Audio QR Transmission*

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***Abstract*— We studied the audio QR transmission problem and investigated the feasibility of this technology by modeling a communication system based on phase shift-keying modulation scheme with 2 (BPSK) and 4 (QPSK) signal points in combination with error-correcting codes, pulse-shaping techniques, and matched filters. Assuming the time limit of 1 second for transmitting a single URL from the provided dataset, we propose a system that achieve a perfect performance at the signal-to-noise ratio $\frac{E_b}{N_0} = 0\,\text{dB}$ and 99% accuracy at $\frac{E_b}{N_0} = -1.5\,\text{dB}$ with an additive white Gaussian noise as the model of the communication channel. As additional results, we investigated the effectiveness of error-correcting codes and pulse-shaping techniques separately. The performance of the system increases by a significant margin when applied with an appropriate error-correcting code.**

I. INTRODUCTION

The audio QR transmission technology is a technology for transmitting messages using an audio signal. Similar to typical QR technology where messages are encoded in images, a sample, direct application of this technology would allow a person to get a URL of a website by simply using a short snippet of audio on their cell phone. The main purpose of this report is to study the feasibility of this audio QR technology using simulation on the provided dataset and propose a prototype communication system for this technology.

To simulate a system for audio QR transmission, we model a communication system consisting of: 1) transmitter for converting a sequence of binary messages to an audio waveform signal, 2) noise channel for modelling the acoustic noise during the transmission, and 3) receiver for recovering an audio waveform back to an original message of the transmission. In our study, we study the system with only an additive noise in the form of additive white Gaussian noise at different signal-to-noise ratios.

II. SYSTEM DESIGN

In designing the system, the following constraints are considered:

1) The audio signal is sampled at 44 100 Hz.
2) The noise channel is modeled by as an ideal bandpass filter with an additive white Gaussian noise with the cutoff frequency 200 Hz and 22 050 Hz.
3) The transmission of a single URL has to be within 1 second.

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In this section, we first study the baseline system of a communication system with BPSK and QPSK modulation scheme applied with only matched filter. Then, we study the effective of adding an error-correcting code and pulse-shaping for the proposed system. The outline of this section starts by explaining the mathematical model of the baseline system and follows by the design rationale of the proposed system.

A. Baseline System

For the baseline system, we model a communication system using BPSK/QPSK modulation scheme and using an additive white Gaussian noise as a channel model. Figure 1 illustrate the proposed baseline system.

During the preprocessing step, a sequence of ASCII characters, or a URL message, m is converted to a list of symbols by first converting a character into a binary number using ASCII table. Note that there are 2 symbols in the BPSK modulation scheme while there are 4 symbols in the QPSK modulation scheme. In the case of QPSK, every two bits are grouped together to create a single symbol.

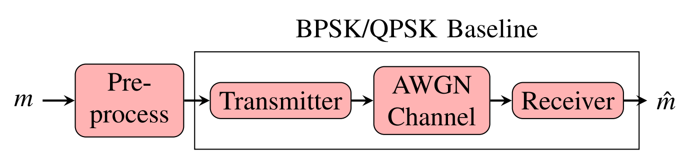

Fig. 1: Baseline system architecture

After preprocessing step, the sequence of symbols to be transmitted is converted to a sequence of waveforms using the following equations for BPSK modulation scheme:

$$\begin{aligned} s_1(t) &= \sqrt{\frac{2}{T}}\cos(2\pi f_c t) \\ s_2(t) &= \sqrt{\frac{2}{T}}\cos(2\pi f_c t + \pi) \end{aligned} \tag{1}$$

and the following equations for QPSK modulation scheme:

$$\begin{aligned} s_1(t) &= \frac{2}{\sqrt{T}}\cos\left(2\pi f_c t - \frac{\pi}{4}\right) \\ s_2(t) &= \frac{2}{\sqrt{T}}\cos\left(2\pi f_c t - \frac{3\pi}{4}\right) \\ s_3(t) &= \frac{2}{\sqrt{T}}\cos\left(2\pi f_c t + \frac{\pi}{4}\right) \\ s_4(t) &= \frac{2}{\sqrt{T}}\cos\left(2\pi f_c t + \frac{3\pi}{4}\right) \end{aligned} \tag{2}$$

where $T = \frac{10}{44100} = 2.267 \times 10^{-4}\,\mathrm{s}$, and $f_c = \frac{1}{T} = 4410\,\mathrm{Hz}$. Note that we used 10 samples to represent a single waveform representing one symbol.

