DC

Q# 4-10 (a)

$$f_1(t) = \begin{cases} 1/2 & 0<t<2 \\ -1/2 & 2<t<4 \\ 0 & \text{o.w} \end{cases} ; \quad f_2(t) = \begin{cases} 1/2 & 0<t<4 \\ 0 & \text{o.w} \end{cases}$$

$$f_3(t) = \begin{cases} 1/2 & 0<t<1,\ 2<t<3 \\ -1/2 & 1<t<2,\ 3<t<4 \\ 0 & \text{o.w} \end{cases}$$

$$\int_{-\infty}^{\infty} f_1(t) f_2(t)\, dt = \int_0^4 f_1(t) f_2(t)\, dt$$

$$= \left(\tfrac{1}{2}\right)^2(2) - \left(\tfrac{1}{2}\right)^2(2) = 0$$

$$\int_0^4 f_1(t) f_3(t)\, dt = \left(\tfrac{1}{2}\right)^2(1) - \left(\tfrac{1}{2}\right)^2(1) - \left(\tfrac{1}{2}\right)^2(1) + \left(\tfrac{1}{2}\right)^2(1)$$

$$= 0$$

$$\int_0^4 f_2(t) f_3(t)\, dt = \left(\tfrac{1}{2}\right)^2(1) - \left(\tfrac{1}{2}\right)^2(1) + \left(\tfrac{1}{2}\right)^2(1) - \left(\tfrac{1}{2}\right)^2(1)$$

$$= 0$$

Hence Orthogonal

$\sum u^2$

$$\int_{-\infty}^{\infty} f_1^2(t)\,dt = \left(\tfrac{1}{2}\right)^2 + \left(-\tfrac{1}{2}\right)^2 = 1$$

$$\int_{-\infty}^{\infty} f_2^2(t)\,dt = \left(\tfrac{1}{2}\right)^2 (4) = 1$$

$$\int_{-\infty}^{\infty} f_3^2(t)\,dt = \left(\tfrac{1}{2}\right)^2 + \left(-\tfrac{1}{2}\right)^2 + \left(\tfrac{1}{2}\right)^2 + \left(-\tfrac{1}{2}\right)^2 = 1$$

Hence also orthonormal.

Q4-10 (b)

$$x(t) = \begin{cases} -1 & 0<t<1 \\ 1 & 1<t<3 \\ -1 & 3<t<4 \end{cases}$$

P4-11 (a)

$$s_1(t) = \begin{cases} 2 & 0<t<1 \\ -1 & 1<t<4 \\ 0 & \text{o.w} \end{cases} ; \quad s_2(t) = \begin{cases} -2 & 0<t<1 \\ 1 & 1<t<3 \\ 0 & \text{o.w} \end{cases}$$

$$s_3(t) = \begin{cases} 1 & 0<t<1,\ 2<t<3 \\ -1 & 1<t<2,\ 3<t<4 \\ 0 & \end{cases} ; \quad s_4(t) = \begin{cases} 1 & 0<t<1 \\ -2 & 1<t<3 \\ 2 & 3<t<4 \end{cases}$$

$\phi_1(t)$:

$$E_{s_1(t)} = (2)^2 + (-1)^2(3) = 4+3 = 7$$

$$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_{s_1(t)}}} = \frac{s_1(t)}{\sqrt{7}}$$

$$\boxed{\phi_1(t) = \begin{cases} 2/\sqrt{7} & 0<t<1 \\ -1/\sqrt{7} & 1<t<4 \\ 0 & \text{o.w} \end{cases}}$$

$\phi_2(t)$

$$g_2(t) = s_2(t) - s_{21}\,\phi_1(t) \quad \text{—— (1)}$$

$$s_{21} = \int_{-\infty}^{\infty} s_2(t)\,\phi_1(t)\,dt$$

$$s_{21} = (-2)\left(\frac{2}{\sqrt{7}}\right)(1) + (1)\left(\frac{-1}{\sqrt{7}}\right)(2)$$

$$= \frac{-4}{\sqrt{7}} - \frac{2}{\sqrt{7}}$$

$$\Rightarrow \boxed{s_{21} = \frac{-6}{\sqrt{7}}}$$

$$s_{21}\,\phi_1(t) = \begin{cases} \frac{-6}{\sqrt{7}} \times \frac{2}{\sqrt{7}} & 0<t<1 \\ \frac{-6}{\sqrt{7}} \times \frac{-1}{\sqrt{7}} & 1<t<4 \\ 0 & \text{o.w} \end{cases}$$

$$\Rightarrow s_{21}\,\phi_1(t) = \begin{cases} \frac{-12}{7} & 0<t<1 \\ \frac{6}{7} & 1<t<4 \\ 0 & \text{o.w} \end{cases}$$

$$g_2(t) = s_2(t) - s_{21}\,\phi_1(t) = \begin{cases} -2 + \frac{12}{7} & 0<t<1 \\ 1 - \frac{6}{7} & 1<t<3 \\ 0 - \frac{6}{7} & 3<t<4 \\ 0 & \text{o.w} \end{cases}$$

$$\Rightarrow g_2(t) = \begin{cases} \frac{-2}{7} & 0<t<1 \\ 1/7 & 1<t<3 \\ \frac{-6}{7} & 3<t<4 \\ 0 & \text{o.w} \end{cases}$$

