MultiFEBE: an open-source multi-domain integrated finite element and boundary element software

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SIANI

Congress on Numerical Methods in Engineering
CMN 2022
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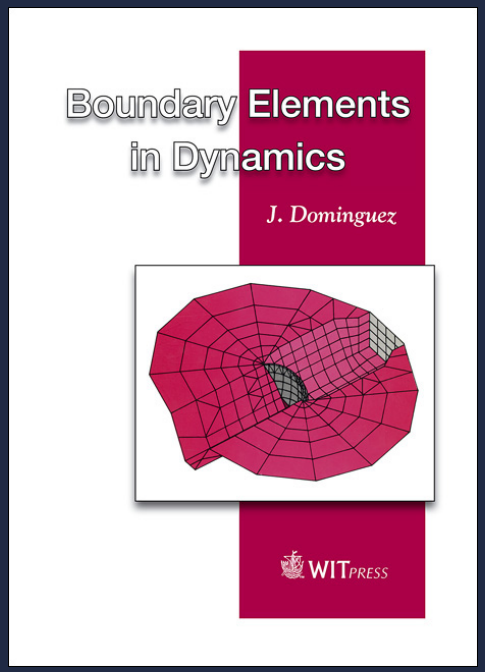
Outline

Outline

A little bit of history: origins

J. Domínguez, Boundary Elements in Dynamics, 1993.



- Time harmonic 2D anti-plane elastod.: `CONDAPEH`, `QUADAPEH`
- Time harmonic 2D plane elastod.: `CONDPLEH`, `QUADPLEH`
- Time harmonic 3D scalar wave prop.: `CONDTHSH`, `QUADTHSH`
- Time harmonic 3D elastod.: `CONDTHEH`, `QUADTHEH`
- and others ...

A little bit of history: in-house codes (dams)

3D BEM–BEM multi-domain multi-material models:

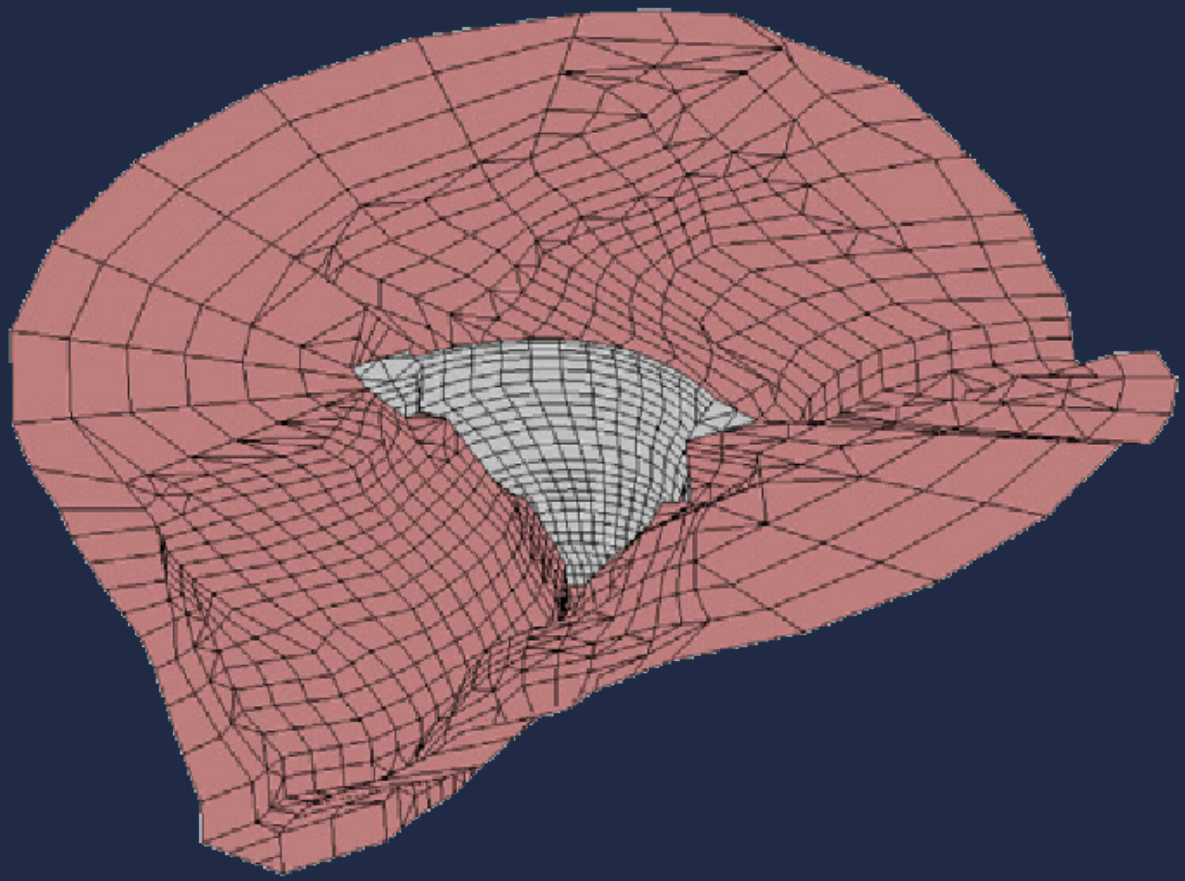


Figure: J.C. Galván et al., Eng Anal Bound Elem 144, 2022.

A little bit of history: in-house codes (cracks)

2D BEM (singular & hypersingular) models for crack analysis:

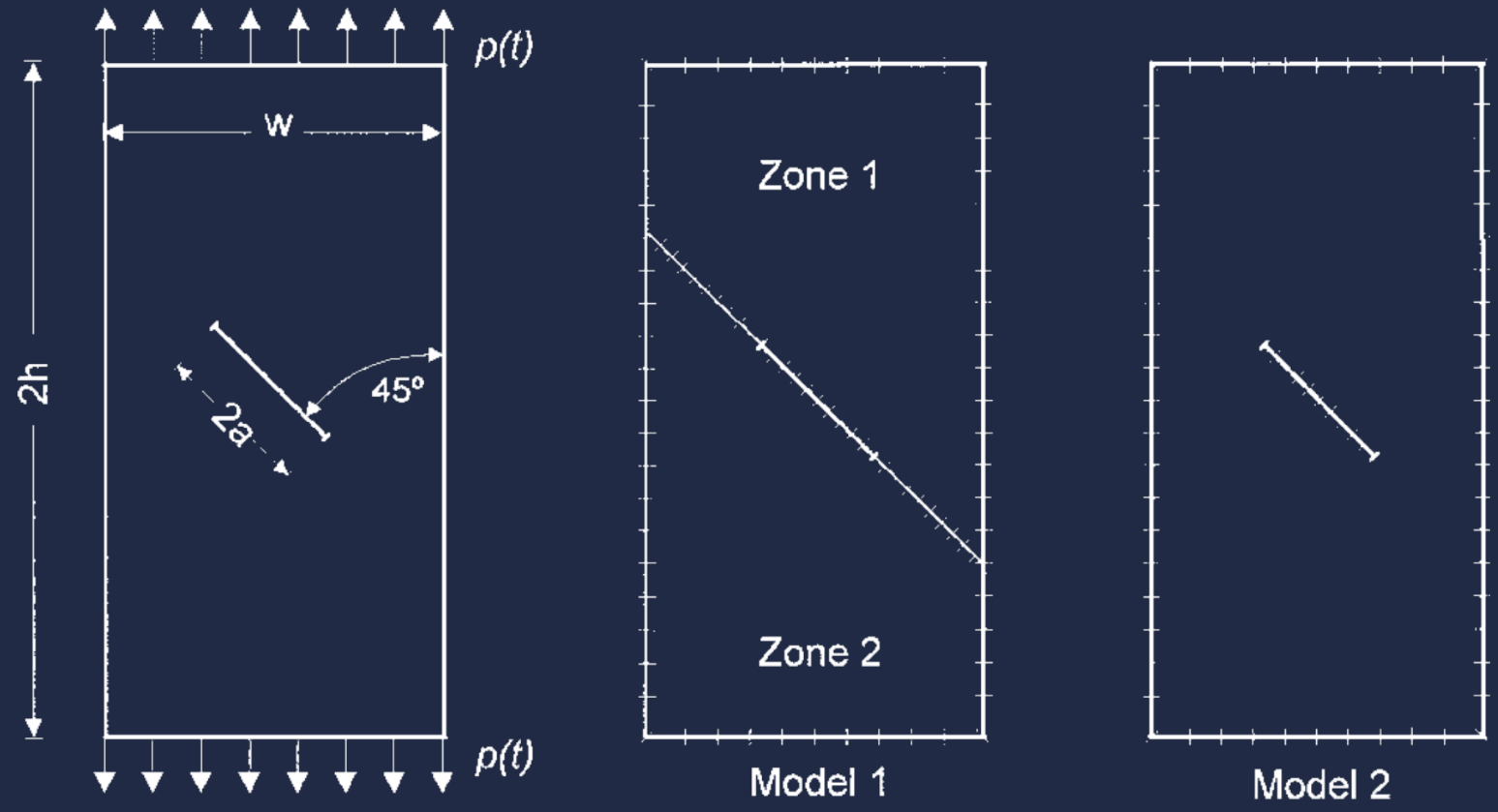


Figure: F. Chirino & R. Abascal, IJNME 43, 1998.

A little bit of history: in-house codes (acoustics)

2D BEM (singular & hypersingular) models for acoustics:

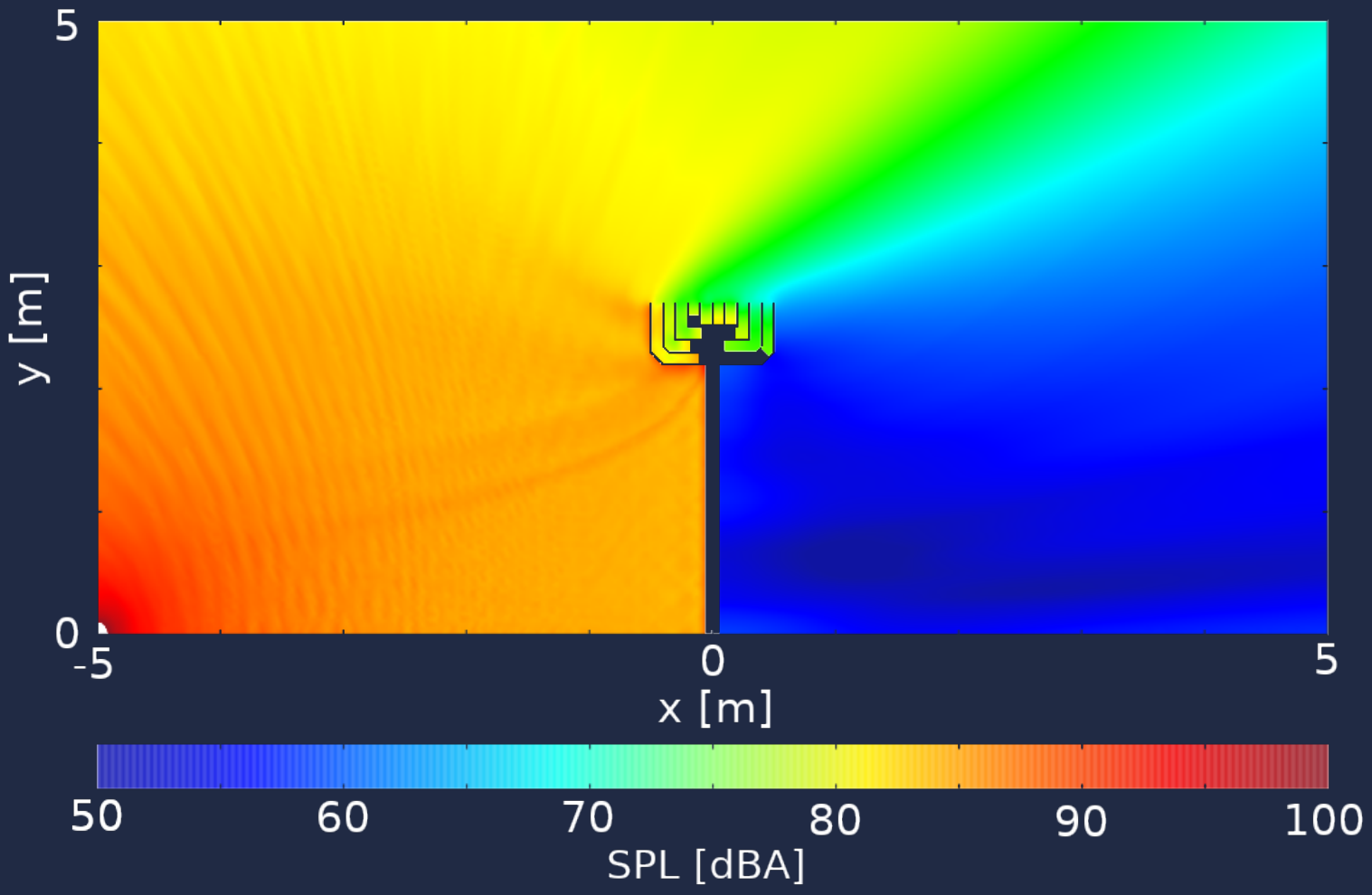


Figure: R. Toledo et al., Eng Anal Bound Elem 63, 2016.

A little bit of history: in-house codes (SSI)

3D BEM–FEM models for analysis of pile and pile groups:

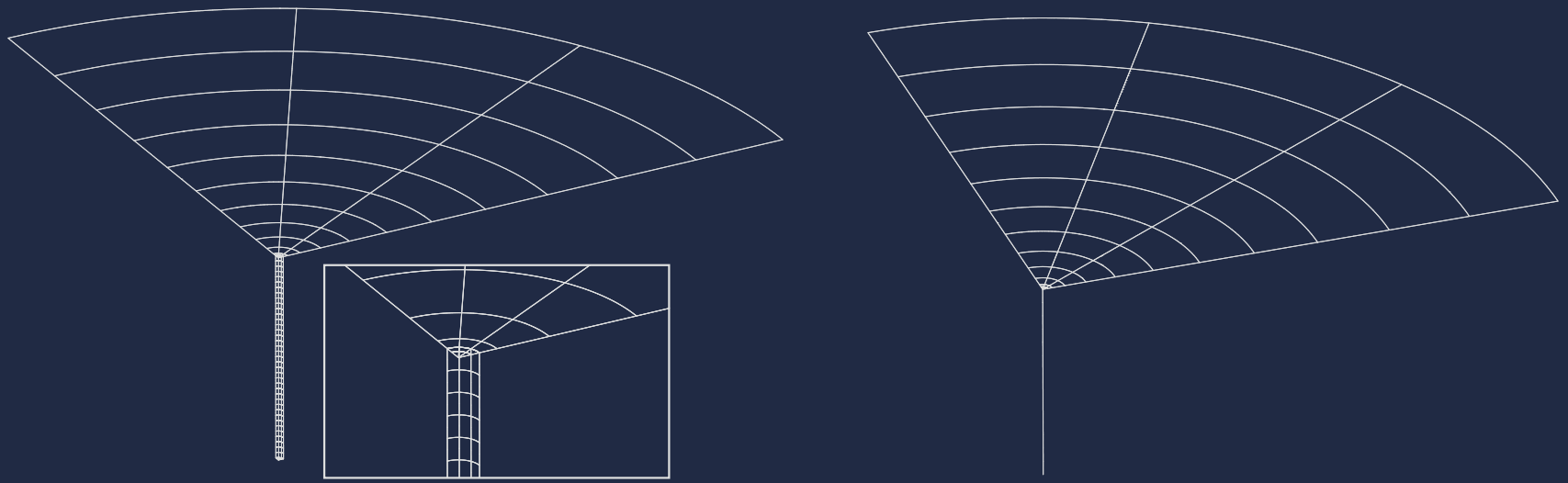


Figure: L.A. Padrón et al., Eng Anal Bound Elem 31, 2007.

