AMAT 503 – Assignment 4 - Lamoureux

Due Thursday, Arpil 5, 2018

1. a) Find the coefficients for a convolutional filter whose frequency response (or Fourier transform) is

$$H_3(\omega) = e^{\pi i \omega} \cos^3(\pi\omega)$$

and also for the filter with response

$$H_5(\omega) = e^{\pi i \omega} \cos^5(\pi\omega).$$

b) Based on these answers, make an educated guess as to what the filter coefficients are for this response

$$H_N(\omega) = e^{\pi i \omega} \cos^N(\pi\omega),$$

where N is odd. (Hint: think Pascal's triangle.)

Note that these filters are often used in the construction of biorthogonal wavelets.

2. Suppose $\phi(t)$ is the piecewise linear spline function defined as

$$\phi(t) = \begin{cases} t & 0 \le t \le 1 \\ 2 - t, & 1 < t < 2 \\ 0, & \text{otherwise.} \end{cases}$$

Sketch this function, and verify that it can be used as a scaling function by finding constants h_0, h_1, h_2 so that

$$\phi(t) = h_0 \cdot \phi(2t) + h_1 \cdot \phi(2t-1) + h_2 \cdot \phi(2t-2).$$

Please sketch the three functions $\phi(2t), \phi(2t-1), \phi(2t-2)$ as well.

3. No doubt you have seen trig identities like

$$\cos(2\omega) = 2\cos^2(\omega) - 1.$$

In this exercise I would like you to show that every function $\cos(k\omega)$ can be written as a sum of powers $\cos^j(\omega)$.
a) Use complex exponentials to show

$$(\cos(\omega) + i\sin(\omega))^k = \cos(k\omega) + i\sin(k\omega).$$

b) Use the binomial theorem to expand the left hand side of equation a).
c) Equate real and imaginary parts, from the binomial sum arising from the left hand side, and the result on the right hand side of a).
d) Do something to get rid of any sines still lying around.
e) State the final result, that $\cos(k\omega)$ is a sum of powers of $\cos(\omega)$.

4. Show me something interesting that you can do with the continuous wavelet transform (in Python or Matlab). For instance, you might try one of these:

- take a short piece of music and use the CWT to identify individual notes, or instruments;
- take some pure tones of a given frequency and see what parameters in the CWT domain correspond to those particular frequencies
- take a complex sound (e.g. a baby crying while a steam kettle is making noise) and see if the CWT can separate these two events in the resulting image;
- anything you might find interesting, that involves a real signal and the CWT.

You might need to play around with different choices of wavelet choices, and different scaling parameters, to get something really useful.