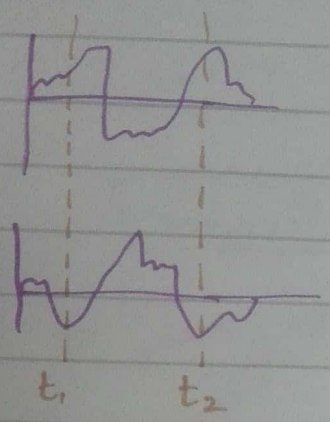
Random Process

collection of random variables over time.

One Realization → getting 1 value from many random variables



$X(t_1)$ → $E[X(t_1)] = m_{t_1}$ mul random variable with pr and integrate over time (range of x)

$X(t_2)$

$\vdots$

$X(t_k)$

$$Var = E[(X(t_1) - m_{t_1})^2]$$

$$= E[X^2(t_1)] - m_{t_1}^2$$

$X(t)$ → m_t dependant on time

$$R_x(t_1, t_2) = E[X(t+t_1)\, X(t+t_2)]$$

auto correlation

$t_1 = t_2$

$$R_x(t_1, t_1) = E[X^2(t+t_1)]$$ 2nd moment

if $m_{t_1} = m_{t_2} = m_{t_k} = m_x$

$R_x(t_1, t_2) = R_x(t_2 - t_1)$ → auto correlation dependant on time diff

WIDE SENSE STATIONARY (WSS)

means at all time instants (10:13 pm) for all days (365) are all same for WSS

avg of all values of 1 day is equal to mean of any specific time instant

$$\lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, P_x \, dt = m_x$$

Ergodic Process

Strict Sense Stationary → Stationary at higher order statistics

Central Limit Theorem

$$\int x(\tau)\, h(t-\tau)\, d\tau$$

$$x(t) = \int X(f)\, e^{j2\pi f t}\, df$$

$$X(f) = \int x(t)\, e^{-j2\pi f t}\, dt$$

→ $H(f) \cdot X(f)$

PCM → Pulse Code Modulation.

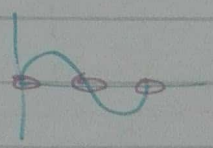
① Sampling. Pulse Amp Modulation (PAM) xplication with impulse train

② Quantization

uniform quantization → equal gaps b/w levels

min rate of change at maxima, minima

max rate of change nearer to t-axis



COMPANDER

Compressor | Expander μ-Law A-Law

uniform quantization

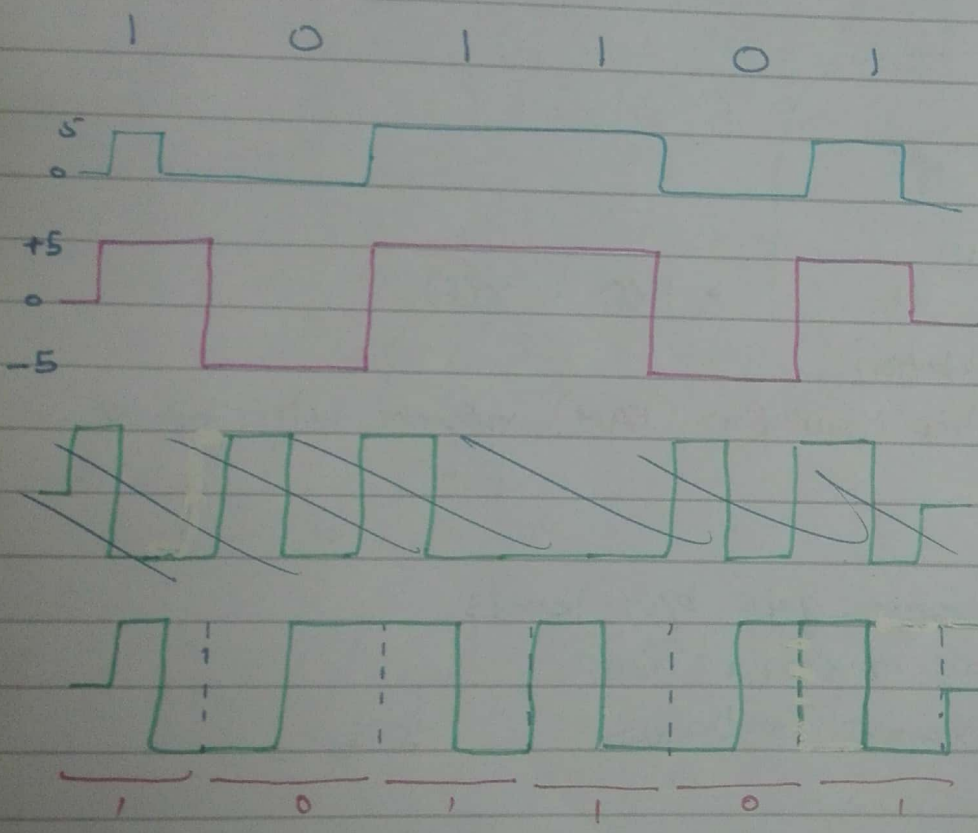
3. Digitization

represent levels in bits

4. Line Coding.

Line Codes

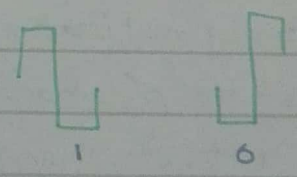
Bitstream to pulses

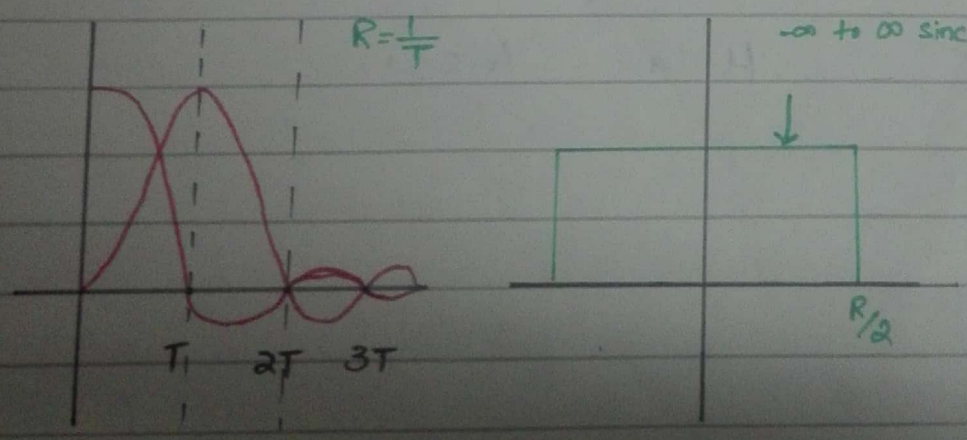


on off signal

polar signal
can differentiate b/w
data and no data

manchester coded signal

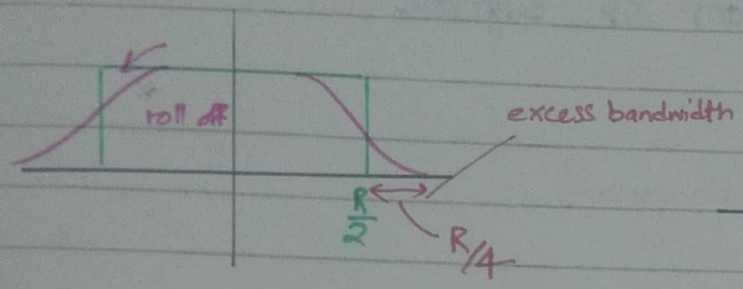


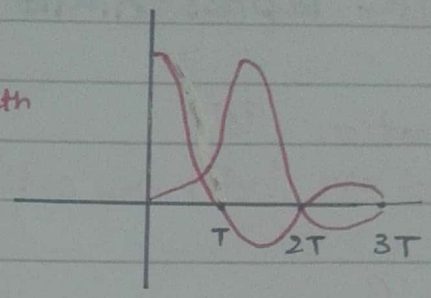


-∞ to ∞ sinc in time domain

inter symbol interference

when we limit sinc in
time, band spread
in f domain





roll off factor $r = \frac{f_x}{R/2}$ excess b/w, max freq

$r = 0.5 \quad f_x = \frac{R}{4}$

Controlled ISI Signalling

1	1	0	1	1	0	0	0	bitstream
1	2	0	0	2	0	-2	-2	encoded
1	1	0	1	1	0	0	0	decoded

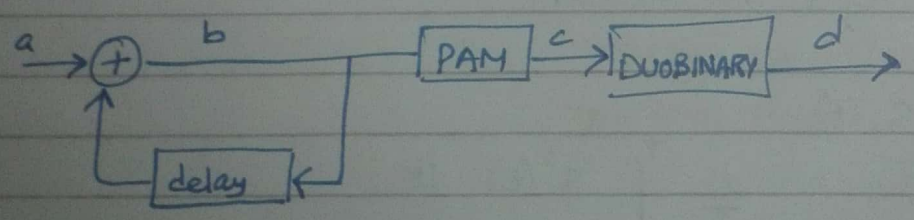
$1 \to 0 \quad 0$
$0 \to 1$
$1 \to 1 \quad 2$
$0 \to 0 \quad -2$