The transmitted signal is then added with white Gaussian noise baseband filtered with the cutoff frequency 200 Hz and 22 050 Hz. Then signal is then filtered with the matched filter using the following impulse responses for the BPSK modulation scheme:

$$\phi(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_c t) \tag{3}$$

and the following impulse responses for the QPSK modulation scheme:

$$\begin{aligned} \phi_1(t) &= \sqrt{\frac{2}{T}} \cos(2\pi f_c t) \\ \phi_2(t) &= \sqrt{\frac{2}{T}} \sin(2\pi f_c t) \end{aligned} \tag{4}$$

The filtered signals are then sampled at every T seconds, and the sampled instances are used to reconstruct the original binary sequence using maximum likelihood decision rule:

$$\arg\max_i \|r - s_i\| \tag{5}$$

where r is the value at sampling instance, s_i is the point in the BPSK/QPSK constellation schemes. The binary sequence is converted back to the original message of URL by using an ASCII lookup table.

B. Proposed System

Motivated by the baseline system, we proposed an improved system shown in Figure 2 that utilizes an error-correcting code (ECC) to allow error correction and a pulse-shaping (PS) filter to eliminate inter-symbol interference. Observing that the bandwidth of QPSK is twice as much as the bandwidth of BPSK, as the QPSK system transmits 2 bits per symbol, whereas the BPSK system transmits only 1 bit per symbol. The same theoretical bound of bit error rate of $Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$ where $\frac{E_b}{N_0}$ is the signal-to-noise ratio motivates the QPSK-based system to send as many bits as possible to take advantages of an error-correcting code.

The error-correcting codes (N, K) where K represents one block of data and N, usually $N > K$, represents one block of code used to represent $K-$bit data is the type of encoding that allows forward-error correction, or correct some of erroneous bits without re-transmission of the message. In this report, we study different kind of error-correcting codes with different ratio of $\frac{N}{K}$ to analyze the effectiveness of using an error-correcting code in the scope of audio QR transmission problem.

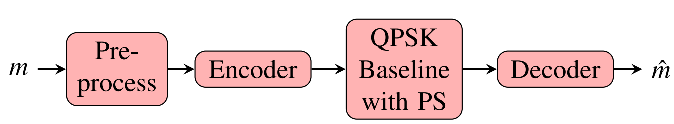

Fig. 2: Proposed architecture

Our proposed system starts off by preprocessing a URL into a sequence of bits using an ASCII table. The bits are divided into groups of K-bit data and converted into a higher dimensional space of N-bit code. Note that $N - K$ bits are then used to enhance the performance of forward-error correction. The groups of N-bit code then transmitted using the QPSK baseline system modified with the pulse-shaping filter to eliminate inter-symbol interference using the transmitter and receiver illustrated in Figure 3. After transmissions, the groups of N-bit code are converted back to groups of K-bit data and later converted back to ASCII characters using the ASCII table similarly to the preprocessing step.

To add the pulse-shaping filter, we modify the transmitter and receiver according to Figure 3. In the transmitter, the sequence of bits are demultiplexed into two sequences of 1 and -1 with half the length of original sequence and then filtered with the root-raised cosine (RRC) filter. Two sequences of filtered signals are modulated at the same carrier frequency as the baseline system's of $f_c = 4410\,\mathrm{Hz}$ using two orthogonal basis functions and added for transmission. We also normalized the signal so that the average energy per bit of the transmitted signal is one. In the receiver, the signal is demodulated and filtered with the matched filter that is the same as the root-raised cosine filter. The filtered signals are sampled and mapped back to 4 possible 2-bit symbols.

