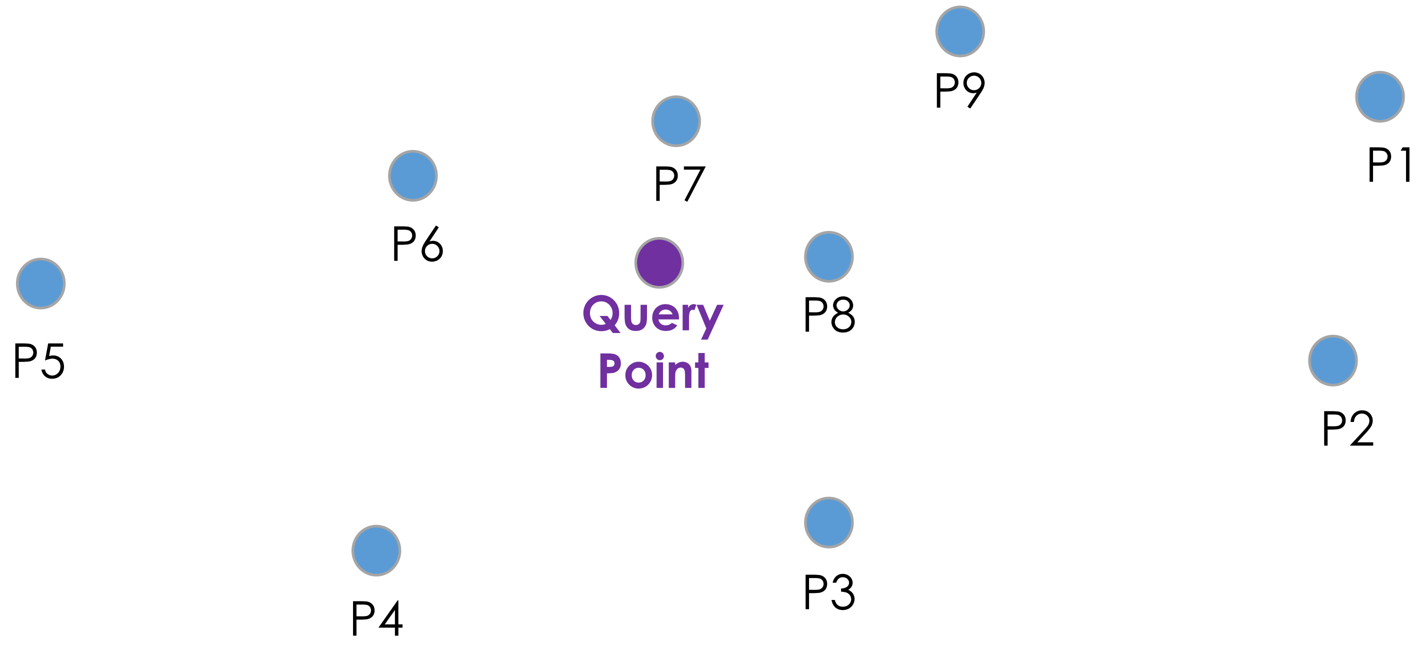
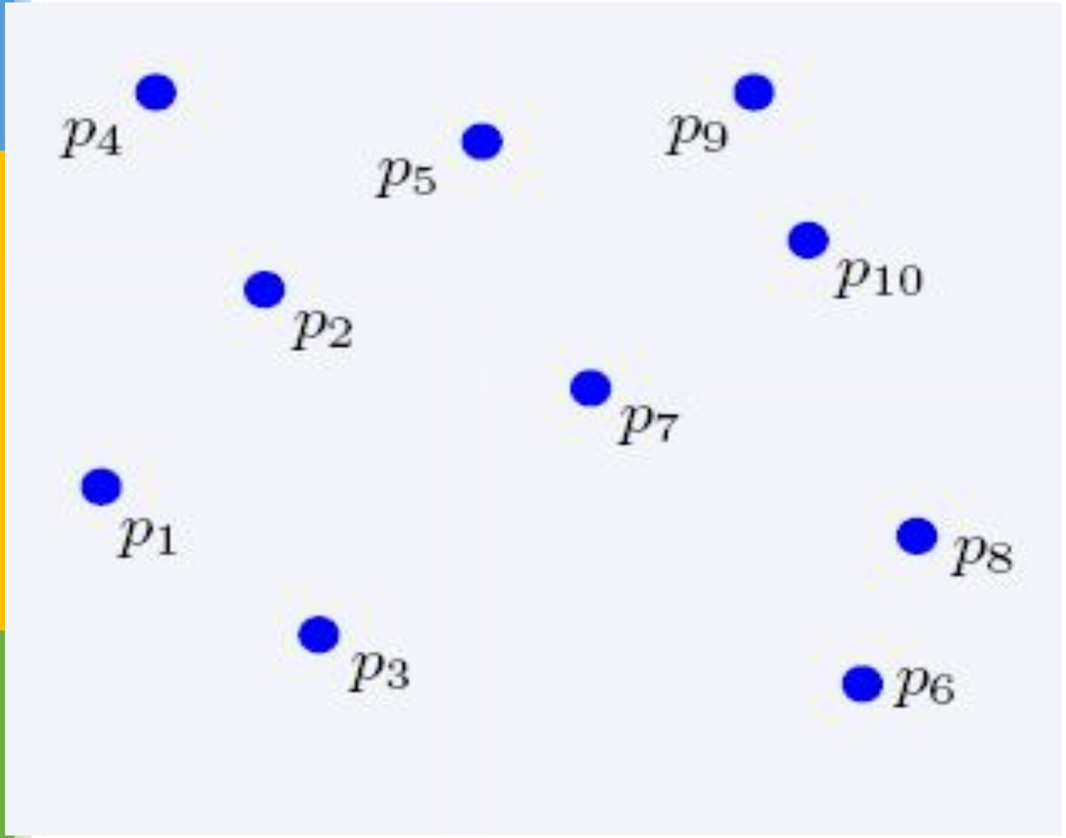
Geometric Data Structures

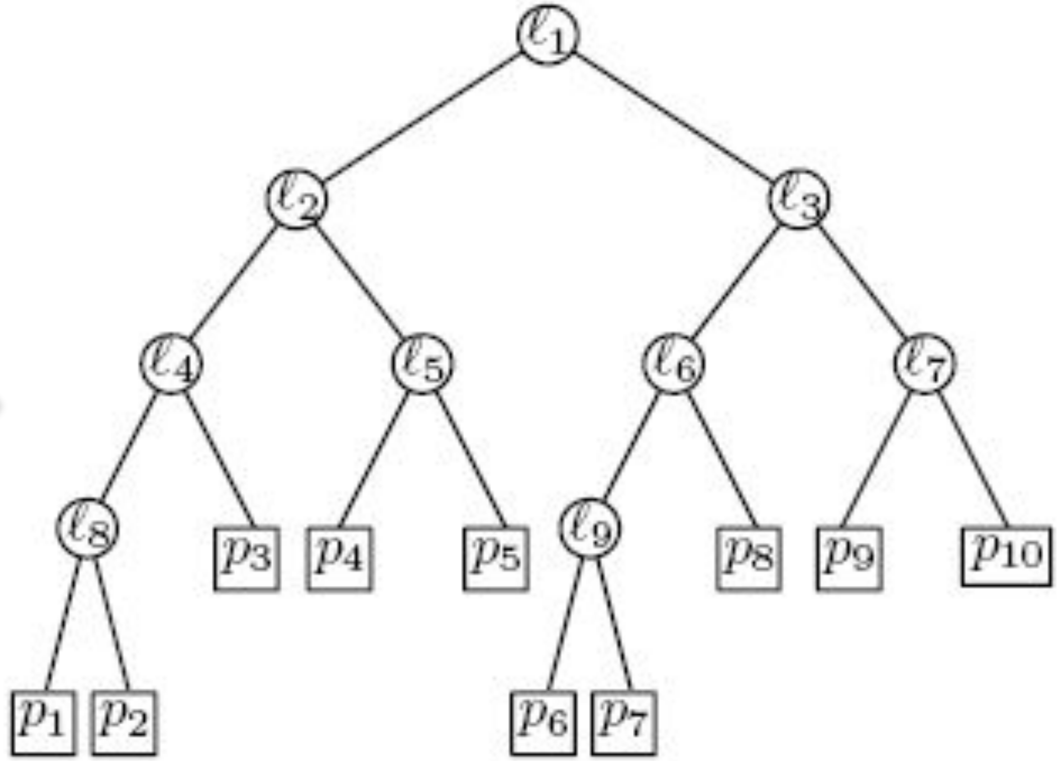
Nearest Neighbor Queries



Retrieve closest two points to the query point

KD Tree





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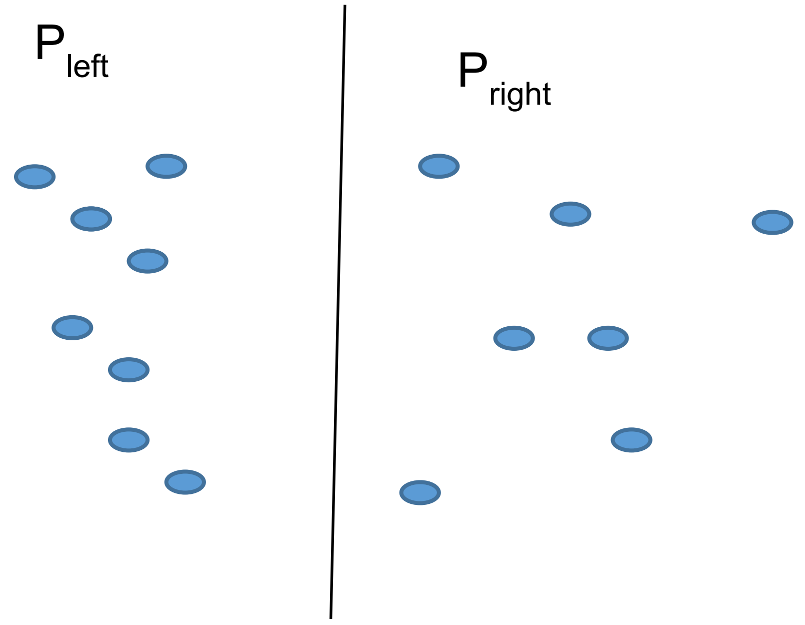
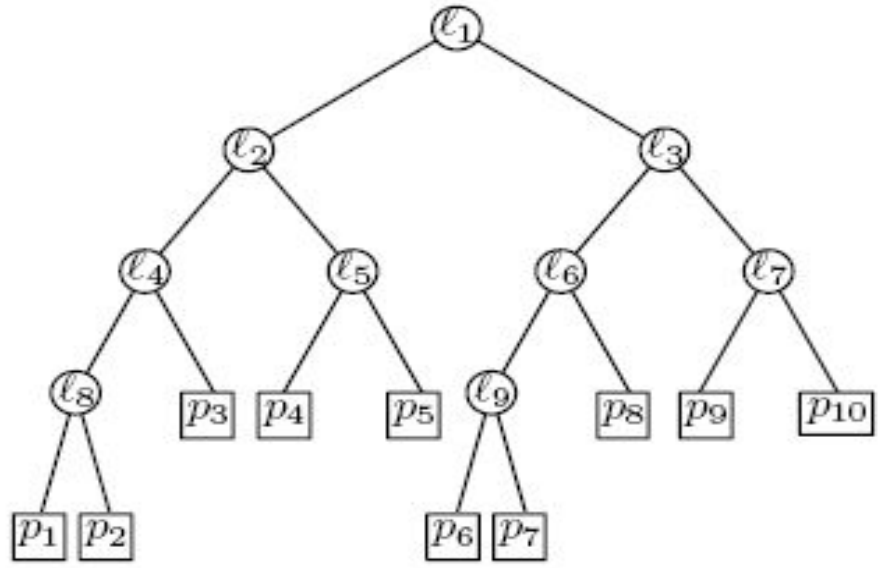
KD Tree

- Every node (except leaves) represents a hyperplane that divides the space into two parts.
- Points to the left (right) of this hyperplane represent the left (right) sub-tree of that node.





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KD Tree

As we move down the tree, we divide the space along alternating (but not always) axis-aligned hyperplanes:

- Split by x-coordinate: split by a vertical line that has (ideally) half the points left or on, and half right.
- Split by y-coordinate: split by a horizontal line that has (ideally) half the points below or on and half above.

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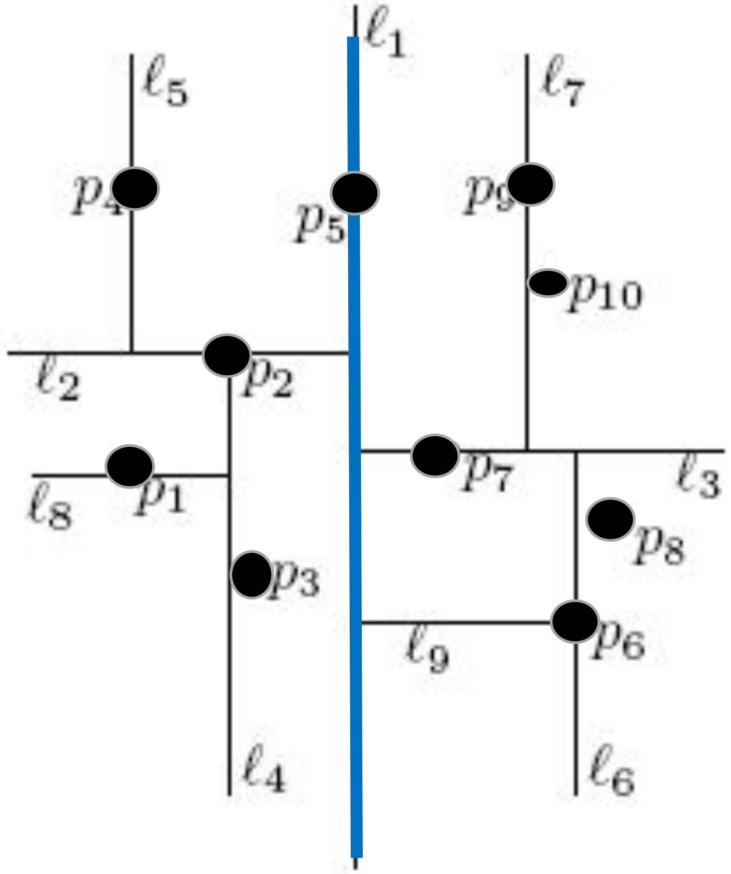
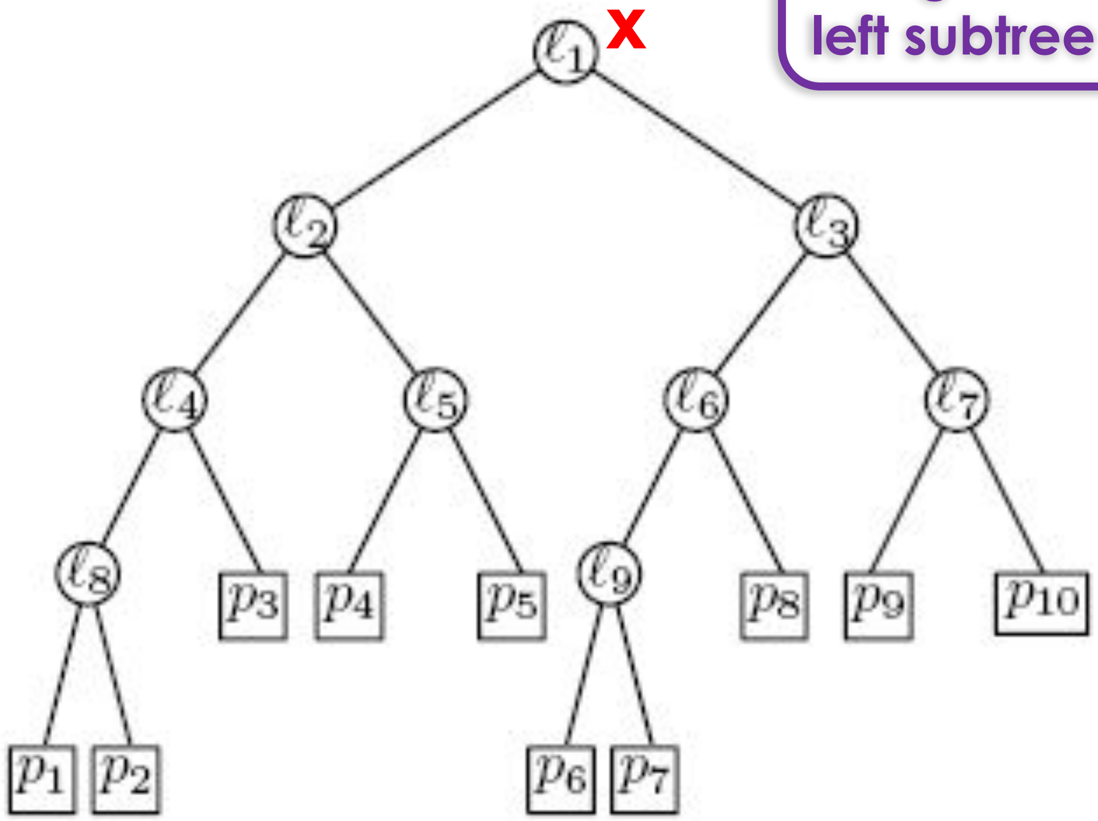
KD Tree Construction

Split by x-coordinate: split by a vertical line that has approximately half the points left or on, and half right.

Points on the line go to the left subtree





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KD Tree Construction

Split by y-coordinate: split by a horizontal line that has approximately half the points left or on, and half right.

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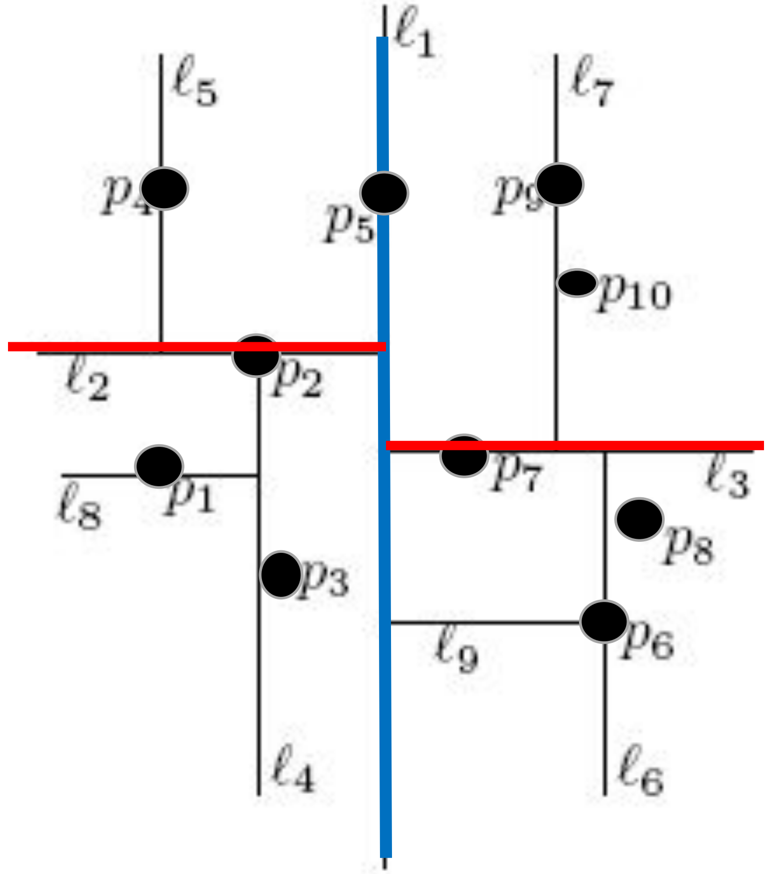

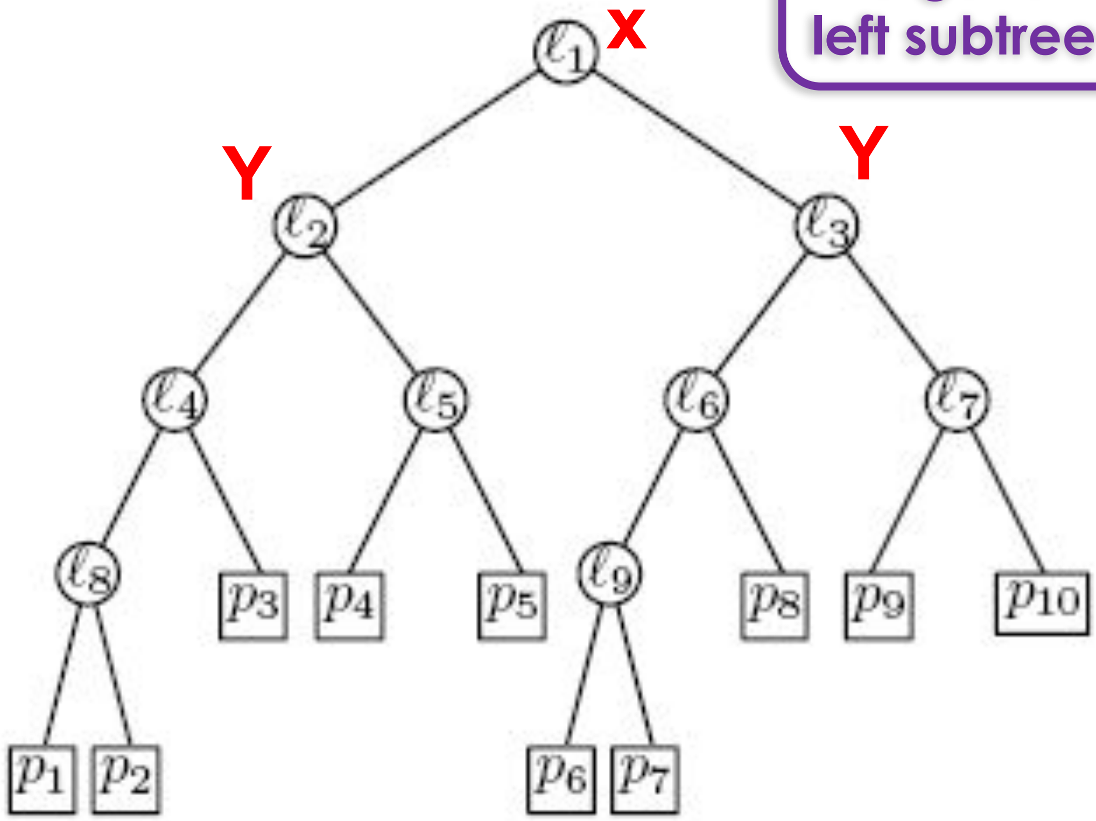

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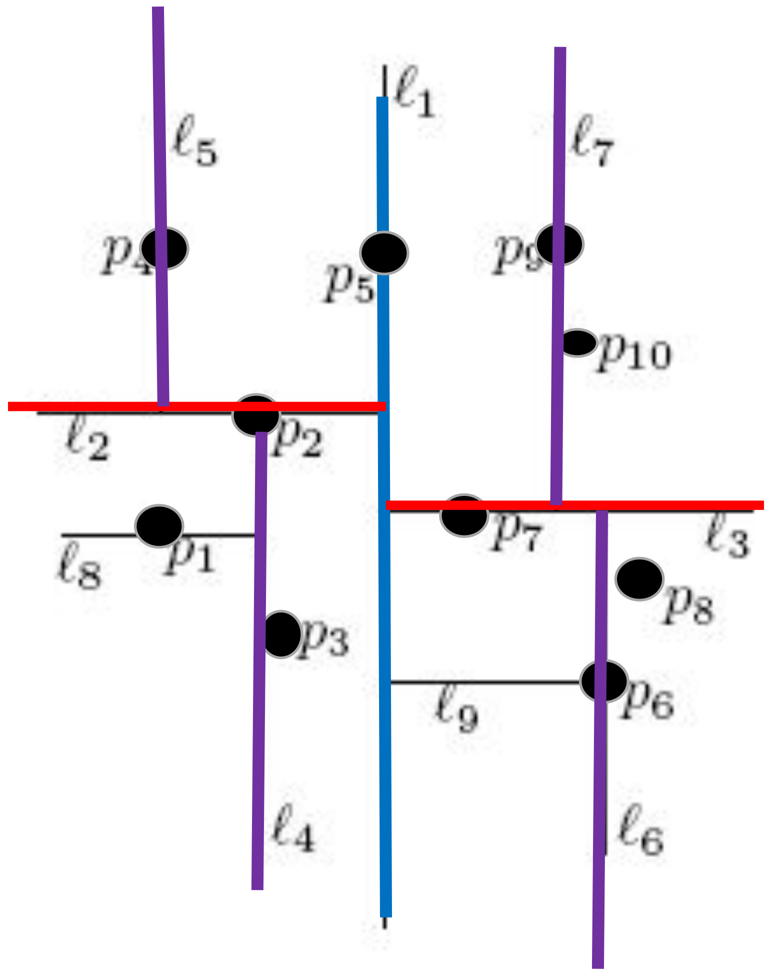
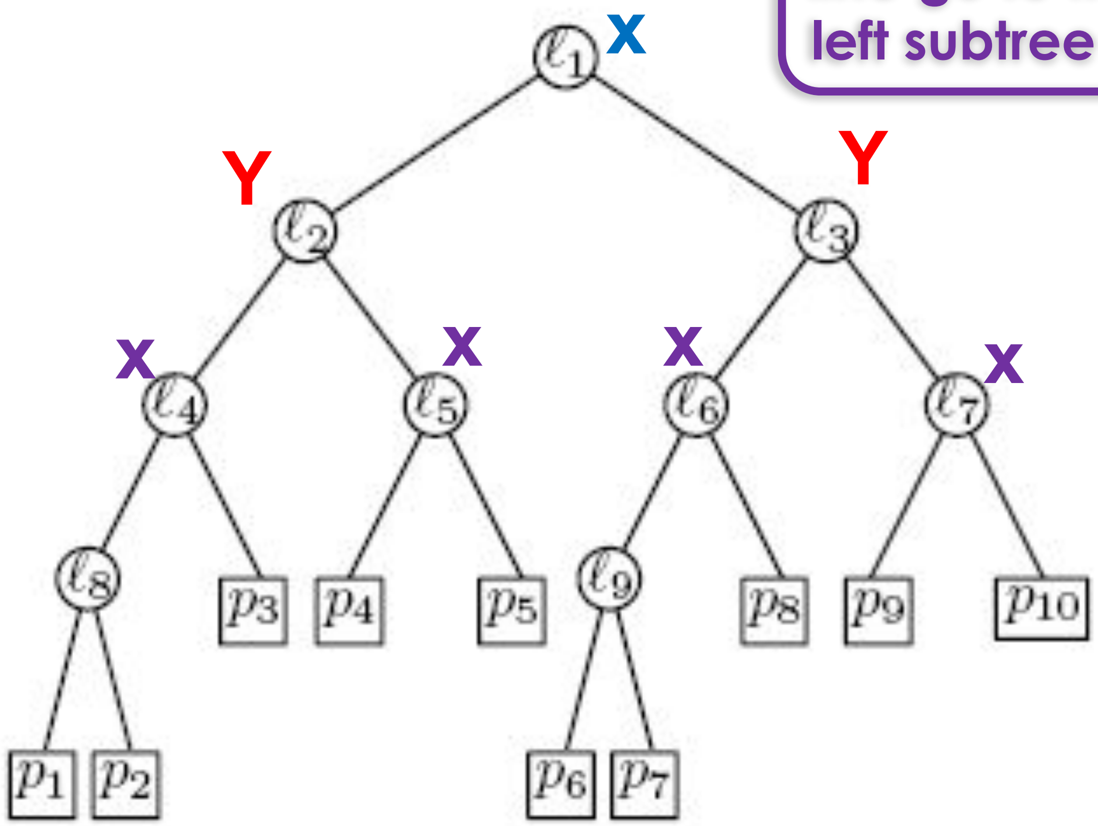
KD Tree Construction

<u>Split by x-coordinate</u>*:* split by a vertical line that has approximately half the points left or on, and half right.





Points on the line go to the left subtree

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KD Tree Construction

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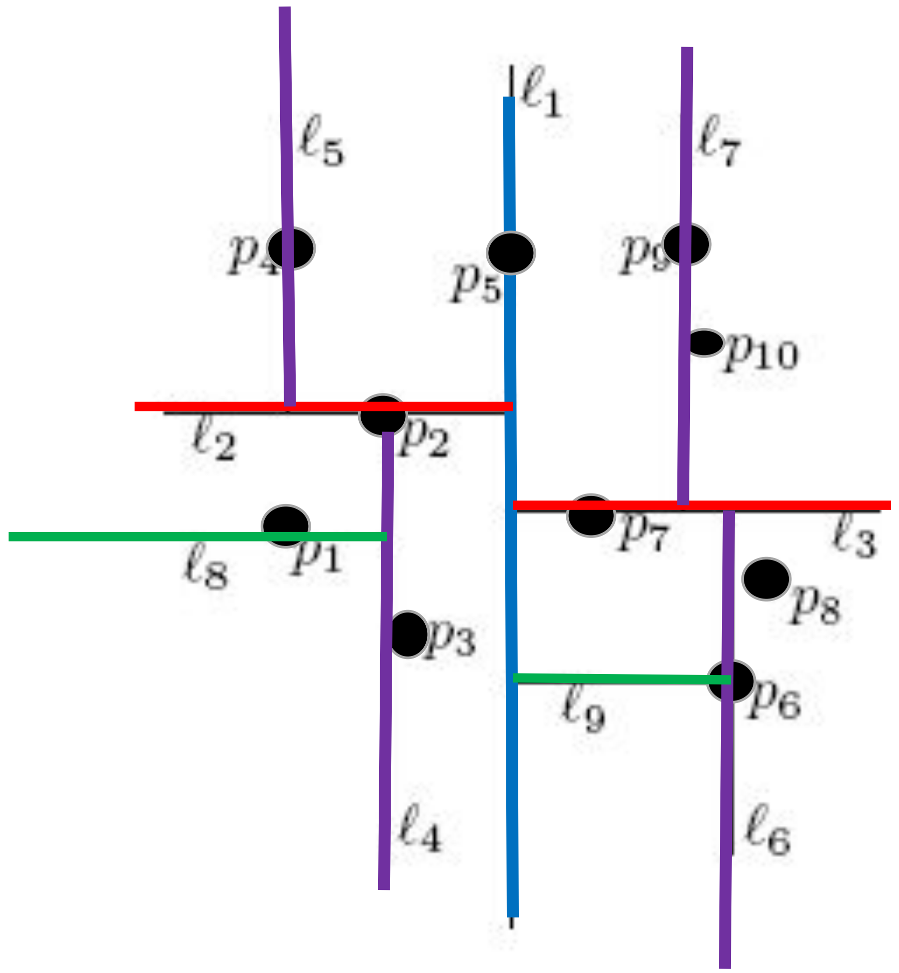
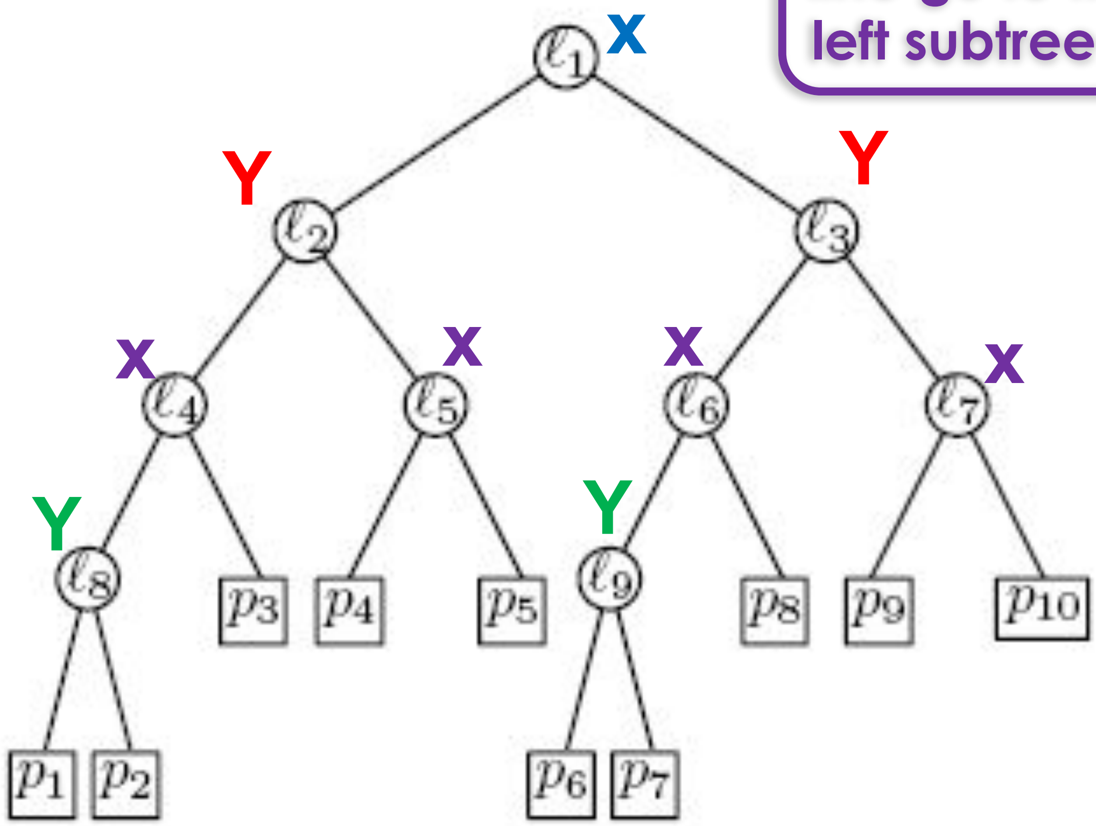
Points on the line go to the left subtree





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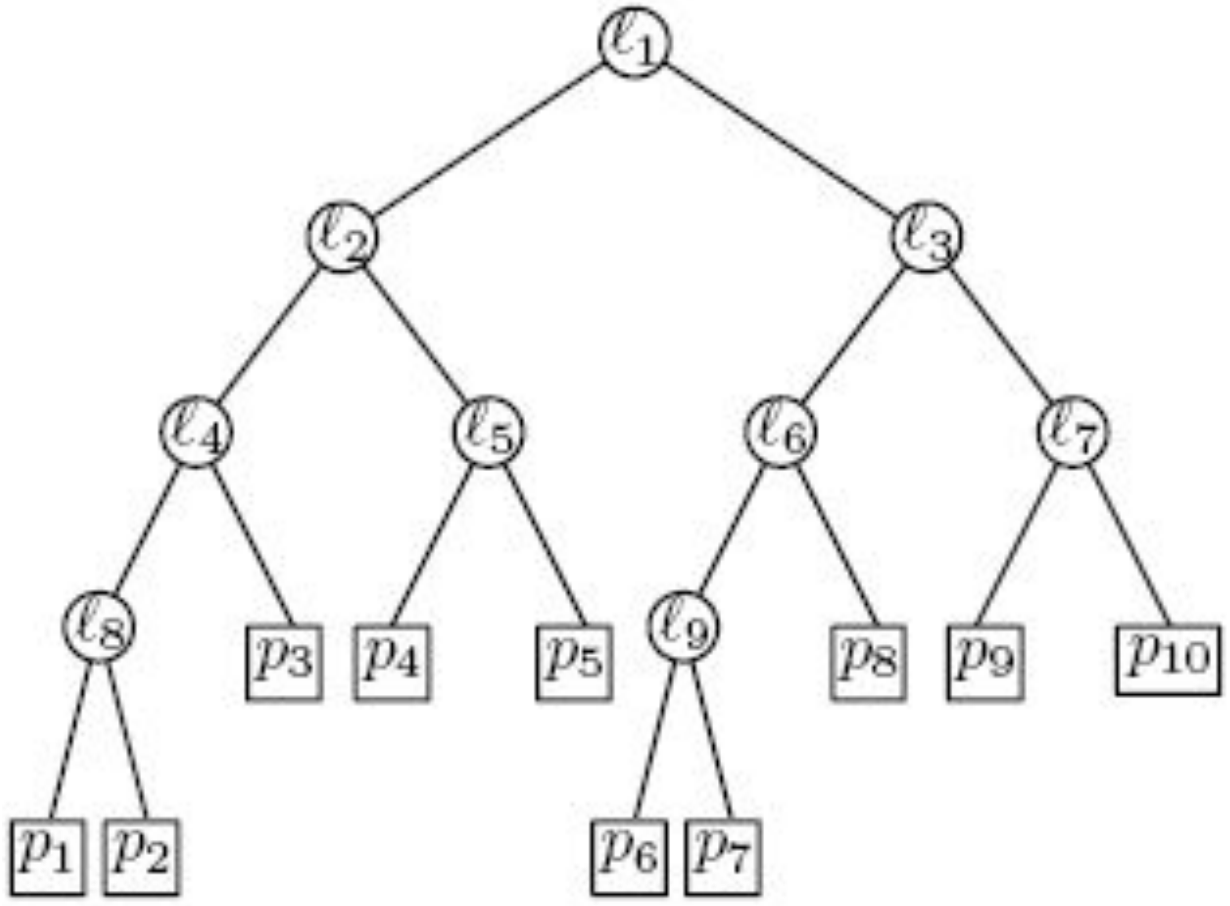
KD Tree Node Structure

- A KD-tree node has 5 fields
  - Splitting axis
  - Splitting value
  - Data
  - Left pointer
  - Right pointer



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KD Tree Splitting Strategies

- **Divide by finding median** Assumes all the points are available ahead of time.
- **Divide perpendicular to the axis with widest spread**
  - Split axes might not alternate
- And many more….

