

# Two-bulb diffusion experiment

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## Introduction

Simulation of the three-component two-bulb diffusion experiment. The experiment consists of two small compartments connected by a tube through which the components can diffuse. The three components considered here are  $H_2$ ,  $N_2$  and  $CO_2$ . The Maxwell-Stefan equations are used to model diffusion.

## Model equations

The Maxwell-Stefan equations are:

$$-\frac{x_i}{RT}\nabla\mu_i = \sum_{j \neq i} \frac{x_j \mathbf{J}_i - x_i \mathbf{J}_j}{c_t D_{ij}} \quad (1)$$

The left side of (1) can be reformulated, giving:

$$-\left(\frac{\partial \ln \gamma_i}{\partial \ln x_i} + 1\right) \nabla x_i = \sum_{j \neq i} \frac{x_j \mathbf{J}_i - x_i \mathbf{J}_j}{c_t D_{ij}} \quad (2)$$

For ideal systems the activity coefficient  $\gamma_i$  of component  $i$  is equal to unity. The left side of (2) then simplifies, resulting in:

$$-\nabla x_i = \sum_{j \neq i} \frac{x_j \mathbf{J}_i - x_i \mathbf{J}_j}{c_t D_{ij}} \quad (3)$$

The change in local composition at any given time is:

$$c_t \frac{\partial x_i}{\partial t} = -\nabla \cdot \mathbf{J}_i \quad (4)$$

## Method

The mole fractions of  $H_2$ ,  $N_2$  and  $CO_2$  in the first compartment are initially 0.0, 0.501 and 0.499, respectively. In the second compartment the mole fractions of  $H_2$ ,  $N_2$  and  $CO_2$  are initially 0.501, 0.499 and 0.0, respectively. The diffusivities are  $D_{12} = 8.33e-5$  ( $m^2/s$ ),  $D_{13} = 6.8e-5$  ( $m^2/s$ ) and  $D_{23} = 1.68e-5$  ( $m^2/s$ ). The volumes of the compartments are  $5e-4$  ( $m^3$ ) and the tube connecting the compartments has a length of  $1e-2$  ( $m$ ) and a diameter of  $2e-3$  ( $m$ ).

To simulate the transient two-bulb diffusion experiment, the model equations are solved. These are solved using the finite volume method. Time discretization is fully implicit. Results are shown in figure 1.

## Appendix

Here the one-dimensional case of the Maxwell-Stefan equations, applicable to the three-component two-bulb diffusion experiment, is elaborated on. The one-