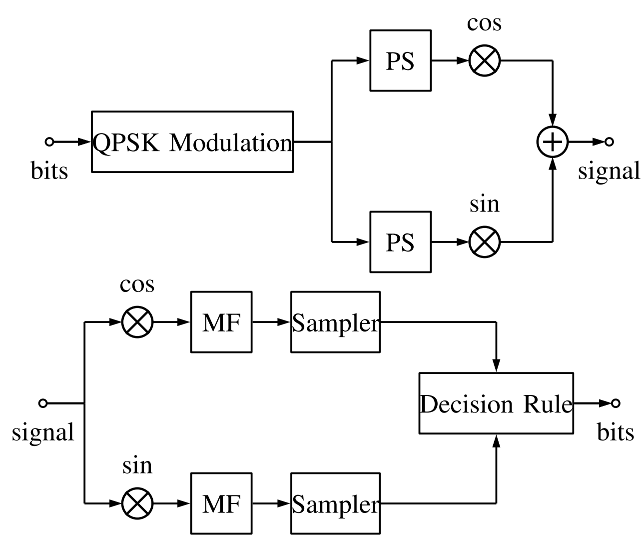

Fig. 3: Top: Modified transmitter with pulse-shaping (PS); Bottom: Modified receiver.

Let T_s represent an amount of time required to transmit one symbol and X_i be the random variable of the length of an i-th URL drawn from the same distribution as the provided dataset. Assume that the sample mean is an unbiased estimate of the population mean:

$$\overline{X} = \frac{1}{N} \sum_{i=1}^{N} X_i. \tag{6}$$

Using the provided dataset, we find that the empirical value of $\overline{X}$ is 29.6, so, based on the 7-bit ASCII table, the average length in bits is $\overline{Y} = 7\overline{X} = 207.2$ bits. Using an (N, K) error-correcting code, the average number of bits to be transmitted is therefore given by

$$M = \binom{N}{K} \overline{Y} \tag{7}$$

Given that the theoretical bound on the maximum number of symbols that can be transmitted within 1 second is

$$N_s = \frac{1}{T_s}, \tag{8}$$

we propose the system that maximizes the ratio of $\frac{N}{K}$ while keeping $M < 0.5N_s$. Using $N_s = 4410\,\mathrm{Hz}$, we have an approximate bound of $N \leq 10K$. Based on our experiment, we choose an error-correcting code $(N,K) = (127,15)$. Note that the maximum length of a single URL in the provided dataset is at most 1.4 times the average length. Hence, we have proposed a system that can transmit a message within 0.5 seconds average and at most 0.7 seconds at the worst case on the provided dataset.

III. Results and Discussion

In this section, we discuss the performance of the proposed system based on two following metrics:

1) Percentage of bits/symbols that are correctly transmitted as a function of the signal-to-noise ratio
2) Percentage of URLs that are correctly transmitted as a function of the the signal-to-noise ratio.

We analyze the effect of using the bandwidth gain from BPSK to QPSK to transmit more bits to enhance the effect of forward-error correction. We then further analyze the effect of different error-correcting codes and the effect of pulse-shaping filter.

Due to the nature of the proposed system that uses about $0.5\,\mathrm{s}$ on average per URL, we limit the number of samples on all analyses in this section to be only the first 10 000 URLs from the provided dataset on different signal-to-noise ratio $\frac{E_b}{N_0}$ from $-10\,\mathrm{dB}$ to $10\,\mathrm{dB}$ at the step of $0.5\,\mathrm{dB}$.

A. Effects of the number of symbols in the modulation scheme

According to Equation 7, the number of transmitted bits is proportional to the ratio $\frac{N}{K}$. We experimented on the proposed system without the pulse-shaping filter on BPSK and QPSK modulation schemes using the error-correcting codes $(N_{\mathrm{BPSK}}, K_{\mathrm{BPSK}})$ and $(N_{\mathrm{QPSK}}, K_{\mathrm{QPSK}})$ respectively such that the ratio $\left(\frac{N_{\mathrm{BPSK}}}{K\mathrm{BPSK}}\right) \approx \frac{1}{2}\left(\frac{N_{\mathrm{QPSK}}}{K\mathrm{QPSK}}\right)$ because QPSK-based system has twice bandwidth compared to the BPSK-based system while maintaining the same bit-error rate at the same signal-to-noise ratio.

Figure 5 demonstrates the result of this experiment. We observe that the performance of the QPSK-based system and the BPSK-based system is relatively similar with the same configuration of the error-correcting code $(127,15)$. Taking into account the amount of bandwidth of QPSK and BPSK systems, we observe that the QPSK-based system is able to transmit more correct URLs at higher noise level. This analysis suggests that high bandwidth of the system is useful when combined with high ratio of $\frac{N}{K}$ error-correcting code (N,K).

