mathbf{R}\mathbf{V}$$

**M** matrix - measured properties  
**R** matrix - response  
**V** matrix - volume fractions

$$\mathbf{V} = \mathbf{R}^{-1}\mathbf{M}$$

$\mathbf{R}^{-1}$  inverse matrix

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## Matrix Inversion: Four Components

$$\begin{bmatrix} \rho \\ \phi_{NC} \\ \Delta t \\ 1 \end{bmatrix} = \begin{bmatrix} 2.71 & 2.87 & 2.65 & 1.00 \\ 0.00 & 0.02 & -0.02 & 1.00 \\ 156 & 143 & 180 & 620 \\ 1.00 & 1.00 & 1.00 & 1.00 \end{bmatrix} \cdot \begin{bmatrix} V_{\text{calcite}} \\ V_{\text{dolomite}} \\ V_{\text{quartz}} \\ \phi \end{bmatrix}$$



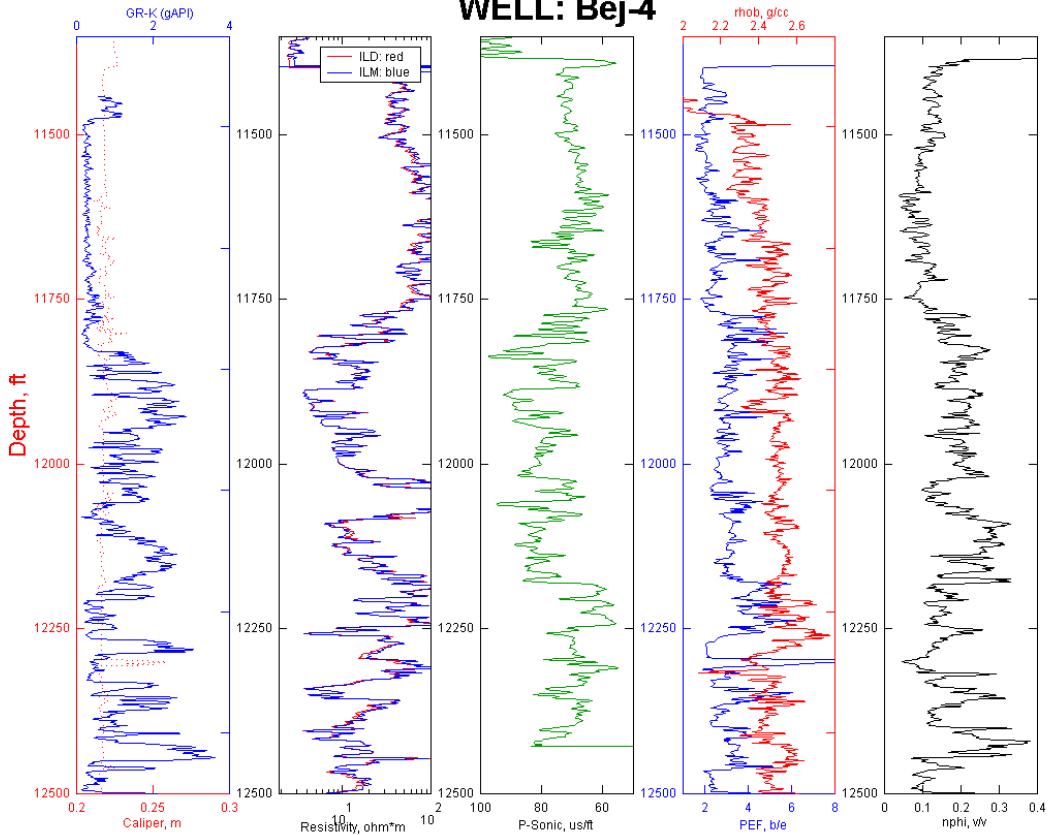
$$\begin{bmatrix} V_{\text{calcite}} \\ V_{\text{dolomite}} \\ V_{\text{quartz}} \\ \phi \end{bmatrix} = \begin{bmatrix} -12.68 & 8.19 & -0.0665 & 45.74 \\ 7.63 & 3.02 & 0.0216 & -24.03 \\ 5.11 & -11.91 & 0.0445 & -20.78 \\ -0.05 & 0.70 & 0.0004 & 0.0651 \end{bmatrix} \cdot \begin{bmatrix} \rho \\ \phi_{NC} \\ \Delta t \\ 1 \end{bmatrix}$$

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**EXAMPLE  
OF  
IMPLEMENTATION**

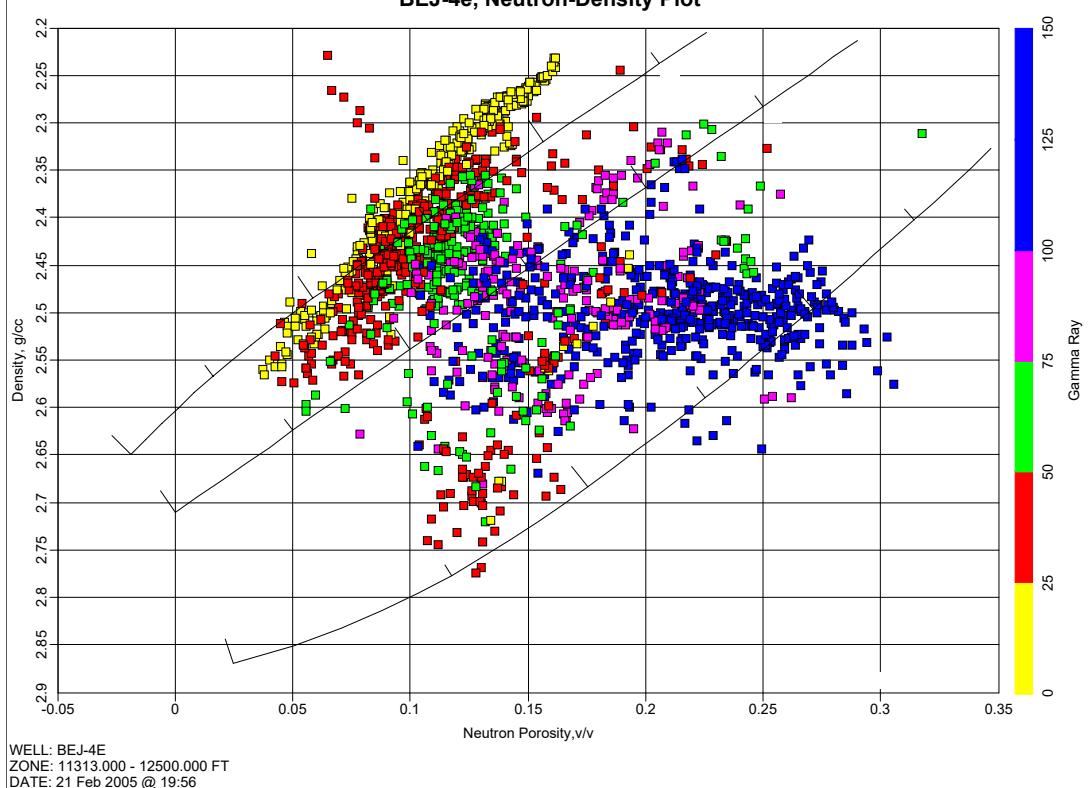
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## WELL: Bej-4

