The GBT Sensitivity Calculator User's Guide

July 1, 2011

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1. Introduction to the GBT Sensitivity Calculator

The new GBT Sensitivity Calculator has been designed to provide observers an easy way to determine the time needed to complete a proposed project or the expected sensitivity achieved by a project of a given length. In comparison with our previous calculator, the replacement is significantly more sophisticated and leads users through a complex web of decisions and choices astronomers make as they think out their sensitivity estimates. The new calculator should simplify the writing of a proposals technical justification since a user can save the input parameters and results to a file which can then be attached to a proposal. Since an attachment will contain very complete details, it will also reduce the change that a reviewer will misinterpret the technical justification.

The calculator has been designed to satisfy 95% or so of 'traditional' spectral line and continuum observations and is not guaranteed to produce results better than 10% in sensitivity or 20% in time. It is up to the user on how best to derive from the calculator's output the total time for their sessions and projects. Thus, we expect those writing proposals to continue to use the technical justification section of their proposals to describe how the results of the calculator were used to derive the time estimates for their projects. Users can also include in their technical justifications as many output logs from the calculator as they feel are needed.

This document has been compiled to guide users through the flow of questions asked by the calculator in order to produce a final sensitivity or time estimate which can then be included in the technical justification of the proposal. The structure and common features of the sensitivity calculator can be found in Chapter 2. The following chapters are devoted to separate frames within the sensitivity calculator that require user input and should each be completed sequentially.

3. General Information
4. Hardware Information
5. Source Information
6. Data Reduction
7. Results

The Appendix provides a detailed discussion of the parameters and equations used by the calculator.

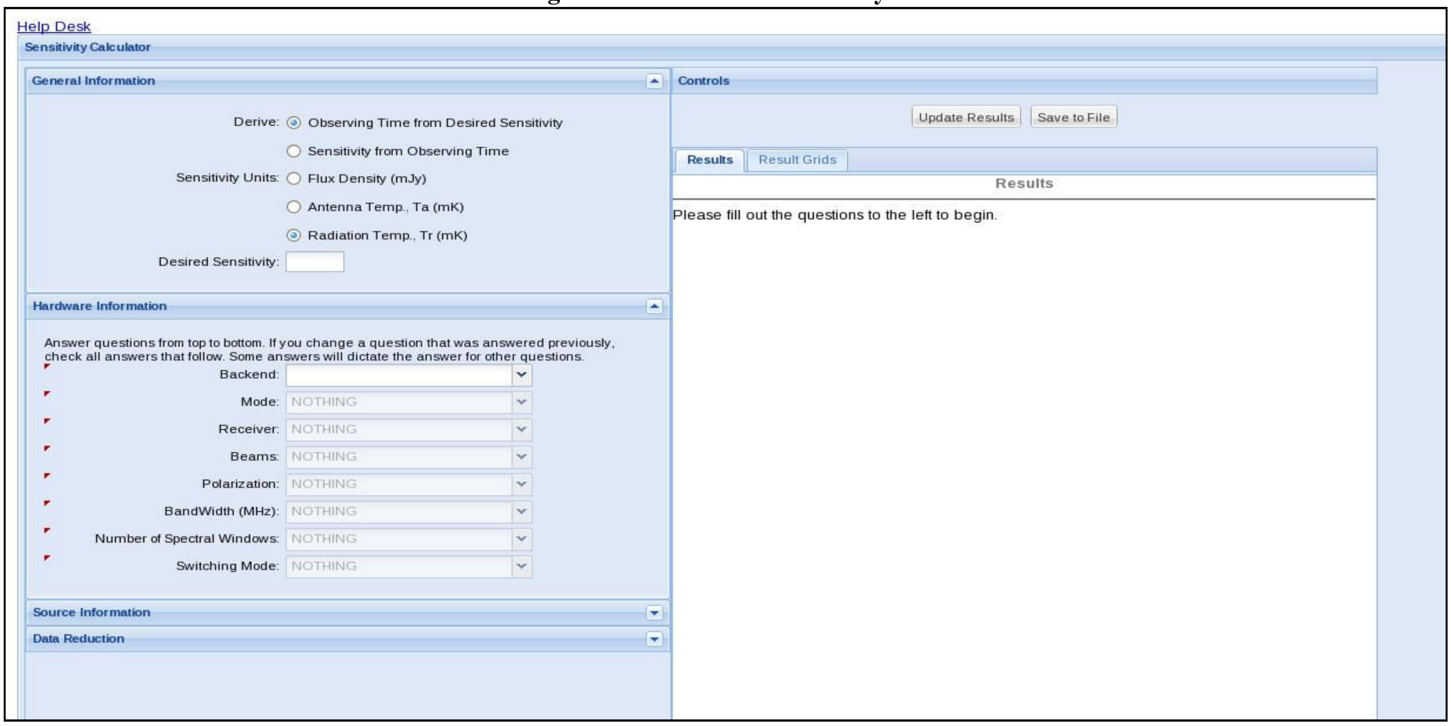
1.1 How Start the GBT Sensitivity Calculator

The Sensitivity Calculator is located at https://dss.gb.nrao.edu/calculator-ui/war/Calculator_ui.html. You will need to supply your 'My NRAO' username and password.

2. Structure and Common Features of the Sensitivity Calculator

When you first start the GBT Sensitivity Calculator you will be presented with the screen shown in Figure 2.1.

Figure 2.1: The GBT Sensitivity Calculator



2.1 Structure

The GBT Sensitivity calculator has been designed in a way that guides the user through various sequential steps. In general you should start by completing boxes from top to bottom, as the questions asked of you will depend on your answers to previous questions. For example, you will not be asked how many beams you wish to use if you have selected a single beam receiver. Any questions that are not applicable to your proposed observations or that cannot be asked at an early stage will be grayed out. As you select your answer to a question, the calculator will automatically update and if necessary, further questions may be asked of you.

Questions asked of the user are organized by topics given in the frames on the left hand side of the screen (General Information, Hardware Information, Source Information and Data Reduction). At any time you may press the 'Update Results' button to view any parameters that the calculator can return to you based on the answers you have given. If the calculator is not able to return a derived sensitivity or total observing time then you still have some questions to answer.

2.2 Features

- Help Desk – You may select this link to send an email to the DSS helpdesk using your default email application.
- – Press the up arrow button to minimize its associated frame.
- – Press the down arrow button to expand its associated frame.
- – Red triangles denote questions that must be answered in order for the calculator to proceed or denote fields that have been altered, requiring you to press the 'Update Results' button on the right side of the calculator.

3. General Information Frame

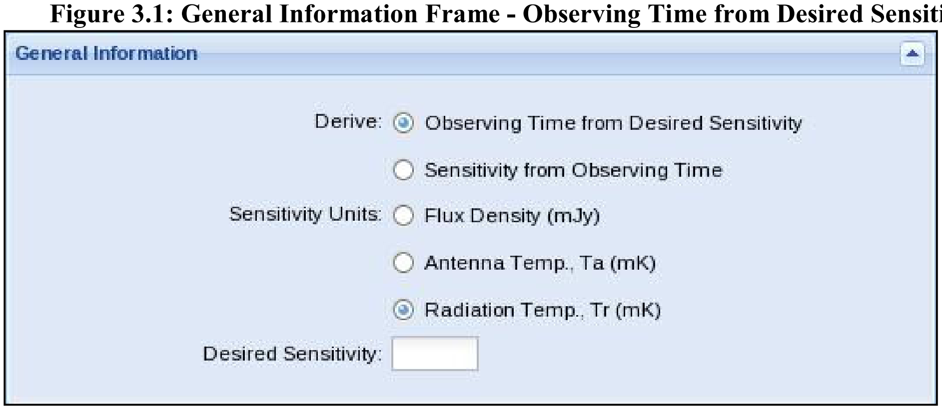
You must select whether you wish to derive the total observing time from a given sensitivity or vice versa. In either case, you must first need to choose your units for sensitivity. The allowed units are:

- **Flux Density (mJy) -** In Jy (10^{-26} Watts $m^2 Hz^{-1}$), and as if measured from above the Earth's atmosphere.
- **Antenna Temp., Ta (mK) -** In mK, and as measured below the Earth's atmosphere.
- **Radiation Temp., Tr (mK)**In K, and as if measured from above the Earth's atmosphere (Default).