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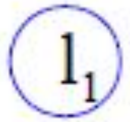
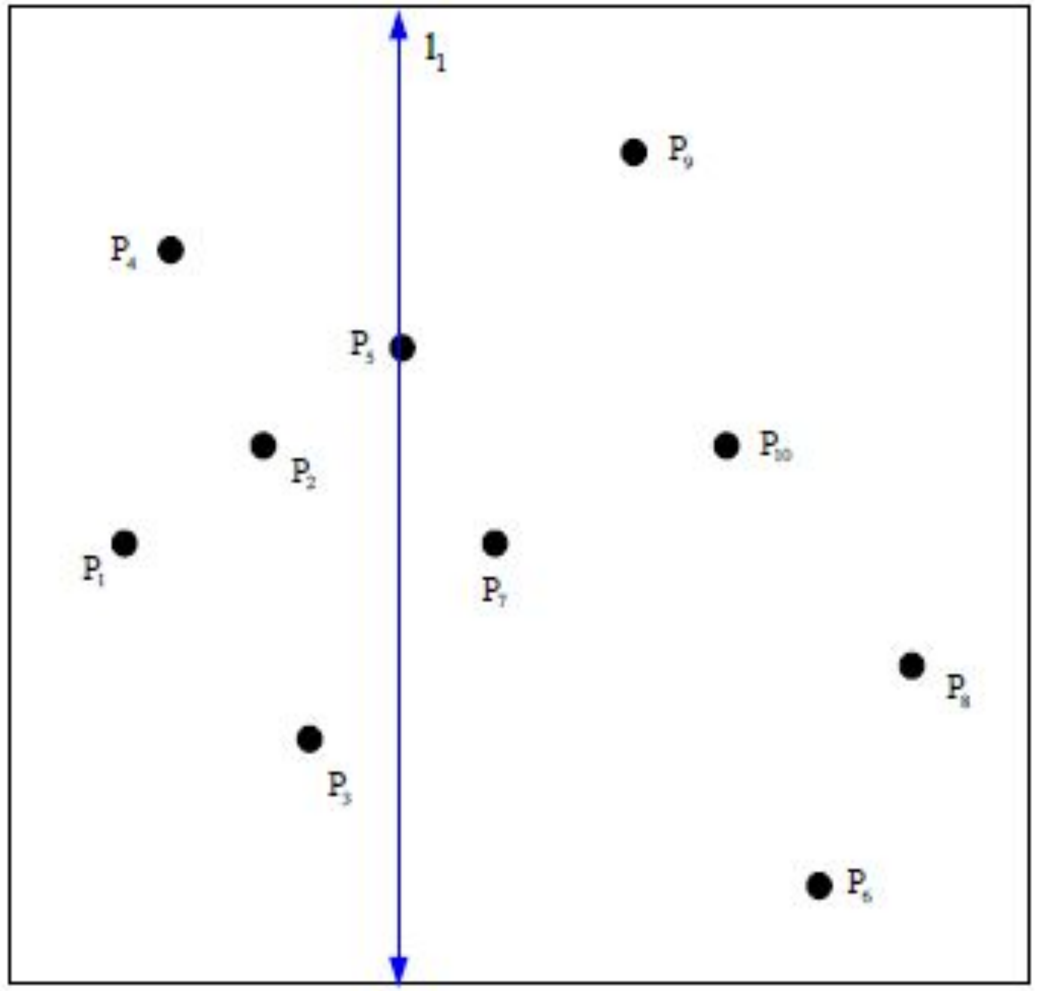
Example – using median (alternating between axis) data stored at the leaves





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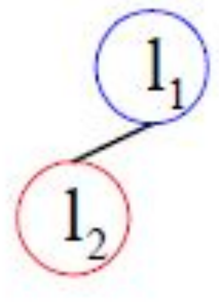

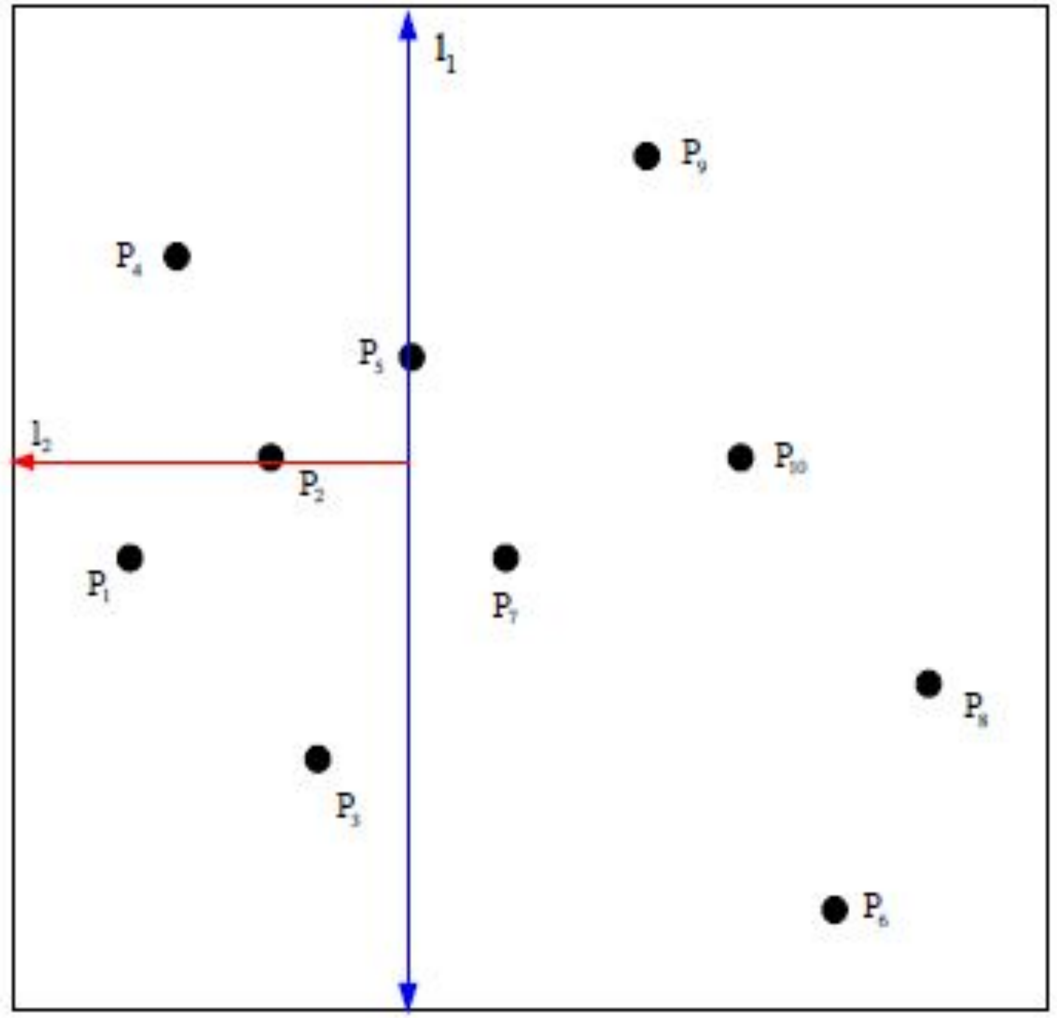

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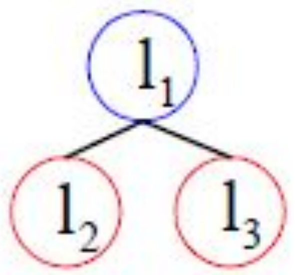

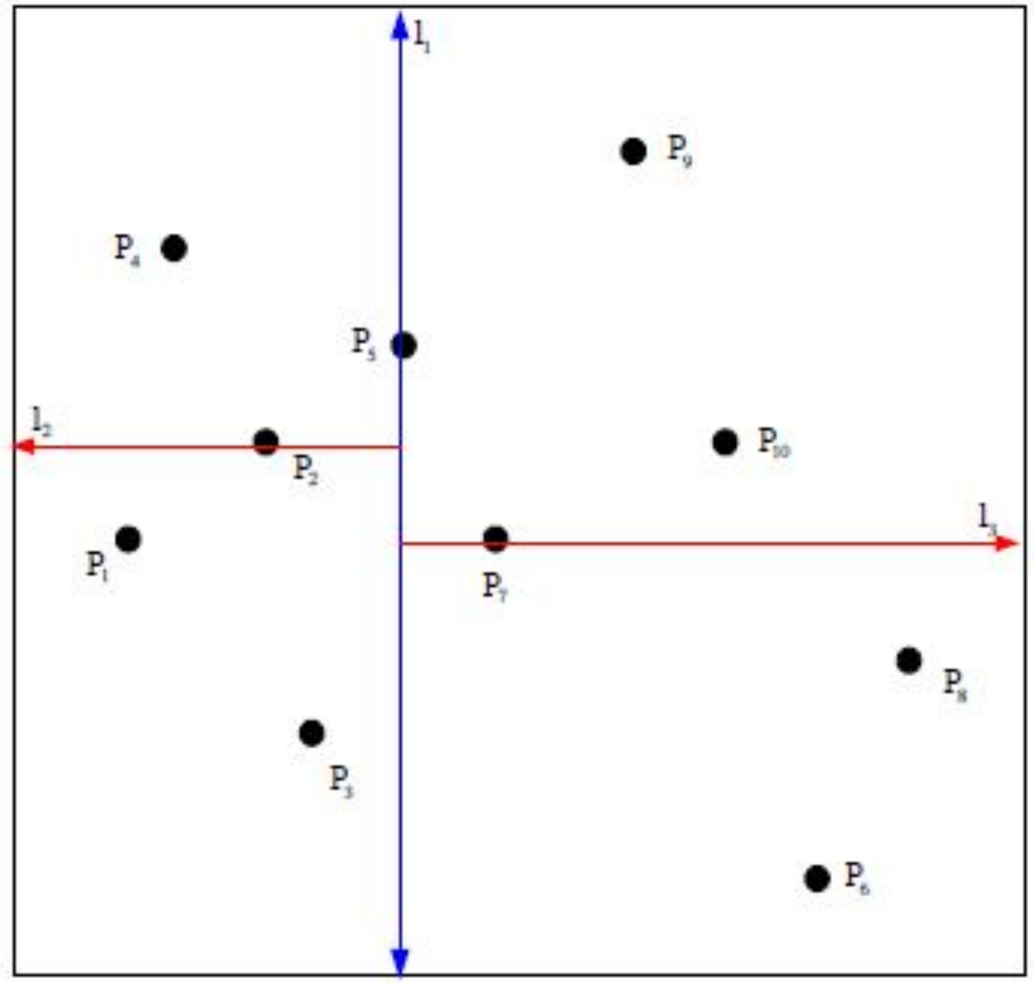

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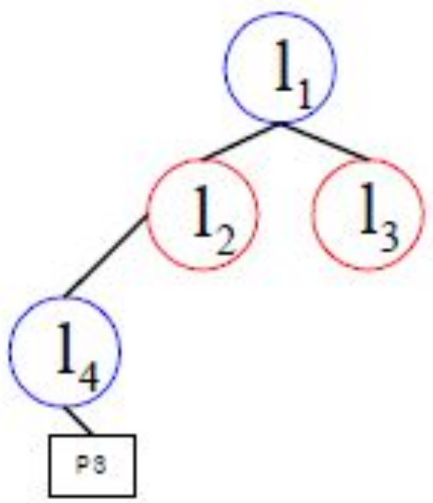

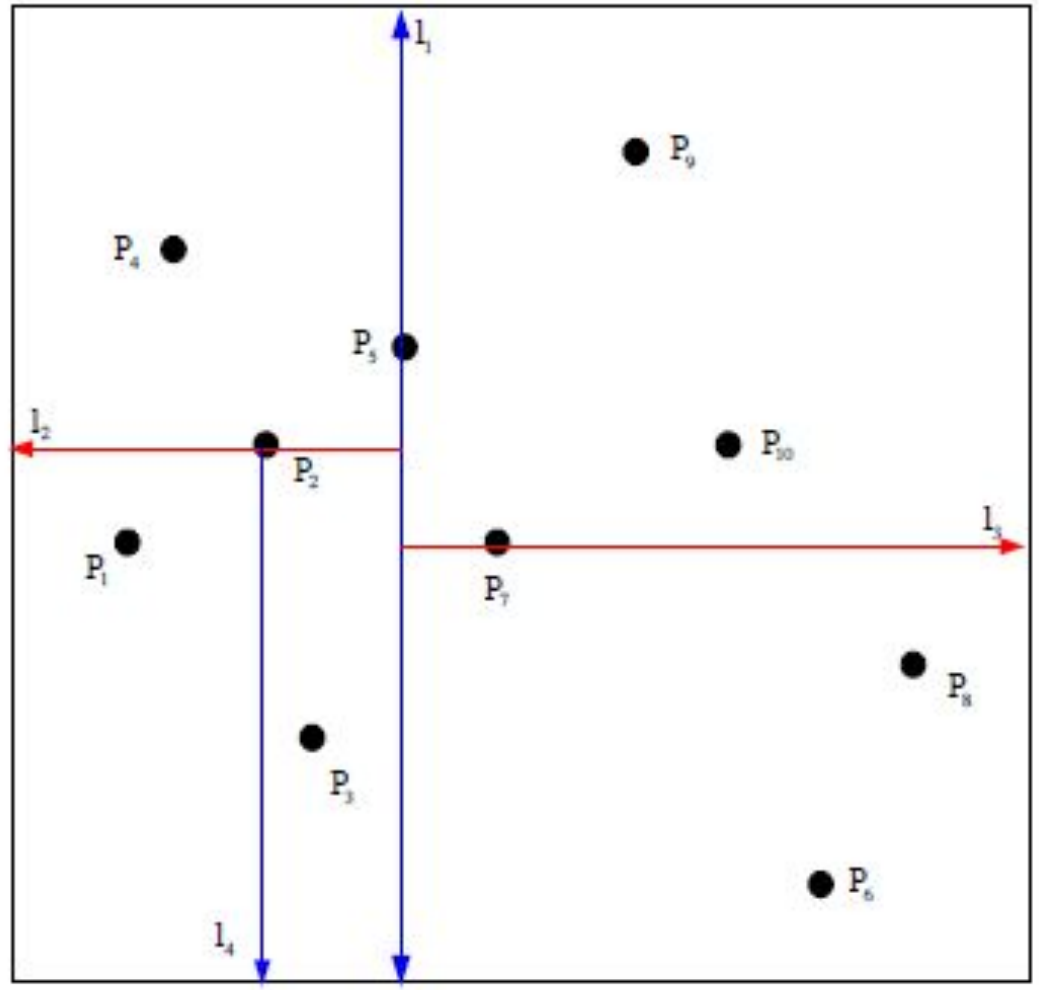

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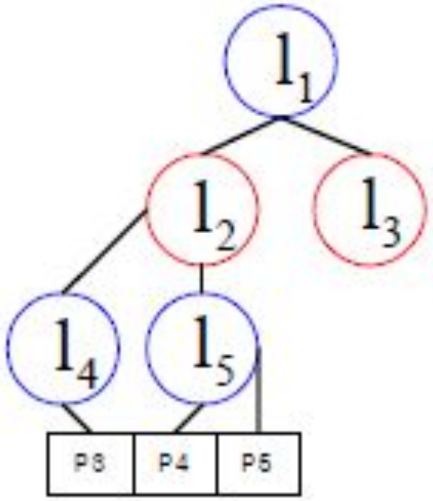

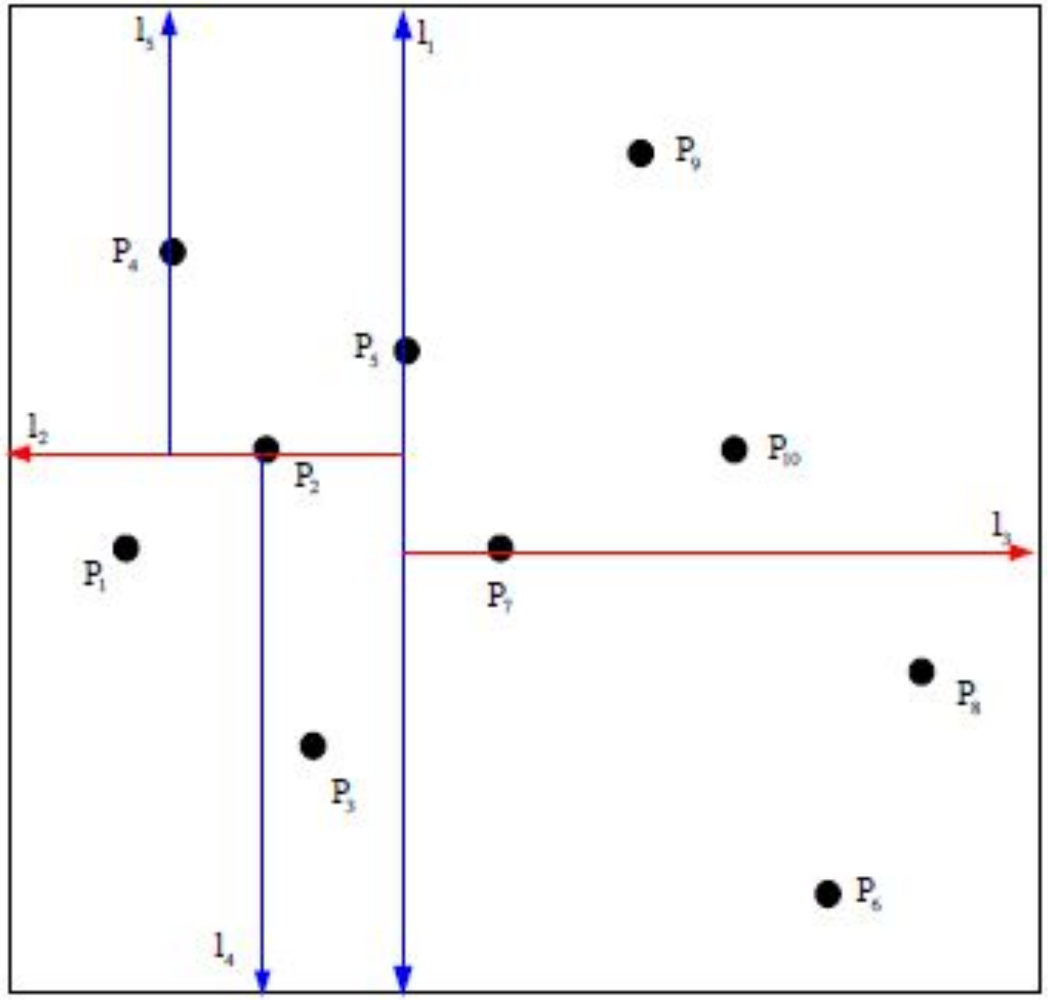

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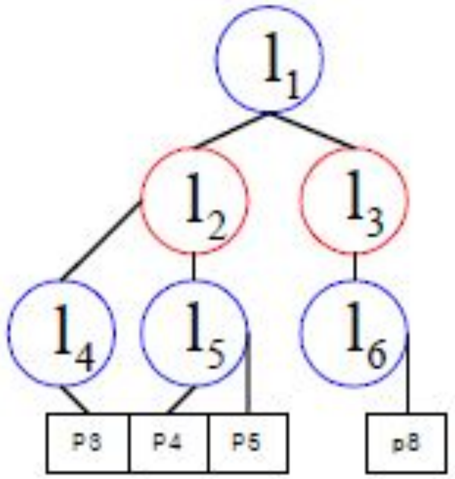

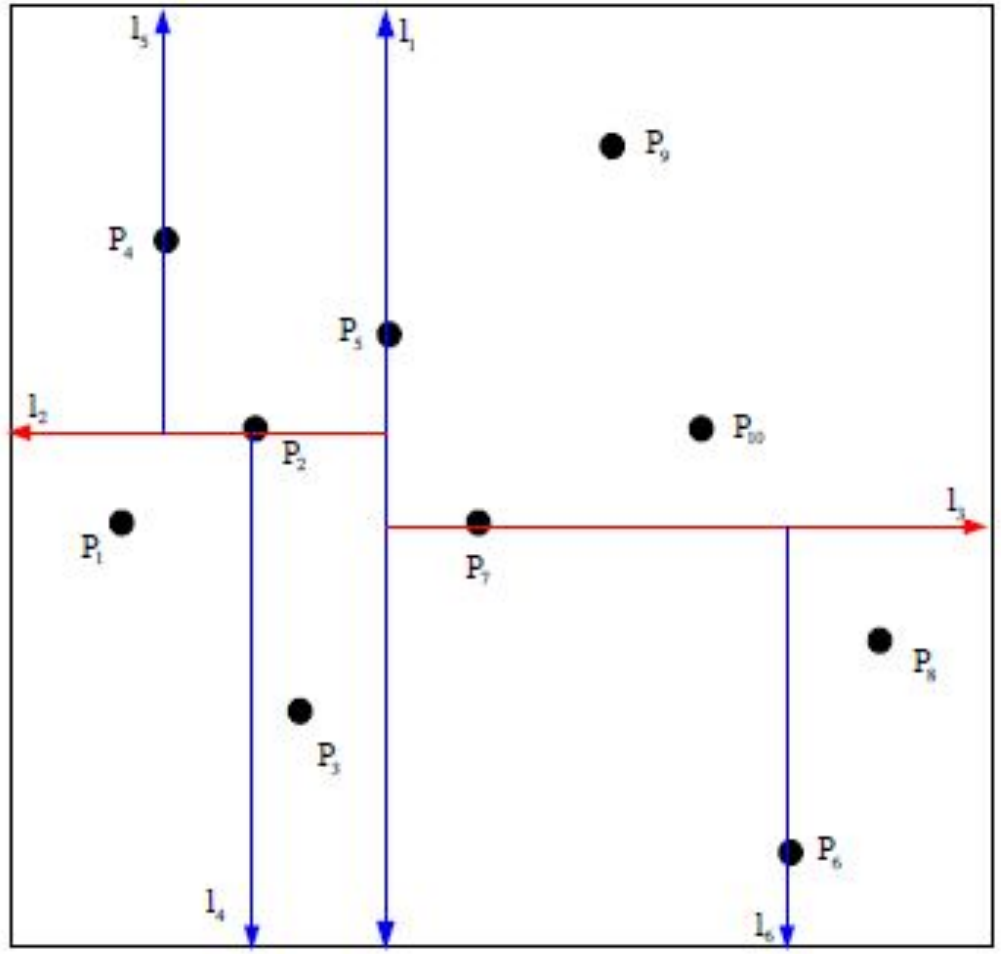

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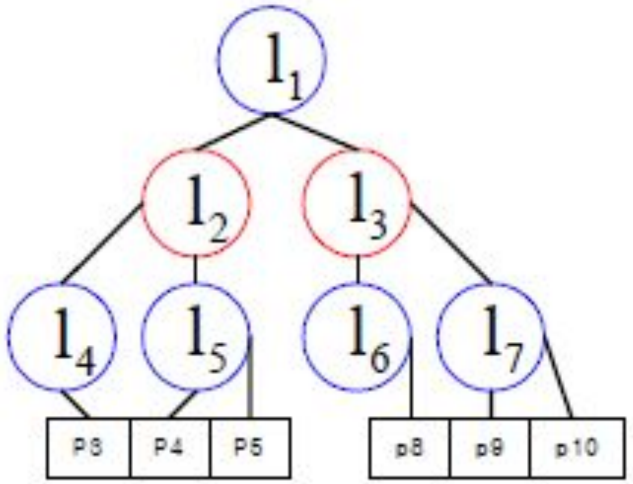

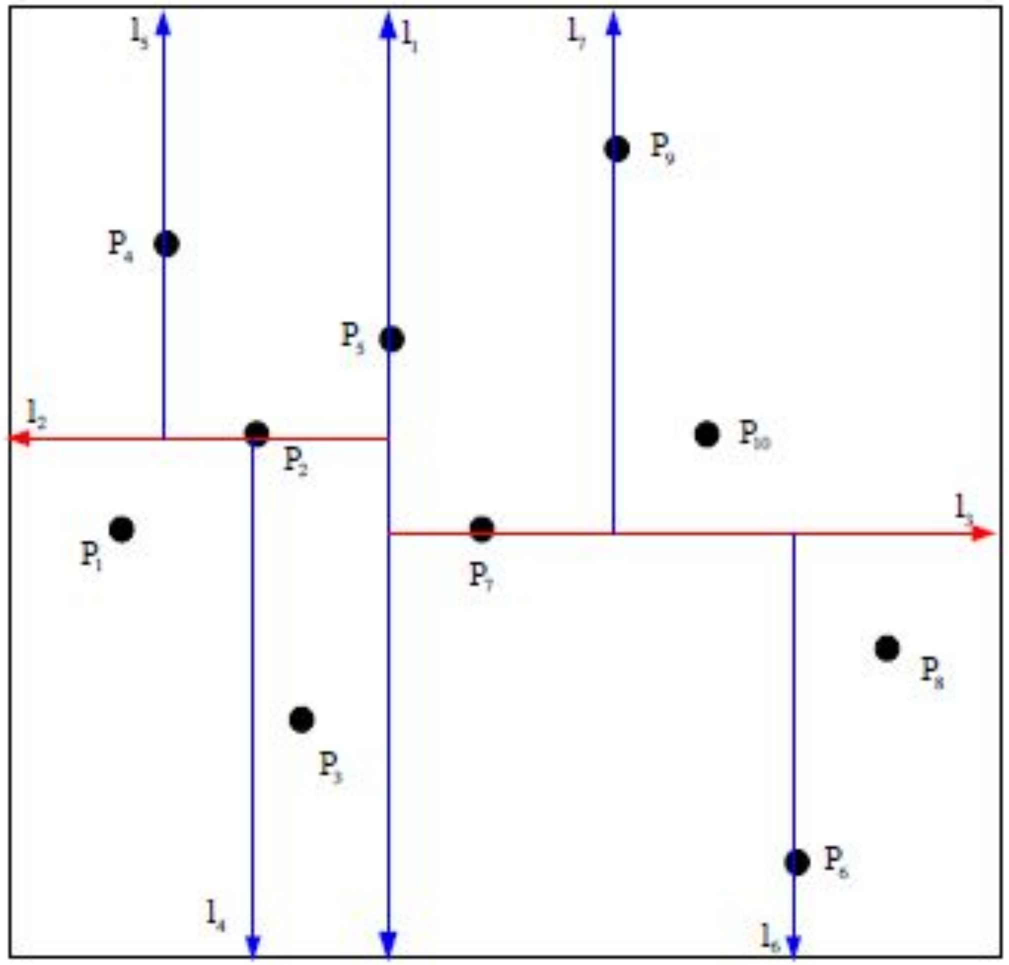

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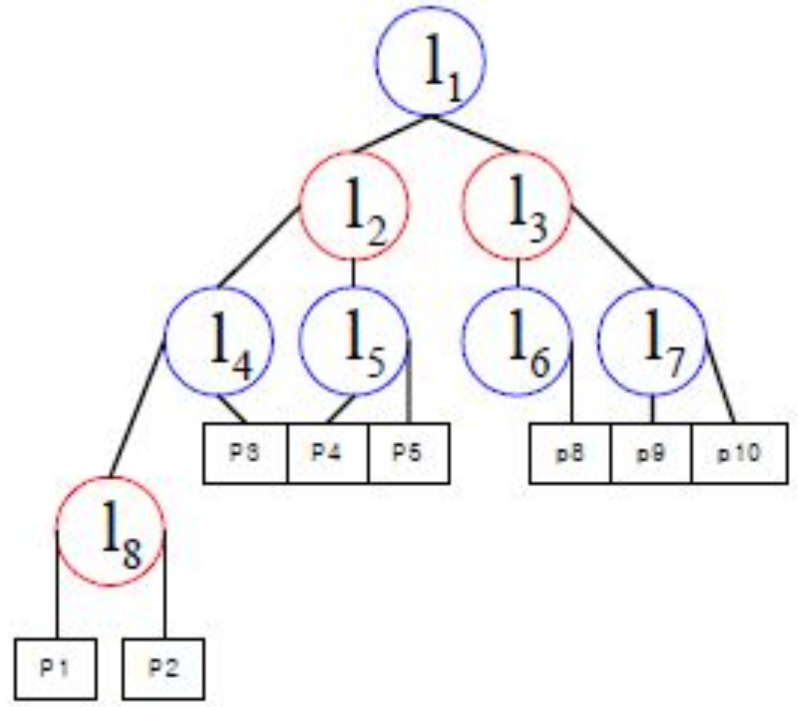

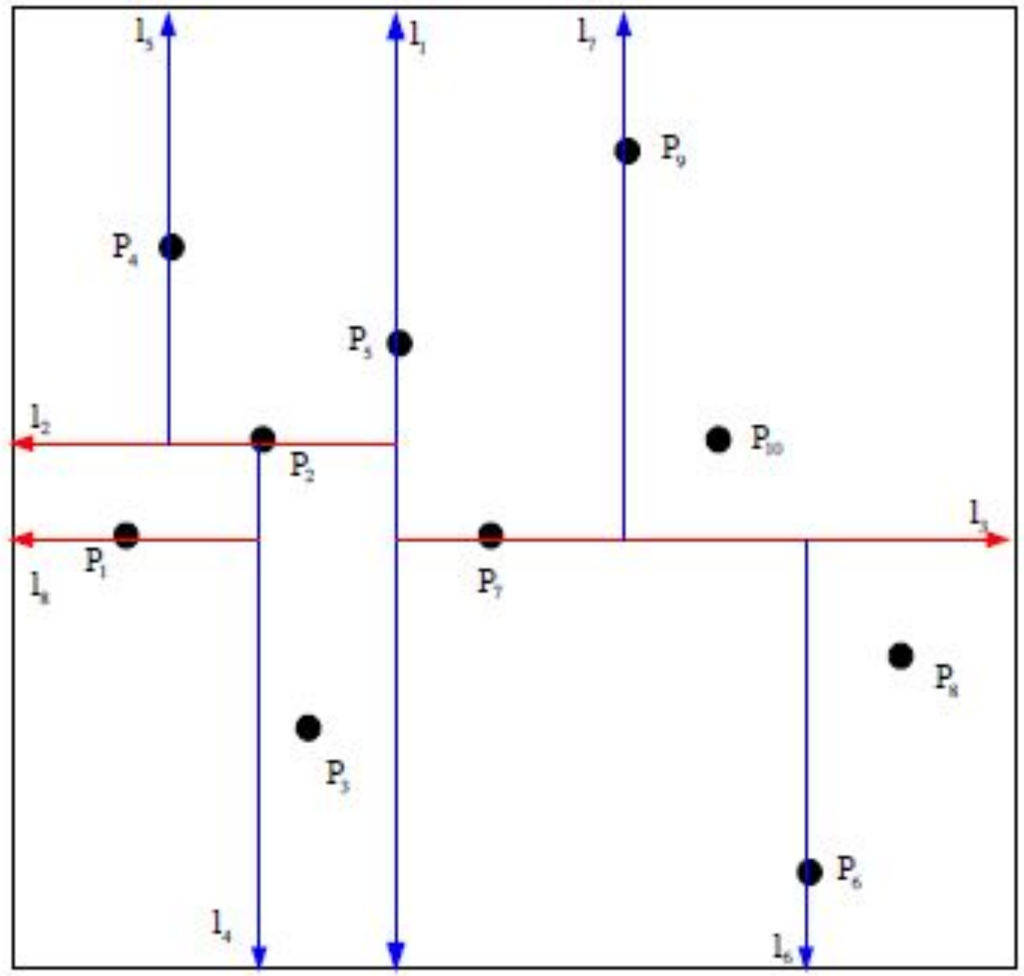

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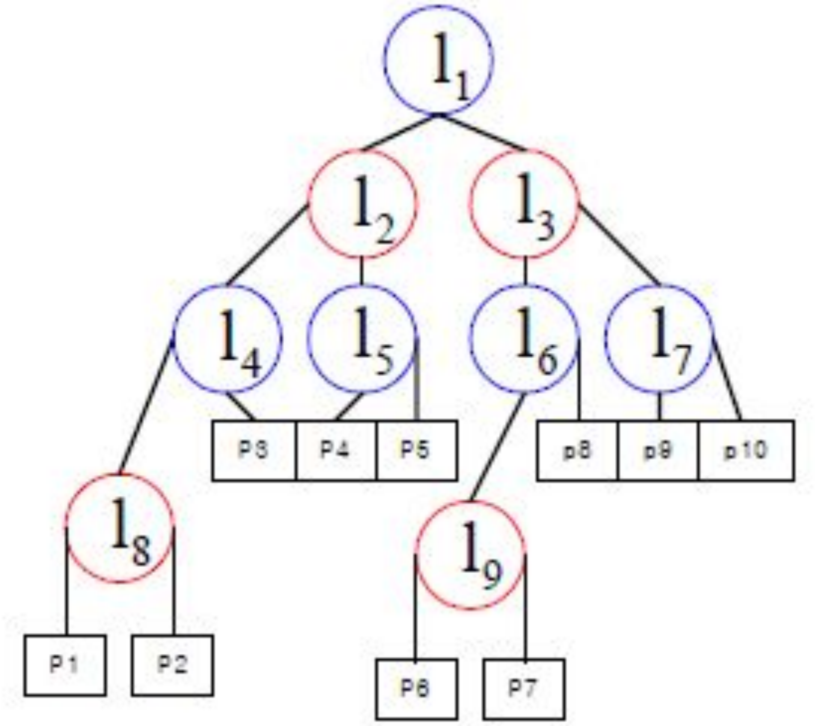