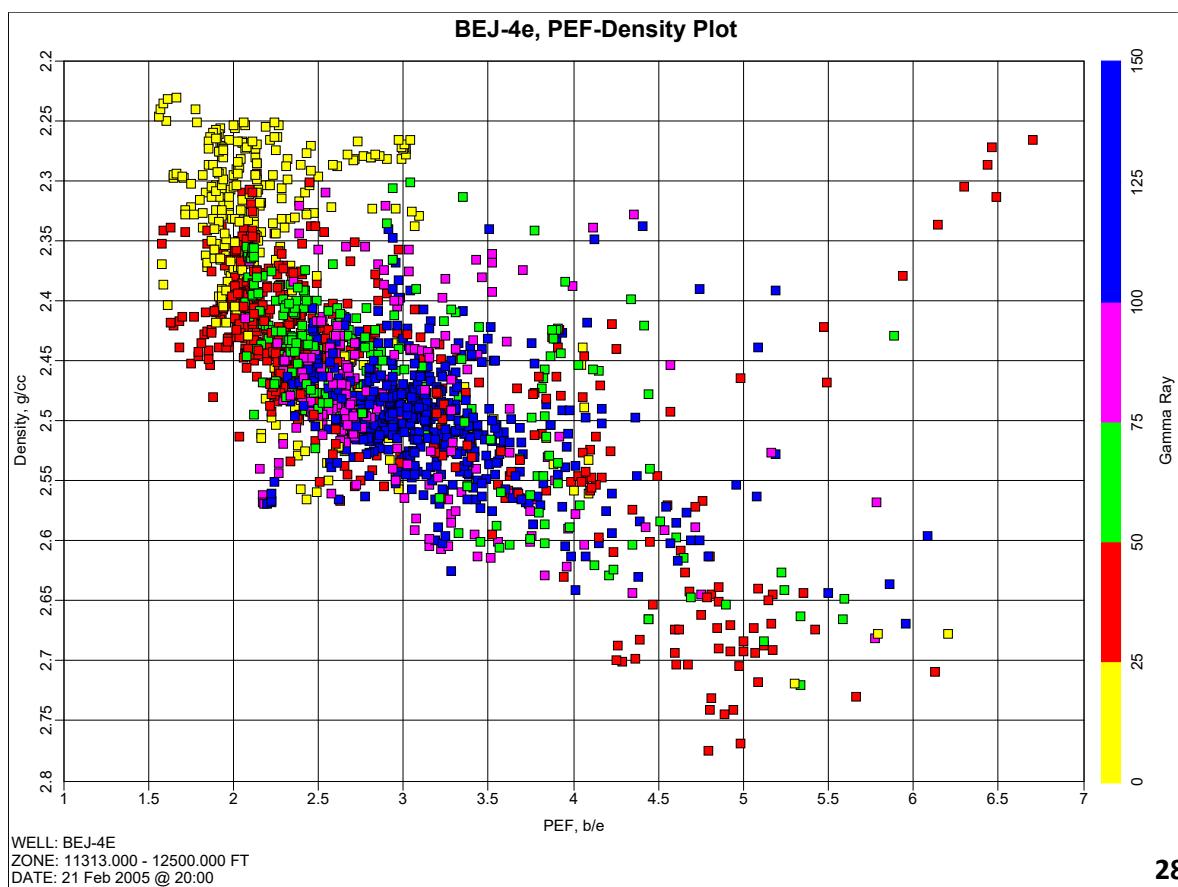
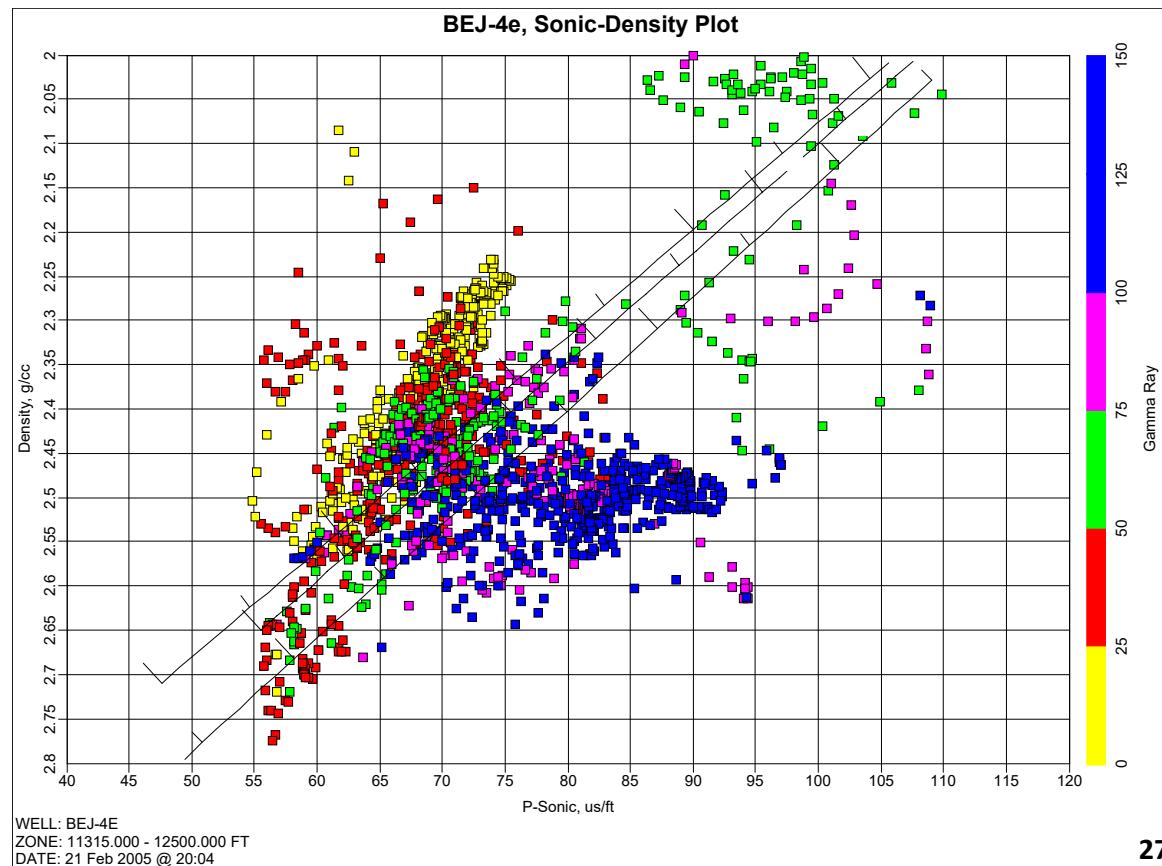


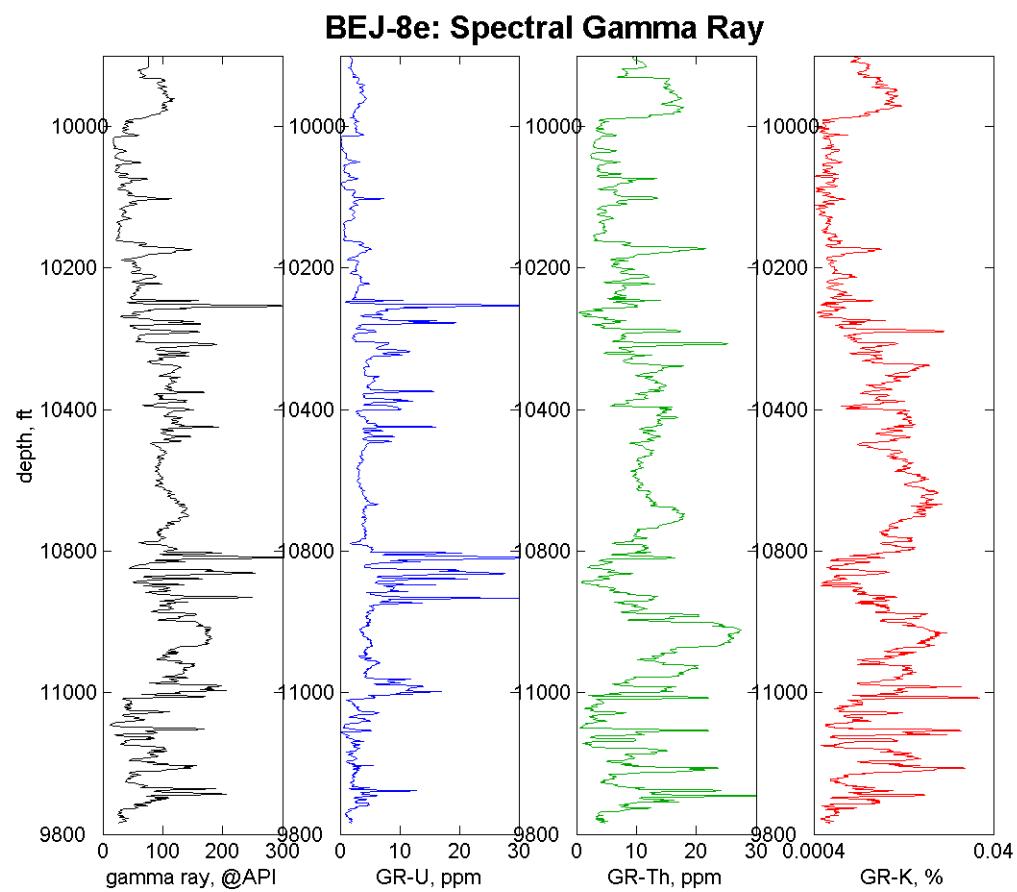
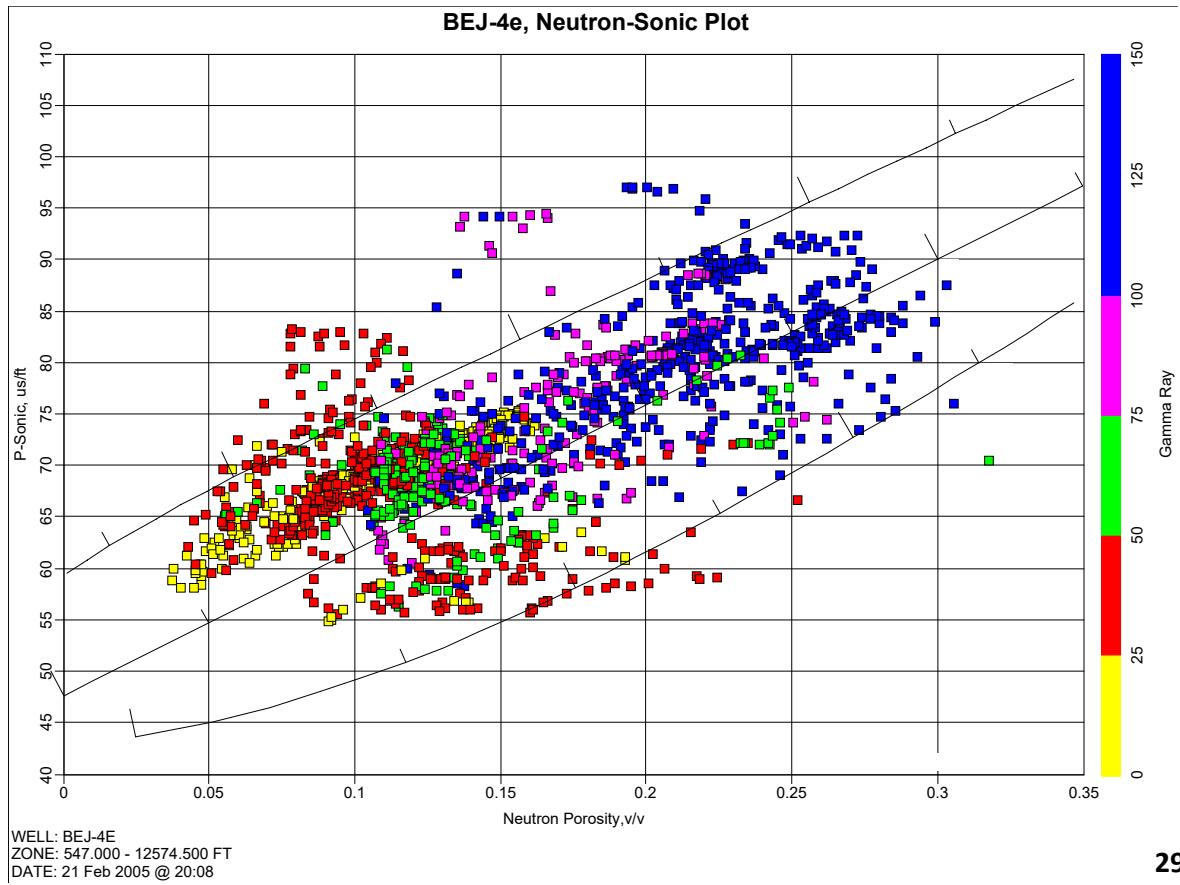
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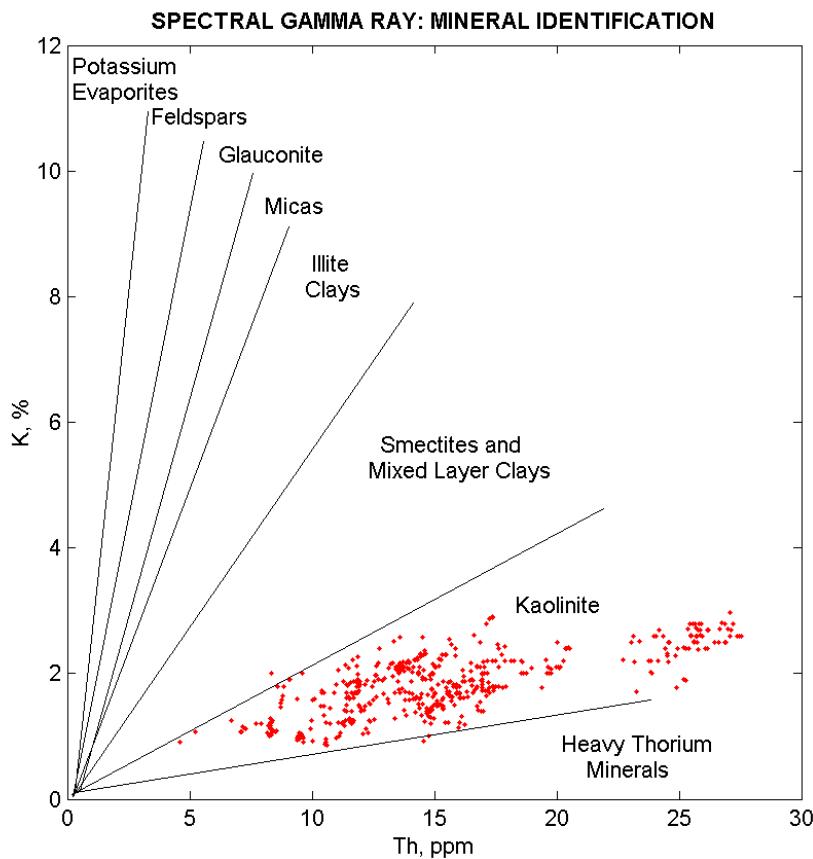
BEJ-4e, Neutron-Density Plot



