جواب تمرینات سری چهارم

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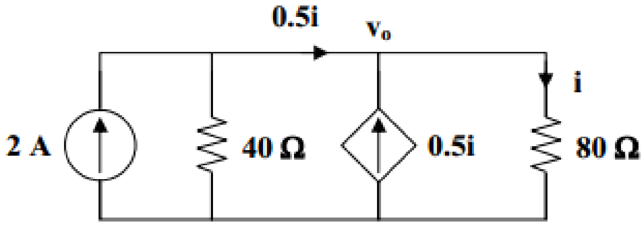
Before t = 0, the circuit has reached steady state so that the capacitor acts like an open circuit. The circuit is equivalent to that shown in Fig. (a) after transforming the voltage source.



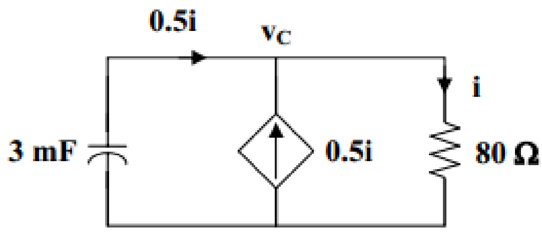
(a)

$$0.5i = 2 - \frac{v_o}{40}, \qquad i = \frac{v_o}{80}$$

Hence, $\frac{1}{2}\frac{v_o}{80} = 2 - \frac{v_o}{40} \longrightarrow v_o = \frac{320}{5} = 64$

$$i = \frac{v_o}{80} = \underline{\mathbf{0.8\ A}}$$

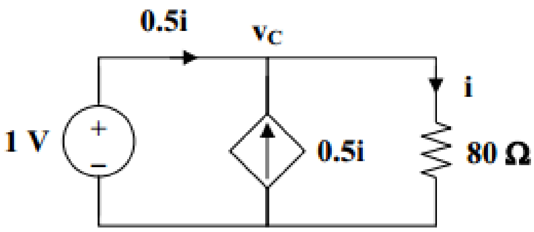
After t = 0, the circuit is as shown in Fig. (b).



(b)

$$v_C(t) = v_C(0)e^{-t/\tau}, \qquad \tau = R_{th}C$$

To find R_{th}, we replace the capacitor with a 1-V voltage source as shown in Fig. (c).



(c)

$$i = \frac{v_C}{80} = \frac{1}{80}, \qquad i_o = 0.5i = \frac{0.5}{80}$$

$$R_{th} = \frac{1}{i_o} = \frac{80}{0.5} = 160\,\Omega, \qquad \tau = R_{th}C = 480$$

$$v_C(0) = 64\text{ V}$$

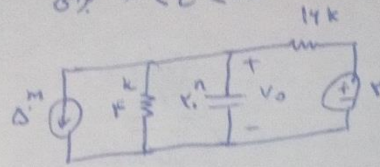
$$v_C(t) = 64e^{-t/480}$$

$$0.5i = -i_C = -C\frac{dv_C}{dt} = -3\left(\frac{1}{480}\right)64e^{-t/480}$$

$$i(t) = \underline{0.8e^{-t/480}\,u(t)\text{A}}$$

-۳

برای $10^{ms} < t < \infty$

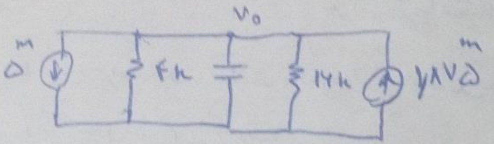


چون ولتاژ خازن تغییر آنی ندارد پس ولتاژ خازن در $V_o(10^-)$ برابر است با ولتاژ خازن در $V_o(10^+)$ پس داریم:

(از معادله V_o از بازه $0<t<10^m$ بدست می‌آید)

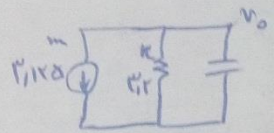
$$V_o(10) = -20\left(1 - e^{-12500 \times 10^m}\right) = -0.248$$