$u(t) = af_1(t) + bf_2(t) + cf_3(t)$

$$E_{f_2} = \left(\frac{-2}{7}\right)^2(1) + \left(\frac{1}{7}\right)^2(2) + \left(\frac{-6}{7}\right)^2(1)$$

$$E_{f_2} = 6/7$$

$$\phi_2(t) = \frac{g_2(t)}{\sqrt{6/7}} = g_2(t)\sqrt{\frac{7}{6}}$$

$$\Rightarrow \phi_2(t) = \begin{cases} -\frac{2}{7} \times \sqrt{\frac{7}{6}} & 0 < t < 1 \\ \frac{1}{7} \times \sqrt{\frac{7}{6}} & 1 < t < 3 \\ -\frac{6}{7} \times \sqrt{\frac{7}{6}} & 3 < t < 4 \\ 0 & \text{o.w} \end{cases}$$

$$\Rightarrow \boxed{\phi_2(t) = \begin{cases} \frac{-2}{\sqrt{42}} & 0 < t < 1 \\ \frac{1}{\sqrt{42}} & 1 < t < 3 \\ -\sqrt{\frac{6}{7}} & 3 < t < 4 \\ 0 & \text{o.w} \end{cases}}$$

Sol: $\phi_3(t)$:

$$g_3(t) = s_3(t) - s_{31}\phi_1(t) - s_{32}\phi_2(t)$$

$$s_{31} = \int_{-\infty}^{\infty} s_3(t)\,\phi_1(t)\,dt$$

$$= (1)\left(\frac{2}{\sqrt{7}}\right)(1) + (-1)\left(-\frac{1}{\sqrt{7}}\right)(1) + (1)\left(-\frac{1}{\sqrt{7}}\right)(1)$$

$$= \frac{2}{\sqrt{7}} + \frac{1}{\sqrt{7}} - \frac{1}{\sqrt{7}}$$

$$\boxed{s_{31} = 4/\sqrt{7}}$$

$$s_{32} = (1)\left(\frac{-2}{\sqrt{42}}\right)(1) + (-1)\left(\frac{1}{\sqrt{42}}\right)(1) + (1)\left(\frac{1}{\sqrt{42}}\right)(1)$$

$$s_{32} = \frac{-2}{\sqrt{42}} - \frac{1}{\sqrt{42}} + \frac{1}{\sqrt{42}}$$

$$\boxed{s_{32} = \frac{-2}{\sqrt{42}}}$$

$$s_{31}\phi_1(t) = \begin{cases} \frac{-2}{\sqrt{42}} \times \frac{2}{\sqrt{7}} & 0<t<1 \\ \frac{-2}{\sqrt{42}} \times \frac{-1}{\sqrt{7}} & 1<t<4 \\ 0 & \text{o.w} \end{cases}$$

$$\boxed{s_{31}\phi_1(t) = \begin{cases} -4/\sqrt{294} & 0<t<1 \\ 2/\sqrt{294} & 1<t<4 \\ 0 & \text{o.w} \end{cases}}$$

$$s_{32}\phi_2(t) = \begin{cases} \frac{-2}{\sqrt{42}} \times \frac{-2}{\sqrt{42}} & 0<t<1 \\ \frac{1}{\sqrt{42}} \times \frac{-2}{\sqrt{42}} & 1<t<3 \\ -\sqrt{\frac{6}{7}} \times \frac{-2}{\sqrt{42}} & 3<t<4 \\ 0 & \text{o.w} \end{cases}$$

$$S_{32}\,\phi_2(t) = \begin{cases} +4/42 & 0<t<1 \\ -2/42 & 1<t<3 \\ \frac{2\sqrt{6}}{\sqrt{294}} & 3<t<4 \\ 0 & o.w \end{cases}$$

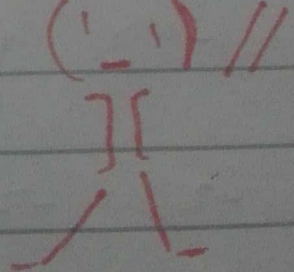
$$g_2(t) = \begin{cases} 1 + \frac{4}{\sqrt{294}} - \frac{4}{42} & 0<t<1 \\ 1 - \frac{2}{\sqrt{294}} + \frac{2}{42} & 1<t< \\ 0 & o.w \end{cases}$$

$g_2(t)$

Donkey work now!

hahaha

"Work smart not hard."



refrence from eng.Project managment slides.

