Exploring the Sun and its effects on the
Earth's atmosphere and physical environment...

HIGH ALTITUDE OBSERVATORY

Planetary Dynamos

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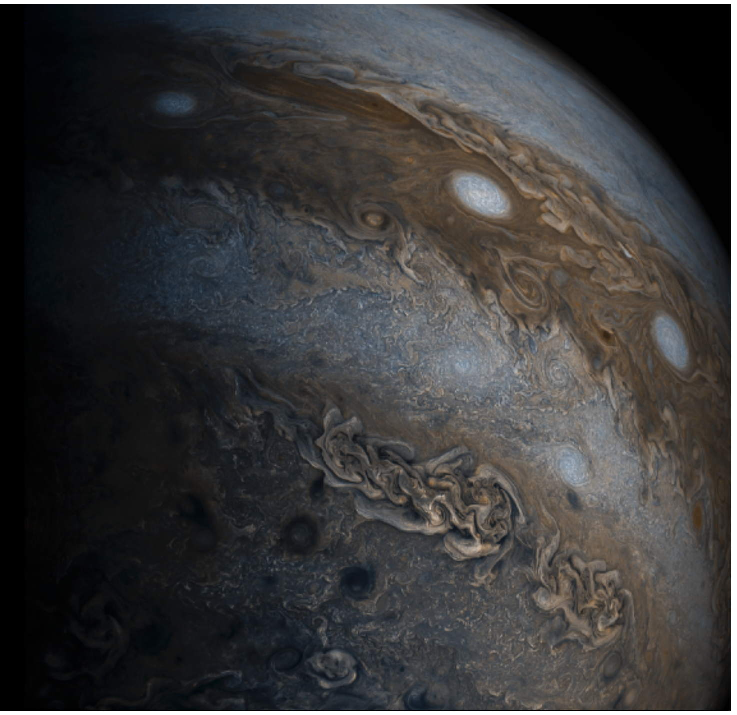


High Altitude Observatory (HAO) – National Center for Atmospheric Research (NCAR)

NCAR

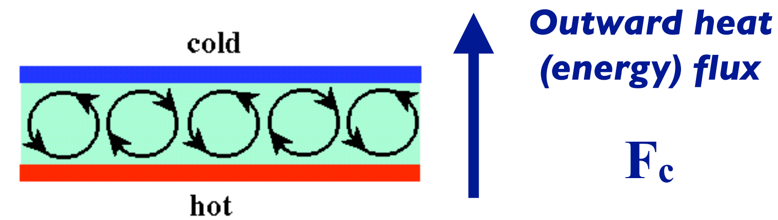
Outline

I) What is a Dynamo?
- Magnetic field creation vs dissipation
- Conditions for a planetary dynamo

II) A whirlwind tour of the Solar System
- Observed magnetic fields
- Internal structure

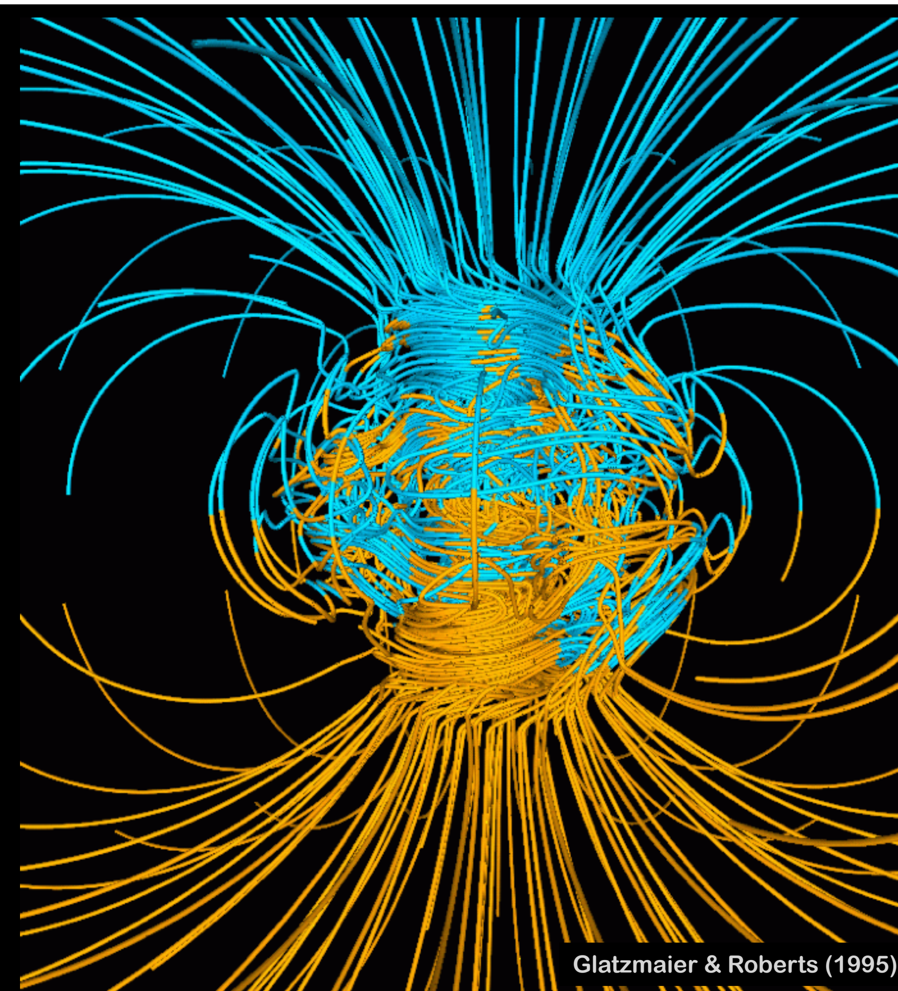
III) Convection in Rotating Spheres
- Dynamical balances
- Columns and waves

IV) Numerical Models
- General trends
- Case Studies (Earth, Jupiter…)



What is a (hydromagnetic) Dynamo?

An object (such as a star or a planet or a lab experiment) that converts the **kinetic energy** *of* **fluid motions** *into* **magnetic energy**



Glatzmaier & Roberts (1995)

MHD Magnetic Induction equation

$$\boxed{\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \times (\mathbf{v} \times \mathbf{B} - \eta \boldsymbol{\nabla} \times \mathbf{B})}$$

Comes from Maxwell's equations (Faraday's Law and Ampere's Law)

$$\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} = -\boldsymbol{\nabla} \times \mathbf{E}$$

$$\boldsymbol{\nabla} \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J}$$ ***(Assumes v << c)***

And Ohm's Law

$$\tilde{\mathbf{J}} = \sigma \tilde{\mathbf{E}}$$

electrical conductivity

Magnetic diffusivity

$$\eta = \frac{c^2}{4\pi\sigma}$$

Creation and destruction of magnetic fields

$$\boxed{\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla}\times\left(\mathbf{v}\times\mathbf{B} - \eta\boldsymbol{\nabla}\times\mathbf{B}\right)}$$

Source of Magnetic Energy

Sink of Magnetic Energy

How would you demonstrate this?

Creation and destruction of magnetic fields

$$\frac{\partial \mathbf{B}}{\partial t} = \mathbf{\nabla} \times (\mathbf{v} \times \mathbf{B} - \eta \mathbf{\nabla} \times \mathbf{B})$$

Source of Magnetic Energy

Sink of Magnetic Energy

How would you demonstrate this?

$$E_m = \frac{B^2}{8\pi}$$

Creation and destruction of magnetic fields

$$\boxed{\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \times (\underbrace{\mathbf{v} \times \mathbf{B}}_{\text{Source of Magnetic Energy}} - \underbrace{\eta \boldsymbol{\nabla} \times \mathbf{B}}_{\text{Sink of Magnetic Energy}})}$$

Source of Magnetic Energy

Sink of Magnetic Energy

How would you demonstrate this?

$$E_m = \frac{B^2}{8\pi}$$

(Hint: have a sheet handy with lots of vector identities!)

Creation and destruction of magnetic fields

$$\boxed{\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla}\times\left(\mathbf{v}\times\mathbf{B} - \eta\boldsymbol{\nabla}\times\mathbf{B}\right)}$$

Source of Magnetic Energy

Sink of Magnetic Energy

$$\frac{\partial E_m}{\partial t} = -\boldsymbol{\nabla}\cdot\mathbf{F}_P - \frac{\mathbf{v}}{c}\cdot\left(\mathbf{J}\times\mathbf{B}\right) - \Phi_o$$

Poynting Flux

$$\mathbf{F}_P = \mathbf{E}\times\mathbf{B} = \left[\frac{\eta}{c}\mathbf{J} - \frac{1}{4\pi}\left(\mathbf{v}\times\mathbf{B}\right)\right]\times\mathbf{B}$$

Ohmic Heating

$$\Phi_o = \frac{4\pi\eta}{c^2}J^2$$

When you dot this with B and integrate over volume you get three terms:
1) conversion of KE into ME (

Creation and destruction of magnetic fields

$$\boxed{\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B})}$$

Source of Magnetic Energy $\sim U\,B / D$

Sink of Magnetic Energy $\sim \eta\,B / D^2$

$$\mathrm{Rm} = \frac{UD}{\eta}$$

If Rm >> 1 the source term is much bigger than the sink term

Creation and destruction of magnetic fields

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B})$$

Source of Magnetic Energy ~ U B / D

Sink of Magnetic Energy ~ η B / D²

$$\mathrm{Rm} = \frac{UD}{\eta}$$

If Rm >> 1 the source term is much bigger than the sink term

....Or is it???

Creation and destruction of magnetic fields

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B})$$

Source of Magnetic Energy $\sim U\,B\,/\,D$

Sink of Magnetic Energy $\sim \eta\,B\,/\,\delta^2$

δ ***can get so small that the two terms are comparable***

It's not obvious which term will "win" - it depends on the subtleties of the flow, including geometry & boundary conditions

Creation and destruction of magnetic fields

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B})$$

Source of Magnetic Energy
~ U B / D

Sink of Magnetic Energy
~ η B / δ^2

What is a Dynamo? (A corollary)

A dynamo must sustain the magnetic energy (through the conversion of kinetic energy) against Ohmic dissipation

The need for a Dynamo

If v = 0 and η = constant then the induction equation becomes

$$\frac{\partial \mathbf{B}}{\partial t} = -\eta \boldsymbol{\nabla} \times \boldsymbol{\nabla} \times \mathbf{B} = \eta \nabla^2 \mathbf{B}$$

The field will diffuse away (dissipation of magnetic energy) on a time scale of $\tau_d \approx \frac{D^2}{\eta}$

A more careful calculation for a planet gives $\tau_d \approx \frac{R^2}{\pi^2 \eta}$

Earth: τ_d ~ 80,000 yrs

Jupiter: τ_d ~ 30 million yrs

Planetary fields must be maintained by a dynamo or they would have decayed by now!

Where does the magnetic energy go? Into internal energy
pi^2 is about an order of magnitude faster than the naive estimate

Conditions for a Planetary (or Stellar) Dynamo

Absolutely necessary

- **An electrically conducting fluid**
  - Stars: plasma
  - Terrestrial planets: molten metal (mostly iron)
  - Jovian planets: metallic hydrogen (maybe molecular H)
  - Ice Giants: water/methane/ammonia mixture
  - Icy moons: salty water
- **Fluid motions**
  - Usually generated by buoyancy (convection)
- **Rm >> 1**
  - Too much ohmic diffusion will kill a dynamo

Rm >> 1 is necessary. It's not technically sufficient but in practice most

The dynamo that produces Jupiter's magnetic field could reside in this metallic hydrogen region of the planet's interior. The molecular hydrogen envelope could be electrically conducting enough near the base of the layer to support dynamo action.

The diffusivity of Jupiter increases rapidly with radius beyond the metallic H region. The bottom part of the molecular H region may be conductive enough to support dynamo action but the upper atmosphere is not

water, methane ammonia at high pressure & temperature can conduct – though not as well as liquid iron

For theorems relating to item 3, see the Encyclopedia of G&P. e.g. for a sphere with a constant eta, Rm has to be bigger than pi^2. The pi^2 comes from the decay rate of a dipole as given on the previous slide

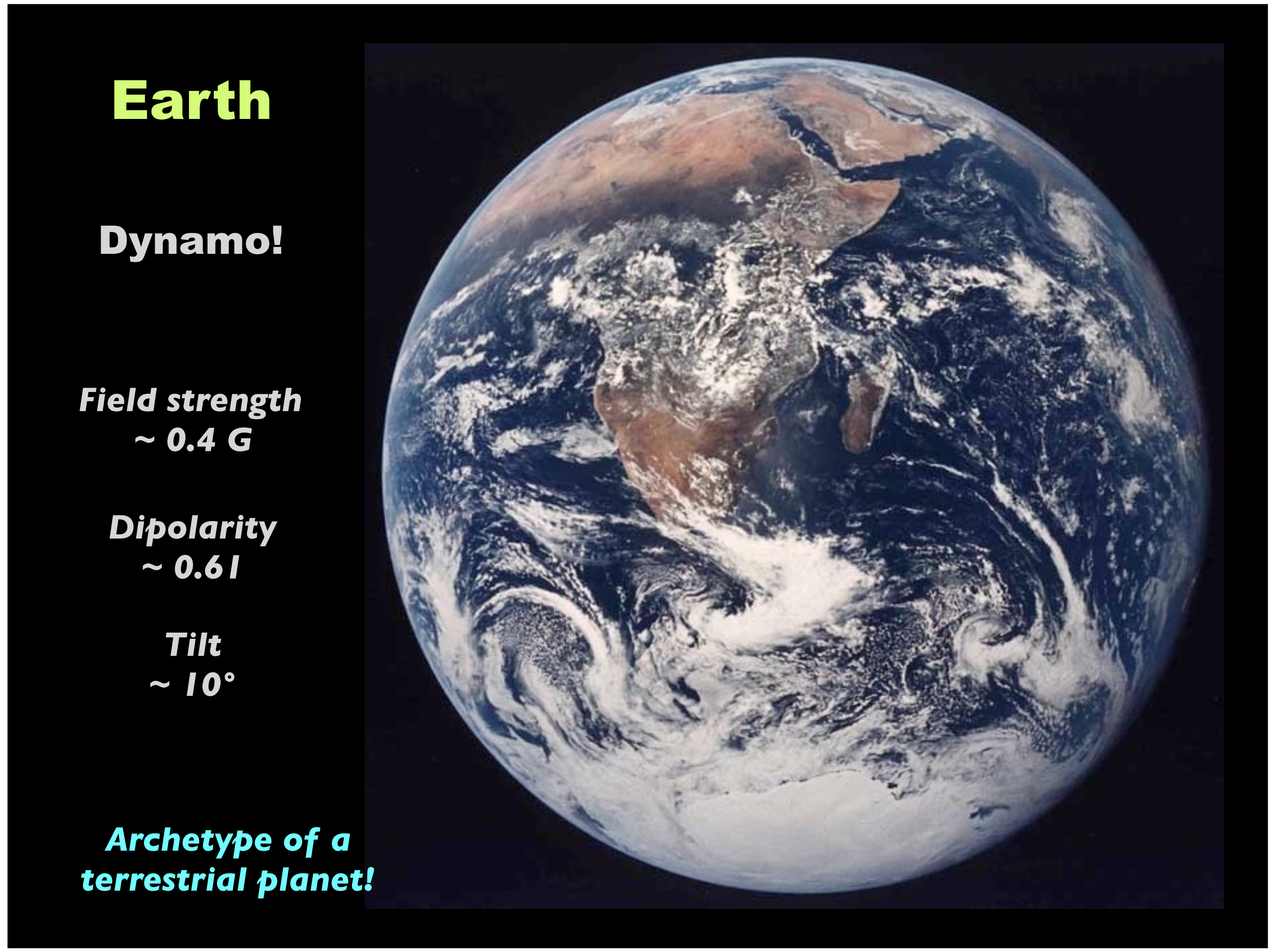
Conditions for a Planetary (or Stellar) Dynamo

Not strictly necessary but it (usually) helps

- **Rotation**
  - **Good**: helps to build strong, large-scale fields (promotes magnetic self-organization)
  - **Bad**: can suppress convection (though this is usually not a problem for planets)
- **Turbulence** (low viscosity / Re >> 1)
  - **Good**: Chaotic fluid trajectories good at amplifying magnetic fields (chaotic stretching)
  - **Bad**: can increase ohmic dissipation

Rm >> 1 is necessary. It's not technically sufficient but in practice most

Rotation helps unless it suppresses convection
Turbulence helps with chaotic trajectories but increases dissipation



Stevenson lists the average field strength as about 0.5 G but 1G is a good rule of thumb for comparing to other planets

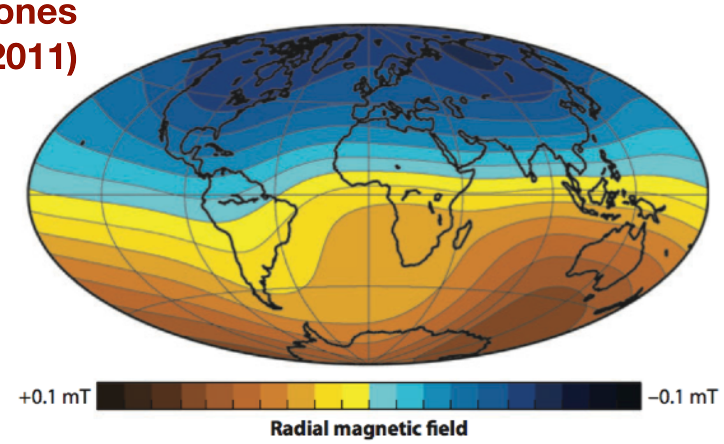
Earth

Magnetometer used by Alexander von Humboldt in his Latin America expedition of 1799-1804

Direct measurements of Earth's magnetic field date back to the early 1500's, with a boost in the early 1800's with the Magnetic Crusade led by Sabine in England and Gauss and Weber in Germany

Today we also have satellite measurements

Jones (2011)



Longer time history can be inferred from measurements of magnetic signatures in crustal rocks

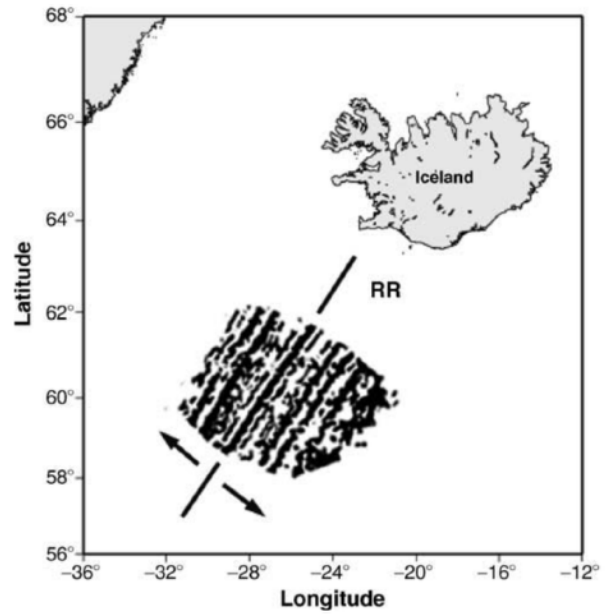
Note that it's dipole-dominated, with a tilt (america to Australia)