A little bit of history: in-house codes (SSI)

2D/3D BEM-FEM models for buried shell structures:

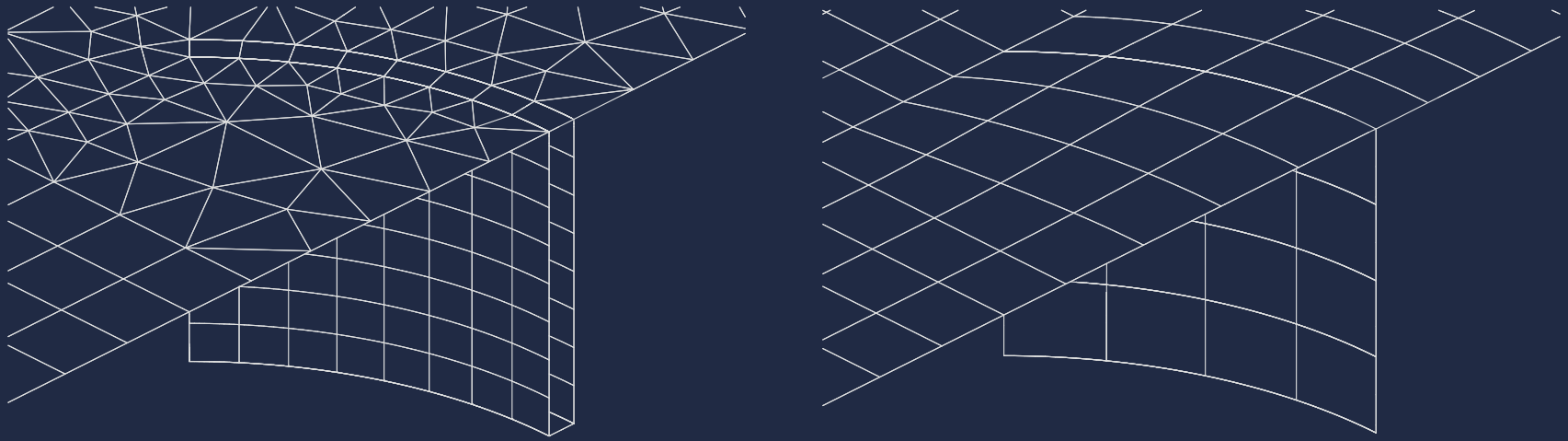
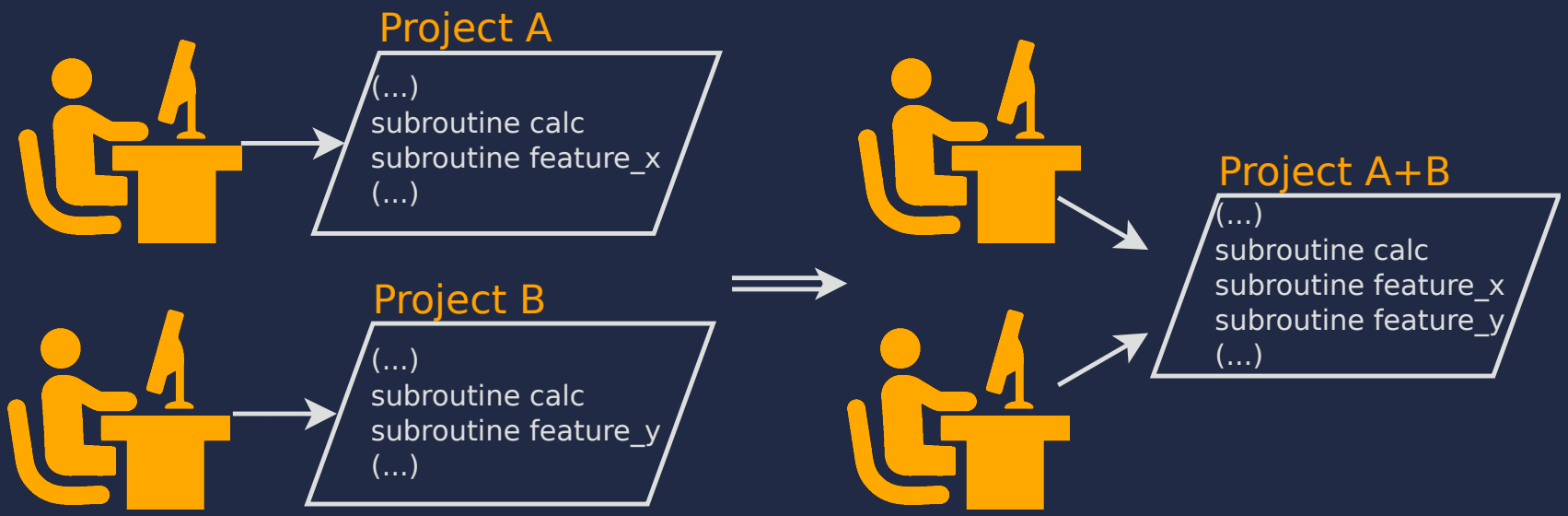


Figure: J.D.R. Bordón et al., Comput Mech 60, 2017.

Why developing MultiFEBE?

- ✗ Repeated work for maintaining several codes
- ✗ Features scattered in different codes
- ✓ Combine features and methodologies
- ✓ Share core algorithms



Why developing MultiFEBE?

Existing projects (not open source):

- BEASY: corrosion, cathodic protection, crack analysis
- BEASTS: elastodynamics, FEM via ANSYS
- ...

Existing projects (open source):

- Domínguez's book codes[1]
- Beer's book codes[2]: elasticity, limited BEM-FEM
- OpenBEM: acoustics
- Bempp: acoustics & EM
- ...

[1] J. Domínguez, Boundary Elements in Dynamics, WIT Press, 1993.

[2] G. Beer et al., The Boundary Element Method with Programming, Springer-Verlag, 2008.

Why developing MultiFEBE?

No previous software combining:

- BEM for acoustics, elastostatics, elastodynamics, poroelastodynamics
- Ordinary and crack boundary elements
- Multi-domain multi-material coupling
- Mixed-dimensional BEM-FEM coupling

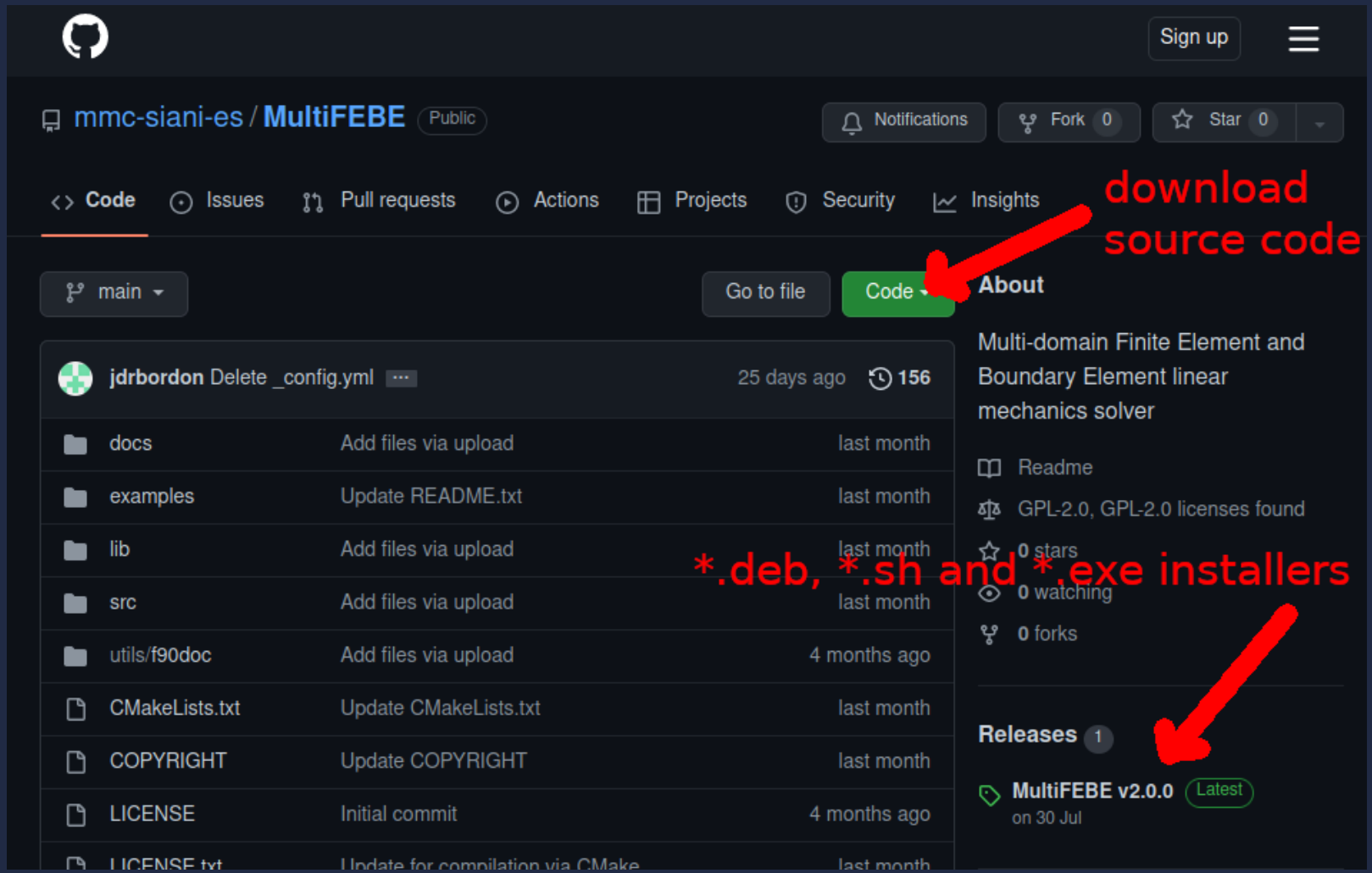
License

GNU GPLv2

Can	Cannot	Must
▶ Commercial Use	▶ Sublicense	▶ Include original
▶ Modify	▶ Hold Liable	▶ Disclose source
▶ Distribute		▶ Include Copyright
▶ Place Warranty		▶ State Changes
		▶ Include License

Repository at GitHub

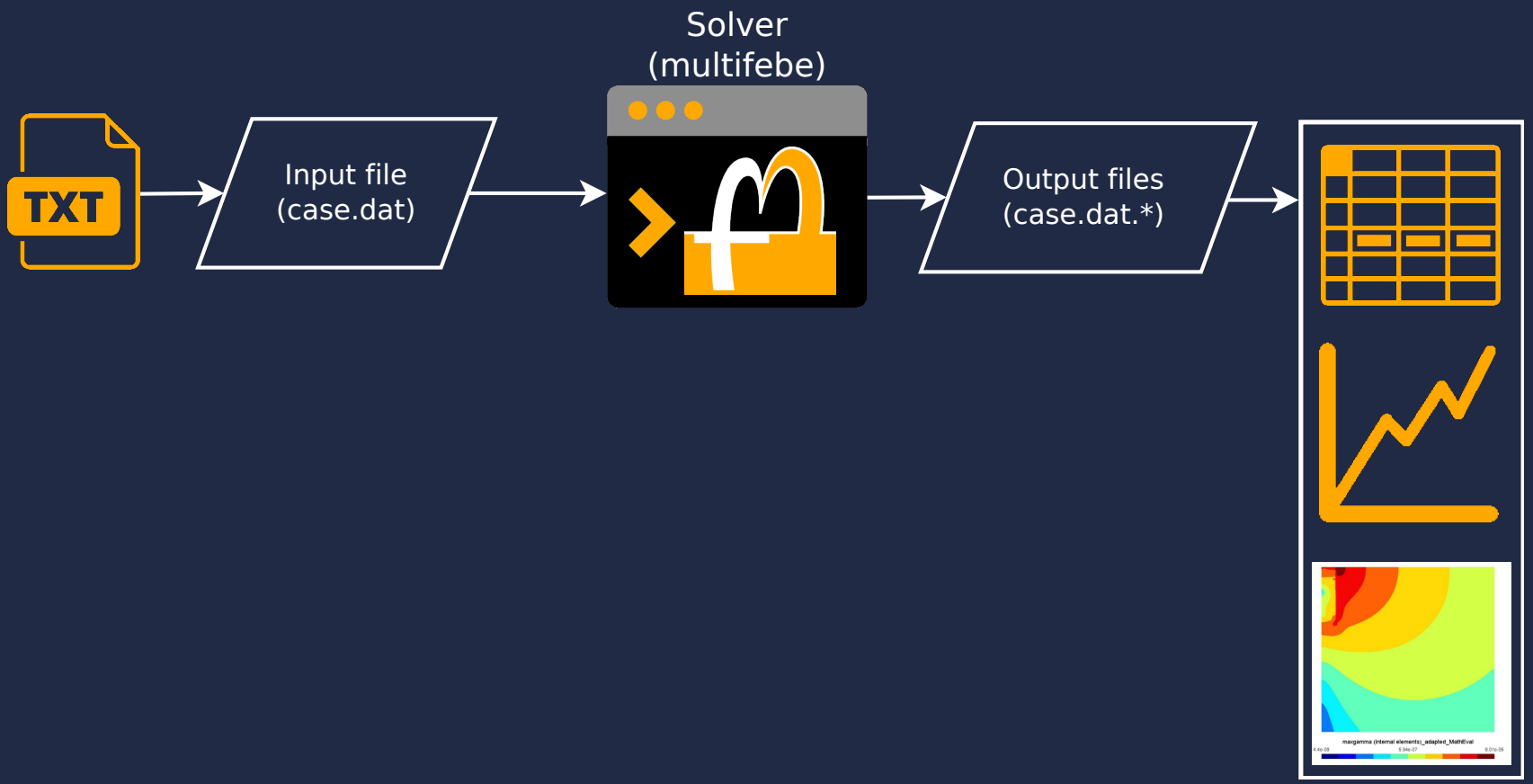
https://github.com/mmc-siani-es/MultiFEBE



Outline

Design choices: USER perspective

MultiFEBE is a command-line solver:



Design choices: USER perspective

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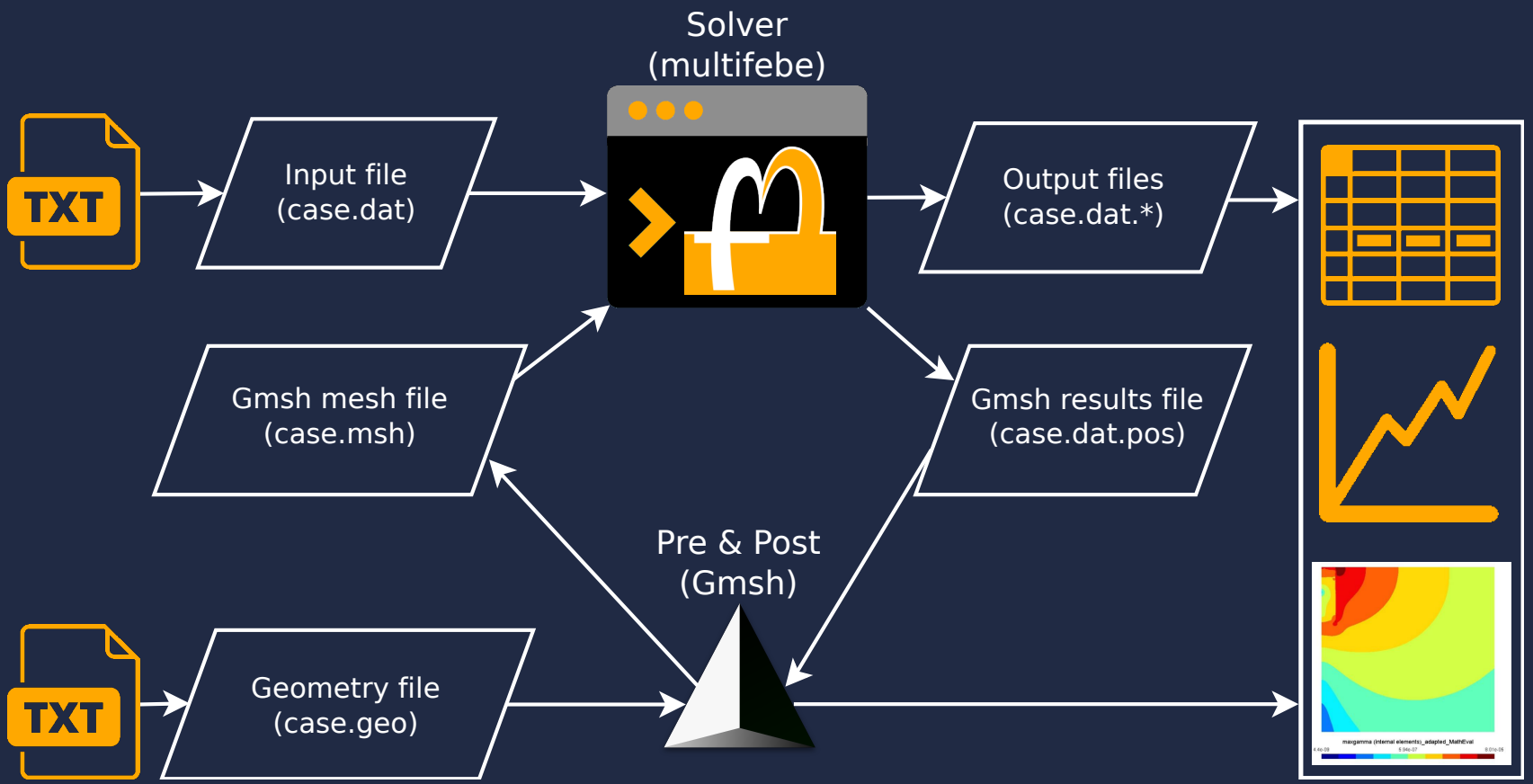