$V_o(\infty) = ?$



$$R_q = 16 \| 4 = 3.2^k$$

$$I_{eq} = 5 - 1.875 = 3.125^m$$

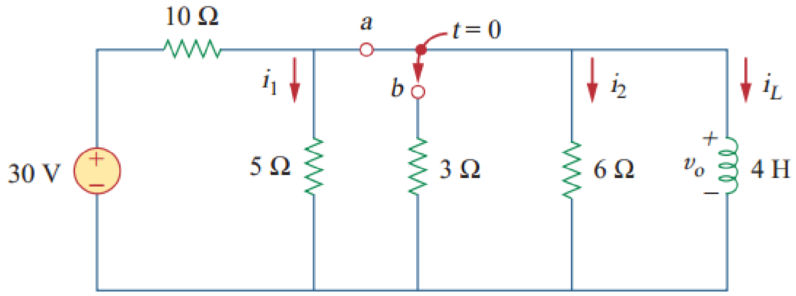


$$V_o(\infty) = -3.2^k \times 3.125^m = -10$$

$$\tau = R_{eq} C = 3.2^k \times 20^n = 64^\mu$$

$$V_o(t) = -10 + (-0.248 + 10)\, e^{-\frac{(t-10^m)}{64^\mu}}$$

$$V_o(t) = \begin{cases} 0 & t<0 \\ -20\left(1-e^{-12500t}\right) & 0<t<10^m \\ -10 + 9.752\, e^{-15625(t-10^m)} & 10^m<t<\infty \end{cases}$$



(a) When the switch is in position A, the 5-ohm and 6-ohm resistors are short-circuited so that

$$\underline{i_1(0) = \;\; i_2(0) = v_o(0) = 0}$$

but the current through the 4-H inductor is $i_L(0) = 30/10 = 3A$.

(b) When the switch is in position B,

$$R_{Th} = 3 // 6 = 2\Omega, \qquad \tau = \frac{L}{R_{Th}} = 4/2 = 2\,\text{sec}$$

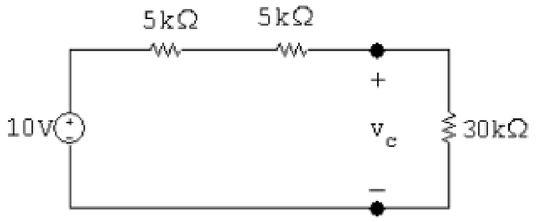
$$i_L(t) = i_L(\infty) + [i_L(0) - i_L(\infty)]e^{-t/\tau} = 0 + 3e^{-t/2} = \underline{3e^{-t/2}\ A}$$

(c) $i_1(\infty) = \dfrac{30}{10+5} = \underline{2\ A}, \qquad i_2(\infty) = -\dfrac{3}{9} i_L(\infty) = \underline{0\ A}$

$$v_o(t) = L\frac{di_L}{dt} \longrightarrow \underline{v_o(\infty) = 0\ V}$$

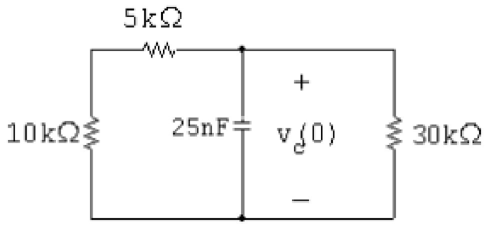
Note that for $t > 0$, $v_o = (10/15)v_c$, where v_c is the voltage across the 25 nF capacitor. Thus we will find v_c first.

$t < 0$



$$v_c(0) = \frac{30}{40}(10) = 7.5\,\text{V}$$

$0 \le t \le 0.2\,\text{ms}$:



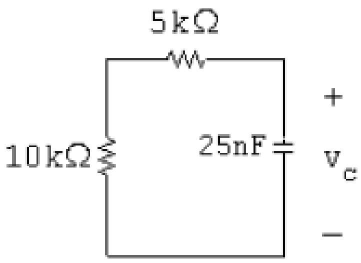
$$\tau = R_eC, \qquad R_e = 15{,}000 \| 30{,}000 = 10\,\text{k}\Omega$$

$$\tau = (10 \times 10^3)(25 \times 10^{-9}) = 0.25\,\text{ms}, \qquad \frac{1}{\tau} = 4000$$

$$v_c = 7.5e^{-4000t}\,\text{V}, \qquad t \ge 0$$

$$v_c(0.2\,\text{ms}) = 7.5e^{-0.8} = 3.37\,\text{V}$$

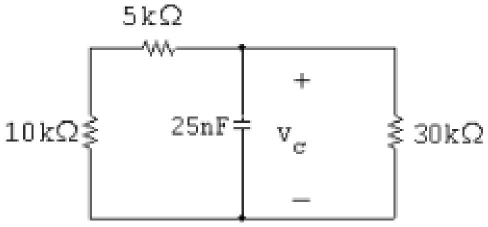
$0.2\,\text{ms} \le t \le 0.8\,\text{ms}$:



$\tau = (15 \times 10^3)(2.5 \times 10^{-9}) = 375\,\mu\text{s}, \qquad \frac{1}{\tau} = 2666.67$

$v_c = 3.37e^{-2666.67(t-200\times10^{-6})}\,\text{V}$

$0.8\,\text{ms} \leq t <$:



$\tau = 0.25\,\text{ms}, \qquad \frac{1}{\tau} = 4000$

$v_c(0.8\,\text{ms}) = 3.37e^{-2666.67(800-200)10^{-6}} = 3.37e^{-1.6} = 0.68\,\text{V}$

