UNIVERSITY OF CALGARY
FACULTY OF SCIENCE
DEPARTMENT OF MATHEMATICS AND STATISTICS
MIDTERM EXAMINATION
AMAT 503, L 01 – Fall 2013

DATE: 22 October 2013 **Time: 1 hour**

STUDENT ID NUMBER

EXAMINATION RULES

1. This is closed book examination.
2. A non-graphing calculator is allowed. No other aids are permitted.
3. Please answer on the sheets attached.
4. The use of personal electronic or communication devices is prohibited.
5. A University of Calgary Student ID card is required to write the Final Examination and could be requested for midterm examinations. If adequate ID isn't present the student must complete an Identification Form.
6. Students late in arriving will not be permitted after one-half hour of the examination time has passed.
7. No student will be permitted to leave the examination room during the first 30 minutes, nor during the last 15 minutes of the examination. Students must stop writing and hand in their exam immediately when time expires.
8. All inquiries and requests must be addressed to the exam supervisor.
9. Students are strictly cautioned against:
   a. communicating to other students;
   b. leaving answer papers exposed to view;
   c. attempting to read other students' examination papers
10. During the final examination, if a student becomes ill or receives word of domestic affliction, the student must report to the Invigilator, hand in the unfinished paper and request that it be cancelled. If ill, the student must report immediately to a physician/counselor for a medical note to support a deferred examination application.
11. Once the examination has been handed in for marking, a student cannot request that the examination be cancelled. Retroactive withdrawals from the course will be denied.
12. Failure to comply with these regulations will result in rejection of the examination paper.

Question	Total Marks	Actual Marks
1	4	
2	4	
2A	2 (bonus)	
3	4	
4	4	
5	4	
6	4	
Total	24+2	

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Questions 1-6 have equal weight. Closed book, no notes. A non-graphing calculator is permitted.

1. Sketch the following curve in the complex plane:

$$\theta \mapsto \theta e^{2\pi i\theta}, \qquad \text{for } \theta \text{ in the range } 0 \leq \theta \leq 2.$$

2. Find non-zero real numbers a, b, c so the three vectors $\begin{matrix}(a, & b, & c)\\ (b, & c, & a)\\ (c, & a, & b)\end{matrix}$ form an orthonormal set.

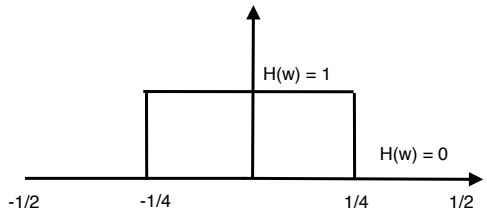
2A: For 2 bonus points, describe ALL such numbers a, b, c. (Hint: think geometrically)

3. Find the inverse of the 4 by 4 matrix $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 2 & 3 & 0 & 1 \\ 4 & 5 & 1 & 0 \end{bmatrix}$ Hint: consider block matrices.

4. Compute the 2D Haar transform of following array, and sketch it and its transform as grayscale images:

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 6 & 7 & 8 & 9 & 10 \\ 6 & 7 & 8 & 9 & 10 & 11 \end{bmatrix}.$$

5. Compute the Fourier coefficients $h_k, k \in \mathbf{Z}$ for an ideal low pass filter, which has frequency response $H(\omega)$ as in this diagram:



6. Find a four-term convolutional filter with coefficients (h_0, h_1, h_2, h_3) with frequency response $H(\omega)$ satisfying

$$\begin{aligned} H(0) &= 1 \\ H'(0) &= 0 \\ H'(1/2) &= 0. \end{aligned}$$

If you can, plot the frequency response $|H(\omega|$.