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### ASSESSMENT OF VOLUMETRIC FRACTIONS ( 4 input logs and estimating porosity/shale concentration)

$$\begin{aligned}
 \delta &= C_q \cdot \delta_q + C_c \cdot \delta_c + \phi \cdot \delta_f + C_{sh} \cdot \delta_{sh} \\
 \phi_n &= C_q \cdot (\phi_n)_q + C_c \cdot (\phi_n)_c + \phi \cdot (\phi_n)_f + C_{sh} \cdot (\phi_n)_{sh} \\
 \Delta t &= C_q \cdot \Delta t_q + C_c \cdot \Delta t_c + \phi \cdot \Delta t_f + C_{sh} \cdot \Delta t_{sh} \\
 v &= C_q \cdot v_q + C_c \cdot v_c + \phi \cdot v_f + C_{sh} \cdot v_{sh} \\
 1 &= C_q + C_c + \phi + C_{sh}
 \end{aligned}$$

$$\left( \begin{array}{cccc}
 \delta_{quartz} & \delta_{carbonate} & \delta_{fluid} & \delta_{shale} \\
 (\phi_n)_{quartz} & (\phi_n)_{carbonate} & (\phi_n)_{fluid} & (\phi_n)_{shale} \\
 \Delta t_{quartz} & \Delta t_{carbonate} & \Delta t_{fluid} & \Delta t_{shale} \\
 v_{quartz} & v_{carbonate} & v_{fluid} & v_{shale} \\
 1 & 1 & 1 & 1
 \end{array} \right) * \begin{pmatrix} C_q \\ C_c \\ phi \\ C_{sh} \end{pmatrix} = \begin{pmatrix} \delta_b \\ \phi_n \\ \Delta t \\ v \\ 1 \end{pmatrix}$$

$\underbrace{\phantom{\left( \begin{array}{cccc} \delta_{quartz} & \delta_{carbonate} & \delta_{fluid} & \delta_{shale} \\ (\phi_n)_{quartz} & (\phi_n)_{carbonate} & (\phi_n)_{fluid} & (\phi_n)_{shale} \\ \Delta t_{quartz} & \Delta t_{carbonate} & \Delta t_{fluid} & \Delta t_{shale} \\ v_{quartz} & v_{carbonate} & v_{fluid} & v_{shale} \\ 1 & 1 & 1 & 1 \end{array} \right)}}_C$ 
 $\underbrace{\phantom{\begin{pmatrix} C_q \\ C_c \\ phi \\ C_{sh} \end{pmatrix}}}_x$ 
 $\underbrace{\phantom{\begin{pmatrix} \delta_b \\ \phi_n \\ \Delta t \\ v \\ 1 \end{pmatrix}}}_d$

$$A = [1 \ 1 \ 1 \ 1]$$

$$b = 1$$

$$A_{eq} = []$$

$$B_{eq} = []$$

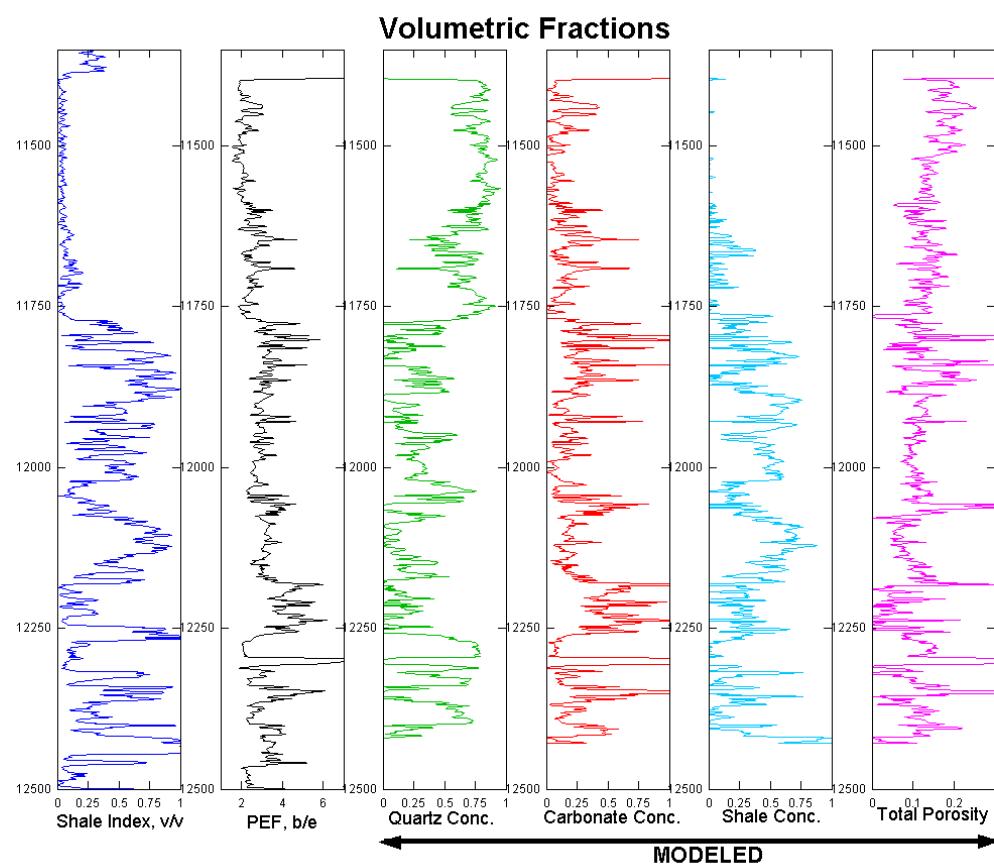
$$x = lsqlin(C, d, A, b, A_{eq}, B_{eq}, 0, 1)$$

