BASIC WELL LOGGING FORMULAE

Temperature Calculation Assuming a Linear Geothermal Gradient

$$T(z) = T(z_1) + \frac{T(z_2) - T(z_1)}{z_2 - z_1}(z - z_1)$$

where:
T = temperature (either °C or °F),
z = true vertical depth (TVD) measured with respect to the logging reference point,
z_1 = TVD of a point in at which the temperature was measured (usually the at the surface), and
z_2 = TVD of a second point at which the temperature was measured (usually at the bottom of the well).

Shale/Clay Index Based on Gamma-Ray Log

$$I_{sh} = \frac{GR - GR_{cn}}{GR_{sh} - GR_{cn}}$$

where:
I_{sh} = shale index,
GR = gamma-ray reading in the depth zone of interest,
GR_{cn} = gamma-ray reading in a depth zone which is considered clean (shale/clay free), also referred to as $GR_{\min}$, and
GR_{sh} = gamma-ray reading in a "pure" shale depth zone, also referred to as $GR_{\max}$.

Shale/Clay Index Based on Gamma-Ray Log Across a Water-Saturated Sandstone

$$I_{sh} = I_{shw} + \frac{1 - I_{shw}}{GR_{sh} - GR_w}\left(GR - GR_w\right)$$

where:
I_{sh} = shale index,
I_{shw} = shale index in the water-saturated sandstone ($I_{shw} \leq 1$),
GR = gamma-ray reading in the depth zone of interest,
GR_w = gamma-ray reading in the water-saturated sandstone ($GR_w \leq GR_{sh}$), and
GR_{sh} = gamma-ray reading in a "pure" shale depth zone, also referred to as $GR_{\max}$.

Spontaneous Electrical Potential

$$SSP \approx -K \times \log_{10}(\frac{C_w}{C_{mf}})$$

where:
SSP = Static spontaneous electrical potential in mV,
C_{mf} = salt concentration of mud filtrate in NaCl ppm,
C_w = salt concentration of connate water in NaCl ppm, and
K is a temperature-independent constant = 71 mV.

Conversion of Electrical Resistivity due to Temperature Change

$$R_2 = R_1 \left(\frac{T_1 + 6.77}{T_2 + 6.77} \right) \quad \deg F$$

or

$$R_2 = R_1 \left(\frac{T_1 + 21.5}{T_2 + 21.5} \right) \quad \deg C$$

where:
R_1 = electrical resistivity (Ohm-m) measured at temperature T_1, and
R_2 = corresponding electrical resistivity (Ohm-m) at temperature T_2.

Conversion of Salt Concentration to Electrical Resistivity of Water Diluted with NaCl

$$R_w \approx \left(0.0123 + \frac{3647.5}{\left[NaCl_{ppm} \right]^{0.955}} \right) \left(\frac{81.77}{T + 6.77} \right)$$

where:
R_w = Electrical resistivity of water in $\Omega\ m$ (Ohm-m),
$NaCl_{ppm}$ = salt (NaCl) concentration of water in ppm, and
T = temperature in ^{o}F.

The error of the above equation is approximately 2% when the concentration of NaCl is between 500 and 100,000 ppm, and between 2 – 10% when the concentration of NaCl is between 100,000 and 230,000 ppm.

It follows that

$$\left[NaCl_{ppm}\right] \approx 10^{\left\{\frac{3.562-\log_{10}\left[\left(\frac{T+6.77}{81.77}\right)\bullet R_w-0.0123\right]}{0.955}\right\}}$$

Archie's Equation

$$R_t \approx R_w \left[\frac{a}{\phi^m}\right]\frac{1}{S_w^{\;n}}$$

where:

R_t = electrical resistivity of a fluid-saturated porous rock,

R_w = electrical resistivity of the water contained in the rock's pore space,

ϕ = interconnected porosity,

S_w = water saturation,

a = Winsauer's multiplier,

m = porosity exponent, and

n = saturation exponent.

General Relationship between Bulk Density and Porosity

$$\rho_b = \phi\ \rho_f + (1-\phi)\ \ \rho_m$$

where:

ρ_b = bulk density of the rock (aka rock's density),

ρ_f = density of the fluid occupying the rock's pore space,

ϕ = total porosity, and

ρ_m = density of the matrix (solid component) contained in the rock.