Design choices: DEVELOPER perspective

Programming languages:

- Fortran 2003 (use PP & some OOP)

```
22 program multifebe
23
24   use iso_fortran_env
25   use fbem_string_handling
26   use fbem_data_structures
27   use fbem_shape_functions
28   use problem_variables
29
30   implicit none
31   integer :: kf
32
33   call date_and_time(timestamp_date_start,timestamp_time_start)
34
35   call process_command_line_options
36
```

Design choices: DEVELOPER perspective

Programming languages:

- Fortran 2003 (use PP & some OOP)
- Fortran preprocessor (FPP)

```
4238          x1(1)=x1_i(1)+rho*costheta
4239          xi(2)=xi_i(2)+rho*sintheta
4240          ! XI->X transformation
4241          ! Geometrical shape functions and first derivatives at xi
4242 #        define etype type_g
4243 #        define delta 0.0d0
4244 #        define phi phi_g
4245 #        define dphidxi1 dphidxi1_g
4246 #        define dphidxi2 dphidxi2_g
4247 #        include <phi_and_dphidxik_2d.rc>
4248 #        undef etype
4249 #        undef delta
4250 #        undef phi
4251 #        undef dphidxi1
4252 #        undef dphidxi2
```

directives

Design choices: DEVELOPER perspective

Programming languages:

- Fortran 2003 (use PP & some OOP)
- Fortran preprocessor (FPP)
- OpenMP (shared-memory parallelism)

```
270  ! REGION BOUNDARIES
271  do kb_int=1,region(kr)%n_boundaries
272    sb_int=region(kr)%boundary(kb_int)
273    sb_int_reversion=region(kr)%boundary_reversion(kb_int)
274    sp_int=boundary(sb_int)%part
275    !$omp parallel do schedule (dynamic) default (shared) private (se_int,se_int_n_nodes
276    do ke_int=1,part(sp_int)%n_elements
277      se_int=part(sp_int)%element(ke_int)
278      se_int_n_nodes=element(se_int)%n_nodes
279      call build_lse_mechanics_bem_harela_element(omega,kr,sb_int,sb_int_reversion,se_ir
280    end do
281    !$omp end parallel do
282  end do
283  ! REGION BODY LOADS
284  do kb_int=1,region(kr)%n_be_bodyloads
```

Design choices: DEVELOPER perspective

Libraries:

- LAPACK (from OpenBLAS or ATLAS)

```
171  if (refine) then
172    allocate (bcopy(n_dof,n_rhs))
173    bcopy=b
174  end if
175  trans='N'
176  call zgetrs(trans,n_dof,n_rhs,A,n_dof,ipiv,b,n_dof,info)
177  if (info.eq.0) then
178    if (verbose_level.ge.2) then
179      call fbem_timestamp_w_message(output_unit,2,'Linear system of equations solved')
180    end if
181  else
182    call fbem_timestamp_message(output_unit,2)
183    write(output_unit,'(a13,i11)') 'zgetrs info: ', info
184    call fbem_error_message(error_unit,0,__FILE__,__LINE__,'solver has failed')
185  end if
```

Design choices: DEVELOPER perspective

Tools:

- 
  Compiler: GNU Fortran
- 
  Build/packaging automation tool: CMake
- 
  GNU/Linux envir. in Windows: MSYS2

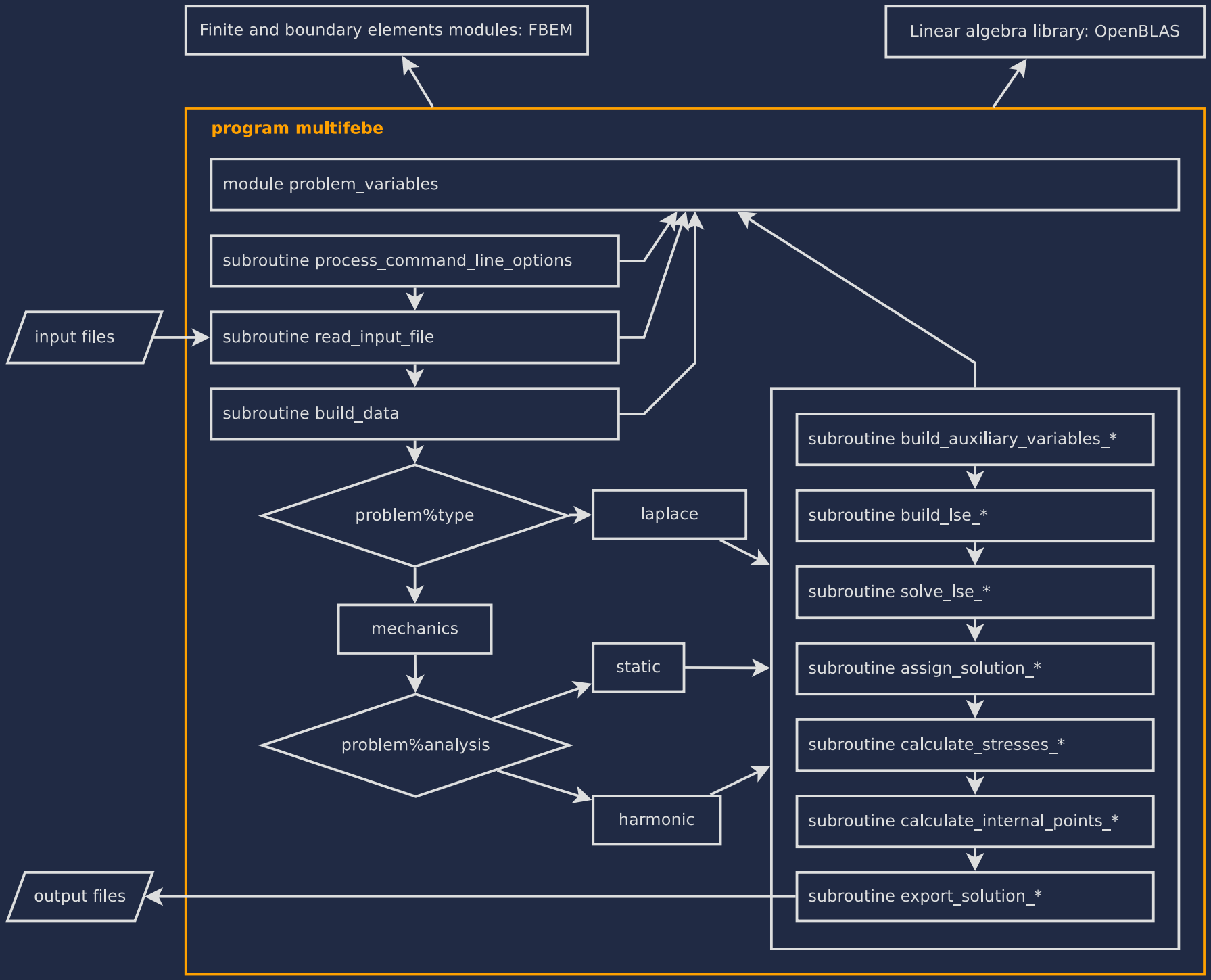
Operating systems (64-bits):

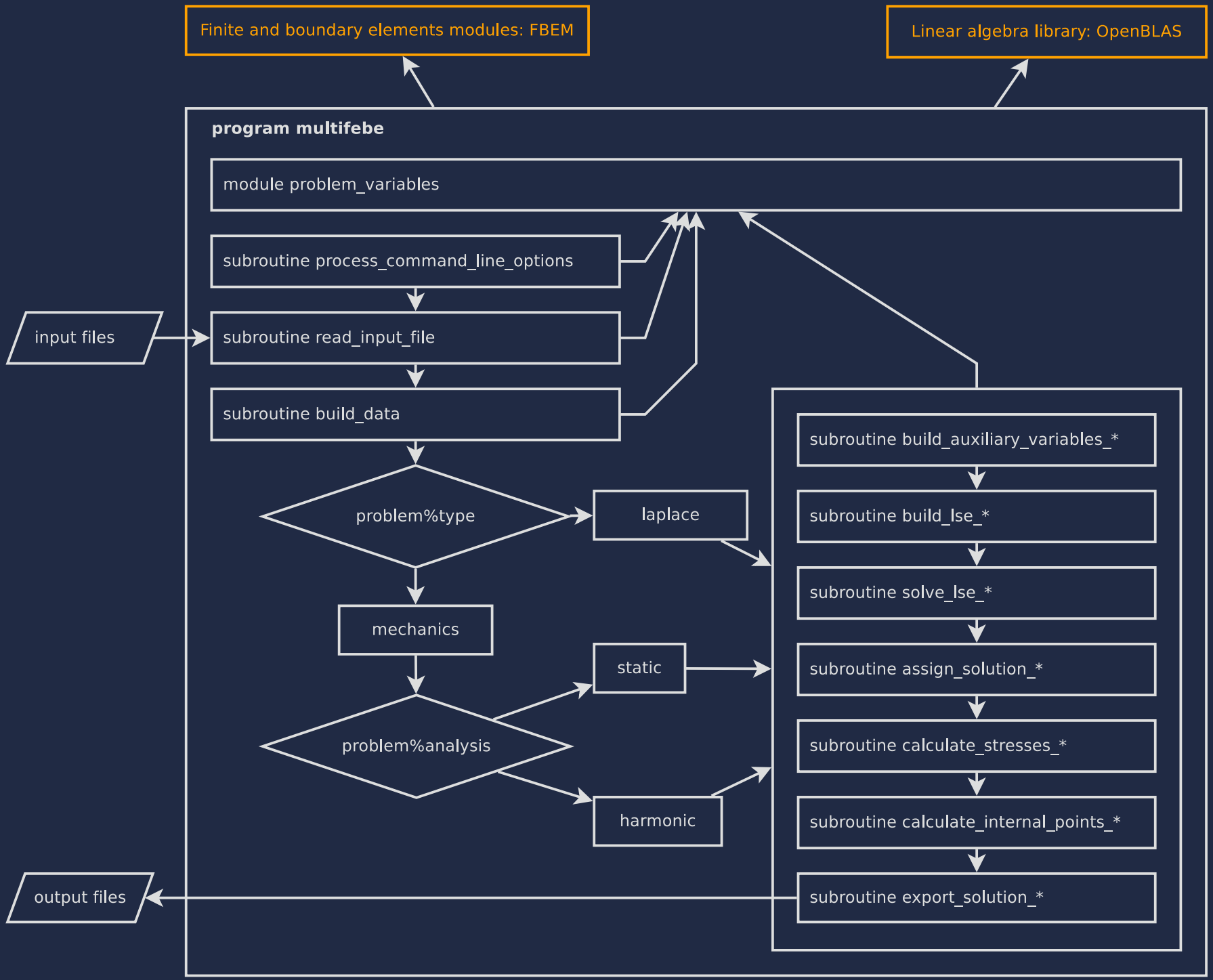
- 
  GNU/Linux
- 
  Windows

Design choices: DEVELOPER perspective

Arquitecture – a two-layer design:

- Application layer
- Computational layer


Finite and boundary elements modules: FBEM
Linear algebra library: OpenBLAS
program multifebe
module problem_variables
subroutine process_command_line_options
input files
subroutine read_input_file
subroutine build_data
problem%type
laplace
mechanics
static
problem%analysis
harmonic
output files
subroutine build_auxiliary_variables_*
subroutine build_lse_*
subroutine solve_lse_*
subroutine assign_solution_*
subroutine calculate_stresses_*
subroutine calculate_internal_points_*
subroutine export_solution_*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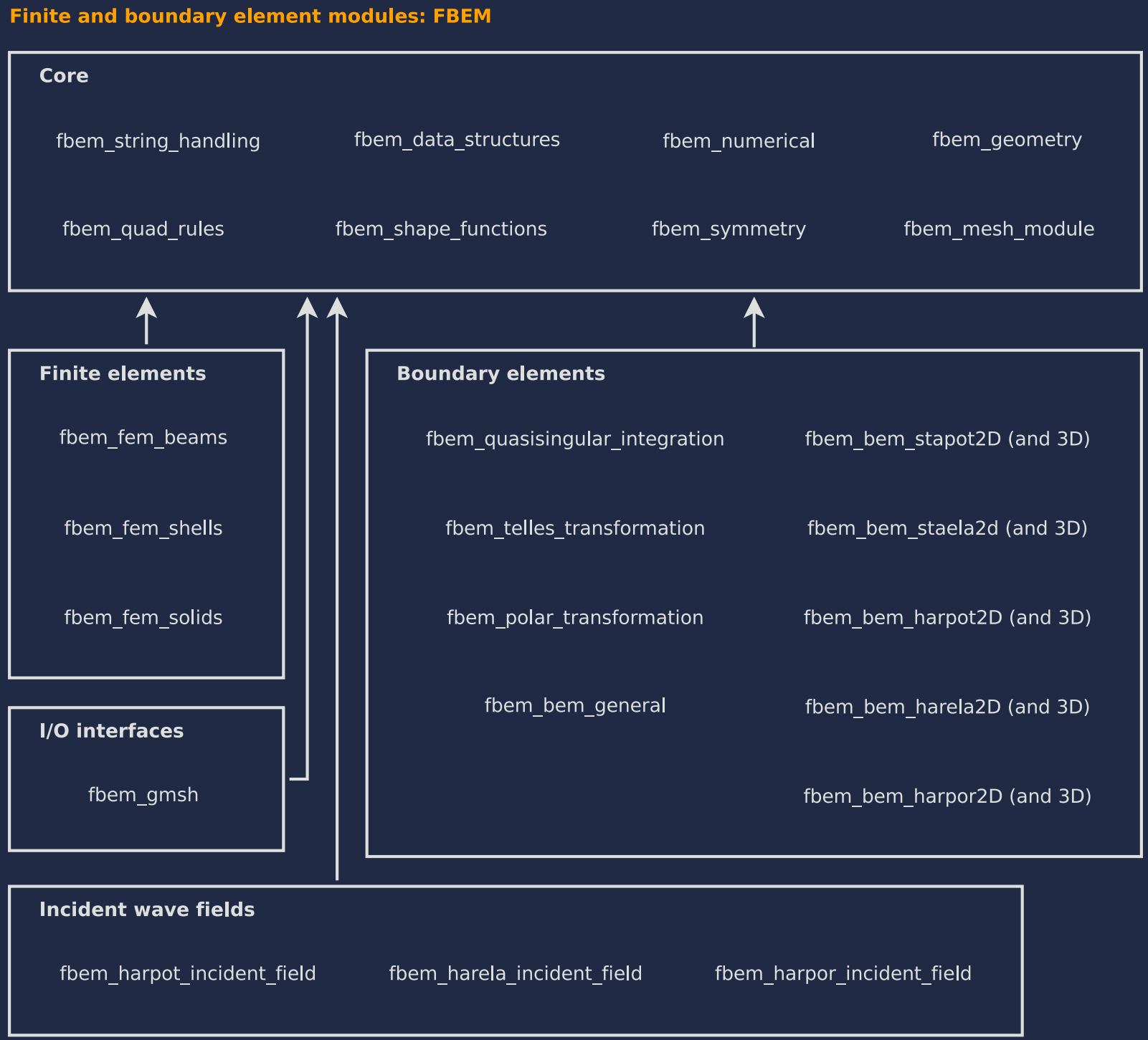
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fbem_shape_functions
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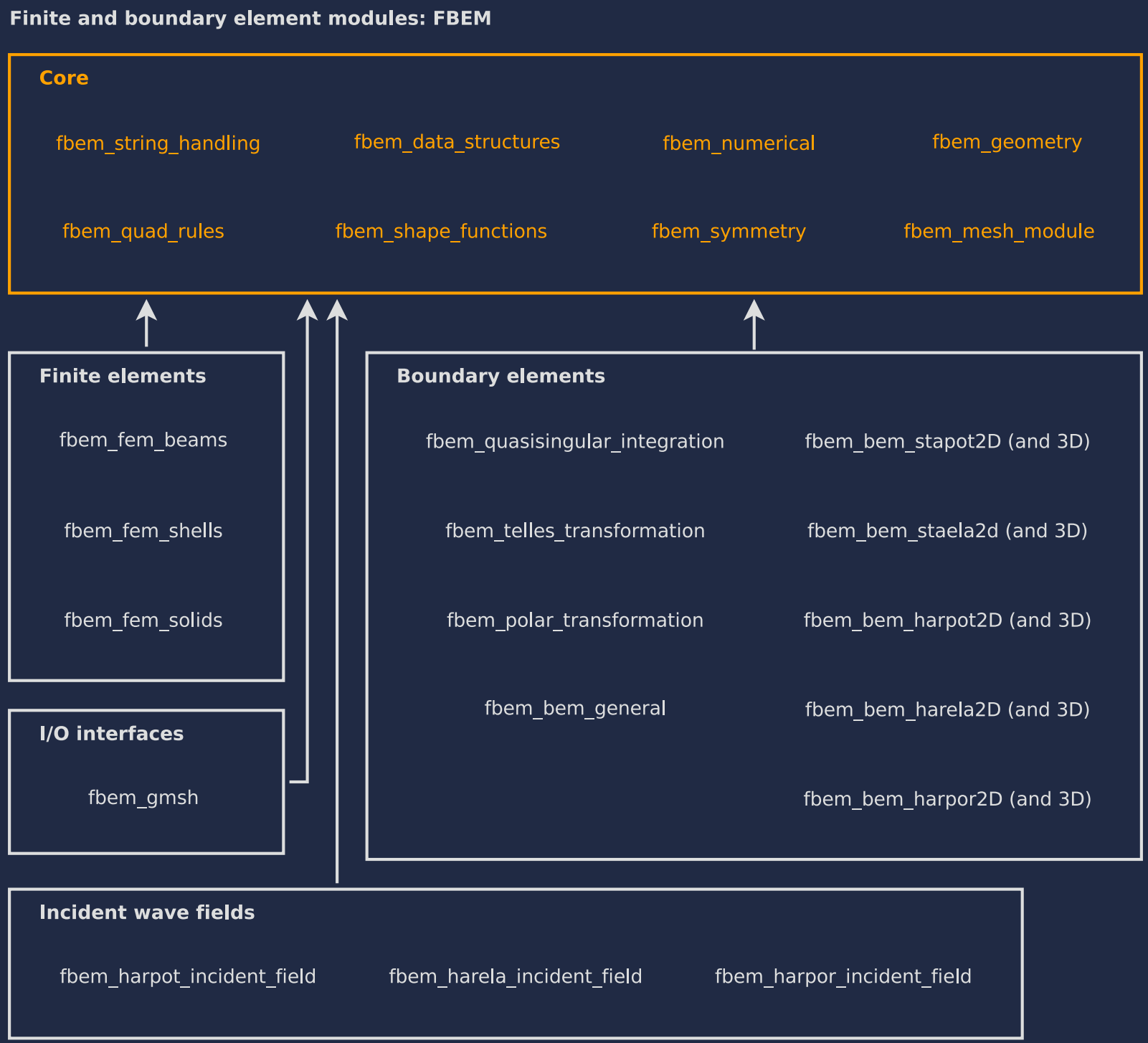
fbem_mesh_module
Finite elements
fbem_fem_beams
fbem_fem_shells
fbem_fem_solids
I/O interfaces
fbem_gmsh
Boundary elements
fbem_quasisingular_integration
fbem_bem_stapot2D (and 3D)
fbem_telles_transformation
fbem_bem_staela2d (and 3D)
fbem_polar_transformation
fbem_bem_harpot2D (and 3D)
fbem_bem_general
fbem_bem_harela2D (and 3D)
fbem_bem_harpor2D (and 3D)
Incident wave fields
fbem_harpot_incident_field
fbem_harela_incident_field
fbem_harpor_incident_field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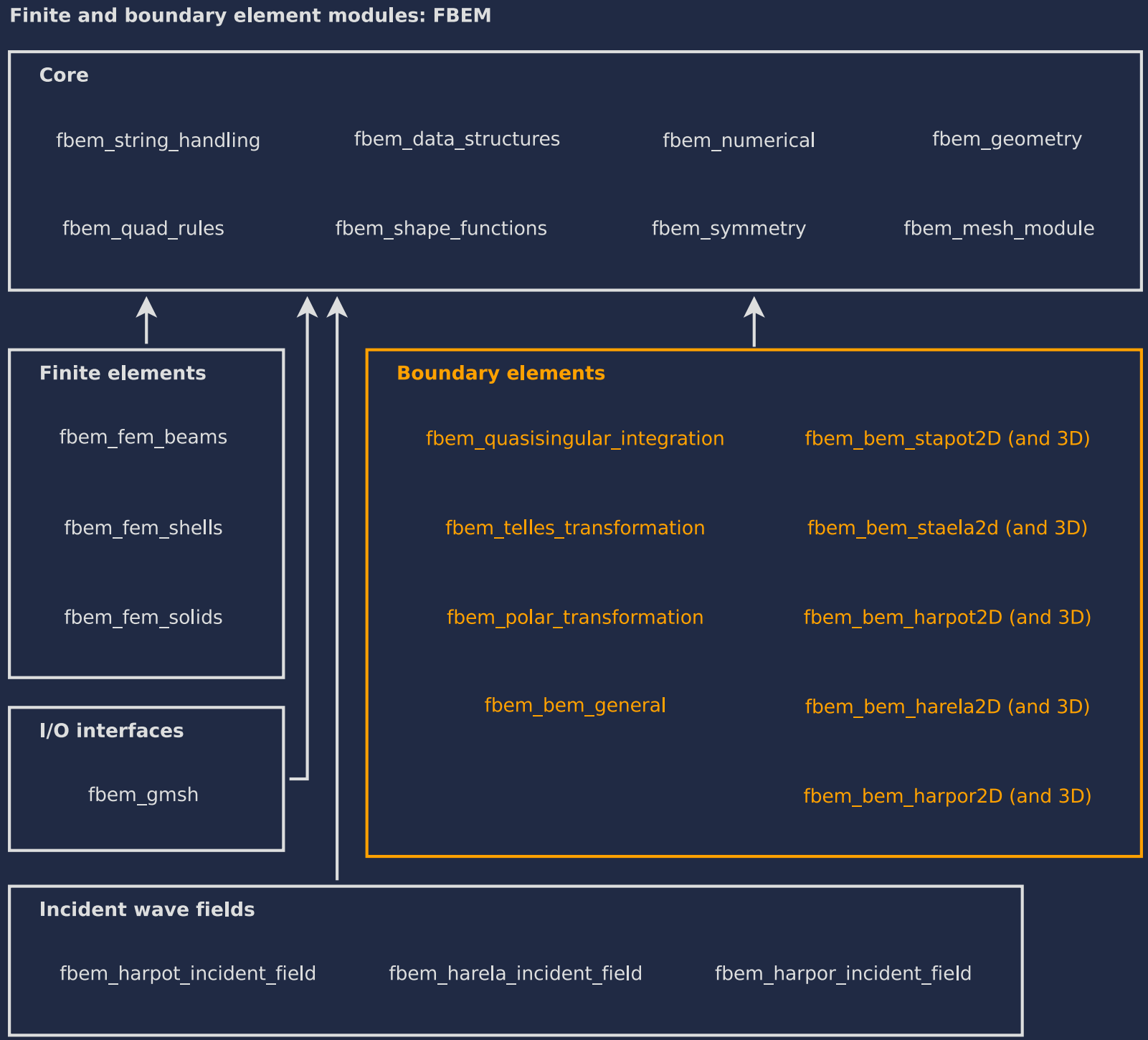
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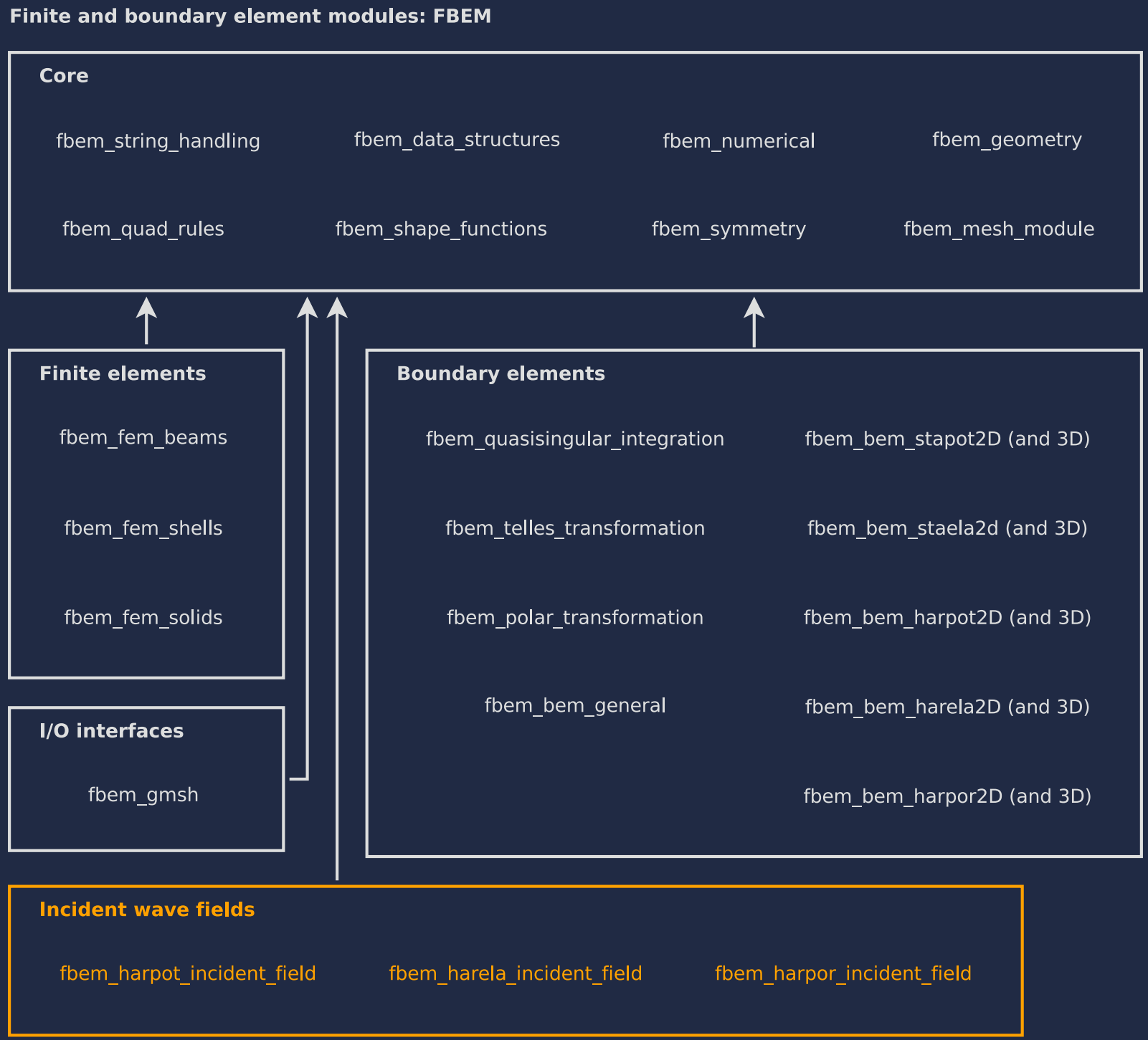
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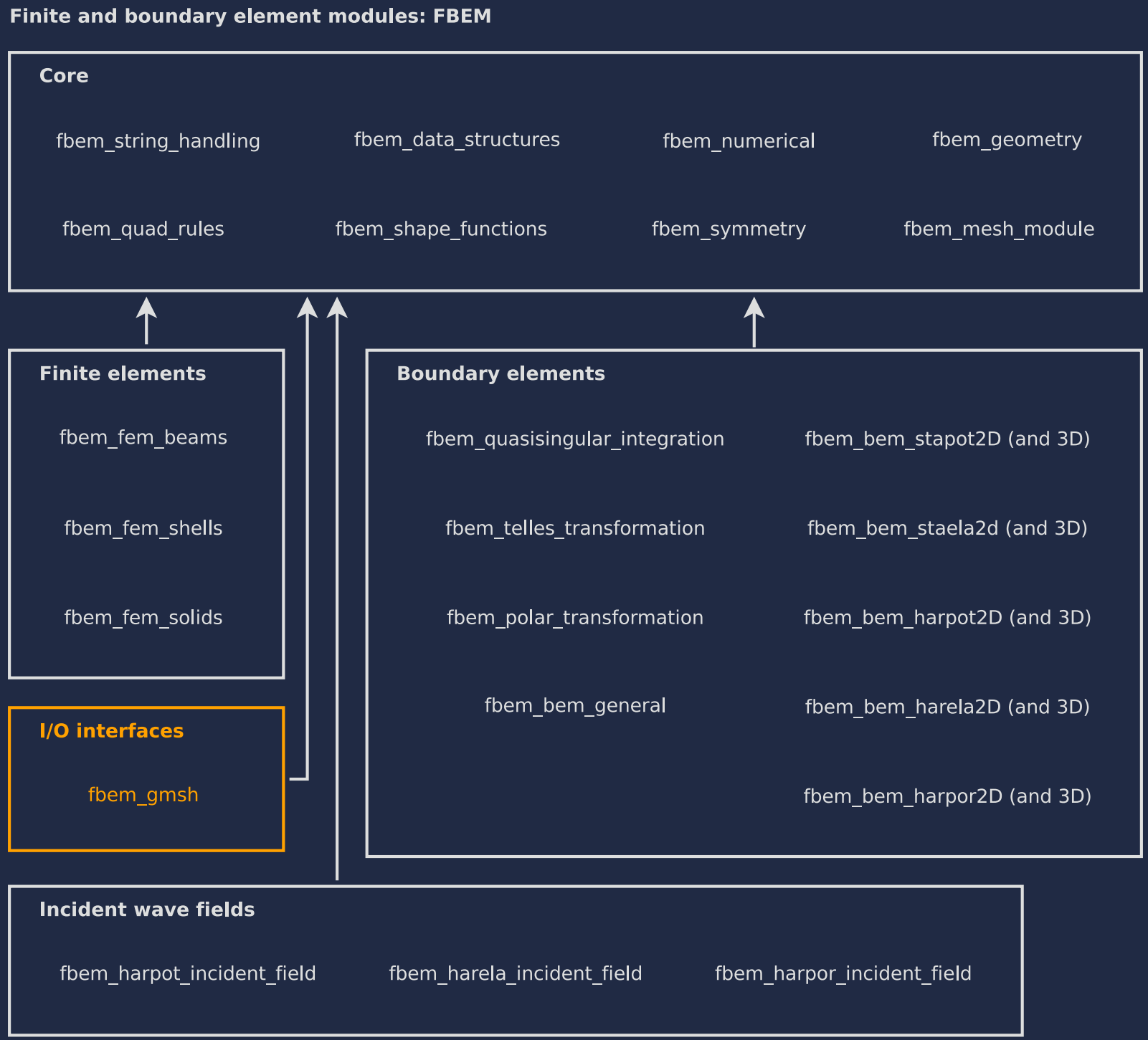
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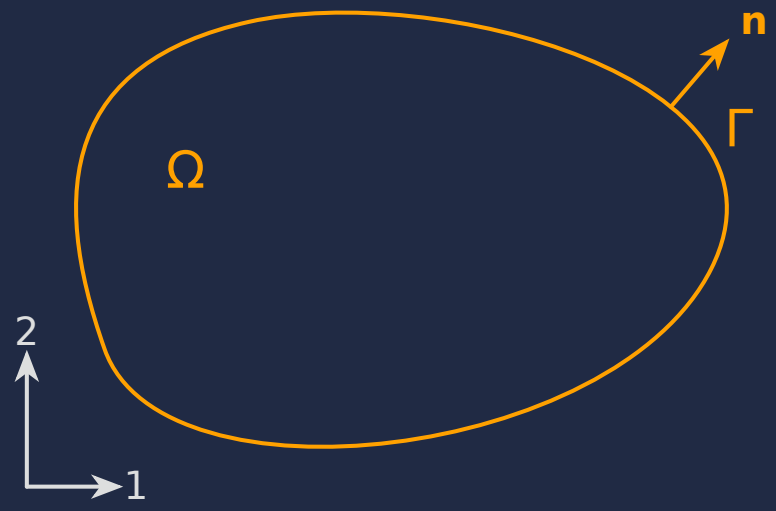
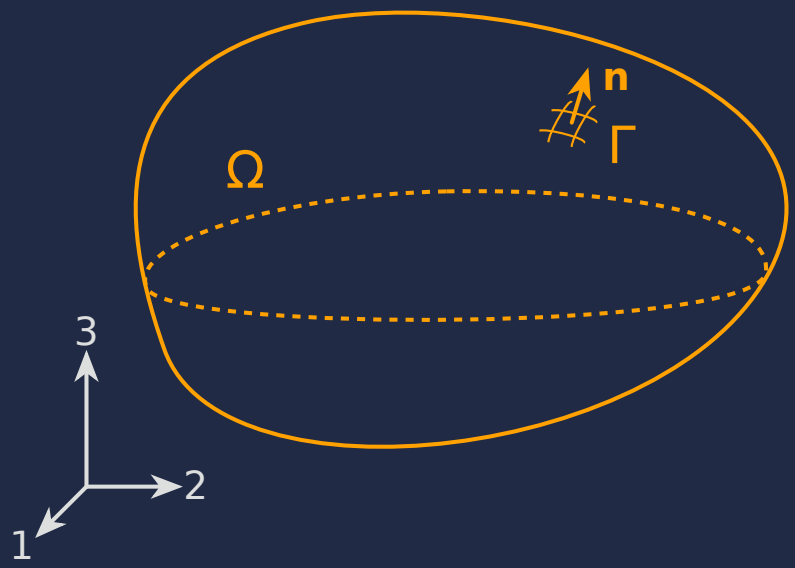
Outline

