Medical Image Processing & Computer Vision BMEN324

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Course Contents:

Day	Date	Description	Chapter	Reference
Lecture 1	05/10/2021	Introduction to Image processing	Ch 1	Digital Image Processing, 4th edition, By Rafael C. Gonzalez, and Richard E. Woods
Lecture 2	12/10/2021	Digital image fundamentals	Ch 2	
Lecture 3	19/10/2021	Arithmetic, logic operators	Ch 2	
Lecture 4	26/10/2021	Intensity transformation	Ch 3	
Lecture 5	02/11/2021	Spacial filtering	Ch 3	
Lecture 6	09/11/2021	Image in Frequency domain	Ch 4	
EXAM	16/11/2021	Mid Term Exam		
Lecture 7	23/11/2021	Frequency domain filtering	Ch 4	
Lecture 8	30/11/2021	Image restoration	Ch 5	
Lecture 9	07/12/2021	Image reconstruction	Ch 5	
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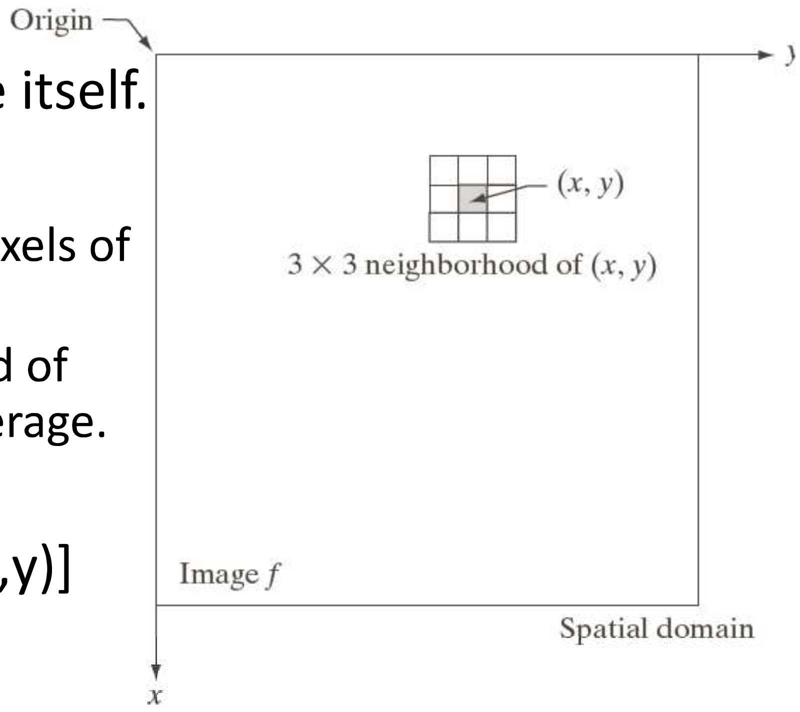
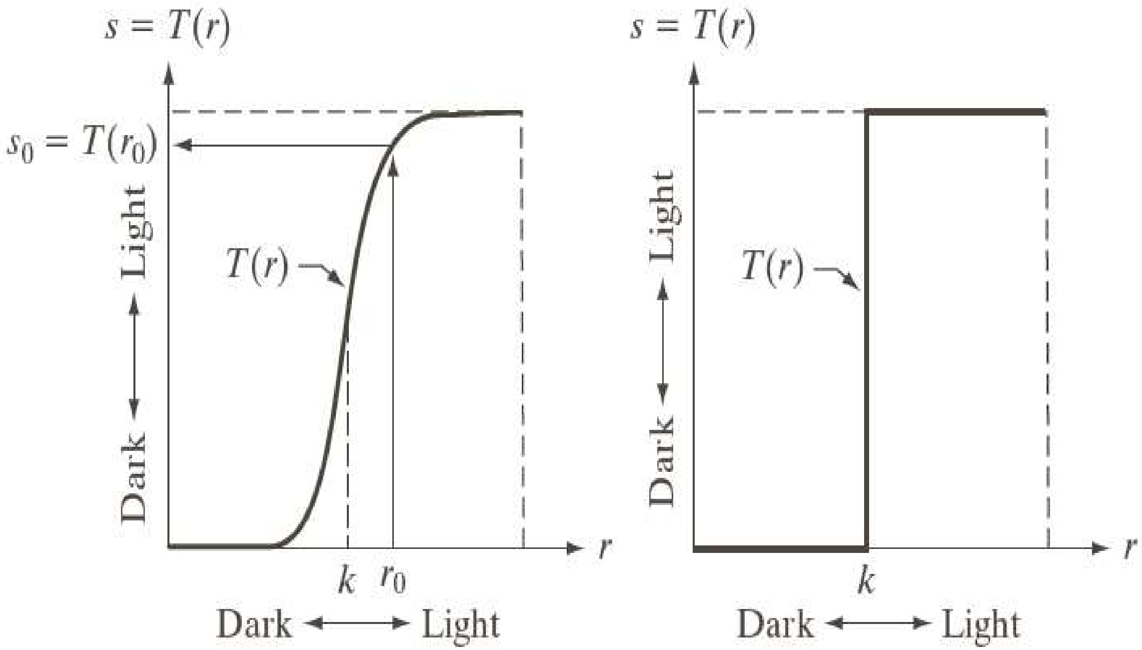
Ch 3: Intensity Transformations and Spatial Filtering

What is Spatial Domain

- The spatial domain refers to the image plane itself.
- Categories of spatial processing:
    1. Intensity transformations: operate on single pixels of an image. E.g; Image negative.
    2. Spatial filtering: operates on the neighborhood of every pixel in an image. E.g; neighborhood average.

- Spatial domain is denoted by: g(x,y) = T[f(x,y)]
    - f(x,y) : input image
    - g(x,y): output image.
    - T is an operator on f defined over a neighborhood of point(x,y).





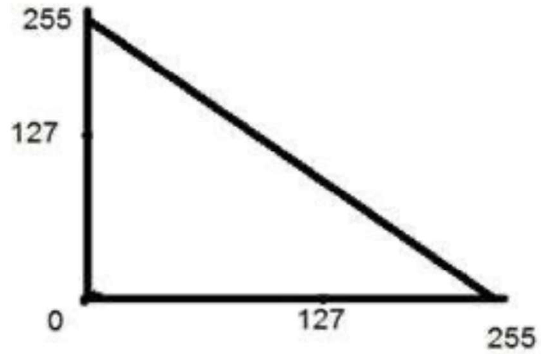
- g(x,y) = T[f(x,y)]
- The smallest possible neighborhood is of size 1x1.
- In this case, g depends on f at a single point (x,y) and T becomes an intensity (gray-level or mapping)
- *Transformation function* is simplified:

$$s = T(r)$$

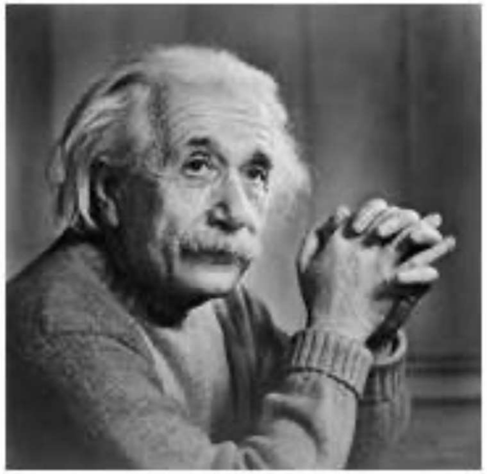
Basic Intensity Transformation Functions

- Image Negatives
- Log Transformations
- Power-Law (Gamma) Transformations
- Piecewise-Linear Transformation Functions
  - Contrast Stretching
  - Intensity-level slicing
  - Bit-Plane slicing
- **Histogram Processing**
  - Histogram Specification
  - Histogram Matching