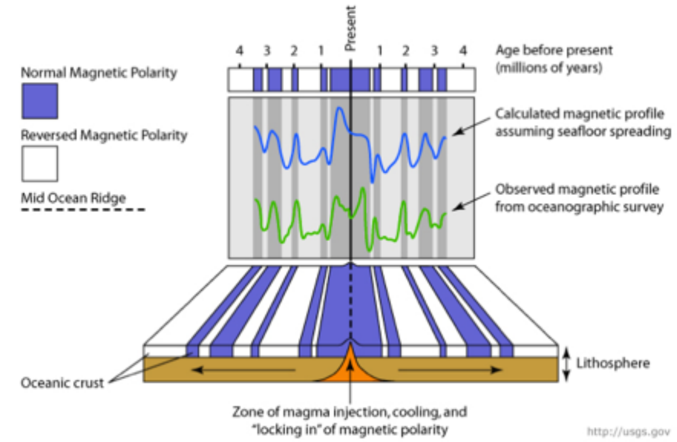
Satellites provide the best global coverage

Magnetic crusade led to the development of a worldwide network of magnetic observatories that still exists today.
This is supplemented by measurement made on ships dating back all the way to 1510.
By the second half of the 16th century compasses were widely employed to measure declination. Inclination, which required measuring the dip of the magnetic vector below the horizontal plane, was determined on a number of vessels in the late 16th and early 17th centuries. However since it never attained a place in standard navigational practice, inclination measurements are much more scarce. Useful relative intensity measurements were made only after 1790 while absolute intensity measurements were first carried out by Gauss in 1832.

paleomagnetism

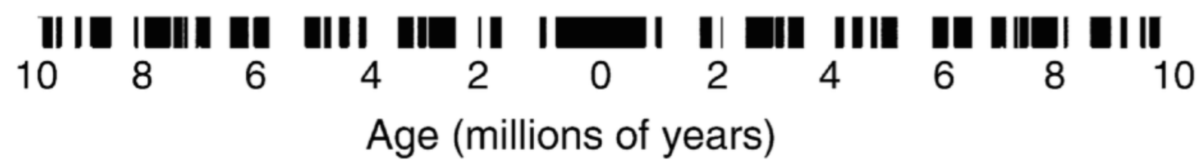
Heirtzler et al (1960's)





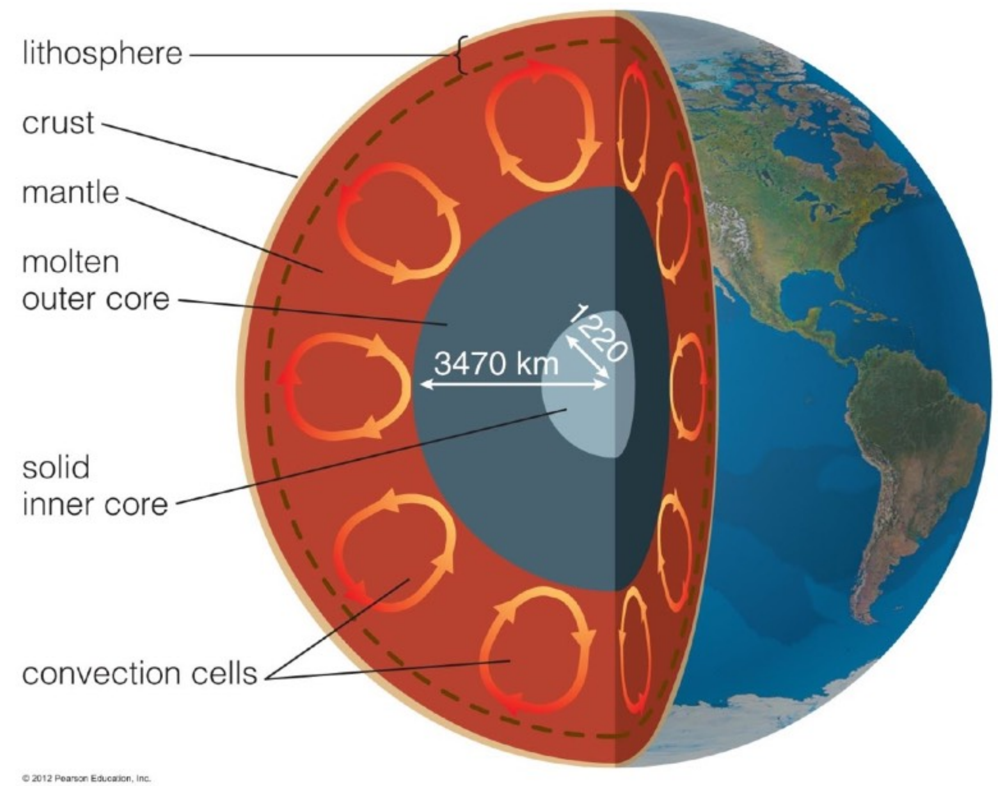
Magnetic poles flip every ~ 200,00 years on average, but randomly

Irregular reversals!



Get a reference for this and better plots - maybe from the USGS?

Earth



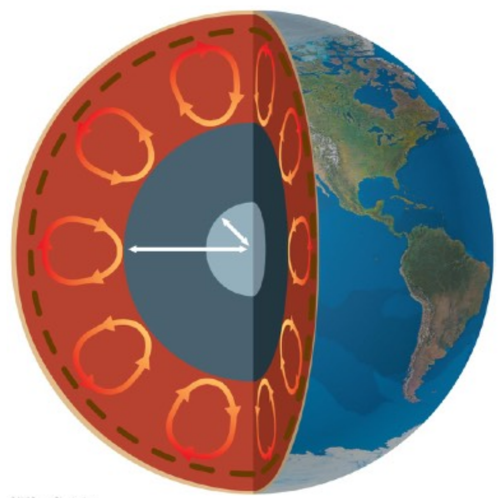
Mantle convection responsible for plate tectonics but not the geodynamo

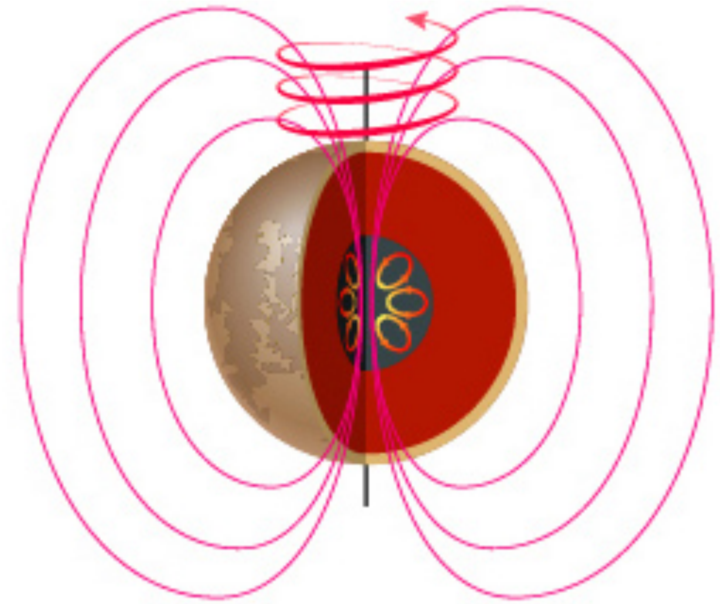
Why?

Earth

Mantle
non-conducting, slow

Overturning time
~100 million years





Outer Core
conducting, fast

Overturning time
~500 years

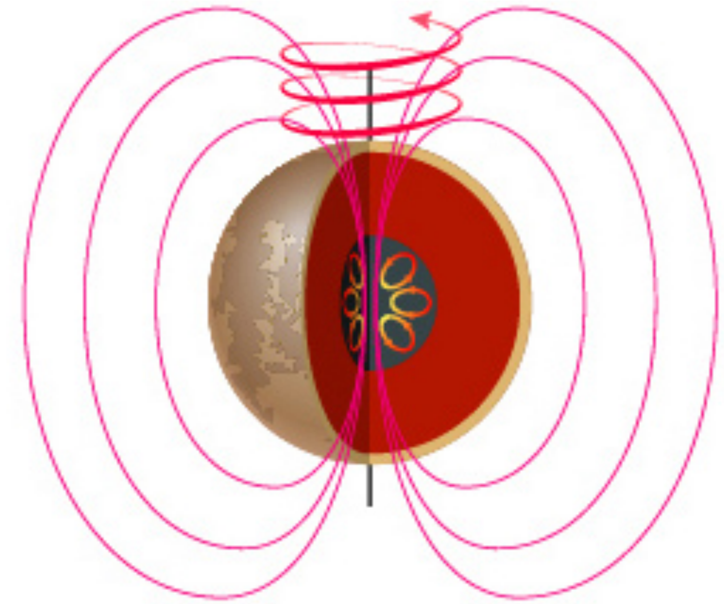
Earth

Rotational influence quantified by

Rossby number

$$\mathrm{Ro} = \frac{U}{2\Omega D} = \frac{1}{4\pi}\,\frac{P_{rot}}{\tau_c}$$

$$\mathrm{Ro} \sim 4 \times 10^{-7}$$

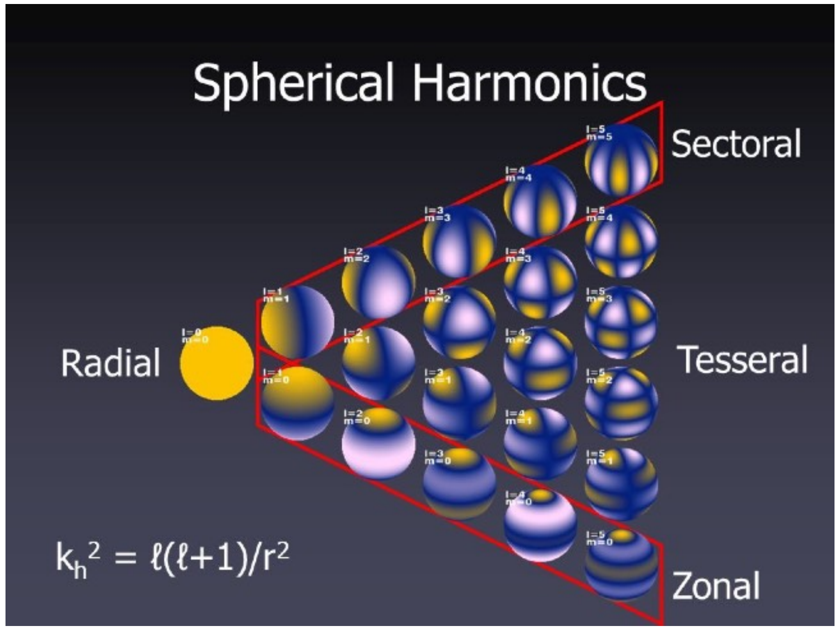


Outer Core
conducting, fast

Overturning time ~500 years

Earth

Spherical Harmonic expansion of the surface field allows for a backward extrapolation to the core-mantle boundary (CMB)



R. Townshend (Wisconsin)

Assuming no currents in the non-conducting mantle & crust

$$B_r \propto r^{-(\ell+2)}$$

If you assume that the mantle is non-conducting, then you can just use a potential field representation to track it back down to the core

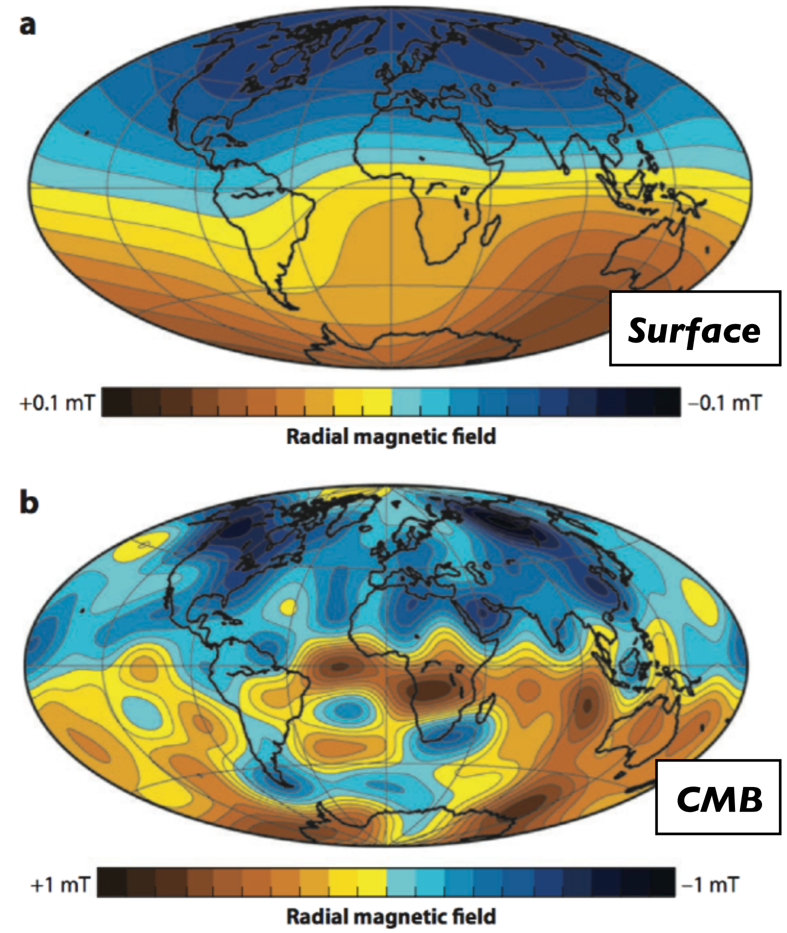
Jones (2011)

Earth

$$B_r \propto r^{-(\ell+2)}$$

Dipole dominates at large distances from the dynamo region ~ r^{-3}

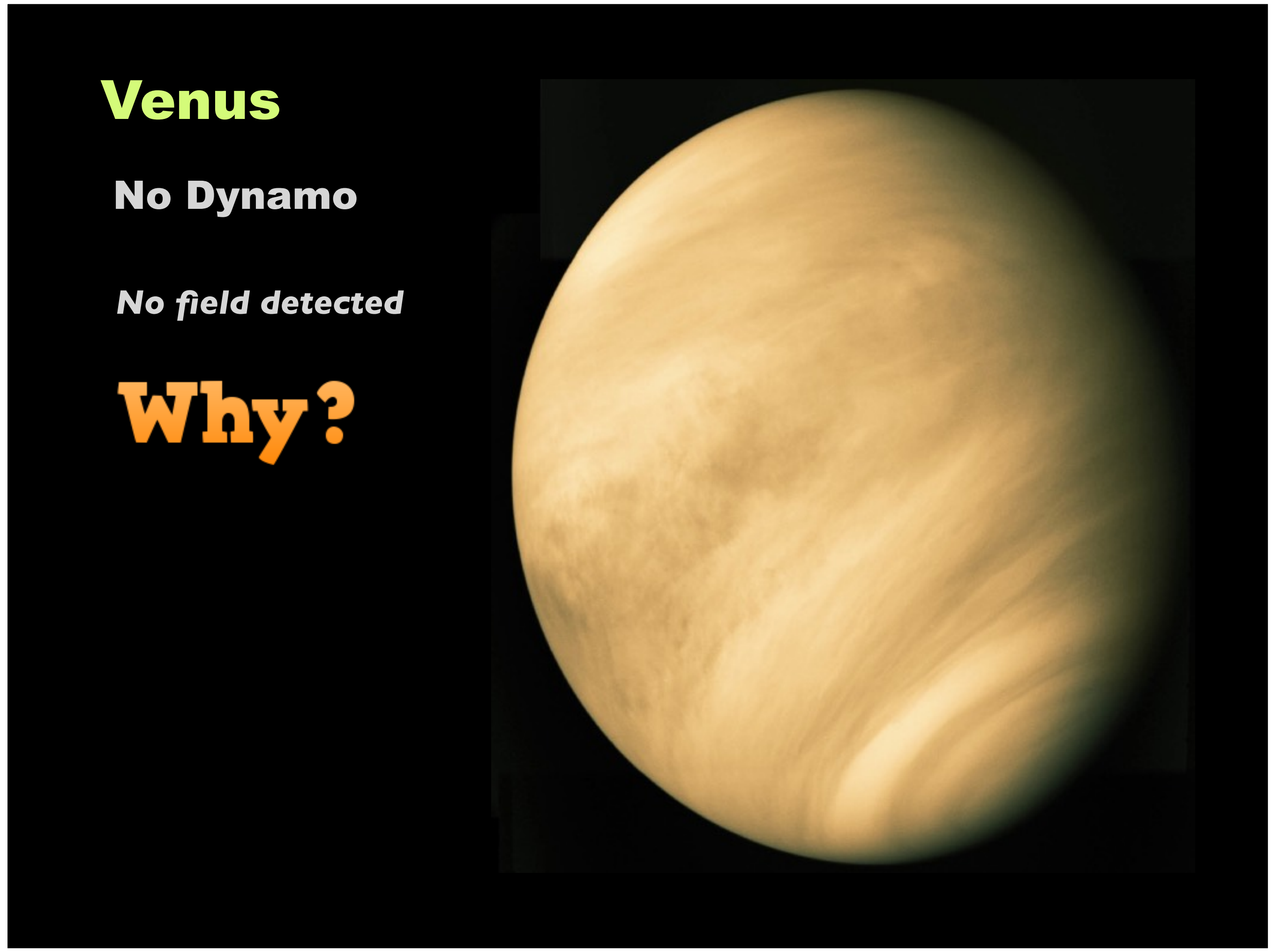
Time evolution of surface field can be used to infer flows at the CMB