~~if received this: 1 2 0 0 2 2 -2 -2~~
~~1 1 0 0~~
~~error~~
~~prev should be 1~~

can detect error but not where

error ruins all seq, also can not detect where

Duobinary



a	1	1	0	1	1	0	0	0
b	1	0	0	1	0	0	0	0
c	+1	-1	-1	+1	-1	-1	-1	-1
d	+1	0	-2	0	0	-2	-2	-2
		1	0	1	1	0	0	0

only 1 bit error
whole seq not affected

where 0, 1
where non zero

Geometric representation of signal

Orthogonalization : 90°
cos and sin both are orthogonal

Ortho-normal
ortho signal when energy is one

Normalization
b/w 0-1 shifting of scale

$\phi_i(t)$, $\phi_j(t)$ → 2 signals

$$\int_0^T \phi_i(t)\, \phi_j(t)\, dt \quad \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$$

Why? we want to represent our signals using bases function
eg i, j, K

M Signals
$1 \leq i \leq M$
$s_i(t)$ → given

Orthonormal Basis Function
$N \leq M$
$1 \leq j \leq N$
$\phi_j(t)$ → unknown

Take 1st signal as $s_1(t)$
energy for s_1 = $E_1 = \frac{1}{T}\int_0^T s_1^2(t)\, dt$

$$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_1}}$$

ϕ_1 is now energy normalized signal

Geometric representation of signal

Orthogonalization : 90°
cos and sin both are orthogonal

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ortho signal when energy is one

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Take 1st signal as $s_1(t)$
energy for s_1 = $E_1 = \frac{1}{T} \int_0^T s_1^2(t)\, dt$

$$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_1}}$$

S_1 is now energy normalized signal

$$S_1(t) = S_{11}\,\phi_1(t) + S_{12}\,\phi_2(t) + S_{13}\,\phi_3(t) + \dots$$

$$= \sum_{j=1}^{N} S_{ij}\,\phi_j(t)$$

$$S_1 = [S_{11} \quad S_{12} \quad \dots \quad S_{1N}]$$ signal vector

$$S_i(t) = \sum_{j=1}^{N} S_{ij}\,\phi_j(t)$$

$$S_i = [S_{i1} \quad S_{i2} \quad \dots \quad S_{iN}]$$

$$S_2(t) = S_{21}\,\phi_1(t) + S_{22}\,\phi_2(t) + \dots$$

$$S_{21} = \int_0^T S_2(t)\,\phi_1(t)\,dt$$

$$S_{ij} = \int_0^T S_i(t)\,\phi_j(t)\,dt$$

$$S_2(t) - S_{21}\,\phi_1(t) = g_2(t)$$

$$E_{g_2} = \frac{1}{T}\int_0^T g_2^2(t)\,dt$$

$$\phi_2(t) = \frac{g_2(t)}{\sqrt{E_{g_2}}}$$

energy normalizing the remainder will give you the second basis function

$$S_3(t) = S_{31}\,\phi_1(t) + S_{32}\,\phi_2(t) + S_{33}\,\phi_3(t)$$

$$g_3(t) = S_3(t) - S_{31}\,\phi_1(t) - S_{32}\,\phi_2(t)$$

$$E_{g_3} = \frac{1}{T}\int_0^T g_3^2(t)\,dt$$

$$\phi_3(t) = \frac{g_3(t)}{\sqrt{E_{g_3}}}$$

Pulse Amplitude Modulation

Amplitude Shift Keying

$$S_i(t) = A_i \cos(2\pi f_c t)$$

$$S_1(t) = \cos(2\pi f_c t)$$

$$S_2(t) = -\cos(2\pi f_c t)$$

$$E_1 = \frac{1}{T}\int_0^T s_1^2(t)\, dt$$

$$= \frac{1}{T}\int_0^T \cos^2(2\pi f_c t)\, dt$$

$$= \frac{1}{T}\int_0^T \frac{1}{2}\left(1 + \cancel{\cos(4\pi f_c t)}^{\,0}\right) dt$$

$$= \frac{1}{2}$$

$$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_1}} = \sqrt{2}\cos(2\pi f_c t)$$

$$s_{11} = \frac{1}{\sqrt{2}}$$

$$s_{21} = \int_0^T s_2(t)\,\phi_1(t)\, dt$$

$$= \int_0^T -\cos(2\pi f_c t)\,\sqrt{2}\cos(2\pi f_c t)\, dt$$

$$= \frac{-1}{\sqrt{2}}$$

$$g_2(t) = s_2(t) - s_{21}\,\phi_1(t)$$
$$= 0$$

$$\phi_1(t) = \sqrt{2}\,\cos(2\pi f_c t)$$

$$s_1 = \frac{1}{\sqrt{2}} \qquad s_2 = \frac{-1}{\sqrt{2}}$$

-3	00
-1	01
1	10
3	11

more bits combined together → higher data rate
but also more power required

-7	-5	-3	-1	1	3	5	7
s_8	s_6	s_4	s_2	s_1	s_3	s_5	s_7

only need 1 basis function (One dimensional data)

Phase Shift Keying

$$S_i(t) = \cos\left[2\pi f_c t + \frac{(i-1)2\pi}{M}\right]$$

M = total signals = 2

$$S_1(t) = \cos\left[2\pi f_c t + \frac{2\pi}{2}(1-1)\right]$$

$$S_1(t) = \cos 2\pi f_c t$$

$$S_2(t) = \cos\left[2\pi f_c t + \underbrace{\frac{2\pi}{2}(2-1)}_{\pi}\right]$$

$$S_2(t) = -\cos(2\pi f_c t)$$

M = 4

$$S_1(t) = \cos 2\pi f_c t$$

$$S_2(t) = \cos\left[2\pi f_c t + (2-1)\underbrace{\frac{2\pi}{4}}_{\frac{\pi}{2}}\right]$$

$$S_2(t) = -\sin 2\pi f_c t$$

$$S_3(t) = \cos\left[2\pi f_c t + (3-1)\underbrace{\frac{2\pi}{4}}_{\pi}\right]$$

$$= -\cos(2\pi f_c t)$$

$$S_4(t) = +\sin(2\pi f_c t)$$

$$\phi_1(t) = \sqrt{2} \cos 2\pi f_c t$$

$$s_{11} = \frac{1}{\sqrt{2}}$$

$$s_{21} = \int_0^T s_2(t)\, \phi_1(t)\, dt$$

$$= \sqrt{2} \int_0^T \sin(2\pi f_c t) \cos(2\pi f_c t)\, dt$$

$$s_{21} = 0$$

$$g_2(t) = s_2(t) - s_{21}\, \phi_1(t)$$

$$= s_{2s}(t)$$

$$= -\sin 2\pi f_c t$$

$$E_{g_2} = \frac{1}{2}$$

$$\phi_2(t) = \frac{g_2(t)}{\sqrt{E_{g_2}}}$$

$$= -\sqrt{2} \sin(2\pi f_c t)$$

$$s_{22} = \int_0^T s_2(t)\, \phi_2(t)\, dt$$

$$= \frac{1}{\sqrt{2}}$$

$$s_{31} = \int_0^T s_3(t)\ \phi_1(t)\ dt$$

$$= -\sqrt{2} \int_0^T \cos(2\pi f_c t) \cos(2\pi f_c t)\ dt$$

$$= -\frac{1}{\sqrt{2}}$$

$$s_{32} = \int_0^T s_3(t)\ \phi_2(t)\ dt$$

$$= \sqrt{2} \int_0^T \cos(2\pi f_c t) \sin(2\pi f_c t)\ dt$$

$$= 0$$

$$g_3(t) = s_3(t) - s_{31}\ \phi_1(t) - s_{32}\ \overset{0}{\phi_2(t)}$$

$$= -\cos(2\pi f_c t) - \left(-\frac{1}{\sqrt{2}}\right) \sqrt{2} \cos(2\pi f_c t)$$