Image Negative



- s = T(r)
- s = L-1-r
- E.g; s=255-r



r

s

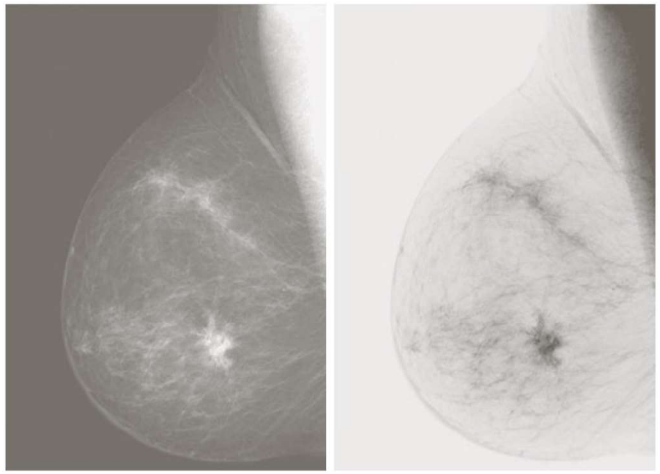
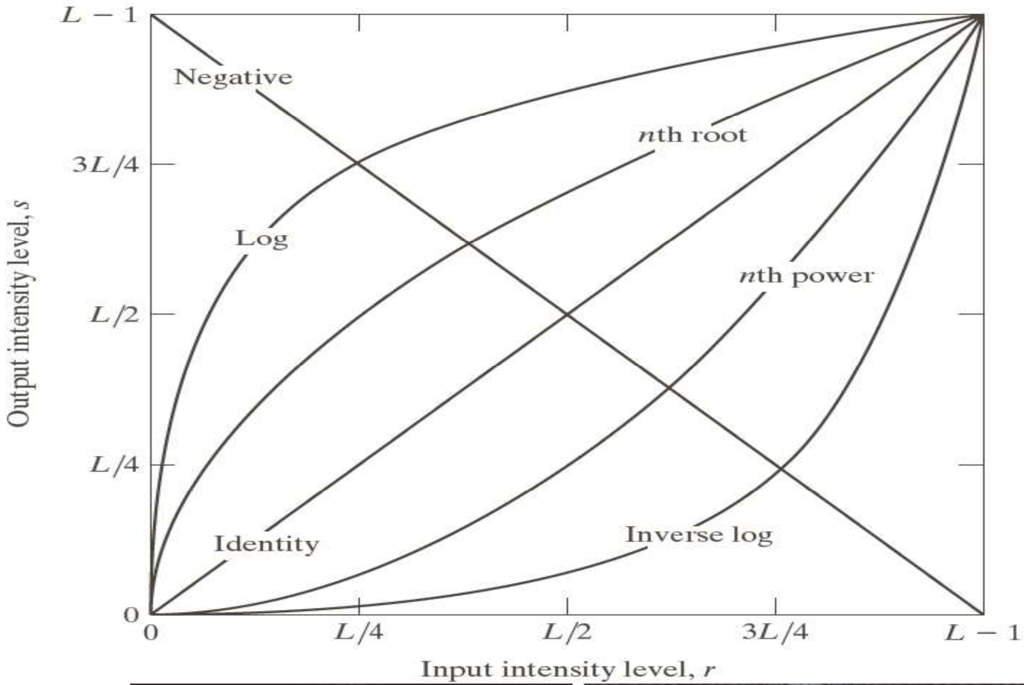
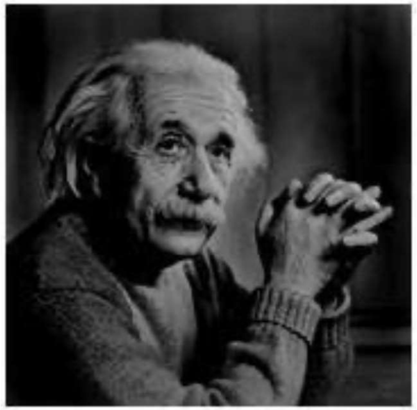
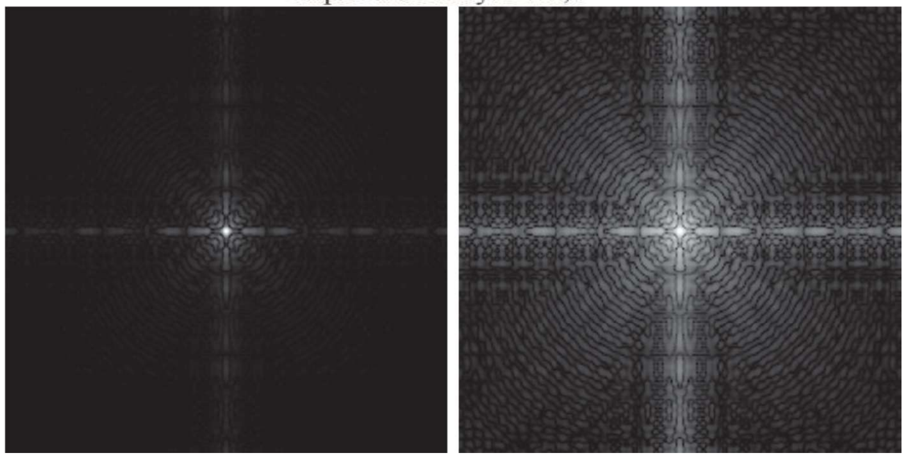


Image Log

- S = T(r)
- s= c.log(1+r), r >=0

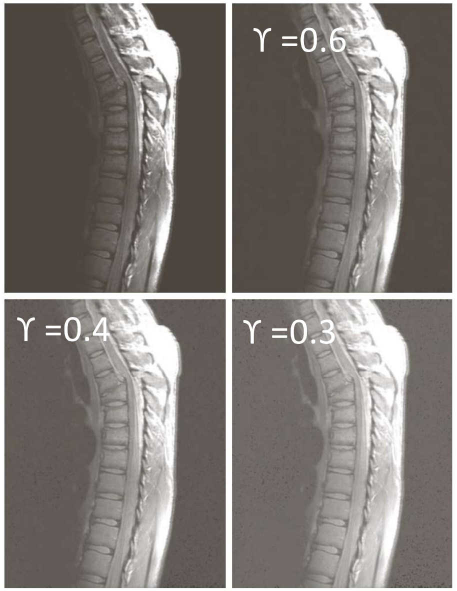


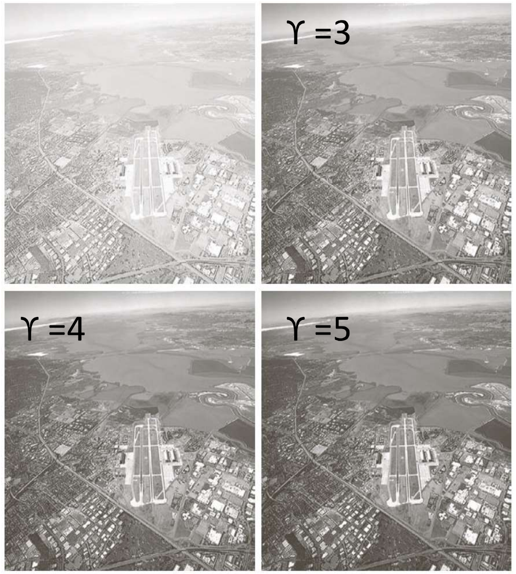


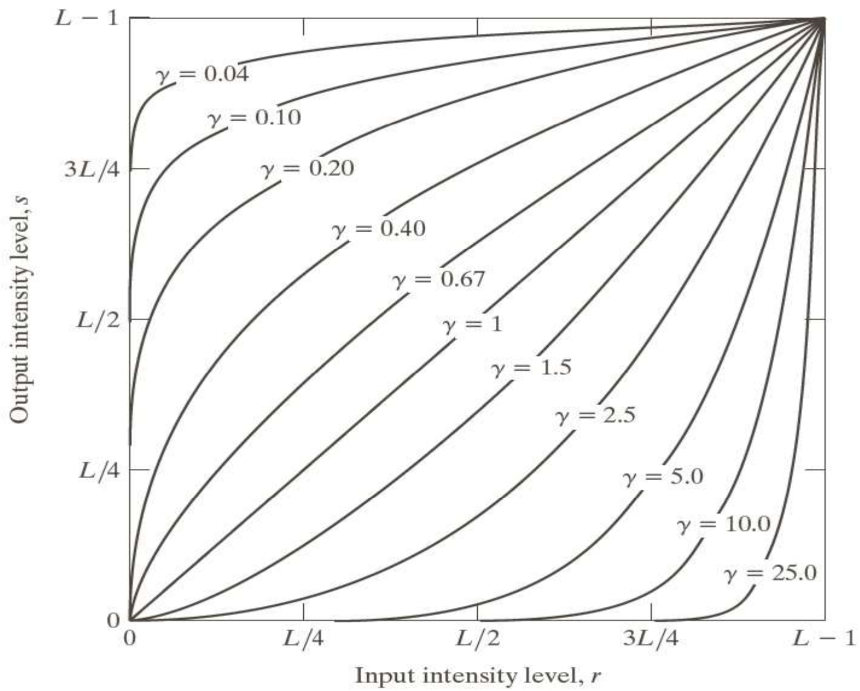


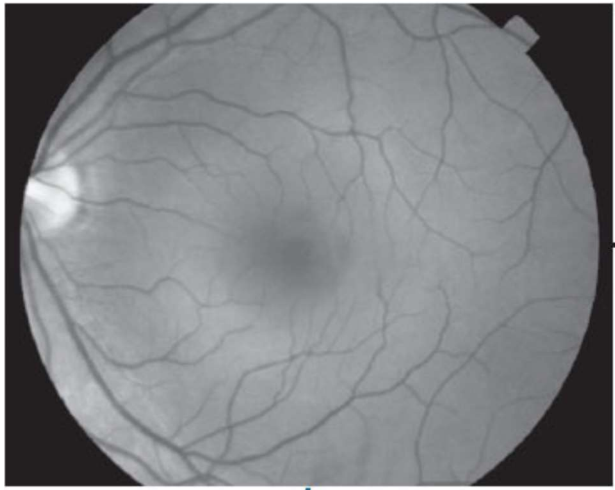
Power-law (Gamma) Transformations (Gamma correction)

- Power-law transformations have the basic form: $s = c.r^{\Upsilon}$
- c and ϒ are +ve const.

