LASER RADAR SYSTEMS

OPTICAL REMOTE SENSING
(F. Rocadenbosch)

PROBLEM 2: LINK-BUDGET (ELASTIC-RAMAN LIDAR)

Consider an elastic-Raman lidar defined by the following system parameters:

LASER

• Quantel (Nd:YAG 2ω)		
• Emission wavelength, λ_0	532 nm	
• Energy, E	(parameter) mJ	
• Pulse-repetition frequency, PRF	(parameter) Hz	
• Beam width, w_0	5×10^{-3} m	(*)
• Divergence (half-width angle), $\theta_{1/2}$	0.5 mrad	(*)

TELESCOPE

• Celestron Schmidt-Cassegrain	
• Primary lens diameter, d_p	(parameter) m
• Shade diameter, d_{sh}	0.06858 m
• Focal length, f	2 m
• Transmissivity, T_1	60 %

ELASTIC-RECEIVING CHANNEL

• Reception wavelength, λ_0	532 nm

INTERFERENCE FILTER

• Bandwidth, $d\lambda_0$	10 nm
• Transmissivity, T_2	65 %

PHOTODIODE

• APD (EGG C30956E)	
• Active area diameter, D_{APD}	3 mm
• Multiplication factor, M	(parameter)
• Excess-noise factor, F	(parameter)
• Dark surface current, I_{ds}	7.64×10^{-8} A
• Dark bulk current, I_{db}	3.10×10^{-10} A
• Intrinsic responsivity, R_{io}	240 mA/W

SIGNAL-CONDITIONING STAGES

• Transimpedance Gain (1st stage), G_t	5750 Ω
• Voltage conditioning Gain (2nd stage), G_{ac}	20.3 V/V
• Noise-equivalent bandwidth, B	10 MHz
• Equivalent input noise (chain input), $\sigma_{th,i}$	5 pA·Hz$^{-1/2}$

ACQUISITION SYSTEM

• Type: Analog-to-digital recorder	ADC
• Sampling frequency, f_s	20 Msps (20×10^6)

RAMAN-RECEIVING CHANNEL

- Reception wavelength, λ_R — 607.4 nm

INTERFERENCE FILTER

- Bandwidth, $d\lambda_R$ — (parameter) nm
- Transmissivity, T_2' — 65 %

PMT (PHOTO-MULTIPLIER TUBE)
(Please note APD-equivalent notation in use)

- Equivalent active area diameter, D — 3 mm
- Multiplication factor, M — (parameter)
- Excess-noise factor, F — 1.8
- Anode dark current, I_{da} (i.e, I_{ds}=0, $I_{db}=I_{da}/M$) — (parameter) nA
- Anode radiant sensitivity, R_i ($R_{io}=R_i/M$) — (parameter) A/W

SIGNAL-CONDITIONING STAGE

- PMT-load resistance, R (i.e, G_t) — 50 Ω
- Noise-equivalent bandwidth, B — 10 MHz

ACQUISITION SYSTEM (See also "Hamamatsu Appl. Note" in pdf.)

- Type: Photon counter — PC
- Temporal resolution, Δt_{PC} — 1 bin
- Sampling ("binning") period — 50 ns/bin

ATMOSPHERE

Aerosol component at λ_0:

- Visibility margin (532 nm), V_M — (parameter) km
- Lidar ratio, $S_M(\lambda_0)=\alpha_{Mie}(\lambda_0)/\beta_{Mie}(\lambda_0)$ — (parameter) sr

Aerosol component at λ_R:

- Wavelength-dependency coef., κ=1.8. I.e., consider $\frac{\alpha_{Mie}(\lambda_R)}{\alpha_{Mie}(\lambda_0)} = \left(\frac{\lambda_R}{\lambda_0}\right)^{-\kappa}$

Molecular components at λ_0, λ_R (Height-dependent, US-standard atmosphere approx.)