Earth

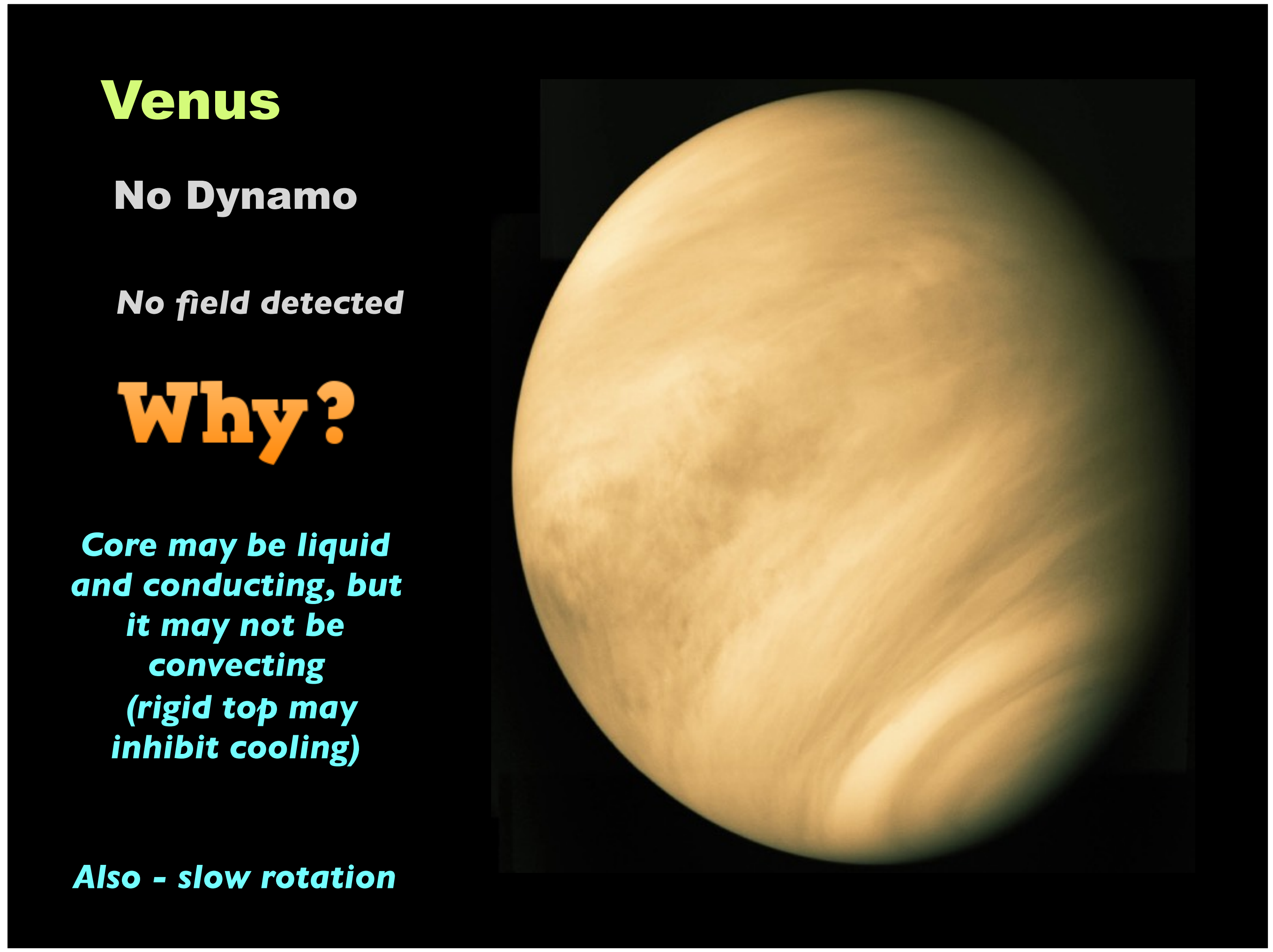
- Energy sources for convective motions
  - **Outward heat transport by conduction**
    - Cooling of the core over time
    - Proportional to the heat capacity
  - **Latent heat**
    - Associated with the freezing (phase change) of iron onto the solid core
  - **Gravitational Differentiation**
    - Redistribution of light and heavy elements, releasing gravitational potential energy
  - **Radioactive Heating**
    - Energy released by the decay of heavy elements

I think the least important is the gravitational differentiation
(more important in the early earth)



Why is it not convecting? It should have all the same energy sources, right? Well, the conduction is less efficient because the crust is rigid (no plate tectonics). This may mean that the interior is fully molten – no core – so the latent heat release is also less efficient. So the first two items in our list on the previous slide are diminished. The fact that Venus does not have a dynamo may be interpreted as evidence that these two terms are more important than the differentiation and radioactivity terms.

The most probable interpretation is that the liquid core of Venus does not convect. This could arise because there is no inner core or because the core is currently not cooling. The absence of an inner core is plausible if the inside of Venus is hotter than the corresponding pressure level of Earth. This can arise because Earth has plate tectonics, which eliminates heat more efficiently than a stagnant lid form of mantle convection. Alternatively (or in addition), Venus' core may not be cooling at present because it is undergoing a transition in convective style following a resurfacing event 700 Ma ago.



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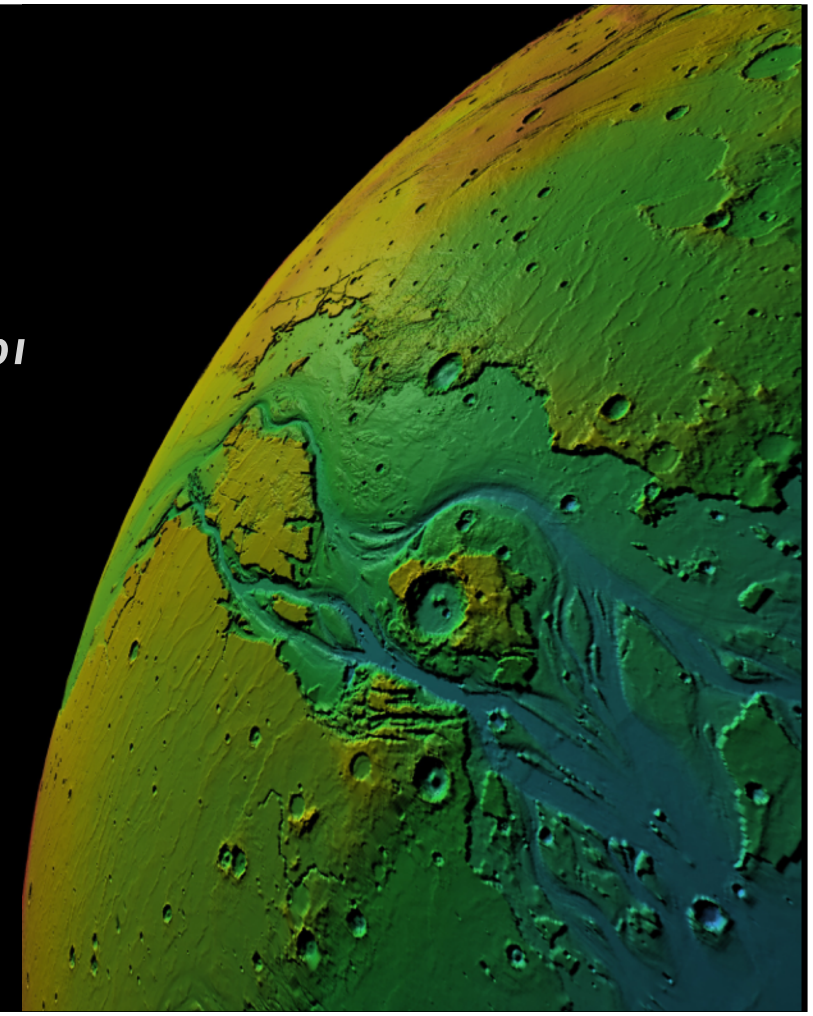
Mars

No Dynamo

Fields patchy, reaching ~ 0.01 G in spots but no dipole

Why?



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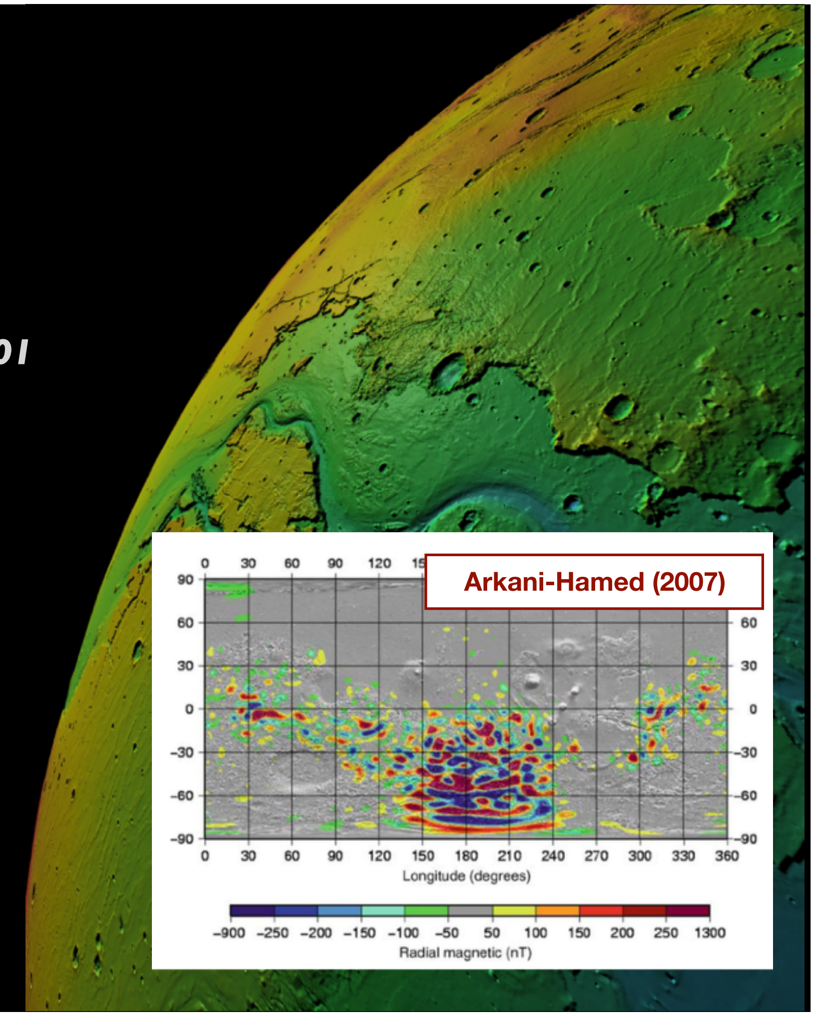


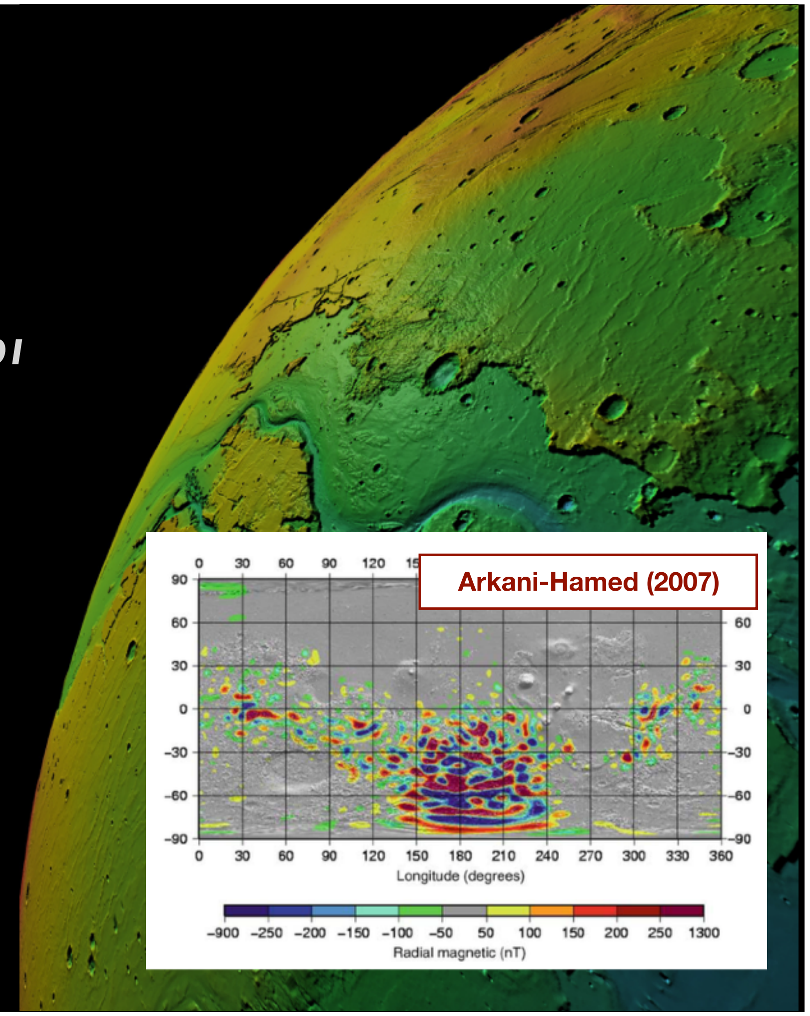
Mars

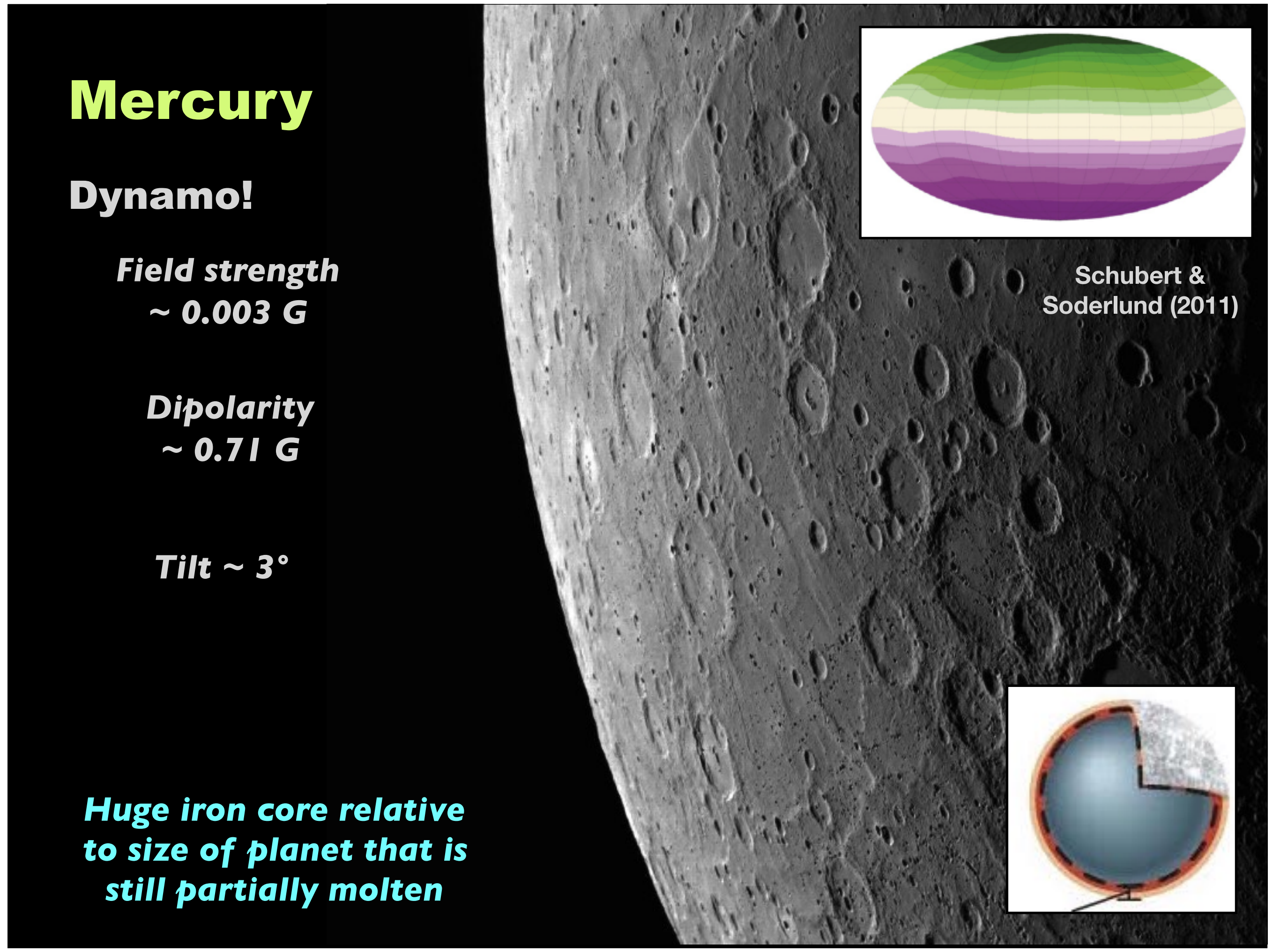
No Dynamo

Fields patchy, reaching ~ 0.01 G in spots but no dipole

Why?

It had a dynamo in the past (remnant crustal magnetism) but it cooled off fast, freezing out its molten core

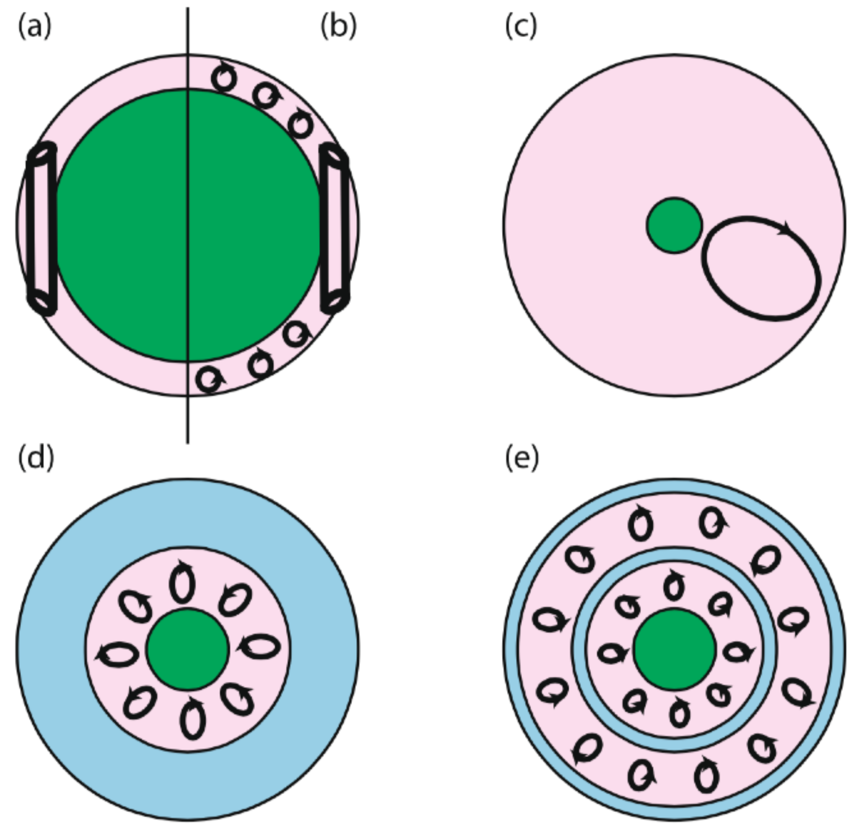




Libration observations demonstrate that Mercury's core is at least partially liquid, suggesting that the magnetic field may be the result of an active dynamo (Margot et al. 2007).

highly symmetric about the rotation axis (Figure 4) and asymmetric about the planetary equator—stronger in the northern hemisphere than in the southern hemisphere [Anderson et al., 2011, 2012; Johnson et al., 2012].

But we're still not really sure what's going on!



