

Corollary: If $f: [a, b] \rightarrow \mathbb{R}$ is continuous, then f has an antiderivative

① ②

substitution: $x = g(y)$

$$\textcircled{1} \quad \int_{g(c)}^{g(d)} f(x) dx = F(g(d)) - F(g(c))$$

$$\textcircled{2} \quad \text{Take } \frac{d}{dy} (F(g(y))) \xrightarrow{\text{chain rule}} = F'(g(y)) \cdot g'(y) = f(g(y)) \cdot g'(y)$$

$$\hookrightarrow \int_c^d \boxed{f(g(y)) g'(y)} dy = F(g(d)) - F(g(c)) = \textcircled{1} \quad \square$$

\hookrightarrow integrable?

① $f, g: [a, b] \rightarrow \mathbb{R}$ are integrable $\Rightarrow f \cdot g: [a, b] \rightarrow \mathbb{R}$ is integrable

② $f: [a, b] \rightarrow \mathbb{R}$ is integrable & $h: [c, d] \rightarrow \mathbb{R}$ is cont.
 $\Rightarrow h \circ f$ is integrable

