جواب سوالات Homwork 6

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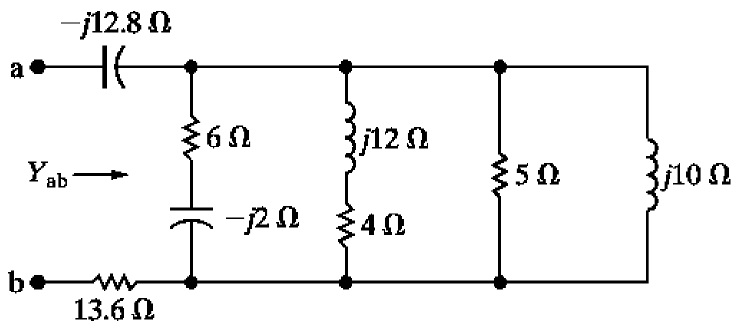

First find the admittance of the parallel branches

$$Y_p = \frac{1}{6-j2} + \frac{1}{4+j12} + \frac{1}{5} + \frac{1}{j10} = 0.375 - j0.125\,\mathrm{S}$$

$$Z_p = \frac{1}{Y_p} = \frac{1}{0.375 - j0.125} = 2.4 + j0.8\,\Omega$$

$$Z_{\mathrm{ab}} = -j12.8 + 2.4 + j0.8 + 13.6 = 16 - j12\,\Omega$$

$$Y_{\mathrm{ab}} = \frac{1}{Z_{\mathrm{ab}}} = \frac{1}{16-j12} = 0.04 + j0.03\,\mathrm{S}$$

$$= 40 + j30\,\mathrm{mS} = 50\underline{/36.87^\circ}\,\mathrm{mS}$$

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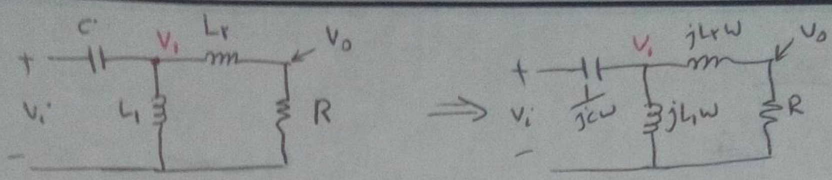


KCL در گره V_1 $\Rightarrow \dfrac{V_1 - V_i}{\frac{1}{jC\omega}} + \dfrac{V_1}{jL_1\omega} + \dfrac{V_1 - V_o}{jL_2\omega} = 0 \qquad (1)$

KCL در گره ۲ $\rightarrow \dfrac{V_o}{R} + \dfrac{V_o - V_1}{jL_2\omega} = 0 \rightarrow V_1 = \dfrac{R + jL_2\omega}{R} V_o \qquad (2)$

(۲) در (۱) $\Rightarrow H = \dfrac{V_o}{V_i} = \dfrac{-\omega^2 R L_1 C}{R(1-\omega^2 L_1 C) + j\omega(L_1 + L_2 - \omega^2 L_1 L_2 C)}$

صورت و مخرج را در مزدوج مخرج ضرب می‌کنیم:

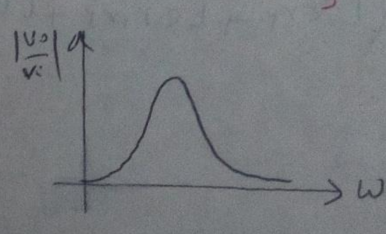
$$\frac{V_o}{V_i} = \frac{-\omega^2 R L_1 C\left[R(1-\omega^2 L_1 C) - j\omega(L_1 + L_2 - \omega^2 L_1 L_2 C)\right]}{R^2(1-\omega^2 L_1 C)^2 + \omega^2(L_1 + L_2 - \omega^2 L_1 L_2 C)^2}$$

$$\frac{V_o}{V_i} = \frac{-\omega^2 R^2 L_1 C(1-\omega^2 L_1 C) + j\omega^3 R L_1 C(L_1 + L_2 - \omega^2 L_1 L_2 C)}{R^2(1-\omega^2 L_1 C)^2 + \omega^2(L_1 + L_2 - \omega^2 L_1 L_2 C)^2}$$

$$\left|\frac{V_o}{V_i}\right| = \frac{\sqrt{\left[-\omega^2 R^2 L_1 C(1-\omega^2 L_1 C)\right]^2 + \left[\omega^3 R L_1 C(L_1 + L_2 - \omega^2 L_1 L_2 C)\right]^2}}{\sqrt{\left[R^2(1-\omega^2 L_1 C)^2 + \omega^2(L_1 + L_2 - \omega^2 L_1 L_2 C)^2\right]^2}}$$