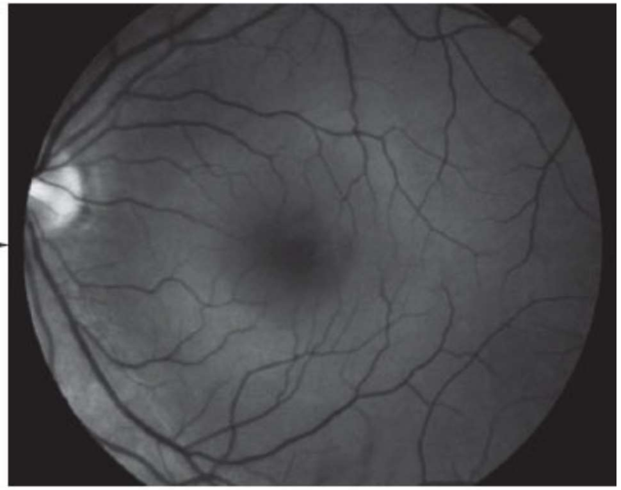






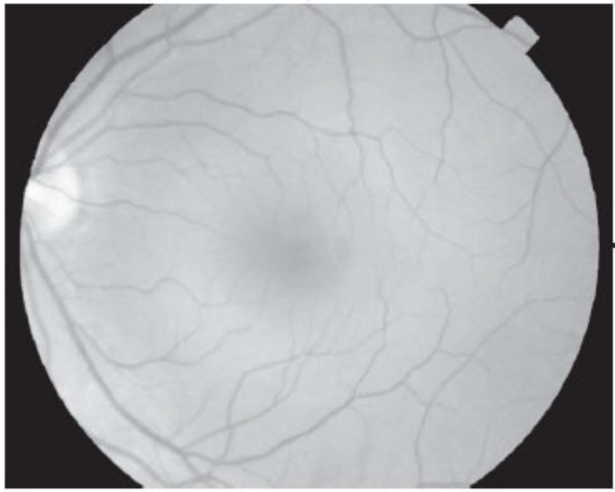


Original image

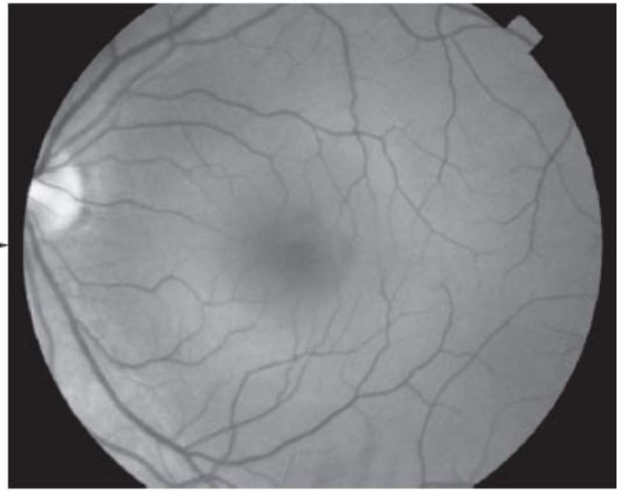


Original image as viewed on a monitor with a gamma of 2.5

Gamma Correction



Gamma-corrected image



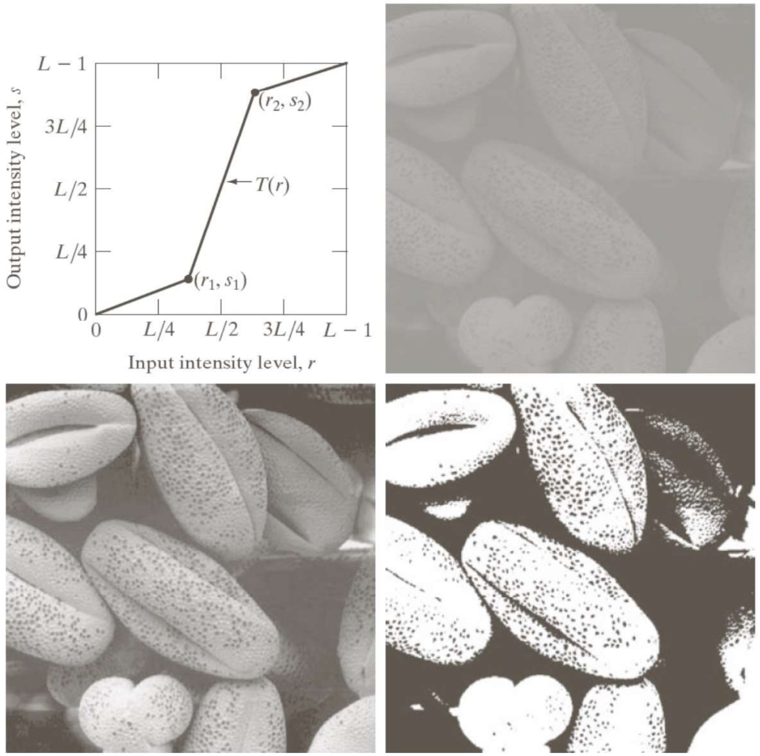
Gamma-corrected image as viewed on the same monitor

Piecewise-Linear Transformation Functions

1. Contrast Stretching
2. Intensity-level slicing
3. Bit-Plane slicing

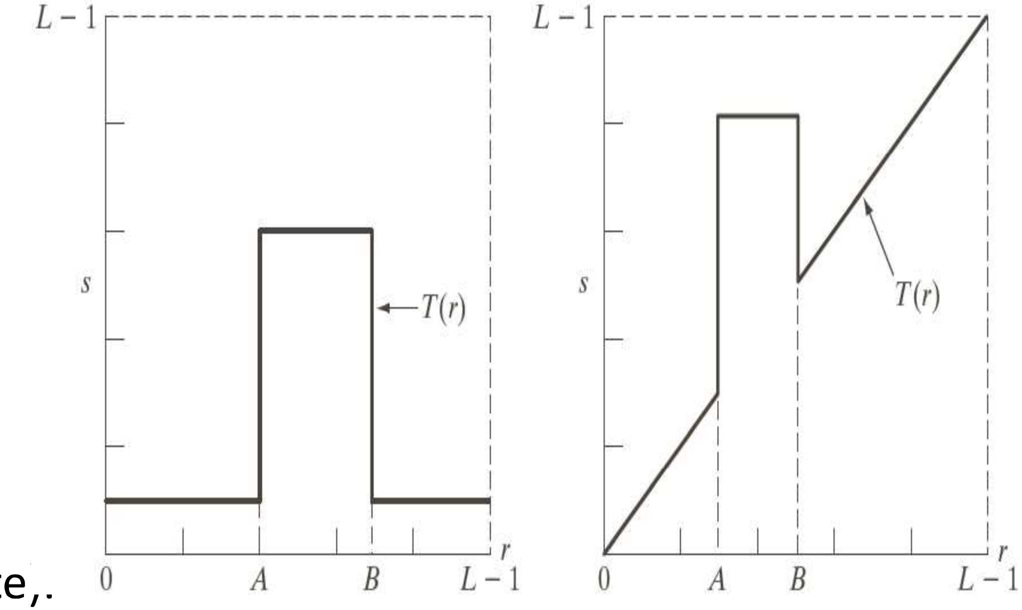
Contrast Stretching Transformations

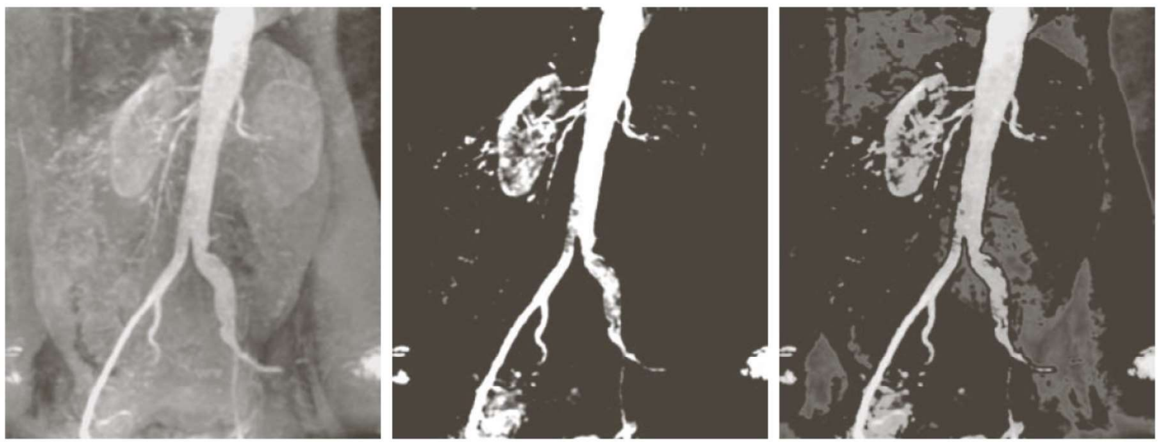
- ***Contrast Stretching**:* a process that expands the range of intensity levels in an image.
- Law-contrast images can result from:
  1. Poor illumination.
  2. Lack of the dynamic range in the imaging sensor.
  3. Wrong setting of the lens aperture during image acquisition.
- The locations of (r1,s1) and (r2,s2) control the shape of the transformation function.
- If r1=s1 and r2=s2 -> linear
- If r1=r2, s1=0, s2=L-1 -> Thresholding



Intensity-level slicing

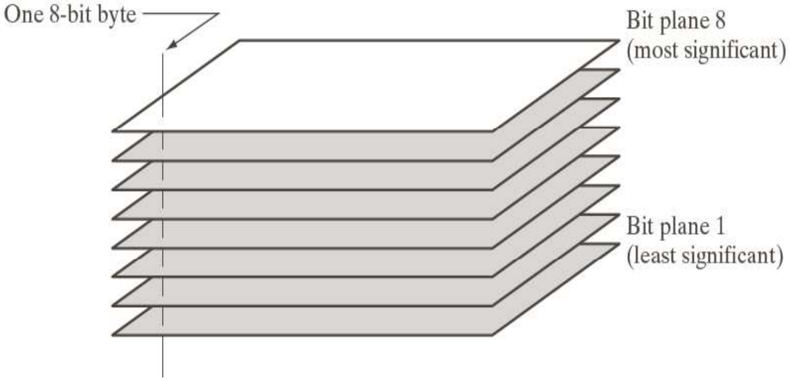
- Highlighting a specific range if intensities in an image often is of interest.
- Has two approaches:
  1. Display two values (black and white,.
  2. brightens or (darkens) the desired range of intensities.

