

Thm 1

$$\begin{array}{c} \bullet M_i \\ \bullet m_i \\ t_{i-1} \quad t_i \end{array}$$

Thm 2

$$\begin{array}{l} M_i = \sup_{x_i, y_i \in [t_{i-1}, t_i]} f([t_{i-1}, t_i]) = f(x_i) \\ m_i = \inf_{x_i, y_i \in [t_{i-1}, t_i]} f([t_{i-1}, t_i]) = f(y_i) \end{array} \Bigg) \leadsto \text{Extreme Value Thm}$$

$$\begin{array}{c} \bullet M_i \\ \bullet m_i \end{array}$$

since $x_i, y_i \in [t_{i-1}, t_i] \Rightarrow |x_i - y_i| < \delta$

(same argument holds for $x = a$)

f is cont. here

$$\rightarrow \begin{cases} M = \sup f([a, b]) \\ m = \inf f([a, b]) \\ M_n = \sup f([c, b]) \\ m_n = \inf f([c, b]) \end{cases}$$

Thm 3

Thm 4