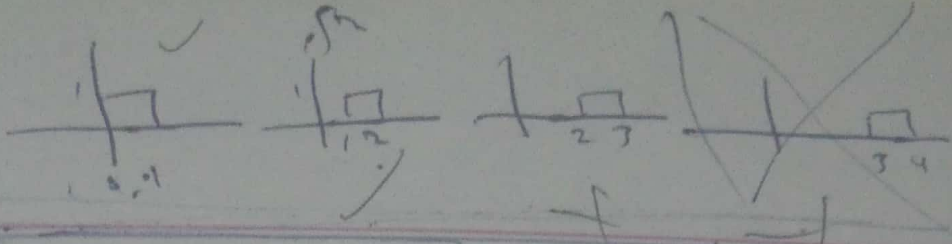
4-11 by inspection

Orthonormal basis function

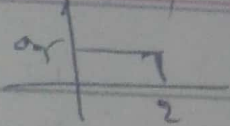
$$f_1(t) = \begin{cases} 1 & 0<t<1 \\ 0 & \text{o.w} \end{cases} ; \quad f_2(t) = \begin{cases} 1 & 1<t< \\ 0 & \text{o.w} \end{cases}$$

$$f_3(t) = \begin{cases} 1 & 2<t<3 \\ 0 & \text{o.w} \end{cases} ; \quad f_4(t) = \begin{cases} 1 & 3<t< \\ 0 & \text{o.W} \end{cases}$$

$$\begin{array}{cc} & \begin{matrix} 0\text{-}1 & 1\text{-}2 & 2\text{-}3 & 3\text{-}4 \end{matrix} \\ s_1 = & \begin{bmatrix} 2 & -1 & -1 & -1 \end{bmatrix} \\ s_2 = & \begin{bmatrix} -2 & 1 & 1 & 0 \end{bmatrix} \\ s_3 = & \begin{bmatrix} 1 & -1 & 1 & -1 \end{bmatrix} \\ s_4 = & \begin{bmatrix} 1 & -2 & -2 & 2 \end{bmatrix} \end{array}$$