$$x = lsqlin(C, d, A, b, A_{eq}, B_{eq}, zeros(4,1), ones(4,1))$$

TABLE 5-1 – Logging Parameters for Some Common Rocks and Minerals

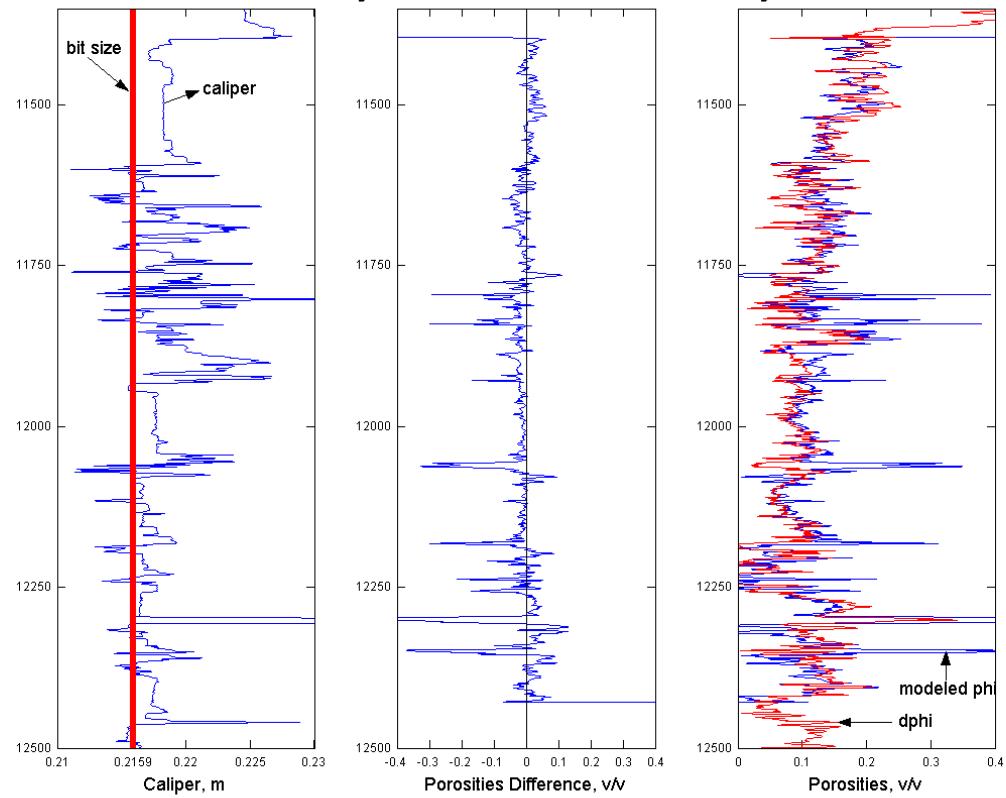
		$\rho_e$	$\rho_b$ (g/cc)	$\Sigma$ (c.u.)	$\Delta t$ ( $\mu\text{s}/\text{ft}$ )	$\phi_{CN}$ (lime)	K (%)	U (ppm)	Th (ppm)
<b>Common Sediment</b>	Quartz	1.8	2.65	8.0 – 13.0	51.3 – 55.5	- 0.04	< 0.15	< 0.4	< 0.2
	Calcite	5.1	2.71	8.0 – 10.0	47.6	0.00	< 0.40	1.5 – 15.0	< 2.0
	Dolomite	3.1	2.87	8.0 – 12.0	43.5	* 0.02	0.1 – 0.3	1.5 – 10.0	< 2.0
	Shales		1.80 – 2.70	25.0 – 45.0	63.0 – 170.0	0.09 – 0.45			
<b>Common Evaporites</b>	Halite	4.7	2.04	748.0	67.0	- 0.02 – 0.03		< 0.3	< 0.2
	Anhydrite	5.0	2.98	12.3	50.0	0.00		0.25 – 0.43	< 0.2
	Gypsum	4.0	2.35	18.8	52.5	0.50 – 0.60			
	Trona	0.7	2.10	18.5	65.0	0.42			
<b>Coals</b>	Lignite	0.16	1.05	12.8	140.0	0.60			
	Bituminous	0.17	1.33	16.4	120.0	0.60			
	Anthracite	0.20	1.57	10.5	105.0	0.40			
<b>Iron Minerals</b>	Limonite	13.0	3.59		57.0				
	Pyrite	17.0	4.99		39.0				
	Siderite	14.7	3.94		48.0				
	Hematite	21.5	5.18		44.0				
<b>Micas</b>	Glauconite	5.5 – 7.1	2.54	23.4		0.19	5.08 – 5.30		
	Biotite	6.2 – 6.4	2.99	30.0	51.0	0.06	6.7 – 8.3		< 0.01
	Muscovite	2.4	2.82	16.9	49.0	0.13	7.9 – 9.8		< 0.01
<b>Clays</b>	Kaolinite		2.61	12.8		0.37	0.42	1.5 – 3.0	6.0 – 19.0
	Chlorite		2.88	25.3		0.32		1.0 – 5.0	< 2.0
	Illite		2.63	15.5		0.09	4.50	2.0 – 5.0	
	Smectite		2.02	14.5		0.17	0.16		6.0 – 19.0

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### Quality Control of Modeled Porosity



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CASE No. 2

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## ASSESSMENT OF VOLUMETRIC FRACTIONS

( 3 input logs and estimating quartz, shale and carbonate concentration)

$$\begin{aligned}\delta &= C_q \cdot \delta_q + C_c \cdot \delta_c + C_{sh} \cdot \delta_{sh} \\ \Delta t &= C_q \cdot \Delta t_q + C_c \cdot \Delta t_c + C_{sh} \cdot \Delta t_{sh} \\ v &= C_q \cdot v_q + C_c \cdot v_c + C_{sh} \cdot v_{sh} \\ 1 &= C_q + C_c + C_{sh}\end{aligned}$$

$$\left( \begin{array}{ccc} \delta_{quartz} & \delta_{carbonate} & \delta_{shale} \\ \Delta t_{quartz} & \Delta t_{carbonate} & \Delta t_{shale} \\ v_{quartz} & v_{carbonate} & v_{shale} \\ 1 & 1 & 1 \end{array} \right) * \left( \begin{array}{c} C_q \\ C_c \\ C_{sh} \end{array} \right) = \left( \begin{array}{c} \delta_b \\ \Delta t \\ v \\ 1 \end{array} \right)$$

$$A = [1 \ 1 \ 1]$$

$$b = 1$$

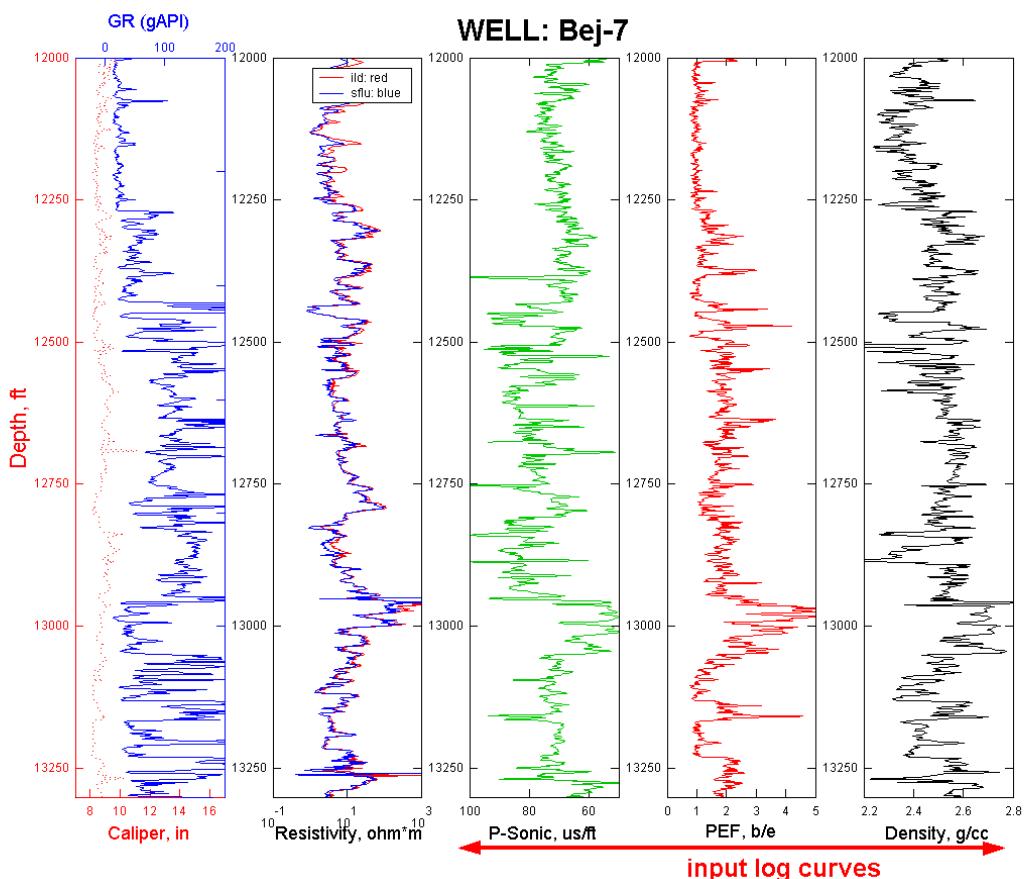
$$A_{eq} = []$$

$$B_{eq} = []$$

```
x = lsqlin(C, d, A, b, Aeq, Beq, 0, 1)
```

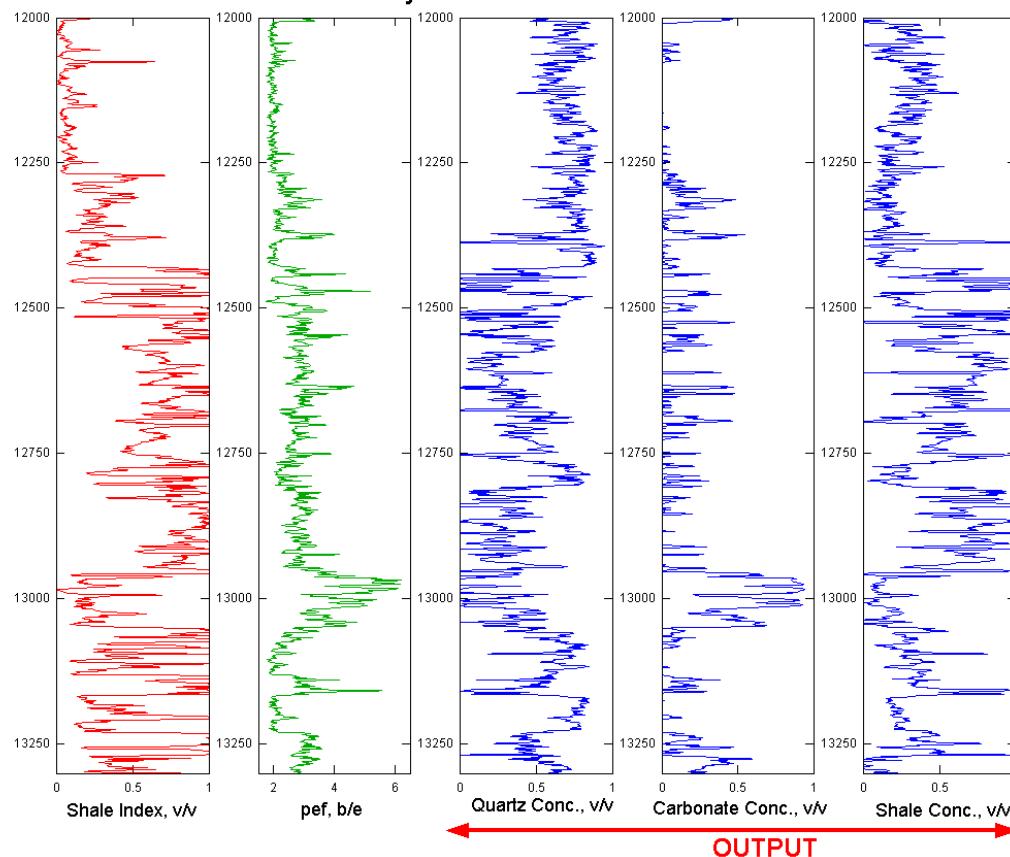
```
x = lsqlin(C, d, A, b, Aeq, Beq, zeros(3,1), ones(3,1))
```