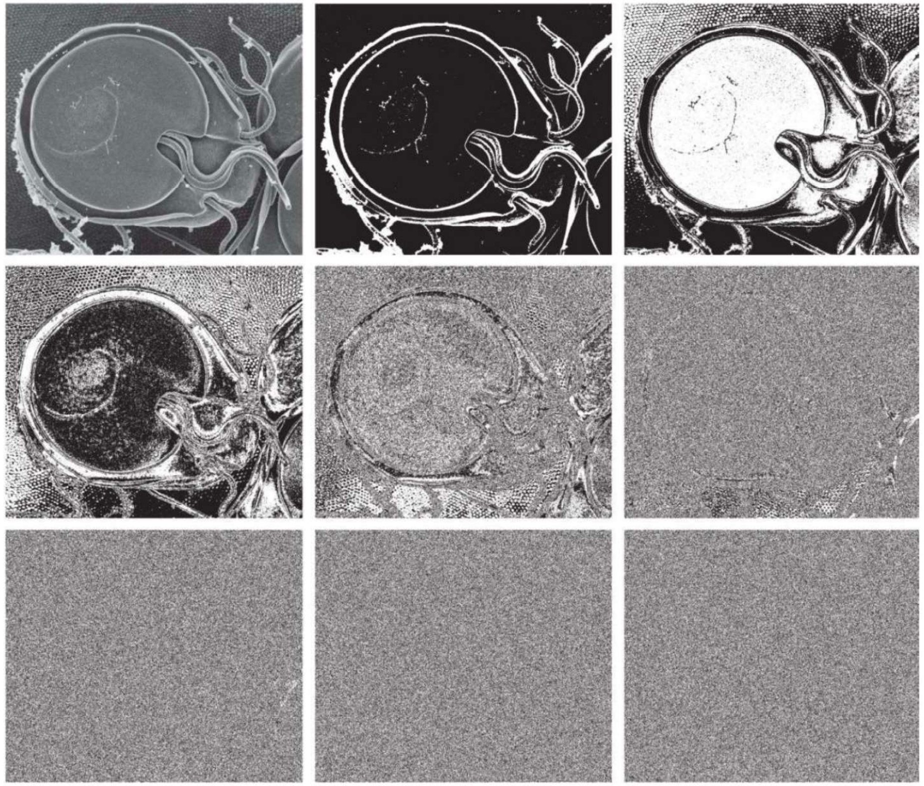


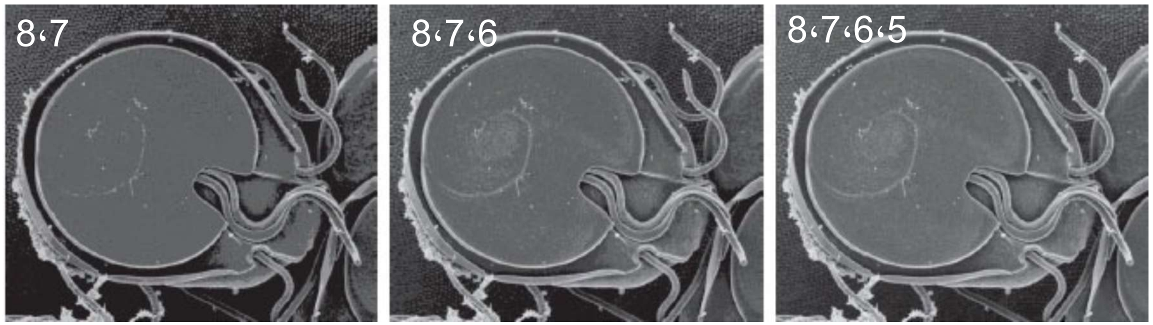


Bit-Plane slicing

- Higher-order bit planes contain significant amount of the visually significant data.
- The lower-order planes contribute to more suitable intensity details in the image.
- The least significant bit contribute to noise.







Histogram

- Unnormalized histogram is the number of times of each grey level occurs in the image

(The frequency of occurrence of gray level values).