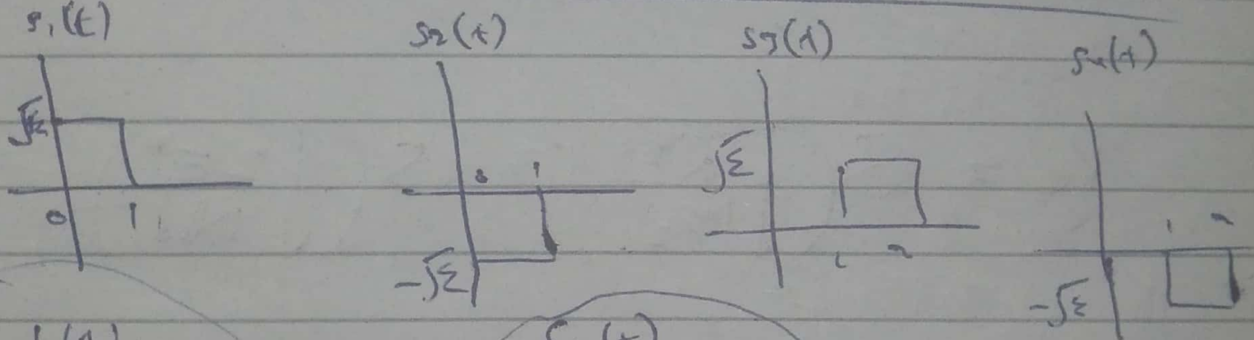
Area = 1

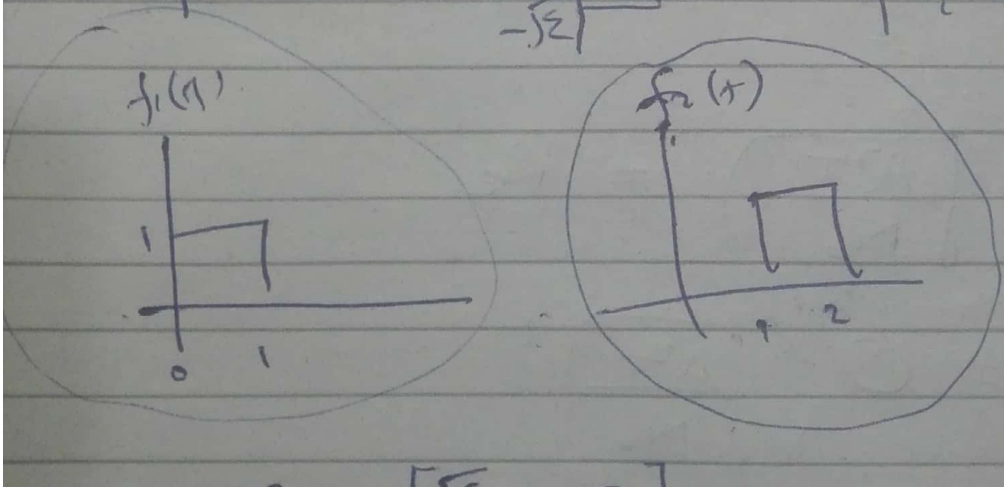


$s_1(t) = [\sqrt{2} \quad -1 \quad -1 \quad -1]$

$s_2(t) = 2f_1(t) - f_2(t) - f_3(t) - f_4(t)$

$\sqrt{(2+2)^2 + (-1-1)^2 + (-1-1)^2 + (-1)^2}$





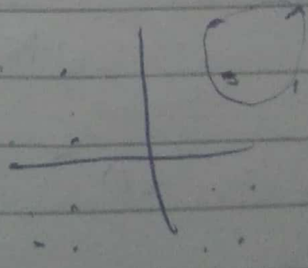
$s_1 = [\sqrt{E}, 0]$

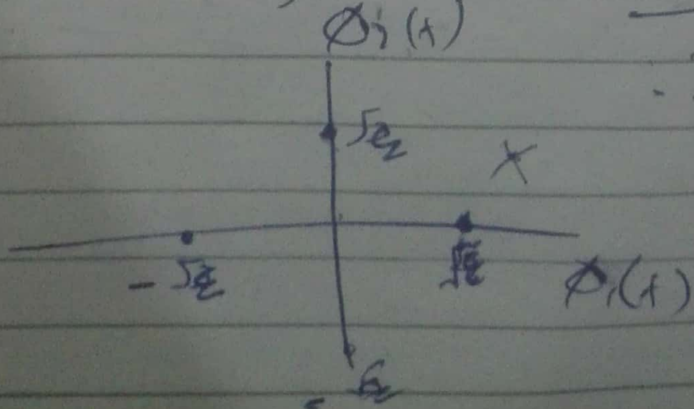
$s_2 = [-\sqrt{E}, 0]$

$s_3 = [0, \sqrt{E}]$

$s_4 = [0, -\sqrt{E}]$

$\sqrt{E} f_1(t) + \_ f_2(t)$





5-18

$$\frac{N_0}{2} = 10^{-10}$$

$$\text{Energy} = \frac{1}{2} A^2 T$$

T = bit interval
A = signal amplitude

$$P_e = 10^{-6}$$

$$P_e = Q\left(\sqrt{\frac{\alpha^2}{N_0/2}}\right) = Q\left(\sqrt{\frac{2E}{N_0}}\right)$$

$$= Q\left(\sqrt{\frac{A^2T}{N_0}}\right) = P_e$$

$$10^{-6} = Q\left(\sqrt{\frac{A^2T}{N_0}}\right)$$

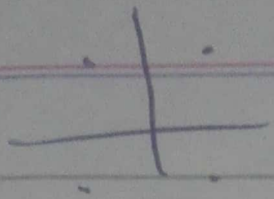
$$T = \frac{1}{10K} = \frac{1}{10^4}$$

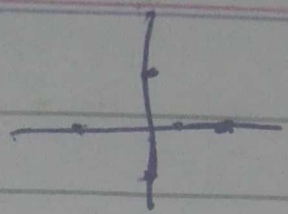
$$T = 10^{-4}$$

x	$Q(x)$
0.01	1 . .
0.07	

$Q(x)$	x

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} \exp\left[-\frac{x^2}{2}\right] dx$$





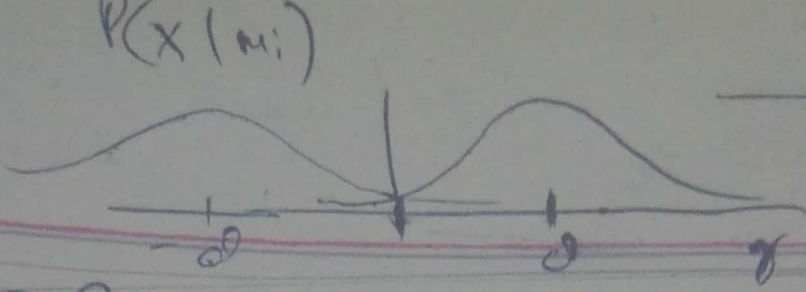
$$\boxed{N_0 = 2 \times 10^{-6}}$$

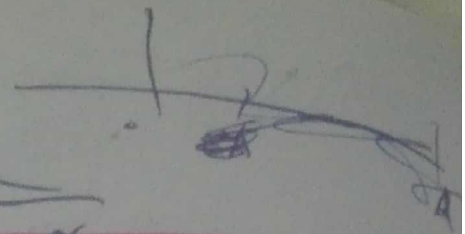
$$10^{-6} = Q\left(\sqrt{\frac{A^2 \times 10^{-4}}{N_0}}\right)$$