Methodologies: types of analyses

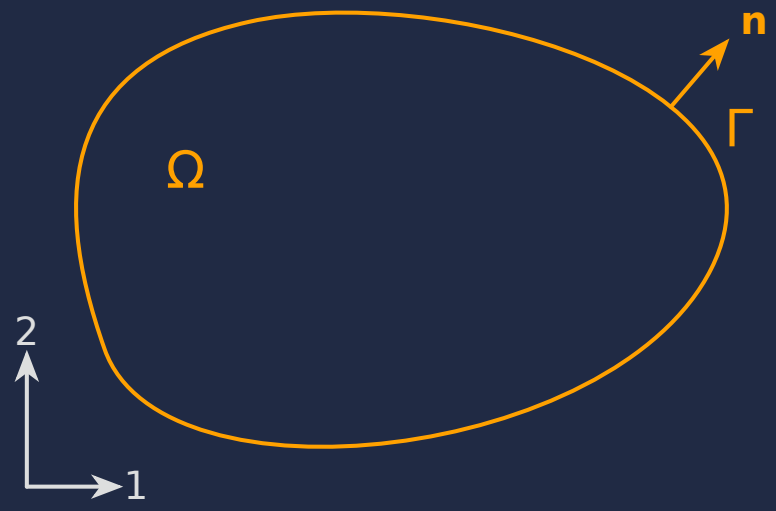
- 2D and 3D

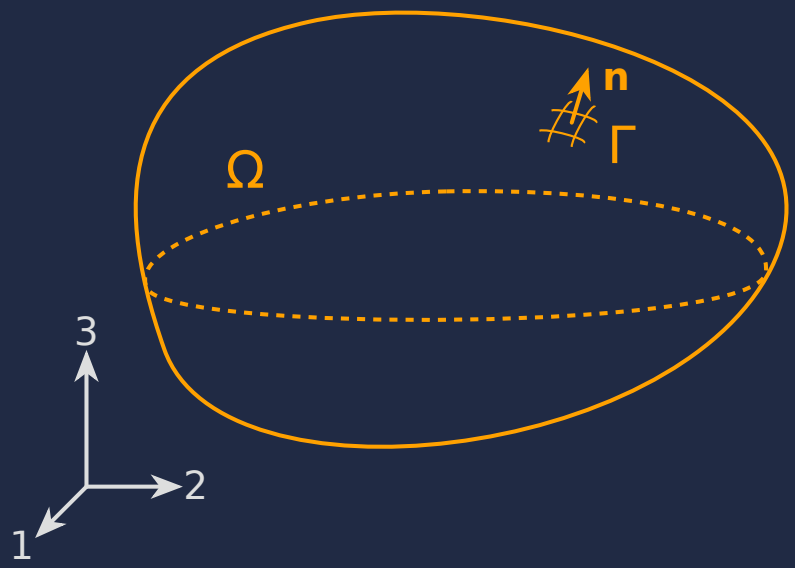




Methodologies: types of analyses

- 2D and 3D
- Static analysis and time harmonic analysis

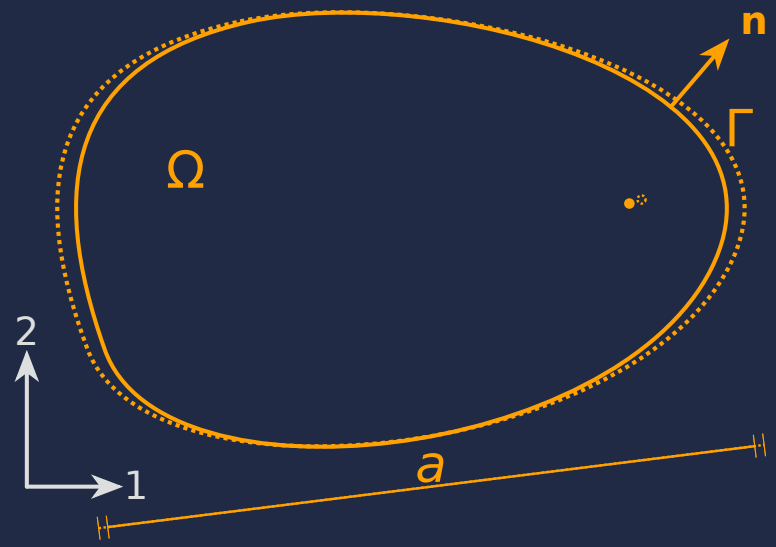


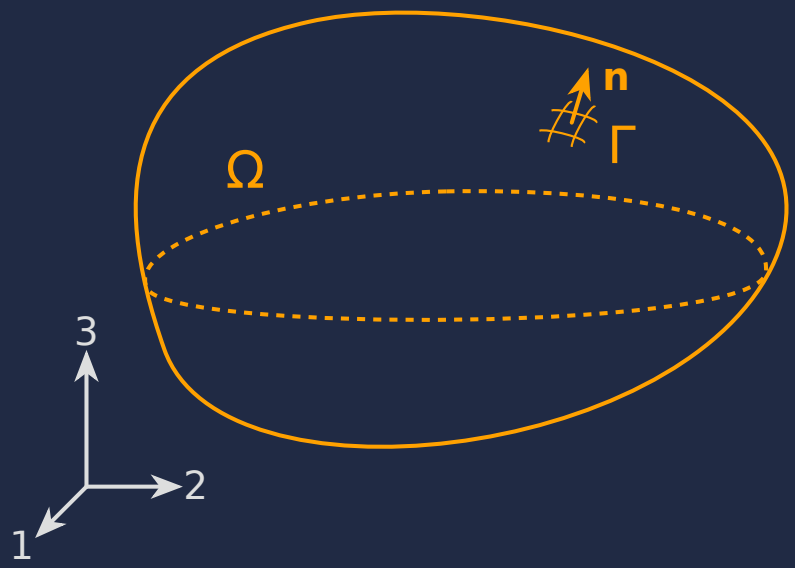


$$\mathbf{u}(\mathbf{x}) \quad \text{or} \quad \mathbf{u}(\mathbf{x}, t) = \tilde{\mathbf{u}}(\mathbf{x}, \omega) e^{i\omega t}$$

Methodologies: types of analyses

- 2D and 3D
- Static analysis and time harmonic analysis
- First-order geometric sensitivity analysis (limited)





$$\mathbf{u}(\mathbf{x}, a) = \mathbf{u}(\mathbf{x}, a_0) + \left(\frac{\partial \mathbf{u}}{\partial a}\right)_{a=a_0} (a - a_0) + \text{higher order terms}$$

Methodologies: Finite Element Method

- Discrete: mass, springs, dashpots.
- Beam elements:
  - Straight with 2 and 3 nodes (Euler-Bernoulli and Tim.[3])
  - Deg. from solid curved with 3 and 4 nodes (Tim.)
- Shell elements (Reissner-Mindlin):
  - Deg. from solid with 3, 4, 6, 8 and 9 nodes
  - MITC4[4] and MITC9[5] quadrilaterals (defect-free)
  - Automatic handling of 5 or 6 DOF nodes

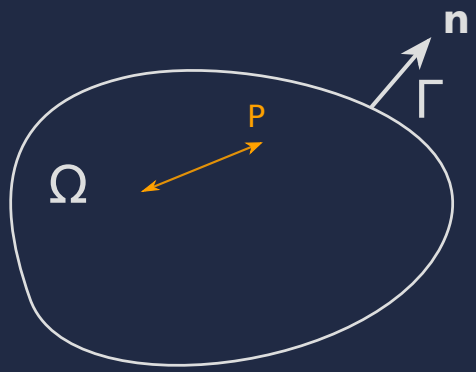
[3] Based on: Friedman & Kosmatka, Comp & Struct 47, 1993

[4] Dvorkin & Bathe, Eng Comput 1, 1984.

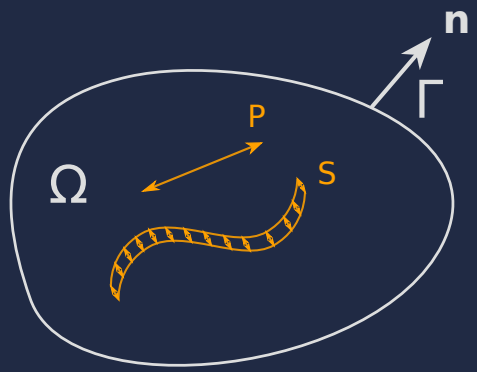
[5] Bucalem & Bathe, Int J Num Meth Eng 36, 1993.

Methodologies: Boundary Element Method

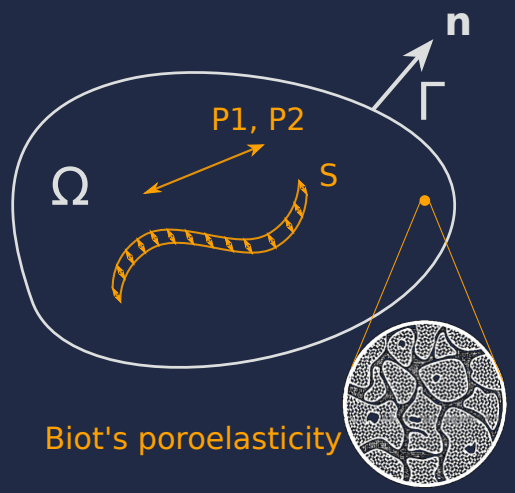
- Material models



Inviscid fluid



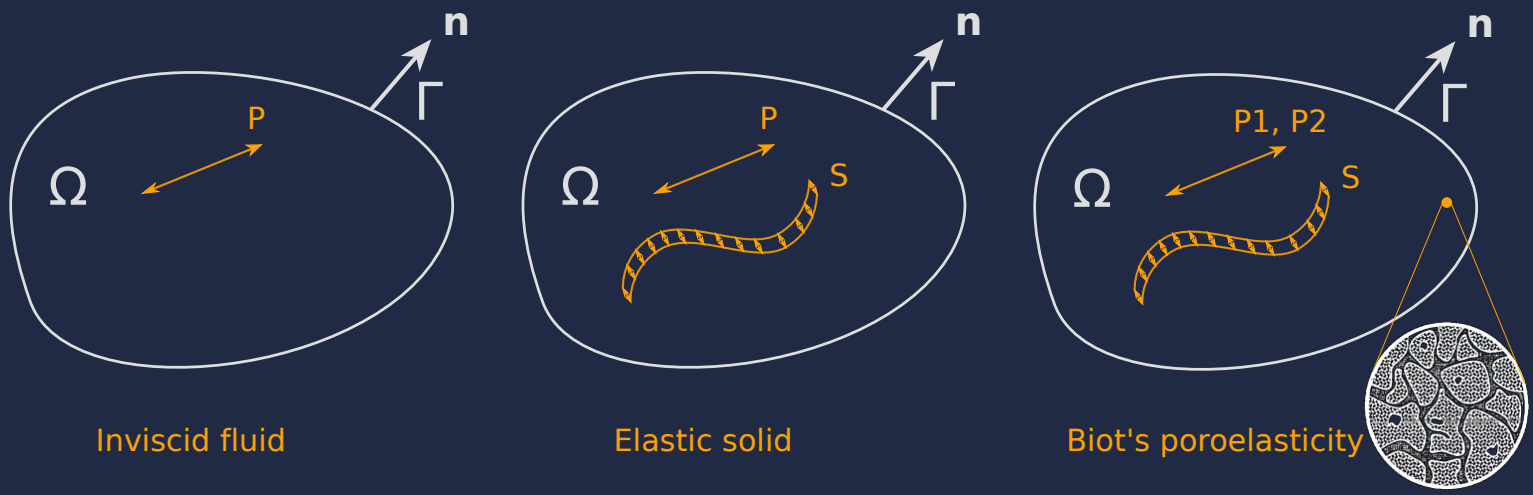
Elastic solid



Biot's poroelasticity

Methodologies: Boundary Element Method

- Material models



- Available Green's functions (u^*_{lk}, t^*_{lk}, d^*_{lk}, s^*_{lk}):
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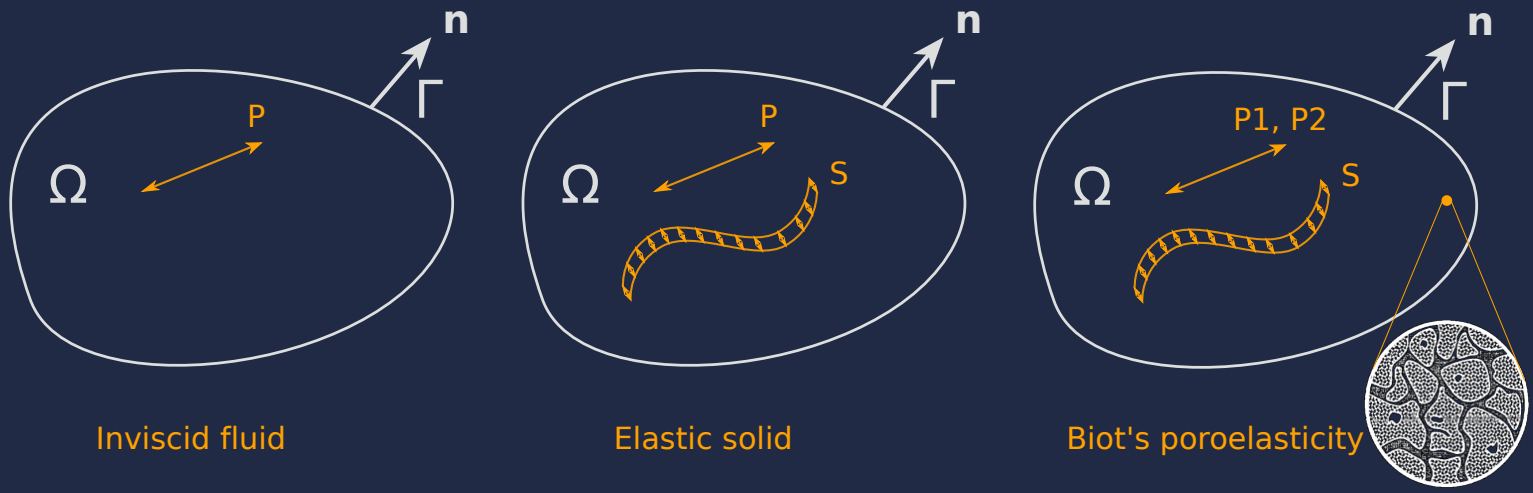