$\omega \rightarrow 0 \qquad \left|\dfrac{V_o}{V_i}\right| = 0$

$\omega \rightarrow \infty \qquad \left|\dfrac{V_o}{V_i}\right| = 0$



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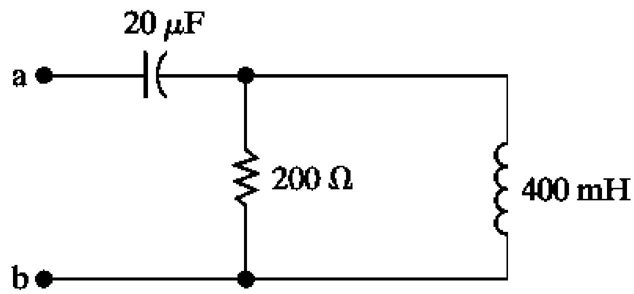

[a] $$\frac{1}{j\omega C} + R \| j\omega L = \frac{1}{j\omega C} + \frac{j\omega RL}{j\omega L + R}$$

$$= \frac{j\omega L + R - \omega^2 RLC}{j\omega C(j\omega L + R)}$$

$$= \frac{(R - \omega^2 RLC + j\omega L)(-\omega^2 LC - j\omega RC)}{(-\omega^2 LC + j\omega RC)(-\omega^2 LC - j\omega RC)}$$

The denominator in the expression above is purely real; set the imaginary part of the numerator in the above expression equal to zero and solve for ω:

$$-\omega^3 L^2 C - \omega R^2 C + \omega^3 R^2 C^2 L = 0$$

$$-\omega^2 L^2 - R^2 + \omega^2 R^2 LC = 0$$

$$\omega^2 = \frac{R^2}{R^2 LC - L^2} = \frac{200^2}{200^2(0.4)(20 \times 10^{-6}) - (0.4)^2} = 250{,}000$$

$$\therefore \qquad \omega = 500 \text{ rad/s}$$

[b] $$Z_{\text{ab}}(500) = -j100 + \frac{(200)(j200)}{200 + j200} = 100\,\Omega$$

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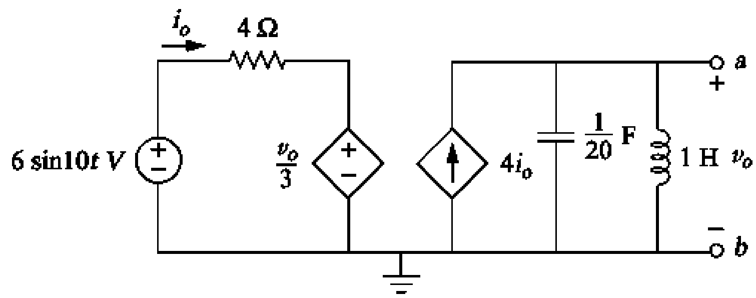
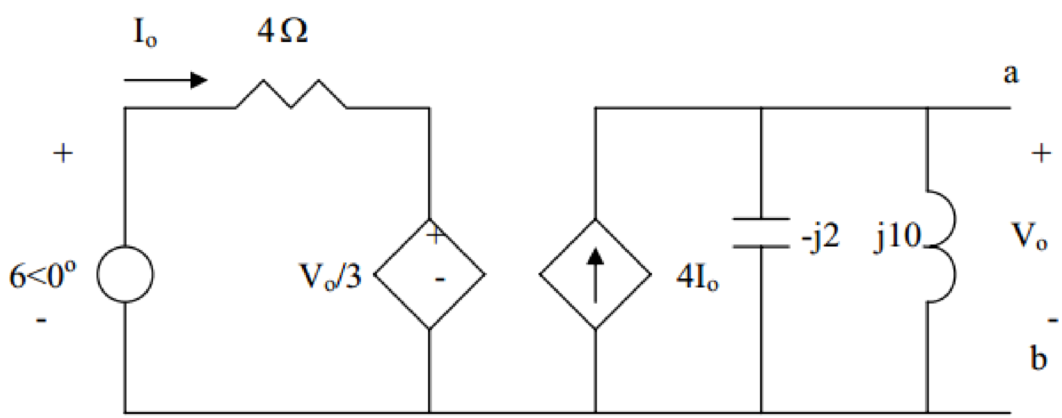


$$1\text{H} \longrightarrow j\omega L = j10x1 = j10$$

$$\frac{1}{20}\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j10x\frac{1}{20}} = -j2$$

We obtain V_{Th} using the circuit below.