$$\sqrt{\frac{A^2 \times 10^{-4}}{N_0}} = 4.74$$

$$\frac{A^2 \times 10^{-4}}{N_0} = 22.4676$$

$$\frac{10^{-4} \times A^2}{N_0} = 22.4676$$





P 5-19

$$\frac{P(e|A) + P(e|-A)}{2}$$

$$= \frac{P(e|A)}{2} + \frac{P(e|-A)}{2}$$

$$= \frac{1}{2}\int_{-\infty}^{0} f(r|A)\,dr + \frac{1}{2}\int_{0}^{\infty} f(r|-A)\,dr \quad \text{— (1)}$$

$$p(n) = \frac{\lambda}{2} e^{-\lambda|n|}$$

$\cancel{r = A + n}$

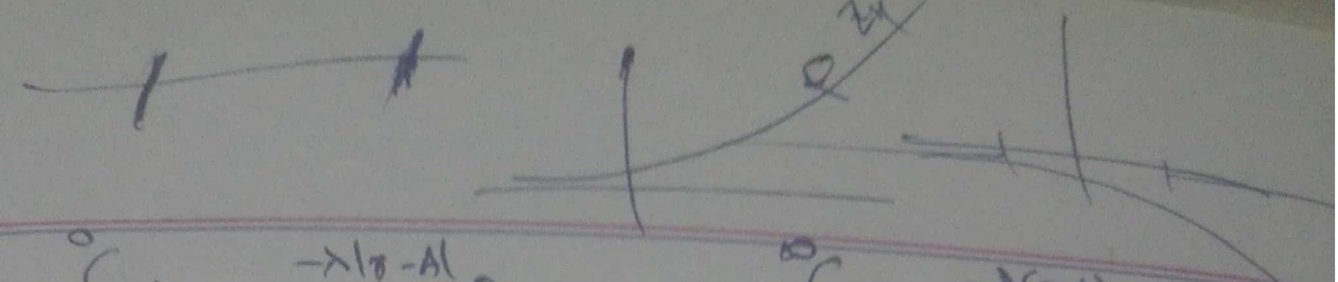
$\cancel{n = r - A}$

$r = A + n \quad | \quad r = -A + n$

$n = r - A \quad | \quad n = r + A$

$$f(r|A) = \frac{\lambda}{2} e^{-\lambda|r-A|}$$

$$f(r|-A) = \frac{\lambda}{2} e^{-\lambda|r+A|}$$



$$\frac{1}{2}\int_{-\infty}^{0} \frac{\lambda}{2} e^{-\lambda|r-A|}\,\partial r + \frac{1}{2}\int_{0}^{\infty} \frac{\lambda}{2} e^{-\lambda(r+A)}\,\partial r$$

$$= \frac{\lambda}{4}\int_{-\infty}^{0} e^{-\lambda|r-A|}\,\partial r + \frac{\lambda}{4}\int_{0}^{\infty} e^{-\lambda(r+A)}\,\partial r$$

$r - A = u \longrightarrow$ $r = 0 \Rightarrow u = -A$

$\Rightarrow \partial r = \partial u$ $\quad$ $r = -\infty \Rightarrow u = -\infty$

$$\underbrace{\frac{\lambda}{4}\int_{-\infty}^{-A} e^{-\lambda|u|}\,\partial u + \frac{\lambda}{4}\int_{A}^{\infty} e^{-\lambda|u|}\,\partial u}_{\text{same}}$$

$r + A = u$ $\quad$ $r = 0 \Rightarrow u = A$

$\Rightarrow \partial r = \partial u$ $\quad$ $r = \infty \Rightarrow u = \infty$

$$\frac{\lambda}{2} \times \frac{1}{-\lambda}\left[e^{-\lambda u}\right]_{A}^{\infty}$$

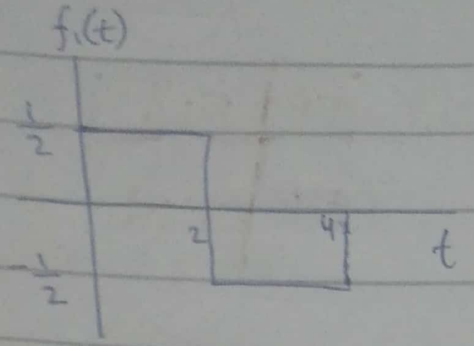
$$-\frac{1}{2}\left(0 - e^{-\lambda A}\right)$$