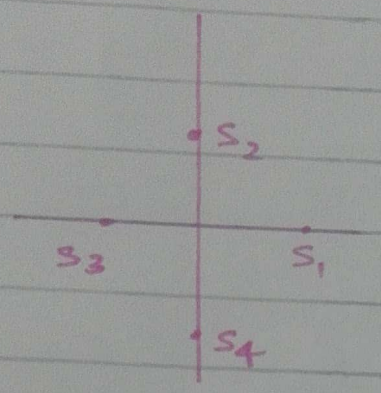
$$= 0$$

$$\phi_1 = \sqrt{2} \cos(2\pi f_c t)$$

$$\phi_2 = -\sqrt{2} \sin(2\pi f_c t)$$

$$s_1 = \left[\frac{1}{\sqrt{2}} \quad 0\right] \qquad s_3 = \left[-\frac{1}{\sqrt{2}} \quad 0\right]$$

$$s_2 = \left[0 \quad \frac{1}{\sqrt{2}}\right] \qquad s_4 = \left[0 \quad -\frac{1}{\sqrt{2}}\right]$$



QAM

Quadrature

$M = 8$ PSK

$$s_i(t) = \cos\left[2\pi f_c t + (i-1)\frac{2\pi}{M}\right]$$

$$s_1(t) = \cos 2\pi f_c t$$

$$\phi_1(t) = \sqrt{2}\cos 2\pi f_c t$$

$$\phi_2(t) = \sqrt{2}\sin 2\pi f_c t$$

$$s_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 \end{bmatrix}$$

$$s_2(t) = \frac{1}{\sqrt{2}}\cos 2\pi f_c t - \frac{1}{\sqrt{2}}\sin 2\pi f_c t$$

$$s_2 = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

$$s_3(t) = -\sin(2\pi f_c t)$$

$$s_3 = \begin{bmatrix} 0 & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

$$s_4(t) = -\frac{1}{\sqrt{2}}\cos(2\pi f_c t) - \frac{1}{\sqrt{2}}\sin 2\pi f_c t$$

$$S_4 = \left[-\frac{1}{2} \quad -\frac{1}{2} \right]$$

$$S_5 = \left[-\frac{1}{\sqrt{2}} \quad 0 \right]$$

$$S_6 = \left[-\frac{1}{2} \quad \frac{1}{2} \right]$$

$$S_7 = \left[0 \quad \frac{1}{\sqrt{2}} \right]$$

$$S_8 = \left[\frac{1}{2} \quad \frac{1}{2} \right]$$

all pts of PSK have same energy, lie on circle, separated only by phase

AWGN

$$x(t) = S_i(t) + w(t)$$

$$f_x(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[- \frac{(x - \overset{0}{\bar{x}})^2}{2\sigma} \right]$$

$$\underset{\text{received}}{x} = \overset{\text{transmitted}}{S_i} + \underset{\text{noise}}{W} \qquad x_j = \overset{\text{new mean value}}{S_{ij}} + W_j$$

$$f_x(\overset{\text{vector}}{\underline{x}} \mid \overset{\text{message}}{m_i}) = \prod_{j=1}^{N} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(- \frac{(x_j - S_{ij})^2}{2\sigma} \right)$$

$$= \frac{1}{(2\pi\delta^2)^{N/2}} \exp\left[-\sum_{j=1}^{N} \frac{(x_j - s_{ij})^2}{2\delta} \right]$$

$$f_x(\underline{x} \mid m_i) = \prod_{j=1}^{N} \frac{1}{\sqrt{2\pi\delta^2}} \exp\left[-\frac{(x_j - s_{kj})^2}{2\delta} \right]$$

$$= \frac{1}{(2\pi\delta^2)^{N/2}} \exp\left[-\sum_{j=1}^{N} \frac{(x_j - s_{kj})^2}{2\delta} \right]$$

$$= \frac{1}{(2\pi\delta^2)^{N/2}} \exp\left[-\sum_{j=1}^{N} \frac{(s_{ij} + w_j - s_{kj})^2}{2\delta} \right]$$

Maximum Likelihood Detector

$$L(\underline{x} \mid m_i) = -\frac{N}{2} \ln(2\pi\delta^2) - \sum_j \frac{(x_j - s_{kj})^2}{2\delta}$$

$$\sum_j (x_j - s_{kj})^2$$

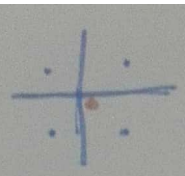
$$= \| \underline{x} - s_k \|^2$$

euclidean distance

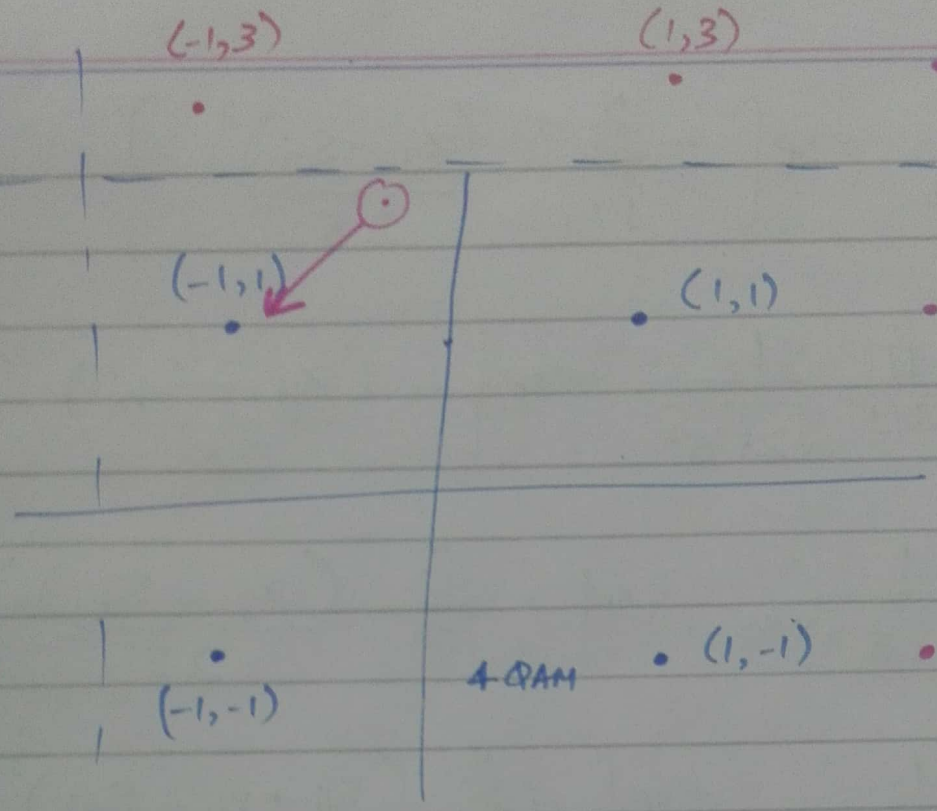
$$= \cancel{\sum x_j^2} - 2\left[\sum x_j s_{kj} + \sum s_{kj}^2 \right]$$

