Single-Tap Equalization for Fast OFDM Signals Under Generic Linear Channels

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***Abstract*—Channel cannot be compensated by single-tap equalizers in fast orthogonal frequency-division multiplexing (OFDM) easily unless the channel impulse response (CIR) is symmetric. In this letter, we propose a low-complexity fast OFDM scheme, which enables efficient single-tap equalization for generic linear channels without the symmetric limitation. Simulations under both wireless multipath fading and multimode fiber channels are provided to verify the feasibility. Compared with other schemes, the proposed technique is simpler and avoids net data rate loss. It is shown that this scheme exhibits better performance than the frequency-domain-equalization-based fast OFDM scheme under the wireless frequency-selective fading channel.**

***Index Terms*—Channel equalization, discrete cosine transform (DCT), fast OFDM.**

I. Introduction

FAST orthogonal frequency division multiplexing (OFDM) is a promising multicarrier transmission technique whose subcarrier space is half of that of the conventional OFDM [1]–[3]. Instead of discrete Fourier transform (DFT), discrete cosine transform (DCT) is commonly adopted in the fast OFDM to multiplex symbols onto subcarriers, and this implementation has been experimentally demonstrated in optic-fiber systems [4]–[6]. Compared with the conventional OFDM, it has been shown that fast OFDM exhibits enhanced tolerance to carrier frequency offset in optical coherent detection, as well as the Doppler shift in wireless communications [7], [8]. In addition, fast OFDM can exhibit better receiver sensitivity in optical full-field detection [9]. These advantages make fast OFDM an attractive multicarrier transmission scheme.

One of the challenges in the fast OFDM technology is that the DCT used for subcarrier MUX/DEMUX does not have the circular convolution property as the DFT does in conventional OFDM. Instead, the DCT possesses symmetric convolution that was studied in the classical work in [10]. In the symmetric convolution, two sequences should be symmetrically extended to make the DCT of the convolution equal to the product of their individual DCTs. In practical implementation, it implies that to enable single-tap equalization in the cosine domain for fast OFDM signal, not only symmetric prefix and suffix are required as guard interval (GI), but also the channel impulse response (CIR) should also be symmetrical inherently. Some channels satisfy this condition, e.g., chromatic dispersion in single-mode fibers (SMFs) [11]. However, in more generic linear channels, such as multipath fading in wireless channels and modal dispersion in multi-mode fibers (MMFs), the condition for symmetric convolution is not satisfied and the channel cannot be easily compensated by simple single-tap equalizers.

To address this issue, Al-Dhahir *et al.* and Mandyam devised different schemes to meet the condition of symmetric convolution [12], [13]. In [12], the fast OFDM signal is symmetrically extended to avoid the needs for channel symmetry. However, the net data rate is reduced by half. The method proposed in [13] avoids the reduction of data rate by using a complicated time-domain finite impulse response (FIR) pre-filter based on minimum mean square error (MMSE) criterion at the receiver to filter CIR to be symmetric. However, it increases system complexity significantly. A more effective method is based on frequency domain equalization (FDE) [14], in which channel impairment is compensated in the frequency domain at the receiver before the signal is transformed to the cosine domain by DCT. Two additional DFTs are required at the receiver. In addition, since the channel is equalized in the frequency domain and symbols are detected in the cosine domain, the noise in the frequency domain would be amplified and spread over other subcarriers after DCT. Even if a MMSE equalizer, instead of a zero-forcing (ZF) equalizer, is applied to mitigate the noise enlargement, the system is still sensitive to the noise at spectral nulls.

In this work, we propose a more computationally-efficient fast OFDM scheme which can efficiently compensate generic linear time-invariant (LTI) channels using single-tap equalizers without the sacrifice of net data rate. In the proposed scheme, zero-padded (ZP) GI is adopted, and the DCT at the receiver is replaced by the DFT of doubled length, i.e., N subcarriers are demultiplexed by the DFT of $2N$-point rather than the DCT of N-point. Simulations under both wireless and optical channels are provided to validate the feasibility. It is also shown that the proposed scheme achieves better performance than the FDE-based fast OFDM scheme under wireless frequency-selective fading channel.