3.1 Observing Time from Desired Sensitivity

Enter the sensitivity that you wish to achieve in the Desired Sensitivity box in your chosen units.

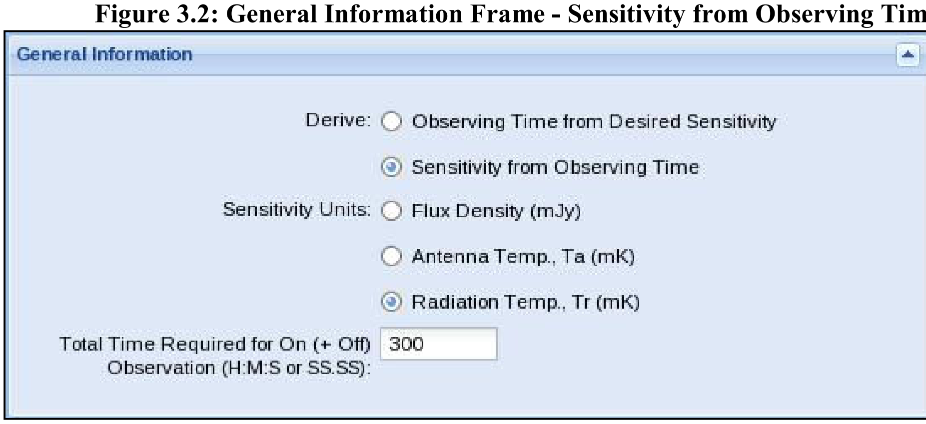
Figure 3.1: General Information Frame - Observing Time from Desired Sensitivity



3.2 Sensitivity from Observing Time

Enter the total time required for your observations. Units of time can be entered in seconds or sexegesimal format.

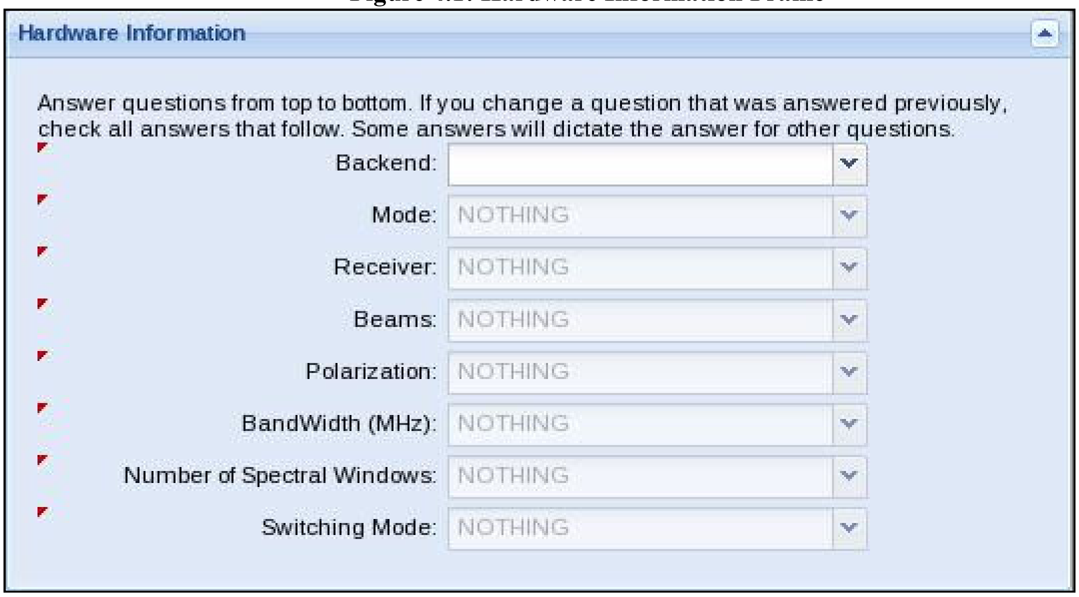
Figure 3.2: General Information Frame - Sensitivity from Observing Time



4. Hardware Information Frame

You will be asked a series of questions concerning your choice of hardware and the calculator will check the answers to make sure that they can be accommodated by the hardware. Only those questions needed for the sensitivity calculator will be asked. For example, if you select MUSTANG as your backend, then all subsequent choices will be filled out for you with the MUSTANG default values, and you will not be required (or able) to answer any further questions within this frame.

Figure 4.1: Hardware Information Frame



The available fields within the Hardware Information Frame are:

- **Backend** – You may currently choose from the following backends:
  - Caltech Continnum Backend (CCB)
  - FPGA Spectrometer
  - GBT Digital Continuum Reciever (DCR)
  - GBT Spectrometer
  - Mustang
  - Spectral Processor
  - Zpectrometer

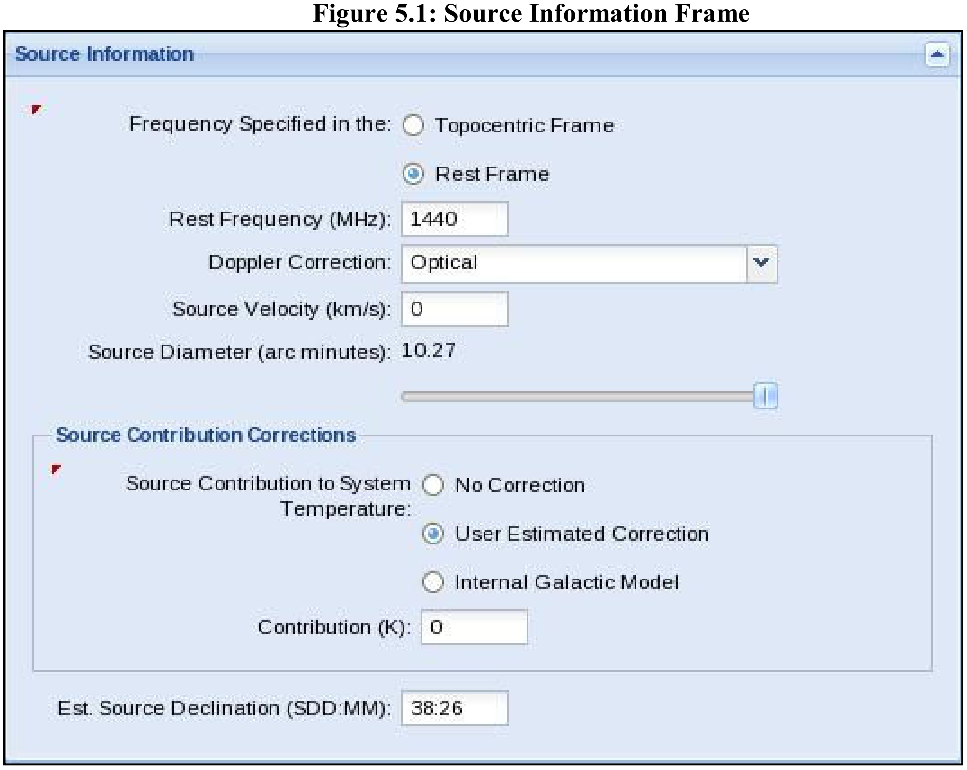
- **Mode –** This will be selected automatically depending on your choice of backend. The available modes currently available for each backend are:
  - Continuum – CCB, DCR and Mustang
  - Spectral Line – FPGA Spectrometer, GBT Spectrometer, Spectral Processor and Zpectrometer