$$\left(\sum x_j s_{kj} - \frac{1}{2} \sum s_{kj}^2 \right)$$

4 QAM = 4 PSK



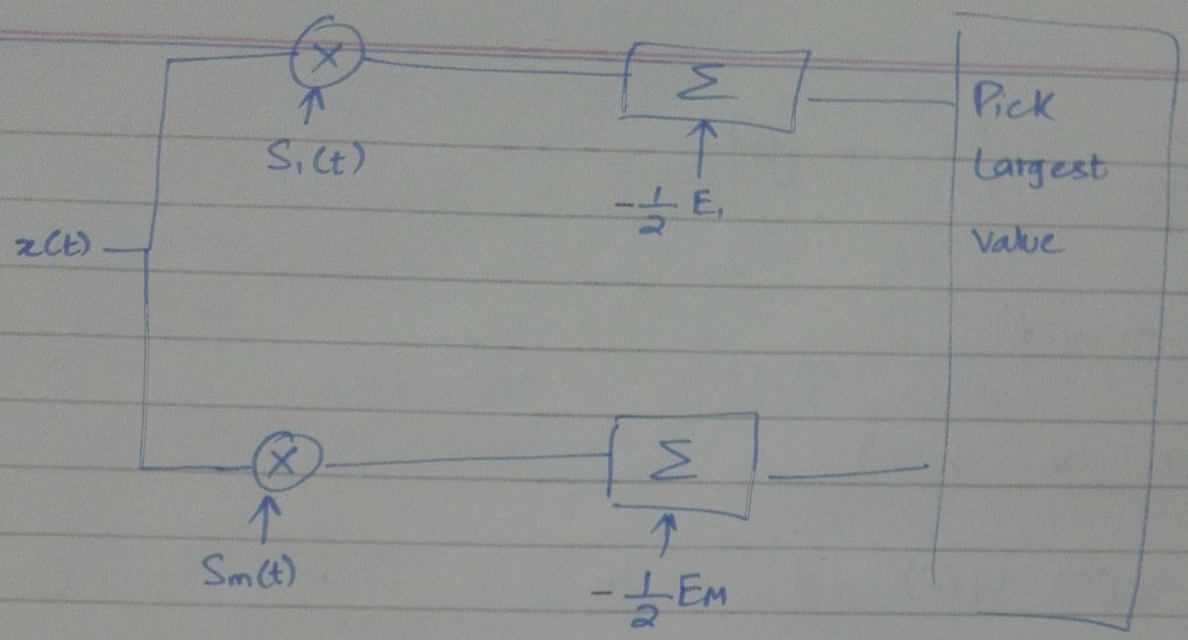
QAM



8 QAM

16 QAM

32 QAM



$$x(t) = S_i(t) + w(t)$$

$$S_i(t) = \sum_j s_{ij}\,\phi_j(t)$$

$$x(t) = \sum_j x_j\,\phi_j(t)$$

$$w(t) = \sum_j w_j\,\phi_j(t)$$

$$\sum_j x_j\,\phi_j(t) = \sum_j s_{ij}\,\phi_j(t) + \sum_j w_j\,\phi_j(t)$$

Correlator Receiver

$$x(t) \longrightarrow \boxed{\begin{matrix}\text{filter} \\ h(t)\end{matrix}} \longrightarrow y(t)$$

$$z(t) = s_i(t) + w(t)$$

$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} X(f)\, H(f)\, e^{j2\pi f T}\, df$$

$$|y(t)|^2 = \left| \int_{-\infty}^{\infty} X(f)\, H(f)\, e^{j2\pi f T}\, df \right|^2$$

<u>Schwartz Inequality</u>

$x_1(t) \qquad x_2(t)$

$$\left| \int x_1(t)\, x_2(t)\, dt \right|^2$$

$$\leq \int |x_1(t)|^2\, dt \quad \int |x_2(t)|^2\, dt$$

$$x_1(t) = K\, x_2^*(t)$$

$x_1 \rightarrow X(f)\, e^{j2\pi f T}$

$x_2 \rightarrow H(f)$

$$\leq \int_{-\infty}^{\infty} |X(f)\, e^{j2\pi f T}|^2 \, df \quad \int_{-\infty}^{\infty} |H(f)|^2 \, df$$

$$= \int_{-\infty}^{\infty} |X(f)|^2 \, df \quad \int_{-\infty}^{\infty} |H(f)|^2 \, df$$

$$w_o(t) = \int_{-\infty}^{\infty} W(f)\, H(f)\, e^{j2\pi f T} \, df$$

$$|w_o(t)|^2 = \left| \int_{-\infty}^{\infty} W(f)\, H(f)\, e^{j2\pi f T} \, df \right|^2$$

$$\leq \int_{-\infty}^{\infty} |W(f)|^2 \, df \quad \int_{-\infty}^{\infty} |H(f)|^2 \, df$$

$$\sigma_w^2 = \frac{N_0}{2} \delta(t)$$

$$\frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 \, df$$

$$\eta = \frac{|y(t)|^2}{|w_o(t)|^2}$$

$$\leq \frac{\int_{-\infty}^{\infty} |X(f)|^2 \, df \cdot \cancel{\int_{-\infty}^{\infty} |H(f)|^2 \, df}}{\frac{N_0}{2} \cancel{\int_{-\infty}^{\infty} |H(f)|^2 \, df}}$$

$$= \frac{2}{N_0} \int_{-\infty}^{\infty} |X(f)|^2 \, df$$

$$x_1 \rightarrow X(f) \, e^{j2\pi fT}$$

$$x_2 \rightarrow H(f)$$

$$K \, X(f) \, e^{j2\pi fT} = H^*(f)$$

$$H(f) = K \, X^*(f) \, e^{-j2\pi fT}$$

$$h(t) = F^{-1}\{H(f)\}$$

$$x(t) \longrightarrow X(f)$$

$$x(-t) \longleftrightarrow X^*(f)$$

$$h(t) = \int X^*(f) \, e^{-j2\pi fT} \, e^{j2\pi ft}$$

$$= \int X^*(f) \, e^{j2\pi f(t-T)} \, df$$

$$= x(T-t)$$

$$Y(t) = \int x(t)\ h(T-t)\ dt$$

$$h(t) = x(T-t)$$

$$h(T-t) = x(t)$$

$$Y(t) = \int x^2(t)\ dt$$

how to convert correlator receiver into matched filter receiver ?

$$S_i(t)$$

$$h(t) = K\ S_i(T-t)$$

$$S_i(t) * h(t)$$

$$= \int S_i(t)\ h(T-t)\ dt$$

$$= K \int S_i(t)\ S_i(t)$$

Probability of error

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{-\frac{x^2}{2}} \, dx$$

erfc

PAM / ASK

no. of symbols

distance bw 2 adjacent pts

$$A_i = (2i - 1 - M)d$$

$M = 2$

$$A_1 = (2 - 1 - 2)d$$
$$= -d$$

$$A_2 = (2 \cdot 2 - 1 - 2)d$$
$$= d$$

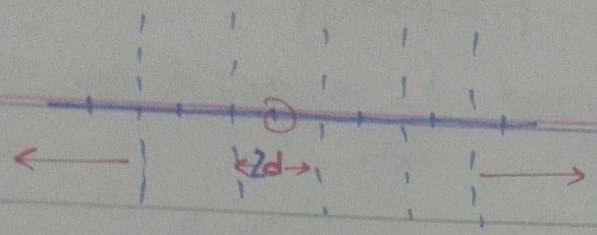
$$E_i = A_i^2$$
$$= [(2i - 1 - M)d]^2$$

$$E_{av} = \frac{1}{M} \sum_{i=1}^{M} [(2i - 1 - M)d]^2$$

$$= \frac{d^2}{M} \sum_{i=1}^{M} (2i - 1 - M)^2$$

$$= \frac{d^2}{M} \frac{1}{3} M(M^2 - 1)$$

$$= \frac{1}{3} d^2 (M^2 - 1)$$

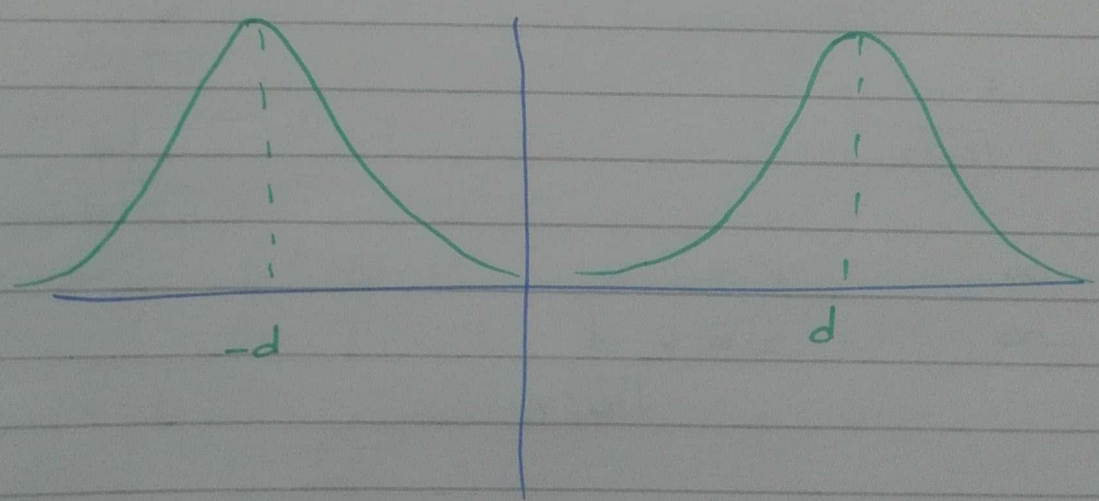


$$r = A_i + n$$

$$P\left(|r - A_i| > d \right)$$

$$\underbrace{M-2}_{\text{between points}} + \underbrace{\frac{2}{2}}_{\text{edge points}}$$

$$P_M = \frac{M-1}{M} P\left(|r - A_i| > d \right)$$



$$\sigma_n^2 = \frac{N_0}{2}$$

$$f_{r|d}(r|d) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(r-d)^2}{2\sigma^2} \right]$$

$$= \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{(r-d)^2}{N_0} \right]$$