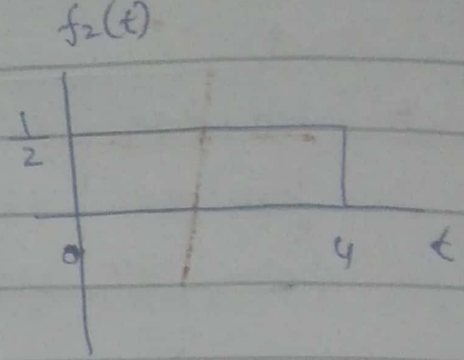
$$= \frac{e^{-\lambda A}}{2}$$

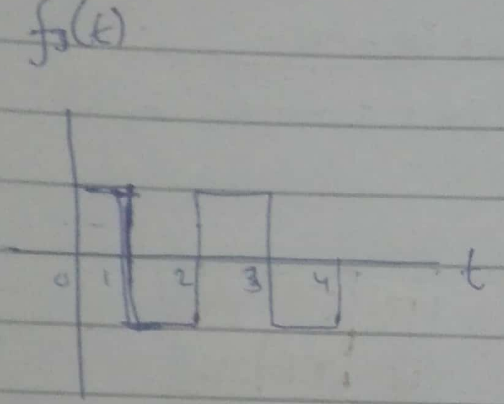
HW

DC

4-10







Energies:

$$\int_0^4 f_1^2(t)\,dt = \int_0^2 (0.5)^2\,dt + \int_2^4 (-0.5)^2\,dt$$

$= 1$

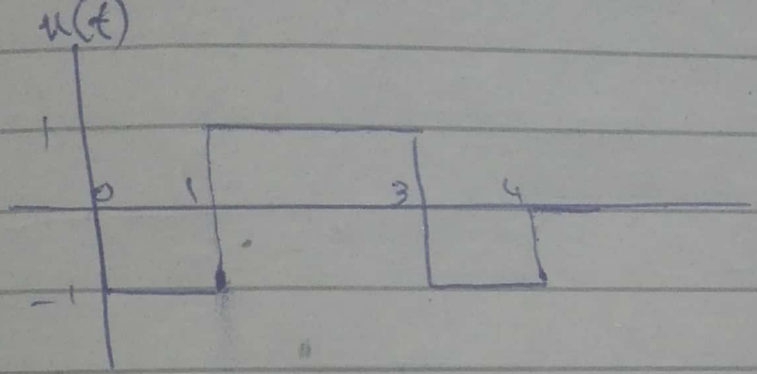
$$\int_0^4 f_2^2(t)\,dt = \int_0^4 (0.5)^2\,dt = 1$$

$$\int_0^4 f_3^2(t)\,dt = 1$$

$$\int_0^4 f_1(t) f_2(t)\, dt = \int_0^2 (0.5)^2\, dt - \int_2^4 (0.5)^2\, dt$$

$$= 0$$

$\cancel{\int_0^4 f_1(t) f_3(t)\, dt = \int}$ Similarly for others

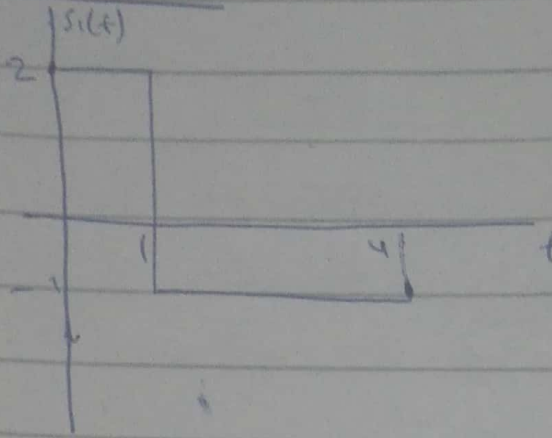


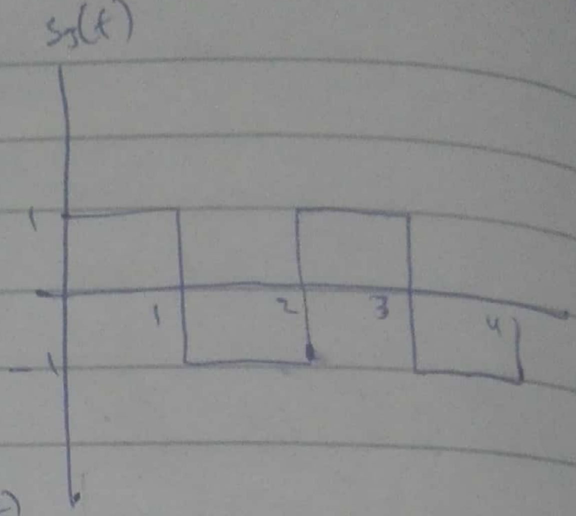
$$x_1 = \int_0^4 x(t) f_1(t)\, dt = \int_0^1 (-1)(0.5)\, dt + \int_1^2 (1)(0.5)\, dt$$