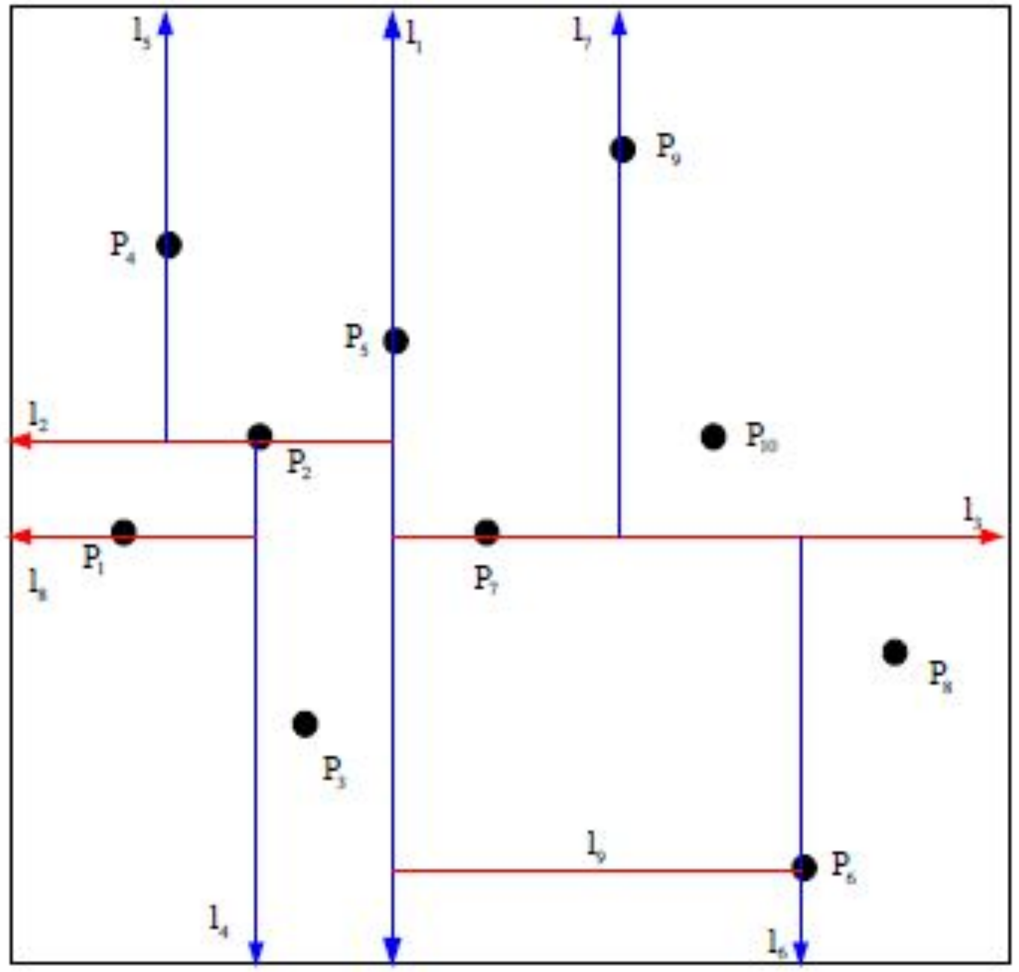

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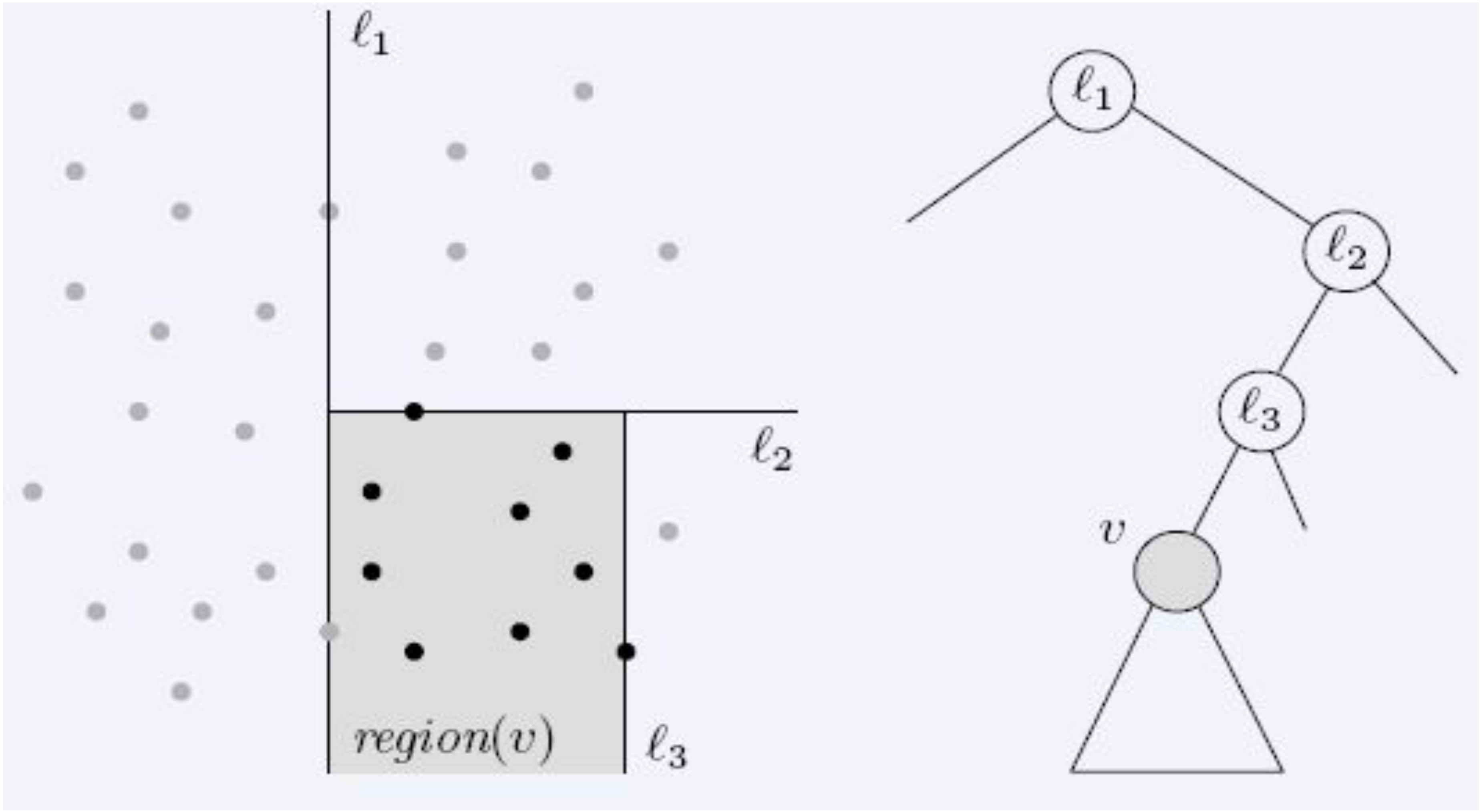
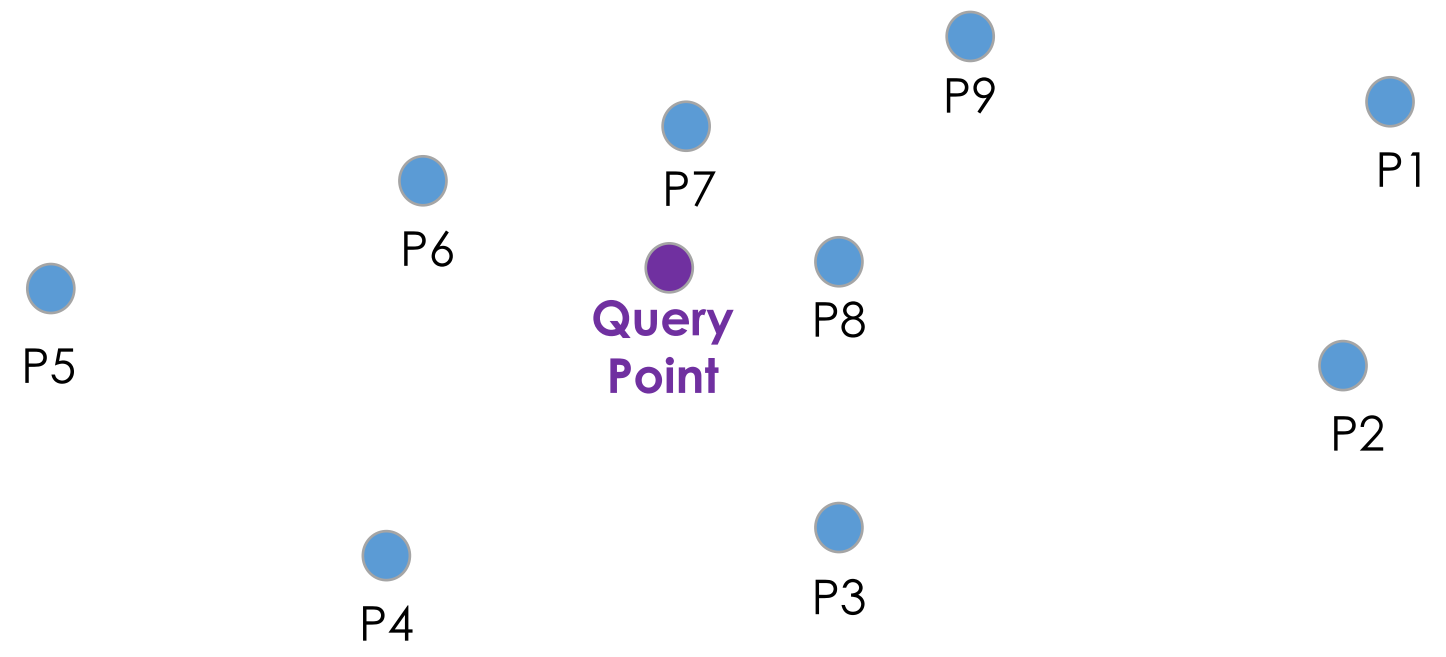
Region of node v



Region(v) : the subtree rooted at **v** stores the points in black dots

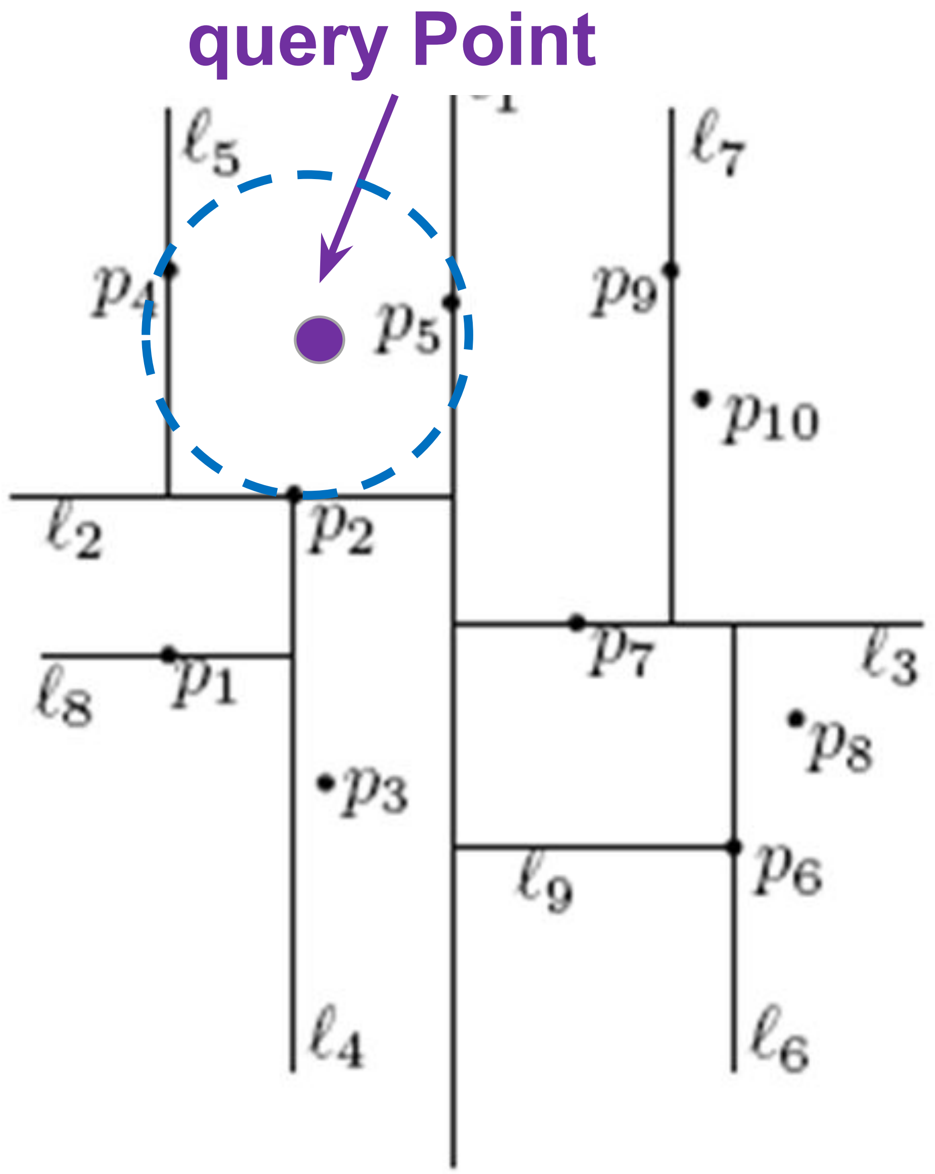
Recall Nearest Neighbor Queries



Retrieve closest k points to the query point

KD Trees – Nearest Neighbor Search

- KNN □ Find k nearest neighbor of the given query point.
- First we will discuss 1-NN and then generalize to K nearest neighbors.
- **Key Idea:** start off with an estimate on nearest neighbor and then keep updating whenever we find a better one.

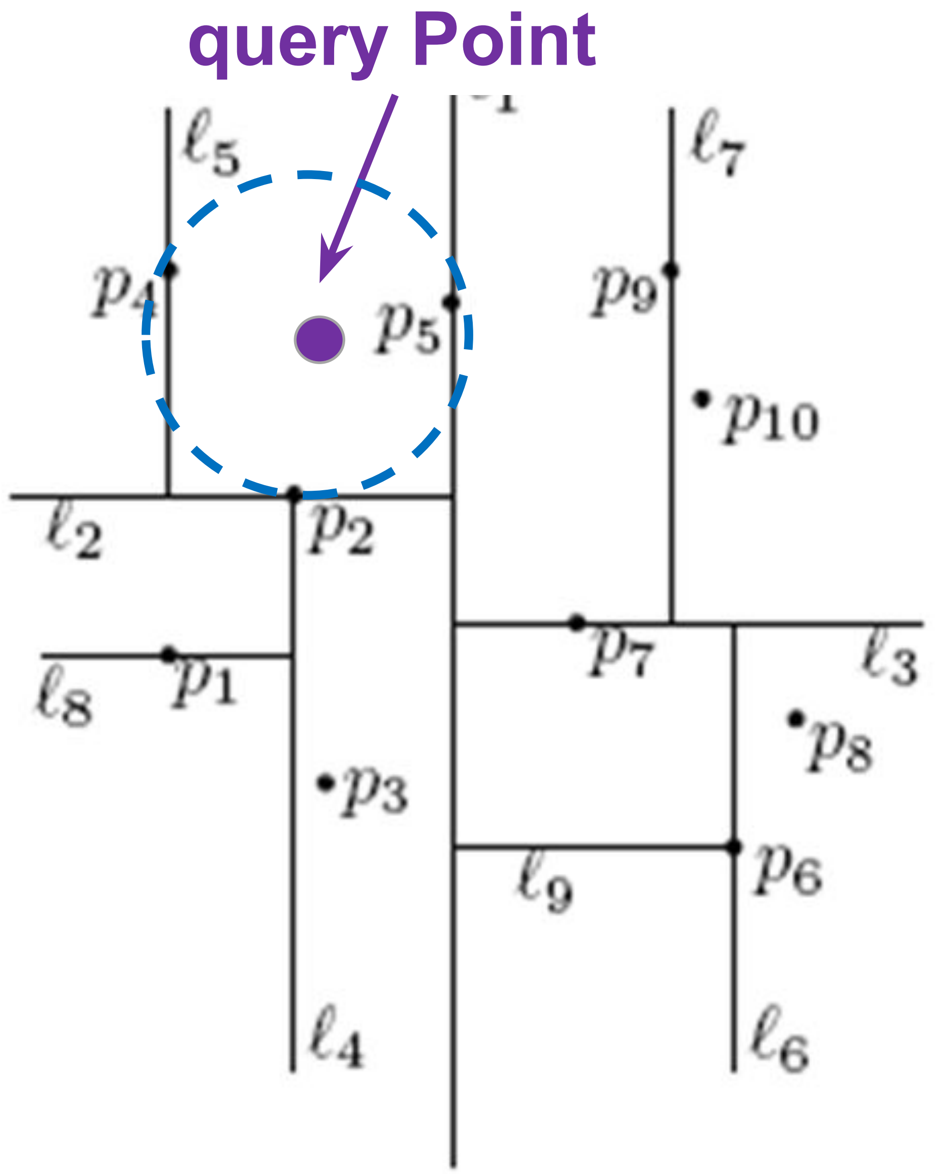


KD Trees – Nearest Neighbor Search

- **Two things to consider for such a approach:**

1) Need a smart way to prune through the data space without actually computing distance to the actual data points

2) Also if my initial estimate is bad, I would end up doing a lot of replacements.



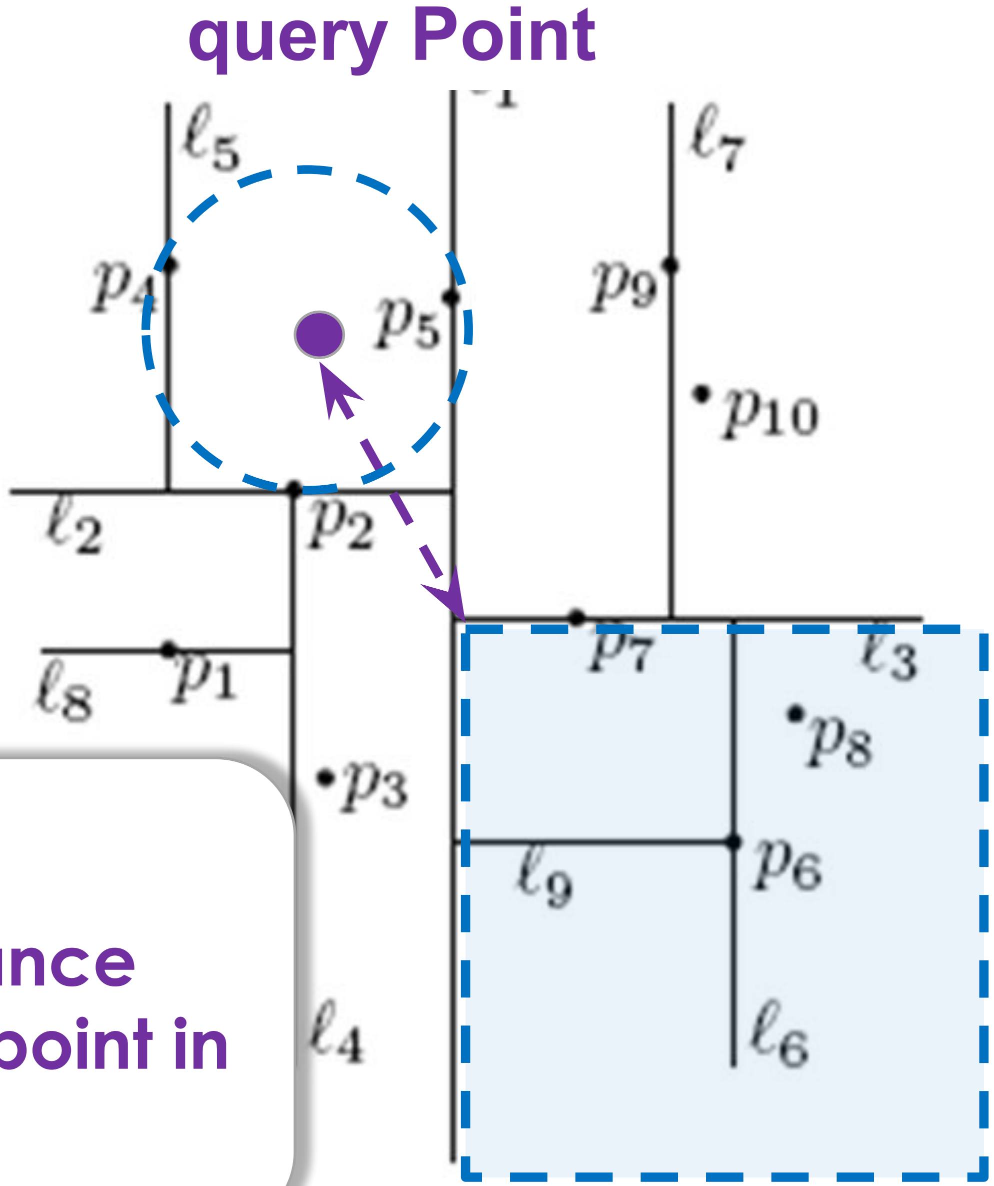
KD Trees – Nearest Neighbor Search

- **Two things to consider for such a approach:**

1) **Need a smart way to prune through the data space**

2) Also if my initial estimate is bad, I would end up doing a lot of replacements.

- **Consider the left child of L3**
- **What is the minimum possible distance between the query point and any point in the region of lc(L3)?**



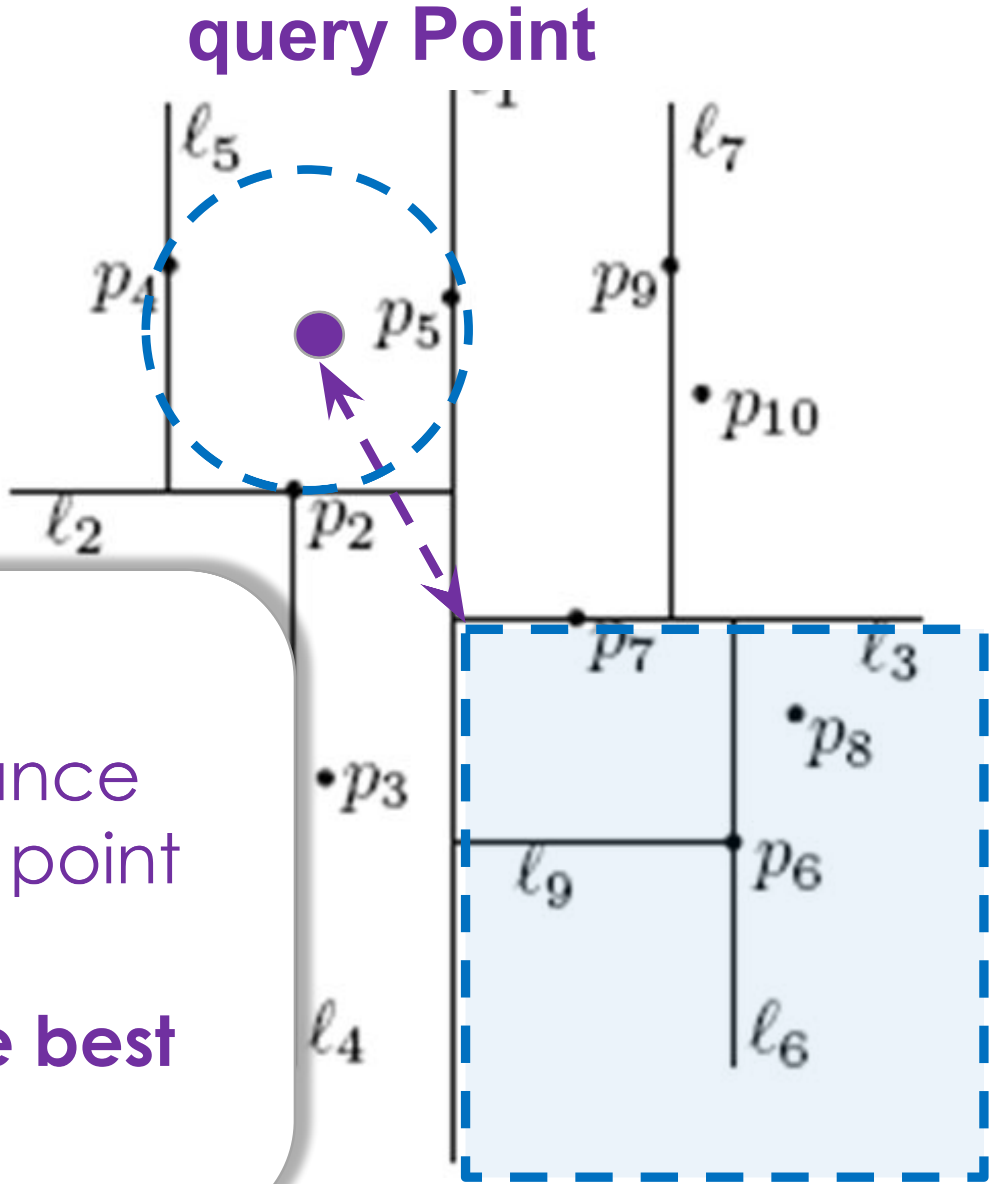
KD Trees – Nearest Neighbor Search

- Two things to consider for such a approach:

1) **Need a smart way to prune through the data space**

2) Also if my initial estimate is bad, I

- Consider the left child of L3
- What is the minimum possible distance between the query point and any point in the region of lc(L3)?
- **If it is less than our current distance best estimate (P2), then what???**



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- **Two things to consider for such a approach:**

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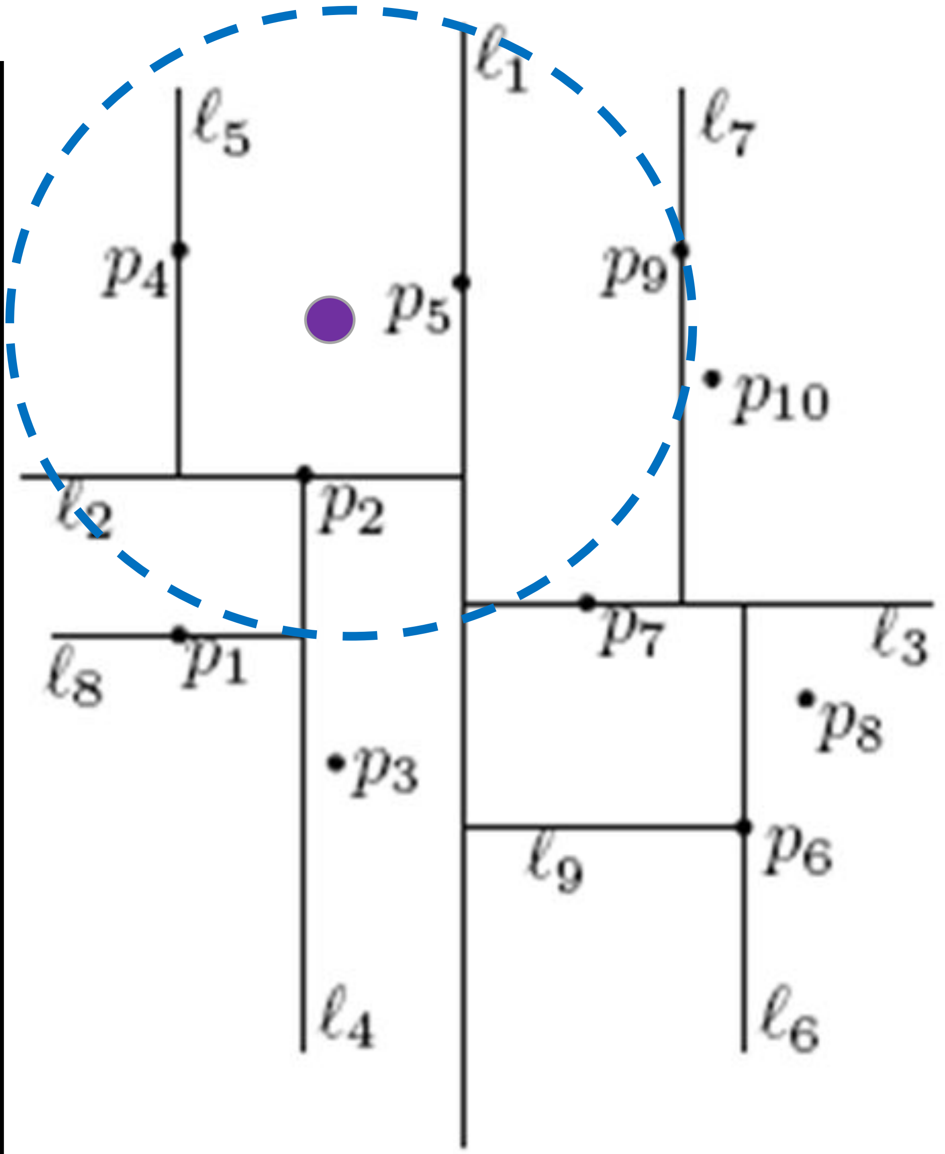
   - **E.g., P9 would be a very bad initial estimate**



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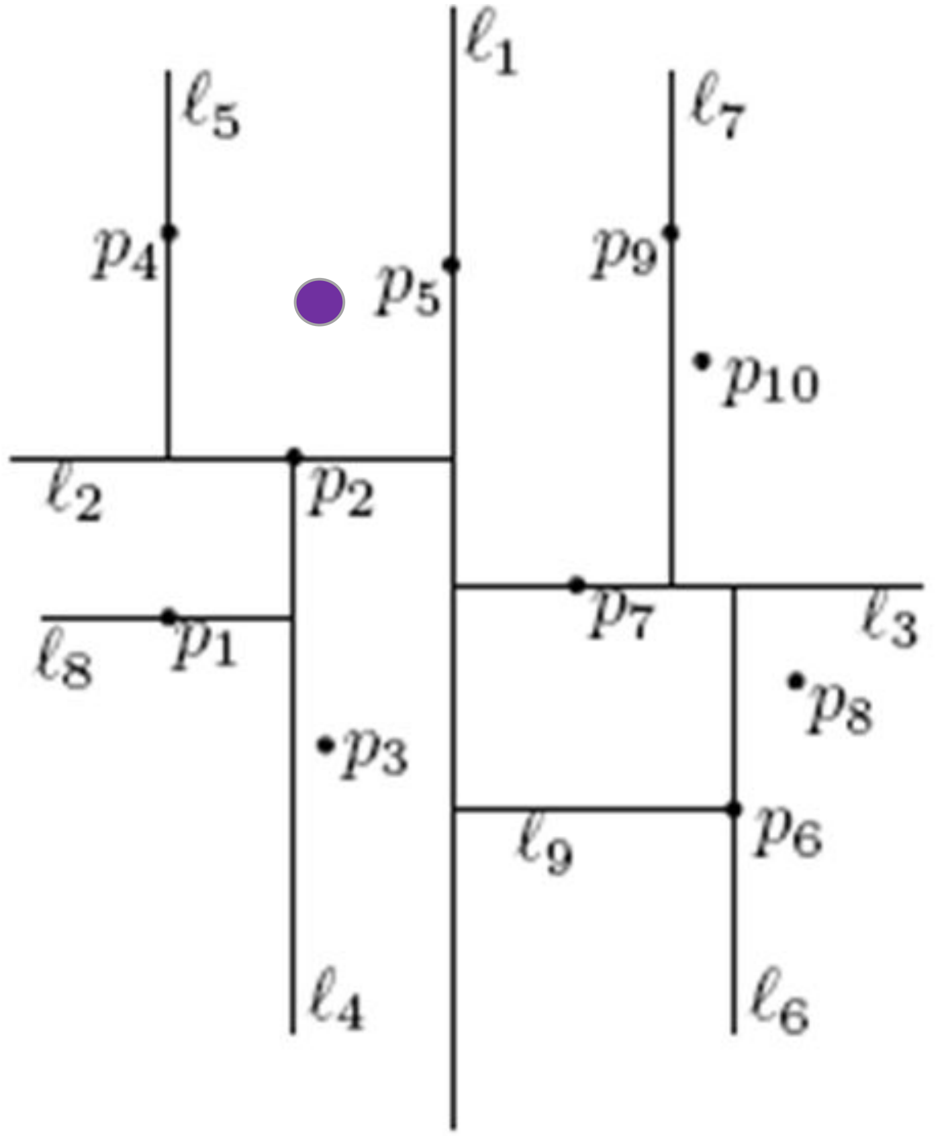
   - **Structure of KD-Tree helps us again.**

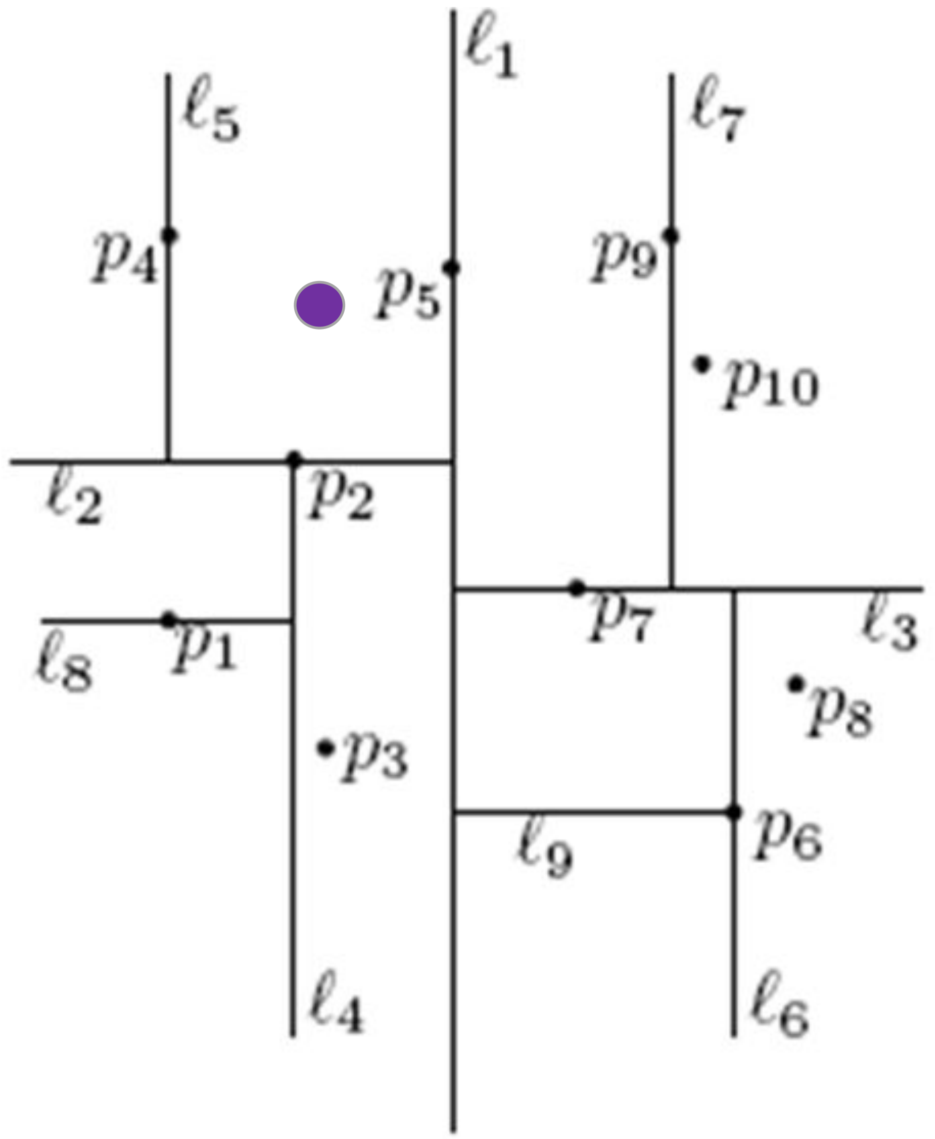


KD Trees – Nearest Neighbor Search

- **Two things to consider for such a approach:**

1) Need a smart way to prune through the data space

2) Need good initial estimates
   - E.g., P9 would be a very bad
   - Structure of KD-Tree helps
   - **“Search” of the query point in the query tree and take the leaf where the search terminates.**
   - **Works ok in practice**



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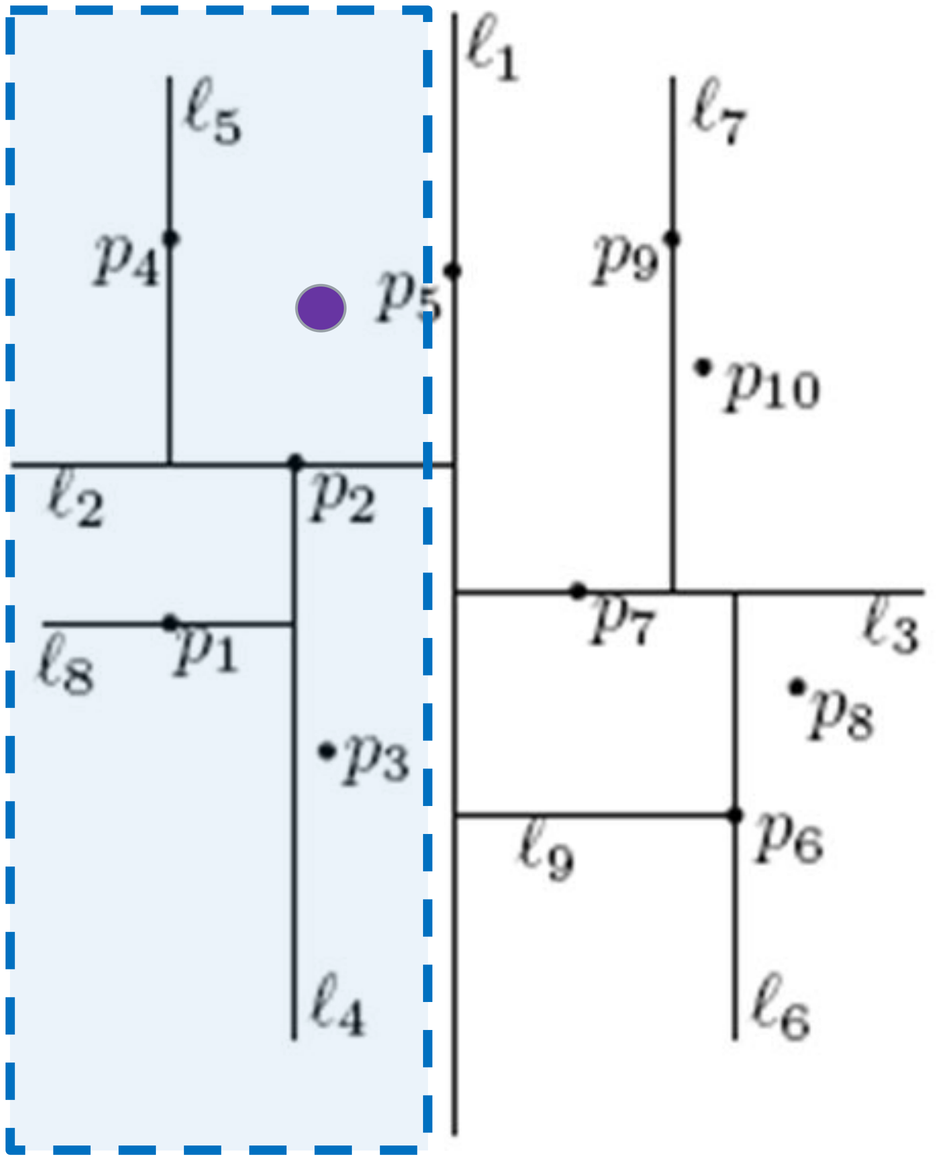