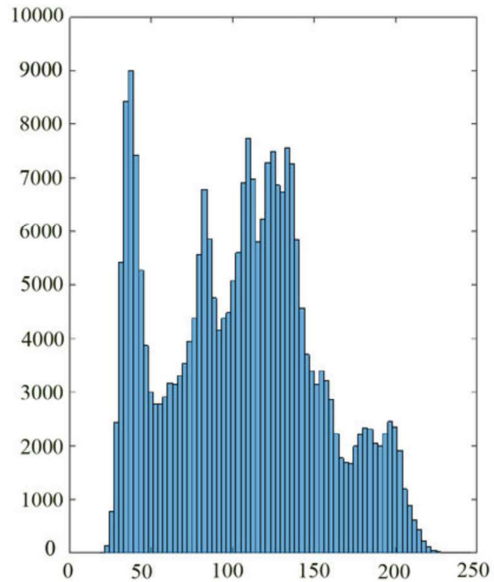
$h(r_k) = n_k$

$\mathrm{k} - 0,1,2, \dots, \mathrm{L} - 1,$

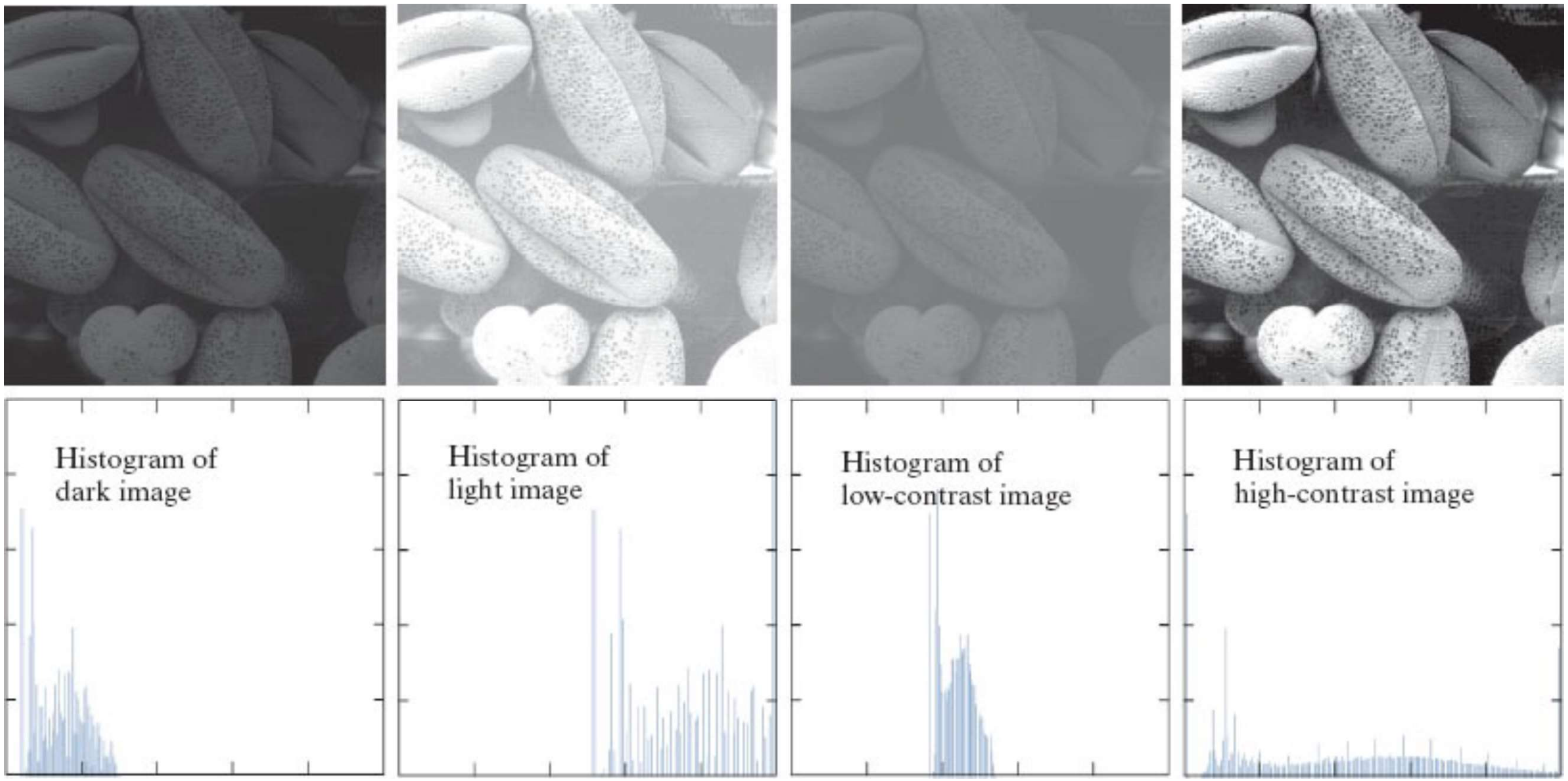
$n_k = number\ of\ pixels\ with\ r_k$

- *Normalized histogram (PDF)*

$$\mathrm{P}(r_k) = \frac{h(r_k)}{MN}$$

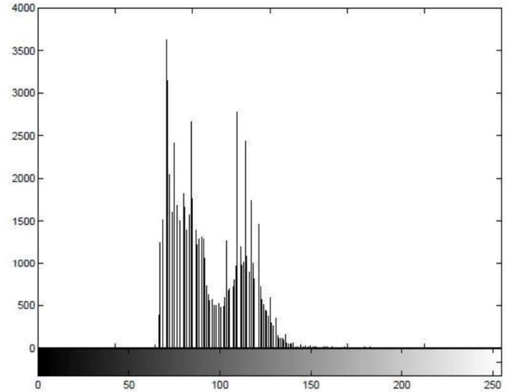


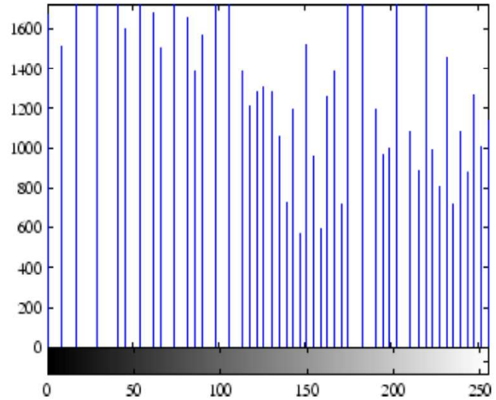
Histogram of different image types



Histogram equalization

- To improve the contrast of the image
- r is the intensity of original image with range [0,L-1]
- s is th eintensity of the result image (uniform distipution)
- Problem : find s=T(r)



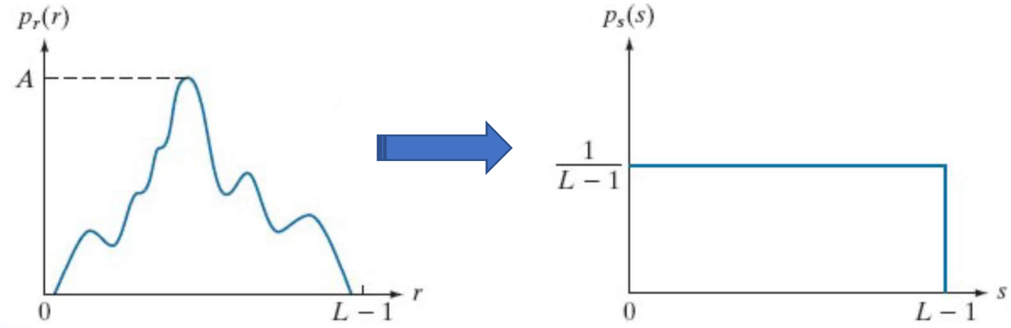


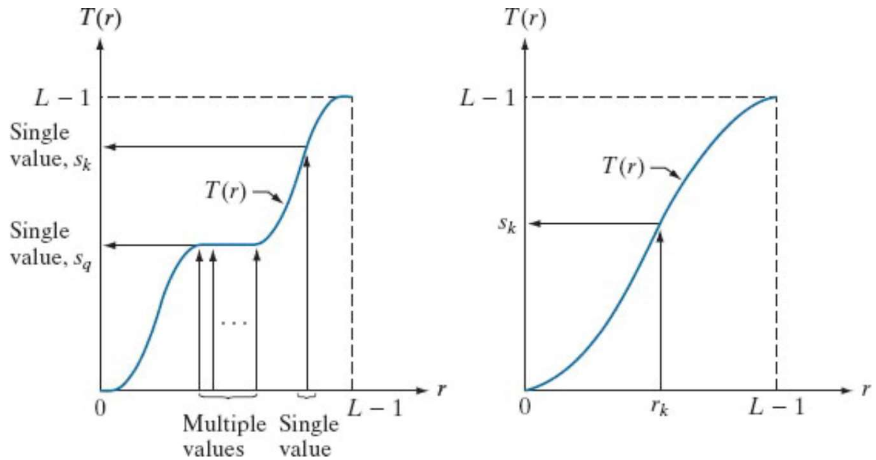
- $P_r(r)$: histogram of input image
- $P_s(s)$: hist. of the resultant image
- $s = T(r)$ monotonically increasing
- Problem : **find** $s = T(r)$
- $P_s(s) = P_r(r)\left|\frac{dr}{ds}\right|$
- $\frac{ds}{dr} = \frac{P_r(r)}{P_s(s)} = (L-1).\,P_r(r)$

$$s = T(r) = (L-1)\int_0^r P_r(w)\,dw$$

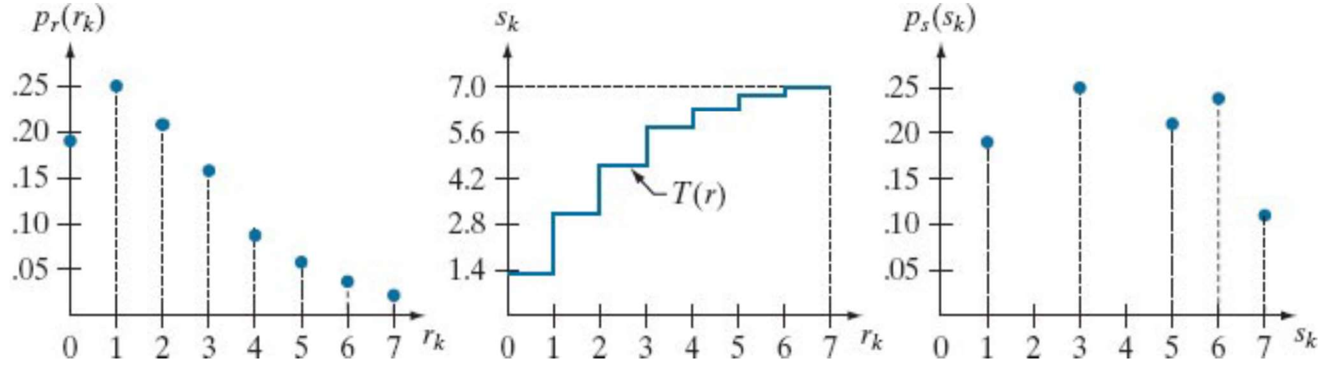
- Discrete form of s

$$s_k = T(r_k) = (L-1)\sum_{j=0}^{k} P_r(r_j)\ ,$$
$$\text{k} = 0,1,2,\cdots,L-1$$





Example



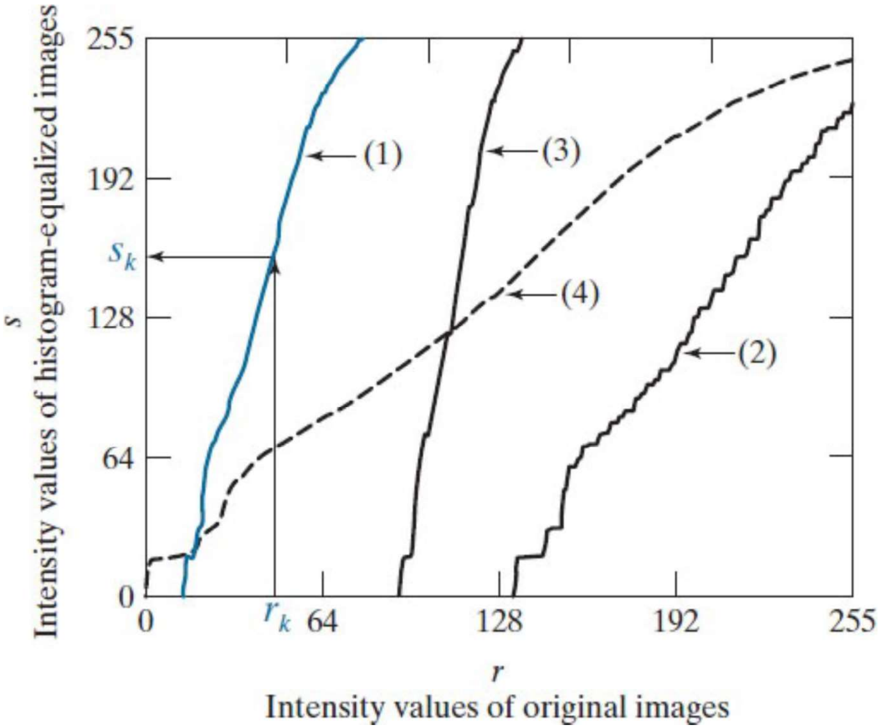
$$s_k = T(r_k) = 7\sum_{j=0}^{k} P_r(r_j) \text{ ,k} = 0,1,2,\cdots,7$$