- **Receiver** – If you have selected the FPGA Spectrometer, DCR, GBT Spectrometer or Spectral Processor as your backend then you will need to select which GBT receiver your wish to use.
- **Beams** – If the receiver you have selected is capable of using more than one beam then you will be prompted to enter how many beams you wish to use here.
- **Polarization -** You will be given a choice between Dual and Full for the GBT Spectrometer and FPGA Spectrometer, and between Cross_QU, Dual and Full with the Spectral Processor.
- **Bandwidth** – You may be given a choice of available bandwidths depending on which backend you have selected.
- **Spectral Windows** – You may be given a choice of how many spectral windows you wish to use for your observations.
- **Switching Mode** – Currently you may choose between 'In-Band Frequency Switching', 'Out-of-Band' Frequency Switching' and 'Position Switching'.

5. Source Information Frame

Since projects usually observe multiple sources, you may want to run the sensitivity calculator for each, but it is also acceptable to run the calculator for a representative source, or a representative set of source parameters. If the latter approach is taken then you will need to infer how that calculation for a single or representative source translates into the time or sensitivity requirements for the body of your sources.

Figure 5.1: Source Information Frame



User input is required for the following fields:

- **Topocentric Frame / Rest Frame** – You will be asked if the observing frequencies will be given in the line's reset frame or in the topocentric frame. The default is 'Rest Frame'

- **Topocentric / Rest Frequency (MHz)** – The default will be the middle of the selected receivers band.

- **Doppler Correction** – If you had previously selected 'Rest Frame' then you will be asked to select between an optical, radio or redshift Doppler correction. The default is 'Optical'.

- **Source Velocity (km/s) / Redshift** - Depending on the selected Doppler correction you will be asked to supply a representative source velocity or redshift. The default in either case is zero.

- **Source Diameter (arc minutes)** – Use the slider to set a representative size for your source. Available values range from 0 (point source) to the FWHM of the GBT beam.

- **Source Contribution Corrections** – You will be asked whether to apply a correction for the representative source's continuum level to the calculators system temperature calculations. The options are:

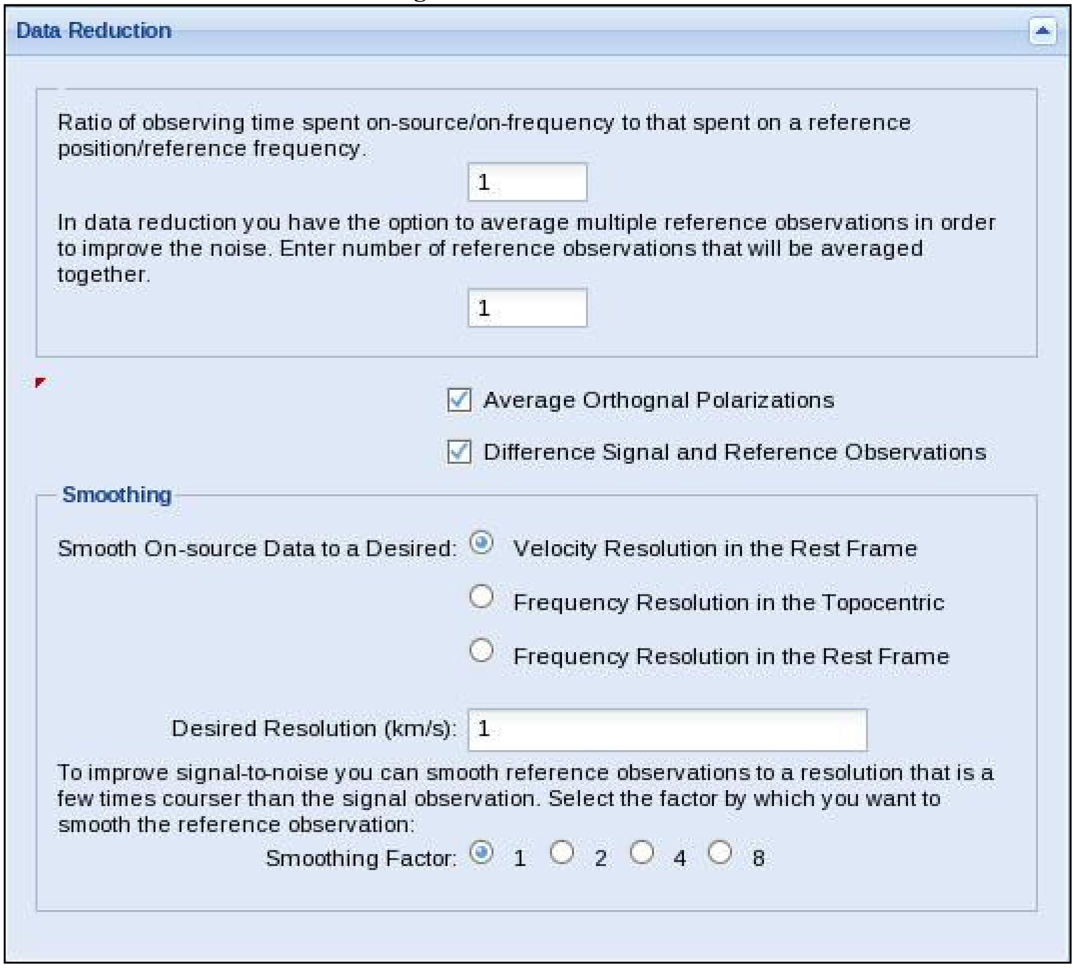
  - **No Correction** – Do not apply the correction
  - **User Correction** – If you select this options then you will be asked to provide the background level in the units that you have selected in the 'General Information' frame.
  - **Internal Galactic Model** – If selected then the calculator will ask for a representative J2000 Right Ascension of the source. The calculator will then augment the system temperature with the approximate continuum background from the Milky Way for the specified position using the 408 MHz survey of Haslam et al (1981, A&A, 100, 209)

- **Est. Source Declination (SDD:MM)** – Supply the calculator with a representative source Declination. The default is the latitude of Green Bank (38:26).

6. Data Reduction Frame

Observers have a number of choices in how they collect and reduce their data that significantly affect the time they will need for an experiment and the corresponding sensitivity they will achieve. Only those that are most common have been included in the calculator.

Figure 6.1: Data Reduction Frame



The available fields available for data reduction depend upon your answers in the hardware frame. Possible questions include:

- **Ratio of observing time spent on-source/on-frequency to that spent on a reference position/reference frequency** – If you have selected any switching mode other than total power, then you will be asked:
  - How many on-source or on-frequency (signal) observations they will use per off (reference) observation. The default is set to 1, that is, an on (signal) for every off (reference).
  - The ratio of time spent on your signal (on-source or on-frequency) observation to their reference observation. The default is set to 1.

- **Average Orthogonal Polarizations** – You will be offered this choice based on your hardware configuration. If applicable then this box will be checked by default.

- **Difference Signal and Reference Observations** – You will be asked whether you plan on differencing signal and reference observations. By default this is set to on.