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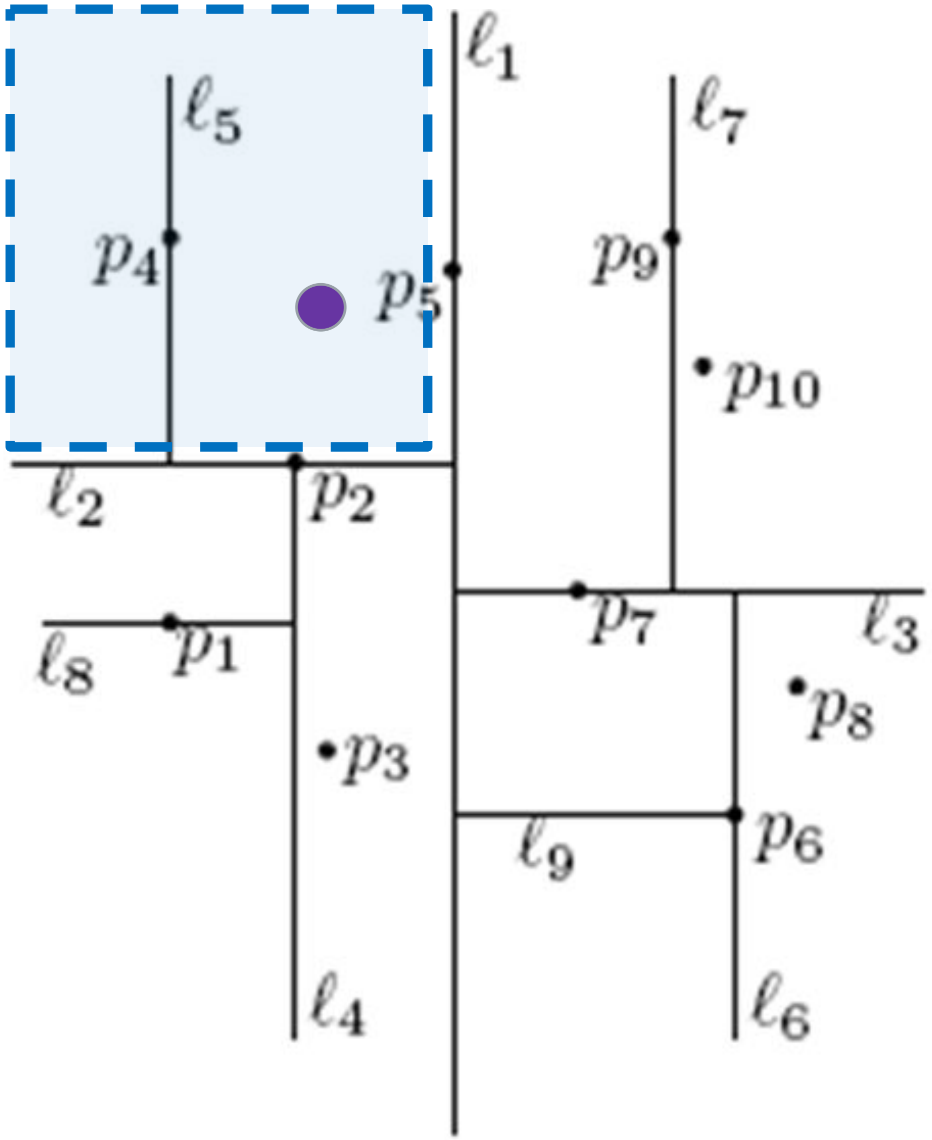

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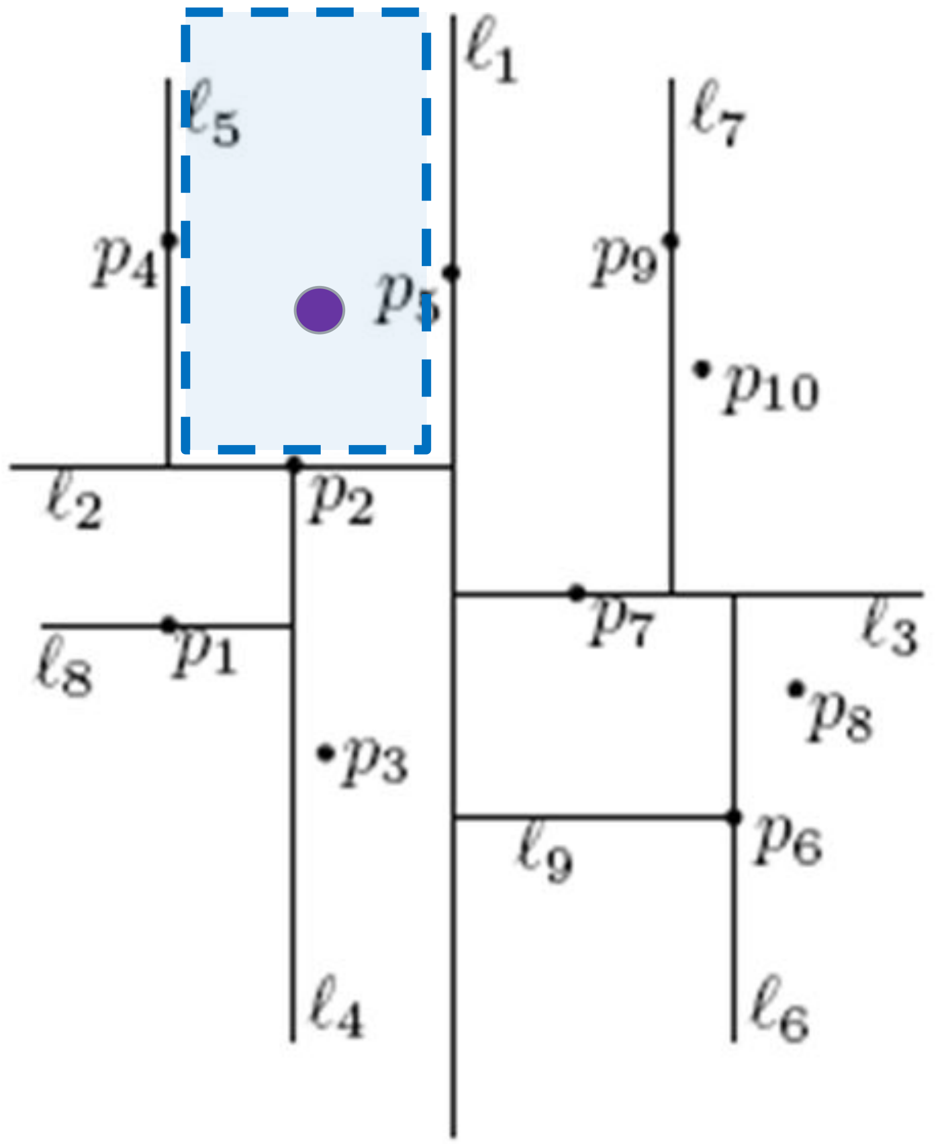

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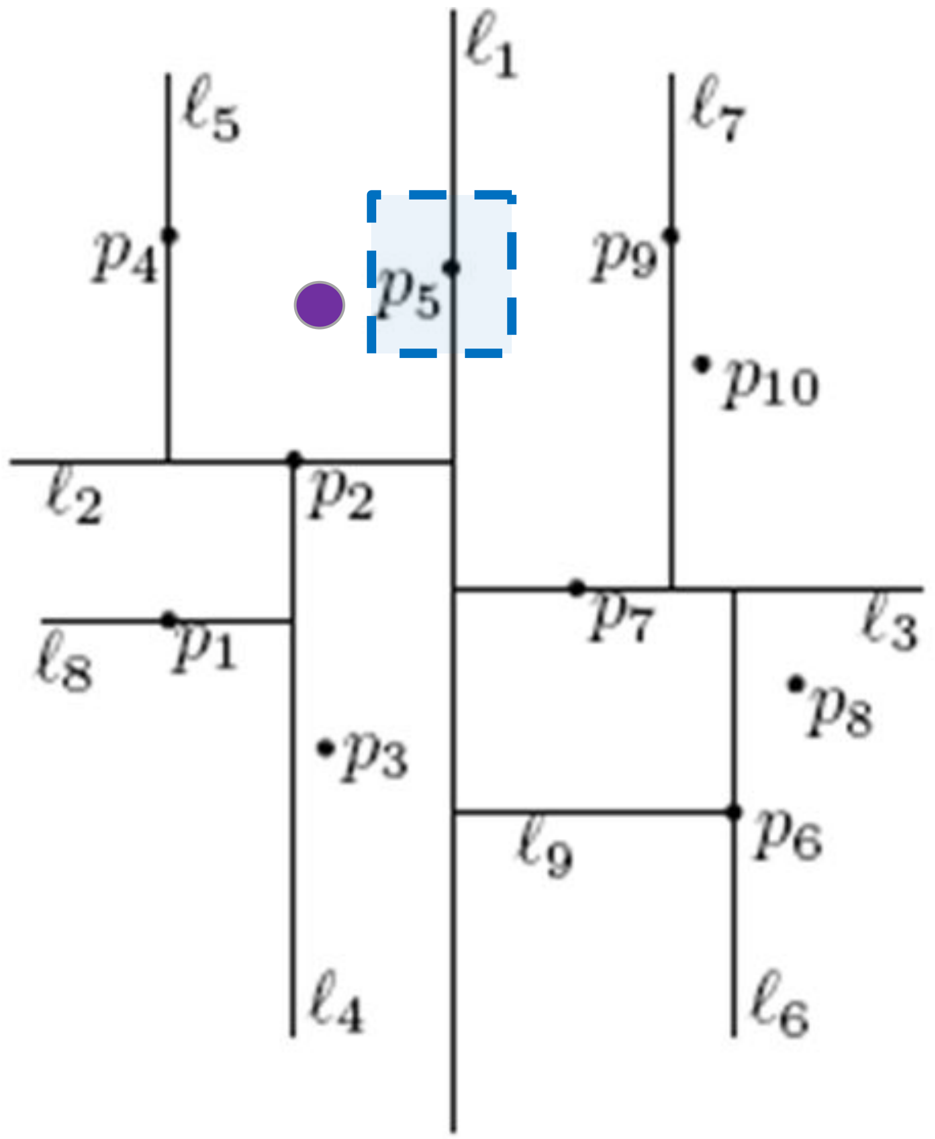
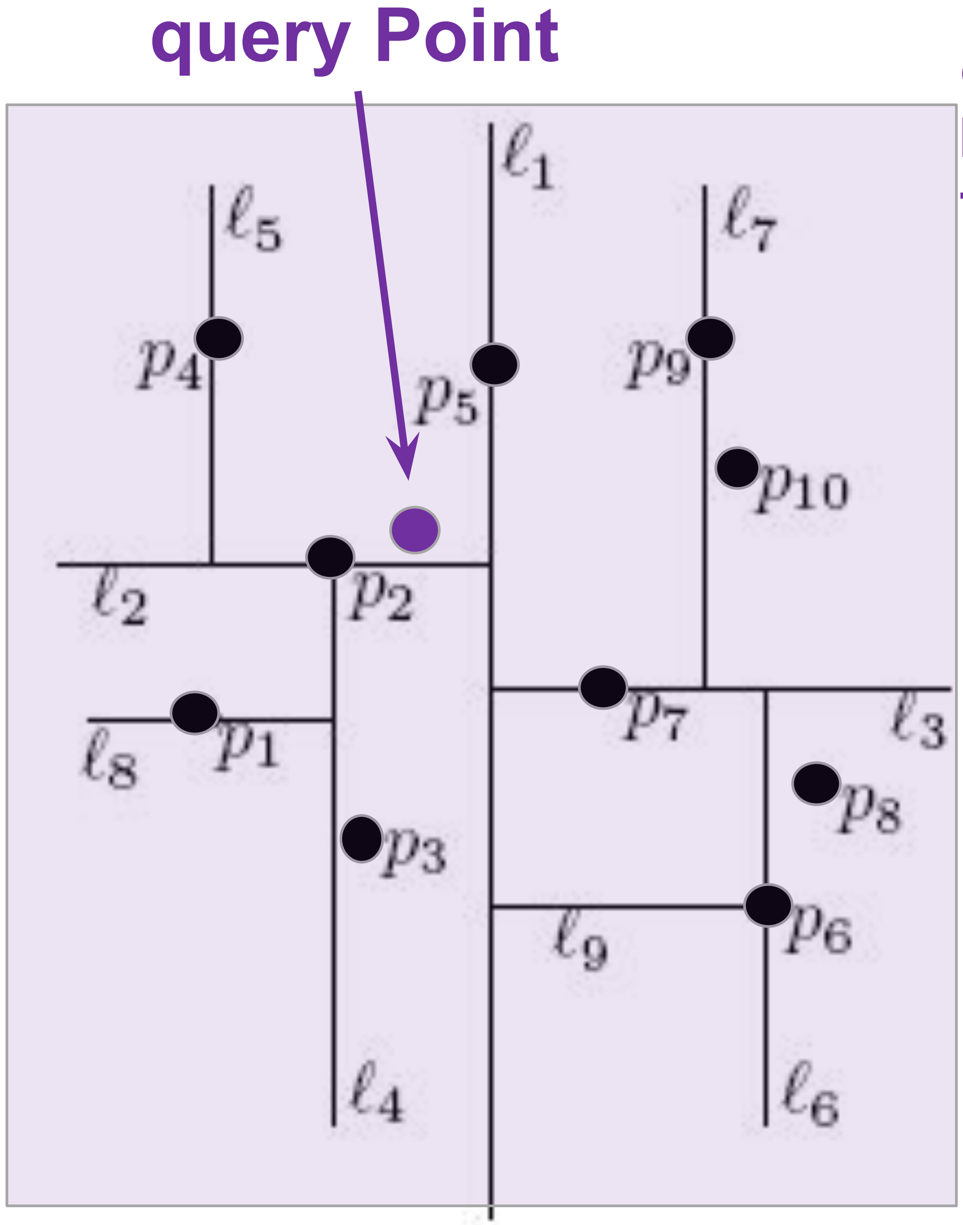
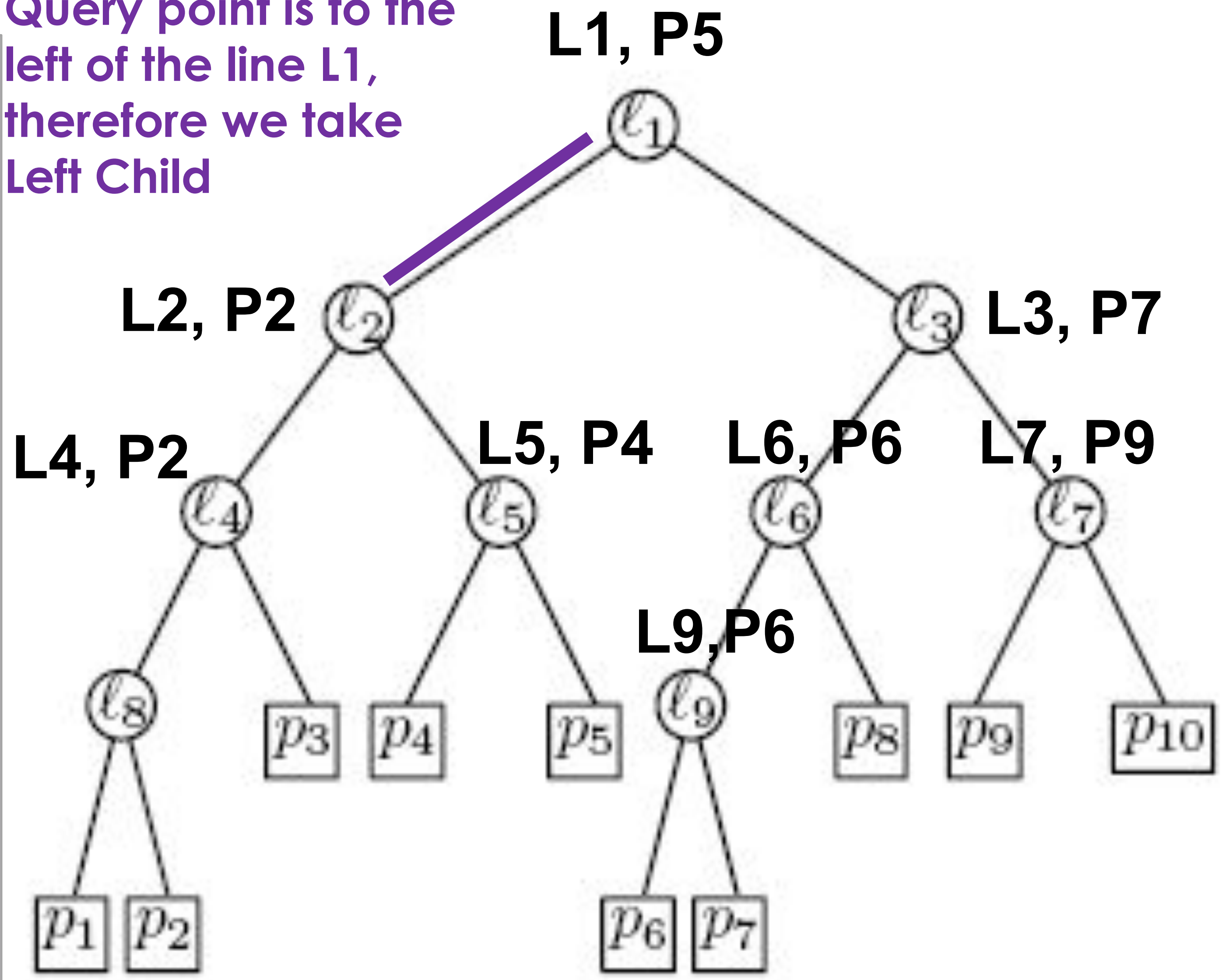


KD-tree: 1-NN Query Running



Query point is to the left of the line L1, therefore we take Left Child



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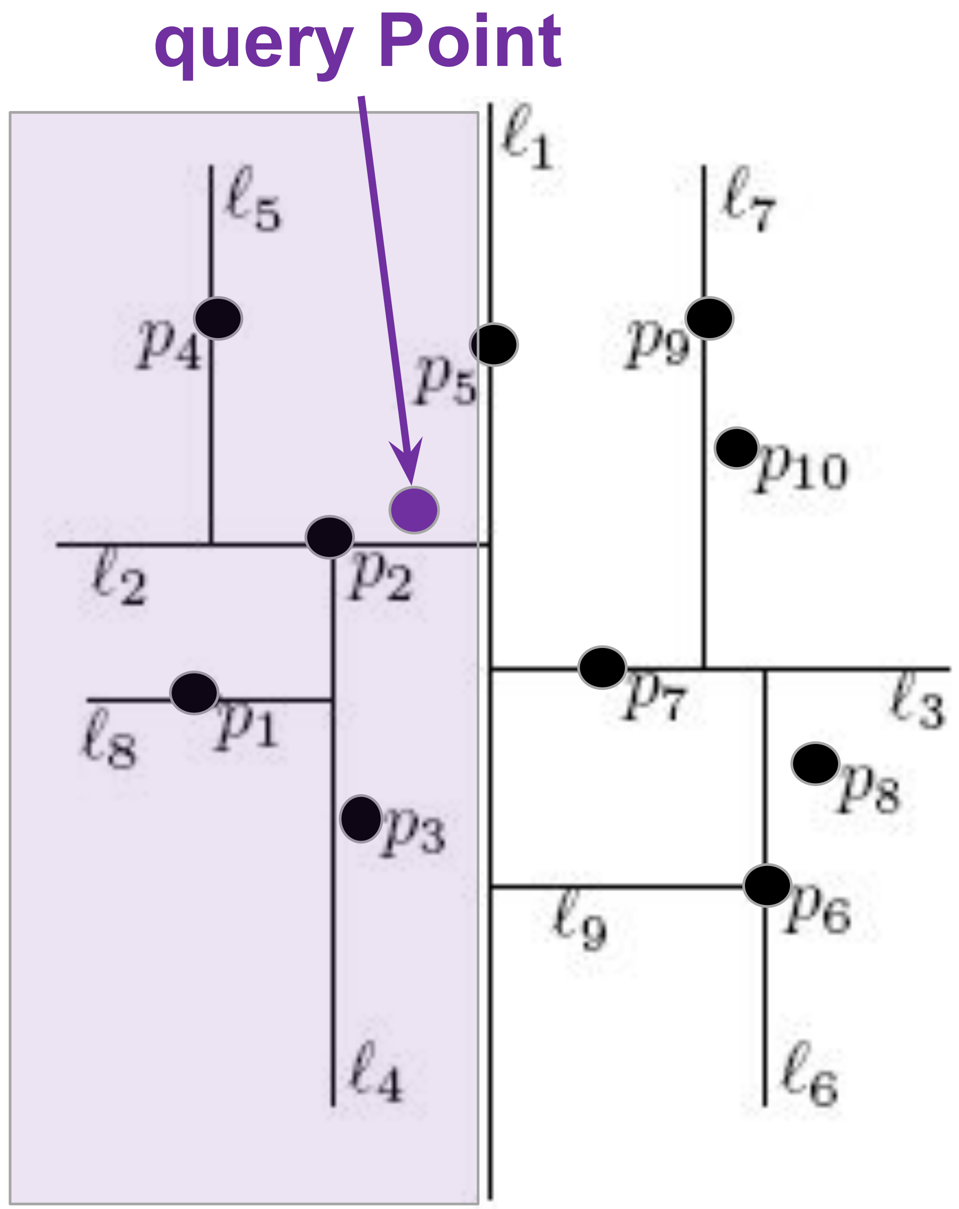
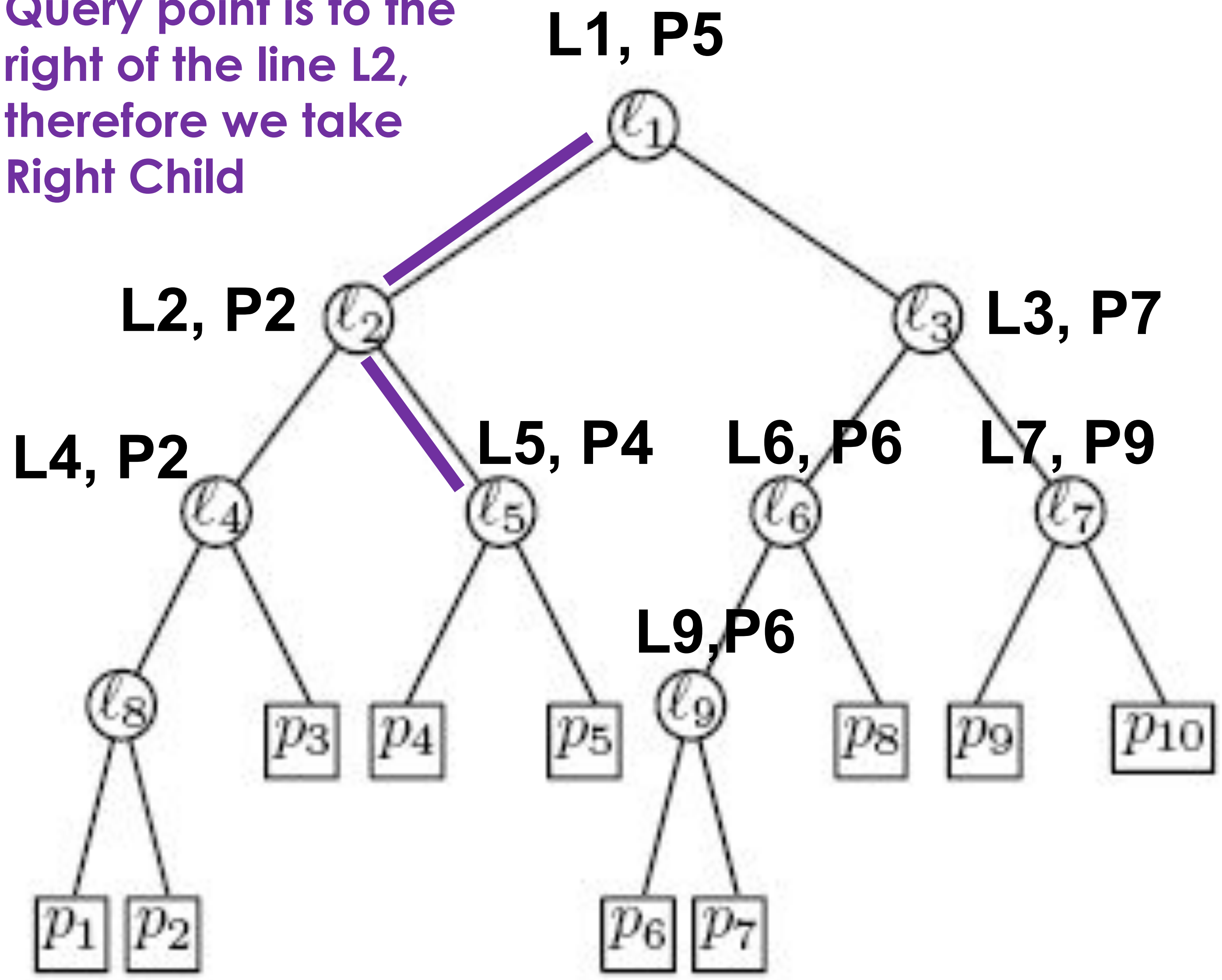
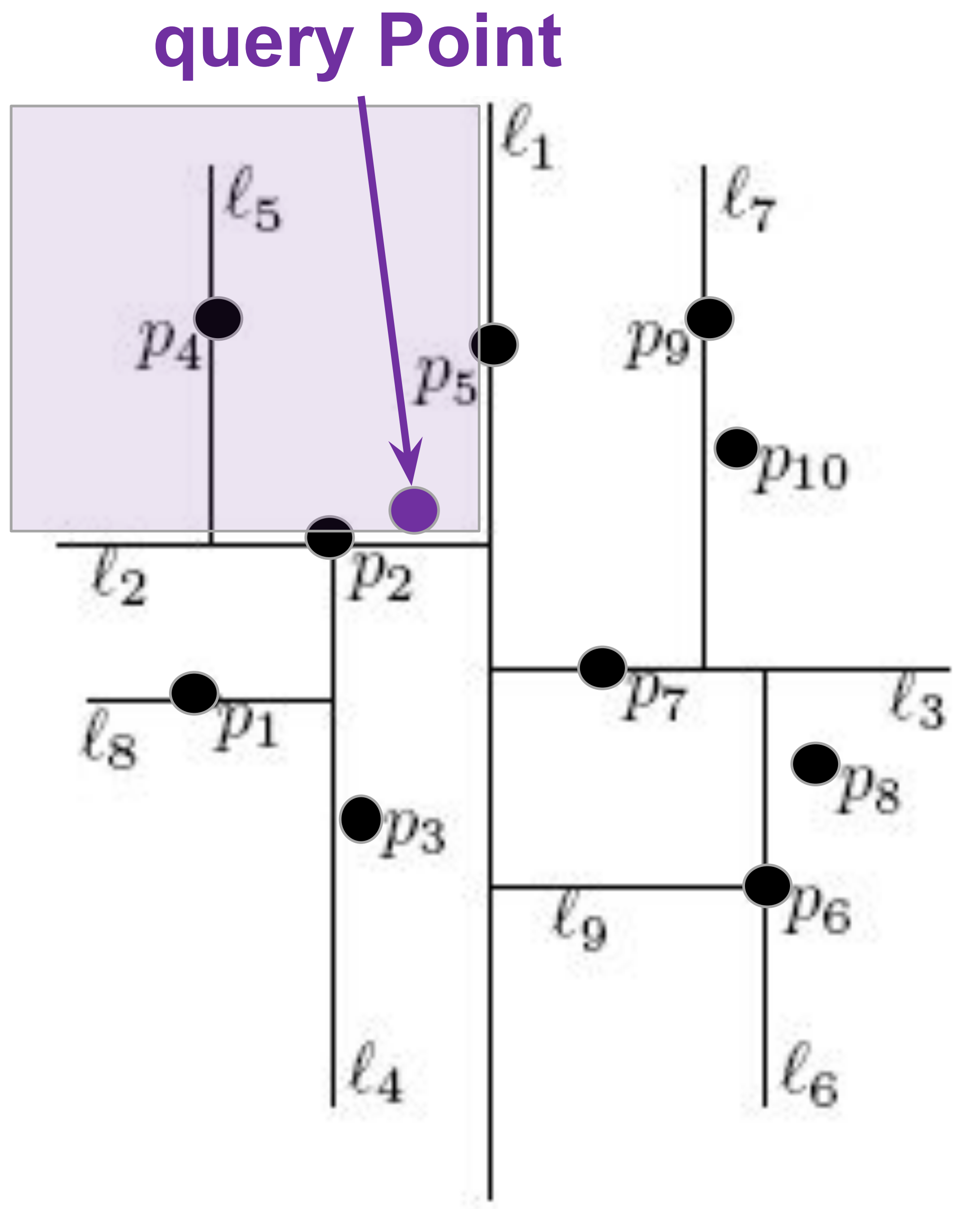


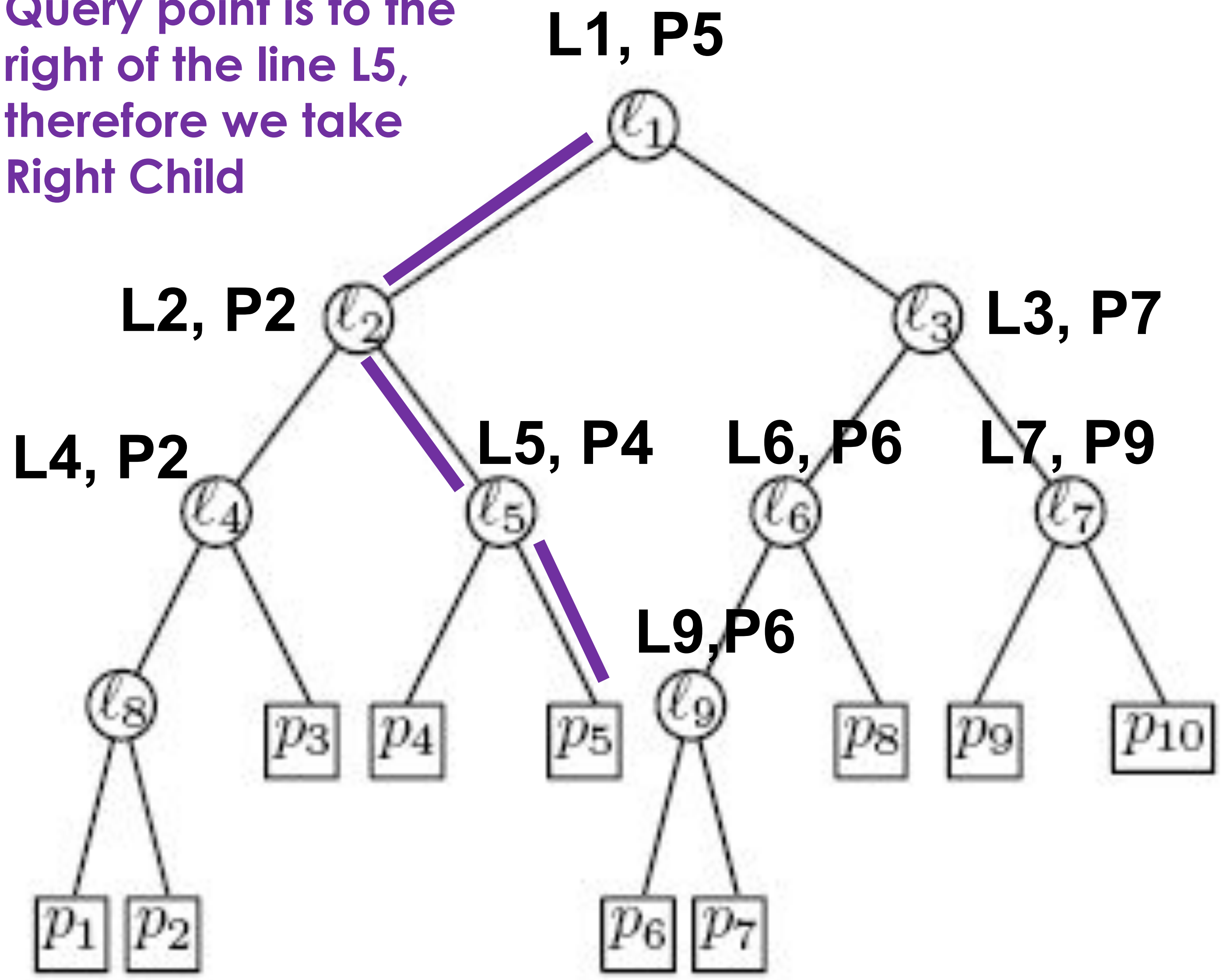
Query point is to the right of the line L2, therefore we take Right Child



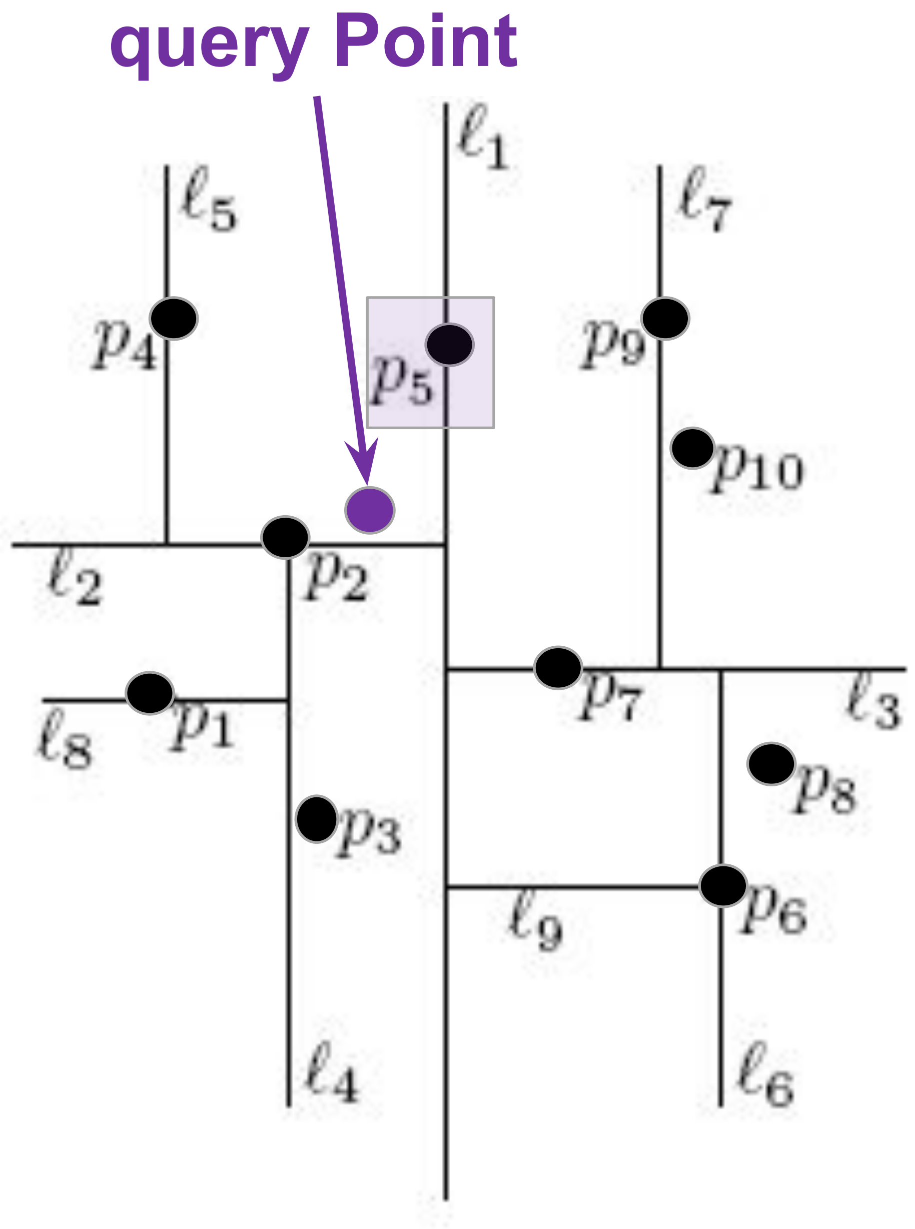
KD-tree: 1-NN Query Running



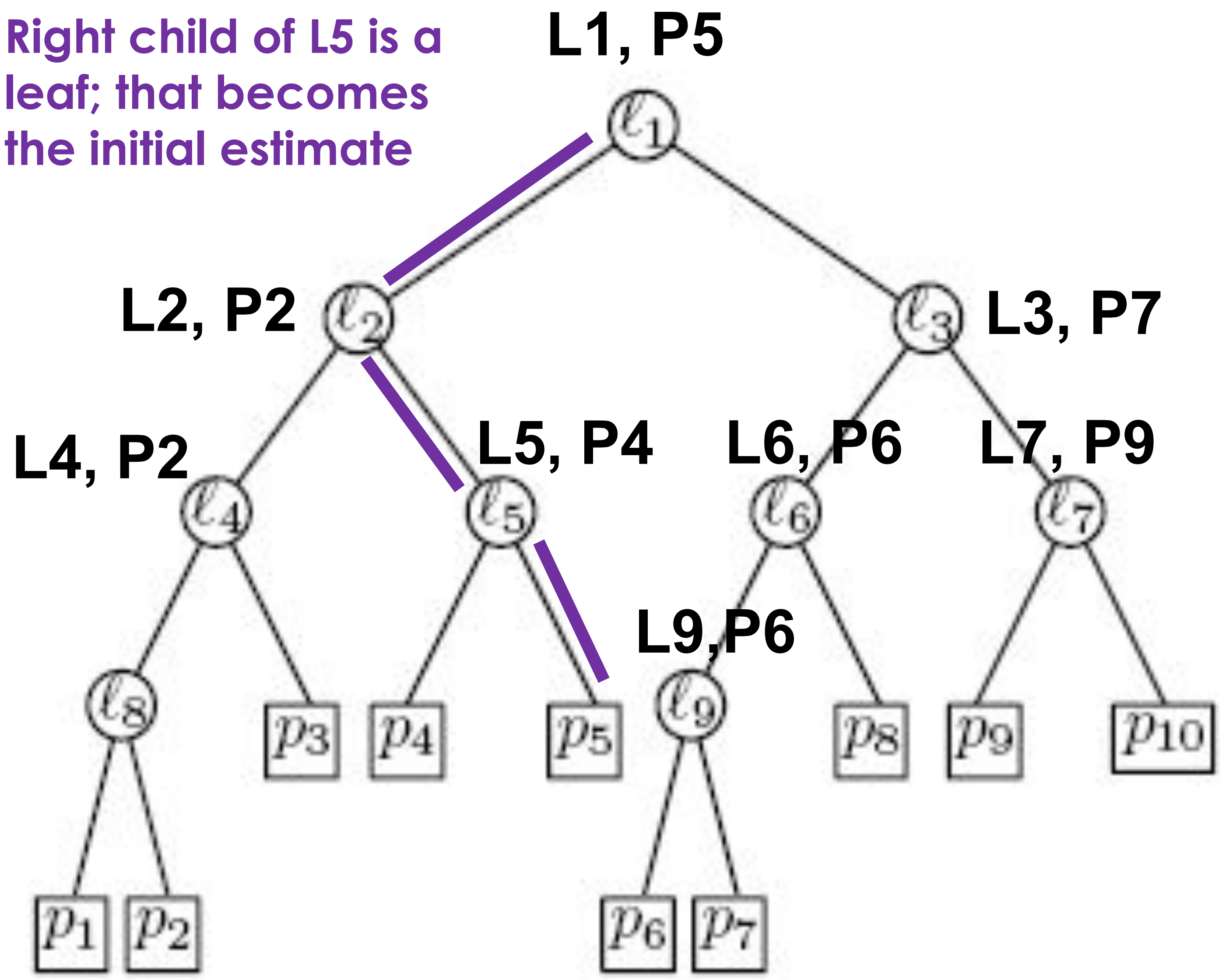
Query point is to the right of the line L5, therefore we take Right Child