$v_c = 0.68e^{-4000(t-0.8\times10^{-3})}\,\text{V}$

$v_c(1\,\text{ms}) = 0.68e^{-4000(1-0.8)10^{-3}} = 0.68e^{-0.8} = 0.306\,\text{V}$

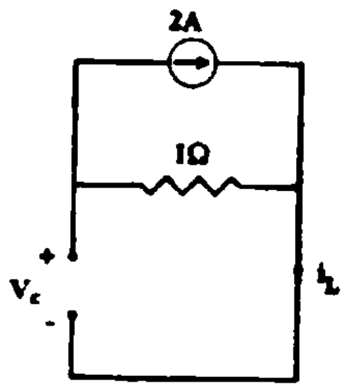
$v_o = (10/15)(0.306) = 0.204\,\text{V}$

۶-

$$\frac{dv_c}{dt}(0^+) = \frac{1}{C} i_c(0^+)$$

$$\frac{di_L}{dt}(0^+) = \frac{1}{L} V_L(0^+)$$

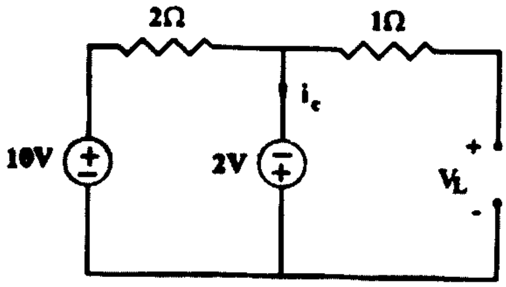
$in \qquad t = 0^-$



$i_L(0) = 0$

$V_c(0) = -2$

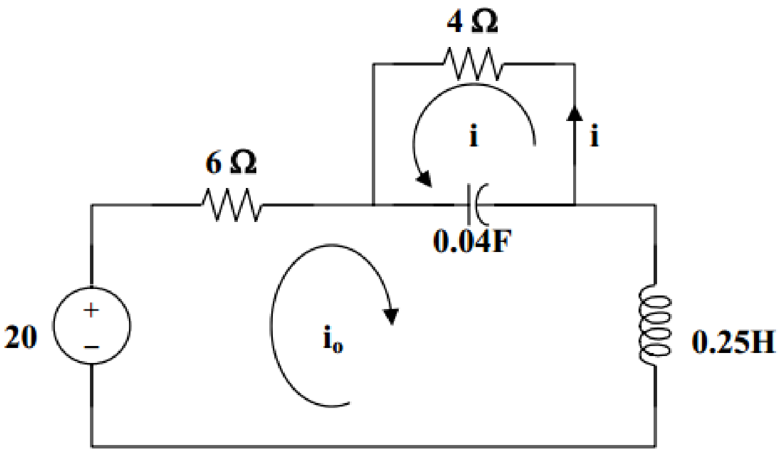
$in \qquad t = 0^+$



$$\begin{matrix} V_L(0^+) = -2V \\ i_C(0^+) = 6A \end{matrix} \Rightarrow \begin{cases} \dfrac{dV_C(0^+)}{dt} = 1 \times 6 = 6 \\ \dfrac{di_L(0^+)}{dt} = \dfrac{1}{1} \times -2 = -2 \end{cases}$$

For $t < 0$, $i(0) = 0$ and $v(0) = 0$.

For $t > 0$, the circuit is as shown below.



Applying KVL to the larger loop,

$$-20 + 6i_o + 0.25di_o/dt + 25\int(i_o + i)dt = 0 \qquad (1)$$

For the smaller loop, $\quad 4i + 25\int(i + i_o)dt = 0$ or $\int(i + i_o)dt = -0.16i \qquad (2)$

Taking the derivative, $\quad 4di/dt + 25(i + i_o) = 0$ or $i_o = -0.16di/dt - i \qquad (3)$

and $\quad di_o/dt = -0.16d^2i/dt^2 - di/dt \qquad (4)$

From (1), (2), (3), and (4), $\quad -20 - 0.96di/dt - 6i - 0.04d^2i/dt^2 - 0.25di/dt - 4i = 0$

Which becomes, $\quad d^2i/dt^2 + 30.25di/dt + 250i = -500$

This leads to, $\quad s^2 + 30.25s + 250 = 0$

$$\text{or } s_{1,2} = \frac{-30.25 \pm \sqrt{(30.25)^2 - 1000}}{2} = -15.125 \pm j4.608$$

This is clearly an underdamped response.

Thus, $i(t) = I_s + e^{-15.125t}(A_1\cos(4.608t) + A_2\sin(4.608t))A$.