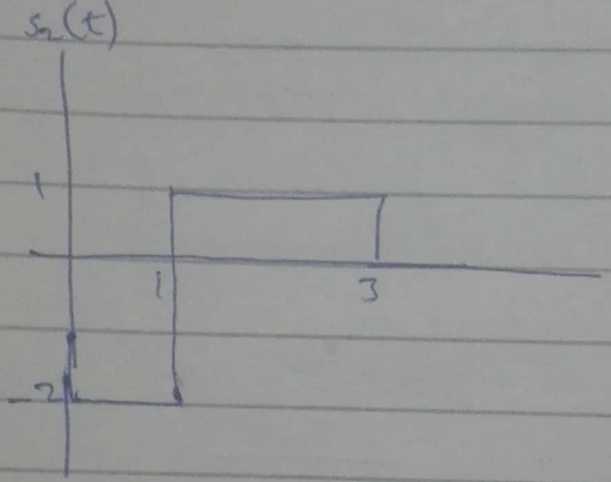
$$+ \int_2^3 (1)(-0.5)\, dt + \int_3^4 (-1)(-0.5)\, dt$$

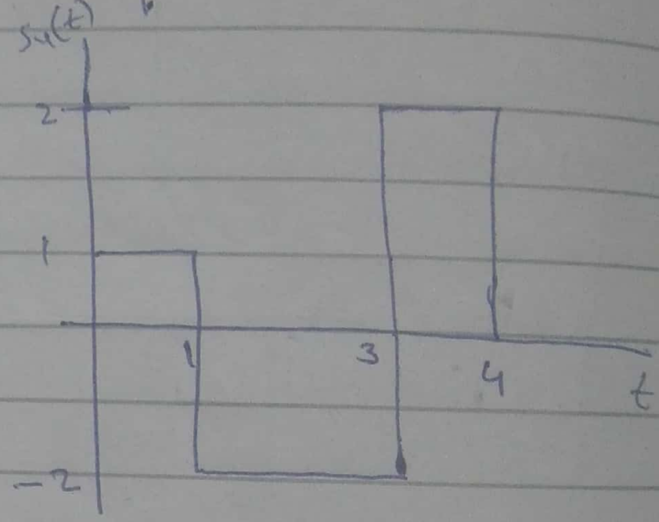
$$= 0$$

P 4-11







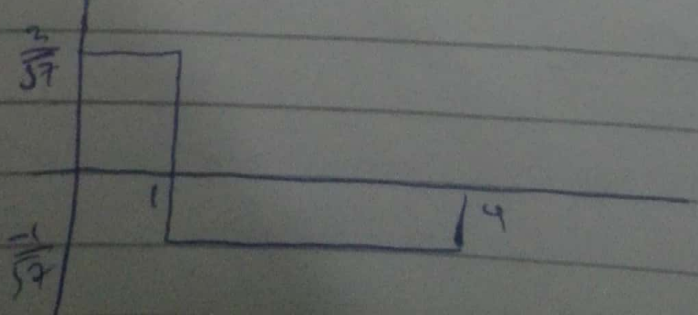


$$E_1 = \int_0^4 s_1^2(t)\,dt = \int_0^1 (2)^2\,dt + \int_1^4 (-1)^2\,dt$$

$$= 4 + 3$$

$$E_1 = 7$$

$$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_1}} = \frac{s_1(t)}{\sqrt{7}}$$



$$\frac{(-4)+(-1\times 2)}{\sqrt{7}} = \frac{-6}{\sqrt{7}}$$

$$g_2(t) = s_2(t) - s_{21}\,\phi_1(t)$$

$$s_{21} = \int_0^4 s_2(t)\phi_1(t)\,dt = \frac{1}{\sqrt{7}}\int_0^4 s_2(t)\,s_1(t)\,dt$$