Alternatively, one can write

$$\phi = \frac{\rho_b - \rho_m}{\rho_f - \rho_m} = \frac{\rho_m - \rho_b}{\rho_m - \rho_f}\ .$$

Likewise,

$$\rho_f = \frac{1}{\phi}\left[\rho_b - (1-\phi)\ \ \rho_m\right]$$

General Relationship between Bulk Density and Non-Shale (Sandstone) Porosity for the Case of Shaly Sands with **Laminated** Shale

$$\rho_b = (1 - C_{sh})\ \rho_s + C_{sh}\ \rho_{sh}$$

where:
ρ_b = bulk density of the rock (aka rock's density),
C_{sh} = volumetric concentration of shale,
ρ_s = sandstone density, and
ρ_{sh} = shale density (includes shale porosity, silt, dry clay, and water).

Likewise,

$$\phi_t = (1 - C_{sh})\ \phi_s + C_{sh}\ \phi_{sh}$$

where:
ϕ_t = total rock porosity,
C_{sh} = volumetric concentration of shale,
ϕ_s = porosity of the sand fraction, and
ϕ_{sh} = shale porosity.

It follows that

$$\phi_s = \frac{\phi_t - C_{sh}\ \phi_{sh}}{1 - C_{sh}}$$

General Relationship between Bulk Density and Non-Shale (Sandstone) Porosity for the Case of Shaly Sandstone with **Dispersed** (Grain-Coating) Shale

$$\rho_b = \phi_s \rho_f + (1 - \phi_s - C_{sh})\ \rho_m + C_{sh}\ \rho_{sh}$$

where:
ρ_b = bulk density of the rock (aka rock's density),
ρ_f = density of the fluid occupying the rock's pore space,
ϕ_s = non-shale (sandstone) porosity,
C_{sh} = volumetric concentration of shale,
ρ_m = grain density (for reference, density of quartz = 2.65 gm/cm^3), and
ρ_{sh} = shale density (includes shale porosity, silt, dry clay, and water).

Alternatively, one can write

$$\phi_s = \frac{\rho_m - \rho_b}{\rho_m - \rho_f} - C_{sh} \frac{\rho_m - \rho_{sh}}{\rho_m - \rho_f} .$$

Likewise,

$$\rho_f = \frac{1}{\phi_s} \left[\rho_b - (1 - \phi_s - C_{sh}) \; \rho_m - C_{sh} \; \rho_{sh} \right]$$

Additionally,

$$\phi_T = \phi_s + C_{sh} \; \phi_{sh} ,$$

where:
ϕ_T = total porosity,
ϕ_{sh} = shale porosity, and
ϕ_s = sandstone (non-shale) porosity.

<u>Fluid Density and Saturations</u>

For the case of water and hydrocarbon, one has

$$\rho_f = S_w \rho_w + (1 - S_w) \; \rho_H$$

where:
ρ_f = density of the fluid occupying the rock's pore space,
ρ_w = density of water (depends on salt concentration),
ρ_H = density of hydrocarbon, and
S_w = water saturation.

It follows that

$$\rho_H = \frac{\rho_f - S_w \rho_w}{1 - S_w}$$

For the case of water, gas, and oil, one has

$$\rho_f = S_w \rho_w + S_o \rho_o + (1 - S_w - S_o) \; \rho_g$$

where:
ρ_f = density of the fluid occupying the rock's pore space,
ρ_w = density of water (depends on salt concentration),
ρ_o = density of oil,
ρ_g = density of gas,
S_w = water saturation, and
S_o = oil saturation.

Density-Neutron Porosity Corrections for Shaly Sandstone (case of **laminated** shale)

$$\phi_D^{©} = \frac{\phi_D - C_{sh}\ \phi_{D,sh}}{1 - C_{sh}},$$

where:
ϕ_D = density (apparent) porosity expressed in water-filled sandstone porosity units,
$\phi_{D,sh}$ = density (apparent) porosity of "pure shale" expressed in water-filled sandstone porosity units,
C_{sh} = volumetric concentration of shale, and
$\phi_D^{©}$ = shale-corrected density porosity expressed in water-filled sandstone porosity units.

$$\phi_N^{©} = \frac{\phi_N - C_{sh}\ \phi_{N,sh}}{1 - C_{sh}},$$

where:
ϕ_N = neutron (apparent) porosity expressed in water-filled sandstone porosity units,
$\phi_{N,sh}$ = neutron (apparent) porosity of "pure shale" expressed in water-filled sandstone porosity units,
C_{sh} = volumetric concentration of shale, and
$\phi_N^{©}$ = shale-corrected neutron porosity expressed in water-filled sandstone porosity units.

It follows that

$$\phi_s \approx \sqrt{\frac{\left(\phi_D^{©}\right)^2 + \left(\phi_N^{©}\right)^2}{2}}$$

where:
ϕ_s = sandstone porosity.