Stanley & Glatzmaier (2009)

Stevenson says Mercury is "not well characterized or understood"

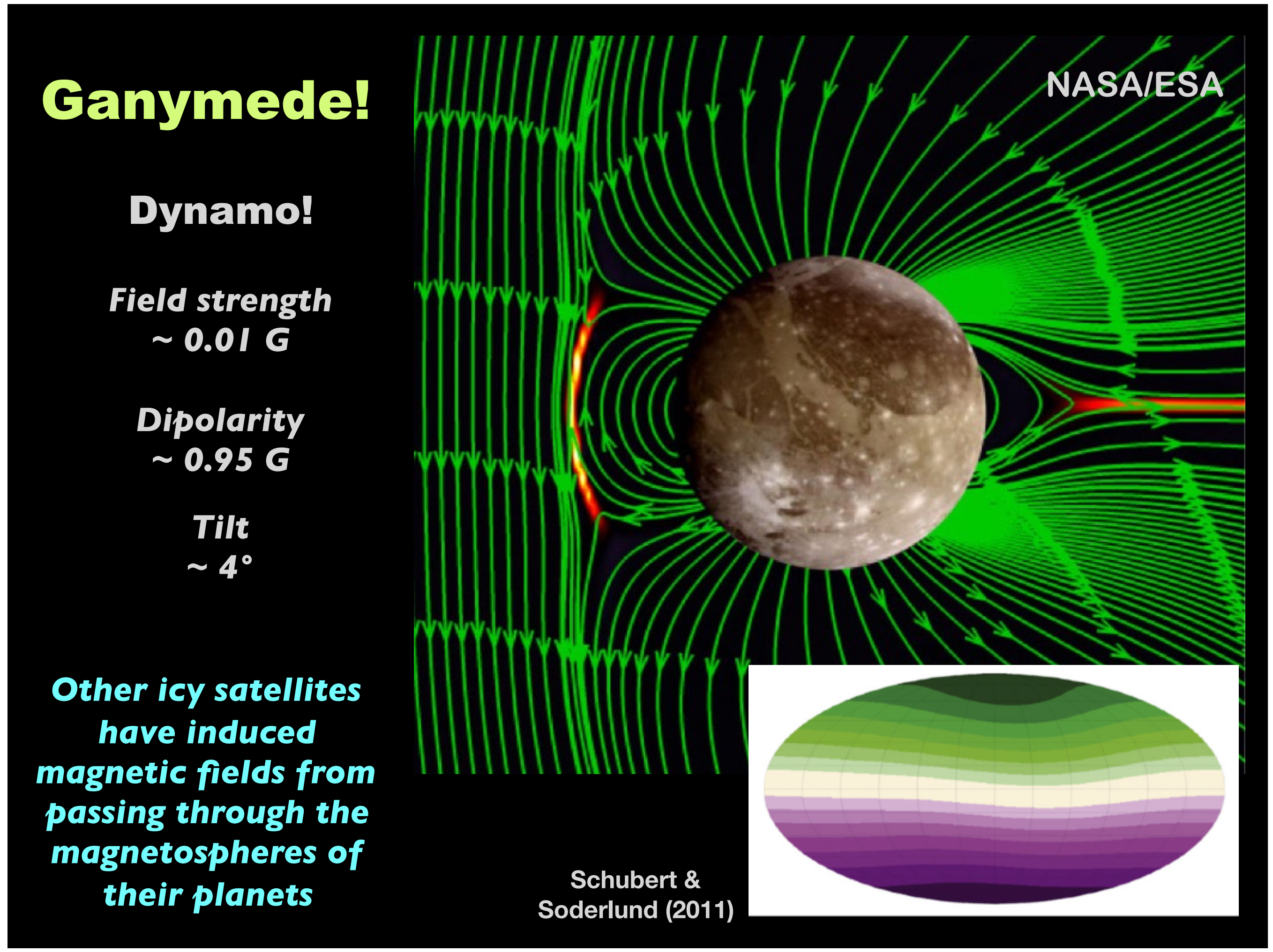
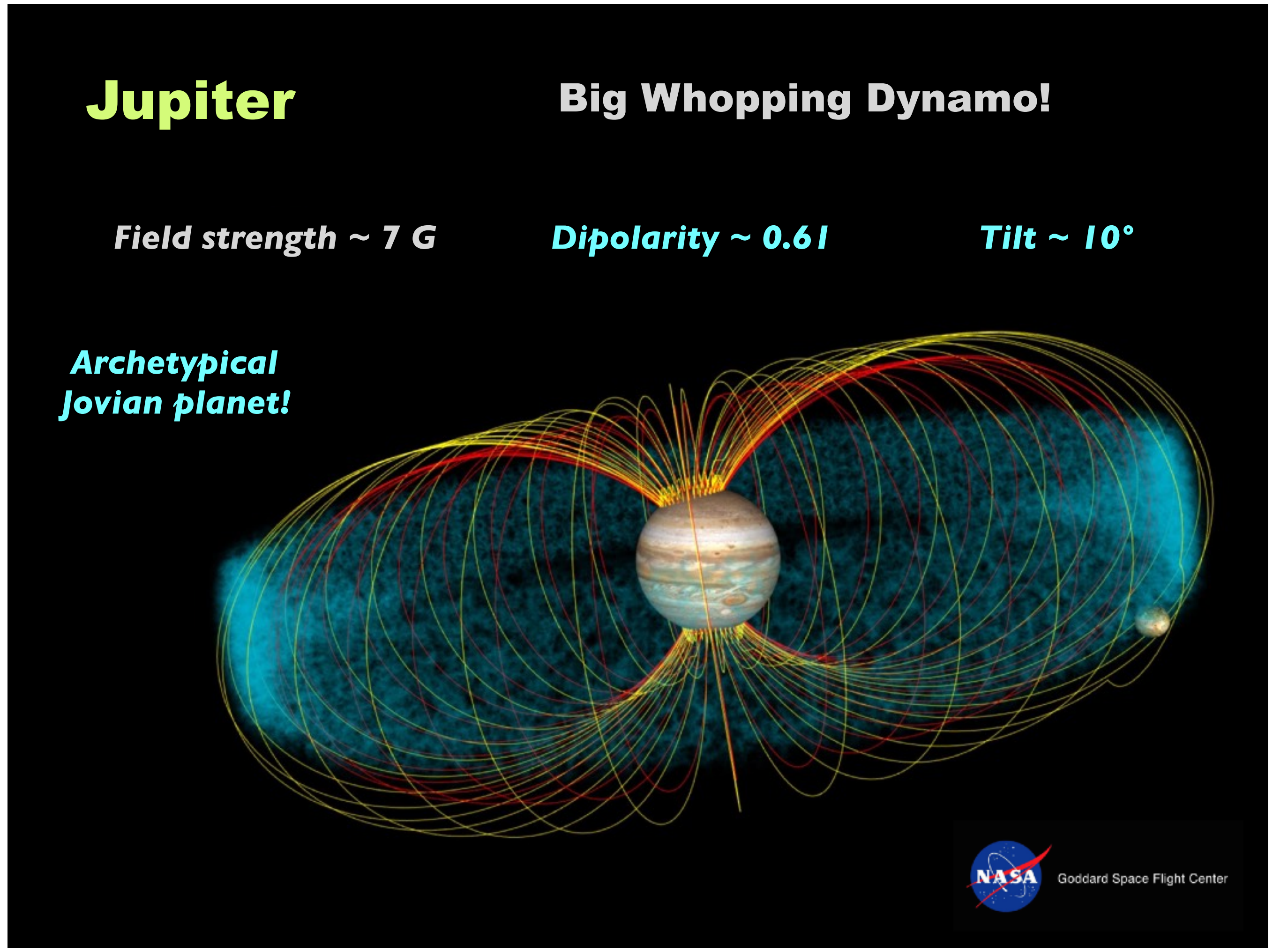


Figure from NASA/ESA

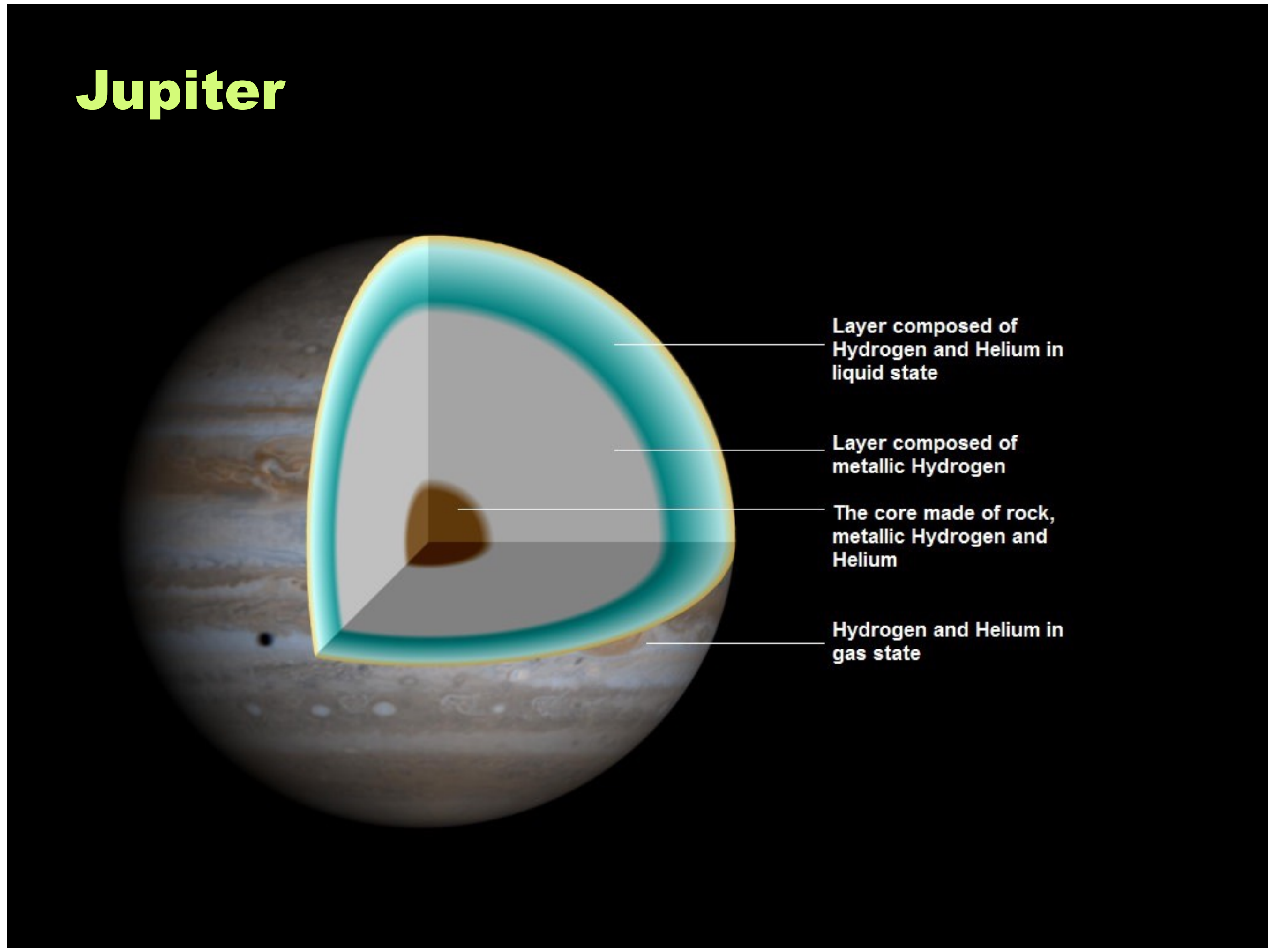
Interesting because the conductivity of salty water is 10 thousand to a million times less than the conductivity of liquid iron. So, lower value of Rm. Harder to get a dynamo but plucky little Ganymede proves that it’s possible!



My colleague Fran Bagenal likes to point out that Jupiter's magnetosphere is the biggest object in the solar system - bigger than the Sun (though there is the heliosphere itself if we solar physicists want to one-up her)

Jupiter may have dynamo generation out to levels where hydrogen is only a semiconductor, perhaps 80% to 85% of the planet radius. This is compatible with the magnitude and harmonic structure of the field Bagenal et al. (2004).

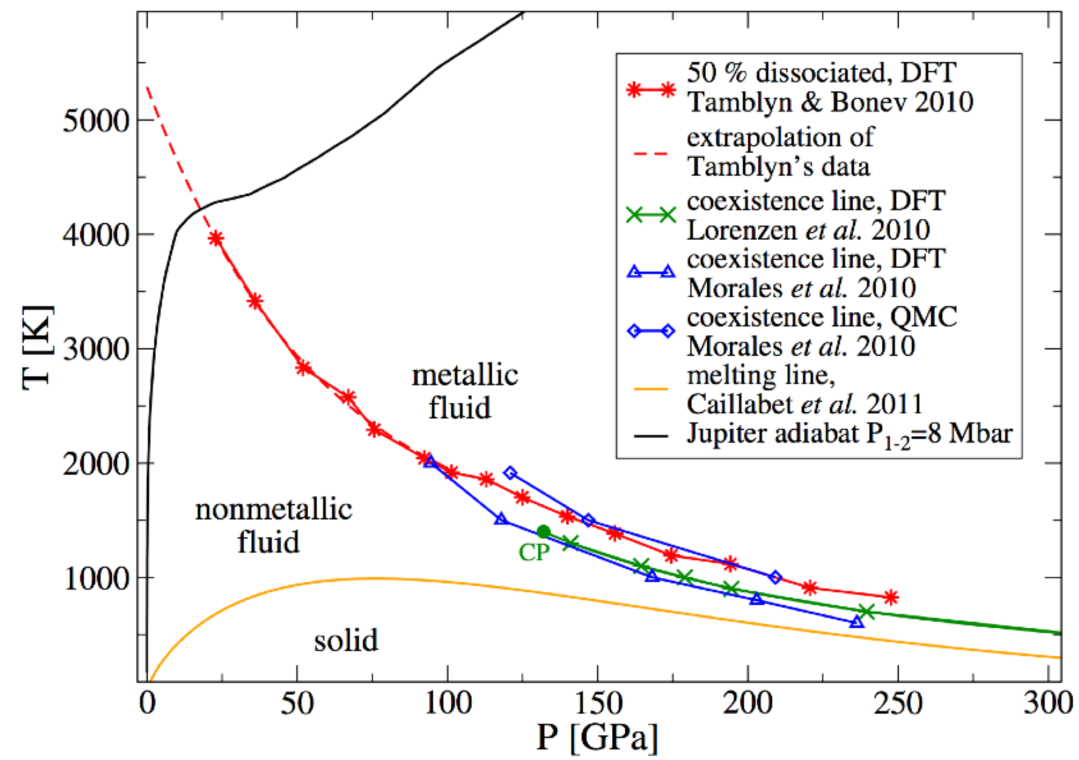
7 G measured; 20 G inferred



source of energy: secular cooling and gravitational contraction
As Jupiter cools, it releases an approximately uniform specific entropy everywhere outside the core, so the driving is different from the geodynamo, where the main buoyancy source is believed to be near the inner core boundary (basal heating).
Still contracting at a rate of 3 cm per year (from Bagenal et al)

Jupiter: Internal Structure

French et al (2012)

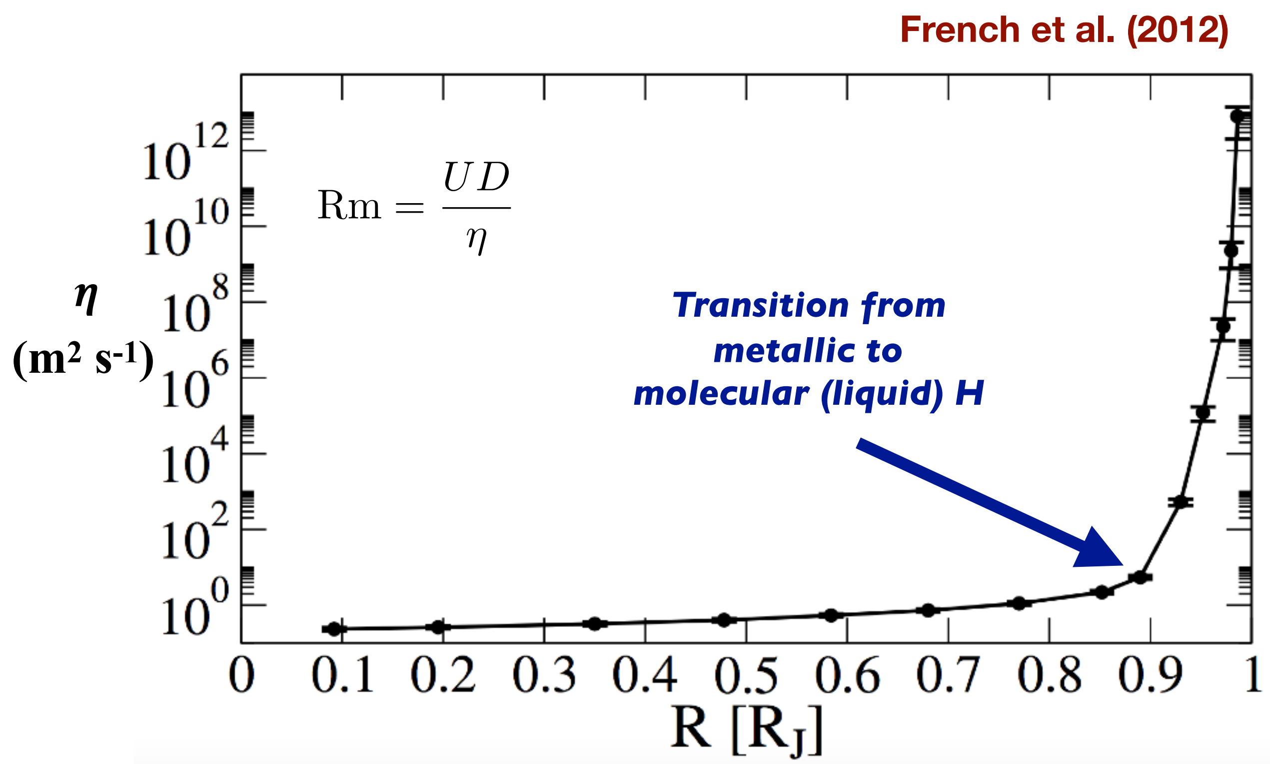


Phase diagram of H

Solid line is Jupiter adiabat
Point out that it becomes metallic above a pressure of about 20 GPa = 0.2 Mbar
P1-2 = 8 Mbar is the pressure jump across the two adiabatic envelopes in their model
In this model, the transition between the two adiabatic envelopes happens at 0.629 R

DFT = Density Functional Theory I believe QMC is also theory – I'm not sure if anyone has achieved the transition to metallic H in the lab