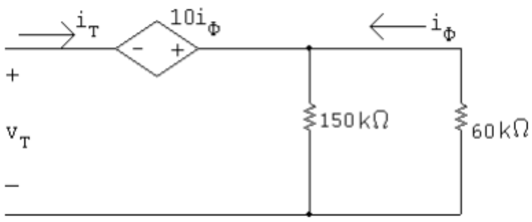
At $t = 0$, $i_o(0) = 0$ and $i(0) = 0 = I_s + A_1$ or $A_1 = -I_s$. As t approaches infinity, $i_o(\infty) = 20/10 = 2A = -i(\infty)$ or $i(\infty) = -2A = I_s$ and $A_1 = 2$.

In addition, from (3), we get $di(0)/dt = -6.25i_o(0) - 6.25i(0) = 0$.

$di/dt = 0 - 15.125\ e^{-15.125t}(A_1\cos(4.608t) + A_2\sin(4.608t)) + e^{-15.125t}(-A_1 4.608\sin(4.608t) + A_2 4.608\cos(4.608t))$. At $t=0$, $di(0)/dt = 0 = -15.125A_1 + 4.608A_2 = -30.25 + 4.608A_2$ or $A_2 = 30.25/4.608 = 6.565$.

This leads to,

$$i(t) = \underline{\mathbf{(-2 + e^{-15.125t}(2\cos(4.608t) + 6.565\sin(4.608t))\ A}}$$



$$v_T = -10i_\phi + i_T\left(\frac{(150)(60)}{210}\right) = -10\frac{-i_T(150)}{210} + i_t\frac{9000}{210}$$

$$\frac{v_T}{i_T} = \frac{1500+9000}{210} = 50\,\Omega$$

$$V_o = \frac{4000}{10{,}000}(50) = 20\,\text{V}; \qquad I_o = 0$$

$$i_C(0) = -i_R(0) - i_L(0) = -\frac{20}{50} = -0.4\,\text{A}$$

$$\frac{i_C(0)}{C} = \frac{-0.4}{8\times10^{-6}} = -50{,}000$$

$$\omega_o = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{(51.2\times10^{-3})(8\times10^{-6})}} = 1562.5\text{ rad/s}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{(2)(50)(8\times10^{-6})} = 1250\text{ rad/s}$$

$\alpha^2 < \omega_0^2$ so the response is underdamped

$$\omega_d = \sqrt{1562.5^2 - 1250^2} = 937.5$$

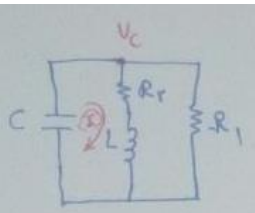
$$v_o = B_1e^{-1250t}\cos 937.5t + B_2e^{-1250t}\sin 937.5t$$

$$v_o(0) = B_1 20\,\text{V}$$

$$\frac{dv_o}{dt}(0) = -\alpha B_1 + \omega_d B_2 = \frac{i_C(0)}{C}$$

$$\therefore \qquad -1250(20) + 937.5B_2 = -50{,}000 \qquad \text{so} \qquad B_2 = -26.67$$

$$v_o = 20e^{-1250t}\cos 937.5t - 26.67e^{-1250t}\sin 937.5t\,\text{V}, \qquad t \ge 0$$



(KCL) $\Rightarrow$ $i_c + i_L + \frac{v_c}{-R_1} = 0$

$$C\frac{dv_c}{dt} + i_L - \frac{v_c}{R_1} = 0 \qquad (1)$$

$(KVL)_I \Rightarrow$ $v_c = R_2 i_L + L\frac{di_L}{dt}$ (2)

۱و۲) $\Rightarrow$ $R_2 C\frac{di_L}{dt} + LC\frac{d^2 i_L}{dt^2} + i_L - \frac{R_2}{R_1} i_L - \frac{L}{R_1}\frac{di_L}{dt} = 0$

$$LC\frac{d^2 i_L}{dt^2} + \left(R_2 C - \frac{L}{R_1}\right)\frac{di_L}{dt} + \left(1 - \frac{R_2}{R_1}\right) i_L = 0$$

$$\frac{d^2 i_L}{dt^2} + \frac{1}{LC}\left(R_2 C - \frac{L}{R_1}\right)\frac{di_L}{dt} + \frac{1}{LC}\left(1 - \frac{R_2}{R_1}\right) i_L = 0$$

شرط نوسانی بودن :
$$\begin{cases} 2\alpha = 0 \\ \omega_0^2 > 0 \end{cases}$$

$2\alpha = \frac{1}{LC}\left(R_2 C - \frac{L}{R_1}\right) = 0$ $\qquad$ $\underline{R_1 = \frac{L}{R_2 C}}$

$\omega_0^2 = \frac{1}{LC}\left(1 - \frac{R_2}{R_1}\right) > 0$ $\qquad$ $\underline{R_1 > R_2}$