$$j10//(-j2) = \frac{j10(-j2)}{j10 - j2} = -j2.5$$

$$V_o = 4I_o x(-j2.5) = -j10I_o \qquad (1)$$

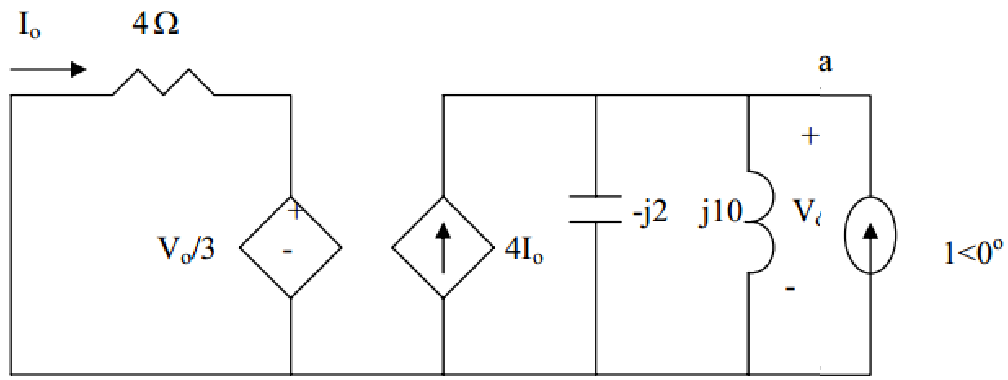
$$-6 + 4I_o + \frac{1}{3}V_o = 0 \qquad (2)$$

Combining (1) and (2) gives

$$I_o = \frac{6}{4 - j10/3}, \qquad V_{Th} = V_o = -j10I_o = \frac{-j60}{4 - j10/3} = 11.52\angle -50.19^o$$

$$\underline{v_{Th} = 11.52\sin(10t - 50.19^o)}$$

To find $R_{Th,}$ we insert a 1-A source at terminals a-b, as shown below.



$$4I_o + \frac{1}{3}V_o = 0 \quad \longrightarrow \quad I_o = -\frac{V_o}{12}$$

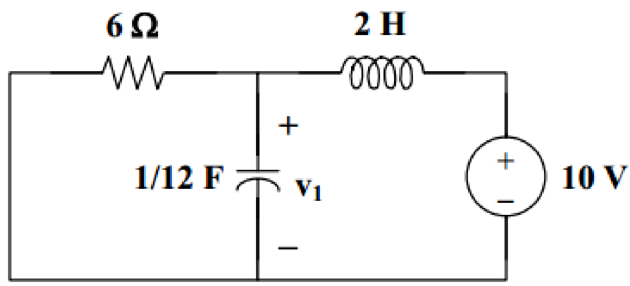
$$1 + 4I_o = \frac{V_o}{-j2} + \frac{V_o}{j10}$$

Combining the two equations leads to

$$V_o = \frac{1}{0.333 + j0.4} = 1.2293 - j1.4766$$

$$Z_{Th} = \frac{V_o}{1} = 1.2293 - j1.4766$$

Let $v_o = v_1 + v_2 + v_3$, where v_1, v_2, and v_3 are respectively due to the 10-V dc source, the ac current source, and the ac voltage source. For v_1 consider the circuit in Fig. (a).



(a)

The capacitor is open to dc, while the inductor is a short circuit. Hence,

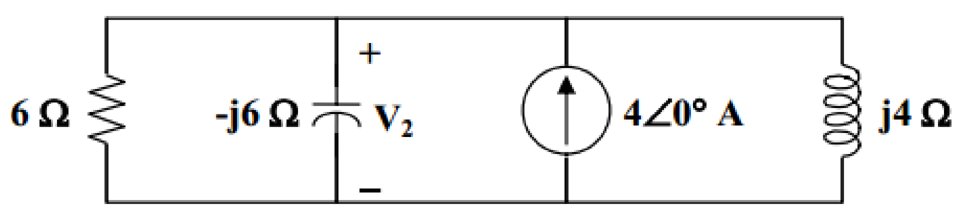
$$v_1 = 10 \text{ V}$$

For v_2, consider the circuit in Fig. (b).

$$\omega = 2$$

$$2 \text{ H} \longrightarrow j\omega L = j4$$

$$\frac{1}{12}\text{F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(2)(1/12)} = \text{-}j6$$



(b)