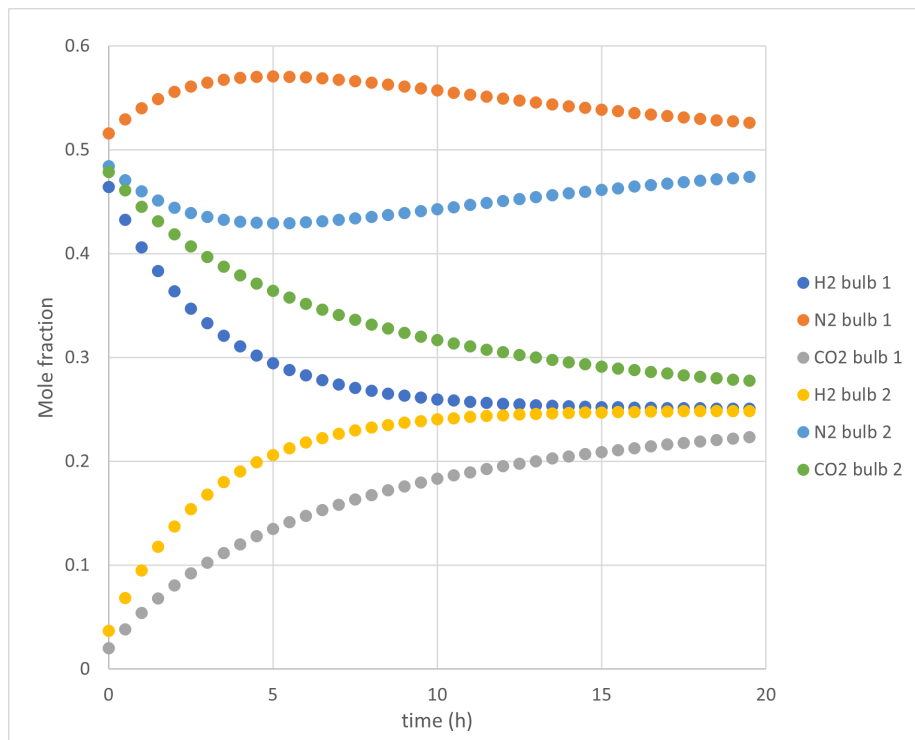


Figure 1: The mole fraction as a function of time (h).

dimensional case of (3) for component 1 is:

$$-c_t \frac{\partial x_1}{\partial z} = \frac{x_2 J_1 - x_1 J_2}{D_{12}} + \frac{x_3 J_1 - x_1 J_3}{D_{13}} \quad (5)$$

The equation for component 2 is:

$$-c_t \frac{\partial x_2}{\partial z} = \frac{x_1 J_2 - x_2 J_1}{D_{12}} + \frac{x_3 J_2 - x_2 J_3}{D_{23}} \quad (6)$$

The change in local composition of component 1 is:

$$c_t \frac{\partial x_1}{\partial t} = -\frac{\partial J_1}{\partial z} \quad (7)$$

The change in local composition of component 2 is:

$$c_t \frac{\partial x_2}{\partial t} = -\frac{\partial J_2}{\partial z} \quad (8)$$

To facilitate the elimination of the fluxes from the equations above equation (5) and (6) are rewritten:

$$-c_t \frac{\partial x_1}{\partial z} = a_1 J_1 + a_2 J_2 \quad (9)$$

$$-c_t \frac{\partial x_2}{\partial z} = b_1 J_1 + b_2 J_2 \quad (10)$$

With  $a_1$ ,  $a_2$ ,  $b_1$  and  $b_2$  given by:

$$a_1 = \left( \frac{1}{D_{12}} - \frac{1}{D_{13}} \right) x_2 + v_1 \quad (11)$$

$$a_2 = x_1 \left( \frac{1}{D_{13}} - \frac{1}{D_{12}} \right) \quad (12)$$

$$b_1 = x_2 \left( \frac{1}{D_{23}} - \frac{1}{D_{12}} \right) \quad (13)$$

$$b_2 = \left( \frac{1}{D_{12}} - \frac{1}{D_{23}} \right) x_1 + v_3 \quad (14)$$

The fluxes can now be written in terms of the composition gradients:

$$J_1 = \beta_1 \frac{\partial x_1}{\partial z} + \beta_2 \frac{\partial x_2}{\partial z} \quad (15)$$

$$J_2 = \alpha_1 \frac{\partial x_1}{\partial z} + \alpha_2 \frac{\partial x_2}{\partial z} \quad (16)$$

With  $\beta_1$ ,  $\beta_2$ ,  $\alpha_1$  and  $\alpha_2$  given by:

$$\beta_1 = -\frac{c_t}{a_1} - \frac{a_2 \alpha_1}{a_1} \quad (17)$$

$$\beta_2 = -\frac{a_2\alpha_2}{a_1} \quad (18)$$

$$\alpha_1 = -\frac{c_t}{\left(a_2 - \frac{a_1b_2}{b_1}\right)} \quad (19)$$

$$\alpha_2 = \frac{a_1c_t}{(a_2b_1 - a_1b_2)} \quad (20)$$

Equations (15) and (16), together with (7) and (8) are the set of equations which are solved to simulate three-component two-bulb diffusion.