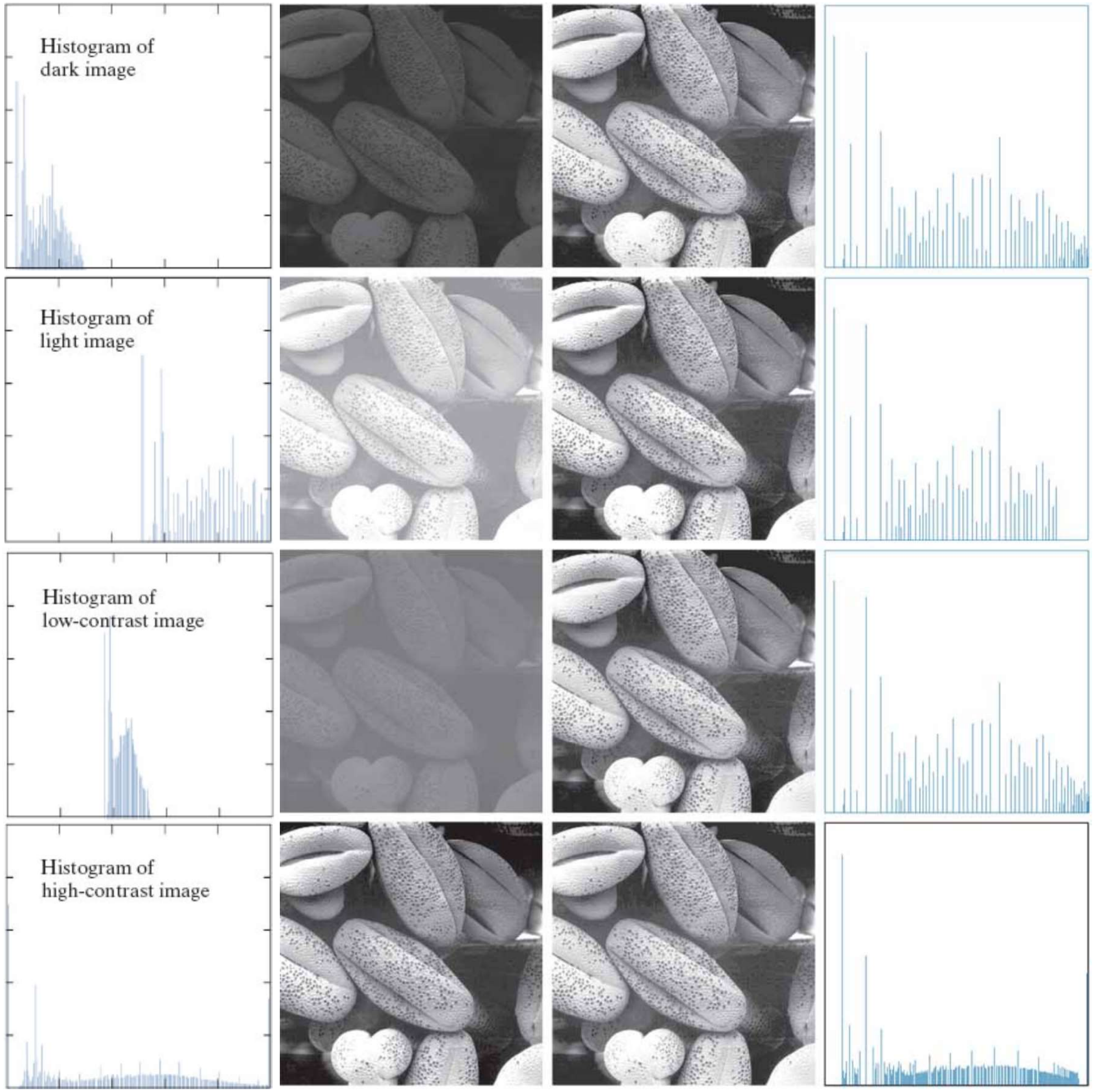
r_k	n_k	$P_r(r_k$	CDF	s_k	s_k		$P_s(s_k)$
0	790	0.19	0.19	Round(7*0.19)=Round(1.33)	1		0.19
1	1023	0.25	0.44	Round(7*0.44)=Round(3.08)	3		0.24
2	850	0.21	0.65	Round(7*0.65)=Round(4.55)	5		0.21
3	656	0.16	0.81	Round(7*0.81)=Round(5.67)	6		
4	329	0.08	0.89	Round(7*0.89)=Round(6.23)	6		0.24
5	245	0.06	0.95	Round(7*0.99)=Round(6.65)	7		
6	122	0.03	0.98	Round(7*0.98)=Round(6.86)	7		0.11
7	81	0.02	1	Round(7*1)=Round(7)	7		

s_k	$P_s(s_k)$
0	0
1	0.19
2	0
3	0.25
4	0
5	0.21
6	0.24
7	0.11

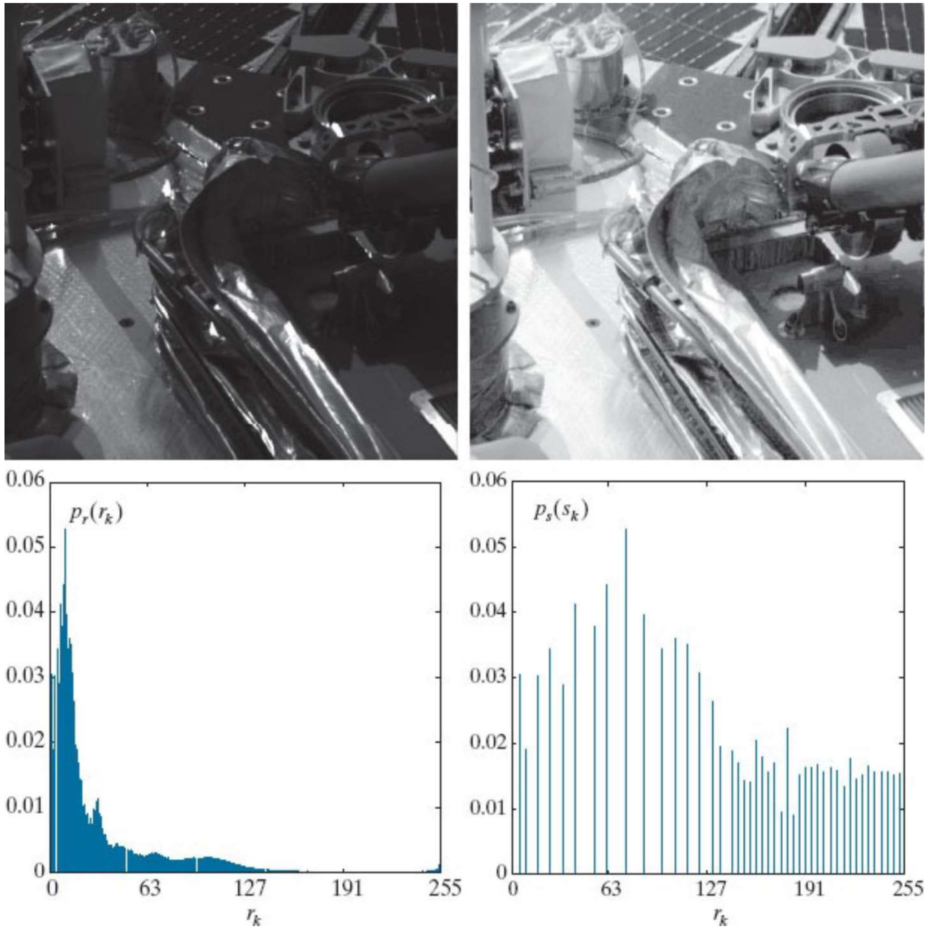
- Although $T^{-1}(s)$ exists and T is strictly monotonically increasing function, Quantization does not help returning back to the original image

Result


255
192
s_k
128
64
0
0
r_k
64
128
192
255
(1)
(2)
(3)
(4)
s
Intensity values of histogram-equalized images
r
Intensity values of original images


Histogram of
dark image
Histogram of
light image
Histogram of
low-contrast image
Histogram of
high-contrast image

