Foundations of General Relativity

The Principle of General Covariance (Invariance?) and Mach's Principle

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Contents

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- summer and fall of 1915: The "Entwurf" theory yielded wrong predictions for the anomalous motion of the mercury. He therefore dropped the theory. Returning to the thinking of 1912 and 1913.

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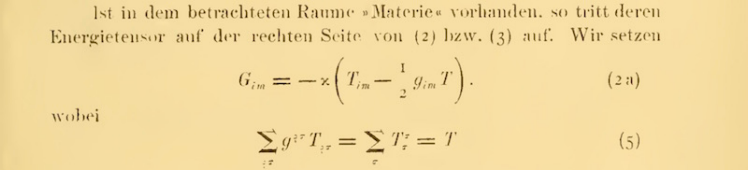
- November 25, 1915: and finally... the corrrect field equation!


Ist in dem betrachteten Raume »Materie« vorhanden, so tritt deren Energietensor auf der rechten Seite von (2) bzw. (3) auf. Wir setzen

$$G_{im} = -\varkappa\left(T_{im} - \frac{1}{2} g_{im} T\right). \qquad (2a)$$

wobei

$$\sum_{\rho\sigma} g^{\rho\sigma} T_{\rho\sigma} = \sum_{\sigma} T_{\sigma}^{\sigma} = T \qquad (5)$$

Figure 1: The apperance of field equations for the first time

General Covariance

General Covariance: The Hole Argument

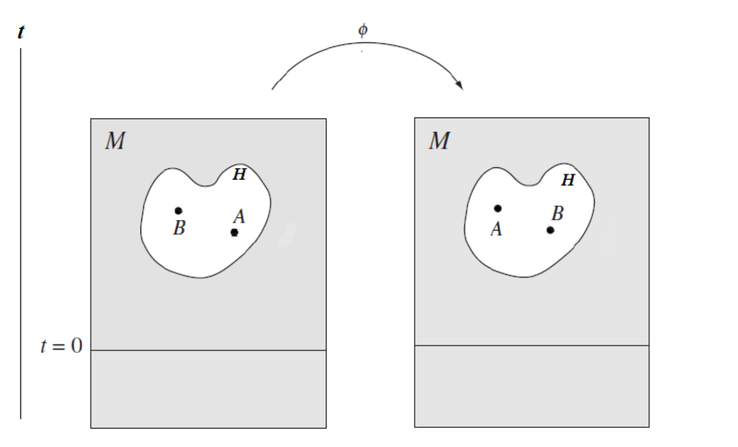


Figure 2: The hole argument

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- Therefore the theory seems non-deterministic.
- Without answering to this argument, Einstein ignored it in 1915 and arrived at the correct generally covariant field equations. What happened to this argument after all?

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- *"All our space-time verifications invariably amount to a determination of space-time coincidences. If, for example, events consisted merely in the motion of material points, then ultimately nothing would be observable but the meetings of two or more of these points. Moreover, the results of our measurings are nothing but verifications of such meetings of the material points of our measuring instruments with other material points, coincidences between the hands of a clock and points on the clock dial, and observed point-events happening at the same place and the same time." (Einstein, 1916)*

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- Thus te physical content of the theory is all about the conincidences between events and such transformations preserve them. Therefore the two diffeomorphically equivalent models we presented represent the same physical content.

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  Principle of Relativity: The laws of nature are only assertions of timespace coincidences; therefore they find their unique, natural expression in generally covariant equations.
- But he was more rebellious than that! He tried to find some physical meaning to general covariance.

Attempts to add Physical Content to General Covariance

— Einstein added the requirement for simplicity.
"Of two theoretical systems compatible with experience, the one is to be preferred that is the simpler and more transparent from the standpoint of the absolute differential calculus. Let one bring Newtonian gravitational mechanics into the form of absolutely covariant equations (four-dimensional) and one will certainly be convinced that [this] principle excludes this theory, not theoretically, but practically!"
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- There is scant historical evidence to justify such a principle.

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- For example, in Newton-Cartan theory which is the generally covariant formulation of Newtonian gravity, we add an absolute time one-form dt representing the absolute time.
- But it's not a well articulated principle and it's not clear that in what sense these new structures are physically baseless.

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- Spacetime looks exactly the same from each of the frames related by such a symmetry transformation.

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- Anderson divides all the geometrical structures of a theory into the absolute $A_1, A_2, \ldots$ and dynamical $D_1, D_2, \ldots$ objects. Thus a model can be written as $(M, A_1, A_2, \ldots | D_1, D_2, \ldots)$.

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- What truely distinguishes general relativity among all generally covariant formulations? This is still a problem.

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- More generally, one can define $G - invariance$ for some $G \subseteq Diff(M)$. But note that if the symmetry group of the fixed fields of a theory is G, then the theory would be $G - invariant$. But it is not true the other way round!

General Covariance Versus General Invariance: Examples

- The standard formulation of special relativity is not generally covariant. But such a formulation exists:

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— But it remains unclear whether this formulation is generally invariant. Some regard the second equation as a dynamical equation and do not count the metric as an absolute structure (Pooley). Some others don't. Friedman's criteria strongly suggests to take the metric as a background structure that the theory is not diffeomorphism invariant.

General Covariance Versus General Invariance: Examples

— Consider the ordinary non-relativistic diffusion equation for a $\mathcal{R}$-value field ϕ:

$$\frac{\partial}{\partial t}\phi = \kappa\Delta\phi$$

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- Again some equations can be suggested for n^μ and $g^{\mu\nu}$ that renders the state of diffeomorphism invariance uncertain.

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- after 1918: He continued emphasising on Machian-seeming effects such as frame-dragging and also demanded that at least one model satisfies the principle rather than the whole theory. He gradually became discouraged with Mach’s principle.

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  - Various people, including Einstein interpreted Mach's ideas as a demand for a new physics. But Mach never correct any one of them. Einstein and Mach exchanged several letters and Mach praised Einstein's revolutionary ideas!

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- Einstein's letter to Felix Pirani (2 February 1954): *"One shouldn't talk at all any longer of Mach's principle, in my opinion. It arose at a time when one thought that 'ponderable bodies' were the only physical reality and that in a theory all elements that are fully determined by them should be conscientiously avoided. I am quite aware of the fact that for a long time, I, too, was influenced by this fixed idea."*

Julian Barbour criticizes this view. Machianity should be stated independent from the underlying ontology.

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- The definition of Mach's Principle (2010): a) direct observations or theoretical considerations suggest that the physical configuration space $\mathcal{Q}$ of a closed dynamical system is to be obtained by a group quotienting of a larger configuration space. b) specification of initial point $q \in \mathcal{Q}$ together with a *direction* or *tangent vector* d at q defines a uniqe dynamical curve in $\mathcal{Q}$.

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- Shape dynamics is a theory that describes the evolution of 3-dimensional conformally equivalent Riemannian metrics through time. It is equivalent to general relativity as a specific gauge fixing of ADM formalism called CMC gauge condition.
- Although Mach's principle created many conceptual problems regarding the development of general relativity, today's Machian programme is very insightful and well-defined.

Thank You!

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