-ve side probability

transmitted d, error if we receive -ve

$$P(|r-d| > d) = \frac{1}{\sqrt{\pi N_0}} \int_{-\infty}^{0} \exp\left[-\frac{(r-d)^2}{N_0}\right] dr$$

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{\lambda}^{\infty} e^{-\frac{x^2}{2}} dx$$

$$\frac{x^2}{2} = \frac{(r-d)^2}{N_0}$$

$$\frac{x}{\sqrt{2}} = \frac{r-d}{\sqrt{N_0}}$$

$$x = \frac{r-d}{\sqrt{N_0/2}}$$

when $r = -\infty$ $\quad$ $r = 0$

$x = -\infty$ $\quad$ $x = \frac{-d}{\sqrt{N_0/2}}$

$$\frac{dx}{dr} = \frac{1}{\sqrt{N_0/2}}$$

$$dr \rightarrow \sqrt{N_0/2}\, dx$$

$$P_e(x) = \frac{1}{\sqrt{\pi N_0}} \int_{-\infty}^{\frac{-d}{\sqrt{N_0/2}}} e^{-\frac{x^2}{2}} \cdot \sqrt{\frac{N_0}{2}}\, dx$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{-d}{\sqrt{N_0/2}}} e^{-\frac{x^2}{2}}\, dx$$

$$P_e = \frac{1}{\sqrt{\pi N_0}} \int_{0}^{\infty} \exp\left[-\frac{(r+d)^2}{N_0}\right] dr$$

$$\frac{x^2}{2} = \frac{(r+d)^2}{N_0}$$

$$\frac{x}{\sqrt{2}} = \frac{r+d}{\sqrt{N_0}}$$

$$x = \frac{r+d}{\sqrt{N_0/2}}$$

$$P_e = \frac{1}{\sqrt{2\pi}} \int_{\frac{d}{\sqrt{N_0/2}}}^{\infty} e^{-\frac{x^2}{2}}\, dx$$

$$Q\left(\frac{d}{\sqrt{N_0/2}}\right) \longrightarrow Q\left(\sqrt{\frac{d^2}{N_0/2}}\right)$$

signal power

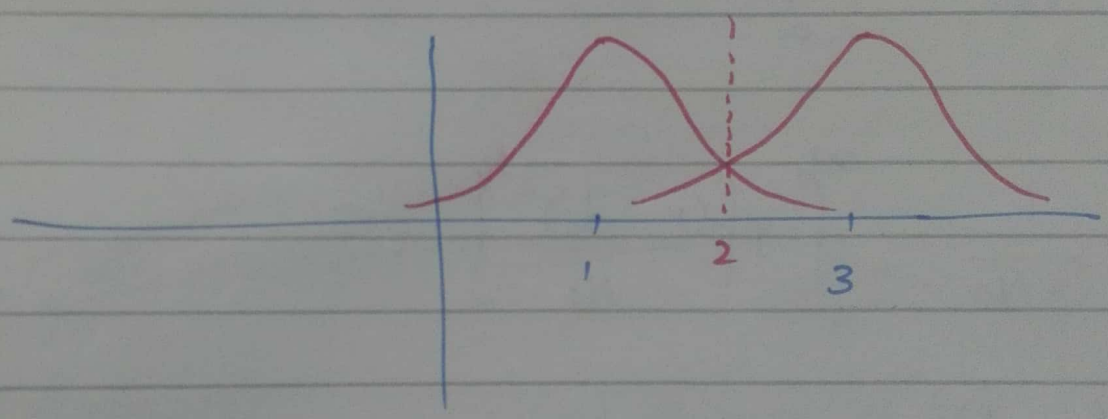
noise power

Signal to noise ratio ka sqrt

$$\sigma^2 = \frac{N_0}{2}$$

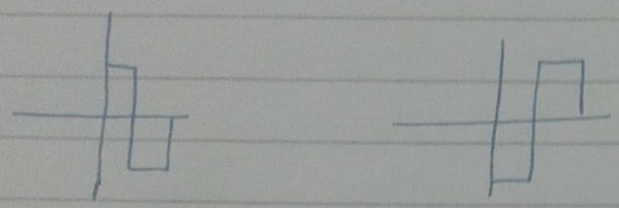
M-ARY PAM / PSK

$$\boxed{P_e = 2\,\frac{M-1}{M}\,Q\left(\sqrt{SNR}\right)}$$



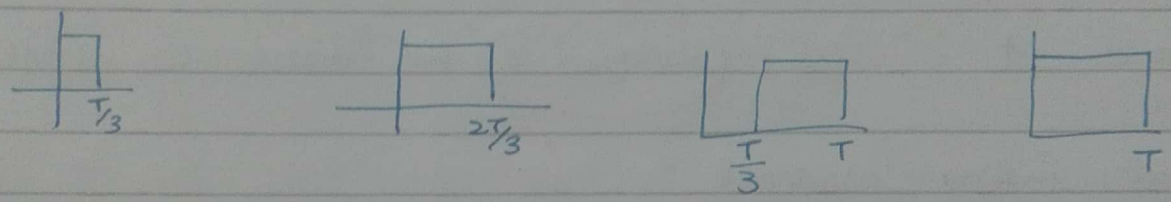
OHT-2

Q.3



$$h(t) = \begin{cases} -\frac{A}{2} & 0<t<\frac{T}{2} \\ \frac{A}{2} & \frac{T}{2}<t<T \\ 0 & \text{ow} \end{cases}$$

Q.1



$$E_1 = \frac{T}{3}$$

$$\phi_1 = \frac{1}{\sqrt{T/3}} = \sqrt{\frac{3}{T}} \quad 0<t<\frac{T}{3}$$

$$S_{11} = \sqrt{\frac{T}{3}}$$

$$S_{21} = \int_0^{T/3} S_2 \cdot \phi_1 = \sqrt{\frac{T}{3}}$$

$$g_2 = S_2 - S_{21}\,\phi_1$$

$$\phi_2 = \frac{1}{\sqrt{T/3}} = \sqrt{\frac{3}{T}} \quad \frac{T}{3}<t<\frac{2T}{3}$$

$$S_{31} = \int S_3 \cdot \phi_1 = 0 \quad \text{no overlap}$$

$$S_{32} = \int_{T/3}^{2T/3} S_3\,\phi_2 = \sqrt{\frac{T}{3}}$$

$$g_3 = S_3 - \cancel{S_{31}\,\phi_1}^{\,0} - S_{32}\,\phi_2$$

$$\phi_3 = \sqrt{\frac{3}{T}} \quad \frac{2T}{3}<t<T$$

$$S_1 = \left[\sqrt{\frac{T}{3}} \quad 0 \quad 0\right]$$

$$S_2 = \left[\sqrt{\frac{T}{3}} \quad \sqrt{\frac{T}{3}} \quad 0\right]$$

$$S_3 = \left[0 \quad \sqrt{\frac{T}{3}} \quad \sqrt{\frac{T}{3}}\right]$$

$$S_4 = \left[\sqrt{\frac{T}{3}} \quad \sqrt{\frac{T}{3}} \quad \sqrt{\frac{T}{3}}\right]$$

received transmitted AWGN

multiple paths → multiple reflections

$$r(t) = s(t) + n(t)$$

① Delay

② Attenuation

fading effect : s_2 + delayed s_1 will distort s_2 (attenuation)

multipath fading channel

PAM / ASK

$$s_i(t) = A_i \, p(t) \cdot \cos 2\pi f_c t$$

reflected signal

$$r(t) = s_i(t) + \alpha_1 \, s_i(t - \tau_1) + n(t)$$

$$= A_i \, p(t) \cos(2\pi f_c t) + \alpha_1 A_i \, p(t - \tau_1) \cos[2\pi f_c (t - \tau_1)] + n(t)$$

remove carrier

delay factor

$$y(t) = A_i \, p(t) + \alpha_1 A_i \, p(t - \tau_1) \underline{\cos(2\pi f_c \tau_1)} + n(t)$$

QAM.