II. Proposed Equalization Algorithm

The block diagram of the proposed fast OFDM is illustrated in Fig. 1. Assuming that there are N subcarriers and the k-th subcarrier is modulated by $x(k)$, the time domain signal $s(n)$, $n = 0, \ldots, N-1$, is obtained by the inverse DCT (IDCT)

$$s(n) = \sqrt{\frac{2}{N}} \sum_{k=0}^{N-1} \varepsilon(k) x(k) \cos\left[\frac{\pi}{N} k \left(n + \frac{1}{2}\right)\right], \quad (1)$$

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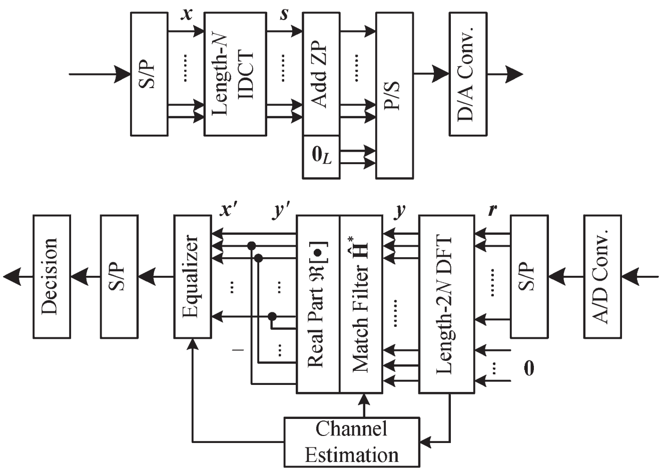

Fig. 1. System diagram of the proposed fast OFDM with single-tap equalizers.

where $\varepsilon(k) = \sqrt{0.5}$ for $k = 0$, and 1 for $k = 1, \ldots, N-1$. To avoid inter-symbol interference, ZP, instead of symmetric prefix and suffix based GI [10], [11], is used. It will be shown that using the ZP-based GI the proposed scheme enables the use of single-tap equalizers for channel compensation in generic LTI channels. The zero-padded signal is represented by $s_{ZP}(n)$, where $s_{ZP}(n) = s(n)$ for $0 \le n \le N-1$, and $s_{ZP}(n) = 0$, for $N \le n \le N+L-1$.

We consider a generic LTI channel without the symmetry limitation and define $h(n)$ as the CIR. For chromatic dispersion in a SMF, $h(n)$ is a symmetric function [11]. In the wireless multipath fading or MMF channels, $h(n)$ is non-symmetric, which is given by $h(0)\delta(0) + h(1)\delta(1) + \cdots + h(L-1)\delta(L-1)$, where $h(n)$, $n = 0, \ldots, L-1$, are the CIR coefficients with L being the channel memory length and $\delta(n)$ being the Kronecker delta function. The values of $h(n)$ are real in the MMF channel and complex in the wireless channel. The received signal is

$$r(n) = h(n) * s_{ZP}(n) + z(n), \; 0 \le n \le N+L-1, \quad (2)$$

where $*$ is the convolution operator, $z(n)$ is the additive noise which is neglected in the following derivations for simplicity.

At the receiver, instead of the use of the DCT, zeros are padded at the end of the received signal $r(n)$ to get $r_{2N}(n)$. A length-$2N$ DFT is applied to obtain the frequency domain signal, $y(m)$. According to the convolution theory, one can obtain $y(m)$ as

$$\begin{aligned} y(m) &= \mathcal{F}_{2N}\left[r_{2N}(n)\right] = \mathcal{F}_{2N}\left[h_{2N}(n)\right] \cdot \mathcal{F}_{2N}\left[s_{2N}(n)\right] \\ &= H(m) \cdot S(m), \end{aligned} \quad (3)$$

where $m = 0, 1, \ldots, 2N-1$, $\mathcal{F}_{2N}[\bullet]$ denotes the normalized DFT of length-$2N$, $h_{2N}(n)$ and $s_{2N}(n)$ are the zero-padded signals of $h(n)$ and $s_{ZP}(n)$ of length-$2N$, respectively. To facilitate the derivation, we re-write Eq. (3) as

$$y(m) = e^{j\frac{2\pi}{2N}m\cdot\frac{1}{2}} H(m) \cdot e^{-j\frac{2\pi}{2N}m\cdot\frac{1}{2}} S(m) = \hat{H}(m) \cdot \hat{S}(m) \quad (4)$$

where $\hat{S}(m)$ can be extended using Eqs. (1) and (3) as

$$\begin{aligned} \hat{S}(m) &= \sqrt{\frac{1}{2N}} \sum_{n=0}^{N-1} s(n) e^{-j\frac{2\pi}{2N}m\left(n+\frac{1}{2}\right)} \\ &= \sum_{k=0}^{N-1} x(k) G(m,k), \end{aligned} \quad (5)$$

and $G(m,k)$ is defined as

$$G(m,k) = \frac{\varepsilon(k)}{N} \sum_{n=0}^{N-1} \cos\frac{\pi}{N}k\left(n+\frac{1}{2}\right) \cdot e^{-j\frac{\pi}{N}m\left(n+\frac{1}{2}\right)}. \quad (6)$$