$$= \frac{1}{\sqrt{7}}\int_0^1 -4\,dt + \frac{1}{\sqrt{7}}\int_1^3 -1\,dt$$

$$= \frac{-4}{\sqrt{7}} - \frac{2}{\sqrt{7}}$$

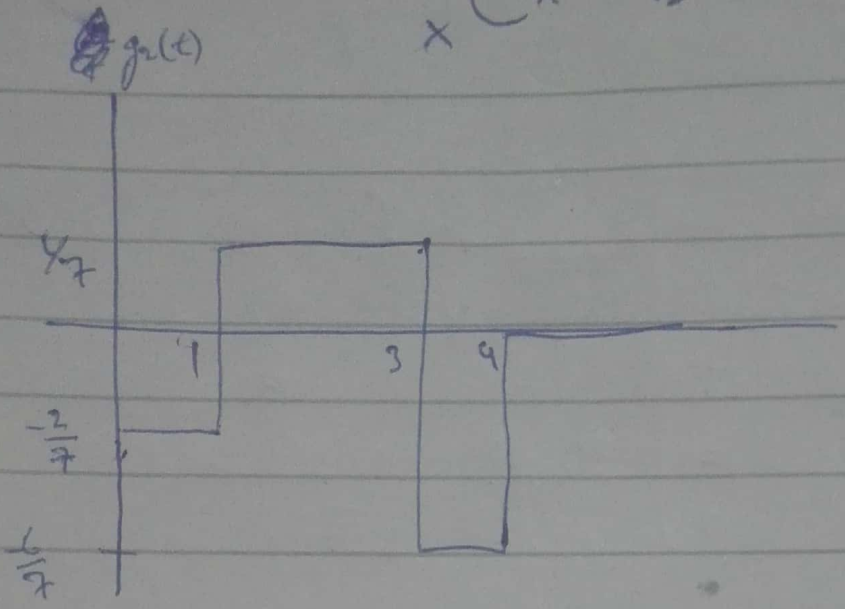
$$= \frac{-6}{\sqrt{7}}$$

$$s_{21} = -\frac{6}{\sqrt{7}}$$

$$g_2(t) = s_2(t) - \left(\frac{-6}{\sqrt{7}} \times \frac{s_1(t)}{\sqrt{7}}\right)$$

$$g_2(t) = s_2(t) + \frac{6}{7}\,s_1(t)$$

$$\left(\frac{-2}{7} \times 2 \times 1\right) \times \left(\frac{1}{7} \times -1 \times 2\right) \times \left(\frac{-6}{7} \times -1 \times 1\right) =$$



$$E_{g_2} = \int g_2^2(t) = \left(\frac{2}{7}\right)^2 + \left(\frac{1}{7}\right)^2 \times 2 + \left(\frac{6}{7}\right)^2$$

$$E_{g_2} = \frac{6}{7}$$

$$\phi_2(t) = \frac{g_2(t)}{\sqrt{\frac{6}{7}}}$$

$$= g_2(t) \frac{\sqrt{7}}{\sqrt{6}}$$

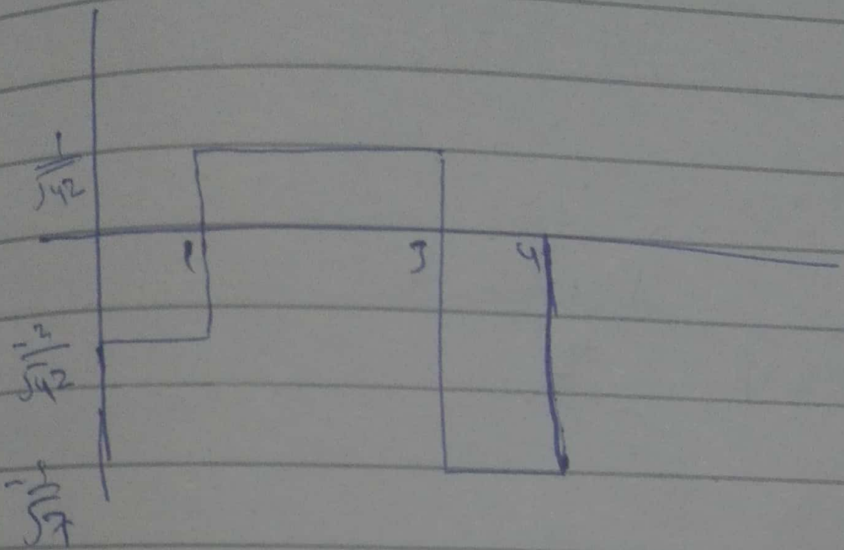
$7+6+5+4+3+2+1$

$\phi_2(t)$

(P_1, P_2)

$(1, 2)$

$(2, 1)$



$\frac{-2}{7} \times \frac{\sqrt{7}}{\sqrt{6}} = \frac{-2}{\sqrt{42}}$

$\frac{1}{7} \times \frac{\sqrt{7}}{\sqrt{6}} = \frac{1}{\sqrt{42}}$

$\frac{-\sqrt{6}}{7} \times \frac{\sqrt{7}}{\sqrt{6}} = -\frac{\sqrt{6}}{\sqrt{7}}$

$$\left(\frac{-2}{\sqrt{42}} \times \frac{2}{\sqrt{7}} \times 1\right) + \left(\frac{-1}{\sqrt{7}} \times \frac{1}{\sqrt{42}} \times 2\right) + \left(\frac{-1}{\sqrt{7}} \times \frac{-\sqrt{6}}{\sqrt{7}}\right) =$$