ISFAHAN UNIVERSITY OF TECHNOLOGY
APPLIED LINEAR ALGEBRA

Fall 2022

Assignment 1

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1. Do the solutions to the following systems of equations yield a vector space? Why or why not? In case it is a vector space, find a basis for it.

   a)
   $$V = \left\{ (x, y) \in \mathbb{R}^2 : \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\},$$

   b)
   $$V = \left\{ (x_1, x_2, x_3) \in \mathbb{R}^3 : \begin{bmatrix} 3 & 2 & 5 \\ 6 & 1 & 1 \\ 3 & 4 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\},$$

   c)
   $$V = \left\{ (y_1, y_2, y_3) \in \mathbb{R}^3 : \exists (x_1, x_2, x_3, x_4) \in \mathbb{R}^4 s.t. \begin{bmatrix} 1 & 3 & 1 & 4 \\ 2 & 1 & 1 & 2 \\ 5 & 3 & 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \right\}.$$

2. Determine which values of "a" render the following set of vectors on $\mathbb{R}^3$ linearly dependent.

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \\ a \end{bmatrix} \right\}$$

3. In each case below determine whether $v \in Span(S)$.

a) In the vector space $\mathbb{R}^3[x]$ of polynomials of degree at most 3.

$$v = x^3 + x + \frac{3}{2}$$

$$S = \{(x^3 + x^2 + 1), (x^3 + 2x^2 - x + \frac{1}{2})\}$$

- b) In the vector space of continuous real functions

$$v = \cos(2x) \quad \text{and} \quad v = \cos x$$

$$S = \{\cos^2 x, \sin^2 x\}$$

4. Which of the following are subspaces of $\mathbf{R^3}$? Find a basis for the ones that are.

a) $V = \left\{ \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} : s.t.\ b_1 + b_2 - b_3 = 0 \text{ and } b_1 + 2b_2 + b_3 = 0 \right\}.$

b) $V = \left\{ \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} : s.t.\ b_1 \cdot b_2 - b_3 = 0 \right\}.$

5. True or false? provide an explanation for each case.

a) Of two parallel planes in $\mathbb{R}^3$ at most one can be a subspace of $\mathbb{R}^3$.

b) The set of vectors $\{v_1, v_2, v_3\}$ in $\mathbb{R}^3$ is linearly independent if and only if they intersect.

c) asymmetric real n by n matrices form a subspace.

d) If W_1 and W_2 are two 2-dimensional subspaces in $\mathbb{R}^4$, then $W_1 \cap W_2$ is a subspace of dimension at least one.

6. Diagonal real matrices are a subspace of 3 by 3 matrices. Find a basis for this subspace.

7. Let A be a matrix in $\mathbb{R}^{3\times3}$ and we define

$$W_A = \{M \in \mathbb{R}^{3\times3} \mid AM = MA\}.$$

a) Prove that W_A is a subspace of $\mathbb{R}^{3\times3}$.
In each of the following cases, find the dimension of W_A.

b)

$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix},$$

where a, b, c are distinct real numbers.

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