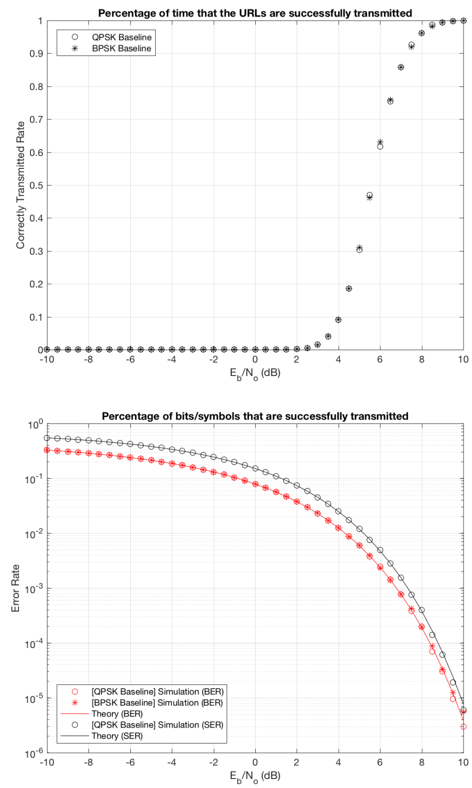

Fig. 4: Baseline system's performance

B. Effects of the rate of error-correcting codes

We experimented on the effectiveness of the ratio $\frac{N}{K}$ from the Equation 7 affects the performance of the system by running simulations on the QPSK-based system with different type of error-correcting codes. Figure 6 and Table I show the results of this experiment, and Figure 1 shows the performance of the baseline systems. We observe that the performance of the system increases significantly when the ratio $\frac{N}{K}$ increases. The system with the error-correcting code $(127,15)$ achieves perfect performance at $\frac{E_b}{N_0} = 0\,\mathrm{dB}$. All systems with the error-correcting codes appear to outperform the baseline system with a significant margin.

System	**Minimum SNR to achieve performance**				
	90%	**95%**	**99%**	**99.9%**	**100%**
Baseline	7.5 dB	8.0 dB	9.0 dB	10.0 dB	N/A
ECC(127,15)	−2.0 dB	−2.0 dB	−1.5 dB	−0.5 dB	0.0 dB
ECC(127,29)	−1.0 dB	−0.5 dB	0.0 dB	0.5 dB	1.0 dB
ECC(127,43)	1.0 dB	1.0 dB	1.5 dB	2.0 dB	3.0 dB
ECC(127,57)	1.5 dB	2.0 dB	2.5 dB	3.0 dB	4.0 dB
ECC(127,71)	2.0 dB	2.5 dB	3.0 dB	3.5 dB	4.0 dB
ECC(127,85)	3.0 dB	3.5 dB	4.0 dB	4.5 dB	5.0 dB
ECC(127,99)	4.0 dB	4.5 dB	5.0 dB	5.5 dB	6.0 dB
ECC(127,113)	5.0 dB	5.5 dB	6.0 dB	7.0 dB	7.5 dB