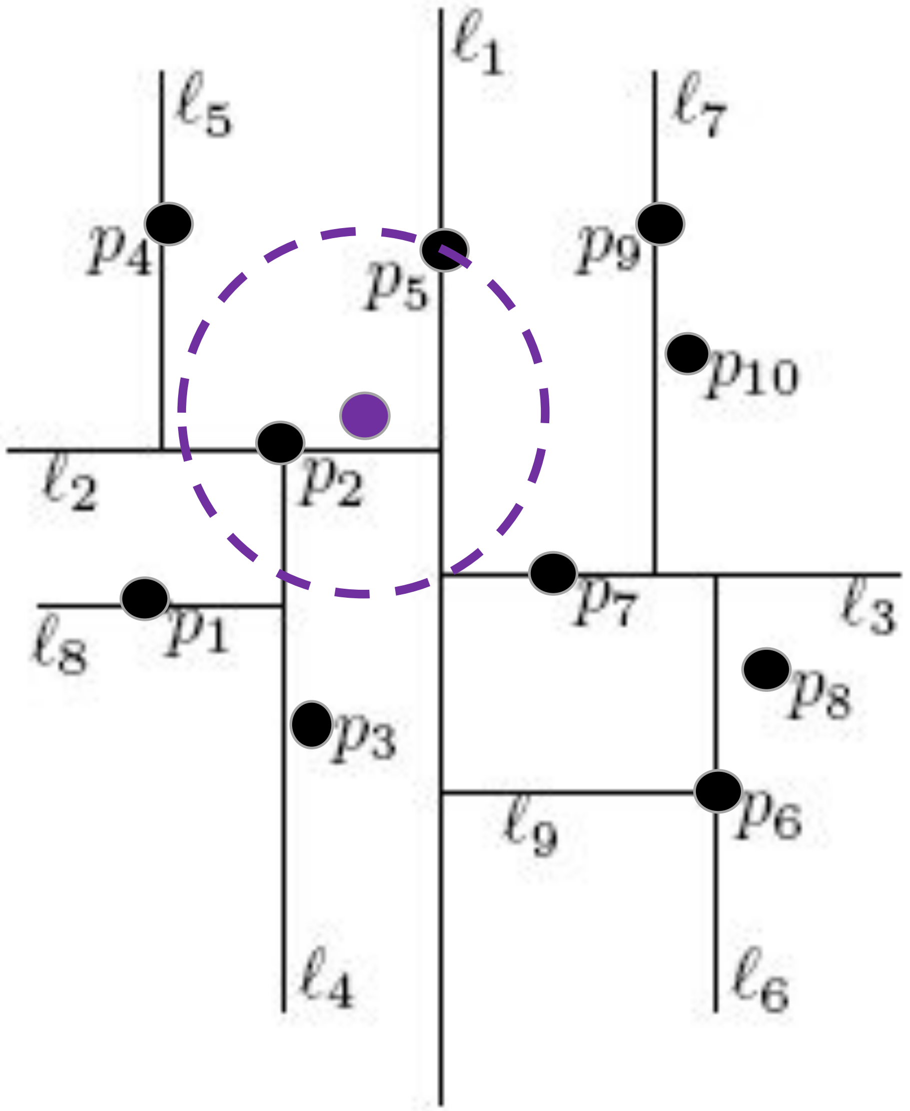
KD-tree: 1-NN Query Running



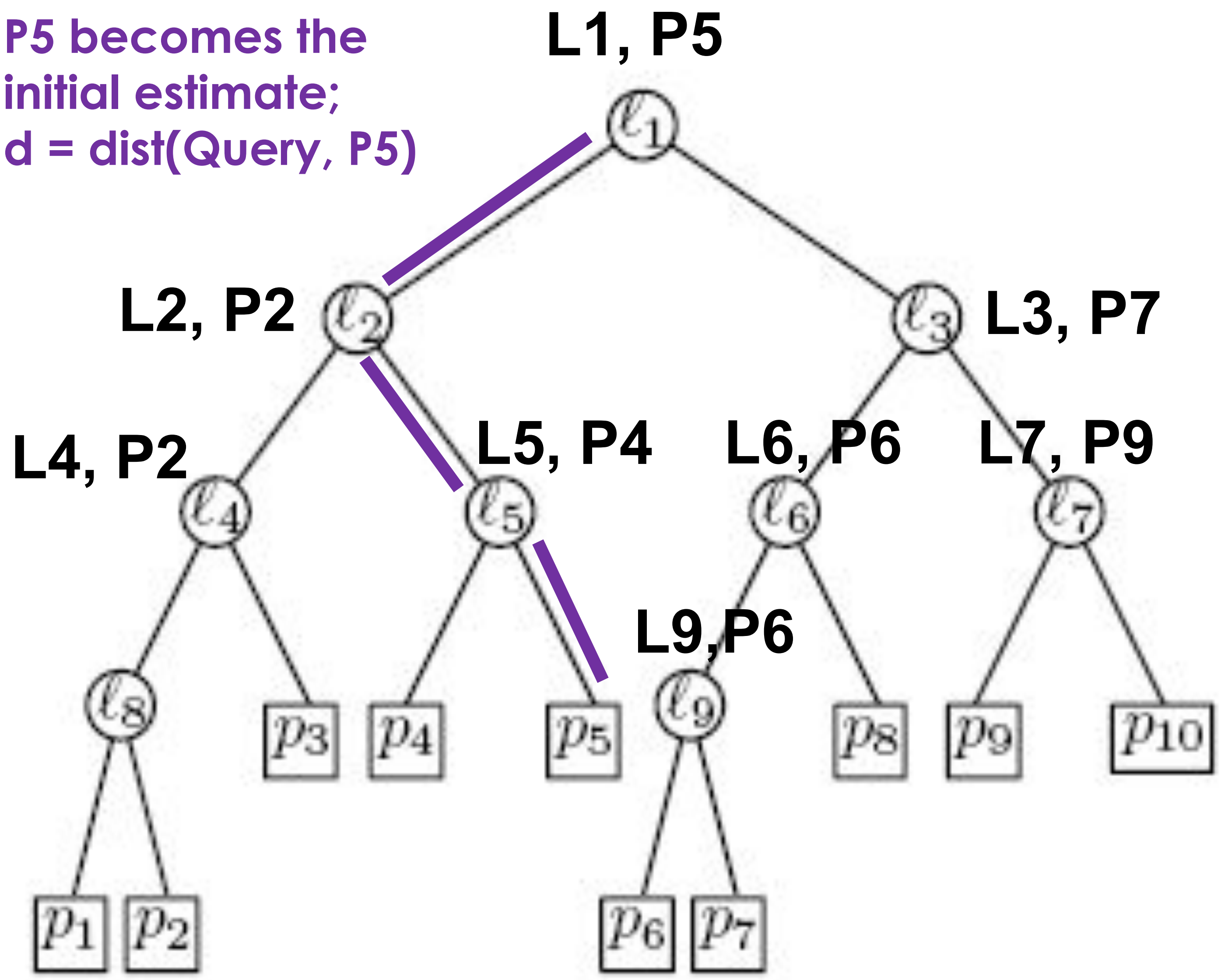
Right child of L5 is a leaf; that becomes the initial estimate



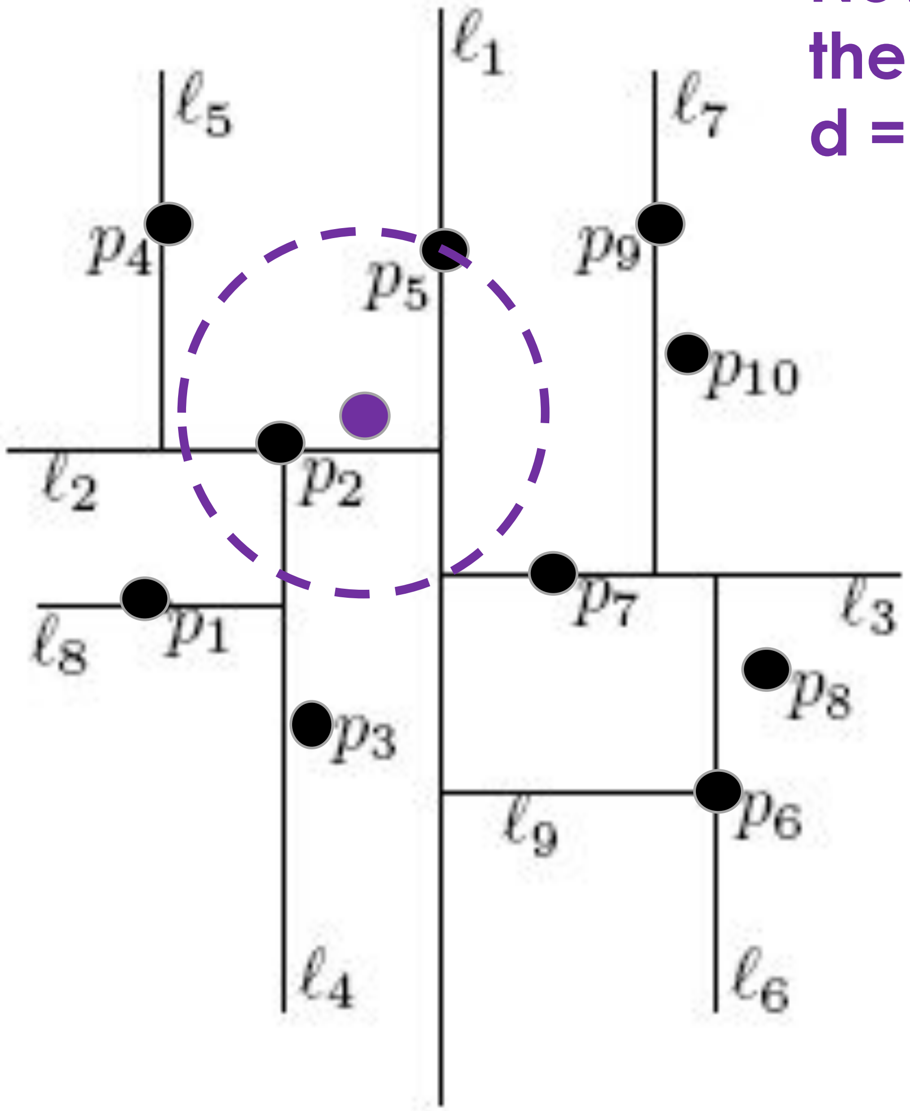
KD-tree: 1-NN Query Running



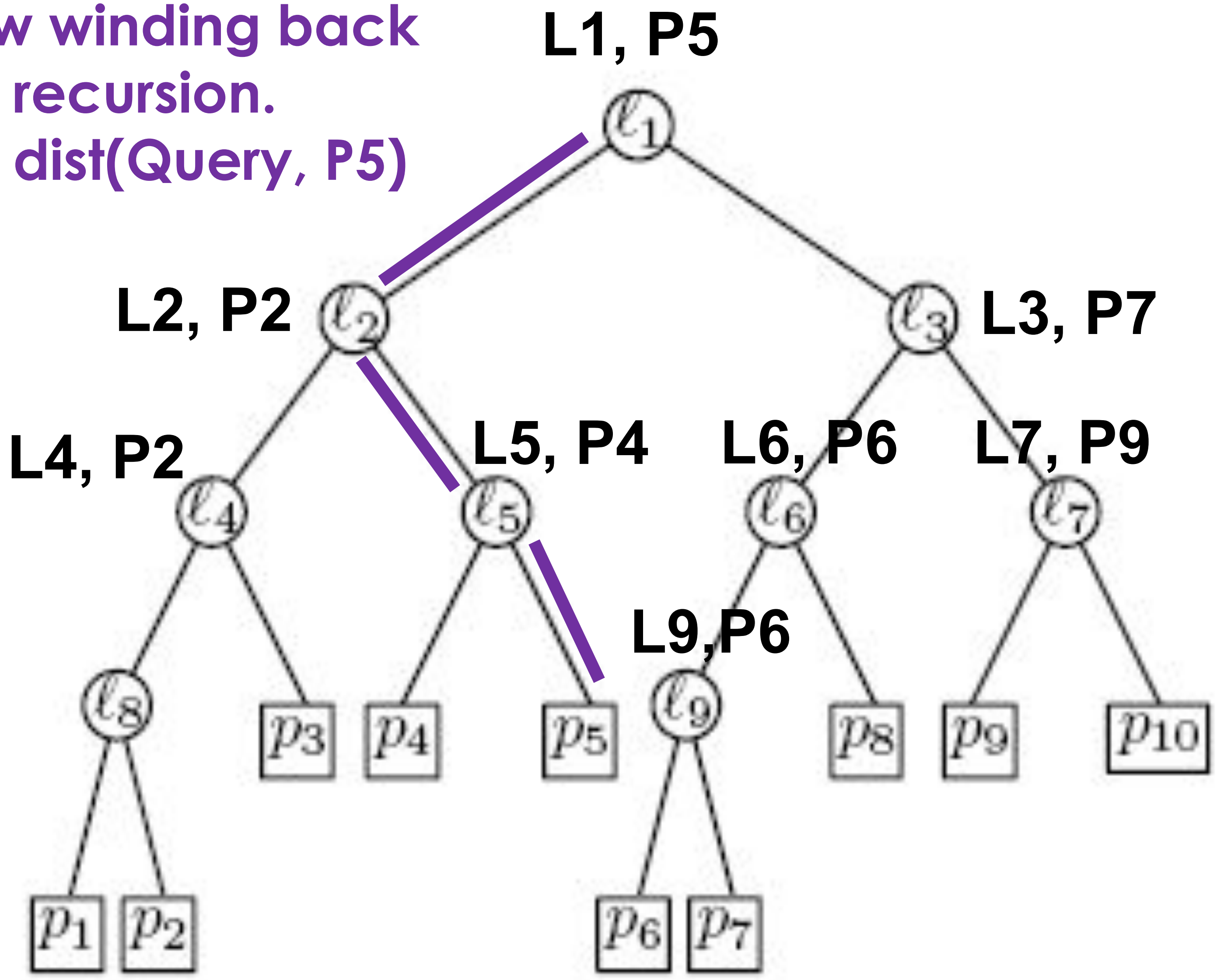
P5 becomes the initial estimate;
d = dist(Query, P5)



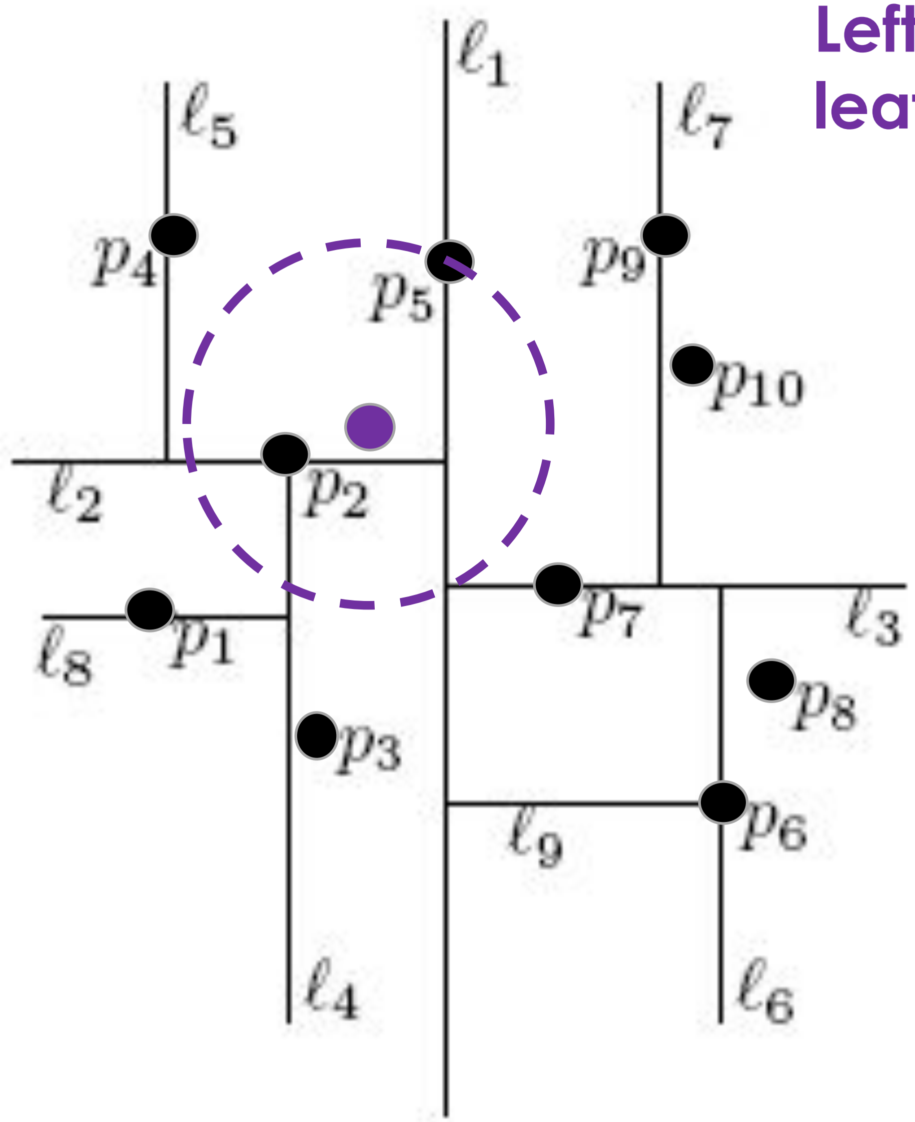
KD-tree: 1-NN Query Running



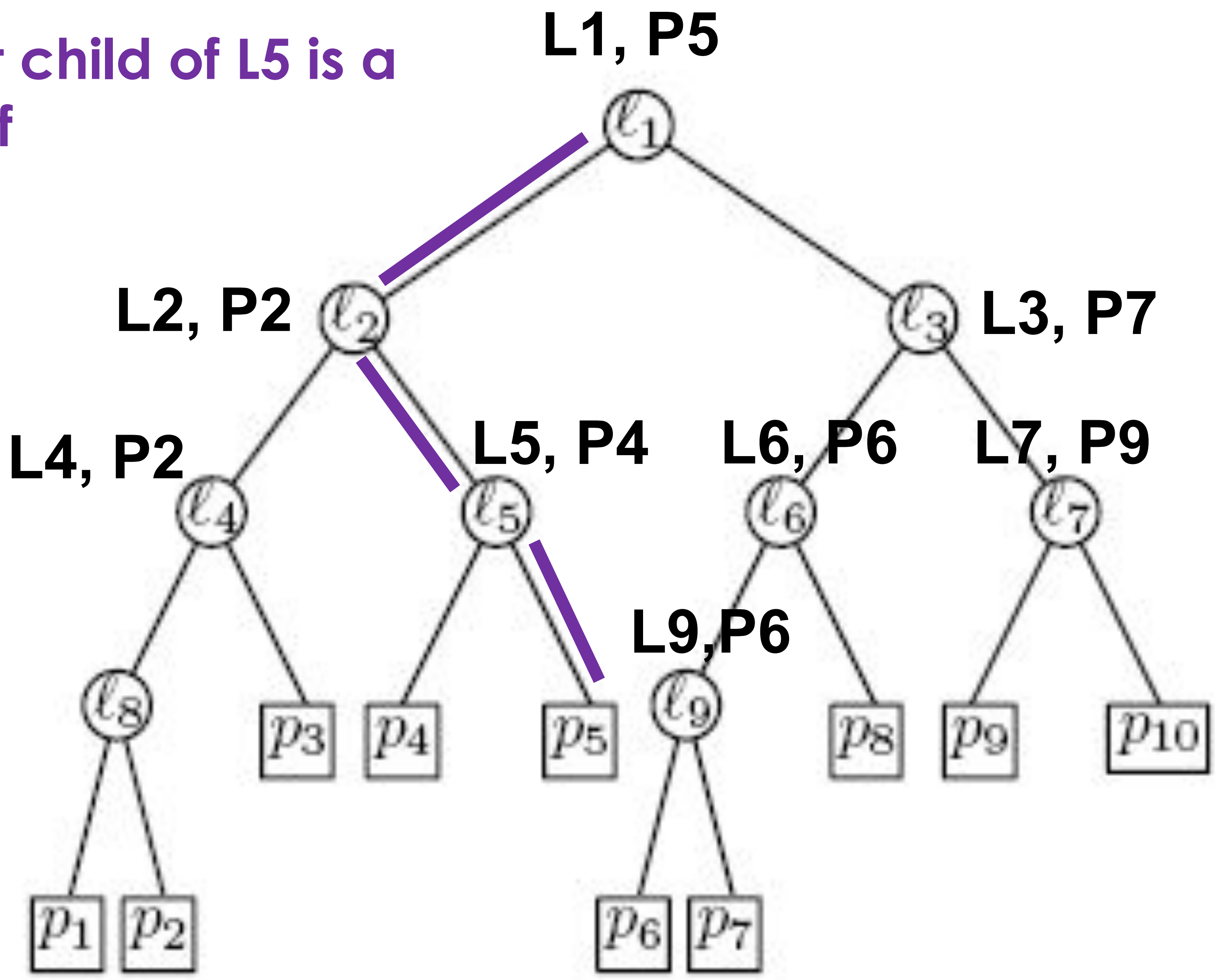
Now winding back the recursion.
d = dist(Query, P5)



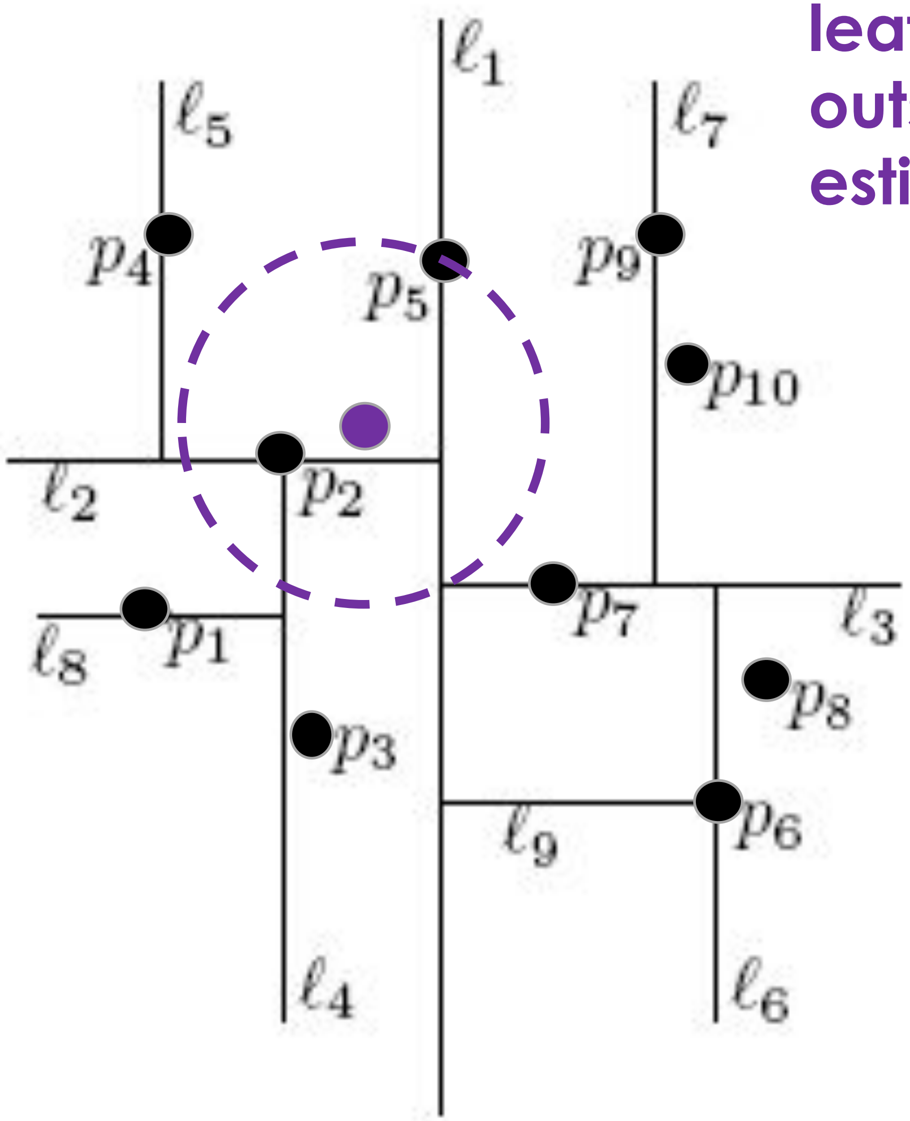
KD-tree: 1-NN Query Running



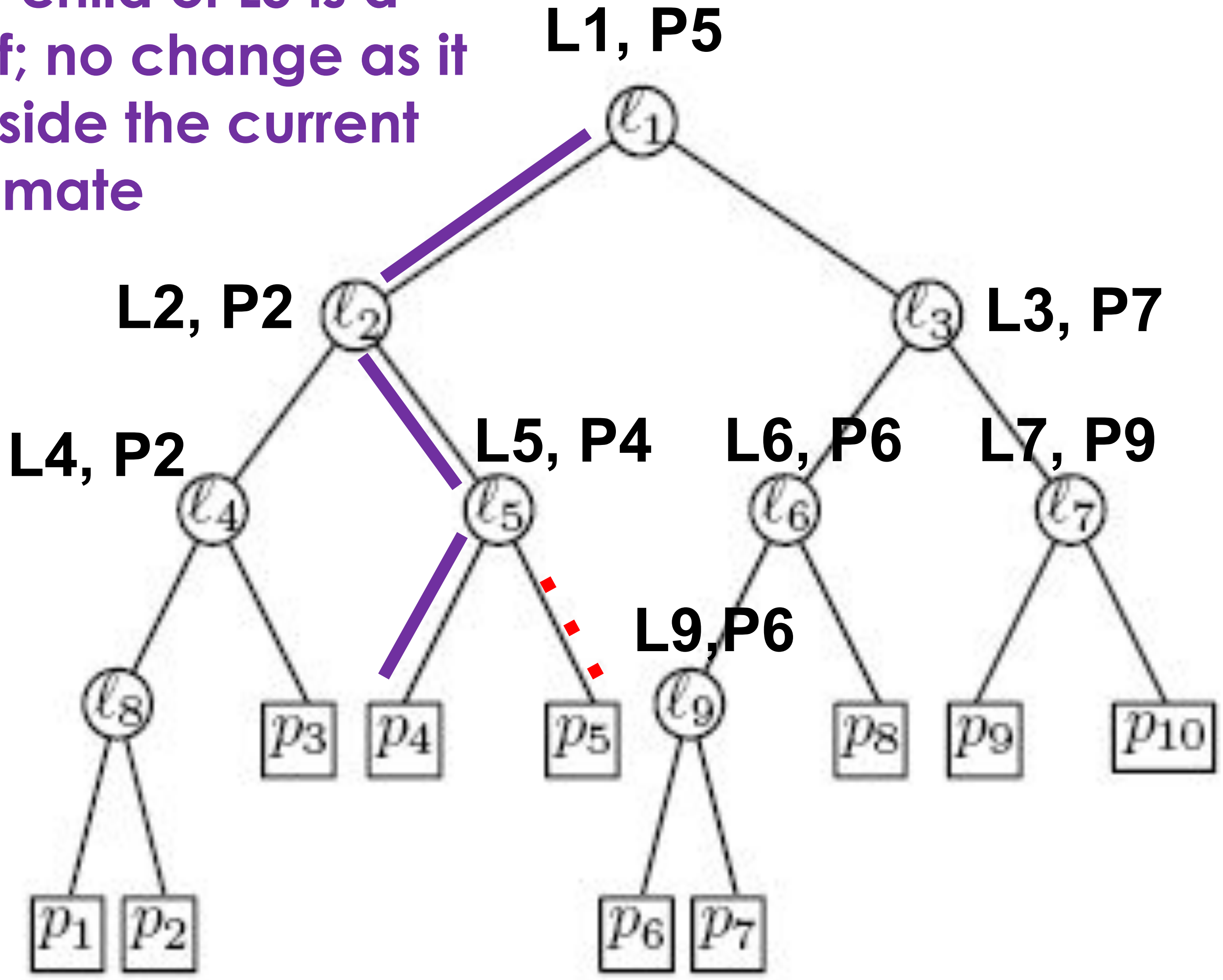
Left child of L5 is a leaf



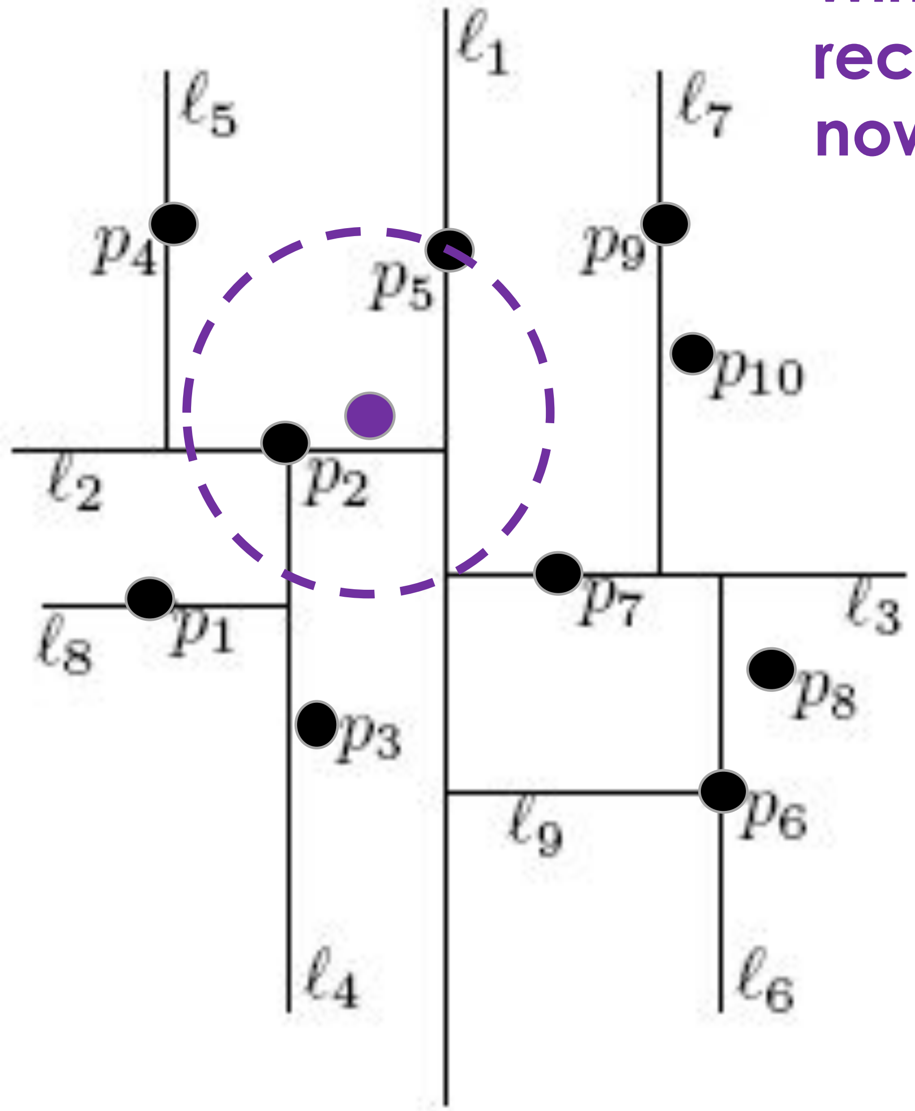
KD-tree: 1-NN Query Running



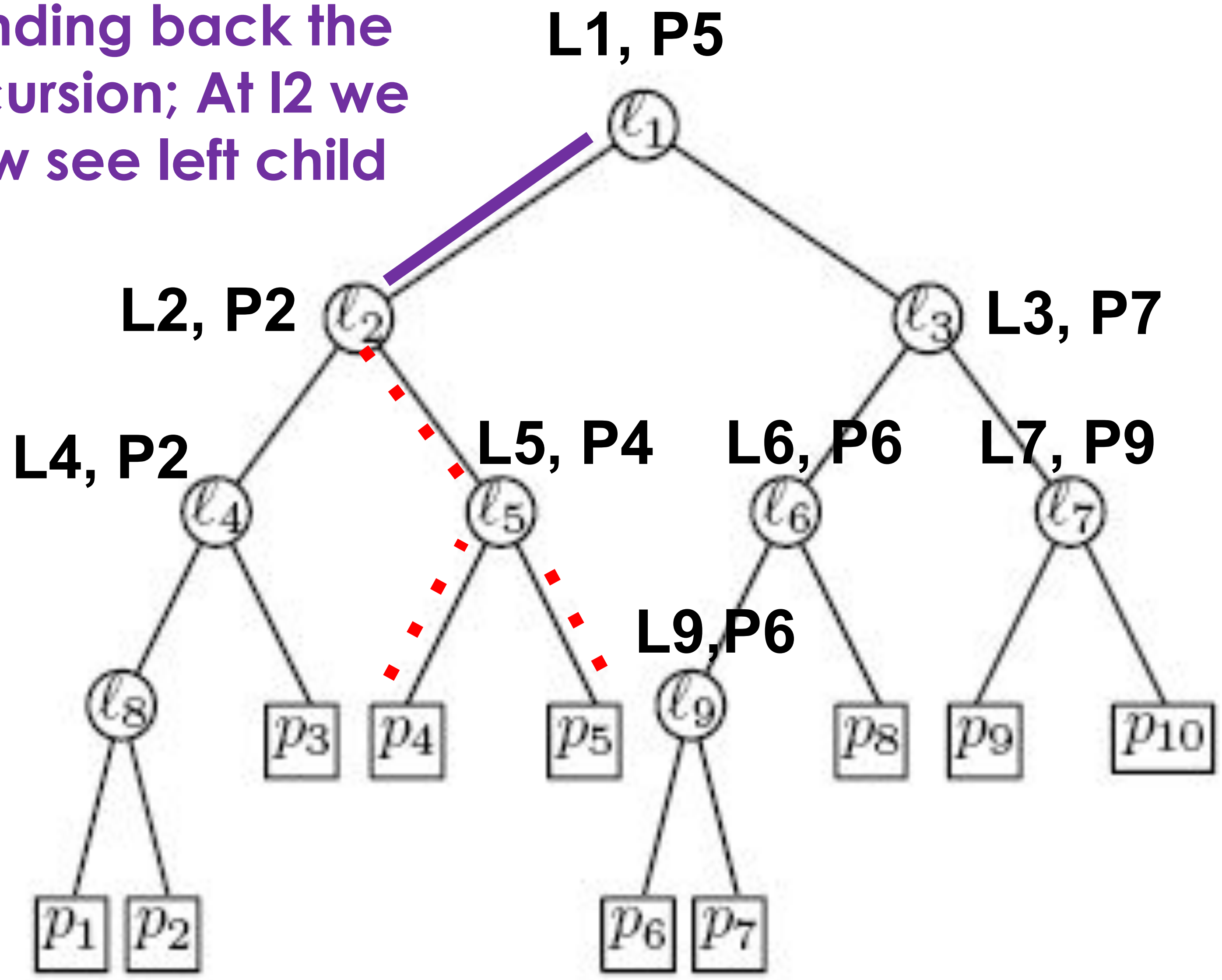
Left child of L5 is a leaf; no change as it outside the current estimate



