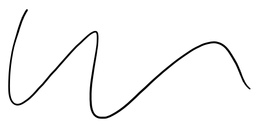
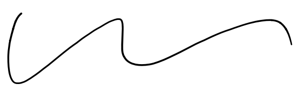
Ex 4: $f : I \xrightarrow{\text{diff}} \mathbb{R}$ diff $\quad f(x) \neq 0 \quad \forall x \in I$

$\hookrightarrow$ 1–1

for any seq





$$\frac{f^{-1}(\overset{= f(x_n)}{y_n}) - f^{-1}(\overset{= f(c)}{d})}{y_n - d} = \frac{x_n - c}{f(x_n) - f(c)} = \frac{1}{\frac{f(x_n) - f(c)}{x_n - c}} = \frac{1}{f'}$$

$a_n \to a$

$b_n \to b$

$b_n \neq 0$, $b \neq 0$

$\frac{a_n}{b_n} = \frac{a}{b}$

$f : (a,b) \longrightarrow \mathbb{R}$ bounded

diff on (a,b)

$\boxed{\lim_{x \to a} f(x)}$ $\searrow$

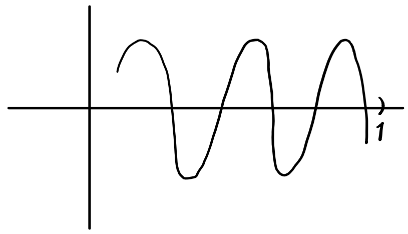
Don't exist

$\lim_{x \to b} f(x)$ $\nearrow$

$\Longrightarrow \forall x \in \mathbb{R}, \exists c \in (a,b)$ with $f'(c) = \alpha$

<u>Ex</u>: $f : (0,1) \longrightarrow \mathbb{R}$

$f(x) = \sin(\frac{1}{x})$



Goal: Prove that for any $R > 0$, we can find $x_1, x_2 \in (a,b)$ s.t.

$f(x_1) > 0$, $f'(x_2) < 0$, $|f'(x_1)|, |f'(x_2)| > R$

$x_1 < x_2$ <u>Dorboux's Thm</u>:

For any $\alpha \in (-R, R)$, there is a $c \in (x_1, x_2)$ with $f(c) = \alpha$