In Eq. (5), one can readily derive the real part of $G(m,k)$ as

$$\begin{aligned} &G_R(m,k) \\ &= \frac{\varepsilon(k)}{N} \sum_{n=0}^{N-1} \cos\frac{\pi}{N}k\left(n+\frac{1}{2}\right) \cdot \cos\frac{\pi}{N}m\left(n+\frac{1}{2}\right) \\ &= \begin{cases} 1 & m = 0 \\ 0.5 \cdot \delta(m-k) & m = 1, \ldots, N-1 \\ 0 & m = N \\ -0.5 \cdot \delta(2N-m-k) & m = N+1, \ldots, 2N-1, \end{cases} \end{aligned} \quad (7)$$

which is skew-symmetrical around N.

On the other hand, the channel frequency response $\hat{H}(m)$ in Eq. (4) can be estimated using various methods. For example, the channel can be obtained by periodically-inserted frequency-domain training sequences [15]. It should be noted that the optimal channel estimation depends on specific channel characteristics.

To further recover the transmitted symbols, we compensate the phase by multiplying the conjugate of the estimated $\hat{H}(m)$ to the received signal $y(m)$, and then extract its real part as

$$\begin{aligned} y'(m) &= \left|\hat{H}(m)\right|^2 \cdot \sum_{k=0}^{N-1} x(k) G_R(m,k) \\ &= \left|\hat{H}(m)\right|^2 \cdot \begin{cases} x(m) & m=0 \\ 0.5x(m) & m=1, \ldots, N-1 \\ 0 & m=N \\ -0.5x(2N-m) & m=N+1, \ldots, 2N-1, \end{cases} \end{aligned} \quad (8)$$

Here, ideal channel estimation is assumed. The transmitted data $x(m)$ can now be extracted using single-tap equalizers based on ZF by removing $|\hat{H}(m)|^2$. However, $x(m)$ can be extracted from either $m = 1, \ldots, N-1$, or $m = N+1, \ldots, 2N-1$, or the combination of both. To make full use of the information in Eq. (8), we further define

$$\hat{H}_c(m) = \frac{1}{2}\left(\left|\hat{H}(m)\right|^2 + \left|\hat{H}(2N-m)\right|^2\right), \quad (9)$$

and obtain the transmitted data as

$$x'(m) = y'(m) - y'(2N-m) = \hat{H}_c(m) x(m). \quad (10)$$

TABLE I
COMPARISON BETWEEN DIFFERENT FAST OFDM SCHEMES

	SNR estimation	Data rate loss	Complexity
Proposed scheme	No	No	Moderate
FDE with MMSE	Yes	No	Moderate
Ref. [12]	No	Yes	Moderate
Ref. [13]	Yes	No	High

In Eq. (10), $\hat{H}_c(m)$ can be readily compensated by ZF-based single-tap equalizers. Equation (10), compared to Eq. (8), is able to optimally combine the signal-to-noise ratio (SNR) at the positive and negative frequencies and may result in improved performance when the power spectral density (PSD) of the channel is non-symmetric. When compared to the FDE-based scheme, the proposed method using Eq. (10) is more tolerable to the noise, especially that at spectral notches, under channels with non-symmetric PSD. Additionally, symbol decision can be made directly after equalization using Eq. (10). In contrast, in the FDE-based scheme, the equalized signal should be transformed from the frequency domain to the cosine domain with additional IFFT and DCT. The noise around spectral nulls in the frequency domain may be amplified and spread over other subcarriers during this process [16].