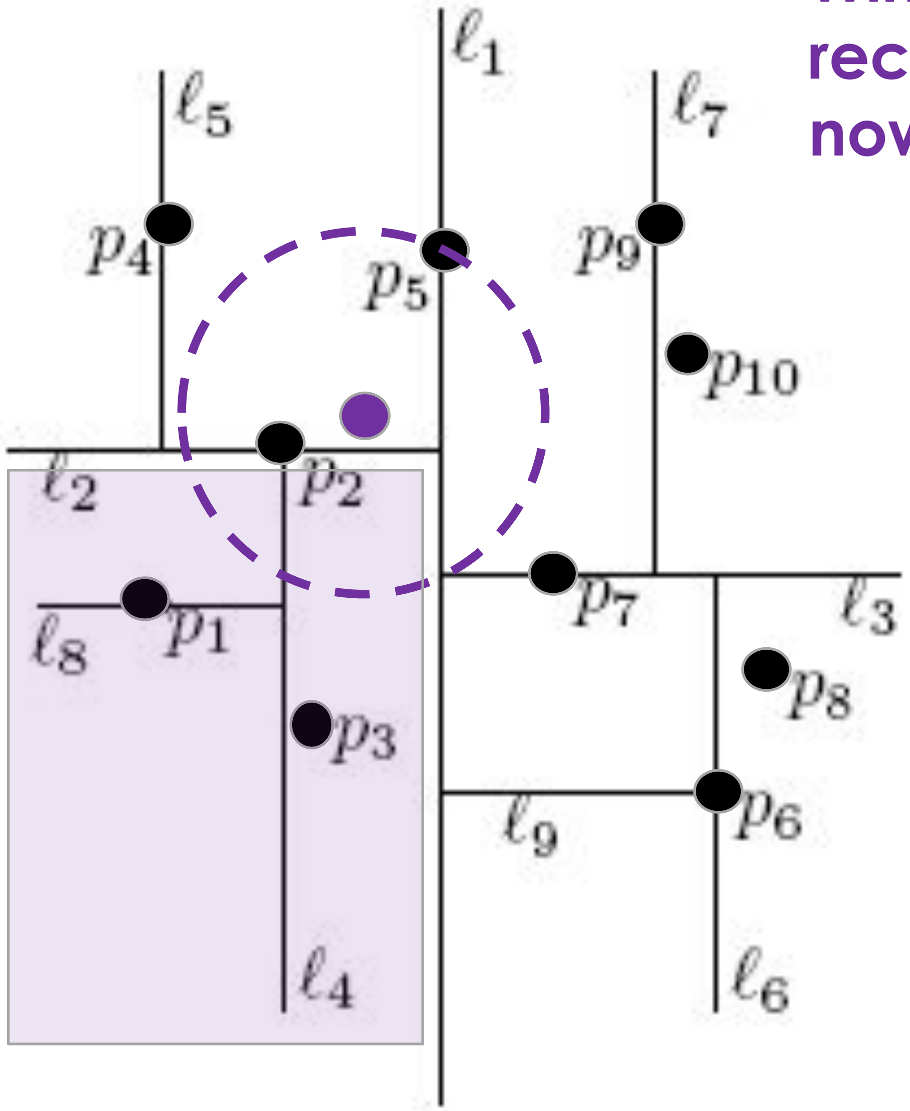
KD-tree: 1-NN Query Running



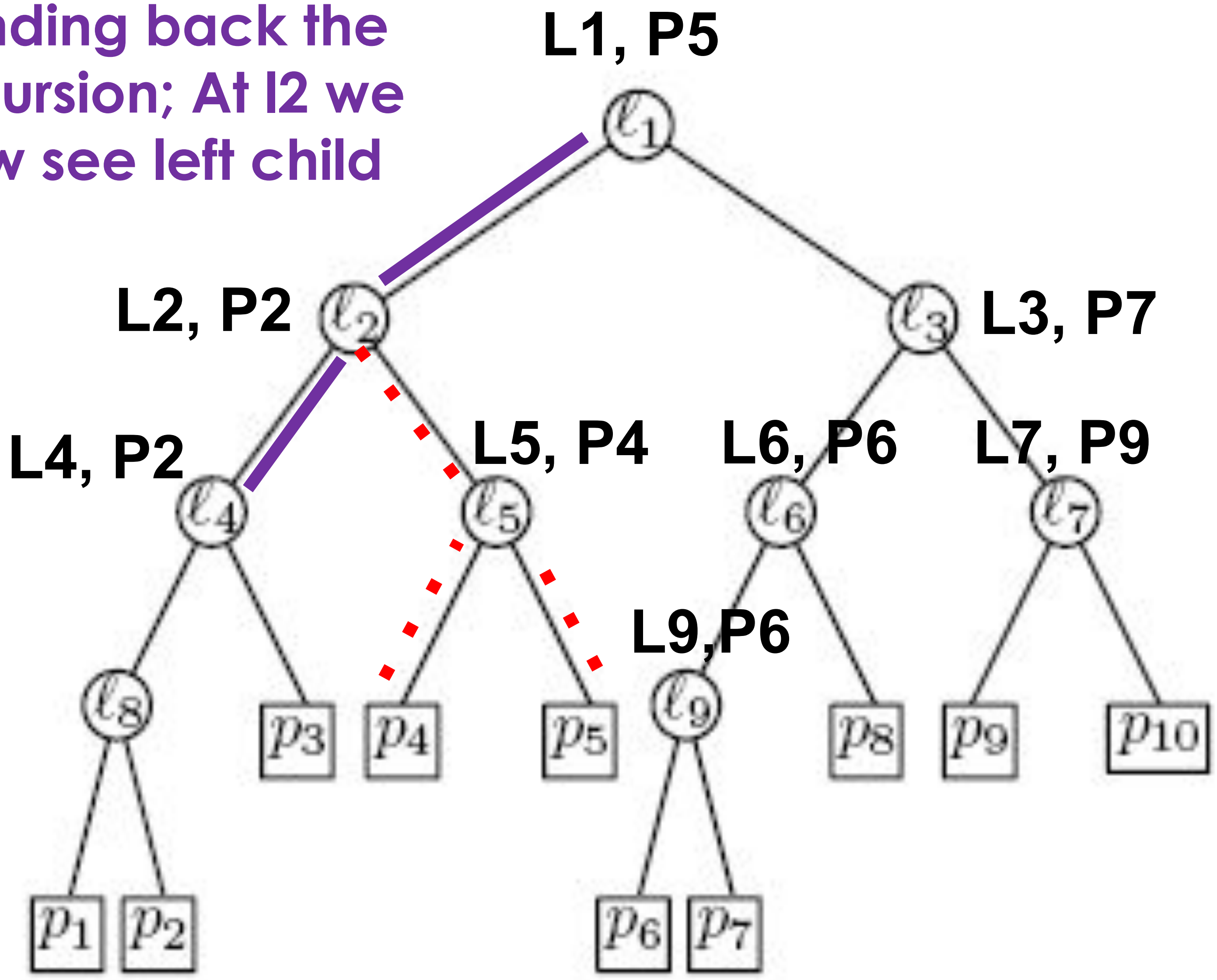
Winding back the recursion; At l2 we now see left child



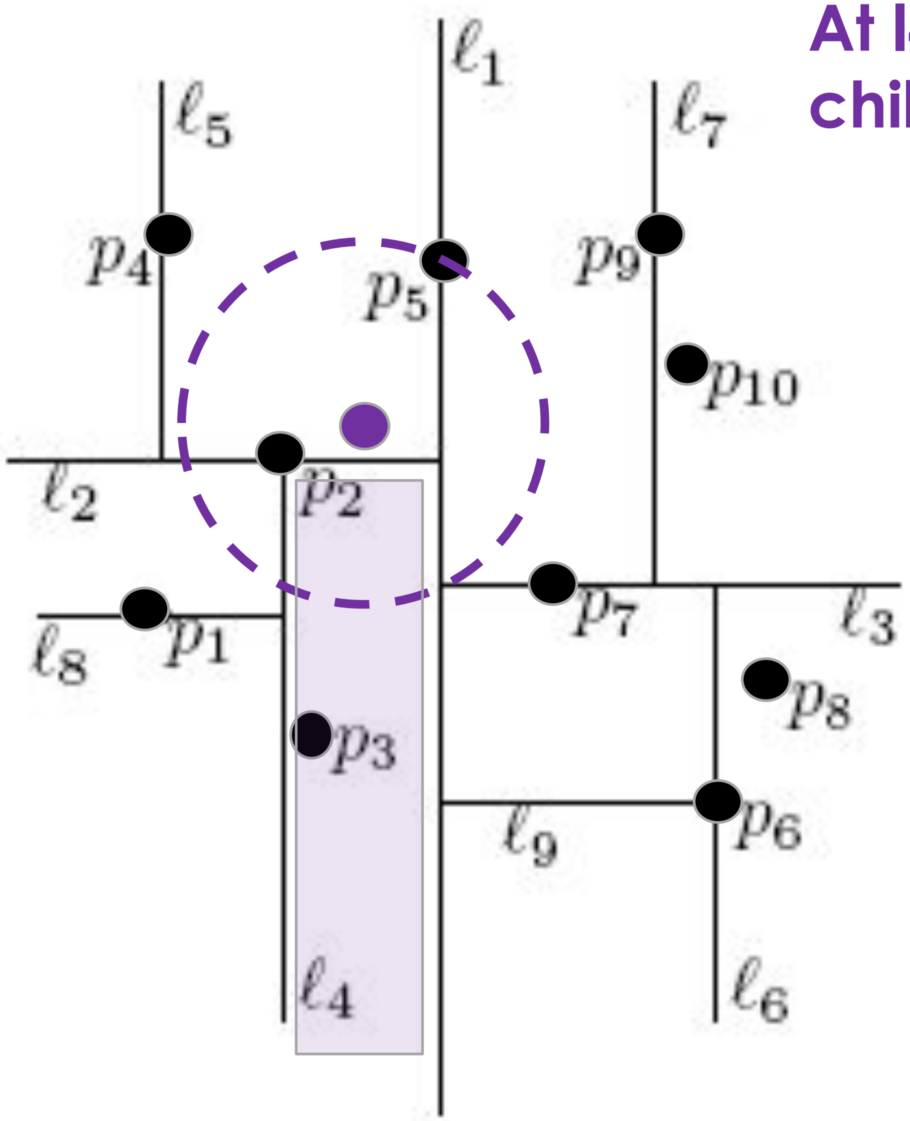
KD-tree: 1-NN Query Running



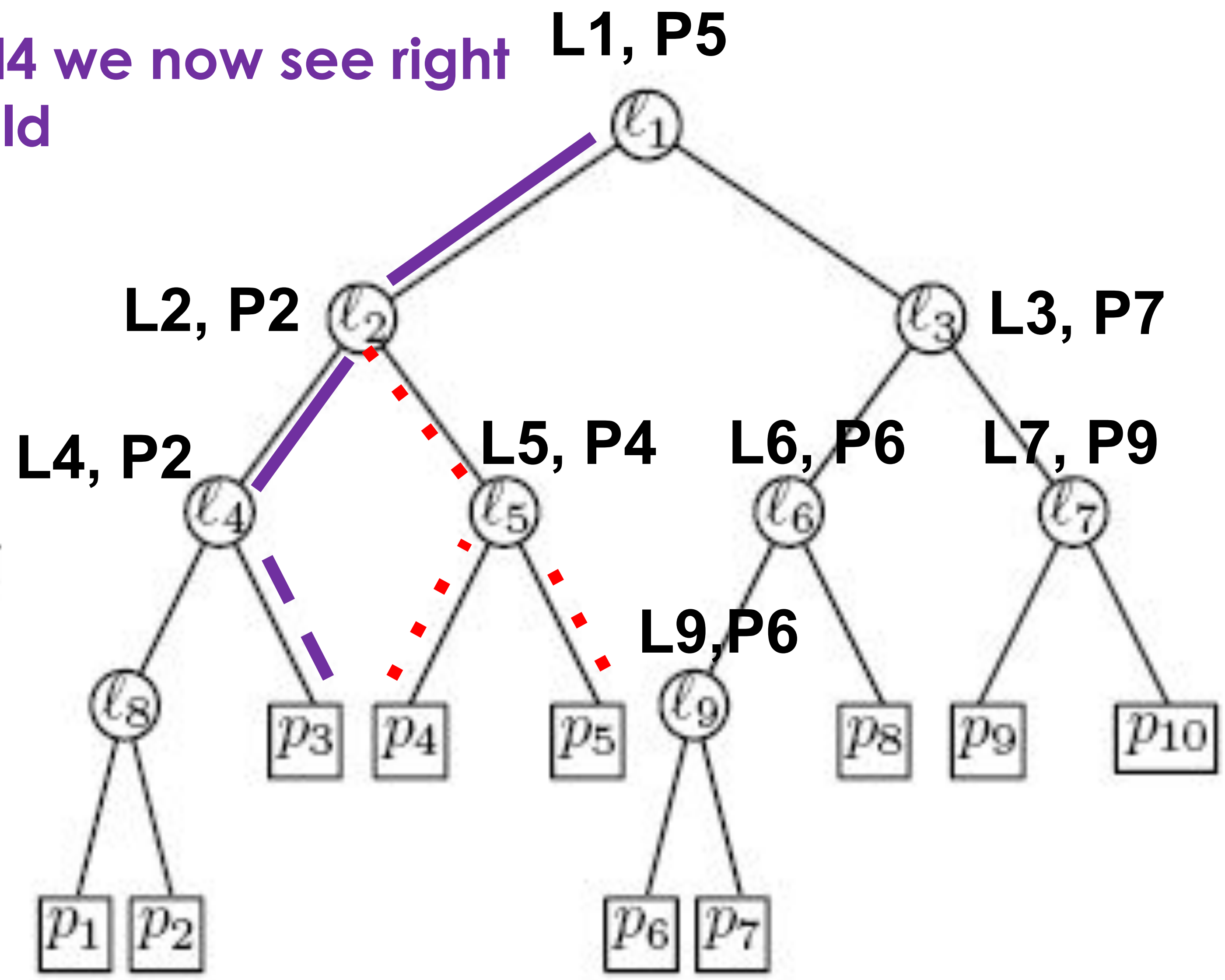
Winding back the recursion; At l2 we now see left child



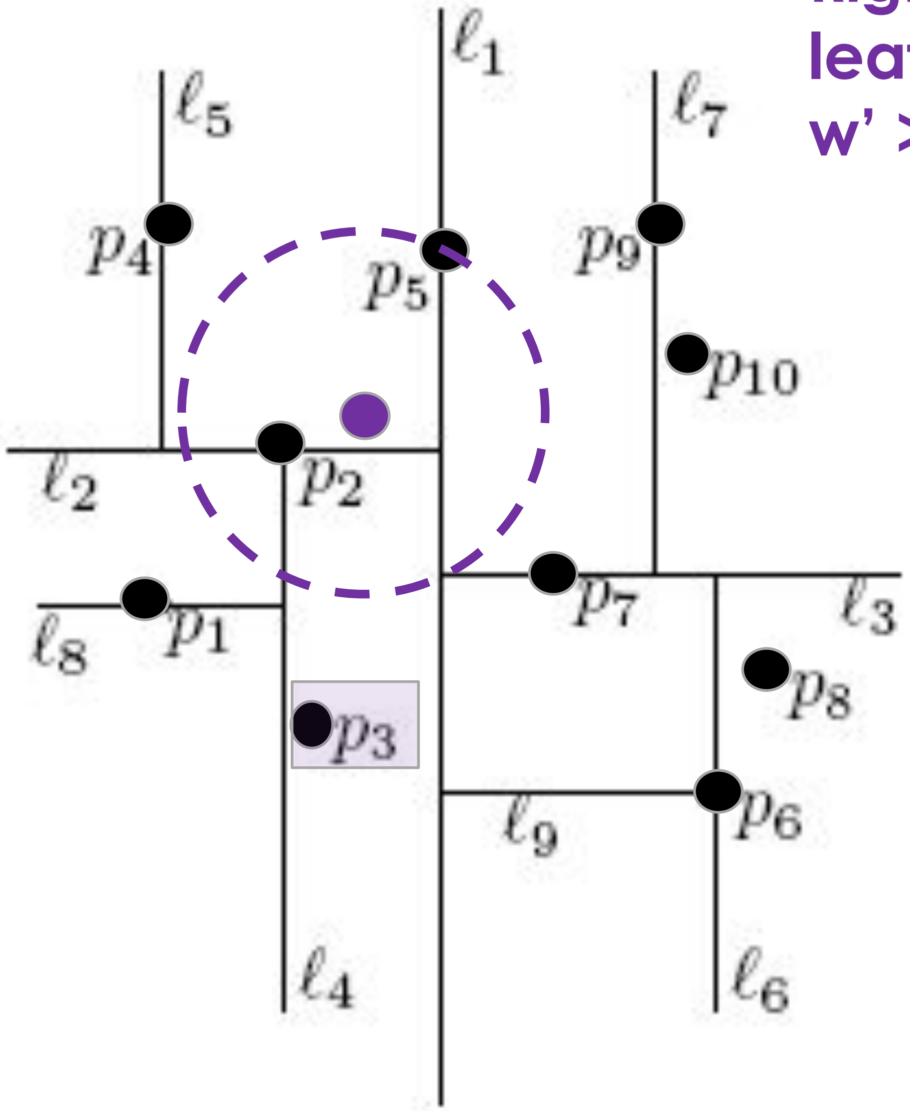
KD-tree: 1-NN Query Running



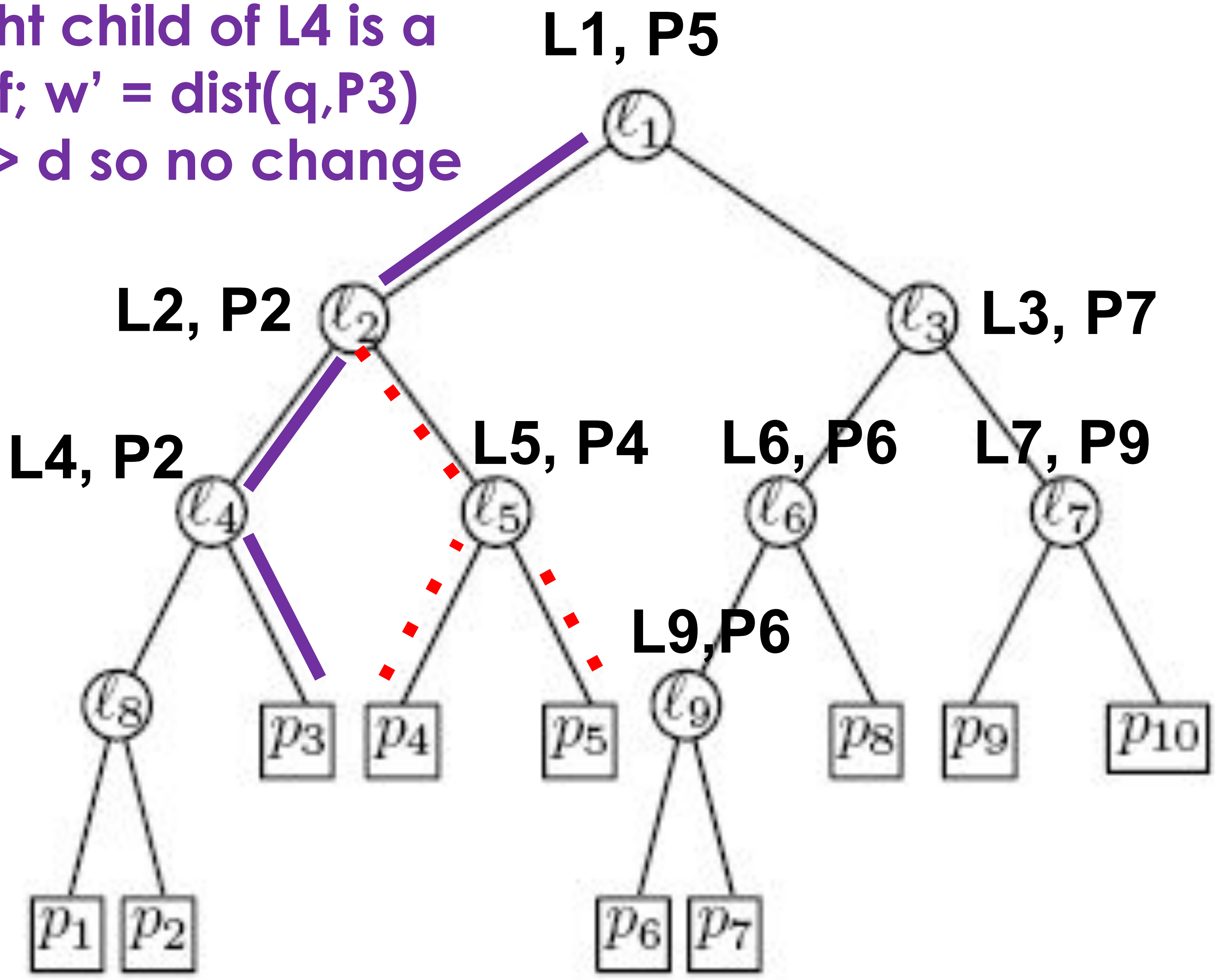
At l4 we now see right child



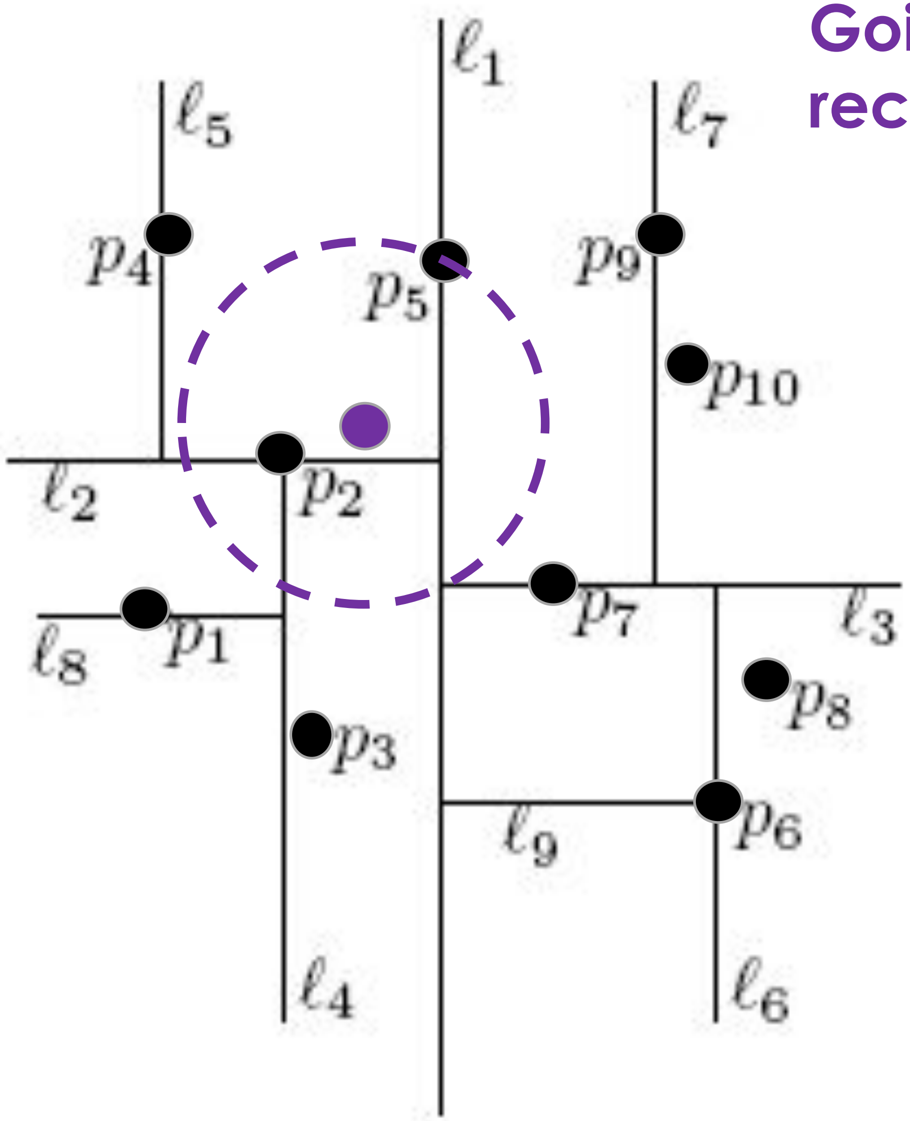
KD-tree: 1-NN Query Running



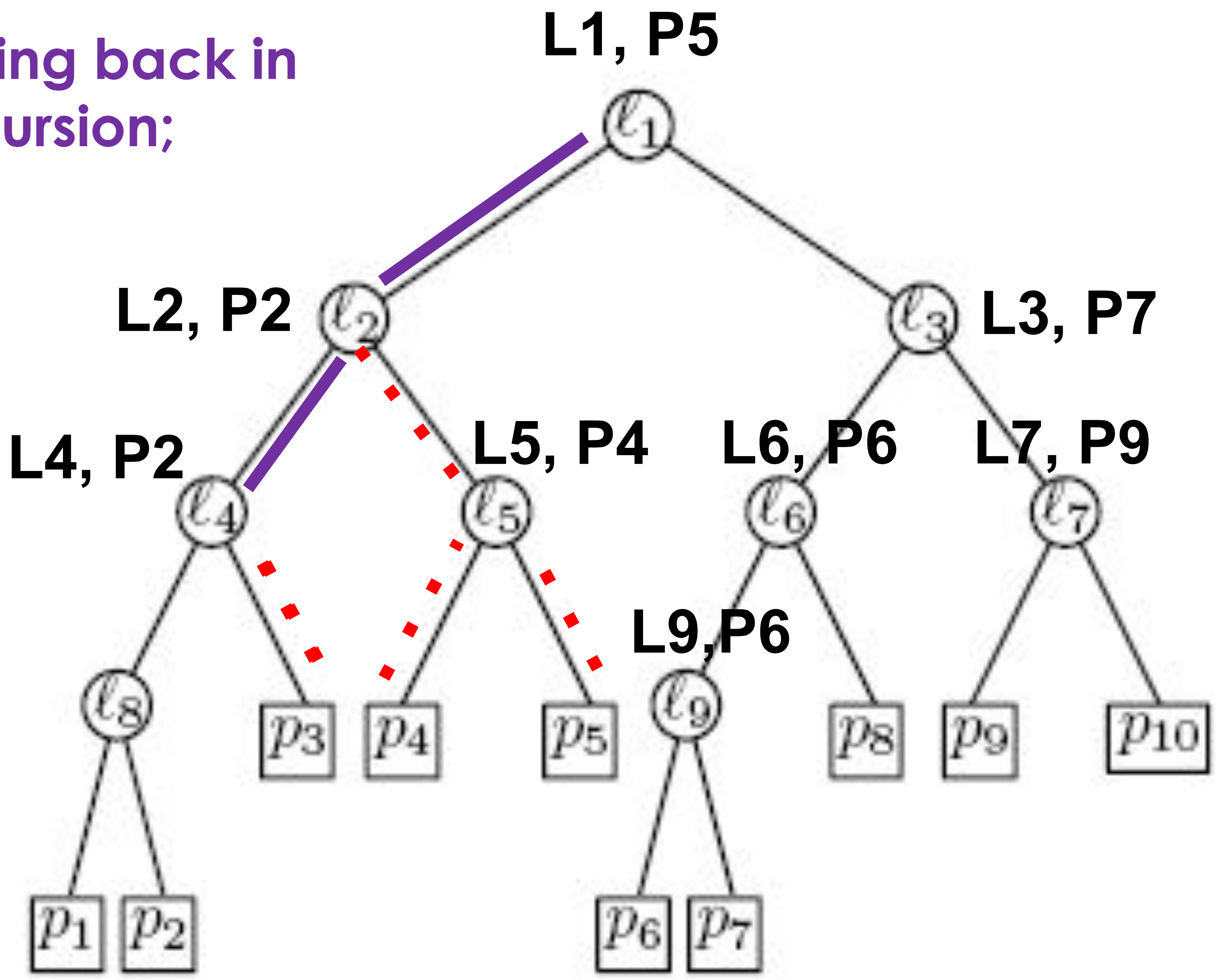
Right child of L4 is a leaf; w’ = dist(q,P3)
w’ > d so no change



KD-tree: 1-NN Query Running

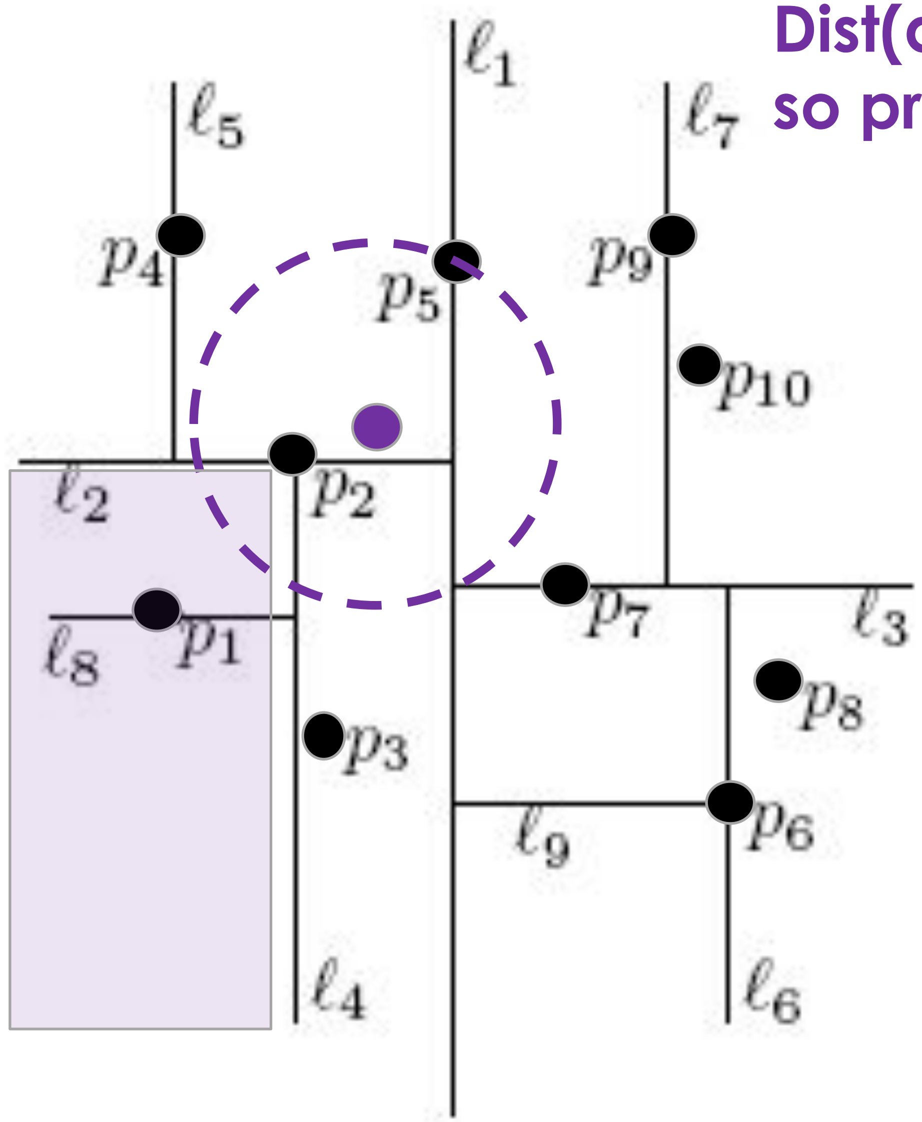


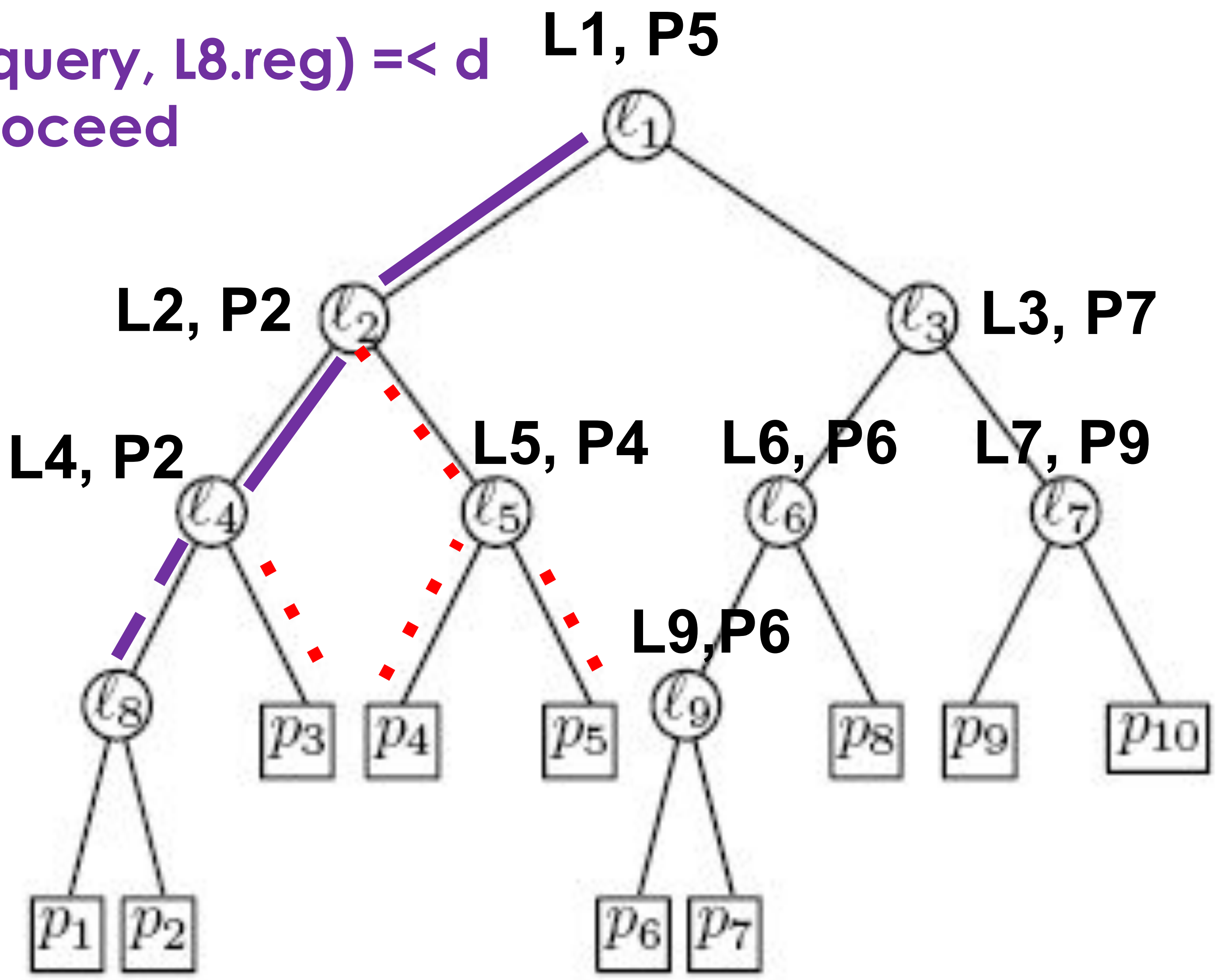
Going back in recursion;