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### Bej-7: Volumetric Fractions



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CASE No. 3

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## ASSESSMENT OF VOLUMETRIC FRACTIONS

(considering Csh as an input log)

$$\begin{aligned}
 \delta - (C_{sh} \cdot \delta_{sh}) &= C_q \cdot \delta_q + C_l \cdot \delta_l + \phi \cdot \delta_f + C_d \cdot \delta_d \\
 \phi_n - (C_{sh} \cdot (\phi_n)_{sh}) &= C_q \cdot (\phi_n)_q + C_l \cdot (\phi_n)_l + \phi \cdot (\phi_n)_f + C_d \cdot (\phi_n)_d \\
 \Delta t - (C_{sh} \cdot \Delta t_{sh}) &= C_q \cdot \Delta t_q + C_l \cdot \Delta t_l + \phi \cdot \Delta t_f + C_d \cdot \Delta t_d \\
 v - (C_{sh} \cdot v_{sh}) &= C_q \cdot v_q + C_l \cdot v_l + \phi \cdot v_f + C_d \cdot v_d \\
 1 - C_{sh} &= C_q + C_l + \phi + C_d
 \end{aligned}$$

$$\underbrace{\begin{pmatrix} \delta_{quartz} & \delta_{limestone} & \delta_{fluid} & \delta_{dolomite} \\ (\phi_n)_{quartz} & (\phi_n)_{limestone} & (\phi_n)_{fluid} & (\phi_n)_{dolomite} \\ \Delta t_{quartz} & \Delta t_{limestone} & \Delta t_{fluid} & \Delta t_{dolomite} \\ v_{quartz} & v_{limestone} & v_{fluid} & v_{dolomite} \\ 1 & 1 & 1 & 1 \end{pmatrix}}_C * \underbrace{\begin{pmatrix} C_q \\ C_l \\ \phi \\ C_d \end{pmatrix}}_x = \underbrace{\begin{pmatrix} \delta - (C_{sh} \cdot \delta_{sh}) \\ \phi_n - (C_{sh} \cdot (\phi_n)_{sh}) \\ \Delta t - (C_{sh} \cdot \Delta t_{sh}) \\ v - (C_{sh} \cdot v_{sh}) \\ 1 - C_{sh} \end{pmatrix}}_d$$

$$A = [1 \ 1 \ 1 \ 1]$$

$$b = 1$$

$$A_{eq} = []$$

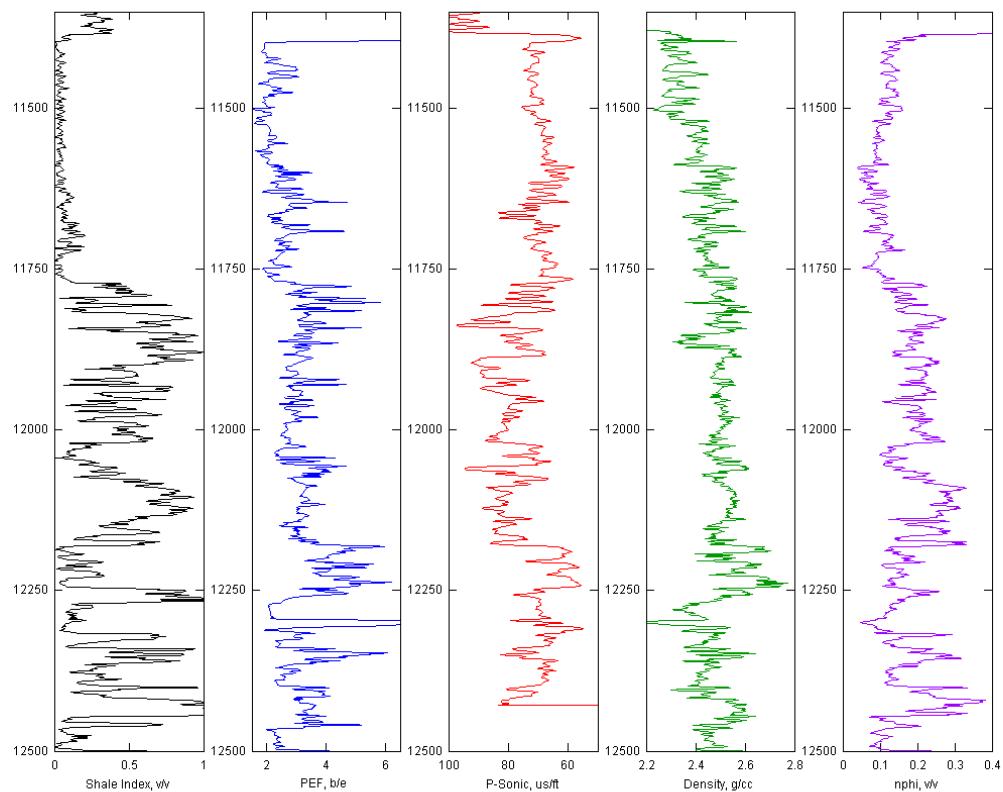
$$B_{eq} = []$$

```
x = lsqlin(C, d, A, b, Aeq, Beq, 0, 1)
```

```
x = lsqlin(C, d, A, b, Aeq, Beq, zeros(4,1), ones(4,1))
```

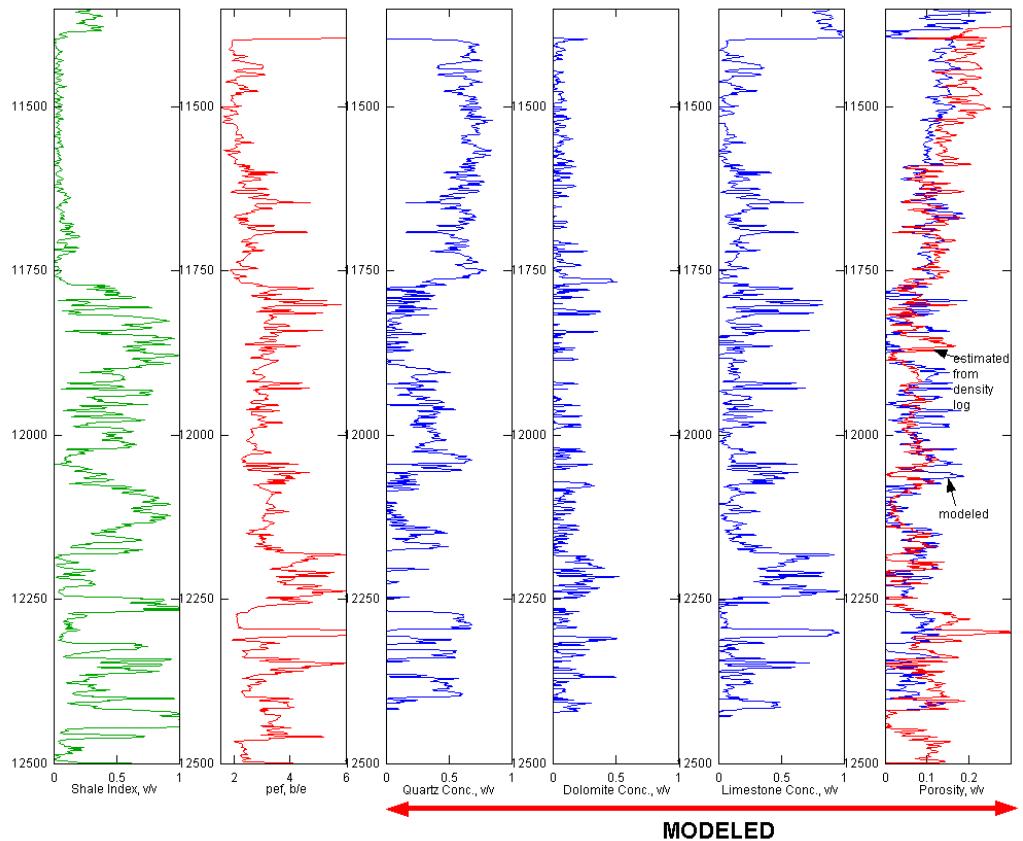
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**Well Logs Used as Input**

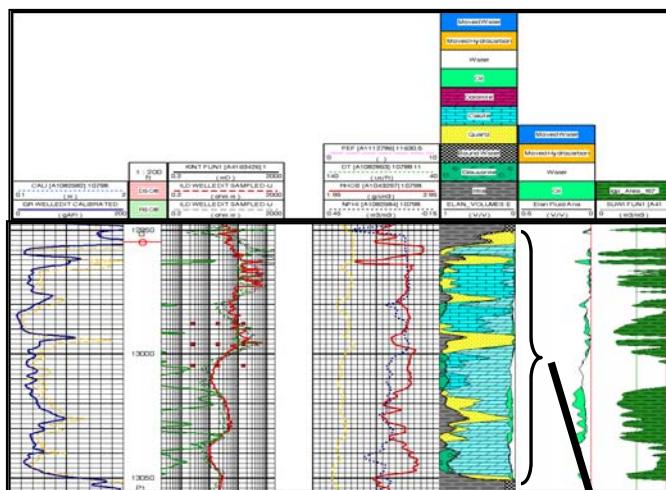


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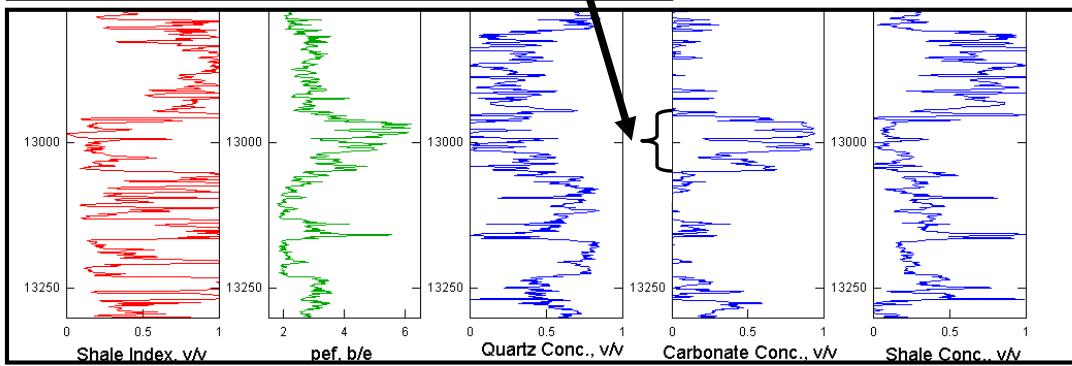
## Volumetric Fractions