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Methodologies: Boundary Element Method

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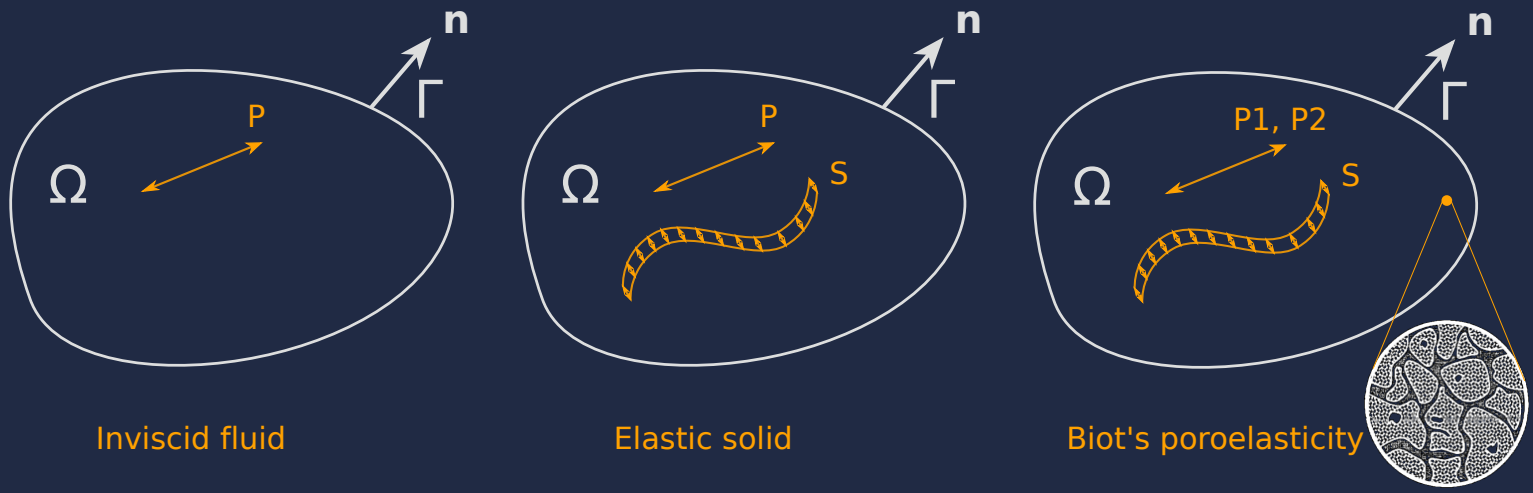

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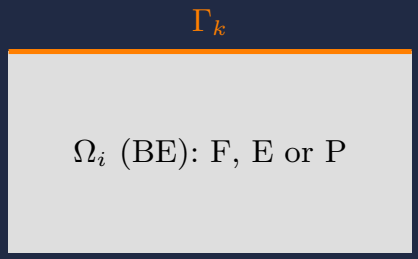
- Mirror symmetry

Methodologies: Boundary Element Method

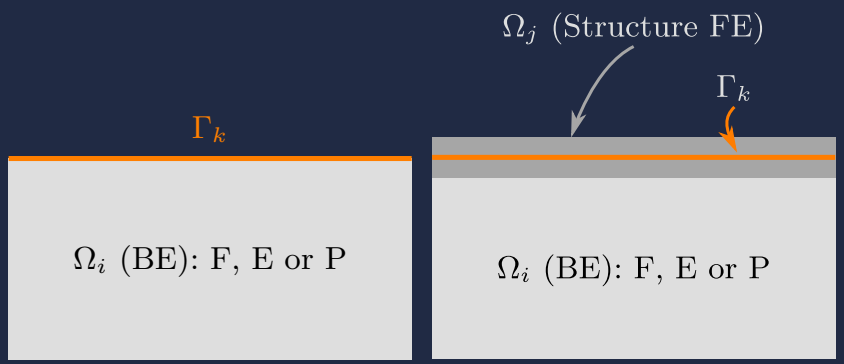
Figure: Inviscid fluid (F), elastic solid (E), poroelastic medium (P).



(a) Ordinary BE

Methodologies: Boundary Element Method

Figure: Inviscid fluid (F), elastic solid (E), poroelastic medium (P).



(a) Ordinary BE (b) Ordinary BE-FE

Methodologies: Boundary Element Method

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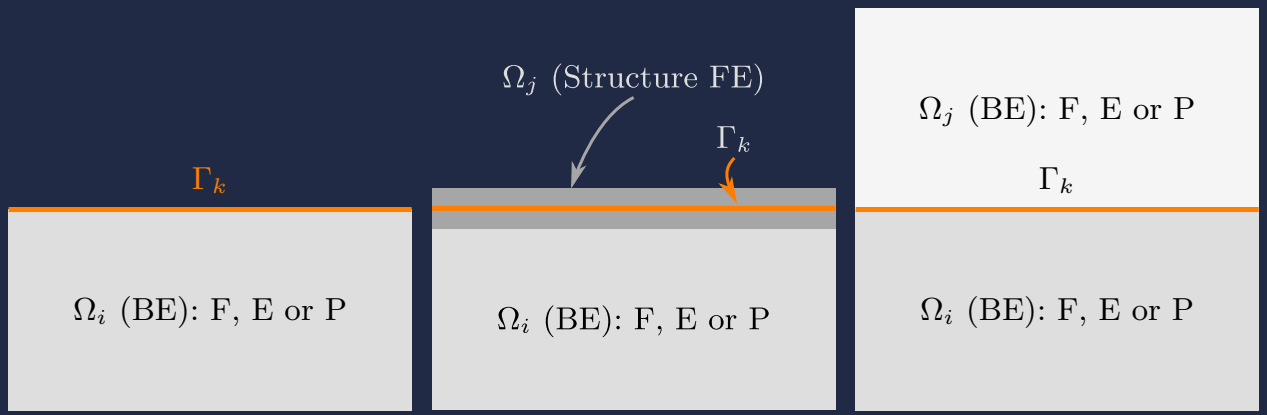

(a) Ordinary BE (b) Ordinary BE-FE (c) BE-BE interface

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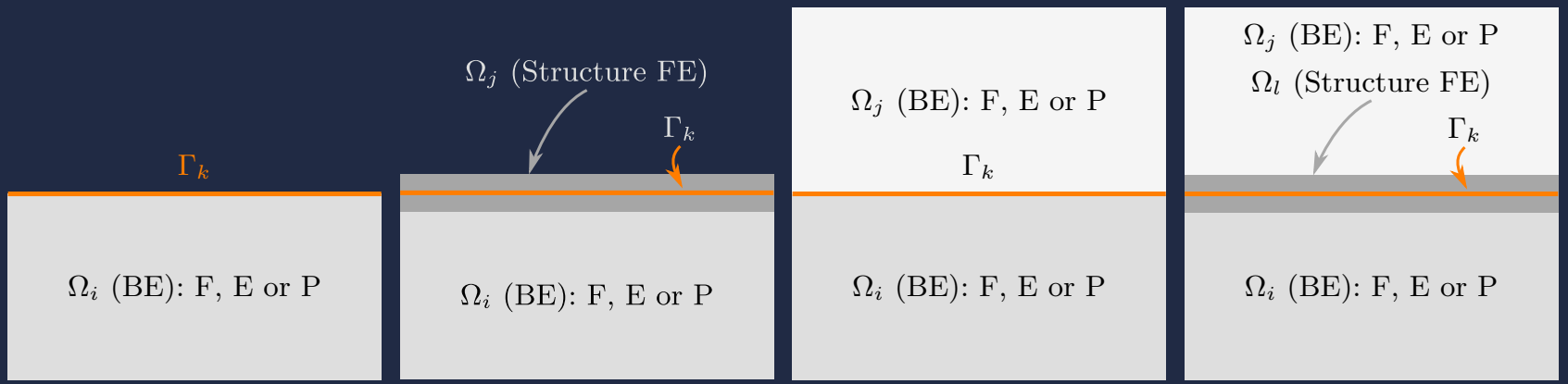

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Methodologies: Boundary Element Method

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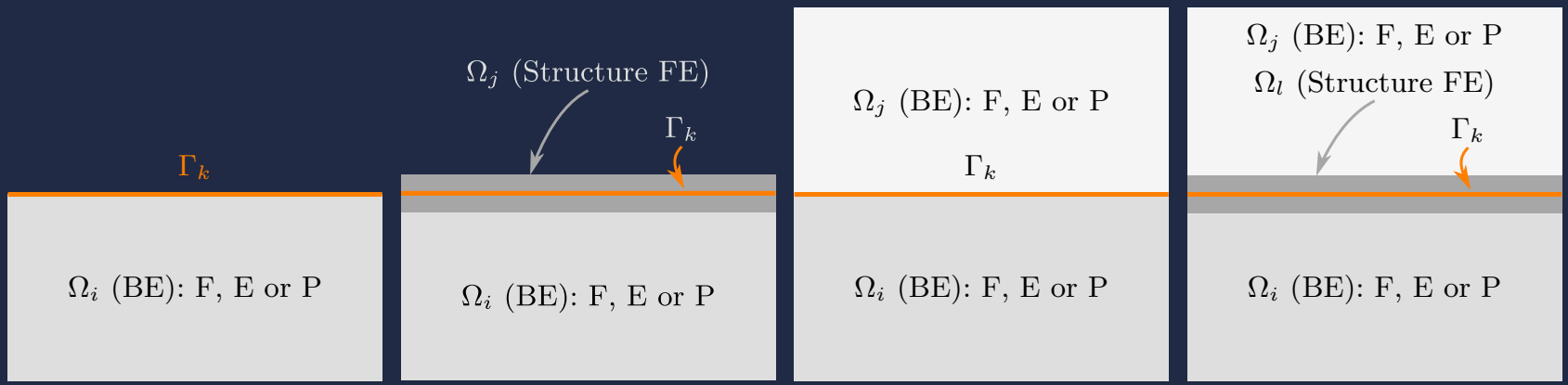

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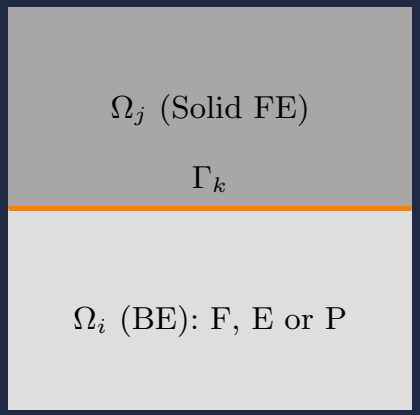

(e) Ordinary BE-FE

Methodologies: Boundary Element Method

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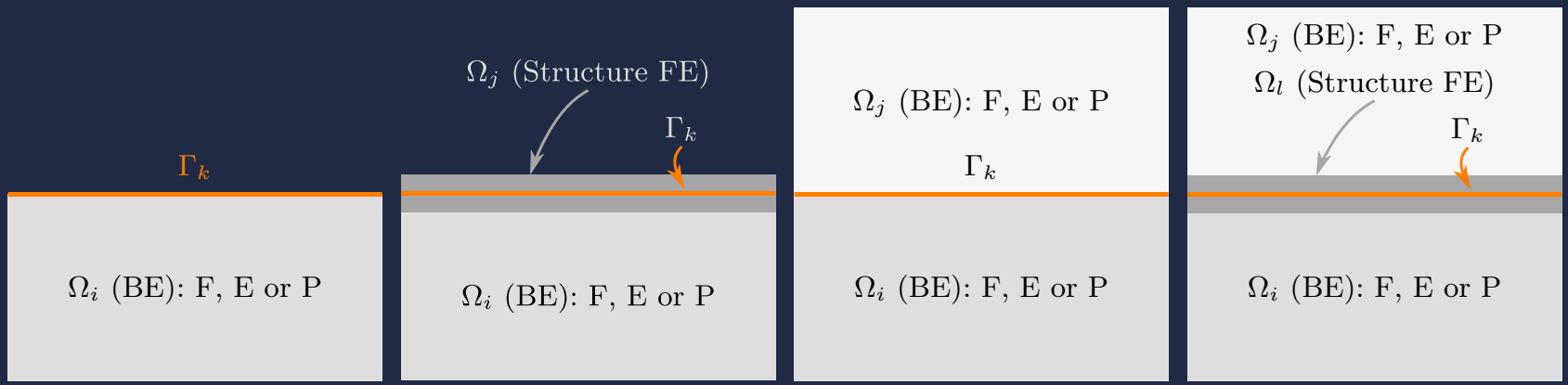

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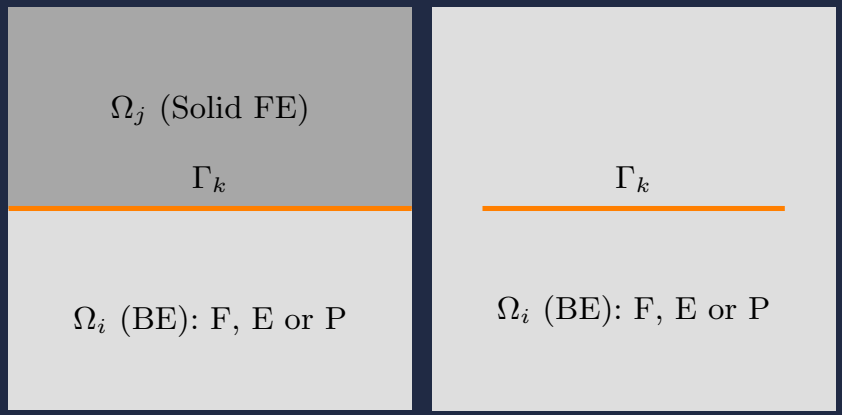

(e) Ordinary BE-FE (f) Crack BE

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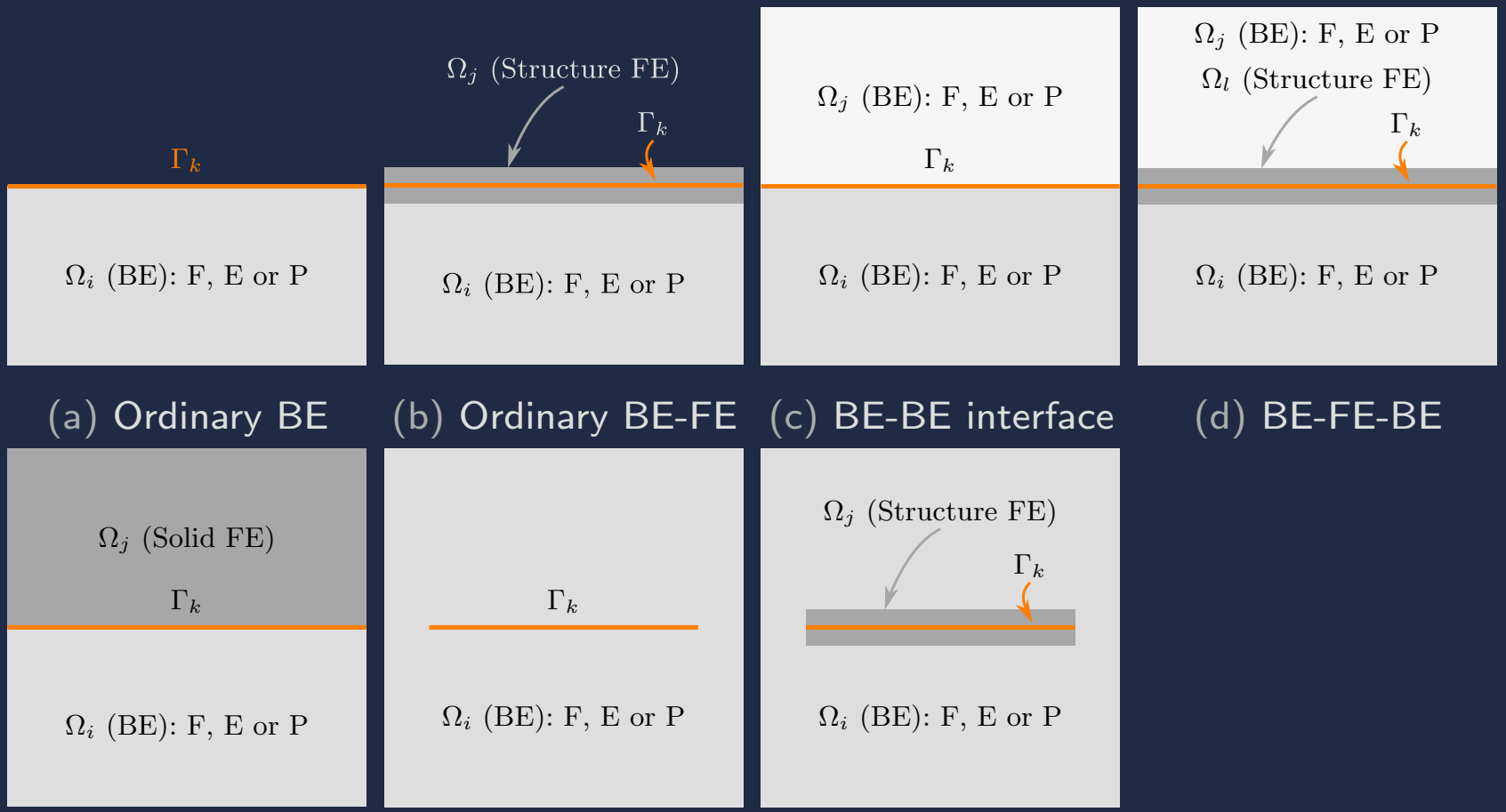