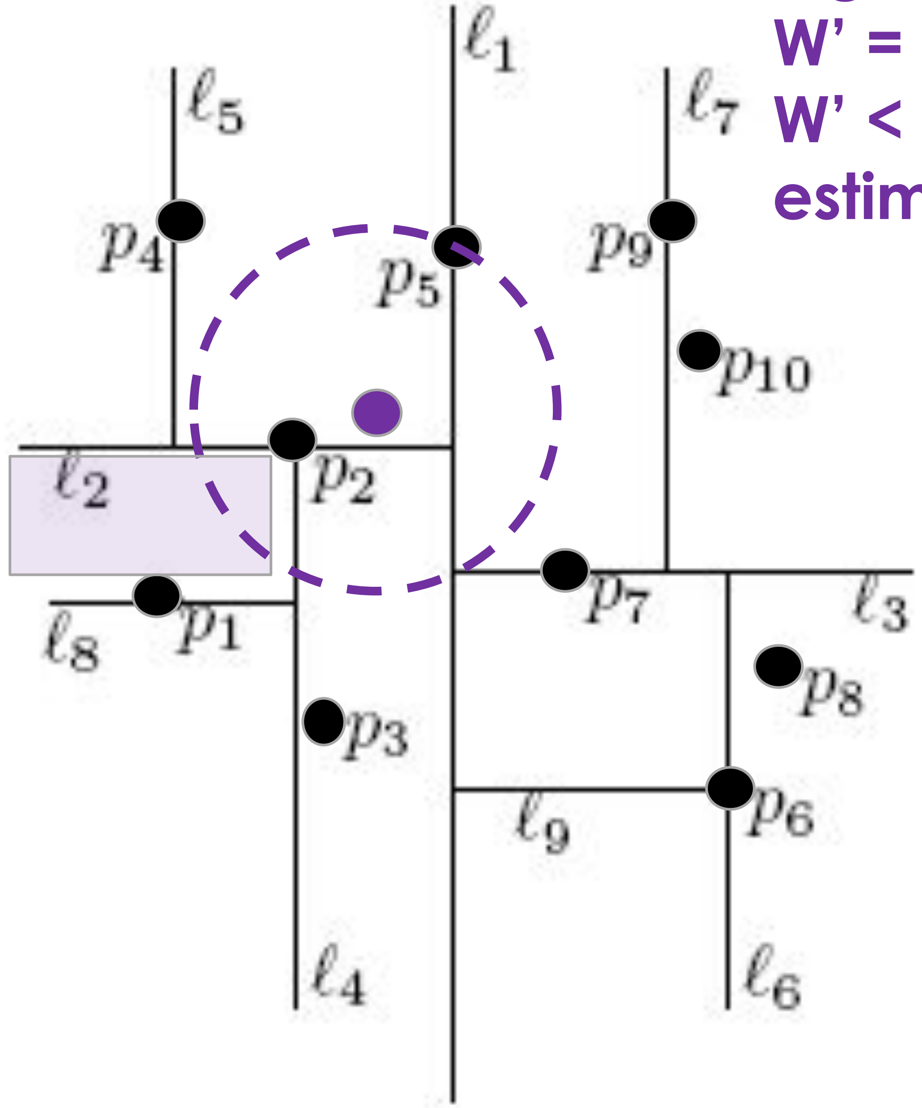
KD-tree: 1-NN Query Running

Dist(query, L8.reg) =< d
so proceed

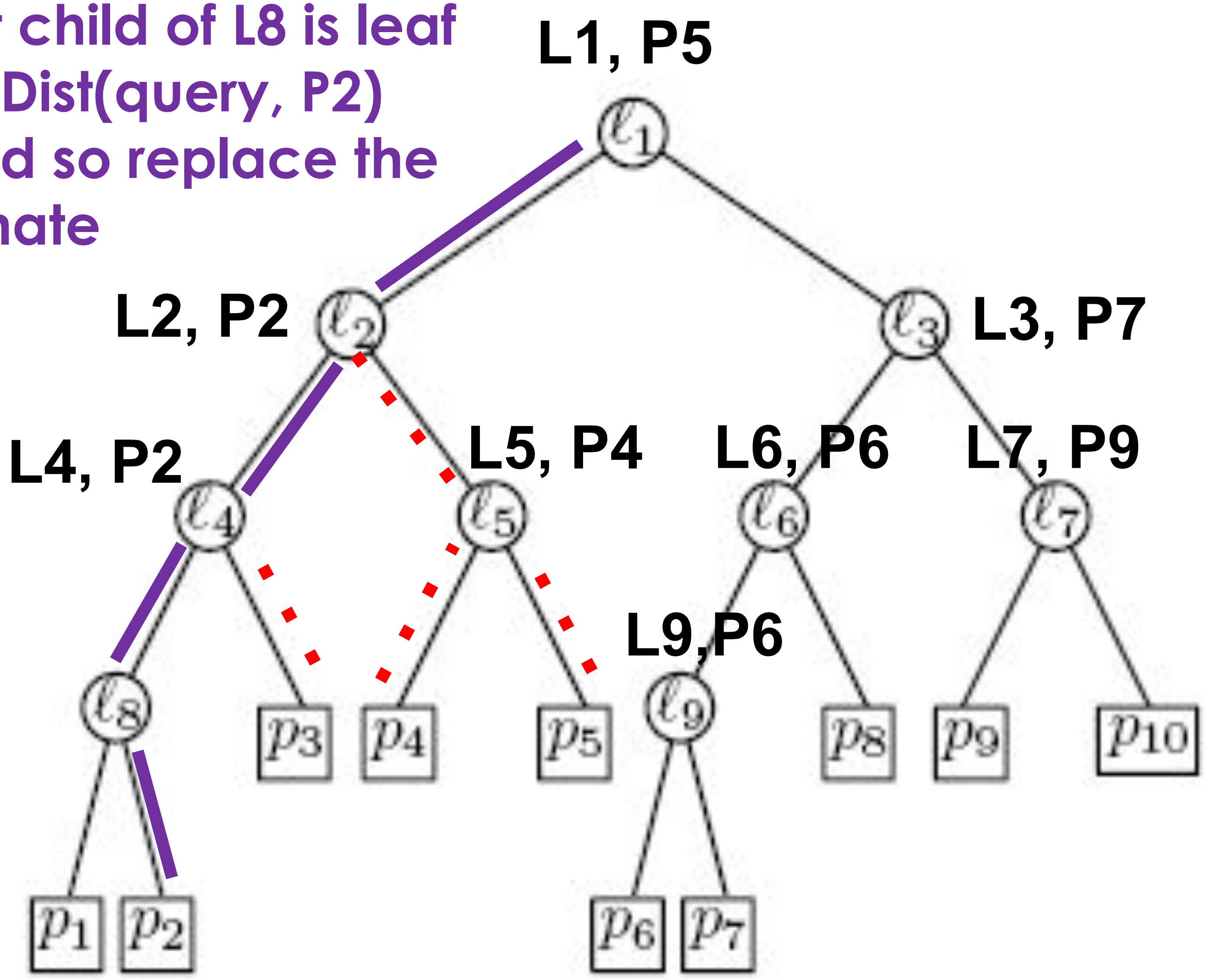




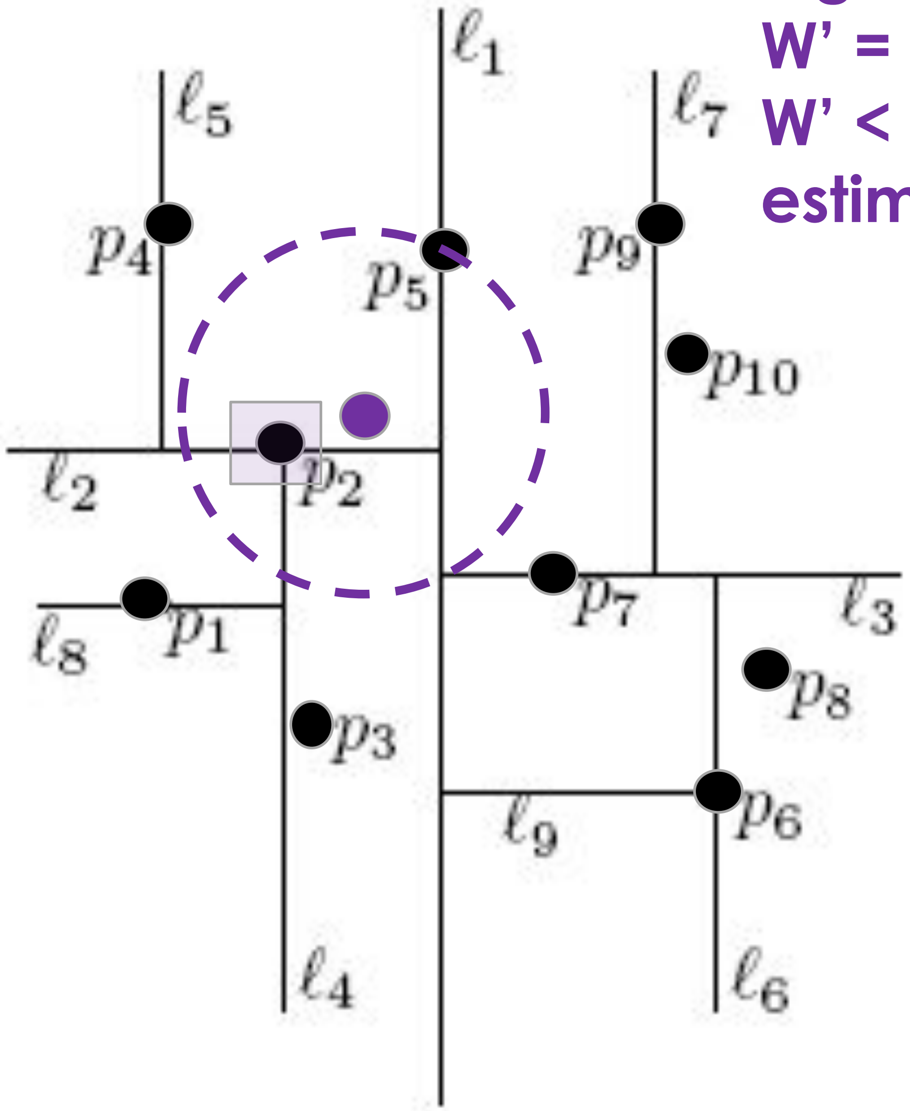
KD-tree: 1-NN Query Running



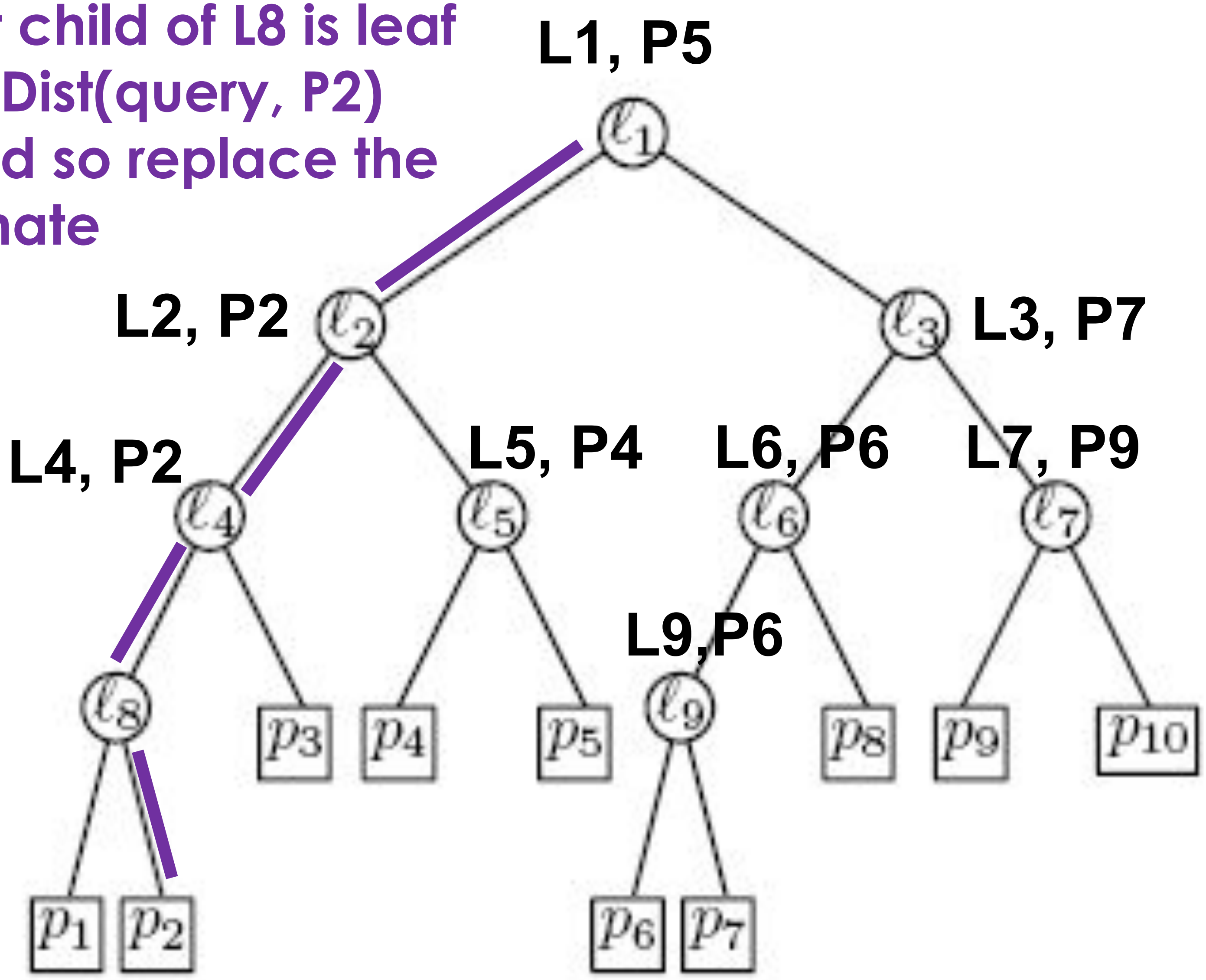
Right child of L8 is leaf
W' = Dist(query, P2)
W' < d so replace the estimate



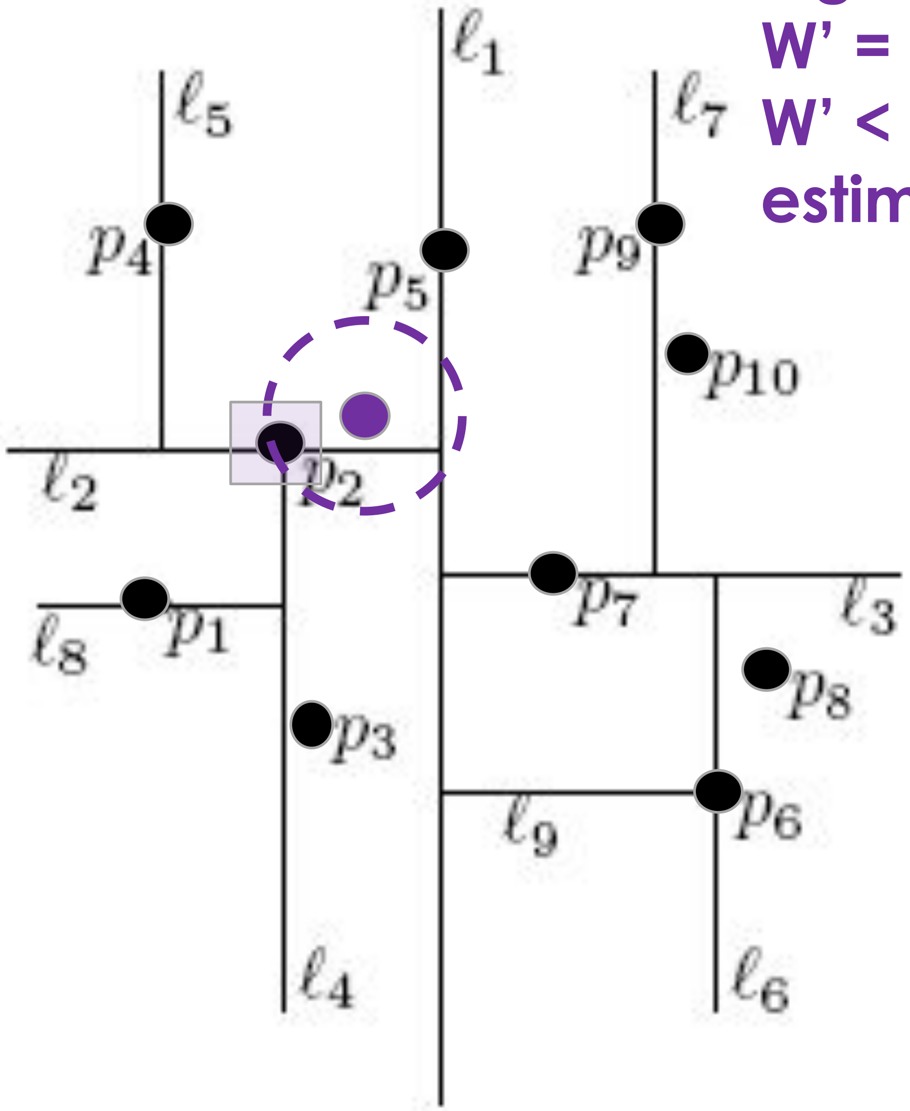
KD-tree: 1-NN Query Running



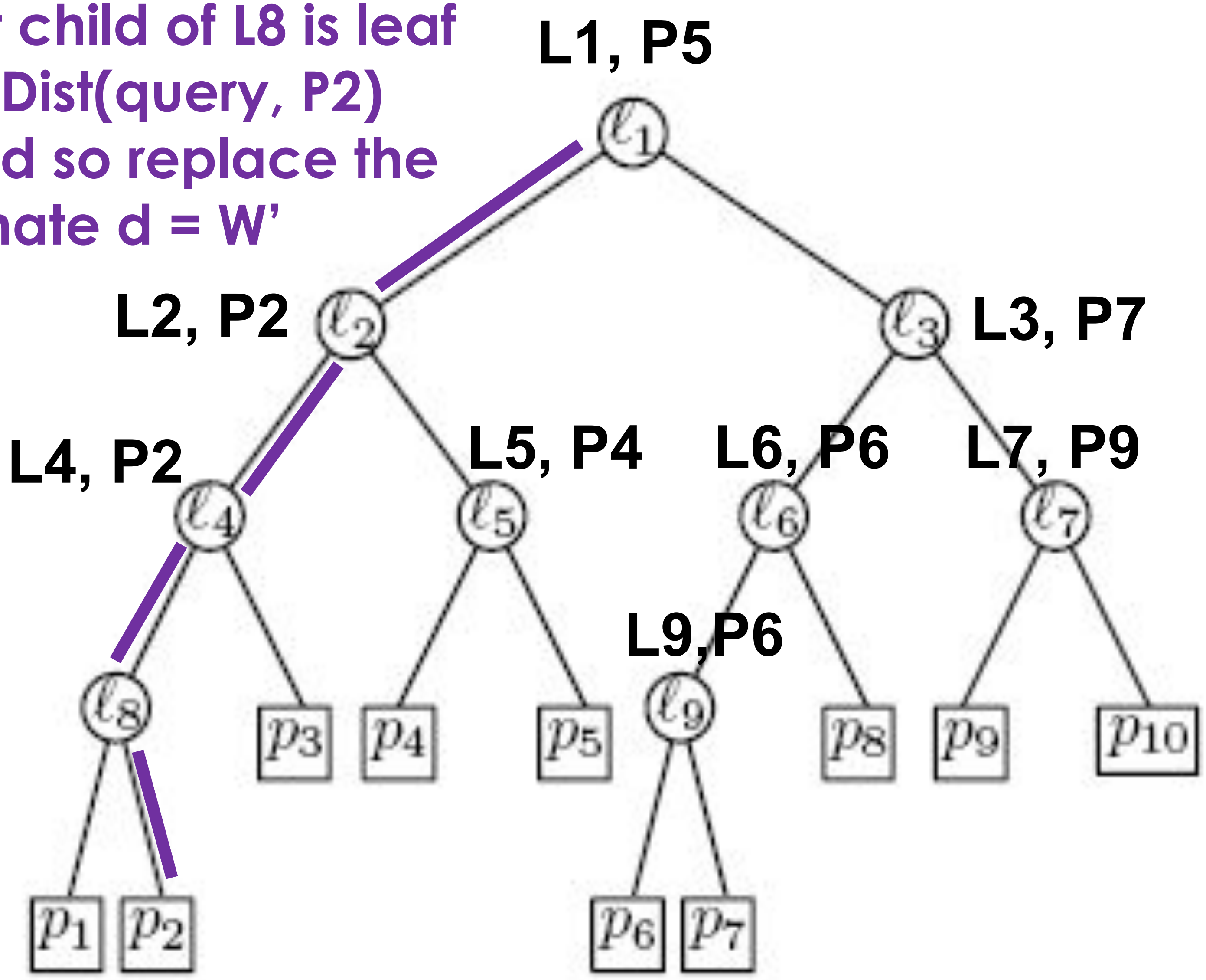
Right child of L8 is leaf
W' = Dist(query, P2)
W' < d so replace the estimate



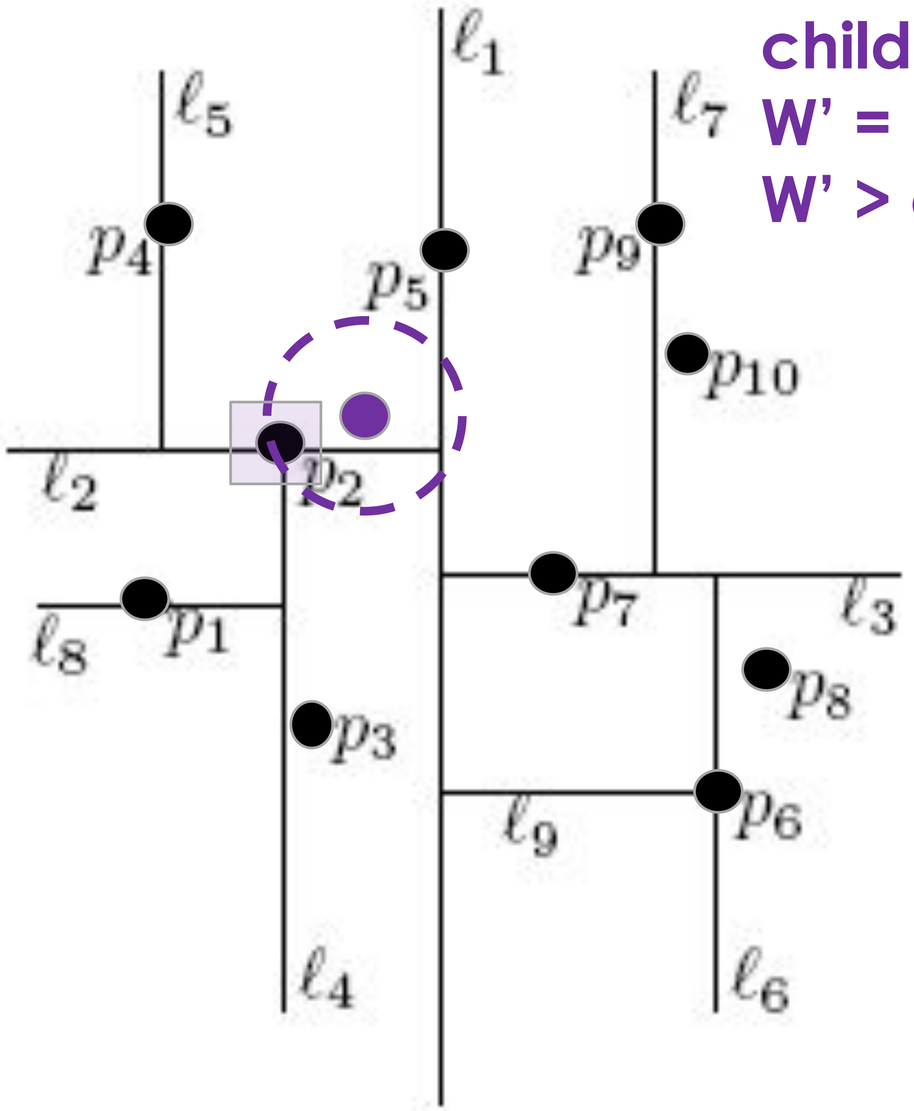
KD-tree: 1-NN Query Running



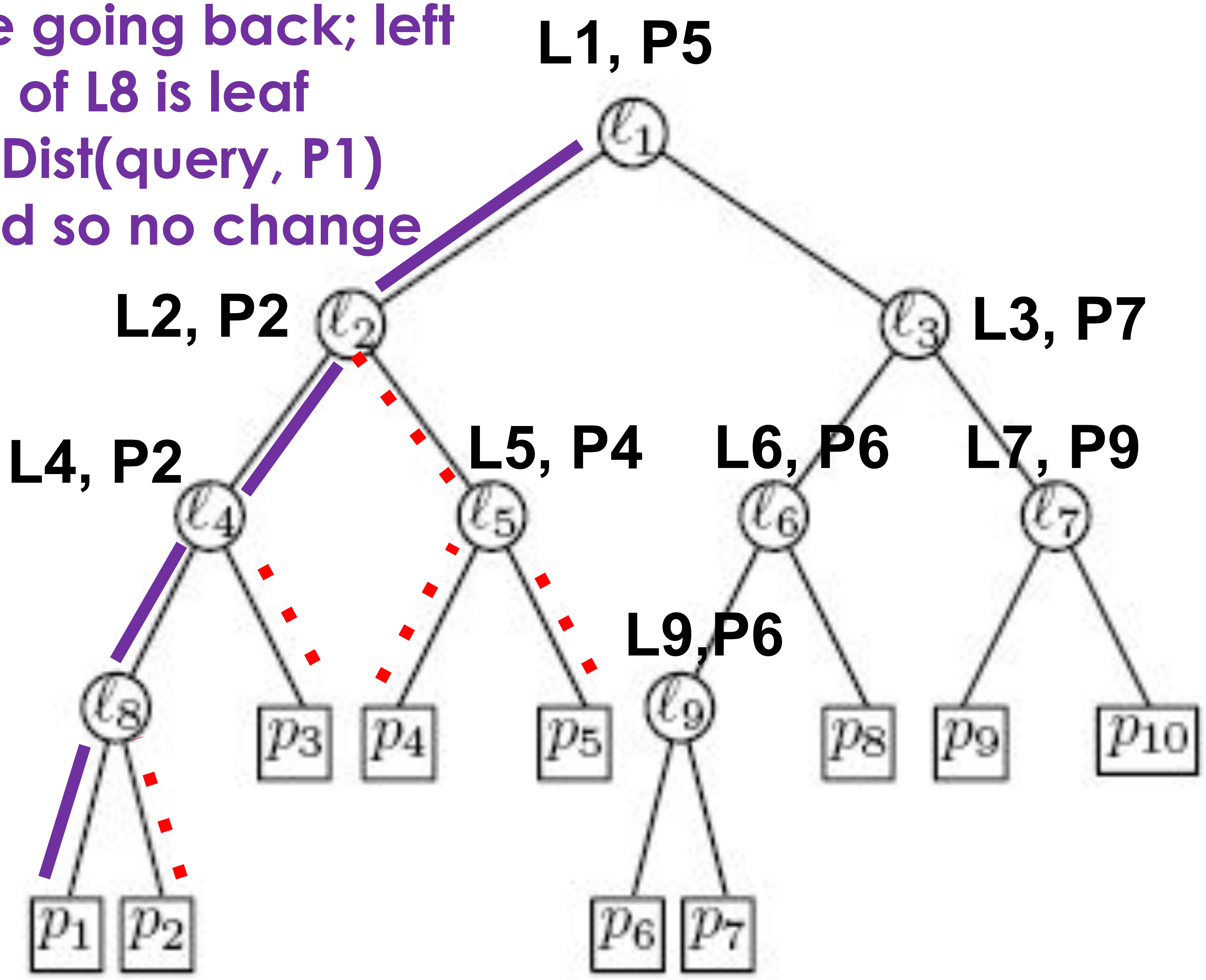
Right child of L8 is leaf
W' = Dist(query, P2)
W' < d so replace the estimate d = W'



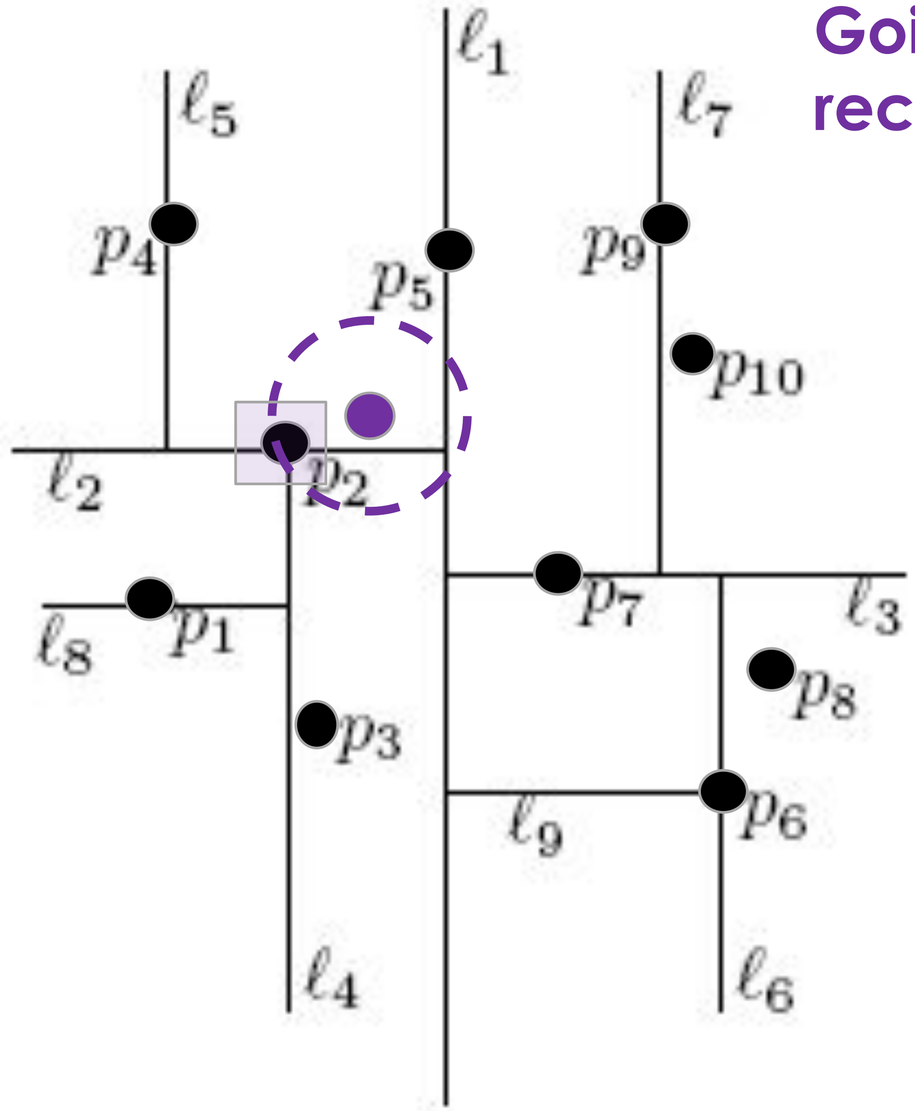
KD-tree: 1-NN Query Running



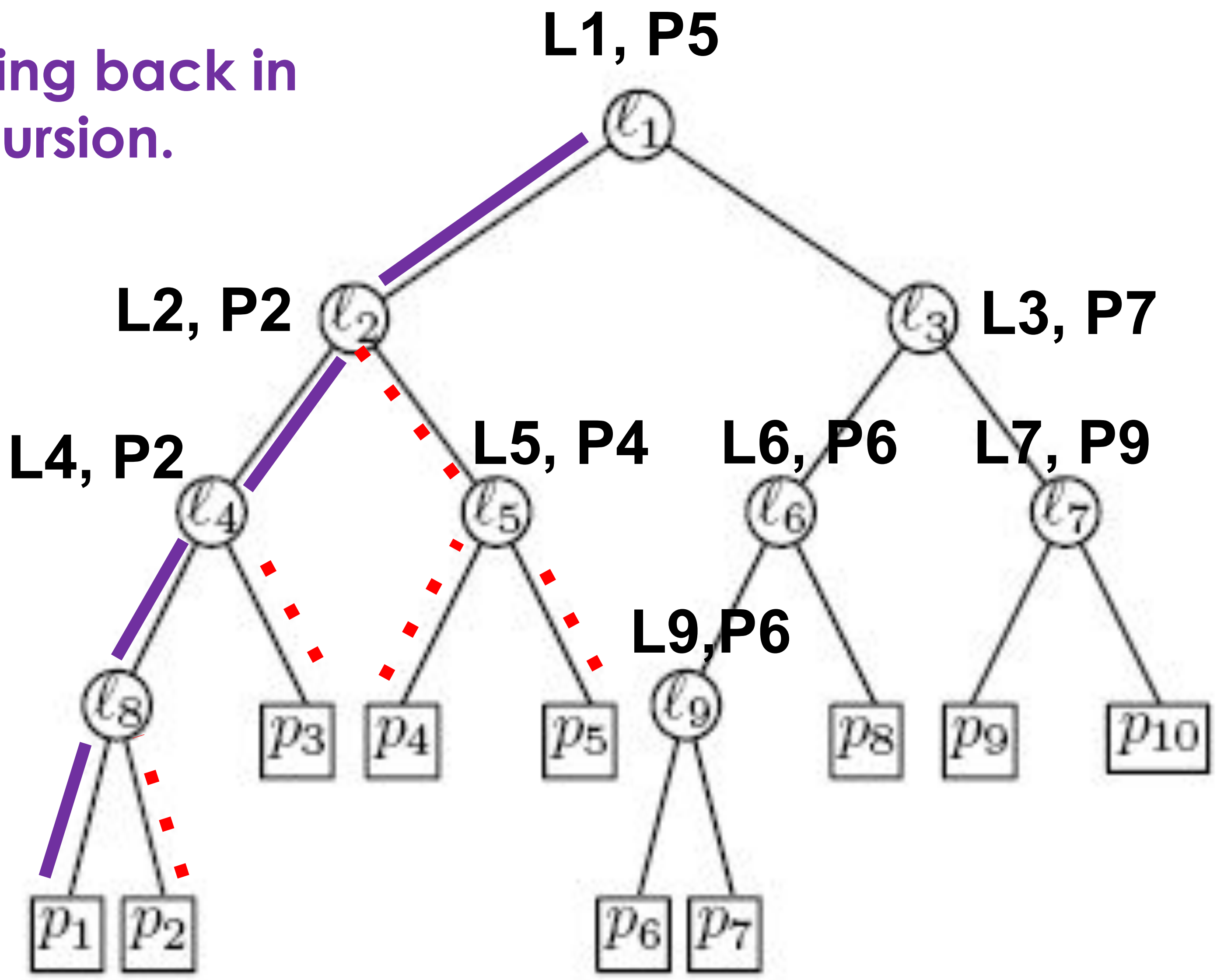
While going back; left child of L8 is leaf
W' = Dist(query, P1)
W' > d so no change



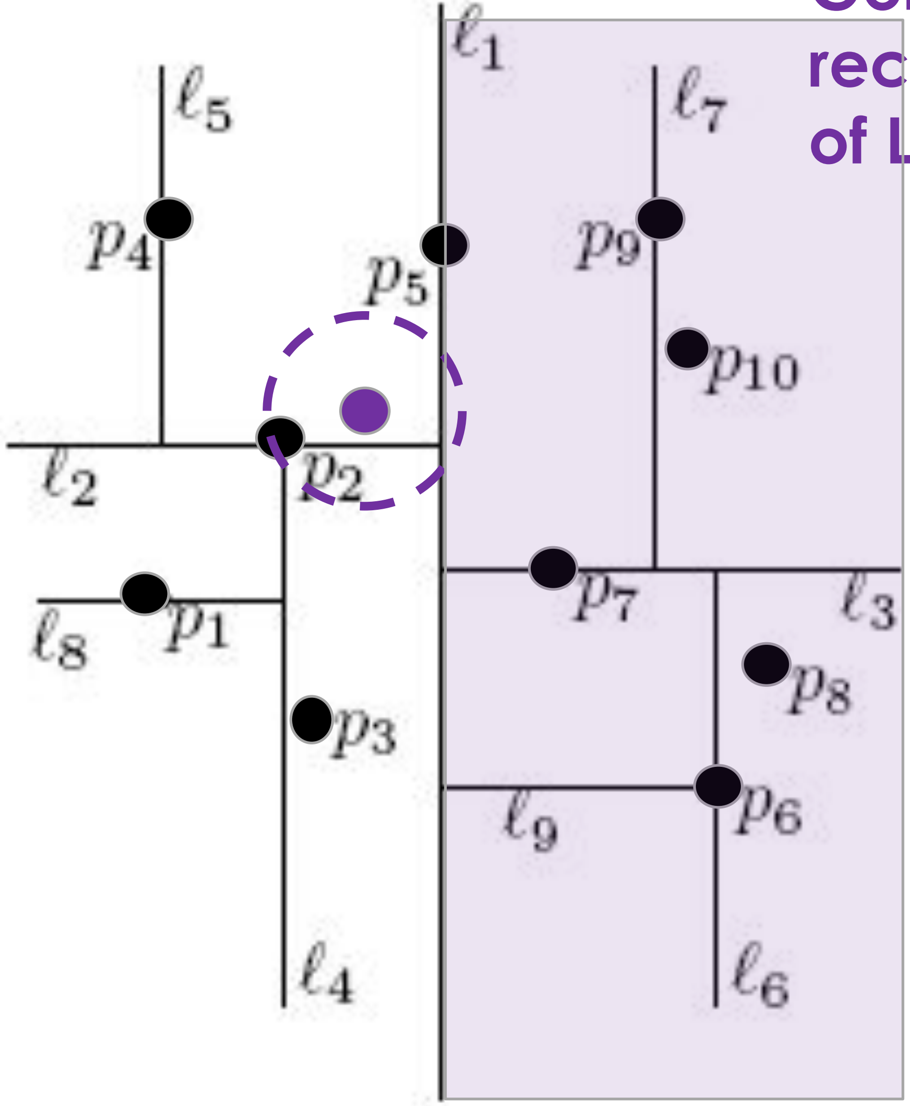
KD-tree: 1-NN Query Running



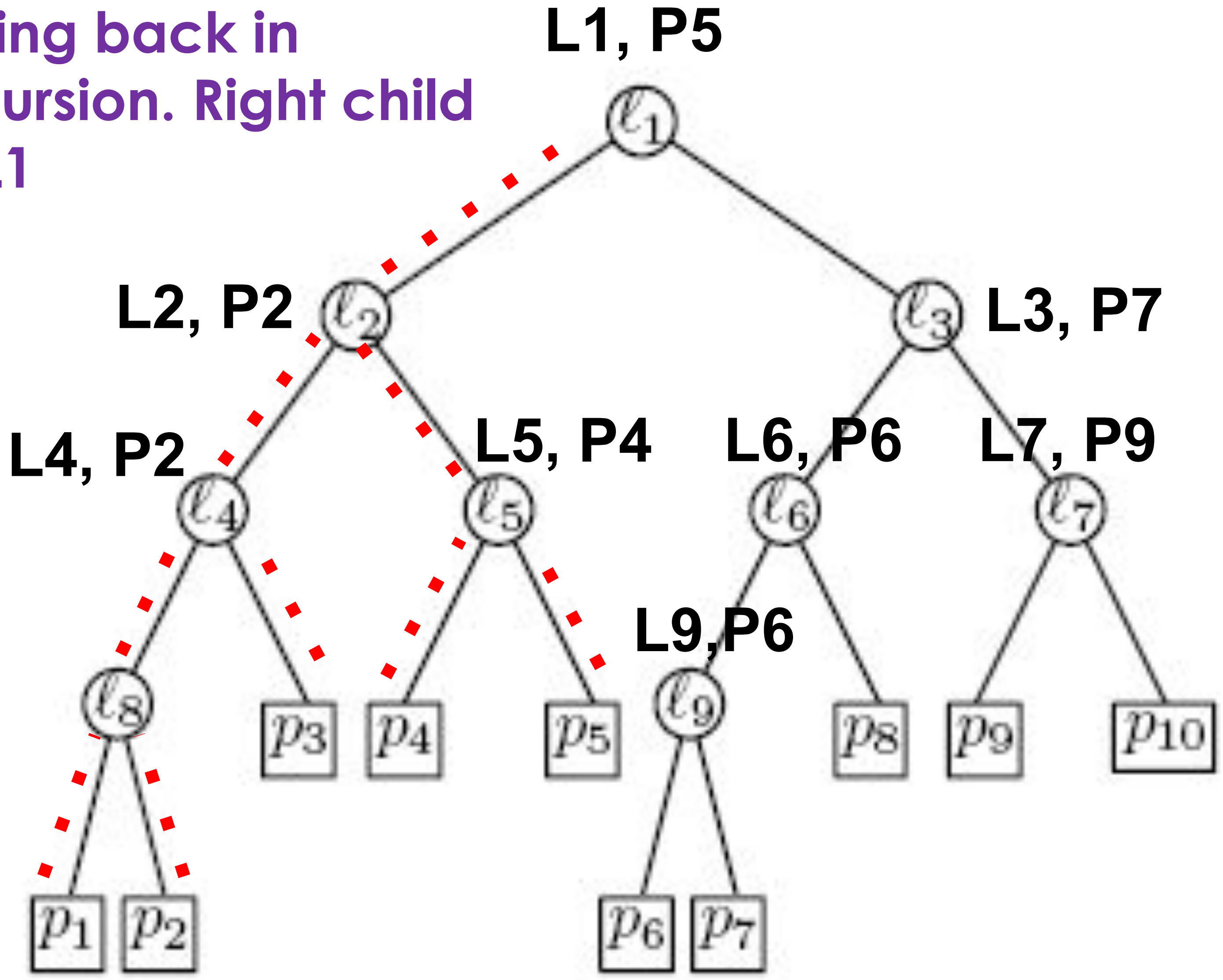
Going back in recursion.