$$s(t) = \sum_K \left[R_e\{s_k\}\, p(t-kT) \right] \cos(2\pi f_c t)$$

$$+ \sum_K \left[I_m\{s_k\}\, p(t-kT) \right] \sin 2\pi f_c t$$

$$s_0\, p(t) + s_1\, p(t-T) + s_2\, p(t-2T)$$

direct path $\delta(t) + \sum_{i=1}^{K} \alpha_i\, s_i(t-\tau_i)$ K multiple reflections

$$r(t) = s(t) + \sum_{i=1}^{K} \alpha_i\, s(t-\tau_i) + n_c(t)$$ K+1 paths

$$= \sum_{i=0}^{K} \alpha_i\, s(t-\tau_i) + n_c(t)$$ $\alpha_0 = 1$, $\tau_0 = 0$

$$= \sum_i \alpha_i \left\{ \sum_K \left[R_e\{s_k\}\, p(t-kT-\tau_i) \right] \cos(2\pi f_c (t-\tau_i)) \right.$$

$$\left. + \sum_K \left[I_m\{s_k\}\, p(t-kT-\tau_i) \right] \sin 2\pi f_c (t-\tau_i) \right\} + n_c(t)$$

$$y(t) = \sum_K s_k \left[\sum_i \alpha_i\, e^{-j2\pi f_c \tau_i}\, p(t-kT-\tau_i) \right] + n(t)$$

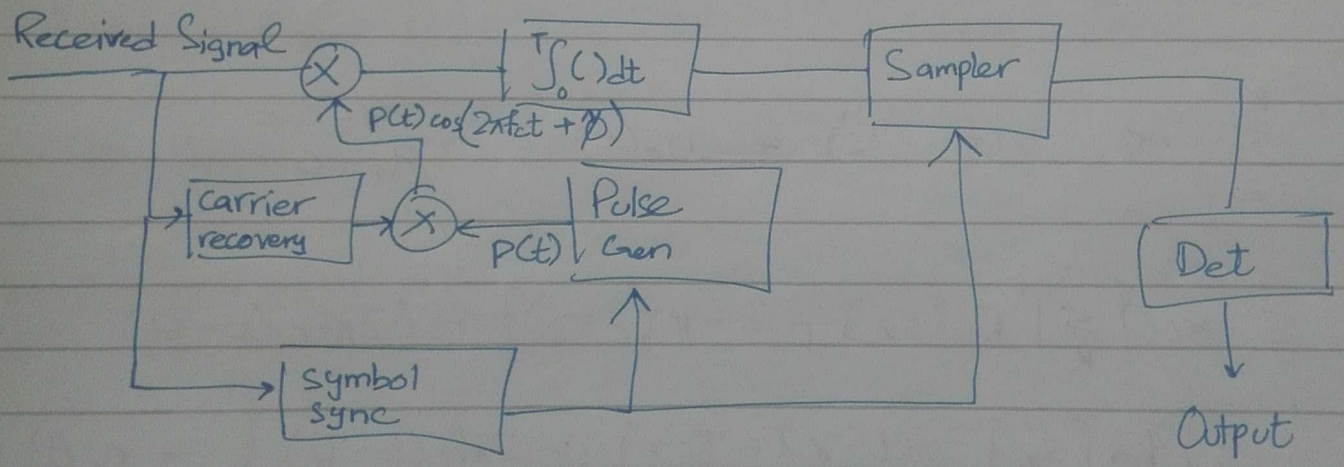
Synchronization

Doppler's Efect → difference in speed b/w sender, receiver is going to be reflected in change in freq of carrier. (Phase difference)

Oscillator → generates signal. can not create perfect oscillator

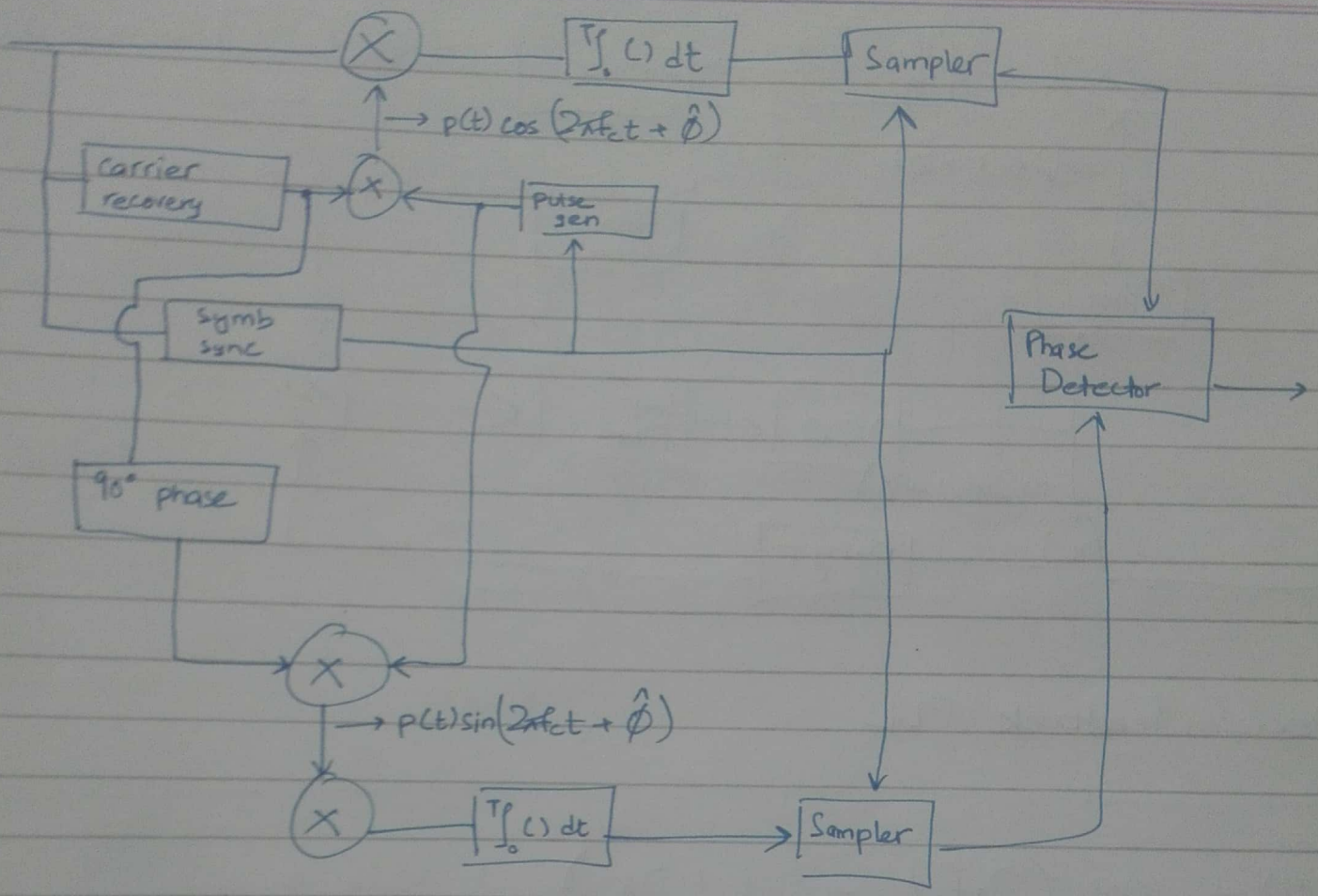
Drift factor → change in freq over time due to wear n tear