- **Smoothing** – For spectral line observations you will be required to provide how you will smooth both the on-source and off-source data and whether you will be smoothing to a specified:

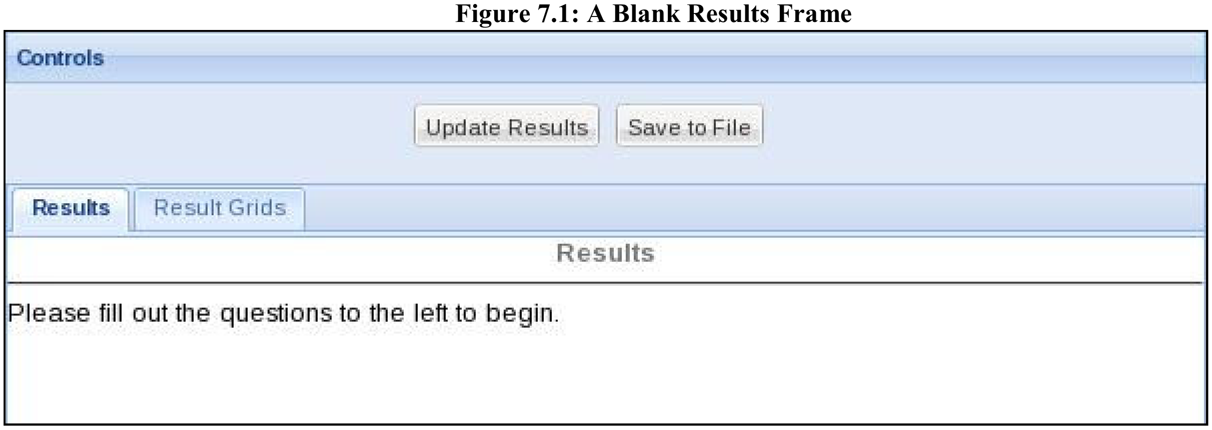
    - *Velocity Resolution* (km/s) in the source's rest frame (default)
    - *Frequency Resolution in the Topocentric* Frame (MHz)
    - *Frequency Resolution in the Rest Frame* (MHz)

- **Smoothing Factor** – For most observing that differences signal and reference observations, one can substantially reduce the noise by smoothing the reference observations more than the signal observations. This is an option provided by all GBTIDL calculation routines. In many cases, narrow lines can withstand more smoothing of the reference observation without compromising baseline shapes. The calculator asks how much one expects to smooth the 'off' (reference), relative to the 'on' (signal). The default is 1, meaning the reference observation will be smoothed to the same resolution as the signal observation.

7. Results Frame

The results frame on the calculator's right side is where all the output from the sensitivity calculator will be displayed. Figure 7.1 shows the results frame prior to any entries by the user.

Figure 7.1: A Blank Results Frame



7.1 Controls

The only interface with the results frame is through the two buttons under 'Control'.

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 - Press the 'Update Results' button to display all results that the sensitivity calculator can currently return based on the information you have given. The calculator will also prompt you to press this button if you have changed any of the entries in the user input frames on the left of the screen by displaying a red arrow next to any field that has been altered.

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 - If you press the 'Save to File' button then you will be given the option of saving the information displayed in the results frame as a text file. The file can then be attached to your proposal.

7.1 The Results Window

The results grid is for debugging as it presents all of the internal variables and values used by the calculator's underlying code. The results tab provides the most user friendly way to look at your results. Figure 7.2 gives an example of the 'Results' tab layout after all information has been provided by the user.

Figure 7.2: Results Frame – 'Results' Tab

Results

Derived Total Observing Time:	16.0 s
Time at Siginal Position or Frequency:	8.0 s
Time at Reference Position or Frequency:	8.0 s
Effective Integration Time:	4.0 s
FWHM Beamwidth:	8.7 '
Aperture Efficiency:	0.70
Extended Source Efficiency:	0.68
Confusion Limit:	119.68
Topocentric Frequency:	1420.406 MHz
Min. Topocentric Channel Width:	0.763 kHz
Desired Freq. or Vel. Resolution:	1.280000
Typical Air Mass:	2.1
Typical Atmospheric Attenuation:	1.015
Typical System Temperature:	16.6 K
Backend Sampling Efficiency (K1):	1.0320
Backend Channel Weighting (K2):	1.2100

Messages

Warning - Since source is extended, the calculated results are approximations.

Other Results

Typical Atmospheric Opacity:	0.007 Nepers
observing_method:	2
eta_dss:	0.97
eta_surf:	1.00
n_uncorr_samp:	4
Maximum Elevation:	90.0 d
max_el_rad:	1.571

Outputs from the sensitivity calculator in this mode are given under the following headings:

- **Results** – This section contains information that will be of use to you when writing your technical justification. If you are unable to derive a final sensitivity or total observing time, then you will need to include additional information in the user input frames to the left of the screen (and also remember to press the 'Update Results' button). You may also find it helpful to look under the 'User Input' heading (Also in the results frame) and scan it for any fields which appear to be blank, in which case you can enter that missing information in the relevant frame.

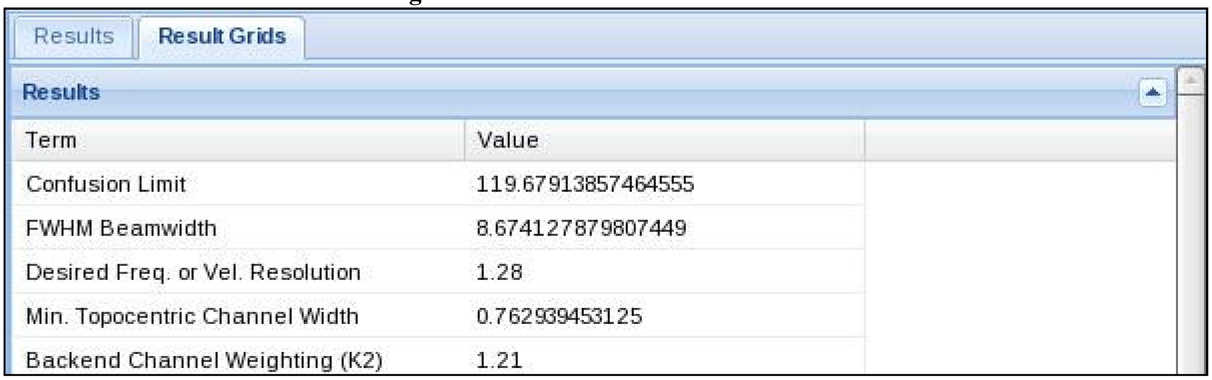
- **Messages** – Miscellaneous information that the calculator may deem important will be displayed under this section.

- **Other Results** – The values of constants used for the sensitivity calculators calculations. Details of these constants and algorithms can be found in the Appendix.

- **User Input** – All of the information entered into the General Information, Hardware Information, Source Information and Data Reduction frames will be displayed under this heading.

7.1.1 The Grid Results Tab

Figure 7.3: Results Frame – 'Result Grids' Tab



Term	Value
Confusion Limit	119.67913857464555
FWHM Beamwidth	8.674127879807449
Desired Freq. or Vel. Resolution	1.28
Min. Topocentric Channel Width	0.762939453125
Backend Channel Weighting (K2)	1.21

If you select the 'Result Grids' tab, then the output will be displayed in table format (Figure 7.3). Unlike the 'Results' tab, there are only two headings:

- **Results** – Contains all of the parameters that are given under the headings 'Results' and 'Other Results' in the Result Tab.

- **User Input** – All of the information entered into the General Information, Hardware Information, Source Information and Data Reduction frames will be displayed under this heading.