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**WELL: Bej-7**



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# Conventional Depth-by-Depth Estimation Method

- Linear system of equations:

– Nonlinear gas correction

– Saturation correction

$$\mathbf{A} = \begin{bmatrix} \rho_{b,1} & \rho_{b,2} & \dots & \rho_{b,p} & \rho_{b,sh} & \rho_{fluid} \\ \phi_{N,1} & \phi_{N,2} & \dots & \phi_{N,p} & \phi_{N,sh} & \phi_{N,fluid} \\ \Delta t_1 & \Delta t_2 & \dots & \Delta t_p & \Delta t_{sh} & \Delta t_{fluid} \\ U_1 & U_2 & \dots & U_p & U_{sh} & U_{fluid} \end{bmatrix}$$

$$\min \| \mathbf{A} \cdot \mathbf{x} - \mathbf{b} \|_2^2$$

↓

Is it a correct assumption?

$$\mathbf{x} = [C_1 \ C_2 \ \dots \ C_p \ C_{sh} \ \phi_s]^T$$

$$\mathbf{b} = [\rho_b \ \phi_N \ \Delta t \ U]^T$$

$$\sum_{i=1}^{n_c} C_i + C_{sh} + \phi_s = 1 \quad 0 \leq x_i \leq 1$$

## Limitations of the Conventional Estimation Method

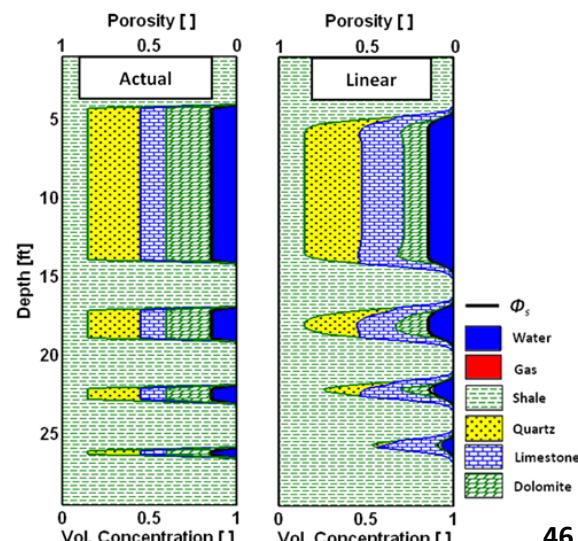
- Nonlinearity between well logs and concentrations, especially in gas-bearing zones and presence of salt