- Rayleigh's extinction, $\alpha_{Ray}(\lambda_i,R)$ — km^{-1} with R [km]

$$\alpha_{Ray}(\lambda_0,R) \approx 1.2569\times10^{-2} - 7.7599\times10^{-4}\ R$$
$$\alpha_{Ray}(\lambda_R,R) \approx 7.3219\times10^{-3} - 4.5204\times10^{-4}\ R$$

- Rayleigh's ratio, $S_R(\lambda)=\alpha_{Ray}(\lambda)/\beta_{Ray}(\lambda)$ — $8\pi/3$

- N_2-Raman backscattering cross-section, $d\sigma_{N2}(\pi)/d\Omega$ — 3.71×10^{-41} [$km^2\cdot sr^{-1}$]

- N_2-molecule number density, $N_{N2}(R)$ (height-dep.) — [km^{-3}] with R [km]

$$N_{N2}(R) \approx 2.1145\times10^{34} - 2.0022\times10^{33}R + 5.4585\times10^{31}R^2$$

Boundary-layer height, R_{PBL} — (parameter) km

Background-radiance component

- Moon's radiance (full Moon), L_{Moon} — $3\times10^{-11}\ W\cdot cm^{-2}\cdot nm^{-1}\cdot sr^{-1}$
- Solar radiance, L_{Sun} (typ.) — $3\times10^{-6}\ W\cdot cm^{-2}\cdot nm^{-1}\cdot sr^{-1}$

OTHER PARAMETERS

• Full-overlap range, R_{ovf}	200 m
• Maximum-range criterion	$SNR(R_{max})$ =1
• Observation time (to simulate), t_{obs}	[1/PRF, 10^4] s

PHYSICAL CONSTANTS

• Electron charge, q	1.602×10^{-19} C	
• Planck's constant, h	6.6262×10^{-34} J·s	
• Light speed, c	2.99793×10^{8} m·s^{-1}	
• Boltzmann's constant, K	1.38×10^{-23} J·K^{-1}	(*)

(*) Parameter not used.

<u>Questions:</u>

1. Determine the system constant, K [W·km^3].

2. Estimate the received background power for both the elastic and Raman channels ($P_{back,0}$ and $P_{back,R}$, respectively) under (see "day-time/night-time" parameter) operation.

3. Plot both elastic- and Raman-return powers ($P_0(R)$ and $P_R(R)$, respectively). Superimpose $P_{back,0}$ and $P_{back,R}$ plots from question 2 results.

4. Compute, for the elastic and Raman channels, receiver-chain voltage responsivities ($R_{v,0}$ and $R_{v,R}$, respectively), and net voltage responsivities (i.e., including spectral optical losses; $R_{v,0}$' and $R_{v,R}$', respectively).

5. a) Assuming <u>analog detection,</u> plot the elastic range-dependent signal-to-noise ratio, $SNR_0(R)$, at the output of the receiver chain (i.e., voltage ratio) and related shot photo-induced, shot-dark and <u>thermal variances</u> ($\sigma^2_{sh,s}$, $\sigma^2_{sh,d}$, σ^2_{th} in units of [V^2]).

   b) Identify where you have different noise-dominant system-operation modes.

6. a) (See Hamamatsu Appl. Note, Sect.1 and Sect. 3.8.2, pp. 14-15) Assuming, <u>photon counting detection,</u> plot the Raman range-dependent photon-count signal-to-noise ratio, $SNR_R(R)$ (Eq.(2), p.15) and related shot photo-induced, shot-dark and <u>shot-background variances</u> (in units of [counts/bin]).

7. (Raman-channel only). Departing from your results of question (6) above, give a plot of the required <u>observation time (Y-axis) versus range (X-axis)</u> in order to ensure a goal SNR of $SNR_{min}=2\times10^3$ (linear units) at each successive range.

8. Compute and compare photodiode's NEP (NEP_{APD}) and PMT's NEP (NEP_{PMT}).

9. (OPTIONAL) Assume that <u>only elastic- and Raman-return powers are known</u> (question (3)). Plot the maximum lidar range (Y-axis) versus required observation time (X-axis; i.e. consider pulse integration) for both elastic and Raman channels (see R_{max} criterion).

Figs.1-4. ANSWER PLOT FORMAT

Q3. Fig.1. Abscissae range 0 to 15 km; linear-X, log-Y. $P_0(R)$, $P_R(R)$, $P_{back,0}$ and $P_{back,R}$ in the same plot.