Appendix

A. Definition and Terms

Term	Definition
A	Representative atmospheric attenuation, $e^{\tau \cdot AirMass}$, for the typical weather condition and elevation of an observation at the user's observing frequency
AirMass	representative air mass through which the observations will be made
BW	Bandwidth in MHz, either native to the backend or the bandwidth to which the user smoothes
BW_{Ref}	Bandwidth in MHz that the reference (off) observation will be smoothed to
c	Speed of light in m s^{-1}
DishRadius	Illumination radius for the selected receiver in m
Δf_{REST}	Frequency resolution in the rest frame in MHz
ΔV_{REST}	Velocity resolution in the rest frame in m s^{-1}
El_{Min}, El_{Max}	Range in observing elevation in °
EST	Effective system temperature in K ($T_{sys} \cdot e^{\tau \cdot AirMass}$)
EST_0	Effective system temperature under the best possible weather conditions
EST_{TS}	Effective system temperature but augmented by the expect loss in efficiency due to tracking and surface errors
FeedTaper	Feed taper illumination of the reflector in dB.
Frequency	Topocentric frequency in MHz
FWHM	Full-width, half-maximum beam width in '
η	Efficiency which takes into consideration surface errors as a function of frequency and source size
η_0	Aperture efficiency at long wavelengths and receiver and frequency dependent
η_A	Aperture efficiency which takes into consideration surface errors as a function of frequency
η_{Dss}	Normalized observing efficiency, in units of time, as suggested by DSS simulations and the product η_{Track} η_{Surf} η_{Atm}
η_{Track}, η_{Surf}, η_{Atm}	The normalized observing efficiency, in units of time, as suggested by DSS simulations due to tracking errors (e.g., wind induced), thermal-induced surface errors, and atmospheric conditions.
θ_{Source}	Source size in '

k	Boltzmann's constant
$\mathbf{K_1}$	Sampling sensitivity of a backend and, thus, is hardware dependent
$\mathbf{K_2}$	Autocorrelation channel weighting factor for spectral line backends or a measure of the independence of samples for continuum observing; hence hardware dependent
$\mathbf{N_{RefAvrg}}$	Number of references observations that will be averaged together in the data reduction and used for every signal (on) observation
$\mathbf{N_{RefSmthAvrg}}$	Amount by which a reference observation is smoothed or the number of reference observations that are averaged together
$N_{Samp}^{\mathrm{Uncorr}}$	Degree to which backend inputs measure uncorrelated signals. (For example, = 2 when averaging orthogonal polarizations or when using nodding or in-band frequency switching)
$\mathbf{R_{SigRef}}$	Ratio of time spent on the signal observation to the time on the reference observatio
σ_S	Sensitivity in units of Jy (above atmosphere)
$\sigma_{T_R^*}$	Sensitivity in K in the T_R^* (above atmosphere) temperature scale
σ_{T_A}	Sensitivity in K in the T_A (below atmosphere) temperature scale
τ	Representative zenith atmospheric opacity in units of Nepers
τ_0	Best possible zenith atmospheric opacity in units of Nepers
$\mathbf{t_{eff}}$	Effective integration time in s - essentially the time that satisfies the radiometer equation and is related to the actual observing time in ways that depend upon the observing tactics
$\mathbf{t_{Sig}}$	Time spent in s on a source or signal position or frequency
$\mathbf{t_{Ref}}$	Time spent in s on a reference position or frequency
$\mathbf{t_{Total}}$	Total time in s needed to complete an observation
$\mathbf{T_{Atm}}$	Temperature of the atmosphere in K to use in an estimate of T_{Sys} -- approximate temperature of the atmospheric layer that is contributing most to the opacity
$\mathbf{T_{CMB}}$	The cosmic microwave background = 2.7 K
$\mathbf{T_{BG}}$	Either the continuum temperature in K of the user's source or the temperature in K of the galactic background in the direction of the observation
$\mathbf{T_{Rcvrl}}$	Contribution to T_{Sys} in K from the receiver
$\mathbf{T_{Sys}}$	Expected approximate system temperature in K at a representative elevation
V	Source velocity in m/s
z	Redshift (V/c)

B. Basic Equations

The algorithms used by the sensitivity calculator revolve around a modified version of the classic radiometer equation. The modified equation describes the theoretical noise one would obtain if observing *above the atmosphere with a given effective system temperature (EST_{TS} in K and, in brief, the classic T_{SYS} augmented by atmospheric attenuation and the loss in efficiency due to tracking and surface errors) and bandwidth (BW in MHz) for a given effective observation duration (t_{eff} in sec)*. If we ignore the contribution from 1/F noise, the radiometer equation in terms of theoretical noise above the atmosphere from a non-ideal telescope can be represented by:

$$\sigma_{T_R^*} = \frac{K_1 \cdot EST_{TS}}{\sqrt{K_2 \cdot N_{Samp}^{\text{Uncorr}} \cdot 10^6 \cdot BW \cdot t_{\text{eff}}}}$$

EST_{TS} is related to T_{SYS} in the way described below. N_{Samp}^{Uncorr} depends upon the details of the observing tactics. K_1 = sampling sensitivity of a backend and K_2 = autocorrelation channel weighting factor for spectral line backends, or a measure of the independence of samples for continuum observing. Both K_1 and K_2 are hardware and observing-mode dependent.

Since the calculator's default is to calculate time from a specified sensitivity, the calculator also uses the inversion of the equation to solve for the effective observing time.

$$t_{\text{eff}} = \left(\frac{K_1 \cdot EST_{TS}}{\sigma_{T_A}}\right)^2 \left(\frac{1}{K_2 \cdot 10^6 \cdot BW \cdot N_{Samp}^{\text{Uncorr}}}\right) \qquad \textbf{B-1}$$

Observers will be specifying (or request) sensitivities in either units of flux density (S in Jy = 10^{-26} Watts m^{-2} s^{-1}), temperature as defined by the T_R^* temperature scale (i.e., as if the observations were made above the Earth's atmosphere), or temperature as defined by the T_A temperature scale (i.e., as observed from the surface of the Earth). The calculator uses the standard conversions between these units of intensity:

$$\sigma_{T_A} = \frac{\eta \cdot \pi \cdot DishRadius^2}{2k \cdot e^{\tau \cdot AirMass}} \cdot \sigma_S \qquad \textbf{B-2}$$

$$\sigma_{T_A} = \frac{\eta}{e^{\tau \cdot AirMass}} \cdot \sigma_{T_R^*} \qquad \textbf{B-3}$$

$e^{\tau \cdot AirMass}$, hereafter called A, is the representative atmospheric attenuation for the elevation of the observation; k = Boltzman's constant; and η = the telescope's instrumental efficiency, as detailed below.

Most observing modes require differencing observations of the source (on or signal) position (or frequency) and a reference (or off) position (or frequency) where one may spend different amount of time on each. Since both the signal and reference observations have noise, the resulting difference will be nosier than the signal observation. To combat this extra noise, it's a common practice to smooth the reference observation or to average multiple reference observations. But, even with reference smoothing/averaging, the effective integration time is always less than the time spent on signal or reference and is even less than the time spent on both signal and reference combined.

Furthermore, the user isn't usually interested in the value for t_{eff} but, instead, is interested in the total time needed for an observation (t_{Total}). From the theory of the propagation of errors:

$$t_{\text{eff}} = \frac{t_{\text{Sig}} \cdot (N_{\text{RefSmthAvrg}} \cdot t_{\text{Ref}})}{t_{\text{Sig}} + (N_{\text{RefSmthAvrg}} \cdot t_{\text{Ref}})} \qquad \textbf{B-4}$$

The calculator starts off with a value of $N_{RefSmthAvrg} = 1$, which the user can then change. If there is no overhead involved in the observing, then the total time needed for an observation is $t_{Sig}+t_{Ref}$.