– Fresh water-bearing zone

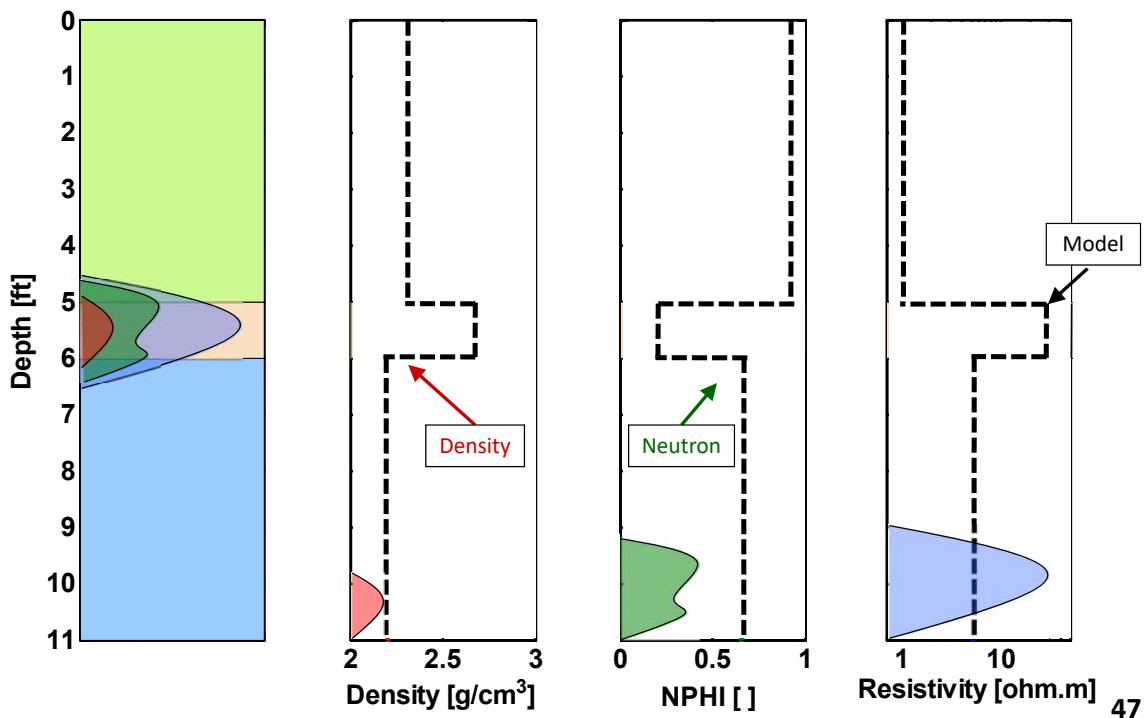
– Gas-bearing zone

- Shoulder-bed effects

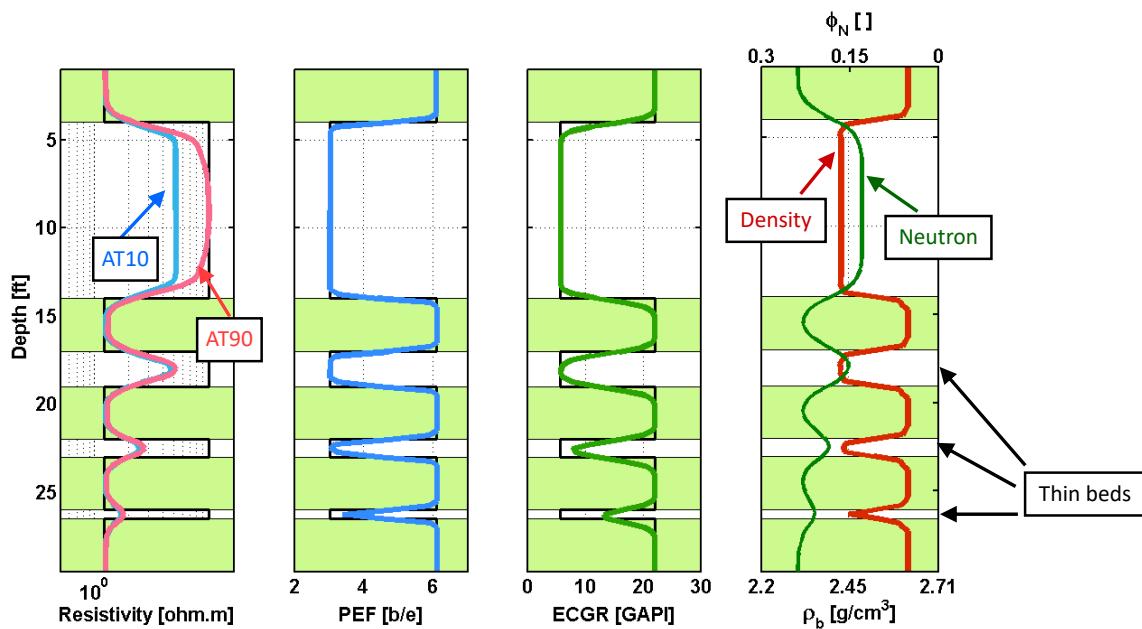
– Thin beds



## Uneven Vertical Resolution and Depth of Investigation

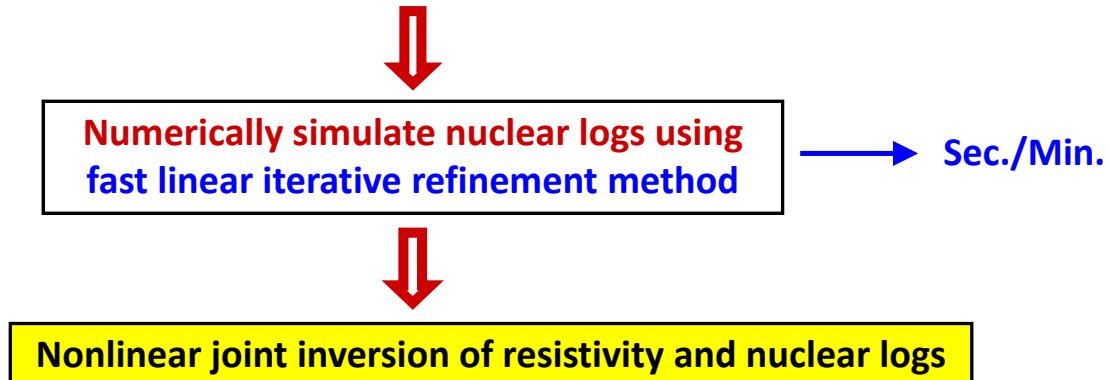


## Thin-Bed Effects on Resistivity and Nuclear Logs



# New Estimation Method

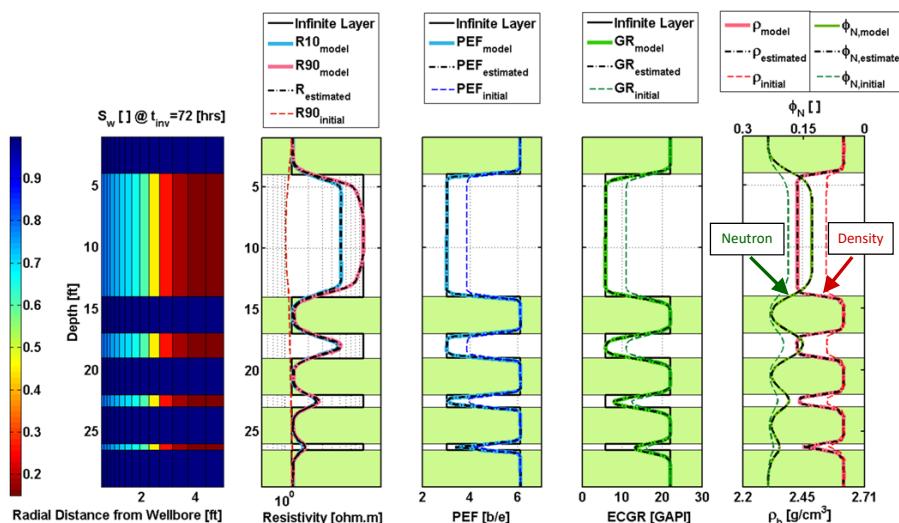
- Limitations of conventional methods:
  - Unlike resistivity logs, nuclear logs were not easy to simulate
    - High CPU processing time → Hours/Days



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## Synthetic Case No. 1

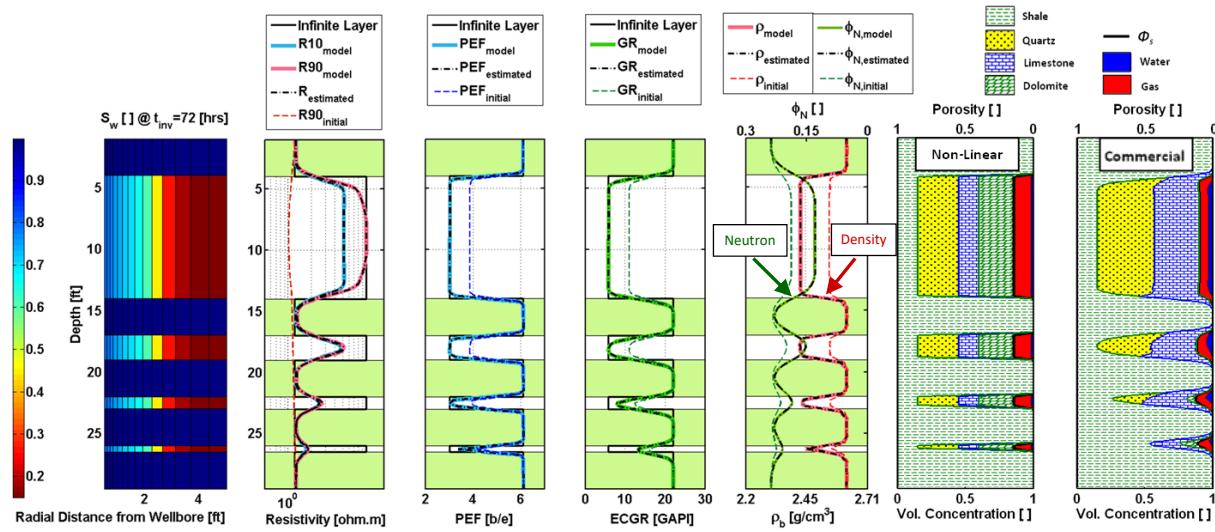
- Gas-Bearing, Single-Layer Formation



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# Synthetic Case No. 1

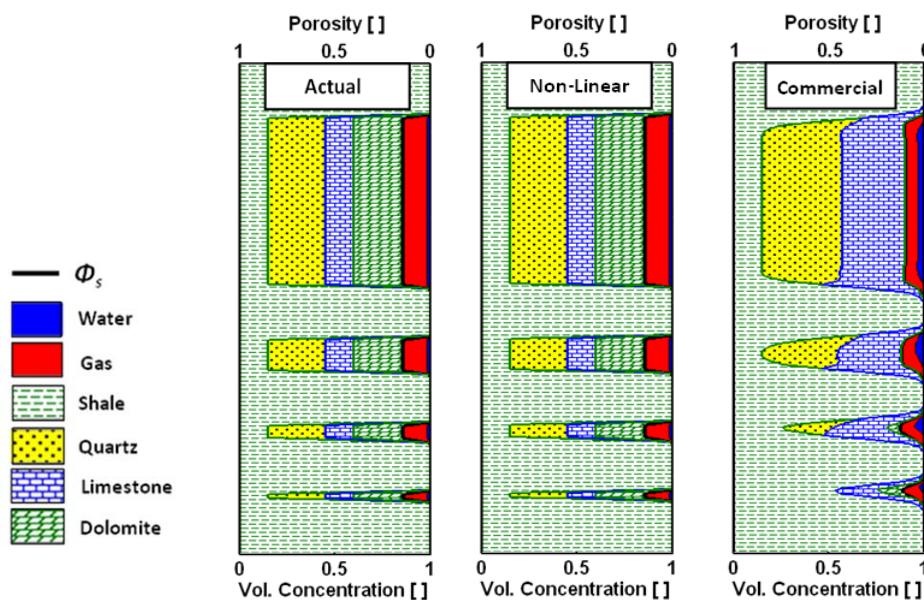
- Gas-Bearing, Single-Layer Formation



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# Synthetic Case No. 1

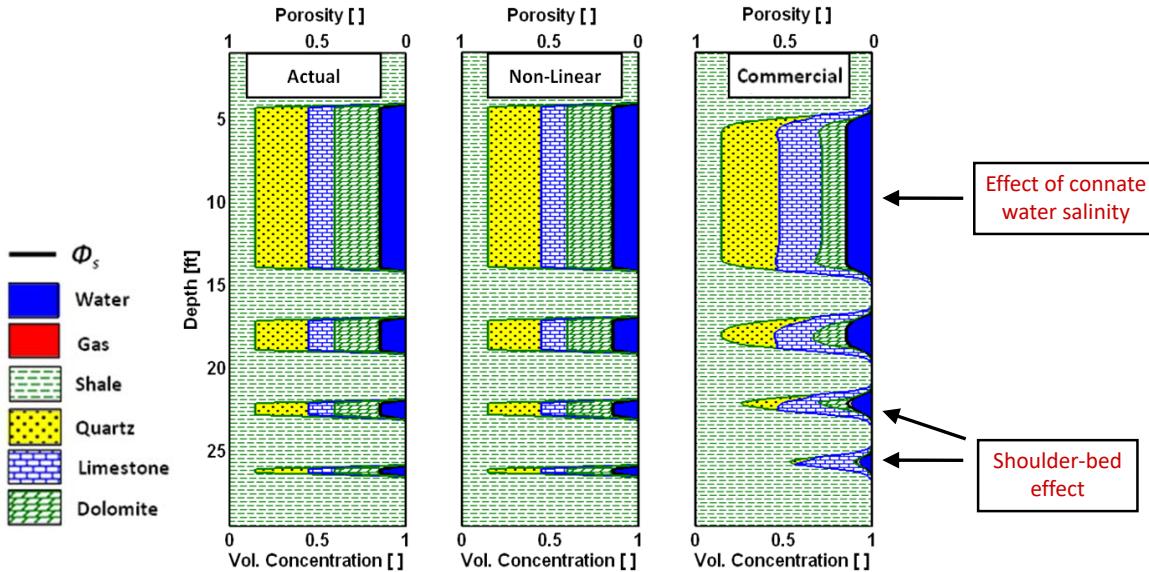
- Synthetic Case No. 1: Gas-Bearing, Single-Layer Formation



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# Synthetic Case No. 1

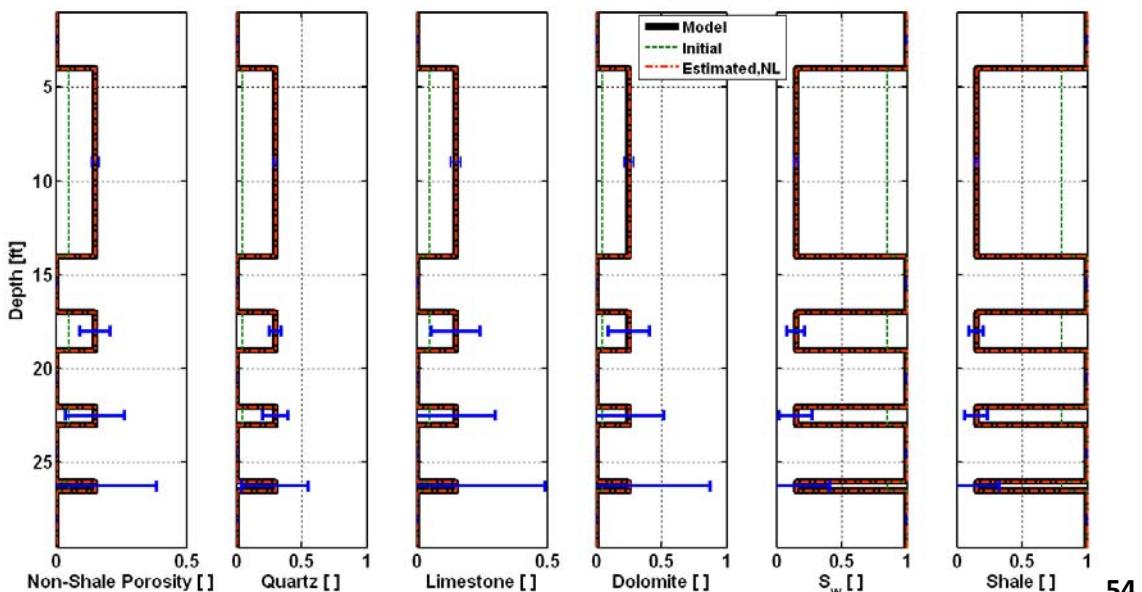
- Synthetic Case No. 1: Water-Bearing, Single-Layer Formation



