1 Gravity Before Relativity

In the first session we talked about the history of gravity and at the end we briefly discuss Mach's ideas on the physical structure of space and time. I followed the first chapter of [1] for the historical part and [2] for Mach's principle.

- For axiomatic approach to Newton's laws I recommend you to visit this page.
- One may think that a sufficiently far point in a rotating disk have a velocity higher than c and that violates special relativity. The point is that you need and infinite amount of energy to speed-up such a disk. Also, this rotation would cause an infinite force acting on a infinitesimal part of the the disk with $v = c$. To see this, we can calculate the proper time of an arbitrary point on the disk. Assume that (t', x', y', z') is the coordinate label of rotating frame.

$$t' = t$$
$$r' = r$$
$$\phi' = \phi - \omega t$$
$$z' = z$$
$$g'_{\mu\nu} = \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu} \eta_{\alpha\beta}$$

  the proper length in rotating frame is

$$ds^2 = (1 - \frac{\omega^2 r'^2}{c^2}) c^2 dt'^2 - 2\omega r'^2 d\phi' dt' - dz'^2 - dr'^2 - r'^2 d\phi'^2$$

  so the proper time of an arbitrary observer at distance r from origin is

$$d\tau = \sqrt{1 - \frac{\omega^2 r^2}{c^2}} dt$$

  Also, the geodesic equation tells us that $\frac{d^2x}{d\tau^2}$ and $\frac{d^2y}{d\tau^2}$ contain a term proportional to $\frac{dt'}{d\tau}$. This means the acceleration is infinite in a point with $r = c/\omega$.

- It was proposed that Hoofman experiment may support Mach's ideas. It seems that this experiment demonstrate frame-dragging effect and this can by fully understood by general relativity using Kerr metric. Therefore, it doesn't justify a "new" interpretation of gravity. You can visit this page for more information about frame-dragging effect.

2 Equivalence Principle

I used the third chapter of [1] and [3] for the statements and consequences of equivalence principle. It is shown in section 2.4 of [4] that why equivalence principle forbid us to describe gravity as an electromagnetic field theory. At the end, Pouya Farokhi talked about different versions of equivalence principle. This talk was based on [5].

3 Manifolds and Tensor Fields I

The basic definitions and theorems of general topology and manifolds is discussed. We introduced tangent space and proved "Pasting Lemma" and arrived at the beginning of topological and differential structure on tangent bundle. I followed part D, chapter 4 of [6].

References

[1] Steven Weinverg. *Gravitation and Cosmology.* Wiley, 1972.

[2] Bahram Mashhon. *Video Lectures on General Theory of Relativity.* Maktabkhone.

[3] Sean Carroll. *Spacetime and Geometry: An Introduction to General Relativity.* Cambridge University Press, 2004.

[4] Norbert Straumann. *General Relativity.* Springer, 2012.

[5] Eolo Di Casola, Stefano Liberati, and Sebastiano Sonego. Nonequivalence of equivalence principles. arXiv:1410.5093.

[6] Siavash Shahshahani. *An Introductory Course on Differentiable Manifolds.* Dover Publications, 2016.