Applying nodal analysis,

$$4 = \frac{\mathbf{V}_2}{6} + \frac{\mathbf{V}_2}{\text{-}j6} + \frac{\mathbf{V}_2}{j4} = \left(\frac{1}{6} + \frac{j}{6} - \frac{j}{4}\right)\mathbf{V}_2$$

$$\mathbf{V}_2 = \frac{24}{1 - j0.5} = 21.45\angle 26.56°$$

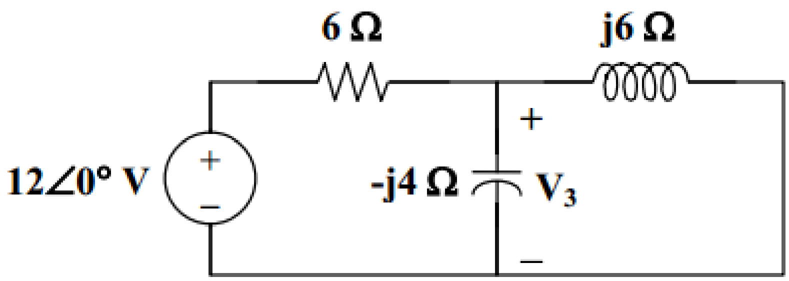
Hence, $$v_2 = 21.45\sin(2t + 26.56°) \text{ V}$$

For v_3, consider the circuit in Fig. (c).

$$\omega = 3$$

$$2\text{ H} \longrightarrow j\omega L = j6$$

$$\frac{1}{12}\text{ F} \longrightarrow \frac{1}{j\omega C} = \frac{1}{j(3)(1/12)} = -j4$$



(c)

At the non-reference node,

$$\frac{12 - \mathbf{V}_3}{6} = \frac{\mathbf{V}_3}{-j4} + \frac{\mathbf{V}_3}{j6}$$

$$\mathbf{V}_3 = \frac{12}{1 + j0.5} = 10.73\angle -26.56°$$

Hence, $v_3 = 10.73\cos(3t - 26.56°)$ V

Therefore, v_o = <u>**10 + 21.45 sin(2t + 26.56°) + 10.73 cos(3t – 26.56°) V**</u>

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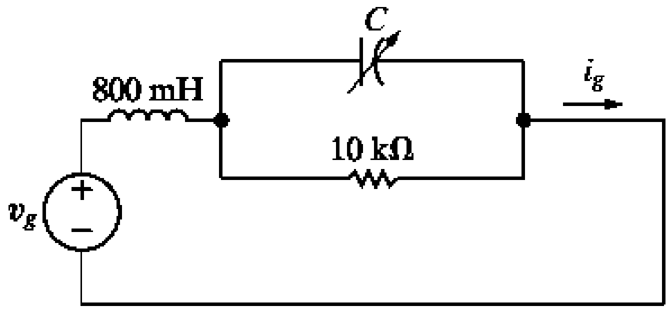

$$\textbf{[a]}\quad Z_p = \frac{\dfrac{R}{j\omega C}}{R + (1/j\omega C)} = \frac{R}{1 + j\omega RC}$$

$$= \frac{10{,}000}{1 + j(5000)(10{,}000)C} = \frac{10{,}000}{1 + j50 \times 10^6 C}$$

$$= \frac{10{,}000(1 - j50 \times 10^6 C)}{1 + 25 \times 10^{14} C^2}$$

$$= \frac{10{,}000}{1 + 25 \times 10^{14} C^2} - j\frac{5 \times 10^{11} C}{1 + 25 \times 10^{14} C^2}$$

$$j\omega L = j5000(0.8) = j4000$$

$$\therefore \quad 4000 = \frac{5 \times 10^{11} C}{1 + 25 \times 10^{14} C^2}$$

$$\therefore \quad 10^{14} C^2 - 125 \times 10^6 C + 1 = 0$$

$$\therefore \quad C^2 - 5 \times 10^{-8} C + 4 \times 10^{-16} = 0$$

Solving,

$C_1 = 40\,\text{nF} \qquad\qquad C_2 = 10\,\text{nF}$

[b] $R_e = \dfrac{10{,}000}{1 + 25 \times 10^{14} C^2}$

When $C = 40\,\text{nF} \qquad R_e = 2000\,\Omega;$

$$\mathbf{I}_g = \frac{80\underline{/0^\circ}}{2000} = 40\underline{/0^\circ}\,\text{mA}; \qquad i_g = 40\cos 5000t\,\text{mA}$$

When $C = 10\,\text{nF} \qquad R_e = 8000\,\Omega;$

$$\mathbf{I}_g = \frac{80\underline{/0^\circ}}{8000} = 10\underline{/0^\circ}\,\text{mA}; \qquad i_g = 10\cos 5000t\,\text{mA}$$