Users of the calculator that want to convert sensitivity into a time typically won't know either t_{Sig} or t_{Ref} but they will know, or their observing tactics will dictate, a value for:

$$R_{\text{SigRef}} = t_{\text{Sig}}/t_{\text{Ref}}, \qquad \textbf{B-5}$$

the ratio of the time spent on and off source (or frequency), a quantity provided by the user and whose default value is 1.

Finally, after a bit of algebra one derives the relationship between t_{eff} and t_{Total}:

$$t_{\text{Total}} = t_{\text{Sig}} + t_{\text{Ref}} = t_{\text{eff}} \cdot \frac{\left(R_{\text{SigRef}} + N_{\text{RefSmthAvrg}}\right)\left(R_{\text{SigRef}} + 1\right)}{R_{\text{SigRef}} \cdot N_{\text{RefSmthAvrg}}} \qquad \textbf{B-6}$$

Once t_{Total} is known, the calculator displays values for $t_{Sig} = t_{Total} \cdot R_{SigRef}/\left(R_{SigRef} + 1\right)$ and $t_{Ref} = t_{Total}/\left(R_{SigRef} + 1\right)$.

Then, one gets our final equations for converting between a user-specified σ_S, $\sigma_{T_R^*}$, or σ_{T_A} and the total time needed for an observation.

$$t_{\text{Total}} = \left(\frac{2k \cdot K_1 \cdot EST_{TS}}{\eta \cdot \pi \cdot DishRadius^2 \cdot \sigma_S}\right)^2 \left(\frac{\left(R_{\text{SigRef}} + N_{\text{RefSmthAvrg}}\right)\left(R_{\text{SigRef}} + 1\right)}{K_2 \cdot 10^6 \cdot BW \cdot N_{Samp}^{\text{Uncorr}} \cdot R_{\text{SigRef}} \cdot N_{\text{RefSmthAvrg}}}\right) \qquad \textbf{B-7}$$

$$t_{\text{Total}} = \left(\frac{K_1 \cdot EST_{TS}}{\eta \cdot \sigma_{T_R^*}}\right)^2 \left(\frac{\left(R_{\text{SigRef}} + N_{\text{RefSmthAvrg}}\right)\left(R_{\text{SigRef}} + 1\right)}{K_2 \cdot 10^6 \cdot BW \cdot N_{Samp}^{\text{Uncorr}} \cdot R_{\text{SigRef}} \cdot N_{\text{RefSmthAvrg}}}\right) \qquad \textbf{B-8}$$

$$t_{\text{Total}} = \left(\frac{K_1 \cdot EST_{TS}}{\sigma_{T_A} \cdot A}\right)^2 \left(\frac{\left(R_{\text{SigRef}} + N_{\text{RefSmthAvrg}}\right)\left(R_{\text{SigRef}} + 1\right)}{K_2 \cdot 10^6 \cdot BW \cdot N_{Samp}^{\text{Uncorr}} \cdot R_{\text{SigRef}} \cdot N_{\text{RefSmthAvrg}}}\right) \qquad \textbf{B-9}$$

Some observing modes do not require taking a reference observation, which implies that $t_{Total}=t_{eff}$. In these cases:

$$t_{\text{Total}} = \left(\frac{2k \cdot K_1 \cdot EST_{TS}}{\eta \cdot \pi \cdot DishRadius^2 \cdot \sigma_S}\right)^2 \left(\frac{1}{K_2 \cdot 10^6 \cdot BW \cdot N_{Samp}^{\text{Uncorr}}}\right) \qquad \textbf{B-10}$$

$$t_{\text{Total}} = \left(\frac{K_1 \cdot EST_{TS}}{\eta \cdot \sigma_{T_R^*}}\right)^2 \left(\frac{1}{K_2 \cdot 10^6 \cdot BW \cdot N_{Samp}^{\text{Uncorr}}}\right) \qquad \textbf{B-11}$$

$$t_{\text{Total}} = \left(\frac{K_1 \cdot EST_{TS}}{\sigma_{T_A} \cdot A}\right)^2 \left(\frac{1}{K_2 \cdot 10^6 \cdot BW \cdot N_{Samp}^{\text{Uncorr}}}\right) \qquad \textbf{B-12}$$

Simple inversions of these equations allow one to go from a user-supplied total time to the sensitivity that time will achieve. The inversions of the four equations are, respectively:

With reference observations:

$$\sigma_S = \left(\frac{2k \cdot K_1 \cdot EST_{TS}}{\eta \cdot \pi \cdot DishRadius^2}\right) \sqrt{\frac{\left(R_{\text{SigRef}} + N_{\text{RefSmthAvrg}}\right)\left(R_{\text{SigRef}} + 1\right)}{K_2 \cdot 10^6 \cdot BW \cdot N_{Samp}^{\text{Uncorr}} \cdot R_{\text{SigRef}} \cdot N_{\text{RefSmthAvrg}} \cdot t_{\text{Total}}}} \quad \textbf{B-13}$$

$$\sigma_{T_R^*} = \left(\frac{K_1 \cdot EST_{TS}}{\eta}\right) \sqrt{\frac{\left(R_{\text{SigRef}} + N_{\text{RefSmthAvrg}}\right)\left(R_{\text{SigRef}} + 1\right)}{K_2 \cdot 10^6 \cdot BW \cdot N_{Samp}^{\text{Uncorr}} \cdot R_{\text{SigRef}} \cdot N_{\text{RefSmthAvrg}} \cdot t_{\text{Total}}}} \quad \textbf{B-14}$$

$$\sigma_{T_A} = \left(\frac{K_1 \cdot EST_{TS}}{A}\right) \sqrt{\frac{\left(R_{\text{SigRef}} + N_{\text{RefSmth}}\right)\left(R_{\text{SigRef}} + 1\right)}{K_2 \cdot 10^6 \cdot BW \cdot N_{Samp}^{\text{Uncorr}} \cdot R_{\text{SigRef}} \cdot N_{\text{RefSmthAvrg}} \cdot t_{\text{Total}}}} \quad \textbf{B-15}$$

Without reference observations:

$$\sigma_S = \left(\frac{2k \cdot K_1 \cdot EST_{TS}}{\eta \cdot \pi \cdot DishRadius^2}\right) \sqrt{\frac{1}{K_2 \cdot 10^6 \cdot BW \cdot N_{Samp}^{\text{Uncorr}} \cdot t_{\text{Total}}}} \quad \textbf{B-16}$$

$$\sigma_{T_R^*} = \left(\frac{K_1 \cdot EST_{TS}}{\eta}\right) \sqrt{\frac{1}{K_2 \cdot 10^6 \cdot BW \cdot N_{Samp}^{\text{Uncorr}} \cdot t_{\text{Total}}}} \quad \textbf{B-17}$$

$$\sigma_{T_A} = \left(\frac{K_1 \cdot EST_{TS}}{A}\right) \sqrt{\frac{1}{K_2 \cdot 10^6 \cdot BW \cdot N_{Samp}^{\text{Uncorr}} \cdot t_{\text{Total}}}} \quad \textbf{B-18}$$

C. 1/F Gain Instabilities

All receivers have a sensitivity limit from 1/F gain instabilities but a few systems that were explicitly designed for continuum observations (e.g., Mustang and the Ka-Band receiver but only when used with the CCB backend) have much less of an issue with 1/F instabilities. These gain instabilities essentially place an upper limit on $t_{\text{Total}} \cdot BW$ – beyond a certain point, increasing the bandwidth or the amount of observing time does not improve one's sensitivity.