TABLE I: Minimum SNR required to attain certain performance of different systems

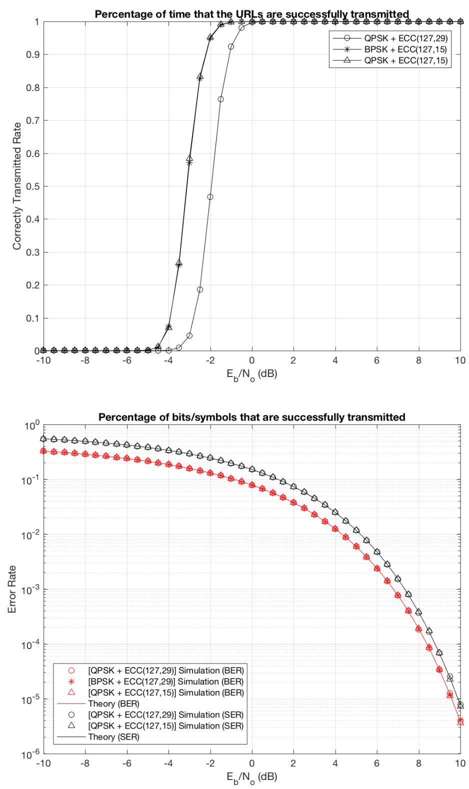

Fig. 5: Comparison between BPSK-based systems and QPSK-based systems

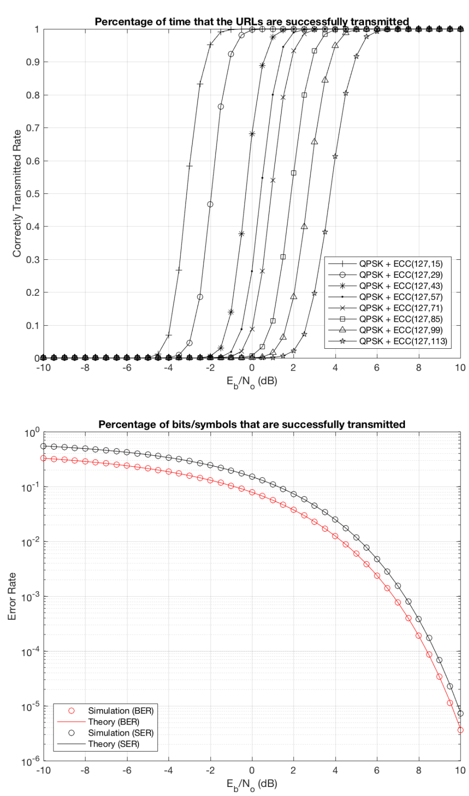

Fig. 6: Comparison among systems with different error-correcting codes (ECC)

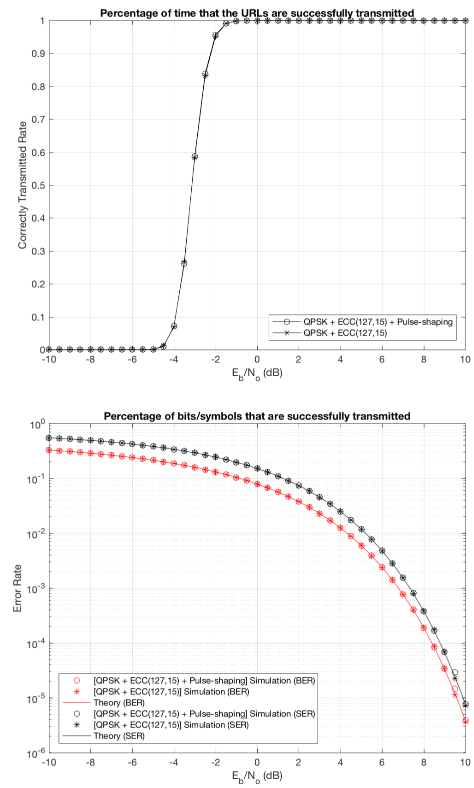

Fig. 7: Comparison among systems with and without pulse-shaping

C. *Effects of pulse-shaping filter*

In this experiment, we measure the performance of the proposed system with and without the pulse-shaping filter. Figure 7 shows the result of this experiment. We observe that the amount of URLs transmitted correctly is almost equal in both systems. The result suggests that is relatively slight in terms of error rate. However, the pulse-shaping filter is useful for transmitting signal in a band-limited channel.

IV. CONCLUSIONS AND RECOMMENDATIONS

In this report, we investigate the feasibility of audio QR technology. Specifically, we study the communication system using BPSK/QPSK modulation scheme in addition to using an error-correcting code. Our analysis shows the significant margin of improvement to the simple baseline communication with QPSK modulation scheme with the significant margin by adding an error-correcting code. Furthermore, our analysis also shows a slight improvement to the system when using pulse-shaping technique.

In conclusion, the results presented in this paper supports the feasibility of the audio QR technology and proposes a solution for transmitting messages using an audio signal. For our recommendations, we recommend a communication system that has an error-correcting code with high number of parity-check bits.

REFERENCES

[1] B. Lathi and Z. Ding, *Modern Digital and Analog Communication Systems*. Oxford Series in Electrical an, Oxford University Press, 2009.