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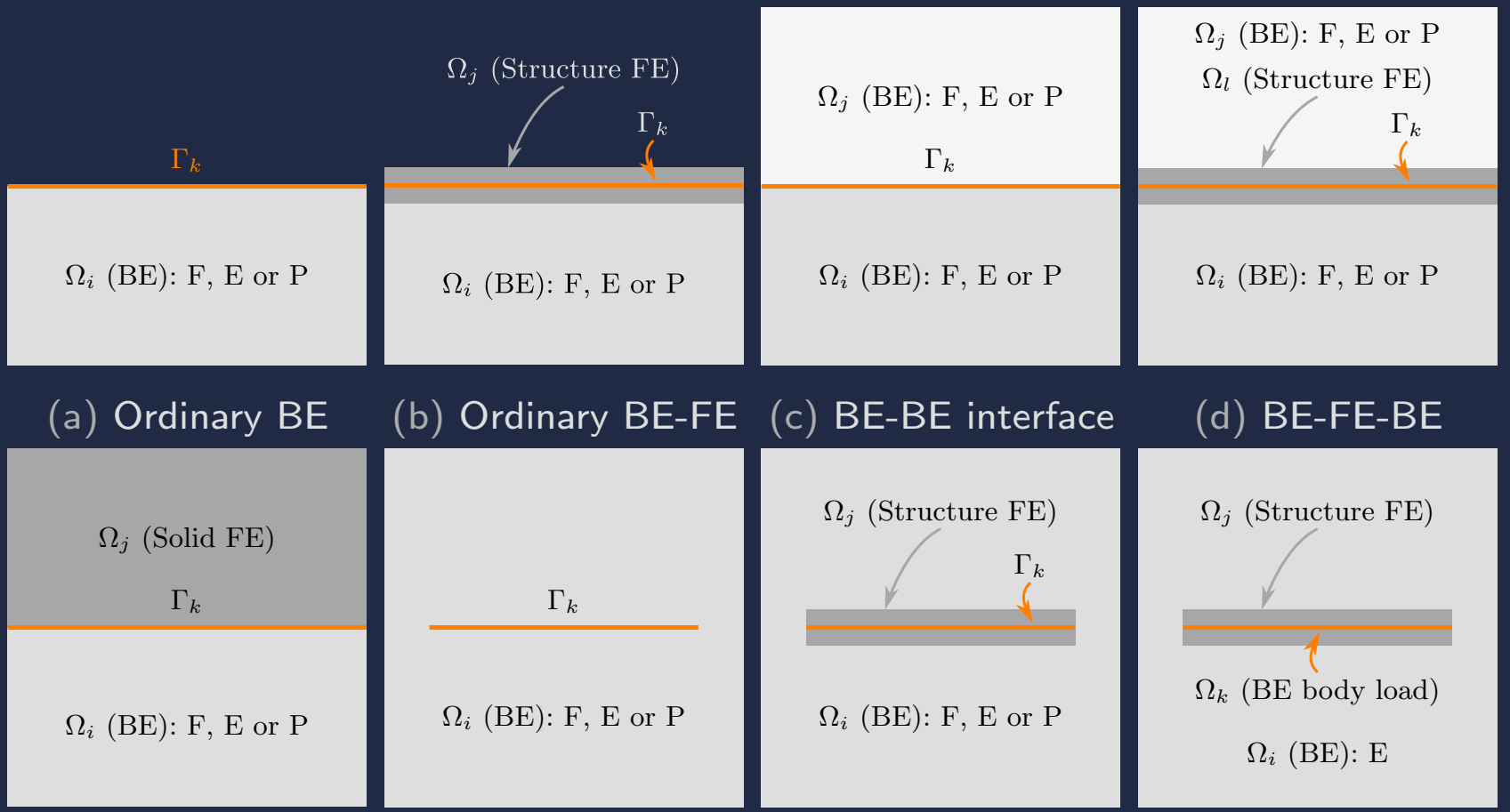
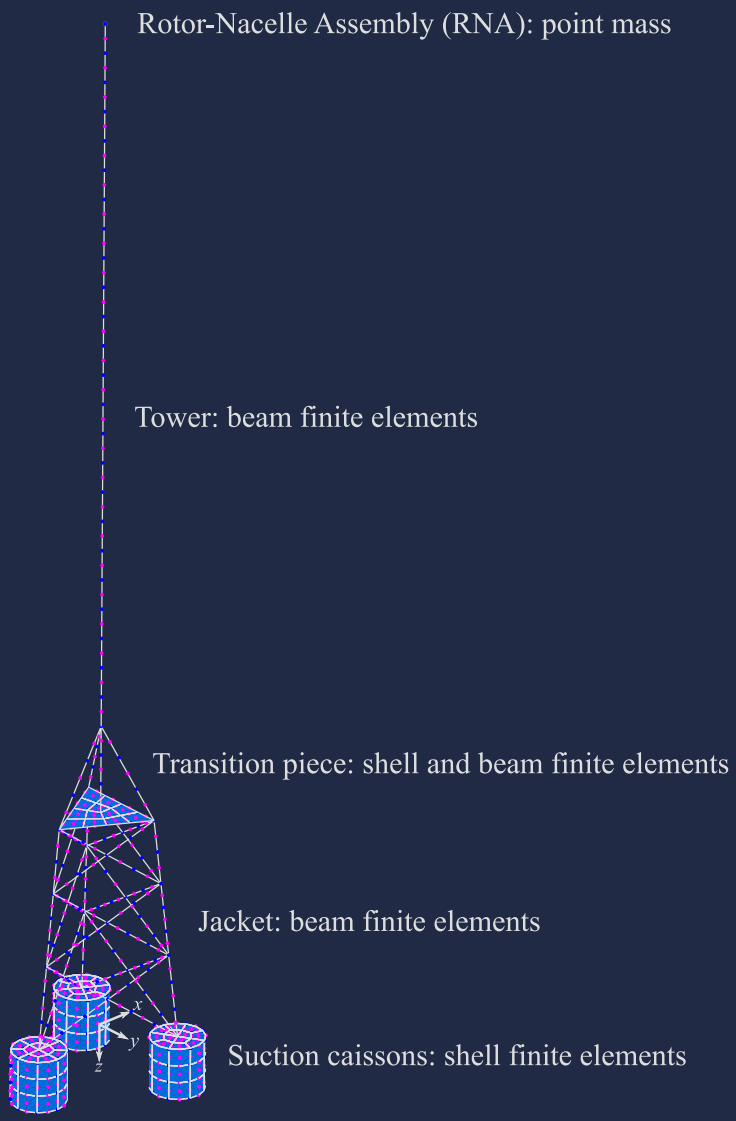


Outline

Offshore Wind Turbine



Finite Element Mesh

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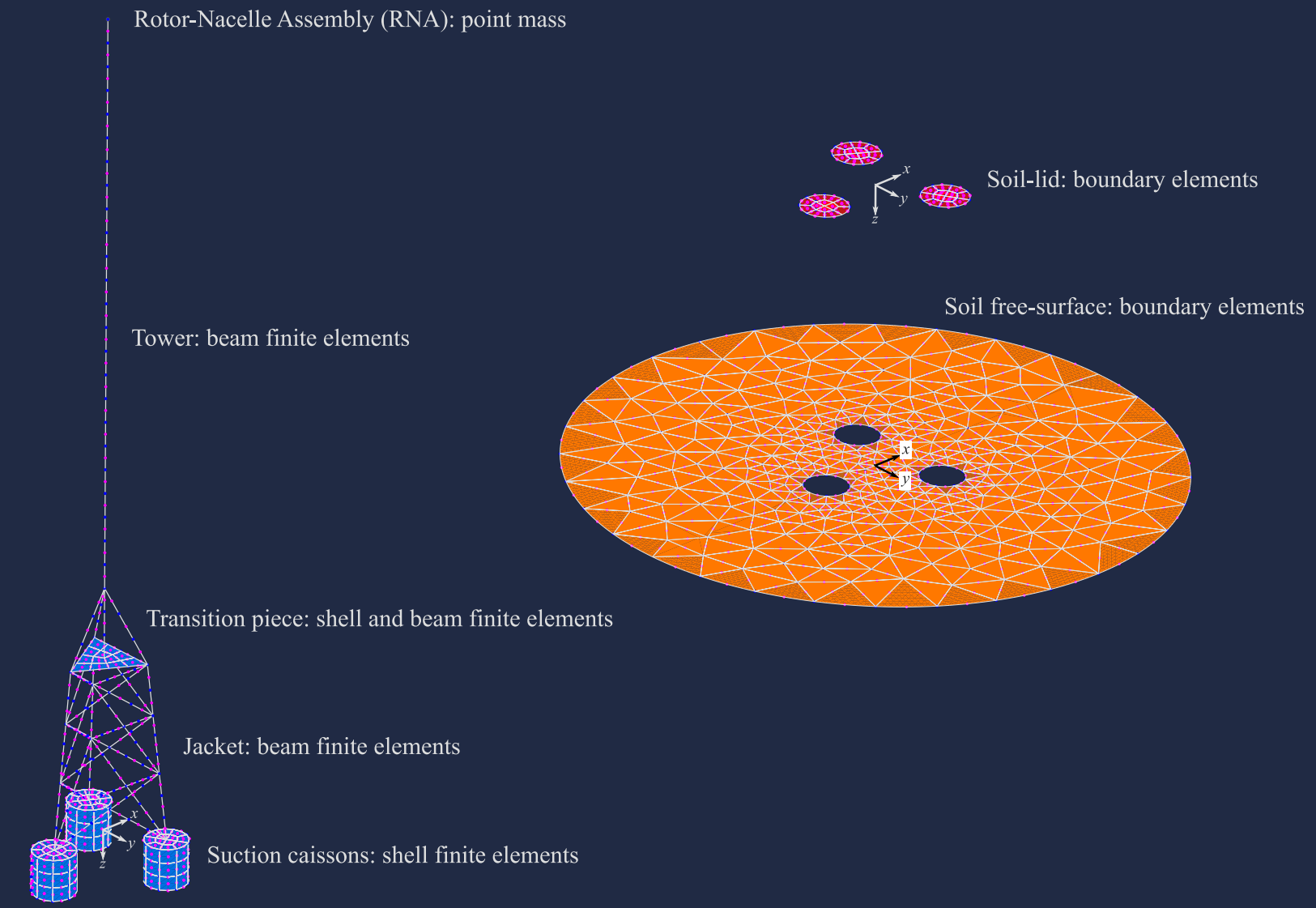

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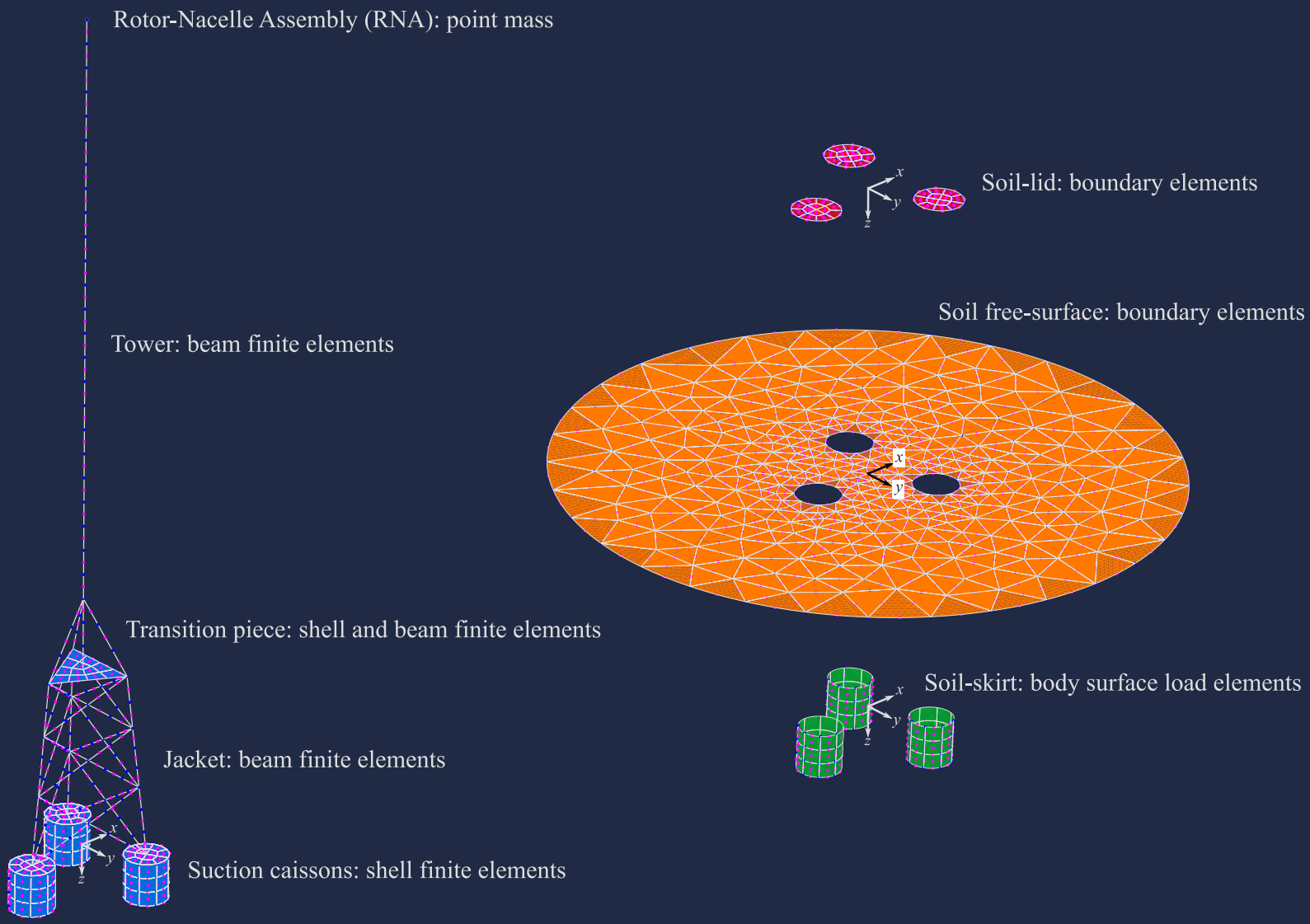