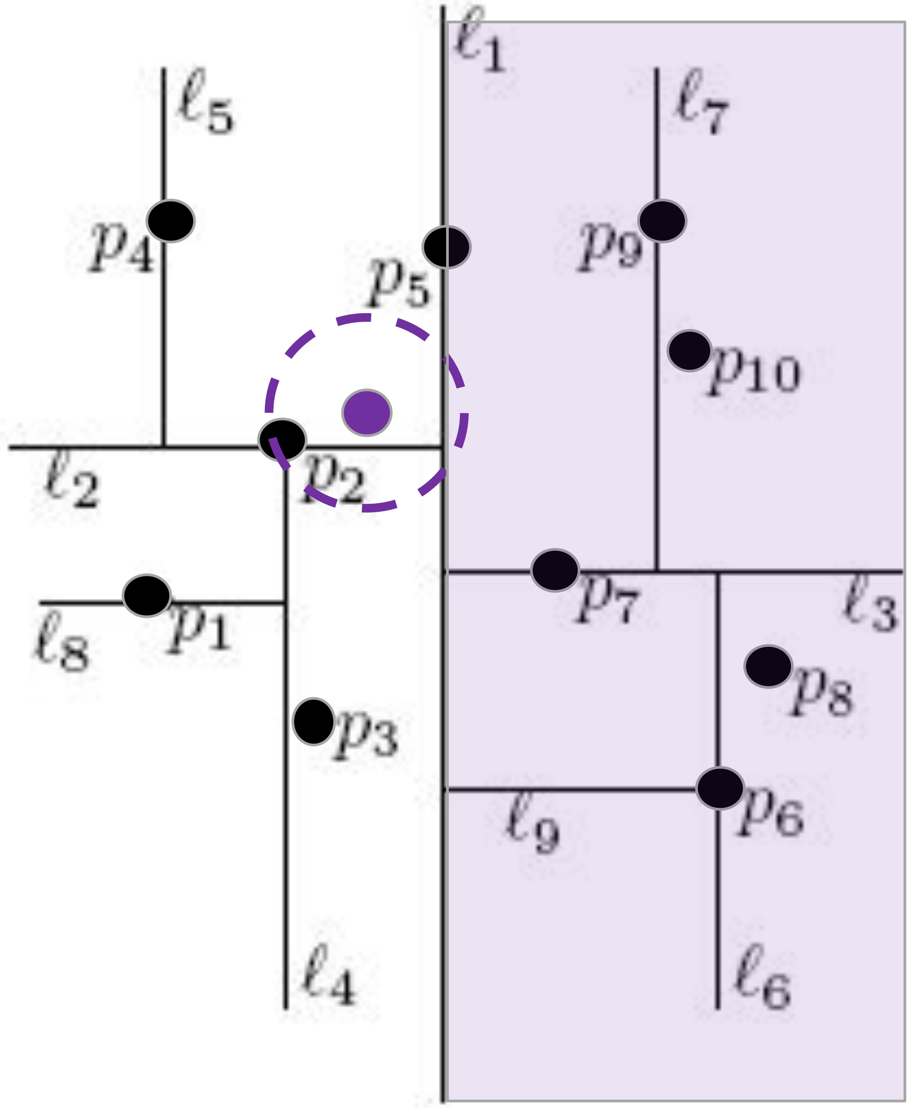
KD-tree: 1-NN Query Running



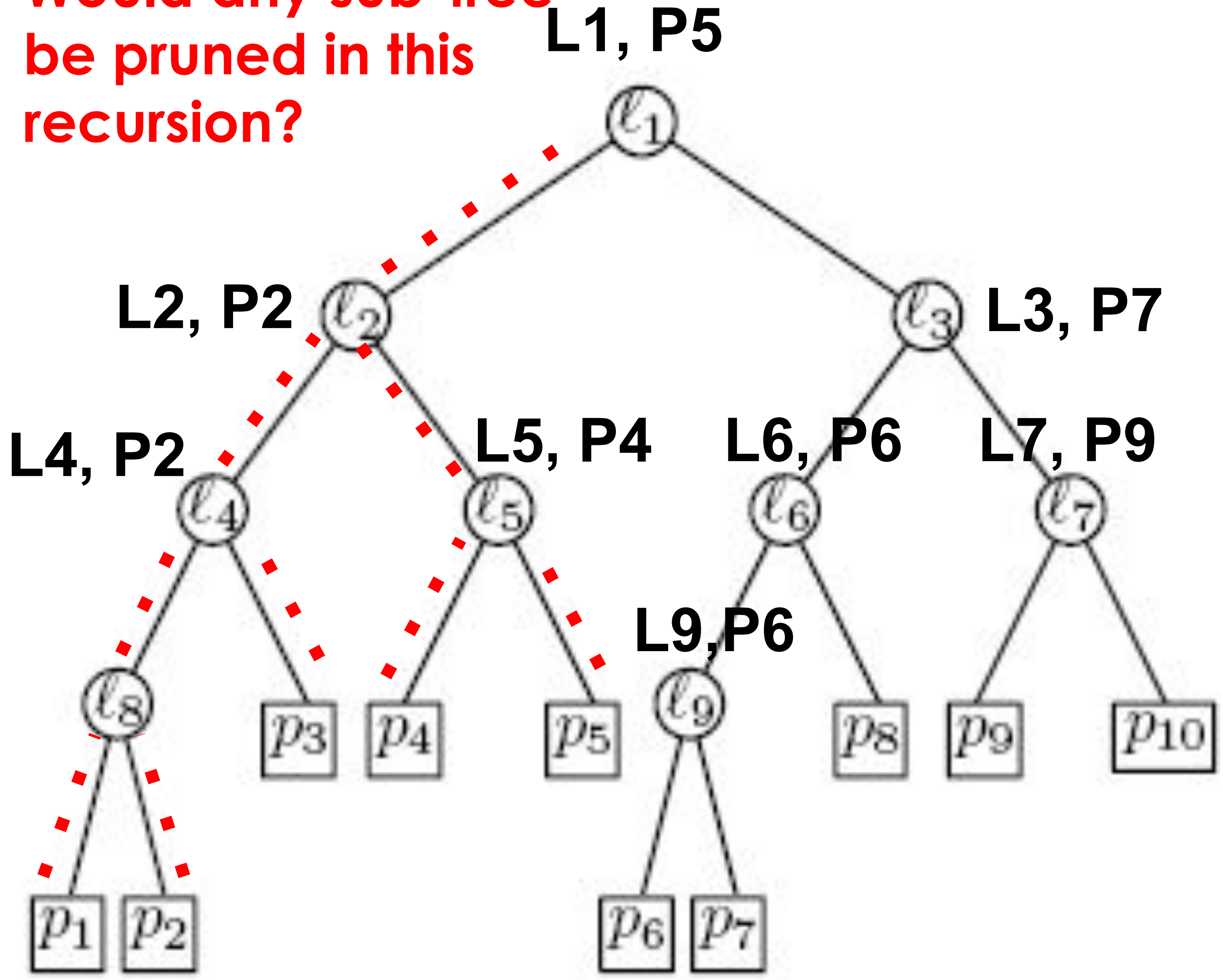
Going back in recursion. Right child of L1



KD-tree: 1-NN Query Running



Would any sub-tree be pruned in this recursion?

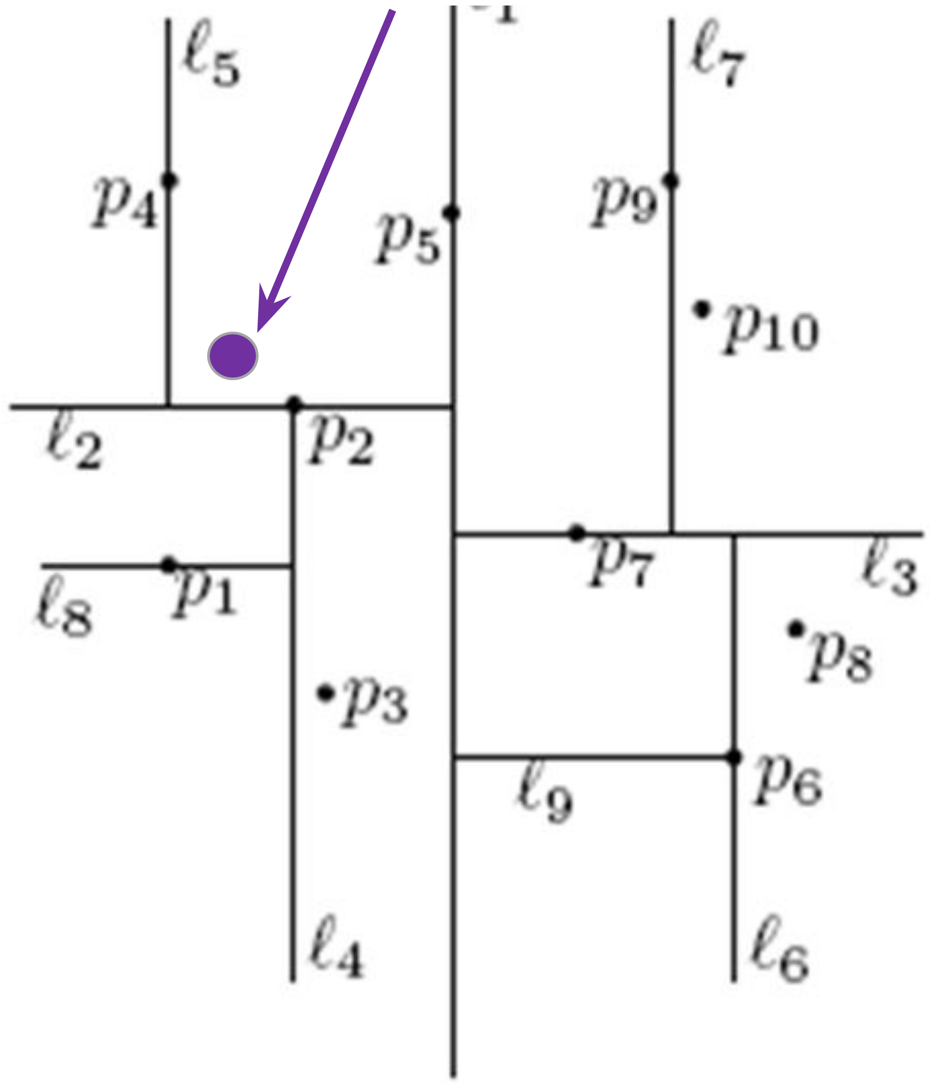


KD Trees – K-Nearest Neighbor Search

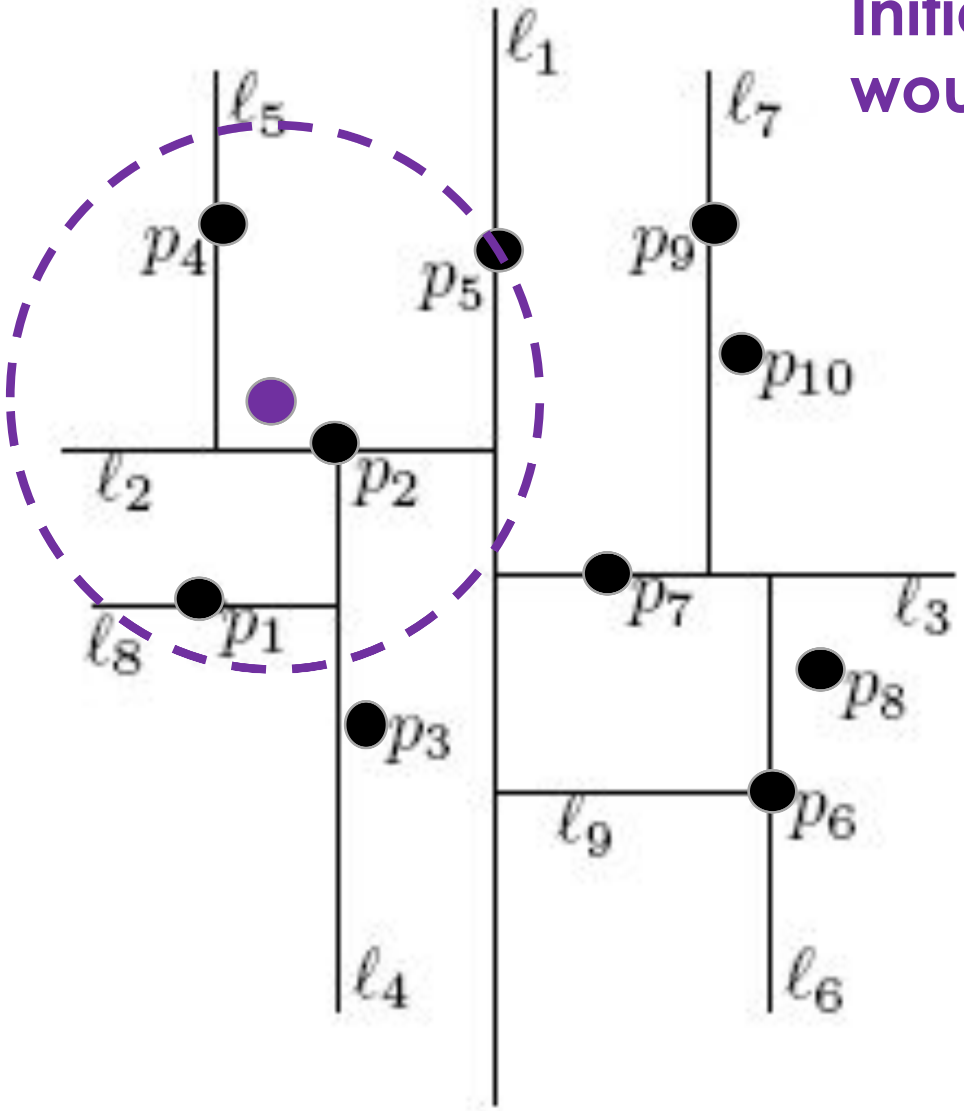
- Similar approach for K-nearest neighbors
- **Three key ideas:**

1) Bounded Priority Queue:
2) "d" thing is defined a distance from q to current farthest point.
3) First we fill up k points in our bounded priority then begin searching the remaining tree while pruning.

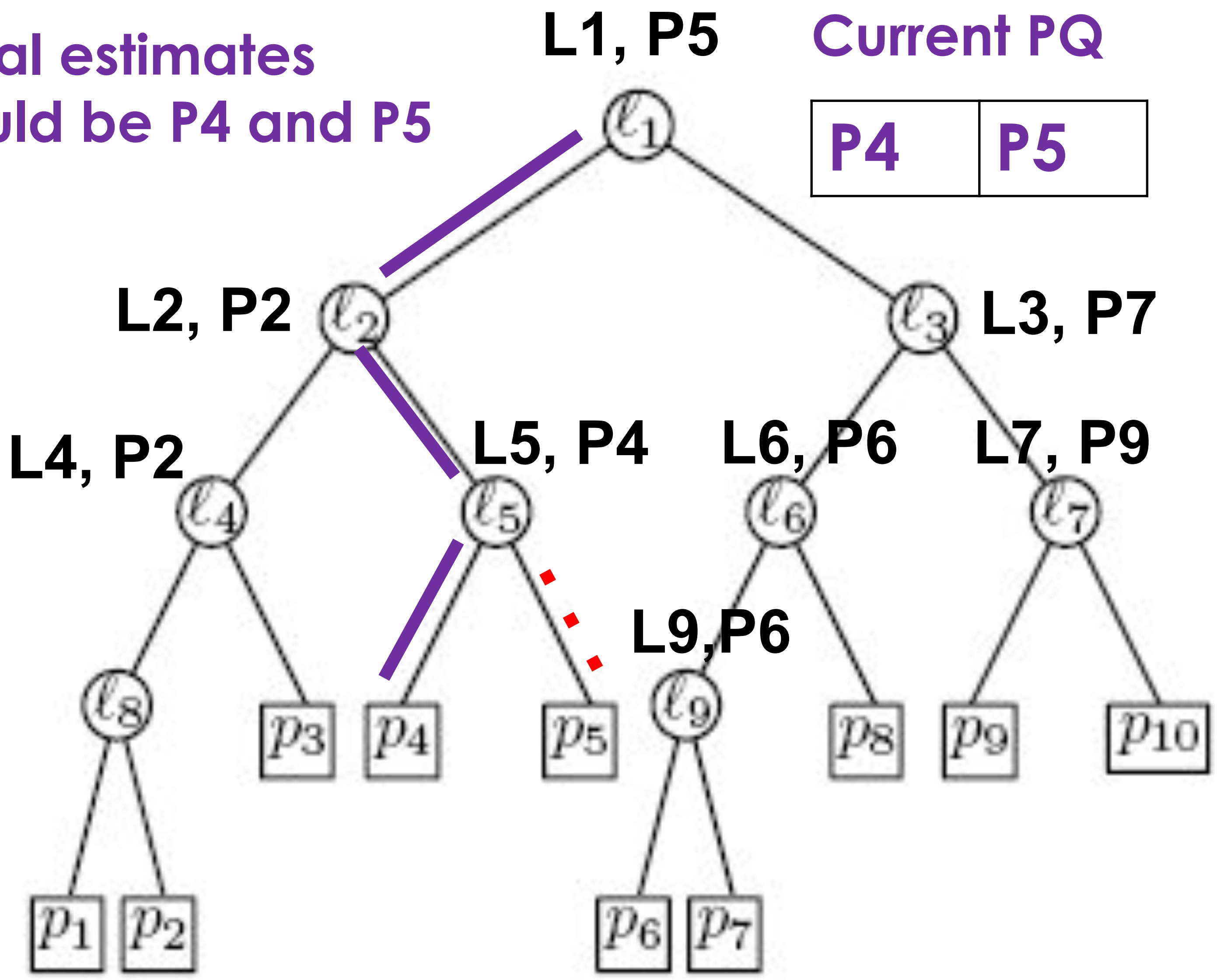
query Point for 2-NN



KD-tree: 2-NN Query Running



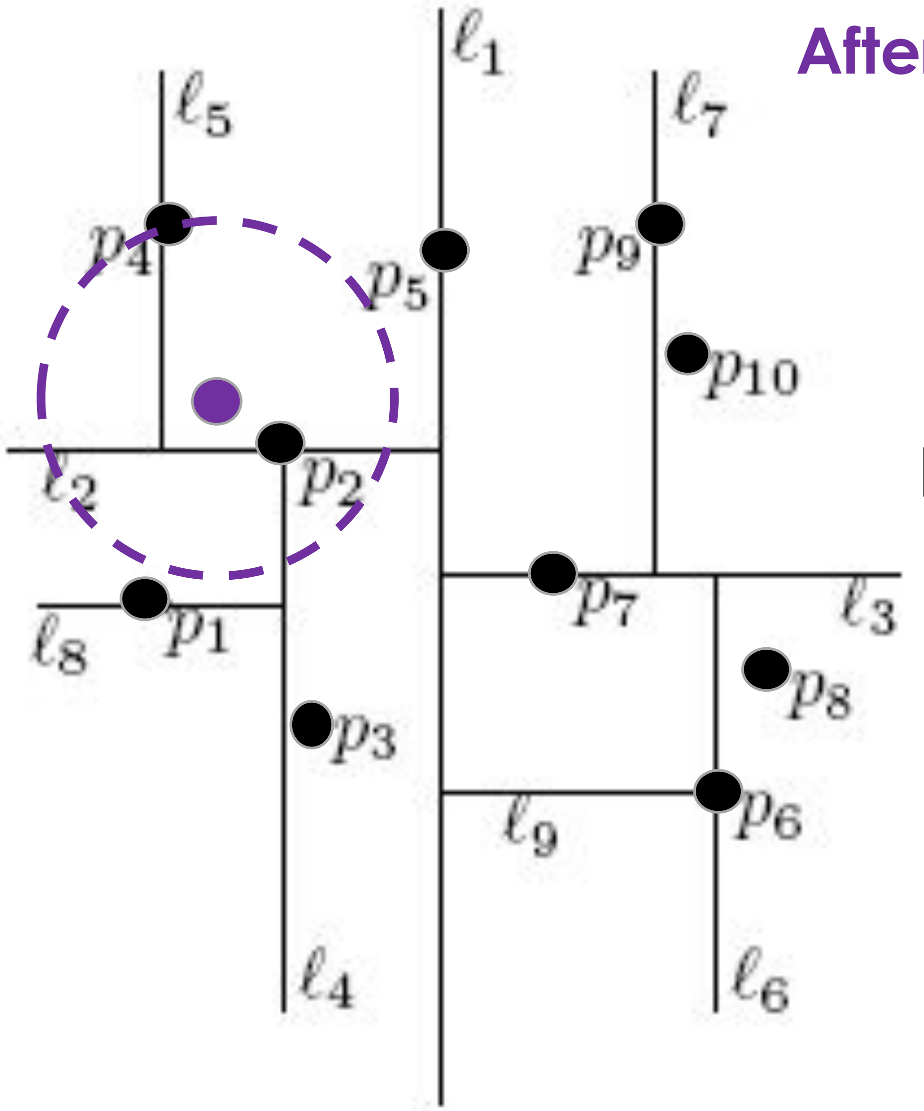
Initial estimates would be P4 and P5



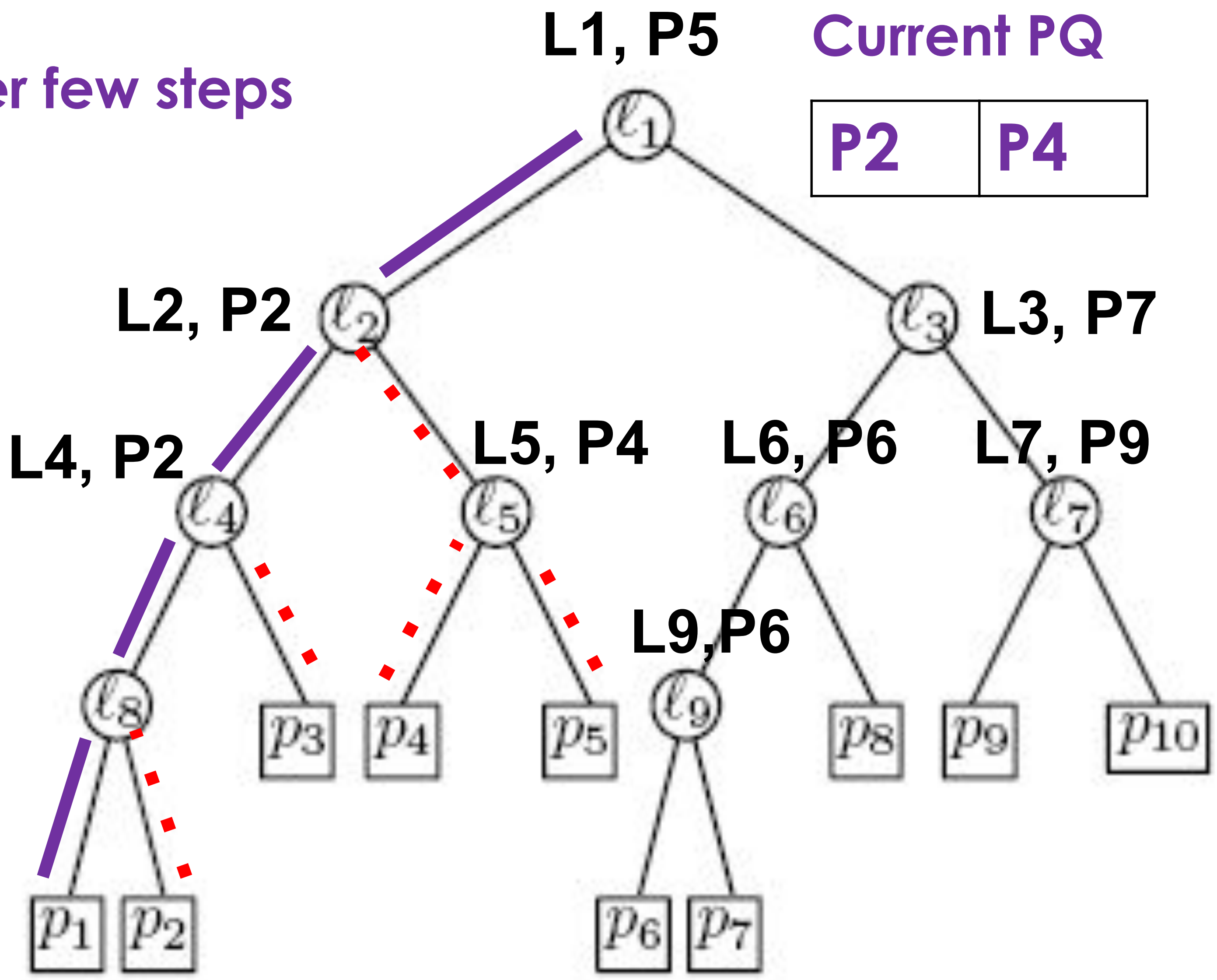
Current PQ

P4	P5

KD-tree: 2-NN Query Running



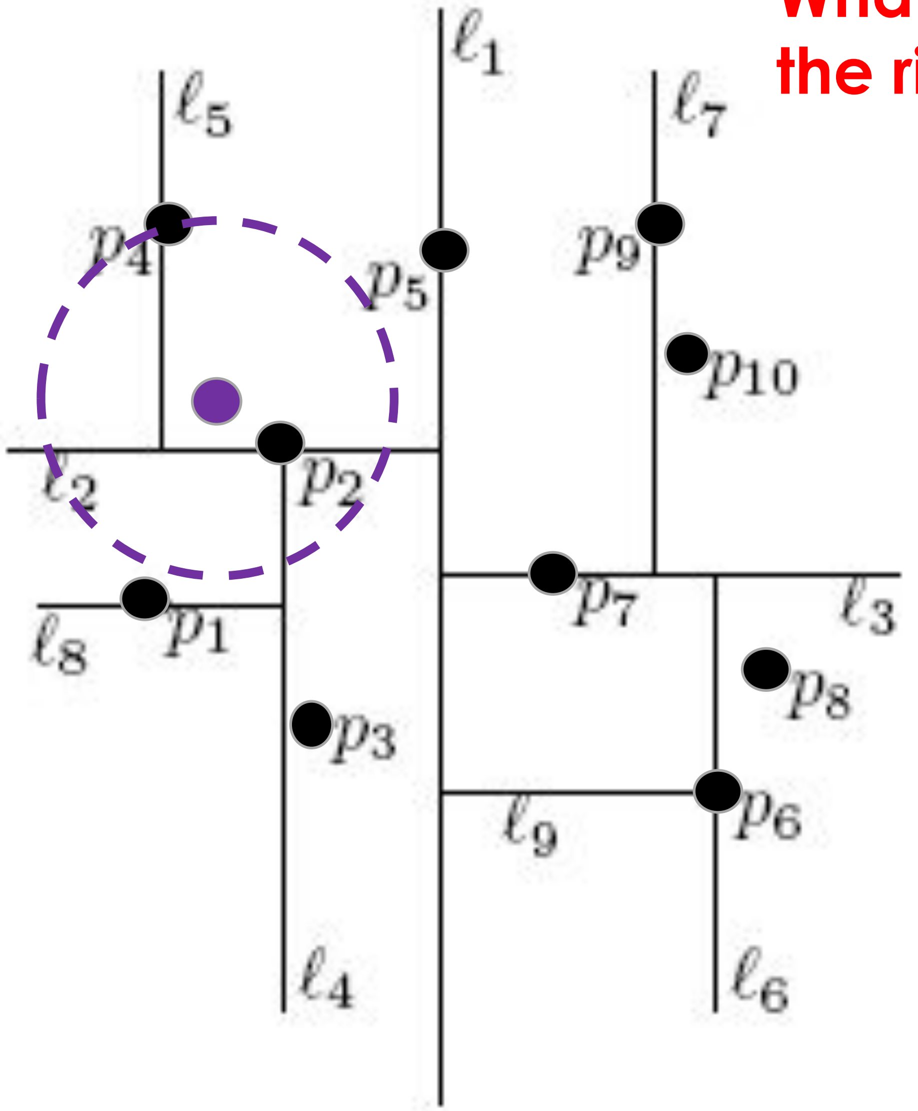
After few steps



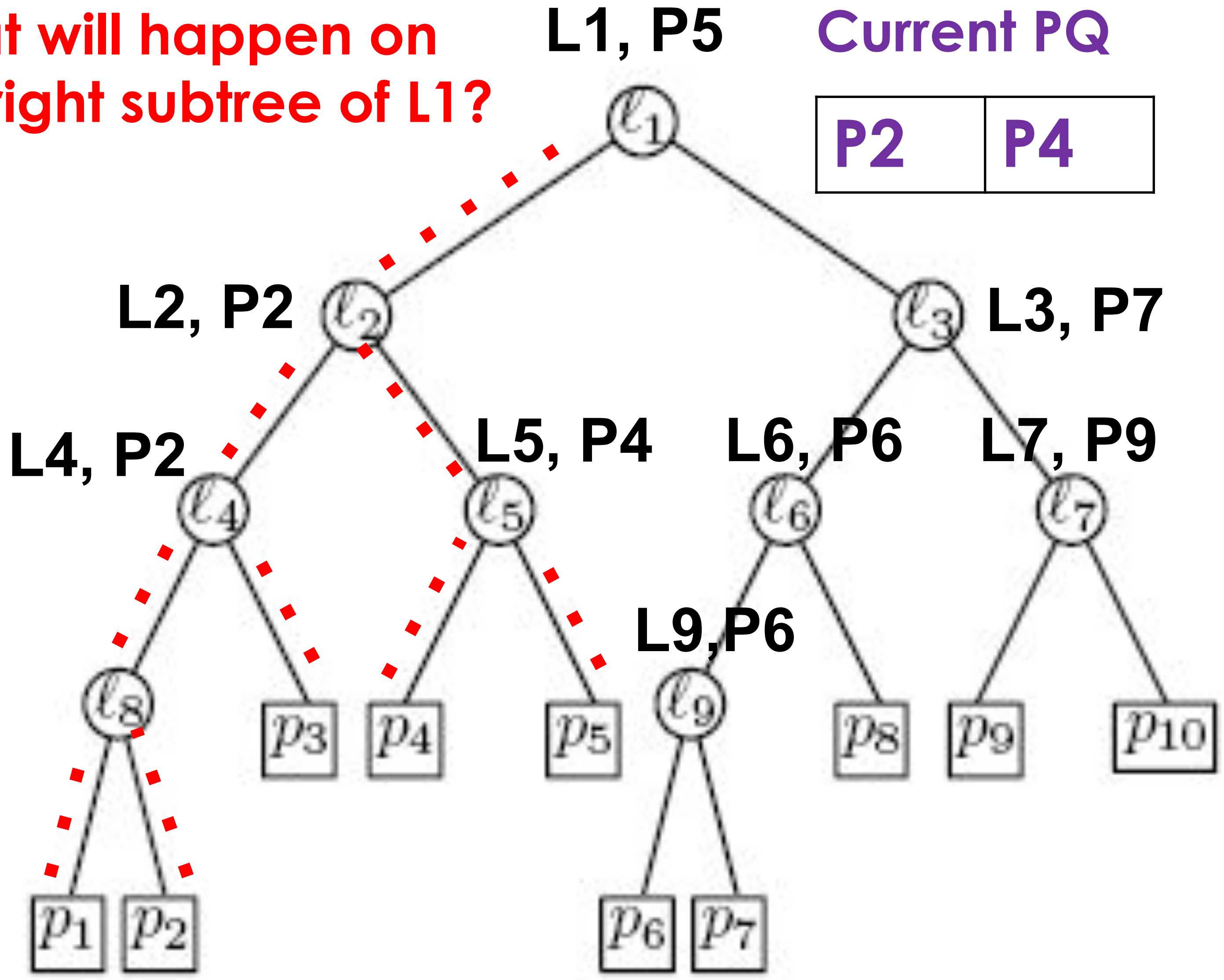
Current PQ

P2	P4

KD-tree: 2-NN Query Running



What will happen on the right subtree of L1?



Current PQ

P2	P4