receiver needs to have generic phase estimator



$$r(t) = \alpha\, s(t-\tau)\cos(2\pi f_c (t-\tau) + \phi)$$

M - PSK



Phase estimation

$\cos(\phi - \hat{\phi})$

$\cos^2(\phi - \hat{\phi})$

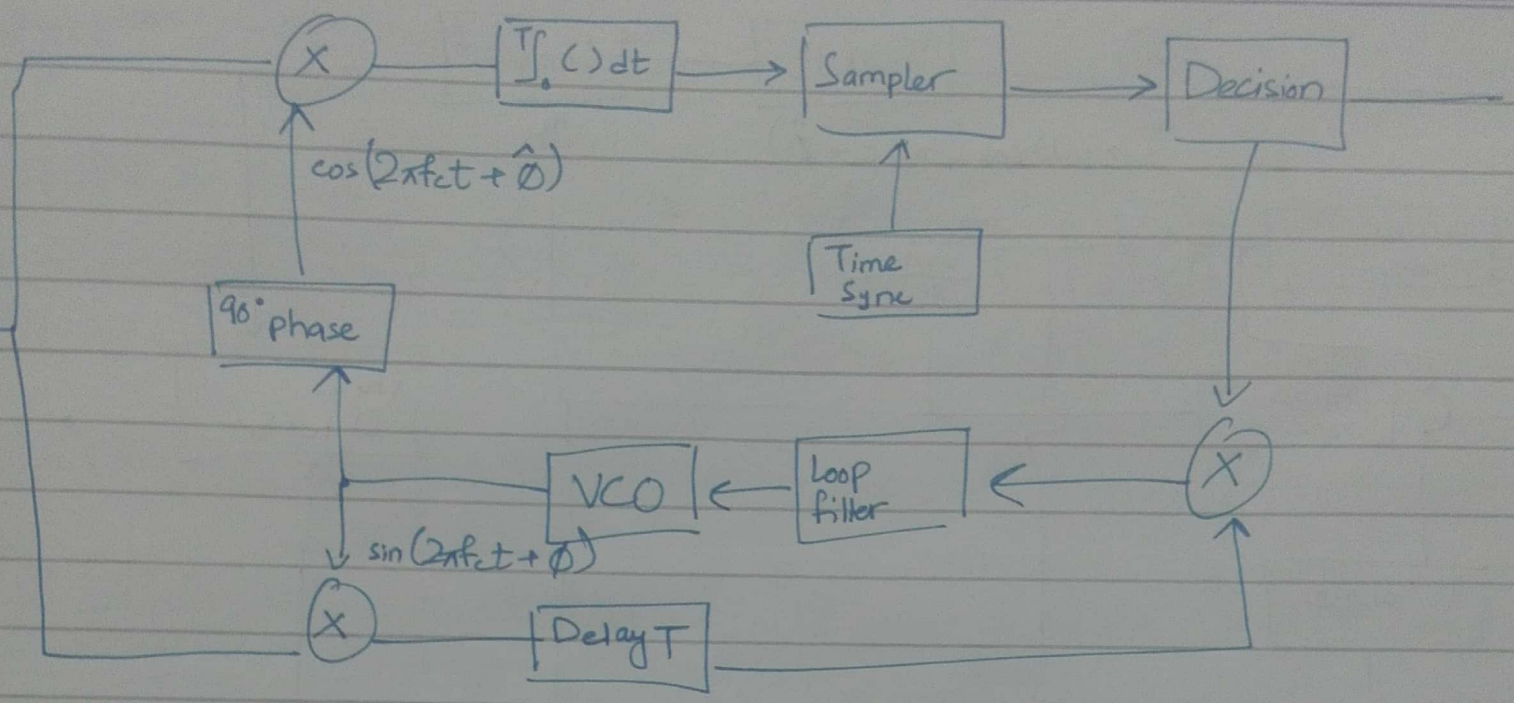
for 30° phase diff, 1.25 dB power loss

3 dB → power doubled or halved

maximum likelihood → to set a benchmark

phase locked loop

PLL (PAM)



Decision feedback PLL

$$s_i(t) = \cos\left(2\pi f_c t + \frac{i-1}{M} 2\pi\right)$$

$$+ \phi + 2\pi f_c \tau$$

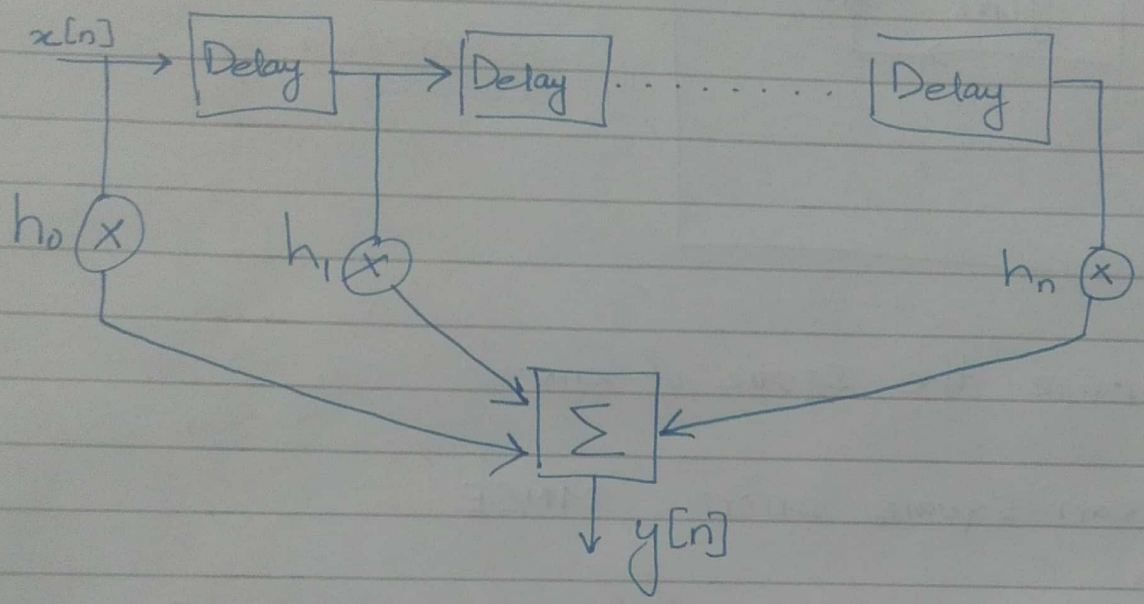
Phase & Time are jointly estimated

Channel Estimation

estimating the effects of channel

Tapped Delay Line

TDL filter

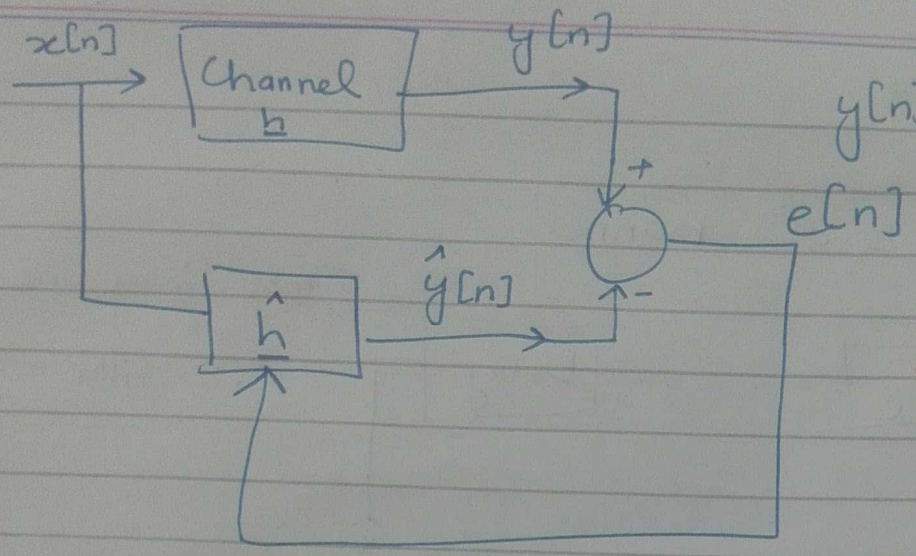


$$y[n] = h_0\, x[n] + h_1\, x[n-1] + \dots h_n\, x[n-N]$$

$$= \underline{h}^T\, \underline{x}[n]$$

$$= x^T[n] \cdot h$$

$$x[n] = \begin{bmatrix} x[n] \\ x[n-1] \\ \vdots \\ x[n-N] \end{bmatrix} \qquad h[n] = \begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_n \end{bmatrix}$$



$y[n] = x^T[n] \cdot h[n] + v[n]$

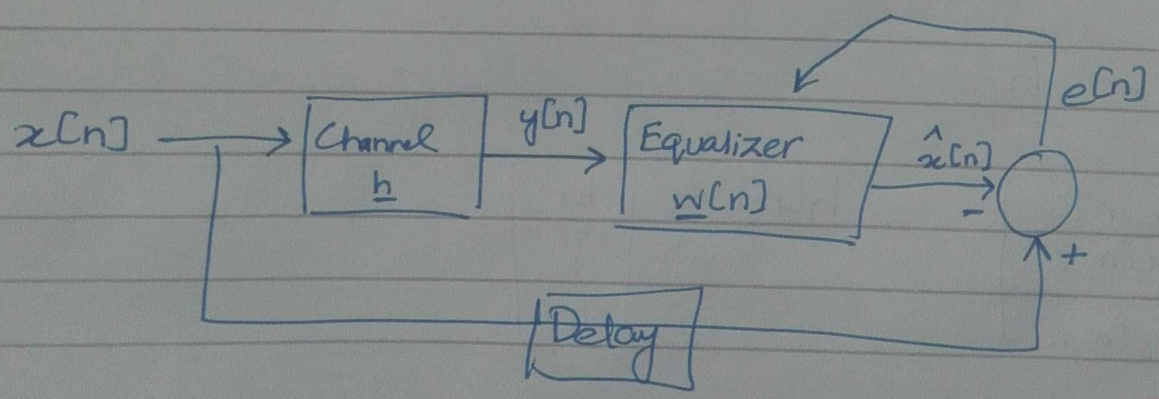
estimating y

We try to minimize the square of error

minimum mean square error MMSE

$\hat{\underline{h}}[n+1] = \hat{\underline{h}}[n] + \mu\, e[n]\, \underline{x}[n]$ Least Mean Square Algorithm