Since we have yet to characterize the 1/F characteristics of most of our receivers, the sensitivity calculator can only provide rough guidelines as to when an observation might hit the 1/F limit. The calculator warns a user that they have probably exceeded the 1/F limitations of the receiver whenever $t_{\text{Total}} \cdot BW$ exceeds certain values. The calculator uses the following preliminary guesses for these limits:

Table 1

System	t_{Total}·BW upper limit
Mustang	10^{11}
Ka-Band with CCB	10^{11}
All Other Systems	10^{9}

There are some techniques for overcoming the 1/F limit, such as making multiple maps with fast slewing or switching. If the user's planned observation exceeds the 1/F limit, the user probably should consider justifying in their proposal how they expect to overcome the 1/F limit and confer with the support staff.

D. Confusion Limit

Desired sensitivities in some observing modes may not be reachable due to confusion within the beam from multiple background sources. The confusion limit depends upon the topocentric observing frequency (in MHz), the FWHM beam width of the telescope (in arc minutes) at that frequency, and the user's chosen units for sensitivity. Standard wisdom recommends that one stay under five times the confusion limit for a reliable detection. The calculator estimates the limit using the following equations (Condon 2002):

Table 2

Units	5x Confusion Limit	
S	$\dfrac{0.13 \cdot FWHM^2}{Frequency^{0.7}}$	**7-19**
T_R^*	$\dfrac{0.13 \cdot \pi \cdot (DishRadius \cdot FWHM)^2}{2k \cdot Frequency^{0.7}}$	**7-20**
T_A	$\dfrac{0.13 \cdot \eta \cdot \pi \cdot (DishRadius \cdot FWHM)^2}{2k \cdot Frequency^{0.7} \cdot A}$	**7-21**

If the user is entering a sensitivity to derive a t_{Total}, then the calculator warns the user whenever the user enters a sensitivity that is smaller than the corresponding value from the above table. If the user enters a time in order to derive sensitivity, then the calculator warns the user whenever the calculated sensitivity is smaller than the corresponding value from the above table. In both cases, the warning presents the value of the confusion limit.

The current release of the calculator cannot distinguish whether or not the specified observing tactics are or are not limited by confusion. Instead, the calculator assumes the user is best qualified to make that judgment. So, it issues a warning for all observing tactics once the above limit is exceeded.

E. Determining Values for Various Quantities

Equations B-7 through B-18 depend upon values for EST_{TS} which in turn depends upon τ, air mass (i.e., elevation), receiver temperature, T_{Atm}, background source temperature, spillover and the cosmic microwave background as well as the yet-to-be defined quantities η_{Track} and η_{Surf}. The calculator, of course, doesn't know the details of the weather conditions or elevation ranges over which the observations will happen. Instead, it makes the assumption that the observations will happen under typical opacity and wind conditions, as determined by DSS simulations, for the chosen semester, topocentric frequency, and receiver. It also assumes observations will be taken symmetrically around the meridian to receiver minimum elevation.

E.1 EST_{TS}

The effective system temperature, augmented for a non-ideal telescope, used in equations B-7 through B-18 depends upon what has been called in various memos the Effective System temperature, EST. EST= $T_{sys} \cdot A$, that is, the system temperature one would use in the radiometer equation to get the same sensitivity as if the observations were taken above the atmosphere.

In addition to atmospheric loses, the calculator also uses estimates for the relative loss of efficiencies due to tracking or surface errors (η_{Track}, η_{Surf}, see below for derivations) from winds or daytime observing. The DSS definitions of these quantities provides the description of how to augment EST for the extra loss in efficiency

$$EST_{TS} = \frac{EST}{\sqrt{\eta_{Track}\eta_{Surf}}} = \frac{T_{sys} \cdot A}{\sqrt{\eta_{Track}\eta_{Surf}}} \tag{7-22}$$

The DSS simulators can in principle provide the calculator with enough information that one can directly estimate a typical value for EST_{TS} for most observing setups. The DSS quantity η_{DSS} is an observing efficiency with respect to time that is normalized to the best conditions that are possible for the observing frequency, receiver, and source elevation. That is, a value of $\eta_{DSS} = 0.5$ suggests one will need twice as much observing time to achieve the same sensitivity as one would under the best opacity, wind, surface, … conditions. η_{DSS} is the product of the observing efficiency for atmospheric conditions (η_{Atm}), tracking errors due to the telescope's pointing accuracy under various wind conditions (η_{Track}) and surface errors (η_{Surf}).

By the DSS definition, $\eta_{Atm} = (EST_0/EST)^2$, where EST_0 is the effective system temperature for the best possible conditions. Thus:

$$\eta_{DSS} = \eta_{Track}\eta_{Surf}\left(\frac{EST_0}{EST}\right)^2 = \left(\frac{EST_0}{EST_{TC}}\right)^2 \tag{E-23}$$

One cannot substitute $EST_{TS} = EST_0/\sqrt{\eta_{DSS}}$ into equations B-7 through B-18 since this would not take into consideration the affects of any strong background source (with $T_{R*=}$ T_{BG}) and, thereby, is too pessimistic and estimate. For example, observing the moon (T_{BG}≈300 K) with our 22 GHz receiver under typical weather conditions (τ=0.06, air mass=2) with no winds should have an EST_{TS}~360 K, yet inverting equation E-23 gives either EST_{TS}~60 K or ~815 K, depending upon assumptions made in how to include background sources. To compensate for a background source one uses:

$$EST_{TS} = \frac{T_{BG}}{\sqrt{\eta_{Track}\eta_{\mathrm{Surf}}}} + \frac{EST_0}{\sqrt{\eta_{DSS}}} \tag{E-24}$$

The DSS simulators supply values for η_{DSS} and the product η_{Track} η_{Surf}, all of which can be assumed to be frequency and receiver dependent. EST_0 is defined in the next section. The user either supplies an estimate for T_{BG} or asks the calculator to provide an estimate of the galactic background for the specified Right Ascension and Declination and observing frequency using the 408 MHz observations of Haslam et al (1981).

E.2 EST₀

EST_0 is obtained from:

$$EST_0 = \left(T_{Rcvr} + T_{Spill} + T_{Atm}\right) \cdot e^{\tau_0 \cdot \mathrm{AirMass}} - (T_{Atm} - T_{CMB}) \quad \textbf{E-25}$$

where τ_0 is the best possible opacity at the observing frequency and AirMass is the ***typical*** air mass for the range of desired elevations.. The calculator assumes T_{Spill} (= 3 K) and T_{CMB} (=2.7 K) are receiver and frequency independent. T_{Rcvr} is receiver and frequency dependent and take on values provided by the receiver engineers. Since τ_0 and T_{Atm} are frequency and weather dependent, the calculator uses values derived from historical weather data averaged over five years year. (Due to the NRAO's semester scheduling system that begin and end in mid winter/summer, using yearly averages provides a sufficiently accurate values. Values for τ_0, T_{Rcvr} , and T_{Atm} are stored with a frequency resolution of 1000 MHz.

E.3 Atmospheric Attenuation (A)

Note that equations B-9, B-12, B-15, B-18, and E-24 require $e^{\tau \text{ AirMass}}$, that is the expected, typical (not best) atmospheric attenuation. With a bit of algebra:

$$A = e^{\tau \cdot Airmass} = \frac{EST_0\sqrt{\frac{\eta_{Track}\eta_{\mathrm{Surf}}}{\eta_{DSS}}} + (T_{Atm} - T_{CMB})}{T_{Rcvr} + T_{Spill} + T_{Atm}} \quad \textbf{E-26}$$

E.4 η

If the user has specified that they will be observing a point source, the value to use for η in the above equations is the telescope's nominal night-time aperture efficiency (η_A), based on an rms surface error of 220 µm:

$$\eta = \eta_A = \eta_0 \cdot \mathrm{e}^{-\left(9.22\times10^{-6} \cdot Frequency\right)^2} \quad \textbf{E-27}$$

In the future η_0 will change with frequency as it depends upon the details of feed illumination. Until then, a value of 0.71 is being used.