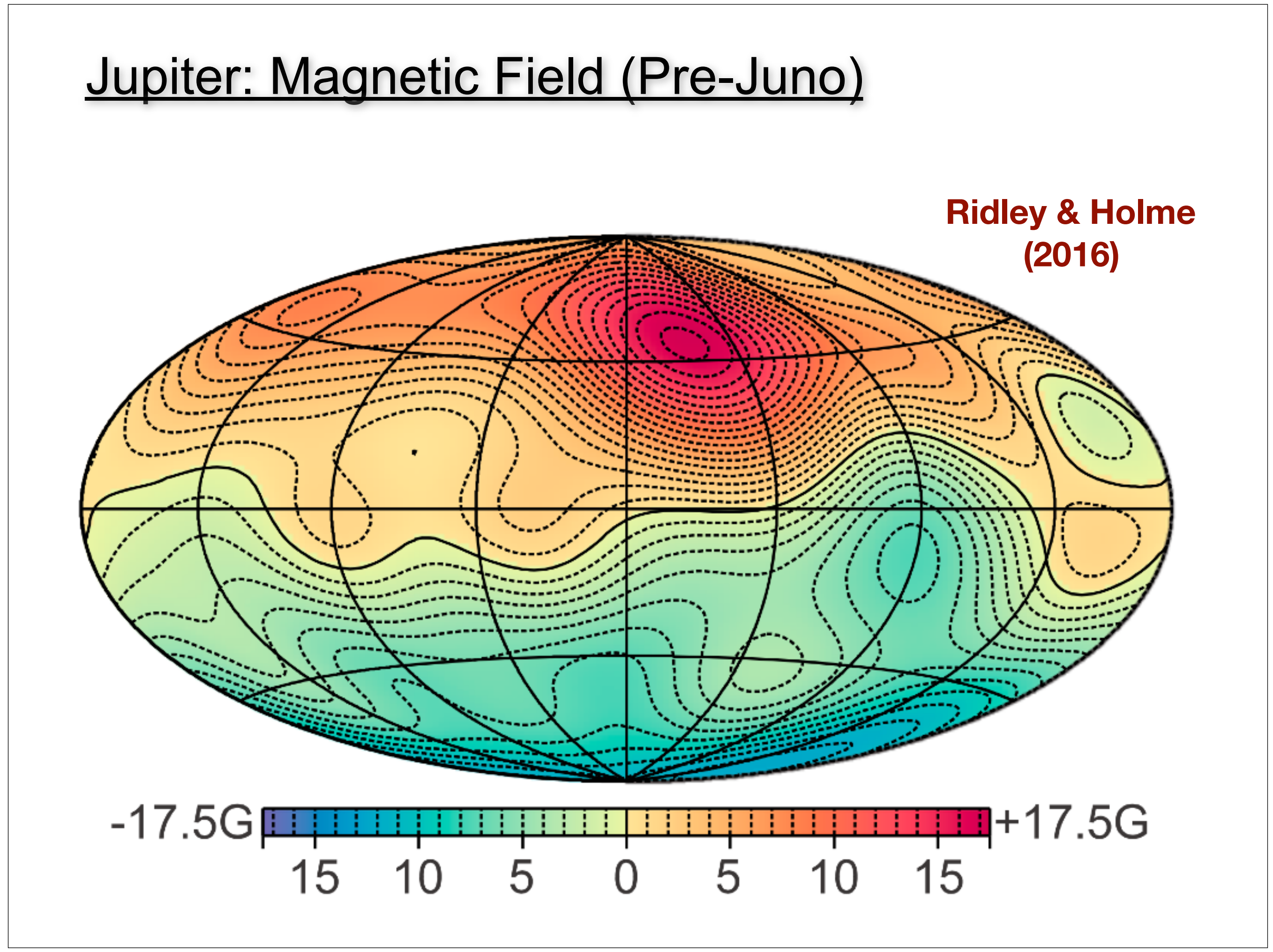
Jupiter: Internal Structure



The transition between the envelopes occurs at 800 GPa = 8 Mbar
Transition point of 25 GPa = 0.25 Mbar

Interior profiles of pressure (blue), temperature (red), and mass density (black) for the two representative Jupiter models J11-4a (dashed) and J11-8a (thin solid). The density has a jump at the transition pressure between the envelopes. Inset: the transit region between the metallic inner region and the molecular outer part.

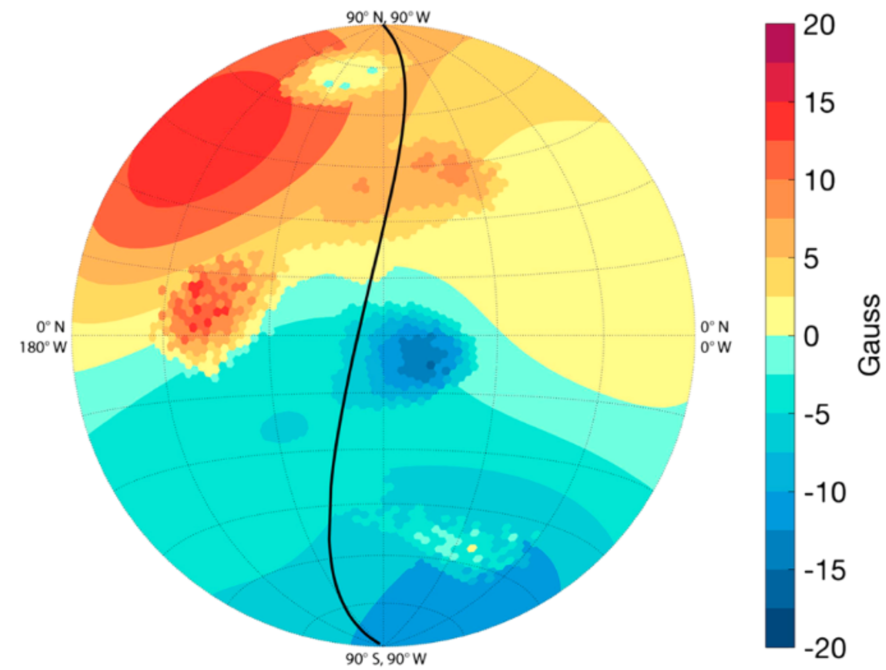
Presumably the other density jump at ~ 0.1R is the rocky core



Data ranging from Pioneer 10 in 1973 to Galileo in 2002

Initial results from Juno

Stronger and more patchy than expected (higher-order multipoles)



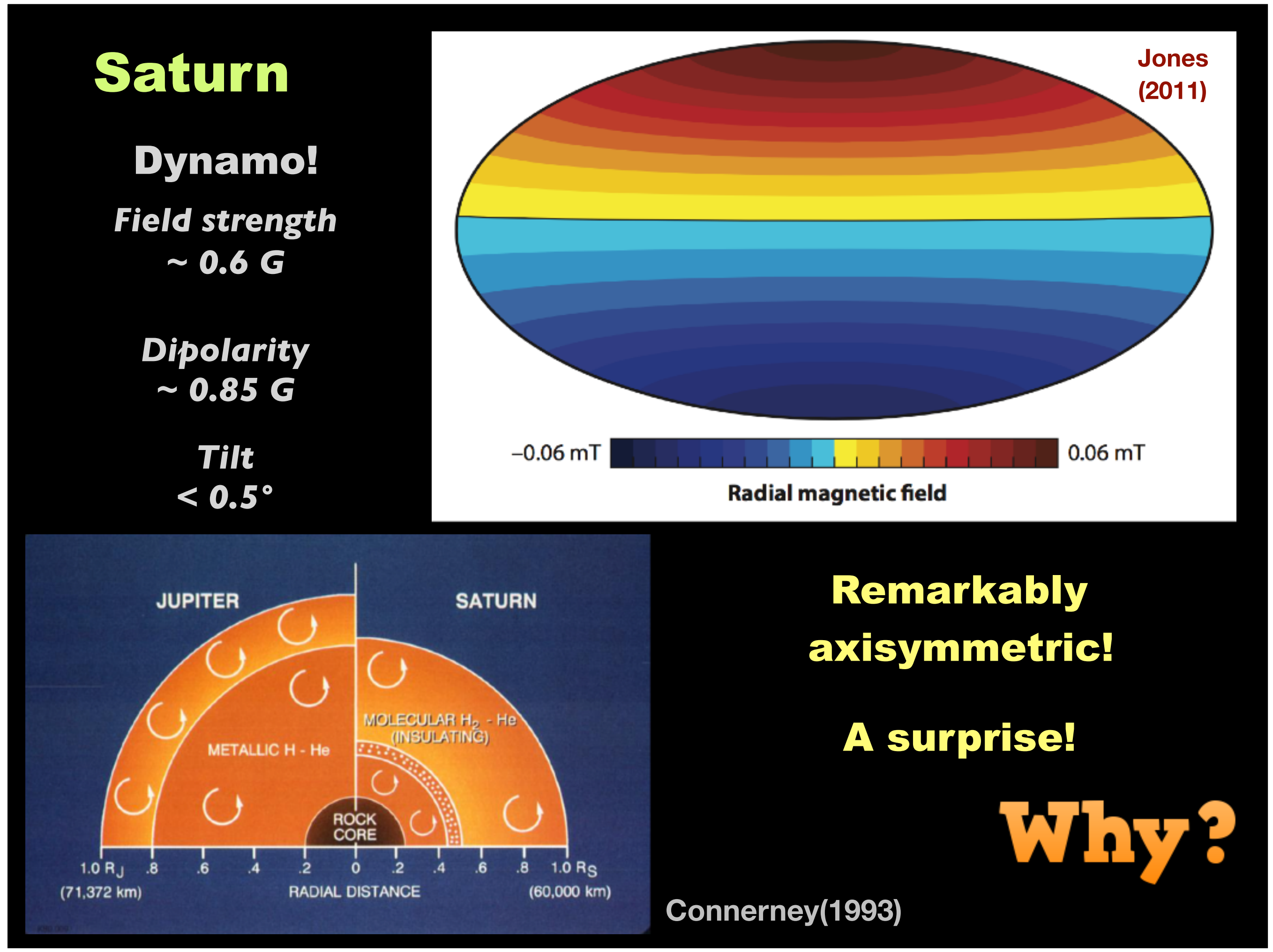
This suggests that dynamo action might exist closer to the surface than previously thought

$$B_r \propto r^{-(\ell+2)}$$

Moore et al (2017)

7 G measured; 20 G inferred
Solid black line is the path sampled by Juno's first periJove pass (PJ1)

This is the modeled field for r = 1 RJ



Smaller metallic H core

Why is this a surprise? There is a theorem that says this shouldn't be possible – a dynamo should not be able to produce a steady, axisymmetric field

Cowling's Theorem

Why is this a surprise?

Assume B is axisymmetric and consider the longitudinally-averaged MHD induction equation:

$$\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla}\times\left(\langle \mathbf{v}\rangle \times \mathbf{B} - \langle \eta \rangle \boldsymbol{\nabla}\times \mathbf{B}\right)$$

Express B as $\mathbf{B} = \boldsymbol{\nabla}\times\left(A\hat{\phi}\right) + B\hat{\phi}$

Evolution eqn for A (after some manipulation)

$$\lambda = r\sin\theta$$

$$\frac{\partial}{\partial t}(\lambda A) = -\mathbf{v}\cdot\boldsymbol{\nabla}(\lambda A) + \eta\lambda\left(\nabla^2 A - \lambda^{-2}\right)A$$

Multiply by λA and integrate over volume: if $\nabla\cdot\mathrm{v} = 0$ ***then the first term on the RHS is zero and the second term is negative***

Note – the gauge (G) cancels out because we're considering the longitudinal average so grad(G) cancels out
Paul says that the advection term "vanishes for incompressible flows and remains negligible for very subsonic compressible flows) but that's not very compelling to me

Cowling's Theorem (cont.)

$$\frac{\partial}{\partial t}(\lambda A) = -\mathbf{v}\cdot\boldsymbol{\nabla}(\lambda A) + \eta\lambda\left(\nabla^2 A - \lambda^{-2}\right)A$$

A decays with time

If A decays with time, then B will decay with time too (Work it out!)

Even if $\nabla\cdot\mathrm{v}\neq 0$ you can show that a steady field ($\partial\mathrm{A}/\partial\mathrm{t}=0$) cannot be maintained

Conclusion: it is not possible to sustain a steady axisymmetric B field against ohmic dissipation

Corollary: It is not possible for a dynamo to produce a steady axisymmetric field!!

Cowling originally assumed div(v) = 0

It's probably true in the time-dependent case as well (see Braginsky 1964 - based on energy arguments)

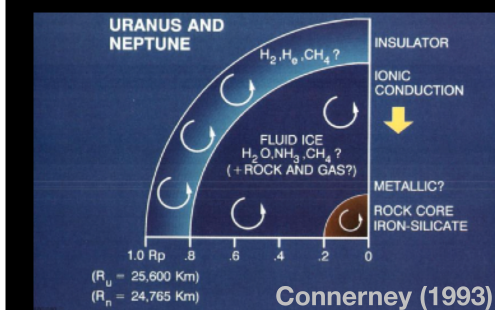
Uranus & Neptune

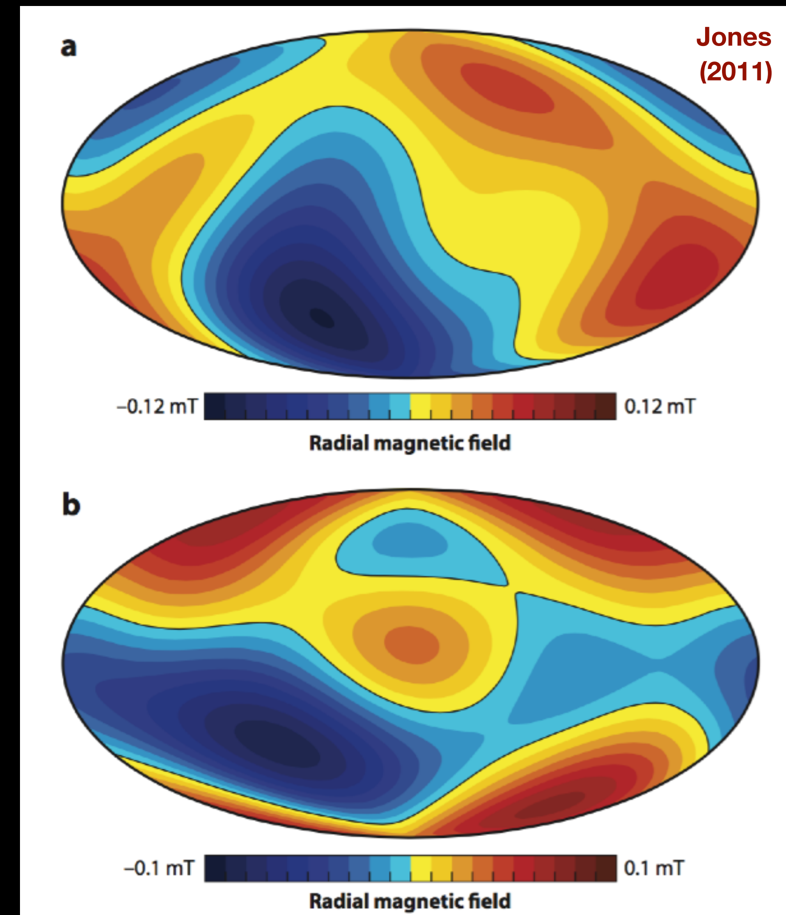
Dynamos!

Field strength ~ 0.3 G

Dipolarity ~ 0.42, 0.31

Tilt ~ 59°, 45°



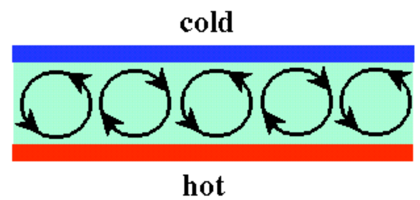


Opposite extreme from Saturn

Understanding the Dynamics

Conservation of momentum in MHD

$$\rho\frac{\partial \mathbf{v}}{\partial t} = -\left(\rho\mathbf{v}\cdot\boldsymbol{\nabla}\right)\mathbf{v} - 2\rho\left(\boldsymbol{\Omega}\times\mathbf{v}\right) - \boldsymbol{\nabla}P + \rho\mathbf{g} + c^{-1}\mathbf{J}\times\mathbf{B} - \boldsymbol{\nabla}\cdot\boldsymbol{\mathcal{D}}$$



Convection established by buoyancy

But rotation exerts an overwhelming influence
Coriolis accelerations happen quickly (days) compared to convection and dynamo time scales (hundreds to thousands of years)

$$\mathrm{Ro} = \frac{U}{2\Omega D} << 1 \qquad\qquad \mathrm{Ek} = \frac{\nu}{2\Omega D^2} << 1$$

Ek very important in modeling dynamos
Maybe someday you too will get a number named after you!

Dynamical Balances

$$\rho\frac{\partial \mathbf{v}}{\partial t} = -\left(\rho\mathbf{v}\cdot\boldsymbol{\nabla}\right)\mathbf{v} - 2\rho\left(\boldsymbol{\Omega}\times\mathbf{v}\right) - \boldsymbol{\nabla}P + \rho\mathbf{g} + c^{-1}\mathbf{J}\times\mathbf{B} - \boldsymbol{\nabla}\cdot\boldsymbol{\mathcal{D}}$$

Result: Flows evolve quasi-statically in so-called

Magnetostrophic (MAC) Balance

$$\boxed{c^{-1}\mathbf{J}\times\mathbf{B} \approx 2\rho\left(\boldsymbol{\Omega}\times\mathbf{v}\right) + \boldsymbol{\nabla}P - \rho\mathbf{g}}$$

Div(rho v) = 0: What goes up comes down

Geostrophic balance is the same but without the magnetic fields
Pressure can usually be eliminated by taking the curl of the equation of motion, which converts it into a vorticity equation

The word “magnetostrophic” is sometimes used as a synonym for MAC balance, though more commonly magnetostrophic balance refers to situations where buoyancy is absent.