III. Discussions and Simulation Results

The proposed scheme does not sacrifice the net data rate as that in [12] or requires a complicated time-domain pre-filter as in [13] while enabling single-tap equalization in generic LTI channels. Compared to the conventional FDE-based scheme [14], the proposed scheme reduces the system complexity while exhibits performance advantages in some application scenarios, as will be shown later. In the FDE-based scheme, one length-N DCT and two length-N DFTs are required at the receiver for channel compensation and demultiplexing. On the other hand, a length-$2N$ DFT is required at the receiver in the proposed scheme. When ZF-based equalizer is adopted in both schemes, the FDE-based scheme and the proposed scheme require about $(3N/2)\log_2(N) + N$ and $N\log_2(N) + 4N$ complex multiplications, respectively. If MMSE is applied in the FDE-based scheme, the complexity increases to $(3N/2)\log_2(N) + 3N$ and the knowledge of SNR is required at the receiver. Thus, the advantage in terms of complexity using the proposed scheme becomes more prominent. Table I provides the implementation complexity of different fast OFDM schemes.

Next, simulations are performed under both a wireless fading channel and a MMF channel to validate the feasibility and to investigate its performance advantage over the FDE-based scheme. Fig. 2 shows the CIRs and their PSD of these two channels. In Fig. 2(a) and (b), the ITU channel model B of pedestrian environment is considered. In Fig. 2(c) and (d), a 300-m MMF is used with intensity modulation and direct detection [17]. The operating wavelength is 1300 nm. The beam of the direct modulated laser is Gaussian with full width at half maximum of 7 μm and radial offset of 20 μm. It can be seen that the CIR of both channels are non-symmetric. In the wireless channel, the PSD is also non-symmetric and many deep notches can be observed in the frequency domain within the 10-MHz system bandwidth. In contrast, $h(n)$ in the MMF

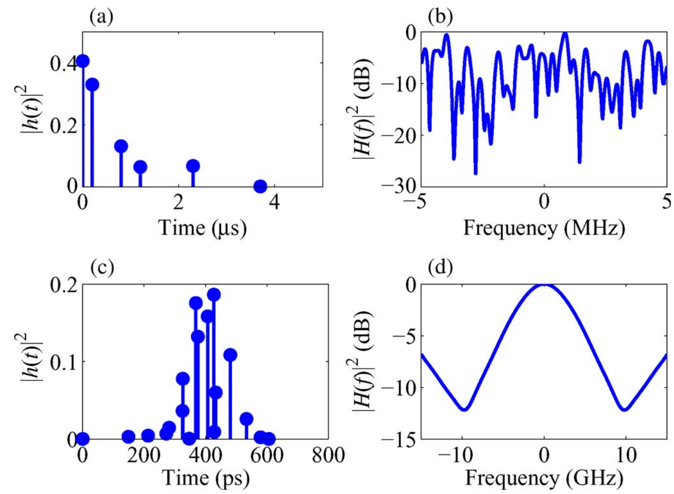

Fig. 2. (a) The power delay profile and (b) power spectral density of the ITU channel model B of pedestrian environment. (c) The power coupling coefficients of modal group delay and (d) its power spectral density of a 300-m 62.5-μm MMF with 20-μm radial offset.

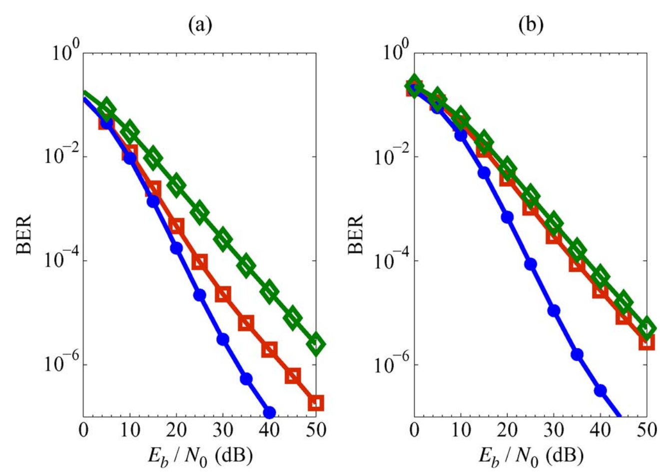

Fig. 3. BER performance of the FDE-based fast OFDM using ZF (diamond) and MMSE (square), and the proposed scheme (dot) for (a) 2-ASK and (b) 4-ASK under the ITU channel model B.

are real values, resulting in symmetric PSD. In addition, less frequency notches occurs in MMF within the system bandwidth of practical interest (commonly $<$ 5 GHz). Thus, different performance characteristics may be observed for the proposed scheme and the FDE-based fast OFDM.

