Index

- HW4 Ex1

  Why $\sup g([a,x_1]) - \inf g([a,x_1]) \leq M - m$? graph ?

- Ex Suppose $f : [0,1] \to \mathbb{R}$ is integrable, $|f|$ is bounded

  Prove that $\int_{1/n}^{1} f\,dx \to \int_0^1 f\,dx$ as $n \to \infty$

  Fix $\varepsilon > 0$, $\sup |f| = M$

- Ex: $f : (-1,1) \to \mathbb{R}$ diff at each $x \in (-1,1) \Rightarrow f' : (-1,1) \to \mathbb{R}$

- Ex. 7.25

<u>Thms</u>: Mean Value Thm

Darboux's Thm

Rolle's Thm

* <u>Thm</u>: If $f : [a,b] \to \mathbb{R}$ attains a max/min at c, then $f'(c) = 0$

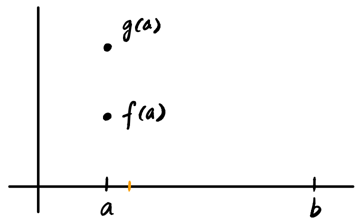
HW4 Ex 1 (a) $f:[a,b]\to\mathbb{R}$ integrable ← bounded

$g:[a,b]\to\mathbb{R}$: $g(x)=f(x)$ everywhere on $[a,b]$ except $x_1, x_2, \cdots, x_p \in [a,b]$ ← bounded

$\Rightarrow$ g is integrable   bounded

(Prove by induction) $\Rightarrow$ $M = \sup g$ $m = \inf g$

Case 1: $x_1 \in [a,b]$, $x_1 = a$ or $x_1 = b$



Recall:

Thm:

$g:[a,b]\longrightarrow\mathbb{R}$ is integrable

$\Longleftrightarrow$ for any $\varepsilon>0$, there is a partition $P=\{x_0, x_1, \cdots, x_n\}$ s.t. $U(g,P)-L(g,P)<\varepsilon$

Fix $\varepsilon > 0$.

Pick an $x_1 \in (a,b)$ s.t.

$\boxed{x_1 - a < \frac{\varepsilon}{2(m-n)}}$

$[x_1, b]$ $f=g$

There is a partition $P' = \{x_1, \cdots, x_{n-1}, x_n\}$ of $[x_1, b]$ s.t.

$$\boxed{U(g,P') - L(g,P') < \frac{\varepsilon}{2}}$$

Take $P = \{a\} \cup P' = \{\overset{x_0}{\overset{\shortparallel}{a}}, x_1, x_2, \cdots, x_n\}$

$$U(g,P) - L(g,P)$$

$$= \sup g([a,x_1])(x_1-a) + U(g,P') - [\inf g([a,x_1])(x_1-a) + L(g,P')]$$

$$= \sup g([a,x_1])(x_1-a) - \inf g([a,x_1])(x_1-a) + [U(g,P') - L(g,P')]$$

$$= \underbrace{(\sup g([a,x_1]) - \underbrace{\inf g([a,x_1])}_{\text{only over } [a,x_1] \Rightarrow < m})}_{\le M-m}(x_1-a) + \underbrace{(U(g,P') - L(g,P'))}_{<\frac{\varepsilon}{2}}$$

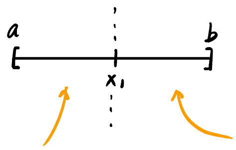
$$< (M-m)(x_1-a) + \frac{\varepsilon}{2}$$

$$< \cancel{(M-m)}\frac{\varepsilon}{2\cancel{(M-m)}} + \frac{\varepsilon}{2} = \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon$$

<u>General Case</u>: $f \neq g$ at $x_1, x_2, \cdots, x_p \in [a,b]$

g_1 g_2 g_3 $\cdots$ $g_p = g$

x_1 x_1, x_2 x_1, x_2, x_3 $x_1, x_2, \cdots, x_p$



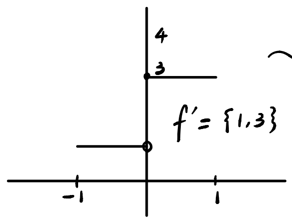
g int. over $[a, x_1]$ g int. over $[x_1, b]$

$\Rightarrow$ g is integrable

g is integrable

$$\int_a^b g\,dx = \int_a^b g\,dx$$

Ex: $f:(-1,1) \longrightarrow \mathbb{R}$ diff at each $x \in (-1,1) \Rightarrow f':(-1,1) \longrightarrow \mathbb{R}$



$\rightsquigarrow$ No!

Darboux Thm

If $f:[a,b] \longrightarrow \mathbb{R}$ is diff

then $f'([a,b])$ is also an interval