Dynamical Balances

$$\rho \frac{\partial \mathbf{v}}{\partial t} = -\left(\rho \mathbf{v} \cdot \boldsymbol{\nabla}\right) \mathbf{v} - 2\rho \left(\boldsymbol{\Omega} \times \mathbf{v}\right) - \boldsymbol{\nabla} P + \rho \mathbf{g} + c^{-1} \mathbf{J} \times \mathbf{B} - \boldsymbol{\nabla} \cdot \boldsymbol{\mathcal{D}}$$

Result: Flows evolve quasi-statically in so-called

Magnetostrophic (MAC) Balance

$$\boxed{c^{-1} \mathbf{J} \times \mathbf{B} \approx 2\rho \left(\boldsymbol{\Omega} \times \mathbf{v}\right) + \boldsymbol{\nabla} P - \rho \mathbf{g}}$$

Conservation of mass

hydrostatic background

Anelastic approximation (valid for small Ma) $\quad \boldsymbol{\nabla} \cdot \left(\hat{\rho} \mathbf{v}\right) = 0$

Boussinesq approximation (valid for small Ma, H_ρ >> D) $\quad \boldsymbol{\nabla} \cdot \mathbf{v} = 0$

Div(rho v) = 0: What goes up comes down

Geostrophic balance is the same but without the magnetic fields
Pressure can usually be eliminated by taking the curl of the equation of motion, which converts it into a vorticity equation

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Dynamical Balances

$$c^{-1}\mathbf{J}\times\mathbf{B} \approx 2\rho\left(\boldsymbol{\Omega}\times\mathbf{v}\right) + \boldsymbol{\nabla}P - \rho\mathbf{g}$$

Now set B = 0 and assume that $\nabla\rho$ is mainly radial

Then the ϕ component of the curl gives (anelastic approximation):

$$\boldsymbol{\Omega}\cdot\boldsymbol{\nabla}\left(\rho\mathbf{v}\right) = \frac{\partial}{\partial z}\left(\rho\mathbf{v}\right) = 0$$

Taylor-Proudman Theorem

Boussinesq version:

$$\frac{\partial\mathbf{v}}{\partial z} = 0$$

Rapidly rotating flows tend to align with the rotation axis

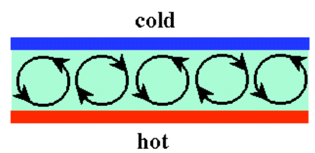
Geostrophic balance is the same but without the magnetic fields
Pressure can usually be eliminated by taking the curl of the equation of motion, which converts it into a vorticity equation

The word "magnetostrophic" is sometimes used as a synonym for MAC balance, though more commonly magnetostrophic balance refers to situations where buoyancy is absent.

Note: this result assumes div(rho v) = 0

Question

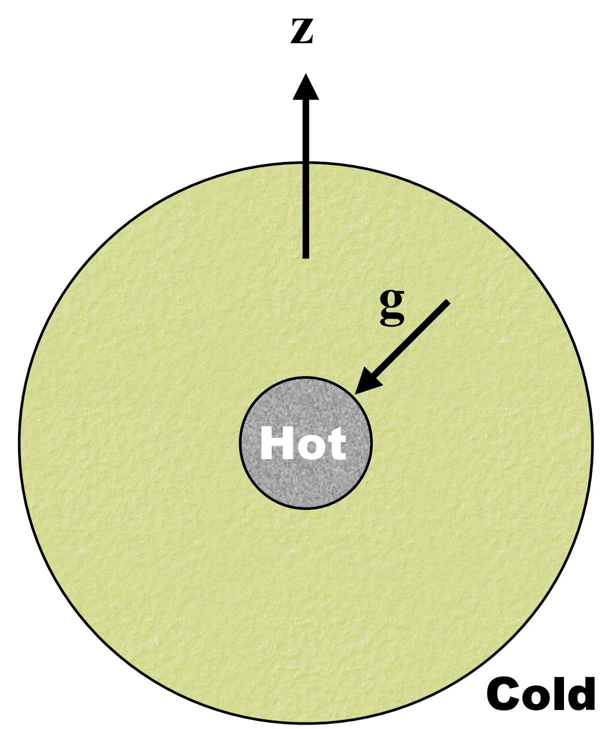
Work with a partner to draw what you think convective motions might look like in a rapidly-rotating spherical shell



How can you get the heat out while still satisfying the Taylor-Proudman theorem

$$\frac{\partial \mathbf{v}}{\partial z} = 0$$

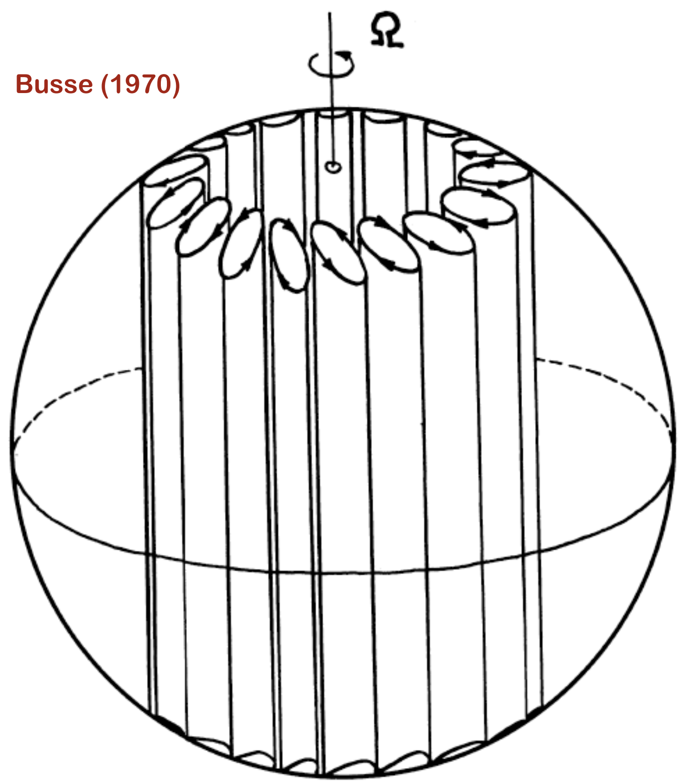
Can you satisfy it everywhere?



warm fluid rises, cool fluid sinks

Linear Theory

Busse (1970)



The most unstable convective modes in a rapidly-rotating, weakly-stratified shell are

Busse columns

aka

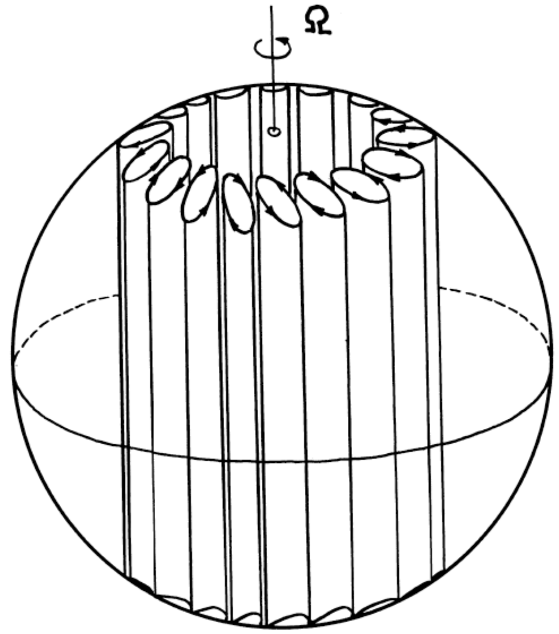
Banana Cells

The preferred longitudinal wavenumber (m) scales as

$Ek^{-1/3}$

Coriolis vs viscous diffusion

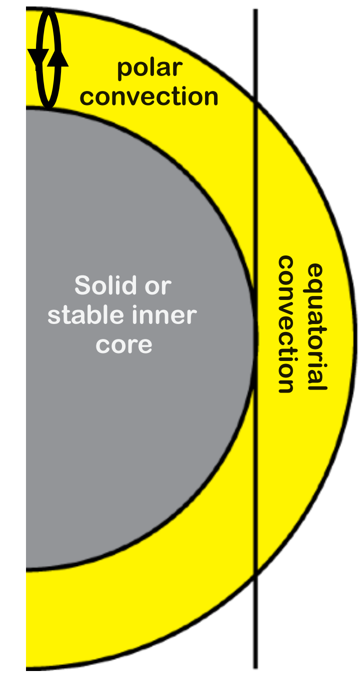
Linear Theory



The

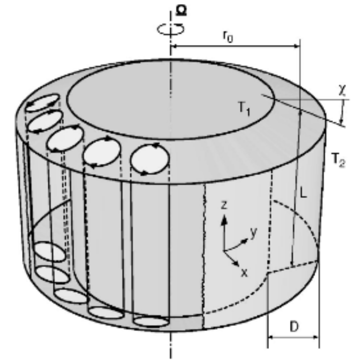
Tangent Cylinder

Delineates two distinct dynamical regimes

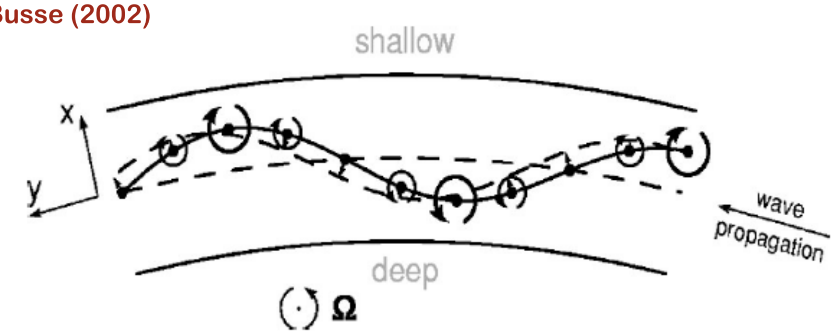


Implication of the Taylor-Proudman theorem

Linear Theory: Traveling Waves



Busse (2002)



Prograde propagation
(thermal Rossby waves)

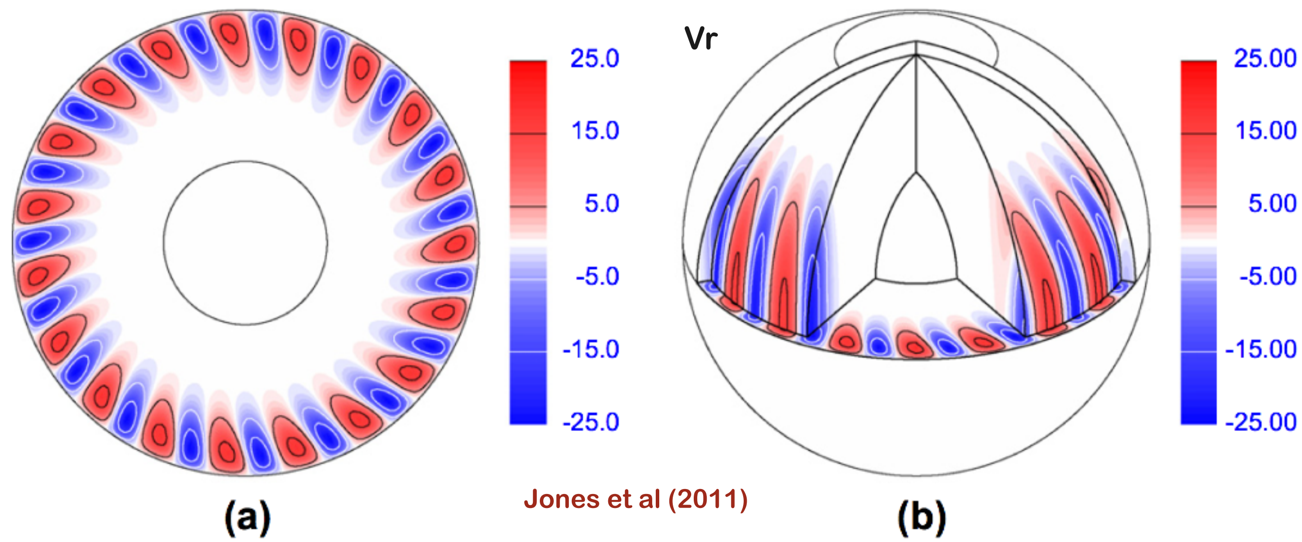
Induced by curvature of outer boundary and/or density stratification

$$\frac{\omega_z}{H} = \text{constant}$$

Simplest example: Boussinesq fluid, centrifugal gravity, local, linear perturbations, small boundary curvature (Busse 2002)

$$v_p = \frac{4\Omega}{L} \frac{\tan\chi}{(1+P_r)(k_y^2+k_x^2)}$$

Nonlinear Regimes require Numerical Models



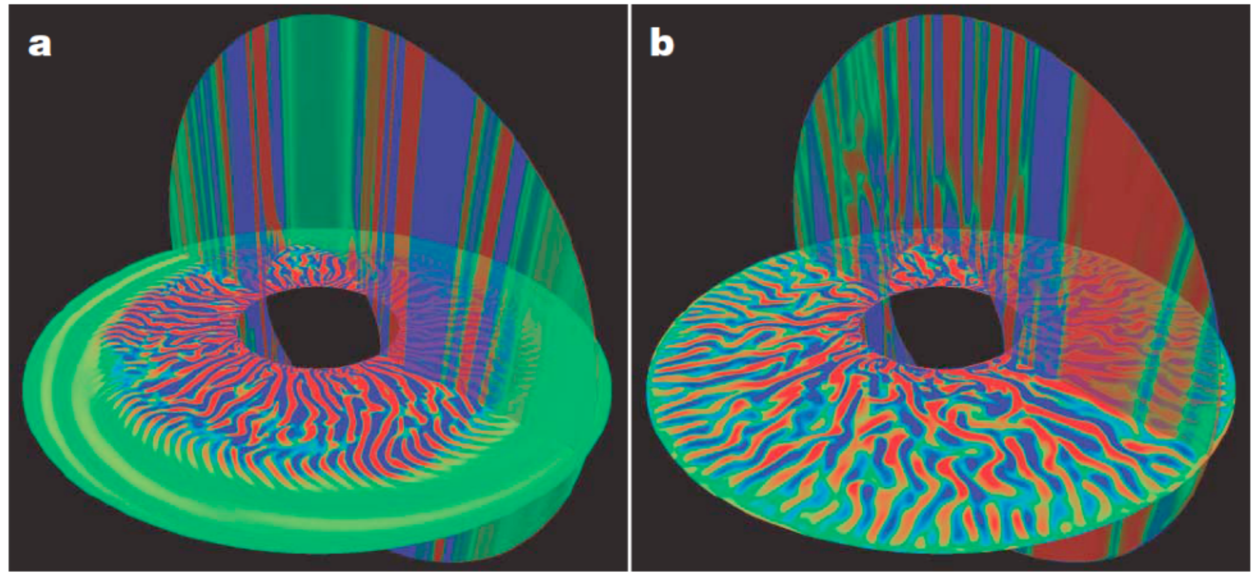
Solve the MHD equations in a rotating spherical shell
Anelastic or Boussinesq approximation
ρ, T, P, S are linear perturbations about a
hydrostatic, adiabatic background state

Convection simulations: heating from below, cooling from above

Axial alignment persists even in turbulent parameter regimes

Kageyama et al (2008)

Axial vorticity $\boldsymbol{\omega} \cdot \boldsymbol{\Omega}$



$\mathrm{Ek} = 2.3 \times 10^{-7}$ $\mathrm{Ek} = 2.6 \times 10^{-6}$

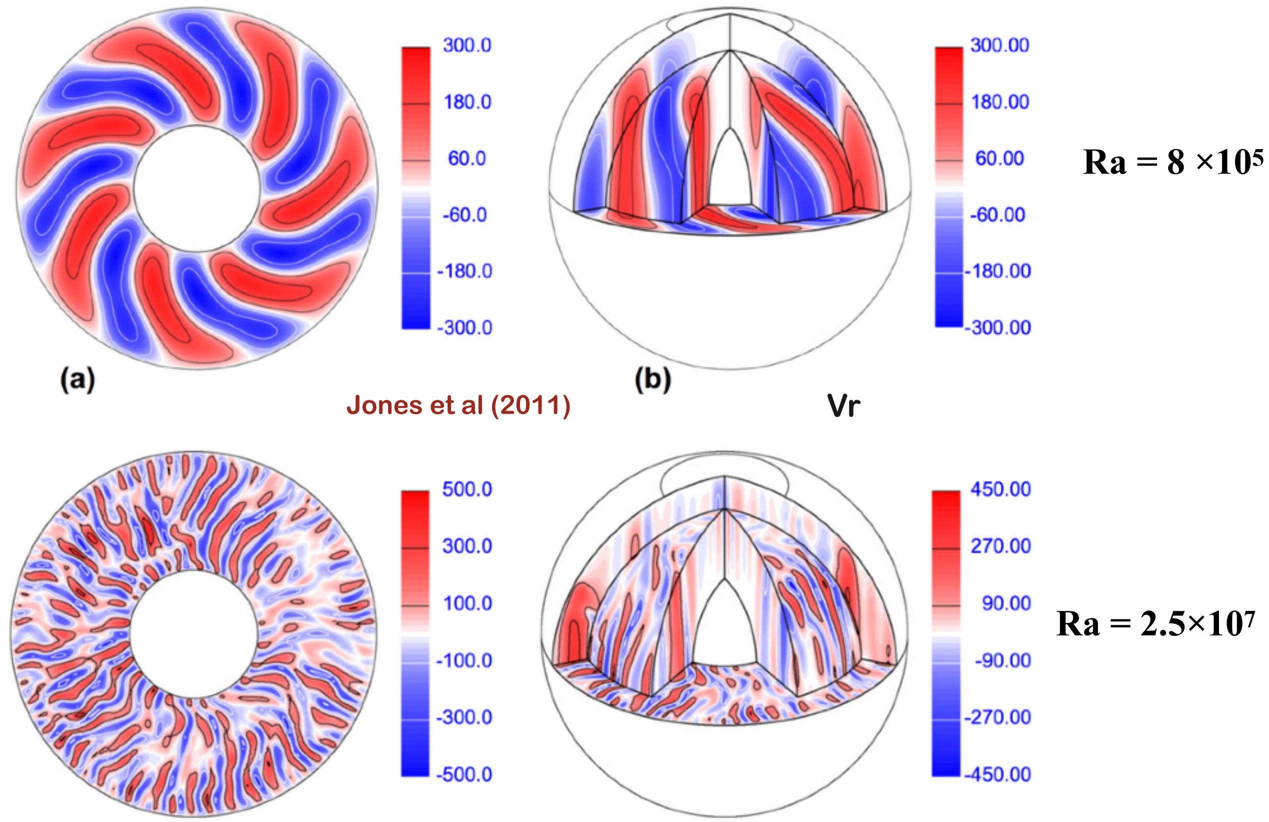
Busse columns give way to vortex sheets but the flow is still approximately 2D

$$\mathrm{Ek} = \frac{\nu}{2\Omega R^2}$$

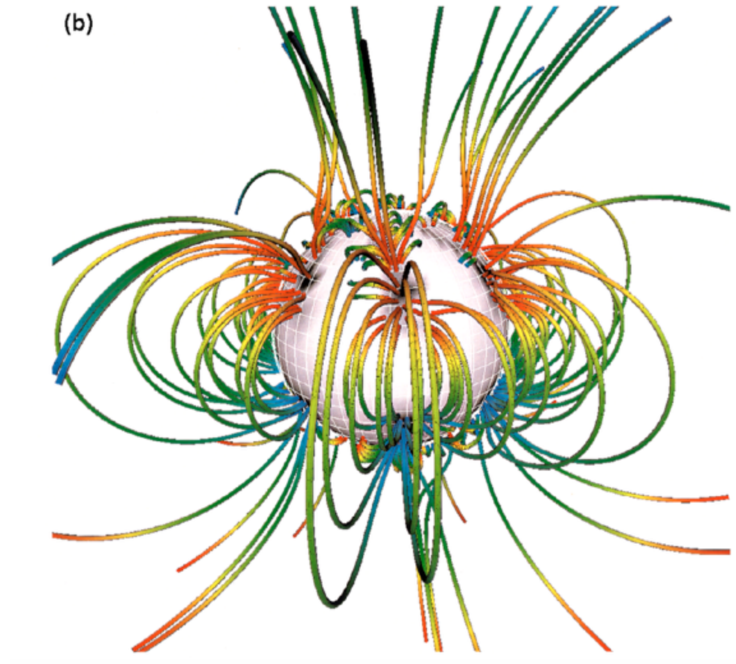
Nonlinear simulations

...and in MHD

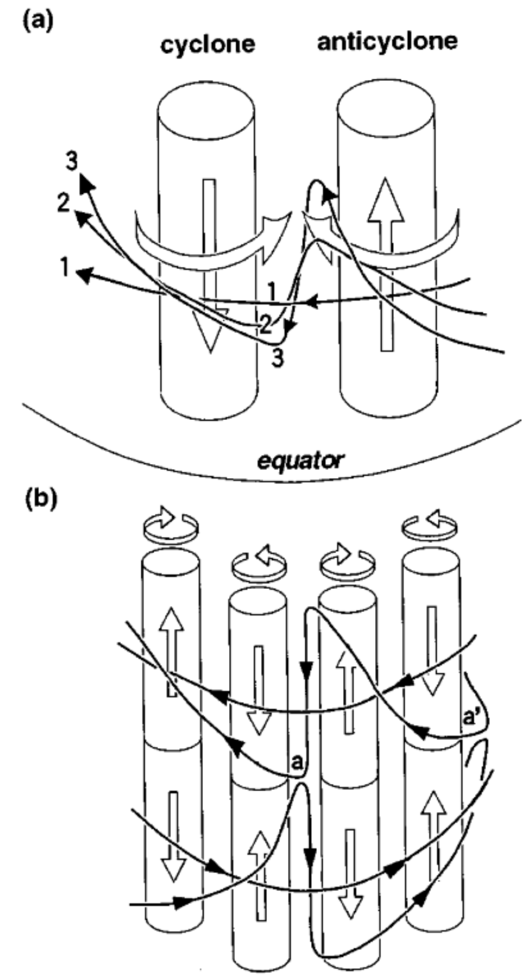
$$\mathrm{Ra} = \frac{GMD\Delta S}{\nu\kappa C_P} = \frac{\text{buoyancy driving}}{\text{dissipation}}$$



