Assign2

February 8, 2018

1 Assignment 2.

AMAT 503 – M. Lamoureux
Posted Feb 7. Due Thursday, Feb 15.

1.1 Q1.

Find a convolutional filter for sampled signals with (normalized) frequency response

$$H(\omega) = \cos^2(\omega\pi).$$

That is, look for coefficients $(h_0, h_1, h_2, h_2, \ldots)$ so that its corresponding frequency response gives the $\cos^2$ response. Try to do this with as few non-zero coefficients as possible.

Is this a lowpass or highpass filter? Or neither?

1.2 Q2.

Suppose that h1 and h2 are lowpass filters and g1 and g2 are highpass filters.

- Show that h = h1 * h2 is a lowpass filter.
- Show that g = g1 * g2 is a highpass filter.
- Is h1 * g1 lowpass, highpass, or neither?

1.3 Q3.

Consider the filter h with only 3 non-zero coefficients $(h_0, h_1, h_2) = (1/5, 3/5, 1/5)$.

- Show that Fourier transform has absolute value $|H(\omega)| =$ a trig function plus a constant. (Be specific.)
- Show that we can write the Fourier transform $H(\omega)$ as $H(\omega) = e^{2\pi i\omega}|H(\omega)|$.
- Can you find another filter g with coefficients such that $G(\omega) = |H(\omega)|$?

(This g is called a zero-phase filter, as it has no complex phase factor.)

1.4 Q4.

Let $v = [2, 6, -4, 2, 400, 402, -8, -6]'$ and use the Haar transform (Algorithm 6.1 in the text, or use the 8x8 matrix from class) to compute the HWT of v. There is a large jump in the values of v from v4 to v5 and from v6 to v7. Is this reflected in the difference block of the transformed data?

1.5 Q5.

Suppose that v in a vector with N entries, N even. Let y denote the Haar wavelet transform of v (i.e., $y = W_N v$, where W_N is the N-dimensional wavelet transform matrix, given by Eqn (6.7) in the text). Show that

- if v is a constant vector (i. e., all $v_k = a$ where a is any real number), then the components of the highpass portion of y are zero.
- if v is a linear vector (i. e., $v_k = ak + b$ for real numbers a and b), then the components of the highpass portion of y are constant. Find this constant value.
- if v is a quadratic vector (i. e., $v_k = ak^2 + bk + c$), then the components of the highpass portion of y form a linear vector. Find this linear vector.

1.6 Q6.

Let $v = [1, 2, 3, 4, 5, 6, 7, 8]'$. Do 3 iterations of the Haar wavelet transform, showing your work at each stage. (i.e. work out by hand. You might want to use the $\frac{1}{2}$ as your normalizing factor, so you avoid square roots in teh answer.) Be very explicit about what your output vectors are at each iteration.

Is it possible to do a 4th iteration?

1.7 Q7.

Compute by hand three iterations of the inverse HWT on the vector y from Question No. 6 above. Show your work at each stage.

Verify that your result is the original vector v from Question 6, showing this was an inverse.

1.8 Q8.

The Z transform for the Haar filters are $H(Z) = (1 + Z)/2, G(Z) = (1 - Z)/2$ which have roots at $-1, +1$ respectively.

Find the roots for the Z transform of the Daubechies filters with 6 non-zero coefficents, lowpass $H(Z) = h_0 + h_1 Z + h_2 Z + \cdots h_5 Z^5$ and highpass $G(Z) = g_0 + g_1 Z + g_2 Z + \cdots g_5 Z^5$. You likely will have to do this numerically. (In Python wavetlet package, the Daubechies filter with 6 coeffcients is called db3.)

Can you see any relationship between those roots?

Verify your answer by checking with the Daubechies filter with 8 coefficients. (In Python wavelets, this is called db4.)

1.9 Q9.

a) Given two n-th order polynomials

$$H(Z) = h_0 + h_1 Z + h_2 Z + \cdots h_n Z^n,$$

$$G(Z) = g_0 + g_1 Z + g_2 Z + \cdots g_n Z^n,$$

find a relationship between their roots, under the assumption that the coefficients of G are the same as H, except in reverse order. That is,

$$g_k = h_{n-k}, \text{ for } 0 \leq k \leq n.$$

b) Same question, except assume the coefficients of G also have alternating signs from H:

$$g_k = (-1)^k h_{n-k}, \text{ for } 0 \leq k \leq n.$$

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