For non-point-like sources, the calculator assumes a disc source distribution with a diameter θ_{Source}. It augments the value for η_A using equation 12 of Baars (1973):

$$\eta = \eta_A \frac{1 - e^{-(\theta_{Source}/1.2FWHM)^2}}{(\theta_{Source}/1.2FWHM)^2} \quad \textbf{E-28}$$

E.5 Average Air Mass

The calculator needs to determine a typical value for the air mass under which one can expect an observation to be run. The air mass is taken as a weighted average over the expected elevation range of an observation. It is sufficiently accurate for the sensitivity calculator to use csc(El) as an approximation for air mass. From the source declination, the calculator determines the elevation at source transit (upper transit if the source is circumpolar). In the future, the calculator will use DSS simulations to determine the minimum elevation under which it will most likely schedule an observation. Until then, the calculator uses the approximate latitude of the GBT (38.43°) and the following estimates:

If Declination < latitude:	El_{Max}= 90 – latitude + Declination El_{Min}=5
If Declination < 90 – latitude:	El_{Max}= 90 –Declination + latitude El_{Min}=5
If Declination > 90 – latitude:	El_{Max}= 90 –Declination + latitude El_{Min}=max(5,latitude – 90 + Declination)

From these, the weighted average air mass displayed and used by the calculator is:

$$AirMass = \frac{57.29 \cdot ln\left(\frac{\tan(El_{Max}/2)}{\tan(El_{Min}/2)}\right)}{El_{Max} - El_{Min}} \qquad \textbf{E-29}$$

Note that if $El_{Min} = El_{Max}$,, then the calculator uses AirMass = $1/\sin(El_{Min})$

E.1 Opacity (τ)

Note that opacity is never directly used by the calculator (only A is used directly) but a value for opacity is supplied as a help for the user in planning observations. An estimate of the expected opacity for the typical weather conditions for the typical observing elevation:

$$\tau = \frac{ln(A)}{AirMass} \qquad \textbf{E-30}$$

E.2 T_{SYS}

Note that T_{SYS} is never directly used by the calculator (only EST_{TS} and EST_0 are used directly) but a value for T_{SYS} is supplied as a help for the user in planning observations. The calculator provides a typical T_{Sys} for the typical weather and elevation, by combining the results of equations E-24 and E-26:

$$T_{Sys} = \frac{EST_{TS} \cdot \eta_{Track} \cdot \eta_{Surf}}{A} \qquad \textbf{E-31}$$

E.3 K_1 and K_2

Values for K_1 and K_2 are backend and backend mode dependent. Values are retrieved from the calculator's internal information according to the answers users have supplied to the hardware questions.

For the GBT Spectrometer (ACS), the value of K_1 depends upon whether the observations are made in 3- or 9-levels modes. The 200 and 800 MHz bandwidth modes of the correlator only use the 3-level mode but the 12.5 and 50 MHz bandwidth allows one to pick either 3 or 9 level sampling. Only those observations that need the highest frequency or velocity resolutions should use 3-level sampling since it is far less efficient than 9-level sampling. The breaking point as to whether an observation should use 3- or 9-level depends upon the user's expected bandwidth after smoothing (BW) and the specified number of spectral windows ($N_{Windows}$) and feeds (N_{Feeds}) in the following way:

Table 3

	$BW/(N_{Windows}\ N_{Feeds})$			
200 or 800 MHz	Any	3-level	1.235	1.21
50 MHz	< 0.76 kHz	3-level	1.235	1.21
	>= 0.76 kHz	9-level	1.032	1.21
12.5 MHz	< 0.19 kHz	3-level	1.235	1.21
	>= 0.19 kHz	9-level	1.032	1.21

For the remaining backends, the calculator uses:

Table 4

Backend	K_1	K_2
Mustang, Radar, CCB, Mark V, GUPPI, DCR, Zpectrometer	1	1
Spectral Processor	1.30	1.21

E.4 $N_{RefSmthAvrg}$

The value for $N_{RefSmthAvrg}$ is obtained from:

$$N_{\mathrm{RefSmthAvrg}} = N_{RefAvrg} \cdot BW_{Ref}/BW \quad \textbf{E-32}$$

where $N_{RefAvrg}$, and BW_{Ref} are answers from the data reduction section of the calculator.

E.5 N_{Samp}^{Uncorr}

$N_{\text{Samp}}^{\text{Uncorr}}$ represents how the sensitivity improves by averaging data from orthogonal polarizations or by use of different observing methods. $N_{\text{Samp}}^{\text{Uncorr}}$ represents the degree to which the data streams from samplers are uncorrelated and is a product of terms that can be represented by:

$$N_{\text{Samp}}^{\text{Uncorr}} = \text{DualPol} \cdot \text{ObervingMethod} \quad \text{E-33}$$

- DualPol = 2 if the user has specified they will be averaging polarizations, otherwise = 1.
- ObservingMethod = 2 if the user has specified in-band frequency switching and any of the 'nodding' observation types. Otherwise = 1.

E.6 FWHM beam size

The FWHM beam width in ' of the GBT depends upon the feed illumination pattern and observing frequency (Goldsmith, 1987, 2002).

$$FWHM = \frac{3437.7 \cdot c \cdot [1.02 + 0.0135 \cdot \text{FeedTaper(dB)}]}{10^6 \cdot Frequency \cdot 2 \cdot DishRadius} \quad \text{E-34}$$

Mustang and future beam forming arrays will have illuminations that are not Gaussian and may not cover the full aperture of the GBT. In the future, the internal database will need to hold some description of each receiver's illumination. Until then, the calculator uses 13 dB for the feed taper for all receivers and a 50-m dish radius. Thus,

$$FWHM \approx {}^{12.3 \cdot 10^3}\!/_{\text{Frequency}} \quad \text{E-35}$$

E.7 Topocentric frequency, frequency resolution, velocity resolution, and velocity coverage

The calculator provides values for the following quantities as a way to help observers plan their observations as well as to provide values it needs for various calculations.

1. To derive the ***highest*** topocentric frequency spacing the backend will allow, the calculator divides the bandpass width selected by the user by the maximum number of channels that the user's configuration will allow.
2. To derive an approximate topocentric frequency from the user-specified rest frequency and velocity or redshift:

Table 5

Velocity Def.	**Topocentric Freq.**
Radio	$RestFreq \cdot (1 - V/c)$
Optical	$RestFreq/(1 + V/c)$
Redshift	$RestFreq/(1 + z)$

3. To convert a rest frame frequency resolution (Δf_{REST}) to a topocentric frequency resolution:

Table 6

Velocity Def.	**Topocentric Freq. Resolution**
Radio	$\Delta f_{REST} \cdot (1 - V/c)$
Optical	$\Delta f_{REST}/(1 + V/c)$
Redshift	$\Delta f_{REST}/(1 + z)$

4. To convert a rest frame velocity resolutions (ΔV_{REST}) to a topocentric frequency resolution:

Table 7

Velocity Def.	**Topocentric Freq. Resolution**
Radio	$RestFreq \cdot \Delta V_{REST}/c$
Optical	$\dfrac{RestFreq \cdot \Delta V_{REST}}{c \cdot (1 + V/c)^2}$
Redshift	$\dfrac{RestFreq \cdot \Delta V_{REST}}{c \cdot (1 + z)^2}$