Example of Histogram Equalization



Histogram matching

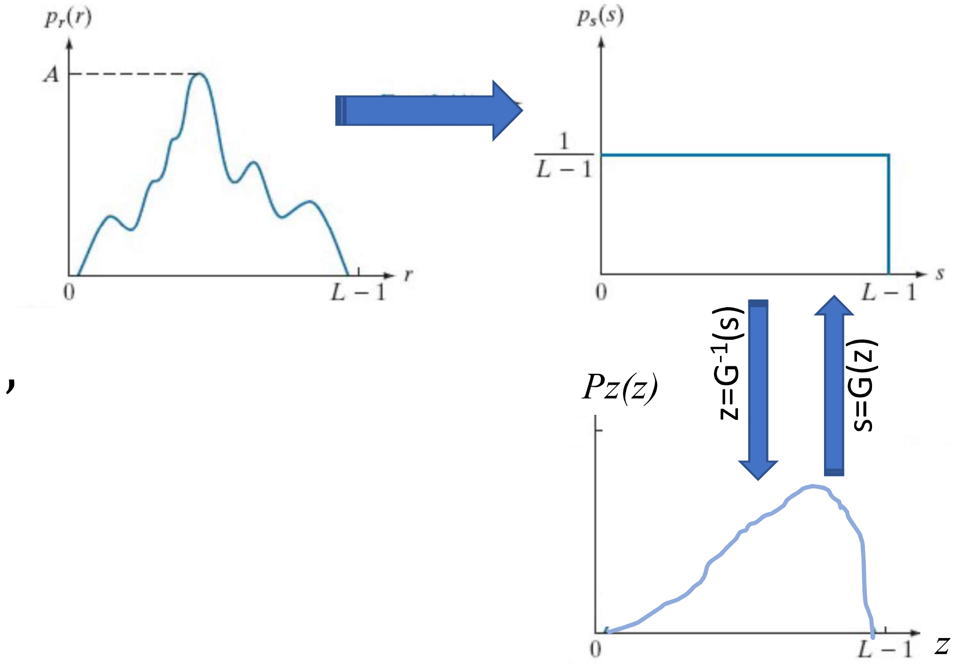
- $s_k = T(r_k) = (L-1)\sum_{j=0}^{k} P_r(r_j),$

  $\mathrm{k} = 0,1,2,\cdots,L-1$

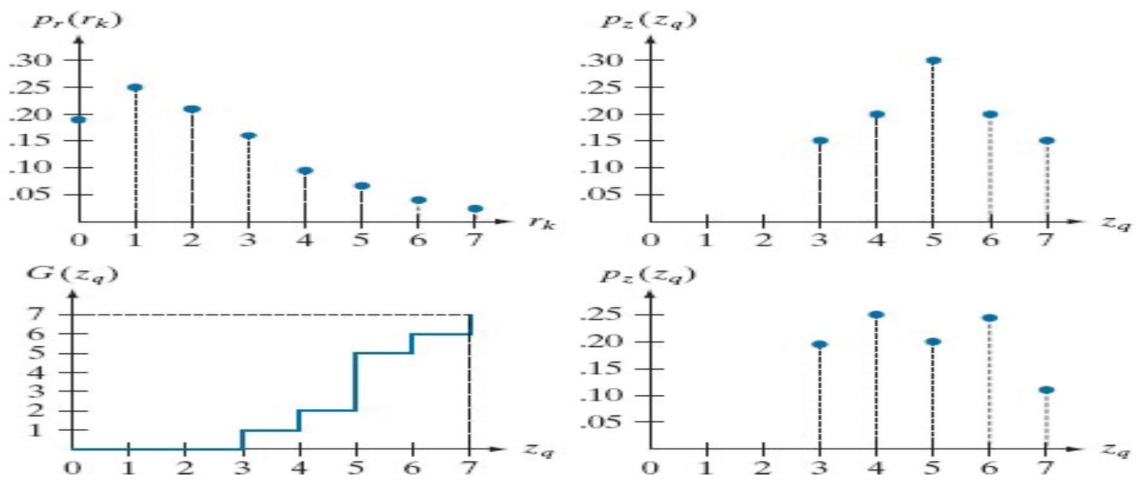
- $s_k = G(z_q) = (L-1)\sum_{i=0}^{q} P_z(z_i)\,,$

  $\mathrm{q} = 0,1,2,\cdots,L-1$

- $z_q = G^{-1}(s_k)$



$$s_k = T(r_k)$$
$$s_k = G(z_q)$$
$$z_q = G^{-1}(s_k) = G^{-1}(T(r_k))$$



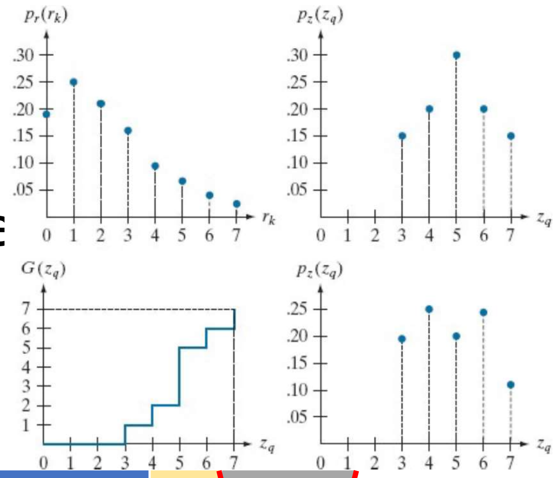
r_k	n_k	$P_r(r_k)$	CDF	s_k	s_k	$P_s(s_k)$
0	790	0.19	0.19	Round(7*0.19)	1	0.19
1	1023	0.25	0.44	Round(7*0.44)	3	0.24
2	850	0.21	0.65	Round(7*0.65)	5	0.21
3	656	0.16	0.81	Round(7*0.81)	6	
4	329	0.08	0.89	Round(7*0.89)	6	0.24
5	245	0.06	0.95	Round(7*0.99)	7	
6	122	0.03	0.98	Round(7*0.98)	7	0.11
7	81	0.02	1	Round(7*1)	7	

z_q	n_q	$P_z(z_q)$	CDF	s_k	s_k	$P_s(s_k)$
0	0	0	0	Round(7*0)	0	0
1	0	0	0	Round(7*0)	0	0
2	0	0	0	Round(7*0)	0	0
3	614	0.15	0.15	Round(7*0.15)	1	0.15
4	819	0.2	0.35	Round(7*0.35)	3	0.2
5	1230	0.3	0.65	Round(7*0.65)	5	0.3
6	819	0.2	0.85	Round(7*0.85)	6	0.2
7	614	0.15	1	Round(7*1)	7	0.15

$$s_k = T(r_k)$$
$$s_k = G(z_q)$$
$$z_q = G^{-1}(s_k) = G^{-1}(T(r_k))$$

Steps of getting the histogram of the resultant image

1) remove last column from B
2) take 1st and last columns form B
3) take last 2 columns from A



r_k	n_k	$P_r(r_k)$	CDF	s_k	s_k	$P_s(s_k)$
0	790	0.19	0.19	Round(7*0.19)	1	0.19
1	1023	0.25	0.44	Round(7*0.44)	3	0.24
2	850	0.21	0.65	Round(7*0.65)	5	0.21
3	656	0.16	0.81	Round(7*0.81)	6	
4	329	0.08	0.89	Round(7*0.89)	6	0.24
5	245	0.06	0.95	Round(7*0.99)	7	
6	122	0.03	0.98	Round(7*0.98)	7	0.11
7	81	0.02	1	Round(7*1)	7	

