Week 6 - Programming Assignment [Optional]

Practice Quiz, 4 questions

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1.

Your goal in this project is to break RSA when the public modulus N is generated incorrectly. This should serve as yet another reminder not to implement crypto primitives yourself.

Normally, the primes that comprise an RSA modulus are generated *independently* of one another. But suppose a developer decides to generate the first prime p by choosing a random number R and scanning for a prime close by. The second prime q is generated by scanning for some other random prime also close to R.

We show that the resulting RSA modulus $N = pq$ can be easily factored.

Suppose you are given a composite N and are told that N is a product of two relatively close primes p and q, namely p and q satisfy

$|p - q| < 2N^{1/4}$ (*)

Your goal is to factor N.

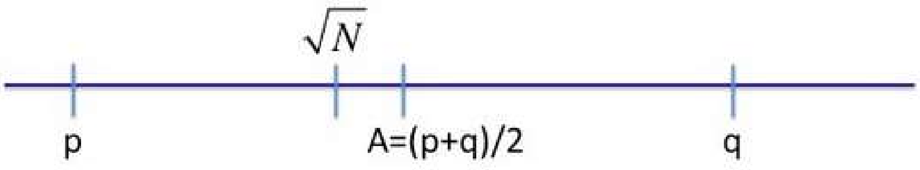
Let A be the arithmetic average of the two primes, that is $A = \frac{p+q}{2}$. Since p and q are odd, we know that $p + q$ is even and therefore A is an integer.

To factor N you first observe that under condition (*) the quantity $\sqrt{N}$ is very close to A. In particular, we show below that:

$A - \sqrt{N} < 1$

But since A is an integer, rounding $\sqrt{N}$ up to the closest integer reveals the value of A. In code, $A = \mathrm{ceil}(\mathrm{sqrt}(N))$ where "ceil" is the ceiling function.

Visually, the numbers $p, q, \sqrt{N}$ and A are ordered as follows:



Since A is the exact mid-point between p and q there is an integer x such that $p = A - x$ and $q = A + x$.

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But then $N = pq = (A - x)(A + x) = A^2 - x^2$ and therefore $x = \sqrt{A^2 - N}$.

Now, given x and A you can find the factors p and q of N since $p = A - x$ and $q = A + x$. You have now factored N !

Further reading: the method described above is a greatly simplified version of a much more general result on factoring when the high order bits of the prime factor are known.

In the following challenges, you will factor the given moduli using the method outlined above. To solve this assignment it is best to use an environment that supports multi-precision arithmetic and square roots. In Python you could use the gmpy2 module. In C you can use GMP.

Factoring challenge #1:

The following modulus N is a products of two primes p and q where $|p - q| < 2N^{1/4}$. Find the smaller of the two factors and enter it as a decimal integer in the box below.

```
N = 179769313486231590772930519078902473361797697894230657273430081157 \
773267580550562068698537944921298295958550138753716401571013985864 \
78337786069255834975410851965916151280575759407526350074759352887 \
710823649949940771895617054361149474865046711015101563940680527540 \
07158456087857766374304008634074285527854909258
```

For completeness, let us see why $A - \sqrt{N} < 1$. This follows from the following simple calculation.

First observe that $A^2 - N = \left(\frac{p+q}{2}\right)^2 - N = \frac{p^2+2N+q^2}{4} - N = \frac{p^2-2N+q^2}{4} = (p-q)^2/4$.

Now, since for all $x, y :\ \ (x - y)(x + y) = x^2 - y^2$ we obtain $A - \sqrt{N} = (A - \sqrt{N})\frac{A+\sqrt{N}}{A+\sqrt{N}} = \frac{A^2-N}{A+\sqrt{N}} = \frac{(p-q)^2/4}{A+\sqrt{N}}$.

Since $\sqrt{N} \leq A$ it follows that $A - \sqrt{N} \leq \frac{(p-q)^2/4}{2\sqrt{N}} = \frac{(p-q)^2}{8\sqrt{N}}$.

By assumption (*) we know that $(p-q)^2 < 4\sqrt{N}$ and therefore $A - \sqrt{N} \leq \frac{4\sqrt{N}}{8\sqrt{N}} = 1/2$ as required.

Enter the answer for factoring challenge #1 in the box below:

Enter answer here

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2.

Factoring challenge #2:

The following modulus N is a products of two primes p and q where $|p - q| < 2^{11} N^{1/4}$. Find the smaller of the two factors and enter it as a decimal integer.

Hint: in this case $A - \sqrt{N} < 2^{20}$ so try scanning for A from $\sqrt{N}$ upwards, until you succeed in factoring N.

```
N =648455842808071669662824265346772278726343720706976263060439070378 \
9730861808111646271401527606141756919558732184025452065542490671989 \
2428844841839353281972988531310511738648965962582821502504990264452 \
1008852816733037111422964210278402893076574586452336833570778346897 \
15838646088239640236866252211790085787877
```

Enter the answer for factoring challenge #2 in the box below:

Enter answer here

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3.

Factoring challenge #3:

The following modulus N is a product of two primes p and q where $|3p - 2q| < N^{1/4}$. Find the smaller of the two factors and enter it as a decimal integer.

Hint: first show that $\sqrt{6N}$ is close to $\frac{3p+2q}{2}$ and then adapt the method in challenge #1 to factor N.

```
N =72006226374735042527956443552558373833808445147399984182665305798191 \
63556901883377904234086641876639384851752649940178970835240791356868 \
77441155132015188279331812309091996246361896836573643119174094961348 \
52463970788523879939683923036467667022162701835329944324119217381272 \
9276147530748597302192751375739387929
```

Enter the answer for factoring challenge #3 in the box below:

Enter answer here

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4.

The challenge ciphertext provided below is the result of encrypting a short secret ASCII plaintext using the RSA modulus given in the first factorization challenge.

The encryption exponent used is $e = 65537$. The ASCII plaintext was encoded using PKCS v1.5 before the RSA function was applied, as described in PKCS.

Use the factorization you obtained for this RSA modulus to decrypt this challenge ciphertext and enter the resulting English plaintext in the box below. Recall that the factorization of N enables you to compute $\varphi(N)$ from which you can obtain the RSA decryption exponent.

```
Challenge ciphertext (as a decimal integer):
22096451867410381776306561134883418017410069787892831071731839143676135600120538
  004282329650473509424343946219751512256465839967942889460764542040581564748988
  013734864120452325229320176487916666402997509188729971690526083222067771600019
  329260870009579993724077458967773697817571267229951148662959627934791540
```

After you use the decryption exponent to decrypt the challenge ciphertext you will obtain a PKCS1 encoded plaintext. To undo the encoding it is best to write the decrypted value in hex. You will observe that the number starts with a '0x02' followed by many random non-zero digits. Look for the '0x00' separator and the digits following this separator are the ASCII letters of the plaintext.

(note: the separator used here is '0x00', not '0xFF' as stated in the lecture)

Enter answer here

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