A. Results Under Wireless Fading Channel

In the wireless simulation, the system bandwidth is 10 MHz and is divided into 256 subcarriers which are modulated by 2-*ary* amplitude shift keying (2-ASK) or 4-ASK. ZP-based GI is used in both schemes with the length 64. ZF equalizer is applied in the proposed scheme, and in the FDE-based fast OFDM, both the MMSE and ZF are adopted for comparison.

Fig. 3(a) and (b) show the BER versus E_b/N_0 for both the FDE-based fast OFDM and the proposed scheme. E_b and N_0 represent the signal power per bit and the noise spectral density, respectively. E_b/N_0 evaluates the SNR in the system. In the FDE-based fast OFDM, the MMSE equalizer requires a SNR

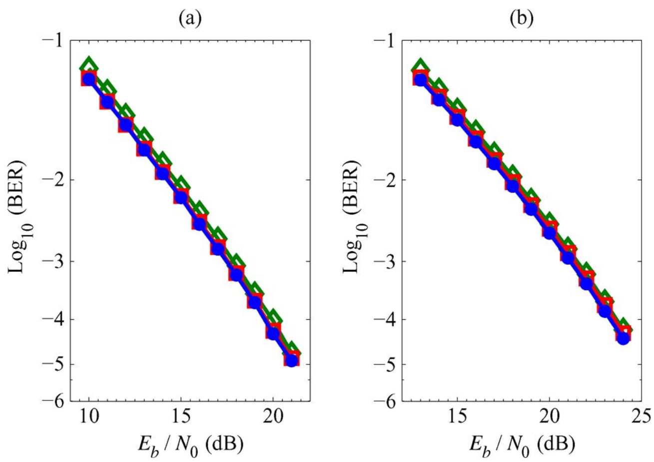

Fig. 4. BER performance of the FDE-based fast OFDM using ZF (diamond) and MMSE (square), and the proposed scheme (dot) for (a) 2-ASK and (b) 4-ASK under the optical MMF channel.

11-dB less than the ZF equalizer to achieve a BER $= 10^{-5}$ for 2-ASK. The difference reduces to 4 dB for 4-ASK due to its higher sensitivity to the noise enlargement. In the FDE-based fast OFDM using either criterion, noise at a spectral null will be amplified after equalization and dominates the performance degradation of the corresponding subcarrier. In addition, the amplified noise also spreads over other subcarriers when the signal is transformed from the frequency domain to the cosine domain using an IFFT and a DCT. In contrast, in the proposed scheme, the SNRs at the positive frequency and the negative frequency are averaged optimally without noise amplification. Even if there is a spectral null, for example, at the m-th subcarrier, unless there is another deep notch at subcarrier $2N - m$, it can be deduced from Eq. (9) that the SNR of $x'(m)$ is only several dB lower than that at frequency $2N - m$ instead of being a value close to zero. In addition, decision can be directly made channel by channel after equalization so that the noise spreading effect does not exist. As a result, the proposed scheme outperforms the MMSE-based FDE scheme by about 5 dB and 13 dB at a BER $= 10^{-5}$ for 2-ASK and 4-ASK, respectively.

B. Results Under Multi-Mode Fiber Channel

In the simulation under the optic MMF channel [17], we use intensity modulation and direct detection. The sampling rates of the digital-to-analog and analog-to-digital converters are 10 GS/s and the number of subcarriers is 256, of which 192 subcarriers are used for modulation. The electrical bandwidth is about 4 GHz. In optical systems, reliable communications is required, and the system commonly operates without spectral nulls. Within the system bandwidth ($\sim$4 GHz), we can see from Fig. 2(d) that the spectral power profile has no deep notches and is also symmetric. The level of the DC bias is optimized to balance the clipping noise and the DC-induced power penalty.

Fig. 4 shows the BER versus the SNR. Because there is no notch in the spectrum, the MMSE-based FDE scheme shows only slightly better performance than the ZF based scheme. It is also shown that in contrast to the wireless channel, the proposed fast OFDM scheme and the MMSE based FDE scheme have similar performance. It is because the PSD is symmetric such that the averaging effect in terms of the SNR in Eq. (10) is the same as that of the FDE-based scheme. In addition, the benefit of avoiding the noise spreading vanishes. Therefore, it can be inferred that the performance advantage of the proposed scheme over the FDE-based scheme occurs under channels with non-symmetric PSD.