Density-Neutron Porosity Corrections for Shaly Sandstone (case of **dispersed** shale)

$$\phi_D^{©} = \phi_D - C_{sh}\ \phi_{D,sh},$$

where:
ϕ_D = density (apparent) porosity expressed in water-filled sandstone porosity units,
$\phi_{D,sh}$ = density (apparent) porosity of "pure shale" expressed in water-filled sandstone porosity units,
C_{sh} = volumetric concentration of shale (by definition, lower than non-shale porosity in this case), and
$\phi_D^{©}$ = shale-corrected density porosity expressed in water-filled sandstone porosity units.

$$\phi_N^{©} = \phi_N - C_{sh}\ \phi_{N,sh},$$

where:
ϕ_N = neutron (apparent) porosity expressed in water-filled sandstone porosity units,
$\phi_{N,sh}$ = neutron (apparent) porosity of "pure shale" expressed in water-filled sandstone porosity units,
C_{sh} = volumetric concentration of shale (by definition, lower than non-shale porosity in this case), and
$\phi_N^{©}$ = shale-corrected neutron porosity expressed in water-filled sandstone porosity units.

It follows that

$$\phi_s \approx \sqrt{\frac{\left(\phi_D^{©}\right)^2 + \left(\phi_N^{©}\right)^2}{2}}$$

where:
ϕ_s = non-shale (sandstone) porosity.

Density-Neutron Calculation of Volumetric Concentration of Shale in a Water-Bearing Sandstone

$$C_{sh} = \frac{\phi_N - \phi_D}{\phi_{N,sh} - \phi_{D,sh}},$$

where:

C_{sh} = volumetric concentration of shale,

ϕ_D = density (apparent) porosity expressed in water-filled sandstone porosity units,

$\phi_{D,sh}$ = density (apparent) porosity of "pure shale" expressed in water-filled sandstone porosity units,

ϕ_N = neutron (apparent) porosity expressed in water-filled sandstone porosity units, and

$\phi_{N,sh}$ = neutron (apparent) porosity of "pure shale" expressed in water-filled sandstone porosity units.

Relationship Between Rock Resistivity and Sandstone Resistivity in **Laminated** Shale-Sandstone Systems (Case of Isotropic Sandstone and Shale)

$$\frac{1}{R_{||}} = \frac{1 - C_{sh}}{R_s} + \frac{C_{sh}}{R_{sh}}$$

where:

C_{sh} = volumetric concentration of shale,

$R_{||}$ = electrical resistivity of the rock parallel to bedding plane,

R_s = electrical resistivity of the sandstone fraction of the rock, and

R_{sh} = electrical resistivity of the shale fraction of the rock.

$$R_{\perp} = \left(1 - C_{sh}\right) R_s + C_{sh} R_{sh}$$

where:

$R_{\perp}$ = electrical resistivity of the rock perpendicular to bedding plane.

Wyllie’s Relationship and Sonic Porosity

$$\Delta t_b \approx \phi_{sonic}\ \Delta t_f + (1-\phi_{sonic})\ \Delta t_m$$

where:
Δt_b = bulk sonic slowness (aka rock’s sonic slowness) [μs/ft],
Δt_f = sonic slowness of the fluid occupying the rock’s pore space [μs/ft],
ϕ_{sonic} = sonic porosity (aka Wyllie’s porosity), and
Δt_m = sonic slowness of the matrix (solid component) contained in the rock [μs/ft].

Alternatively,

$$\phi_{sonic} = \frac{\Delta t_b - \Delta t_m}{\Delta t_f - \Delta t_m}.$$

Timur-Tixier Equation

$$k \approx \alpha\ \phi^{\beta}\ \left(\frac{1-S_{wirr}}{S_{wirr}}\right)^{\gamma}$$

where:
k = permeability,
ϕ = total porosity, and
S_{wirr} = irreducible water saturation.

Electrical Resistivity of **Dispersed** (Grain-Coating Clay) Shaly Sandstone

$$\frac{1}{R_t} \approx \frac{1}{R_w}\left[1 + B\ R_w\ \frac{S_b}{S_w}\right]\left[\frac{\phi^{m^*}}{a^*}\right] S_w^{\ n^*},$$

where:

$$S_b = C_{sh}\ \frac{\phi_{sh}}{\phi},$$

$$B = \frac{-5.41 + 0.133\ T - 0.0001253\ T^2}{1 + R_w^{1.23}(0.025\ T - 1.07)},$$

and

R_t = electrical resistivity of the sandstone with grain-coating clay,

R_w = electrical resistivity of the water contained in the rock's pore space,

ϕ = interconnected porosity,

ϕ_{sh} = shale porosity,

C_{sh} = volumetric concentration of shale ($C_{sh} < \phi$),

S_w = water saturation,

a^* = modified Winsauer's multiplier,

m^* = modified porosity exponent,

n^* = modified saturation exponent, and

T = temperature in ^{o}F.