1a $R \sim B(100, 0.2)$

$P_r(R=k) = \binom{100}{k} 0.2^k 0.8^{100-k}$

Explanation: R follows Binomial distribution with parameters $n = 100$ and $p = 0.2$

1b $E(R) = 100 \times 0.2 = 20$

$SD(R) = 100 \times 0.2 \times 0.8 = 16$

1c True.

$SD(W) = 100 \times 0.8 \times 0.2 = 16$.

Explanation: $SD(R) = SD(-R) = SD(100-R) = SD(W)$

1d $E(S) = 4E(R) - E(W)$

$= 4 \times 20 - 80$

$= 0$

1e $SD(S) = 4\,SD(R) + SD(W)$

$= 5 \times 16$

$= 80$

2a $E(V) = E(X_{sn}^2 + Y_{sn}^2 + 2X_{sn}Y_{sn})$

$= 2 + 2E(X_{sn}Y_{sn})$

$= 2 + 2r(X,Y)$

≥ 0

$\therefore r(X,Y) \geq -1$

2b $E(W) = E(X_{sn}^2 + Y_{sn}^2 - 2X_{sn}Y_{sn})$

$= 2 - 2E(X_{sn}Y_{sn})$

$= 2 - 2r(X,Y)$

≥ 0

$\therefore r(X,Y) \leq 1$

3. H_0 : modification did nothing

H_1 : modification made robots faster

$X \sim B(12, 0.5)$

$P(X=9) = \binom{12}{9} 0.5^{12} = \frac{220}{2^{12}} = 0.0537 > 0.05$

$\therefore$ Reject H_0. Accept H_1

Conclusion: Modification made robots faster.

4a. $Var(Z) = p(1-p) = -(p-\frac{1}{2})^2 + \frac{1}{4} \leq \frac{1}{4}$, "=" if and only if $p = \frac{1}{2}$.

4b. $P(X \in \left[\frac{a\sqrt{np(1-p)} + np}{n}, \frac{b\sqrt{np(1-p)} + np}{n}\right]) = \int_a^b \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$

modified according to the way the question asks:

$P(X \in p \pm a\sqrt{\frac{p(1-p)}{n}}) = \int_{-a}^{a} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \approx 0.95$

$\Rightarrow \int_{-\infty}^{a} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \approx 0.975$

$\Rightarrow a \approx 1.96$

$\therefore$ Blank should be filled with '$1.96\sqrt{\frac{p(1-p)}{n}}$'

4c. $1.96\sqrt{\frac{p(1-p)}{n}} \overset{ba}{\leq} 0.98\sqrt{\frac{1}{n}}$

$\therefore P(X \in p \pm 0.98\sqrt{\frac{1}{n}}) \geq 0.95$.

$\therefore$ Blank should be filled with '$0.98\sqrt{\frac{1}{n}}$'

4d. According to 4c. $w = 0.98\sqrt{\frac{1}{10000}} = 0.0098$