Busse columns are really good at making roughly dipolar fields



Kageyama & Sato (1997)



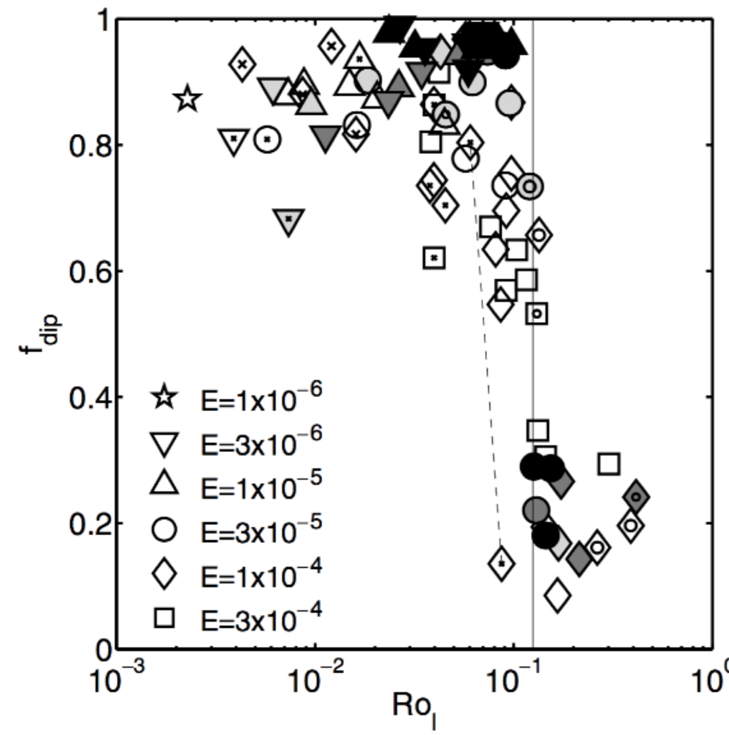
Similarly for other planets

General trends

Complexity of magnetic field depends mainly on the rotational influence

Rapid rotators tend to be more dipolar

Christensen & Aubert (2006)



Nonlinear simulations

Question

$$c^{-1}\mathbf{J}\times\mathbf{B} \approx 2\rho\left(\mathbf{\Omega}\times\mathbf{v}\right) + \nabla P - \rho\mathbf{g}$$

Assuming MAC balance, compute the ratio of ME/KE
How does it scale with Ro?

$$\nabla\times\mathbf{B} = \frac{4\pi}{c}\mathbf{J}$$

$$\mathrm{ME} = \frac{B^2}{8\pi}$$

$$\mathrm{Ro} = \frac{U}{2\Omega D}$$

$$\mathrm{KE} = \frac{1}{2}\rho v^2$$

Work it out on your own or with a partner

Subtlety: v also depends on Omega, and maybe B
The length scale may as well

If UD decreases at 1/Omega, then B should be independent of Omega

Question

$$c^{-1}\mathbf{J}\times\mathbf{B} \approx 2\rho\left(\mathbf{\Omega}\times\mathbf{v}\right) + \mathbf{\nabla}P - \rho\mathbf{g}$$

Assuming MAC balance, compute the ratio of ME/KE
How does it scale with Ro?

$$\frac{ME}{KE} \sim \mathrm{Ro}^{-1}$$

>>1 *if* Ro << 1*!*

But how do KE and Ro (and thus, ME) depend on observable*
global parameters like Ω and F_c?

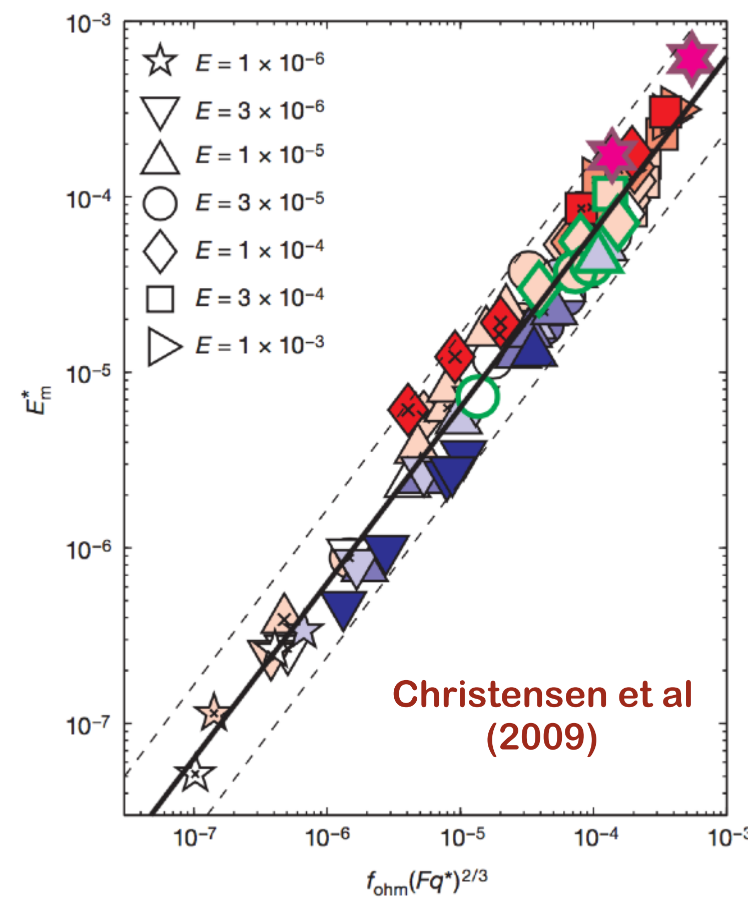
****in principle***

Searching for scaling relationships like this is comm

General trends

Field strength scales with the heat flux through the shell (independent of Ω!)

Rapid rotators seem to operate at maximum efficiency, tapping all the energy they can

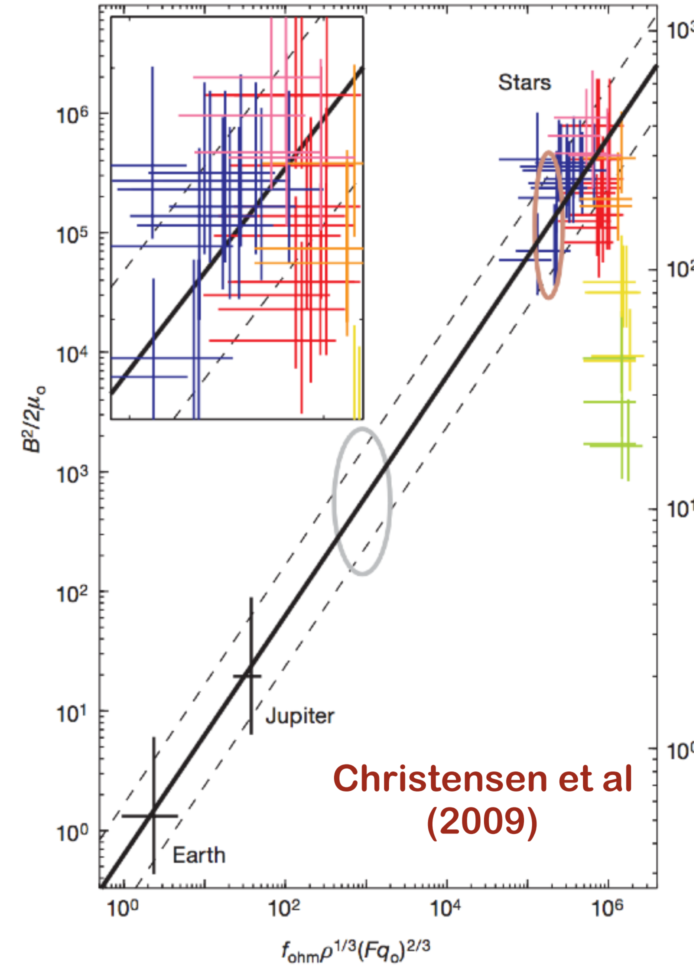


fohm = fraction of dissipation due to Ohmic dissipation
F = efficiency factor that takes into account the radial dependence of the various quantities like length scale L, rho, H_T, and qc. Should be independent of Omega, but not q*
q0 is a reference flux, e.g. the total flux at the outer boundary
q* = q0 / (rho Omega^3 R^3)
Em* = Em / (rho Omega^2 R^2)
So, q0 and Em are nondimensionalized based on a rotational velocity of R Omega

General trends

Rapid rotators seem to operate at maximum efficiency, tapping all the energy they can

This may apply to rapidly-rotating stars as well as planets!



Numerical Models: The Challenge

$$\mathrm{Pm} = \frac{\nu}{\eta}$$

	Earth	Jupiter	Simulations
Ra	10^{31}	10^{37}	10^{6}-10^{7}
Ek	3×10^{-15}	10^{-9}	10^{-6} - 10^{-7}
Rm	300-1000	400-3×10^{4}	50-3000
Pm	5-6×10^{-7}	6×10^{-7}	0.1-0.01

Similarly for other planets

Numerical Models: The Hope

Realistic simulations might be possible if you can achieve the right dynamical balances (e.g. MAC balance)

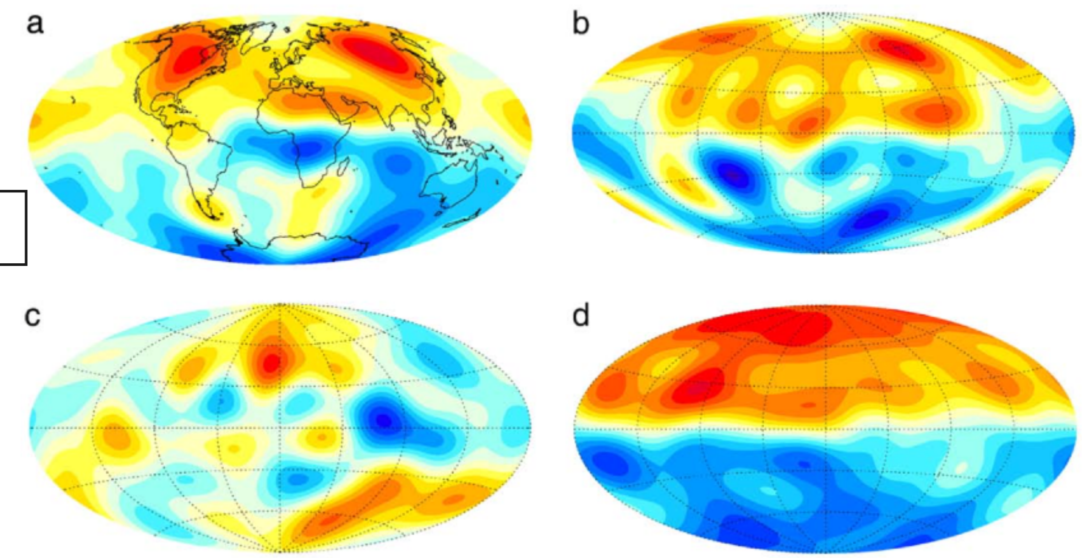
- **The most important parameters to get right**
  (or as right as possible)
  - **Ro**
    - **Appropriate rotational influence on the convection**
  - **Rm**
    - **Reasonable estimate of the ohmic dissipation**
  - **Ek**
    - **At least get it small enough that viscosity isn't part of the force balance**

Nonlinear simulations

Example: The Geodynamo

Points of comparison: Field strength, morphology (spectrum, symmetry, etc), Reversal timescale

Inferred from observations

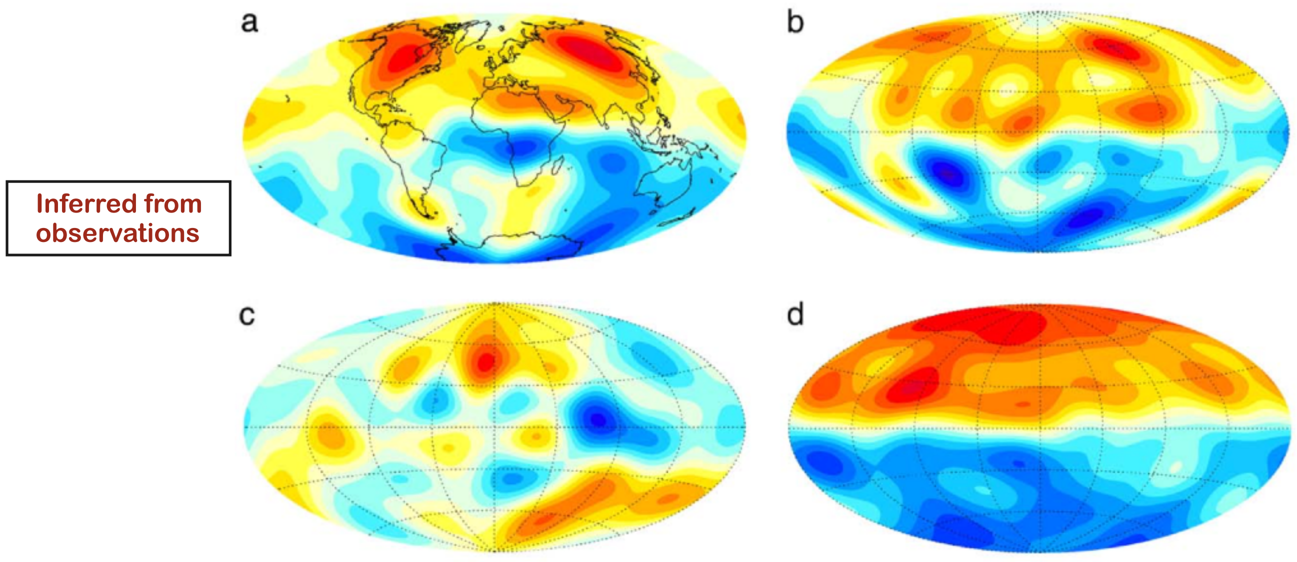


Christensen et al (2010)

Best matches are those with Ek < 10^{-4} and Rm "large enough"

This paper focuses on the morphology

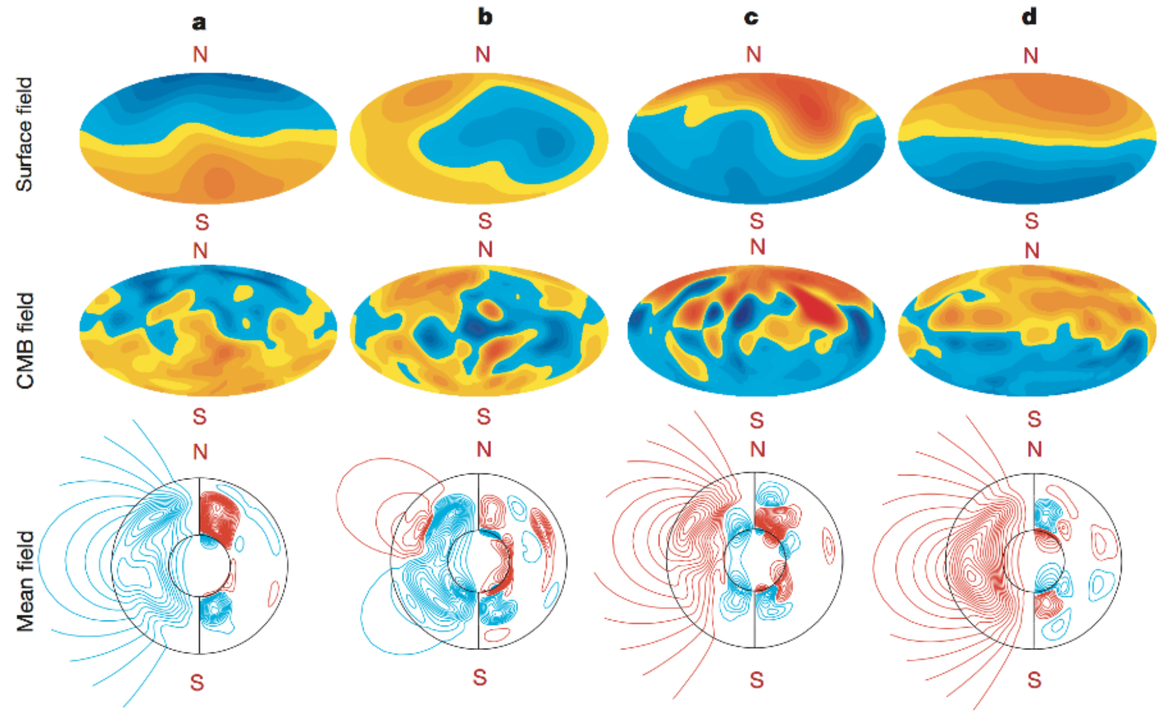
Example: The Geodynamo



But be careful! They could be right for the wrong reasons!
For example, both c and d have a higher Ra and lower Ek than b
they should be more realistic, right?