IV. Conclusion

This letter presents a computationally-efficient fast OFDM scheme which can efficiently compensate generic LTI channels without the symmetric constraint on the CIR. Simulations are performed to validate its feasibility under both the wireless multipath fading and optical MMF channels. Compared to previous methods, the proposed scheme is simpler and avoids the sacrifice of the net data rate. We have particularly compared this scheme with the FDE-based fast OFDM and shown that, besides lower complexity, the proposed scheme can achieve better performance than the FDE-based fast OFDM under channels with non-symmetric PSD.

References

[1] M. Rodrigues and I. Darwazeh, "Fast OFDM: A proposal for doubling the data rate of OFDM schemes," in *Proc. IEEE ICT*, Beijing, China, 2002, pp. 484–487.
[2] W. Long *et al.*, "Mitigation of the interference between odd and even terms in optical fast OFDM scheme based on interleaved multiplexing," *IEEE Photon. Technol. Lett.*, vol. 24, no. 13, pp. 1160–1162, Jul. 2012.
[3] X. Ouyang, J. Jin, G. Jin, Z. Wang, and Y. W. Park, "Interleaved multiplexing optical fast OFDM without the interference between subchannels," *IEEE Photon. Technol. Lett.*, vol. 25, no. 4, pp. 378–381, Feb. 2013.
[4] F. Xiong, "M-ary amplitude shift keying OFDM system," *IEEE Trans. Commun.*, vol. 51, no. 10, pp. 1638–1642, Oct. 2003.
[5] J. Zhao and H. Shams, "Fast dispersion estimation in coherent optical 16QAM fast OFDM systems," *Opt. Exp.*, vol. 21, no. 2, pp. 2500–2505, 2013.
[6] E. Giacoumidis *et al.*, "Experimental and theoretical investigations of intensity-modulation and direct-detection optical fast-OFDM over MMF-links," *IEEE Photon. Technol. Lett.*, vol. 24, no. 1, pp. 52–54, Jan. 2012.
[7] P. Tan and N. C. Beaulieu, "A comparison of DCT-based OFDM and DFT-based OFDM in frequency offset and fading channels," *IEEE Trans. Commun.*, vol. 54, no. 11, pp. 2113–2125, Nov. 2006.
[8] J. Zhao and A. D. Ellis, "Advantage of optical fast OFDM over OFDM in residual frequency offset compensation," *IEEE Photon. Technol. Lett.*, vol. 24, no. 24, pp. 2284–2287, Dec. 2012.
[9] J. Zhao, "Intensity-modulation full-field detection optical fast OFDM," *J. Opt. Commun. Netw.*, vol. 5, no. 5, pp. 465–474, May 2013.
[10] S. A. Martucci, "Symmetric convolution and the discrete sine and cosine transforms," *IEEE Trans. Signal Process.*, vol. 42, no. 5, pp. 1038–1051, May 1994.
[11] J. Zhao and A. D. Ellis, "Transmission of 4-ASK optical fast OFDM with chromatic dispersion compensation," *IEEE Photon. Technol. Lett.*, vol. 24, no. 1, pp. 34–36, Jan. 2012.
[12] G. D. Mandyam, "Sinusoidal transforms in OFDM systems," *IEEE Trans. Broadcast.*, vol. 50, no. 2, pp. 172–184, Jun. 2004.
[13] N. Al-Dhahir, H. Minn, and S. Satish, "Optimum DCT-based multicarrier transceivers for frequency-selective channels," *IEEE Trans. Commun.*, vol. 54, no. 5, pp. 911–921, May 2006.
[14] D. Falconer, S. L. Ariyavisitakul, A. Benyamin-Seeyar, and B. Eidson, "Frequency domain equalization for single-carrier broadband wireless systems," *IEEE Commun. Mag.*, vol. 40, no. 4, pp. 58–66, Apr. 2002.
[15] S. Coleri, M. Ergen, A. Puri, and A. Bahai, "Channel estimation techniques based on pilot arrangement in OFDM systems," *IEEE Trans. Broadcast.*, vol. 48, no. 3, pp. 223–229, Sep. 2002.
[16] F. Adachi, H. Tomeba, and K. Takeda, "Frequency-domain equalization for broadband single-carrier multiple access," *IEICE Trans. Commun.*, vol. E92-B, no. 5, pp. 1441–1456, May 2009.
[17] M. Webster, L. Raddatz, I. H. White, and D. G. Cunningham, "A statistical analysis of conditioned launch for gigabit ethernet links using multimode fiber," *IEEE J. Lightw. Technol.*, vol. 17, no. 9, pp. 1532–1541, Sep. 1999.