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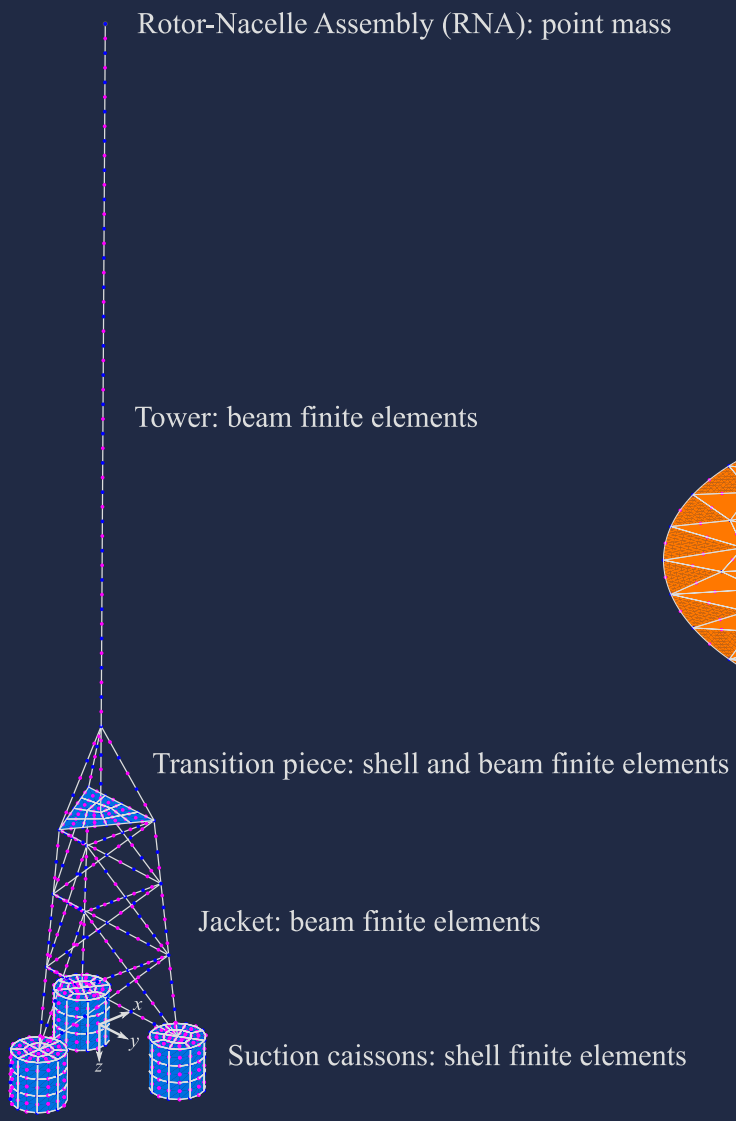

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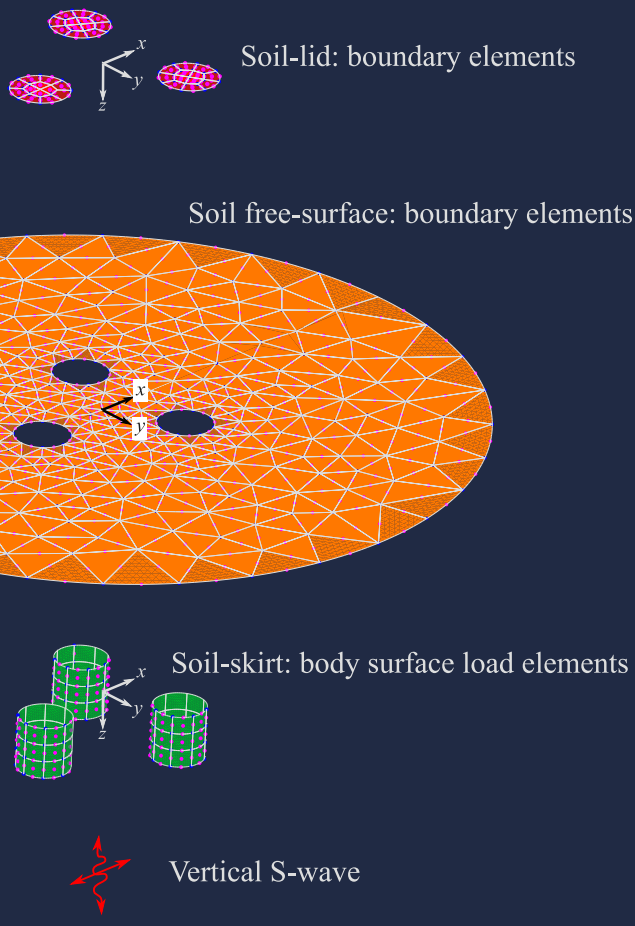
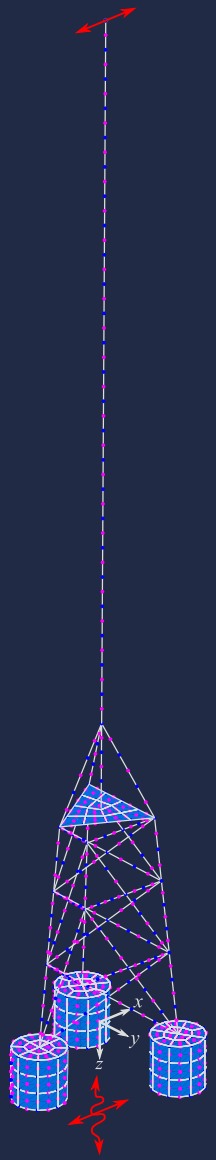


Boundary Element Mesh

Results: tower top displacement



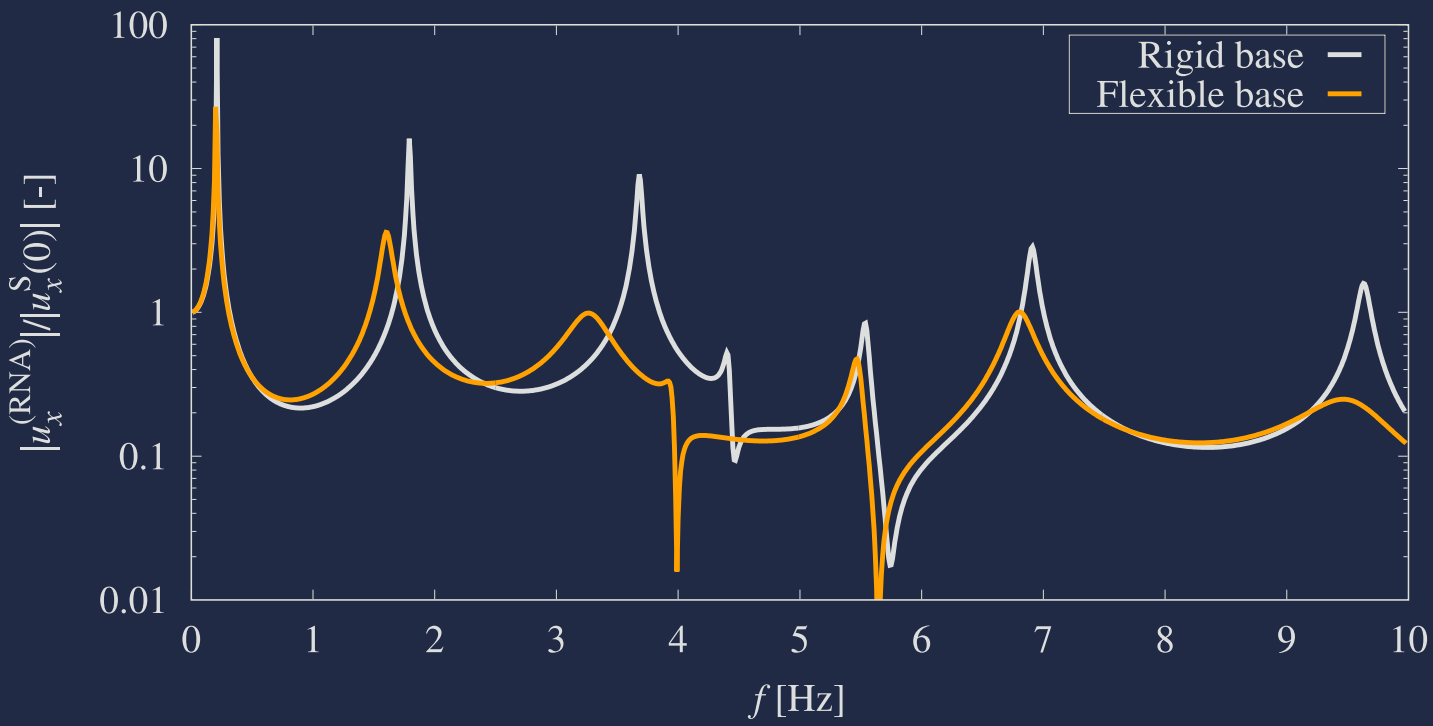
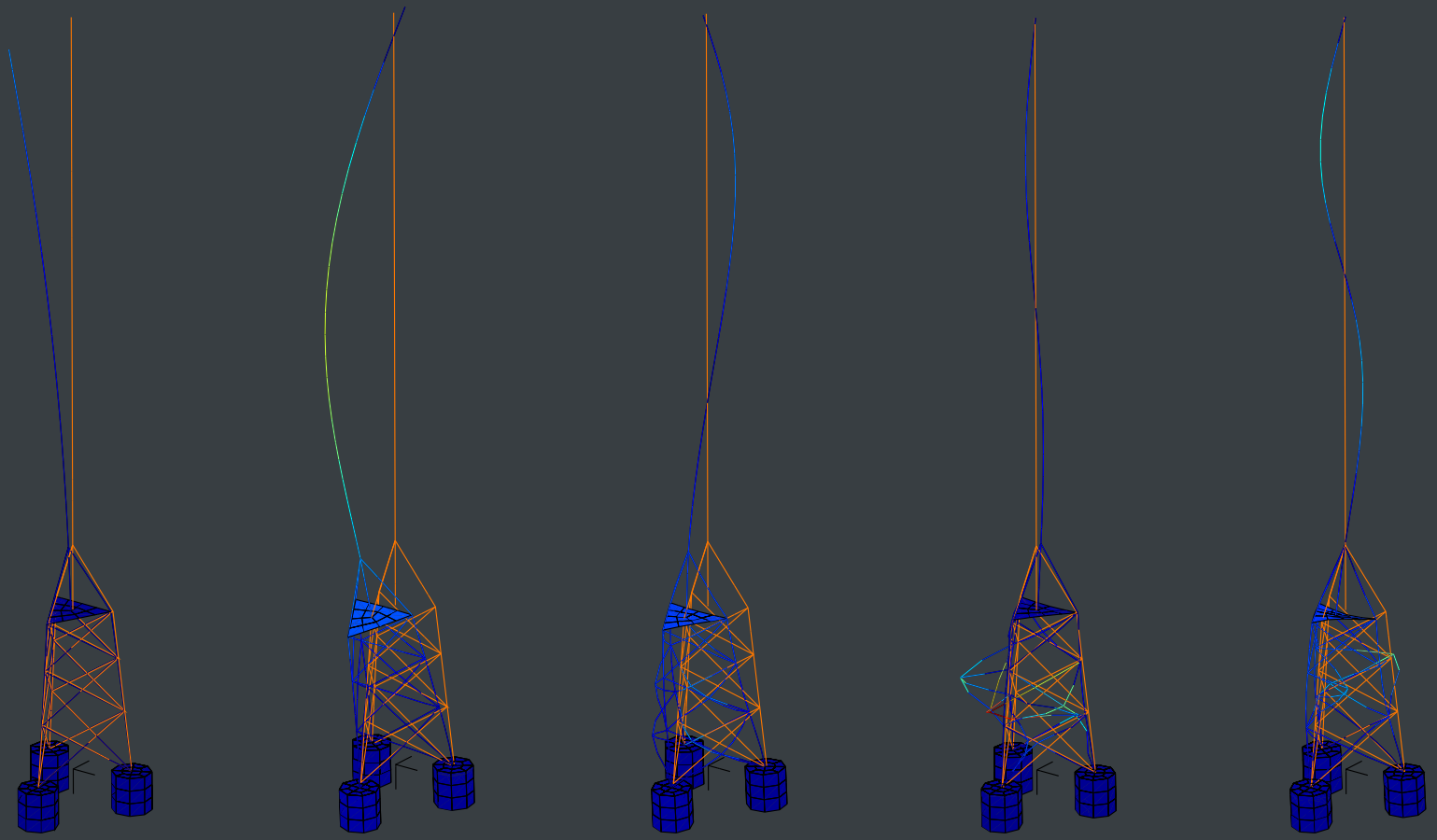


Figure: Normalized horizontal displacement response at the tower top

Results: deformed shapes



(a) $f = 0.2$ Hz (b) $f = 1.6$ Hz (c) $f = 3.26$ Hz (d) $f = 5.47$ Hz (e) $f = 6.81$ Hz

Outline

Conclusions

- Presentation of MultiFEBE project:
  - Why?
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Acknowledgments

Research Project PID2020-120102RB-I00, funded by the Agencia Estatal de Investigación of Spain, MCIN/AEI/10.13039/501100011033:

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Numéricas en Ingenierías

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`www.mmc.siani.es`

Methodologies: Boundary Element Method

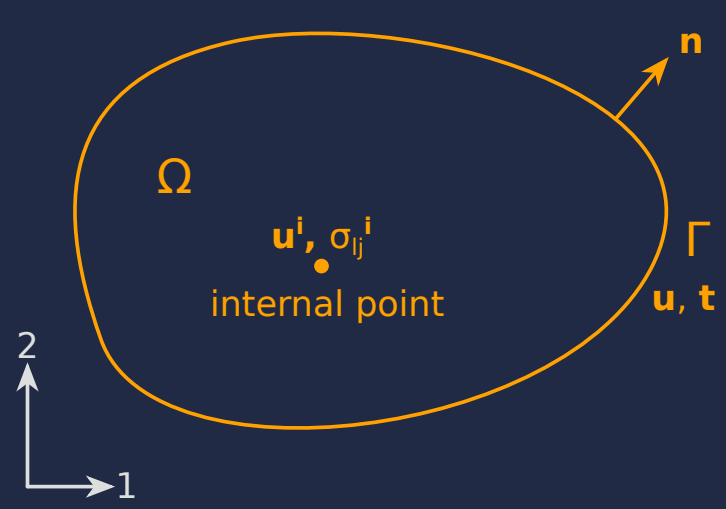
- Boundary elements. Boundary classes:
  - Ordinary:
    - Uncoupled: BC imposed
    - Coupled: BE-BE, BE-FE, BE-FE-BE
  - Crack:
    - Uncoupled: BC imposed
    - Coupled: BE-FE
- Body load elements (only coupled BE-FE):
  - Line load
  - Surface load

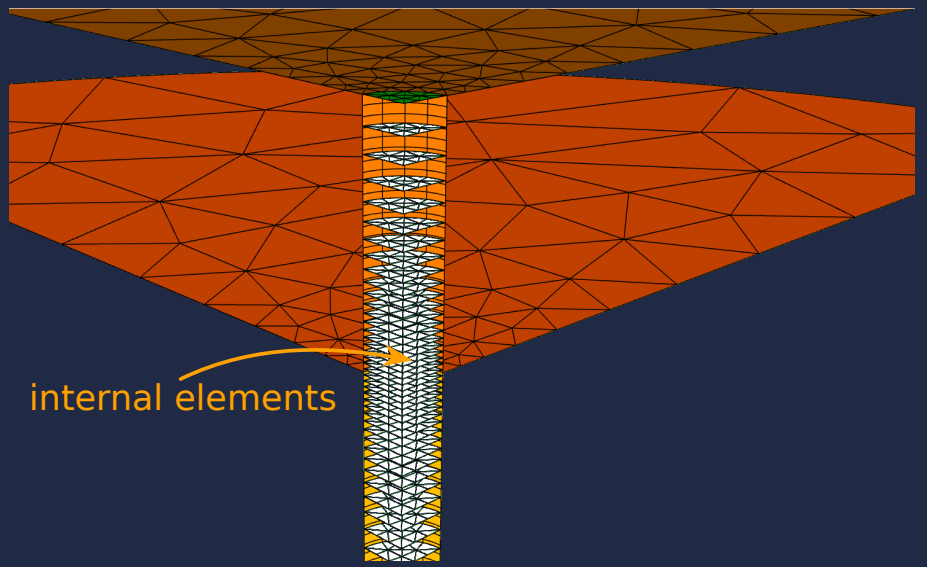
Methodologies: Boundary Element Method

- Internal points & internal elements (post-processing)

$$u_l^i = \int_\Gamma u_{lk}^* t_k \, \mathrm{d}\Gamma - \int_\Gamma t_{lk}^* u_k \, \mathrm{d}\Gamma + \int_\Omega u_{lk}^* b_k \, \mathrm{d}\Omega$$

$$\sigma_{lj}^i = \int_\Gamma d_{lkj}^* t_k \, \mathrm{d}\Gamma - \int_\Gamma s_{lkj}^* u_k \, \mathrm{d}\Gamma + \int_\Omega d_{lkj}^* b_k \, \mathrm{d}\Omega$$





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  - Collocation strategies (automatic):
    - Nodal (SBIE)
    - Non-nodal multiple collocation (SBIE, HBIE) approach[6].

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Methodologies: Boundary Element Method

Numerical algorithms:

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  - Quasi-singular integrals: combination of Telles' transformation & recursive uniform subdivision.

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