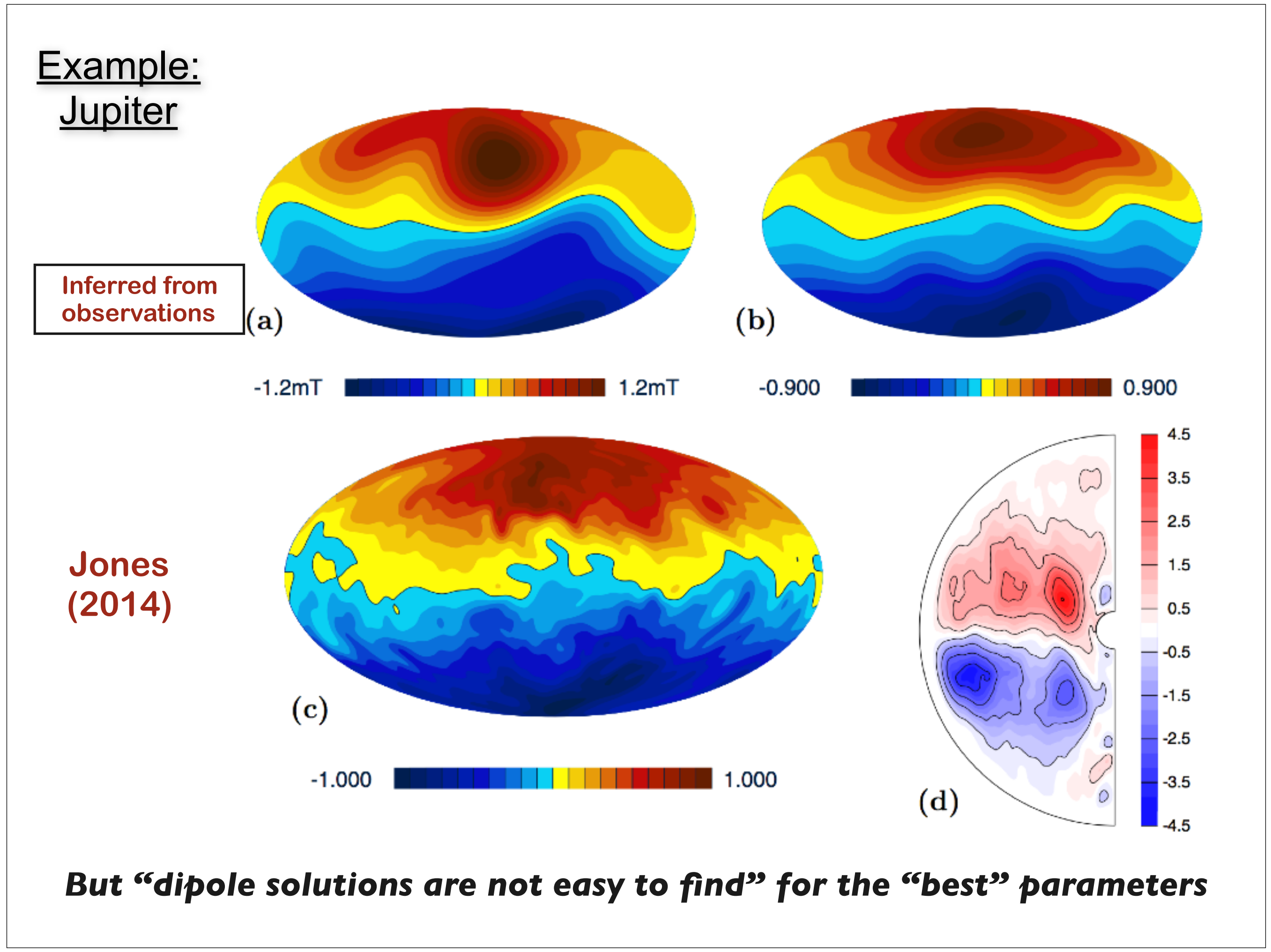
Nonlinear simulations

Example: The Geodynamo



Coupling to inner core needed to get the reversal time scale right (Glatzmaier & Roberts 1995; Glatzmaier et al 1999)

Nonlinear simulations

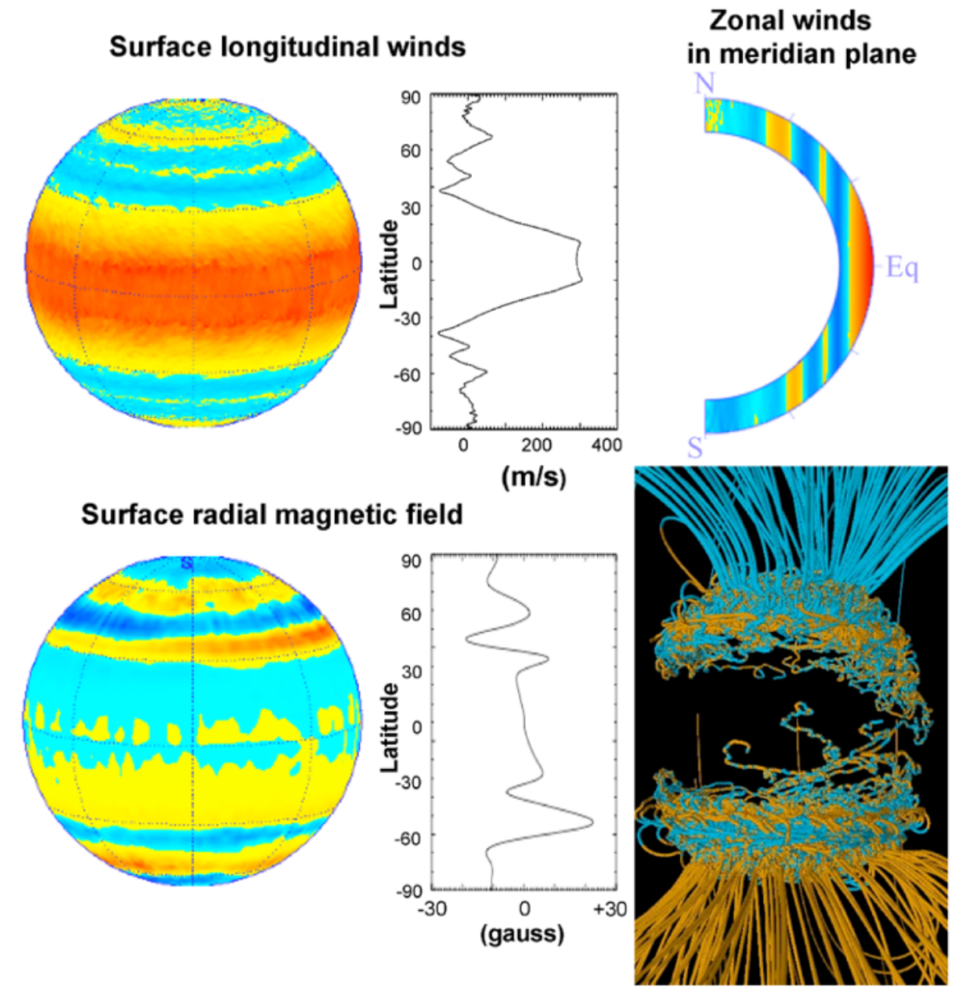


Key to getting good solutions: state-of-the-art reference state and plausible heating model. Large enough Ra and Rm.

We also found that the Rayleigh number had to be large enough to give a magnetic Reynolds number of order 1000 in order to generate fields strong enough to suppress the differential rotation in the dynamo region.

This is run D. Run H has the highest Ra and lowest Ek. This one is reversing

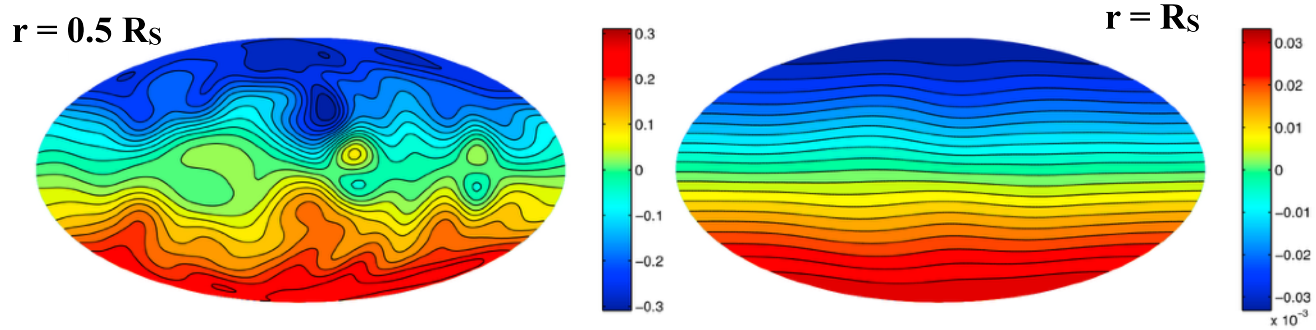
Another example of a Gas Giant dynamo highlighting banded zonal flows



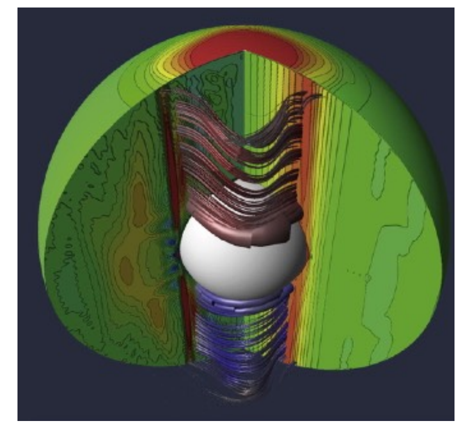
Stanley & Glatzmaier (2009)

Snapshots of an anelastic simulation of a gas giant dynamo. Top row: The longitudinal component of velocity at the surface, its zonal average vs. latitude and the zonal velocity displayed in the meridian plane. Reds and yellows are eastward up to 300 m/s; blues are westward relative to the rotating frame up to 100 m/s. Bottom row: The radial component of the magnetic field at the surface, its zonal average vs. latitude and the 3D field illustrated with lines of force. Reds and yellows are outward directed; blues are inward. Orange lines of force are directed outward; blue are directed inward. Typical intensity within the convection zone is a few hundred gauss. (Glatzmaier 2005)

So what's going on with Saturn?



Maybe the field is "axisymmeterized" by an overlying stable layer that has differential rotation but no convection (Stevenson 1982, Stanley 2010)



Or, maybe it's running a different type of dynamo, driven more by shear than buoyancy (Cao et al 2012)

Shear case would still be subject to Cowling's theorem but it may be generated by time-dependent, non-axisymmetric instabilities of the shear

Numerical Models: Summary

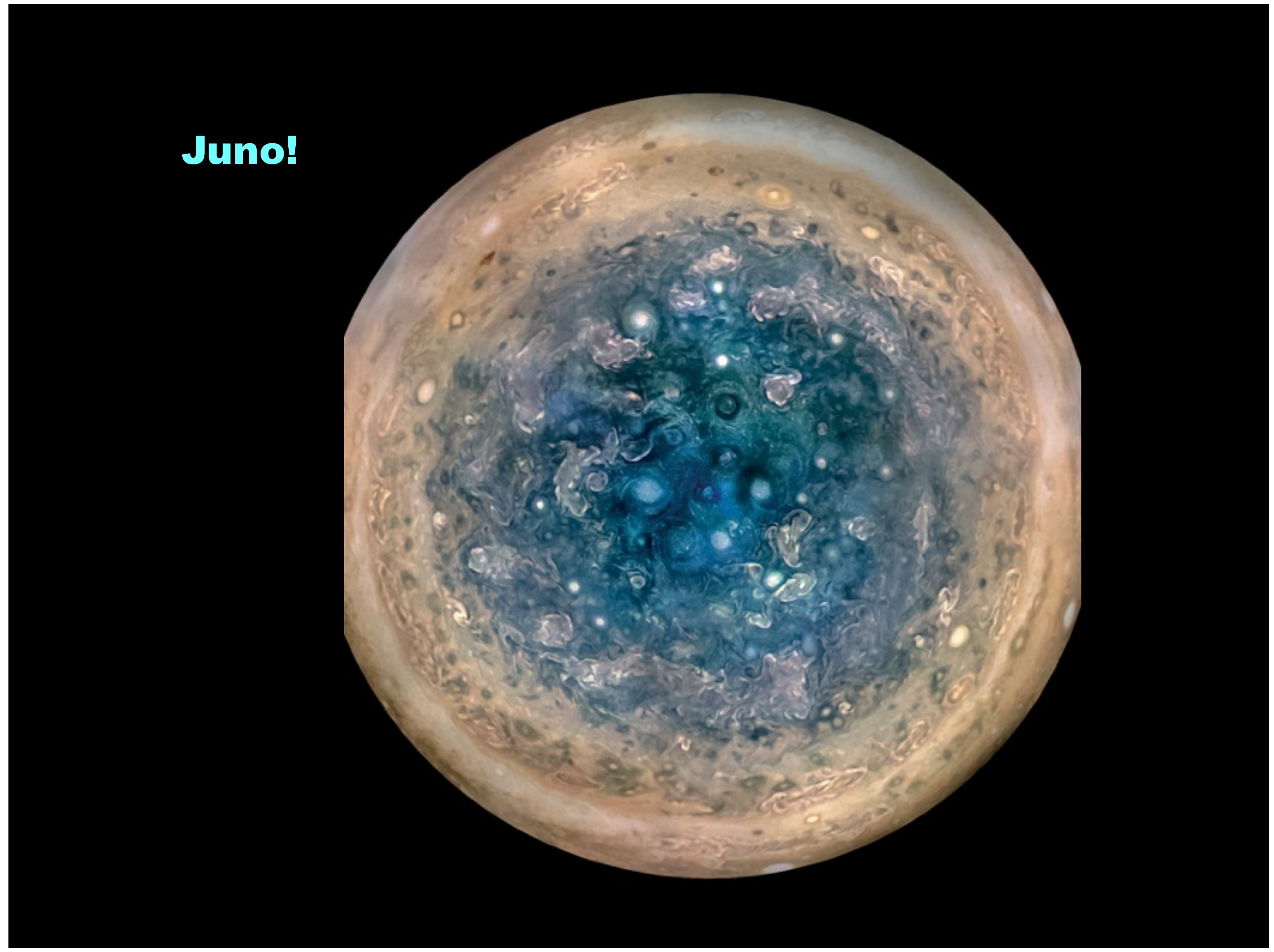
Lessons Learned

- ***Rapid Rotation has a profound influence on the dynamics***
- ***Success attributed to correct dynamical balances and (when possible) realistic Rm***

Future challenges

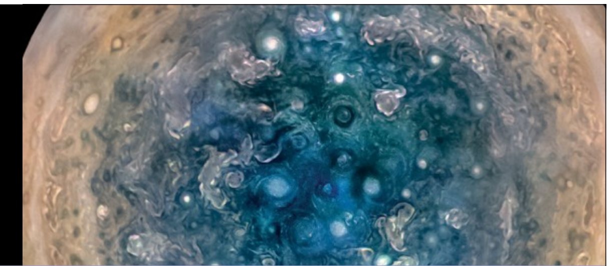
- ***What happens at really low Ek (tiny ν)?***
- ***Peculiarities of particular planets (Saturn, Mercury, Uranus, Neptune…)***
  - Boundary conditions (adjacent layers)
  - Rapid variations of η
  - Energy sources
  - Compositional convection
- ***Moving to more realistic parameters doesn't always improve the fidelity of the model***
- ***Exoplanets!***

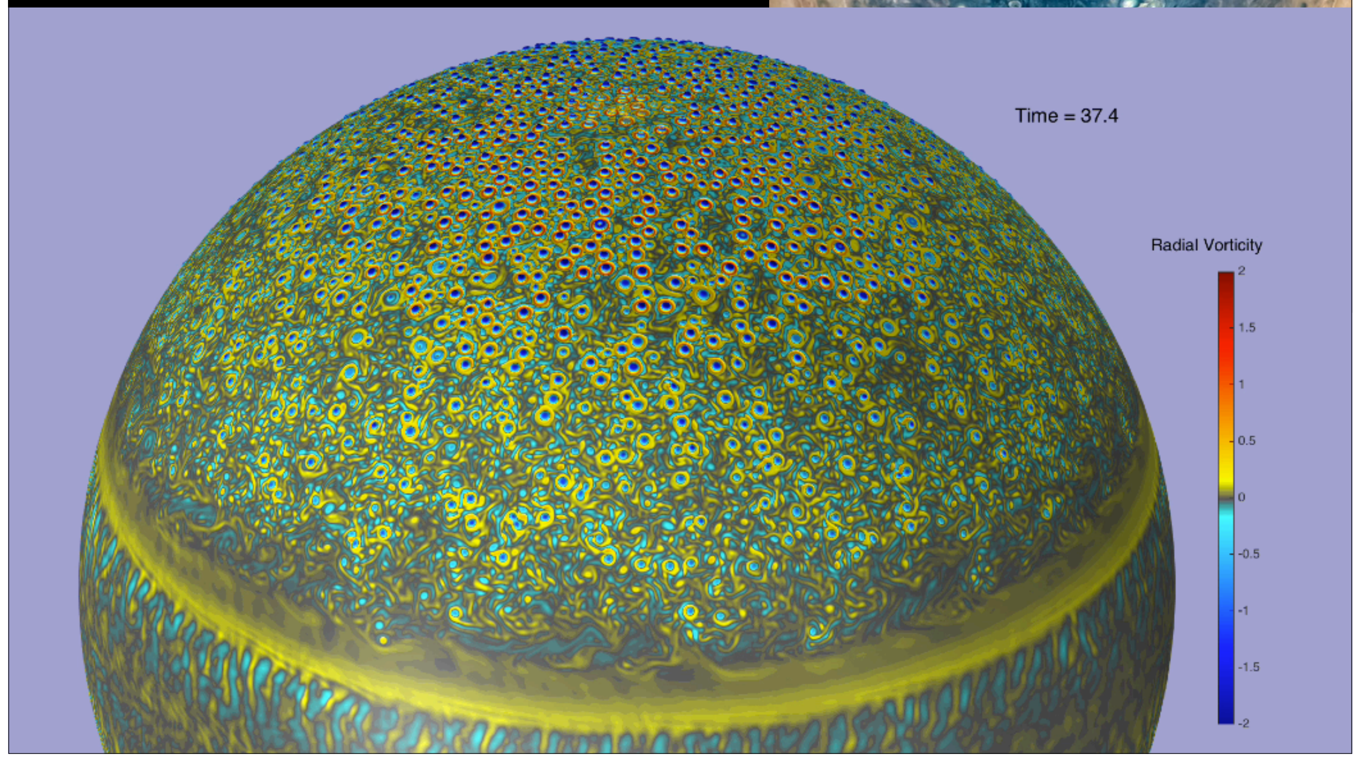
The main puzzle with Uranus and Neptune is how they are producing fields with such a small heat flux



I started with Juno, I'll end with Juno

Featherstone & Heimpel 2017





Featherstone & Heimpel 2017