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## Effect of Layer Thickness

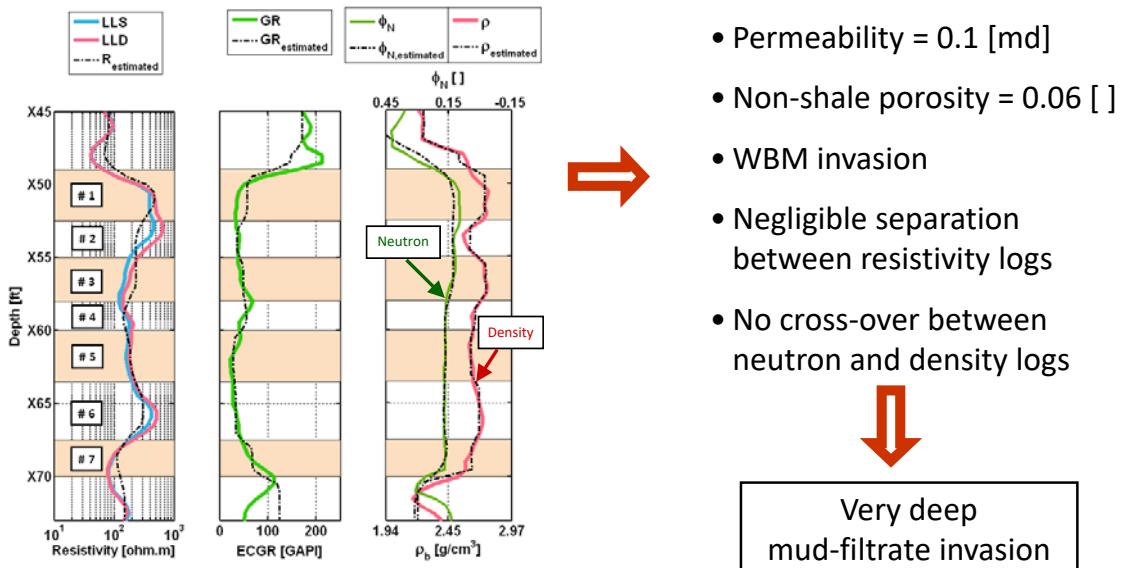
- Synthetic Case No. 1: 5% zero-mean Gaussian random perturbations on the original synthetic well logs



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# Field Example No. 1

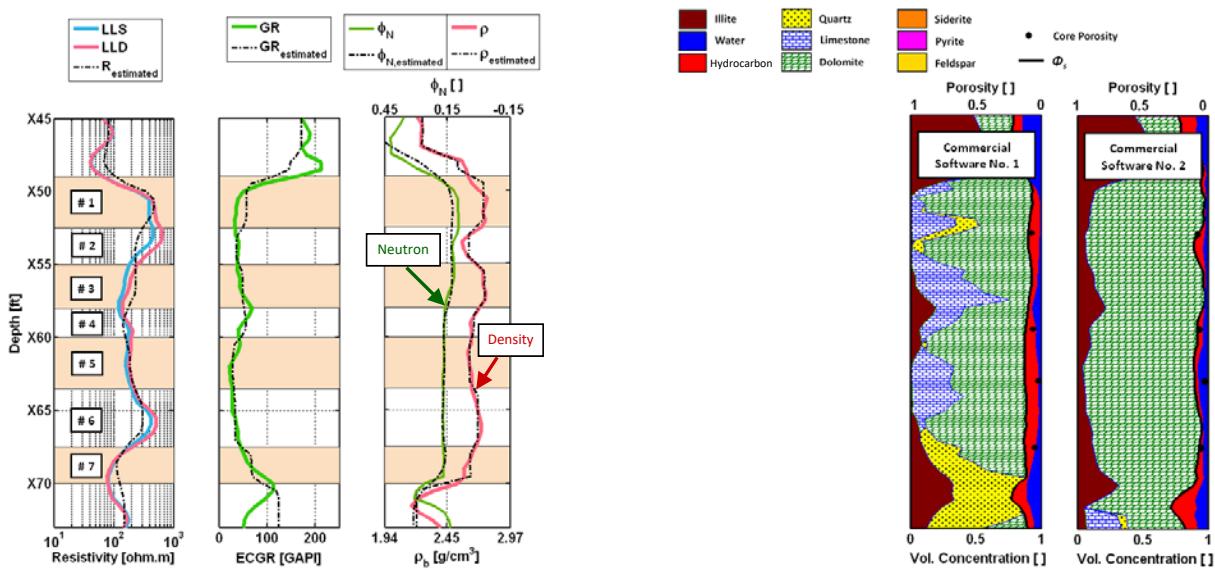
- Hydrocarbon-Bearing Carbonate Formation



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# Field Example No. 1

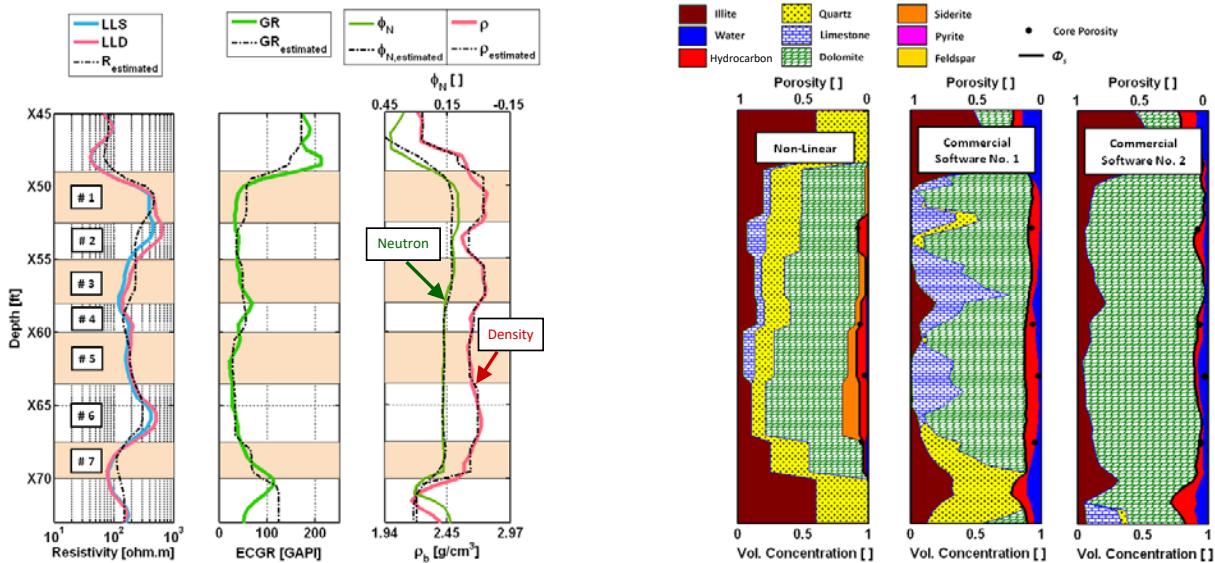
- Hydrocarbon-Bearing Carbonate Formation



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# Field Example No. 1

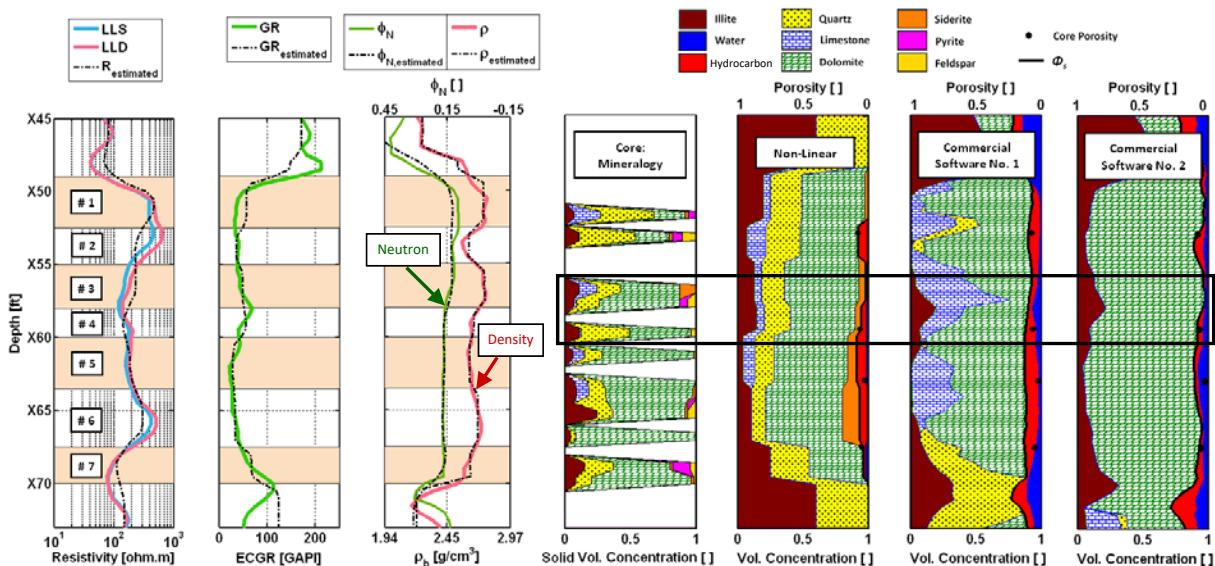
- Hydrocarbon-Bearing Carbonate Formation



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# Field Example No. 1

- Hydrocarbon-Bearing Carbonate Formation



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## Acknowledgements:

- Baker Atlas