Q5 & Q6. Fig.2. Abscissae range 0 to 15 km; linear-X, log-Y. $SNR_0(R)$ and $SNR_R(R)$ superimposed. Fig.3. Abscissae range 0 to 15 km; linear-X, log-Y, 2 subplots: Fig.3(a) Superimposed elastic variances; Fig.3(b) Superimposed Raman variances.

Q7. Fig.4. Abscissae range 0 to 15 km; linear-X, log-Y.

Q9. Fig.5. Abscissae range 10^{-2} to 10^4 s; log-X, log-Y.

COMMENTS AND SUPPORT FORMULAE

In this problem an elastic-backscatter and a Raman lidar channel operate simultaneously at λ_0 and λ_R. The elastic-backscatter lidar equation

$$P_{\lambda_0}(z) = \frac{K_{\lambda_0}}{z^2}\left[\beta_{\lambda_0}^{aer}(z) + \beta_{\lambda_0}^{mol}(z)\right] \times \exp\left\{-2\int_0^z \left[\alpha_{\lambda_0}^{aer}(\xi) + \alpha_{\lambda_0}^{mol}(\xi)\right] d\xi\right\}\xi(z)$$

has already been addressed in Chap.3 with z=R the vertical direction.

In the case of the Raman channel, the reception channel is spectrally tuned to receive backscattered radiation from atmospheric nitrogen molecules. In Chap. 7, we will see that since nitrogen is an atmospheric abundant species, it is of advantage to calibrate the co-operative elastic channel and hence, to obtain independent estimations of both extinction and backscatter atmospheric optical components. In comparison with the elastic-backscatter lidar equation, in which both the optical emission path (i.e., from the laser source to the atmosphere) and return path (i.e., from the atmosphere back to the telescope) were operating at the same wavelength, λ_0, now, in the Raman case, the emission path operates at λ_0 while the return path operates at λ_R. This translates into a two-way path Raman transmittance

$$T(\lambda_0, z)T_R(\lambda_R, z); \quad T(\lambda_i, z) = \exp\left[-\int_0^z \alpha_{\lambda_i}^{mol}(\xi) + \alpha_{\lambda_i}^{aer}(\xi)\right]$$

instead of the well-known two-way path elastic transmittance,

$$T(\lambda_0, z)^2$$

In addition, since the Raman receiver is specifically tuned to receive backscattered radiation from N_2 atmospheric molecules at the Raman-N_2 shifted wavelength (λ_R=607.4 nm), the Raman backscatter coefficient is computed as,

$$\beta_{\lambda_R}(z) = N_R(z)\frac{d\sigma_{\lambda_R}(\pi)}{d\Omega}$$

where $N_{\lambda_R}(z)$ is the nitrogen molecule number density at λ_R, and $d\sigma_{\lambda_R}(\pi)/d\Omega$ is the range-independent nitrogen Raman backscatter cross-section per solid angle unit (see also Chap.2).

Thus, the Raman lidar equation takes the form

$$P_{\lambda_R}(z) = \frac{K_{\lambda_R}}{z^2}\left[N_R(z)\frac{d\sigma_{\lambda_R}(\pi)}{d\Omega}\right] \times \exp\left\{-\int_0^z \left[\alpha_{\lambda_0}^{mol}(\xi) + \alpha_{\lambda_0}^{aer}(\xi) + \alpha_{\lambda_R}^{mol}(\xi) + \alpha_{\lambda_R}^{aer}(\xi)\right] d\xi\right\}\xi(z)$$

where all the variables have already been defined either here or in Chap.3.

In the proposed problem, $N_{\lambda_R}(z), \alpha_{\lambda_i}^{mol}(z), \alpha_{\lambda_i}^{aer}(z)$ magnitudes have been approximated from a US-standard atmosphere model. Note, all these magnitudes progressively decrease with height in the troposphere.

Computation of the Raman-channel range-dependent signal-to-noise ratio is completely analogous to the link-budged relations presented in Chap.4 when Analog Detection is used. Yet, in this problem (as well as in practice), Photon Counting Detection is used. Please refer to Sect. 3.8.2, pp. 14-15 of Hamamatsu's Application Note and use Eq.(2).