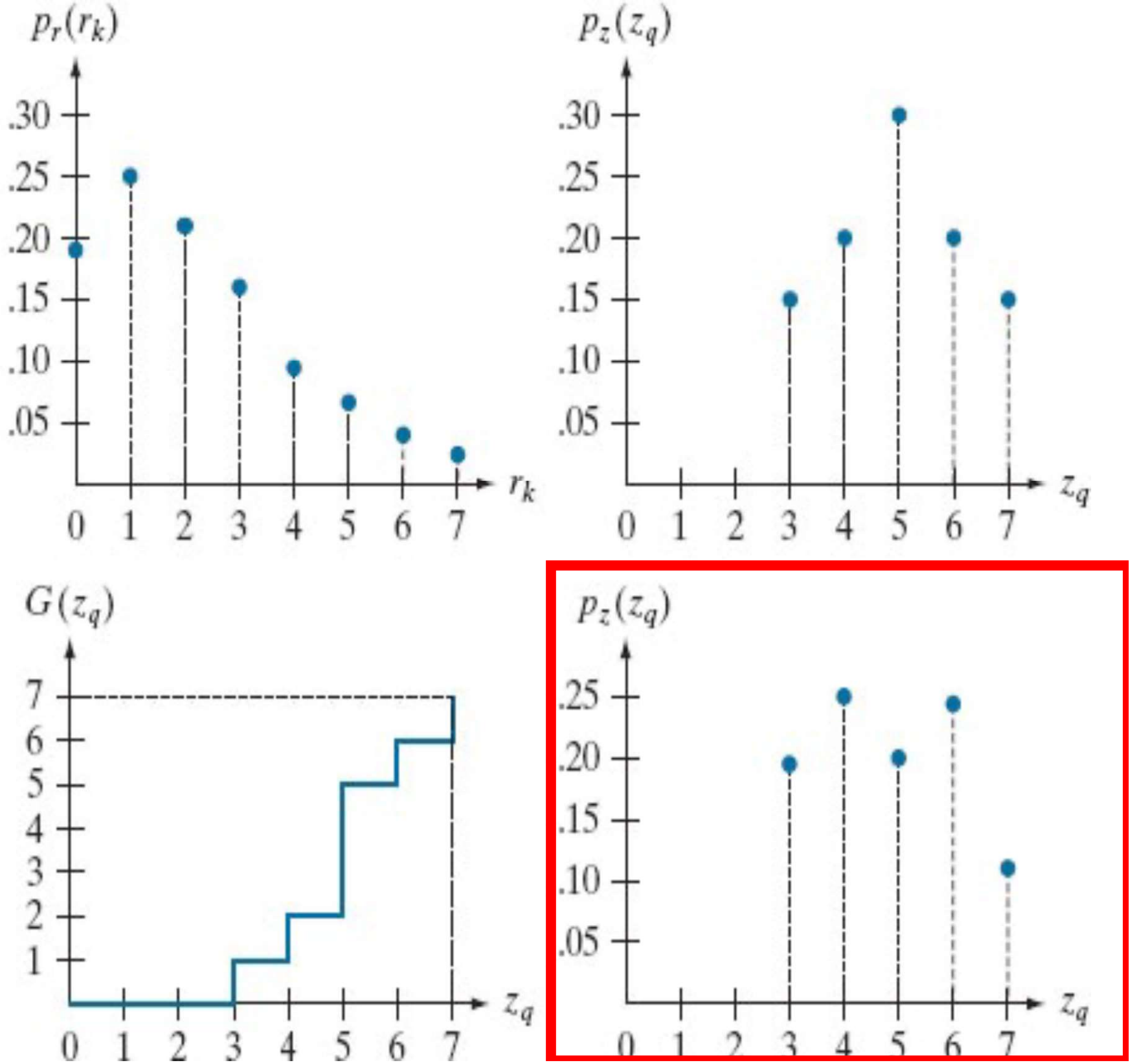
z_q	n_q	$P_z(z_q)$	CDF	s_k	s_k	$P_s(s_k)$
0	0	0	0	Round(7*0)	0	0
1	0	0	0	Round(7*0)	0	0
2	0	0	0	Round(7*0)	0	0
3	614	0.15	0.15	Round(7*0.15)	1	0.15
4	819	0.2	0.35	Round(7*0.35)	3	0.2
5	1230	0.3	0.65	Round(7*0.65)	5	0.3
6	819	0.2	0.85	Round(7*0.85)	6	0.2
7	614	0.15	1	Round(7*1)	7	0.15

Histogram of resultant image

z_q	s_k
0	0
1	0
2	0
3	1
4	3
5	5
6	6
7	7

s_k	$P_s(s_k)$
1	0.19
3	0.24
5	0.21
6	
6	0.24
7	
7	0.11
7	

z_q	$P_z(z_q)$
0	0
1	0
2	0
3	0.19
4	0.24
5	0.21
6	0.24
7	0.11



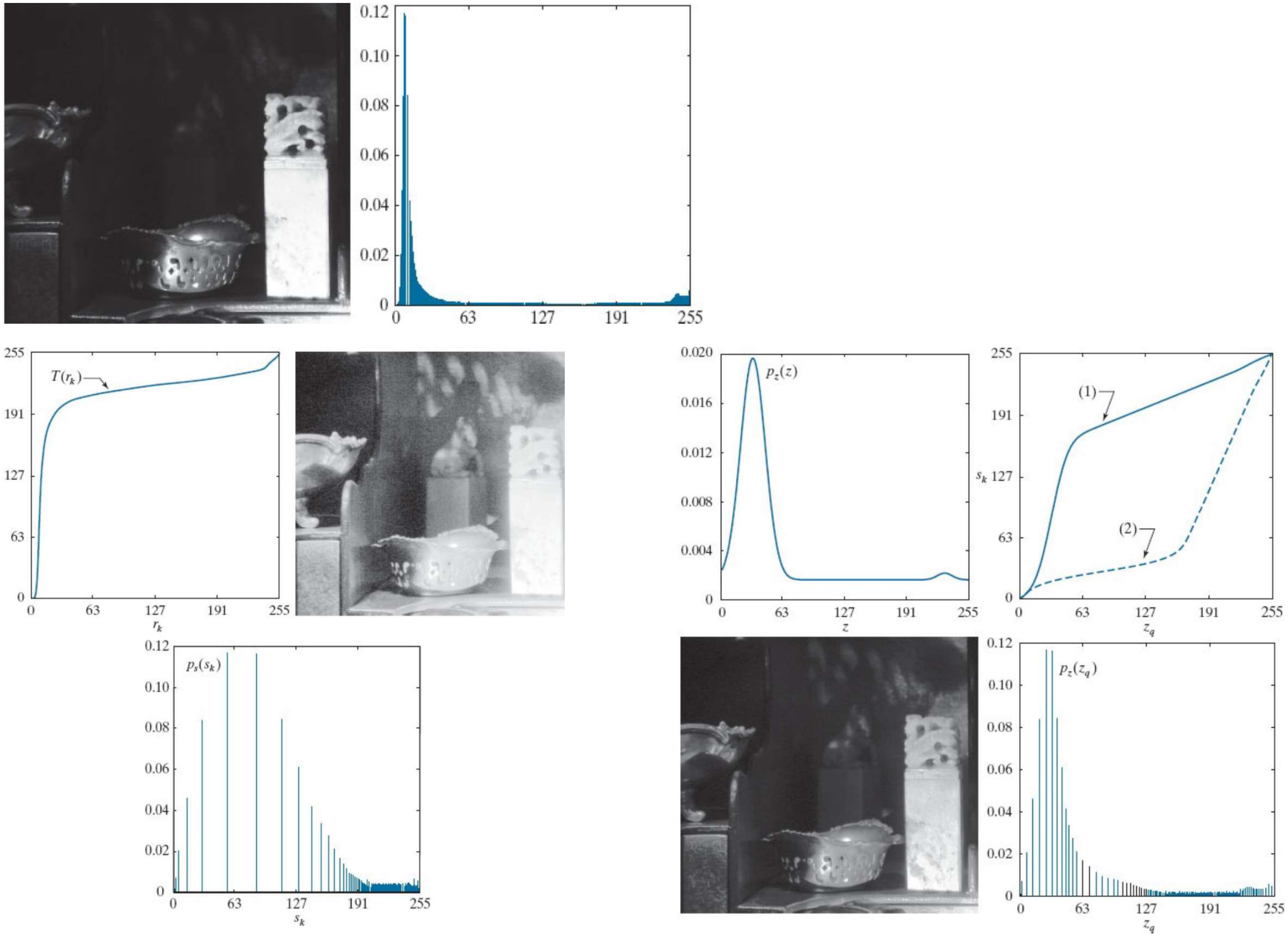
r_k	n_k	$P_r(r_k)$	CDF	s_k	s_k	~~$P_s(s_k)$~~
0	790	0.19	0.19	Round(7*0.19)	1	~~0.19~~
1	1023	0.25	0.44	Round(7*0.44)	3	~~0.24~~
2	850	0.21	0.65	Round(7*0.65)	5	~~0.21~~
3	656	0.16	0.81	Round(7*0.81)	6	
4	329	0.08	0.89	Round(7*0.89)	6	~~0.24~~
5	245	0.06	0.95	Round(7*0.99)	7	
6	122	0.03	0.98	Round(7*0.98)	7	~~0.11~~
7	81	0.02	1	Round(7*1)	7	

z_q	n_q	$P_z(z_q)$	CDF	s_k	s_k	~~$P_s(s_k)$~~
0	0	0	0	Round(7*0)	0	~~0~~
1	0	0	0	Round(7*0)	0	~~0~~
2	0	0	0	Round(7*0)	0	~~0~~
3	614	0.15	0.15	Round(7*0.15)	1	~~0.15~~
4	819	0.2	0.35	Round(7*0.35)	3	~~0.2~~
5	1230	0.3	0.65	Round(7*0.65)	5	~~0.3~~
6	819	0.2	0.85	Round(7*0.85)	6	~~0.2~~
7	614	0.15	1	Round(7*1)	7	~~0.15~~

r_k	s_k
0	1
1	3
2	5
3	6
4	6
5	7
6	7
7	7

s_k	z_q
0	0
0	1
0	2
1	3
3	4
5	5
6	6
7	7

r_k	z_q
0	3
1	4
2	5
3	6
4	6
5	7
6	7
7	7



0.12
0.10
0.08
0.06
0.04
0.02
0
0
63
127
191
255
$T(r_k)$
r_k
$p_z(z)$
0.020
0.016
0.012
0.008
0.004
z
(1)
(2)
s_k
z_q
$p_s(s_k)$
$p_z(z_q)$

Notes on Matching

- Discrete histogram equalization and specification do not produce exact specified histogram **because:**
  - Intensities distribution does not match the desired distribution.
  - Rounding could cause merging two intensities into one.

Assignment #4

- Using the GUI you created in assignment 1:

1. After browsing for an image in a new tab, you are required to:
    1. Display the image
    2. Calculate (from scratch) its normalized histogram and display it
    3. Perform histogram equalization (from scratch)
    4. Display the equalized image
    5. Calculate and display the equalized image's normalized histogram
2. Attached with the statement are sample images to use for equalization
3. You are not allowed to use built-in libraries for histogram and histogram equalization. The code should be written from scratch