Rake Receiver



estimating x

$y[n] = \underline{x}^T[n] \cdot h + v[n]$

$\hat{x}[n] = y^T[n] \cdot w[n]$

$e[n] = x[n] - \hat{x}[n]$

$$w[n+1] = w[n] + \mu e[n] y[n]$$

zero forcing equalizer

$$\begin{bmatrix} y_0 & y_1 & y_2 \\ y_{-1} & y_0 & y_1 \\ y_{-2} & y_{-1} & y_0 \end{bmatrix} \begin{bmatrix} w_{-1} \\ w_0 \\ w_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$y_{-2} = 0.1$
$y_{-1} = -0.3$
$y_0 = 0.9$
$y_1 = 0.2$
$y_2 = 0$

$$\begin{bmatrix} 0.9 & 0.2 & 0 \\ -0.3 & 0.9 & 0.2 \\ 0.1 & -0.3 & 0.9 \end{bmatrix} \begin{bmatrix} w_{-1} \\ w_0 \\ w_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$w_{-1} = -0.2140$
$w_0 = 0.9631$
$w_1 = 0.3445$

$\hat{x}_{-1} = 0$
$\hat{x}_0 = 1$
$\hat{x}_1 = 0$

$\hat{x}_{-3} = 0$
$\hat{x}_{-2} = -0.0428$
$\hat{x}_2 = -0.0071$
$\hat{x}_3 = 0.0345$

residual values

LMS provides better solution as compared to ZFE but LMS is iterative so slow

Orthogonal Frequency Division Multiplexing (OFDM)

$$\underset{\text{received}}{r[k]} = \sum_{i=0}^{L} \underset{\text{Channel}}{h[i]} \; \underset{\text{Signal}}{s[k-i]} + \text{AWGN}$$

$$= \begin{bmatrix} h[0] & \dots & h[L] \end{bmatrix} \begin{bmatrix} s[k] \\ s[k-1] \\ \vdots \\ s[k-L] \end{bmatrix}$$

N = no of symbols in block

$$\begin{bmatrix} r[N] \\ \vdots \\ r[1] \end{bmatrix} = \begin{bmatrix} h[0] & \dots & h[L] & & \\ & \ddots & & \ddots & \\ & & & h[0] & \dots & h[L] \end{bmatrix} \begin{bmatrix} s[N] \\ \vdots \\ s[1] \\ s[0] \\ \vdots \\ s[-L+1] \end{bmatrix}$$

replace

$s[N]$

$\vdots$

$s[N-L+1]$

$$r[N] = \begin{bmatrix} h[0] & \dots & h[L] \end{bmatrix} \begin{bmatrix} s[N] \\ \vdots \\ s[N-L] \end{bmatrix}$$

$$\begin{bmatrix} h[0] & \cdots & h[L] & 0 & \cdots & 0 \\ \vdots & & & & & \\ h[1] & \cdots & h[L] & 0 & \cdots & 0 \quad h[0] \end{bmatrix} \begin{bmatrix} s[N] \\ \vdots \\ s[1] \end{bmatrix}$$

H_{cp} cyclic prefix

$$W_N = \exp\left(-j\frac{2\pi}{N}\right)$$

$$S[k] = \sum_{0}^{N-1} s[n]\, e^{\frac{-j2\pi kn}{N}}$$

$$= \sum_{0}^{N-1} s[n]\, W_N^{kn}$$

$$W_N = \begin{bmatrix} W_N^{0\cdot 0} & \cdots & W_N^{0\cdot(N-1)} \\ W_N^{1\cdot 0} & \cdots & W_N^{1(N-1)} \\ \vdots & & \\ W_N^{(N-1)0} & \cdots & W_N^{(N-1)^2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \cdots & 1 \\ 1 & \cdots & W_N^{(N-1)} \\ \vdots & & \\ 1 & \cdots & W_N^{(N-1)^2} \end{bmatrix}$$

$$W_N^{-1} = \frac{1}{N}\begin{bmatrix} 1 & \cdots & 1 \\ \vdots & & \\ 1 & \cdots & W_N^{-(N-1)^2} \end{bmatrix}$$

$$H_{cp} \cdot W_N^{-1}$$

$$= \frac{1}{N}\begin{bmatrix} h[0] & \cdots & h[L] & & \\ & \ddots & & & \\ h[1] & \cdots & h[L] & \cdots & h[0] \end{bmatrix}\begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \\ 1 & & W_N^{-(N-1)^2} \end{bmatrix}$$

$$= \frac{1}{N}\begin{bmatrix} H[0] & H[-1] & \cdots & H[-N+1] \\ H[0] & H[-1]W_N^{-1} & \cdots & H[-N+1]W_N^{-1} \\ \vdots & & & \\ H[0] & \cdots & & H[-N+1]W_N^{-(N+1)^2} \end{bmatrix}$$

$$= \frac{1}{N}\begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \\ 1 & & W_N^{-(N-1)^2} \end{bmatrix}\begin{bmatrix} H[0] & & \\ & \ddots & \\ & & H[-N+1] \end{bmatrix}$$

$$= \frac{1}{N} \begin{bmatrix} 1 & \cdots & 1 \\ \vdots & & \\ 1 & \cdots & W_N^{=(N-1)^2} \end{bmatrix} \underbrace{\begin{bmatrix} H[N] & & \\ & \ddots & \\ & & H[1] \end{bmatrix}}_{D_H}$$

$$= W_N^{-1} D_H$$

$$H_{cp} W_N^{-1} = W_N^{-1} D_H$$

$$H_{cp} W_N^{-1} W_N = W_N^{-1} D_H W_N$$

$$H_{cp} = W_N^{-1} D_H W_N$$

$$r = H_{cp} S + AWGN$$

$$= W_N^{-1} D_H W_N \cdot S + AWGN$$

$$R = W_N \cdot r$$

$$R = \boxed{W_N \quad W_N^{-1}} D_H \boxed{W_N \cdot S} + AWGN$$

$$\downarrow$$

$$S$$

diagonal

$$R = D_H S \rightarrow \text{vector}$$

$$R[N] = H[N]\, S[N]$$

$$R[K] = H[K]\, S[K]$$

$$R[1] = H[1]\, S[1]$$

error correcting codes

Block Codes

$\underline{c} = \underline{d}\, G$

generator matrix

$\underline{c}\, H = 0$

parity check matrix

Reed Solomon Hamming

location of 1's should be maximum different between 2 code words

Convolutional Codes

↓ Decoding

① Viterbi

② Sequential

Turbo Codes derived from conv codes (serial or parallel combination)

Low Density Parity Check no. of ones < 10% in parity check matrix

LDPC codes.

Claude Shannon

Shannon's Limit 0.0045 dB

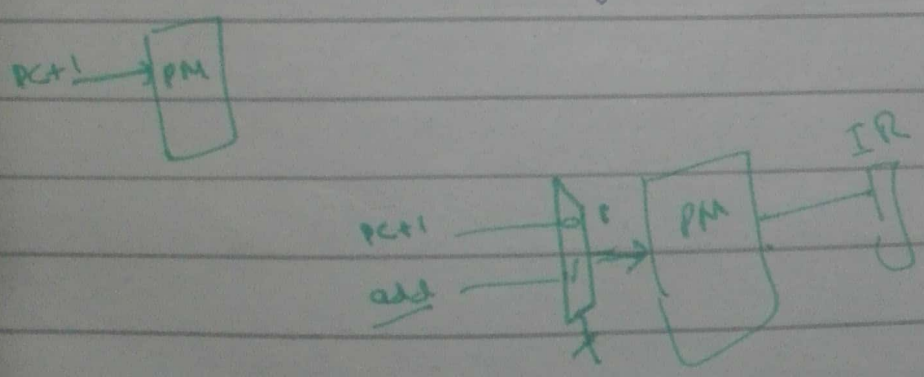
Synchronization

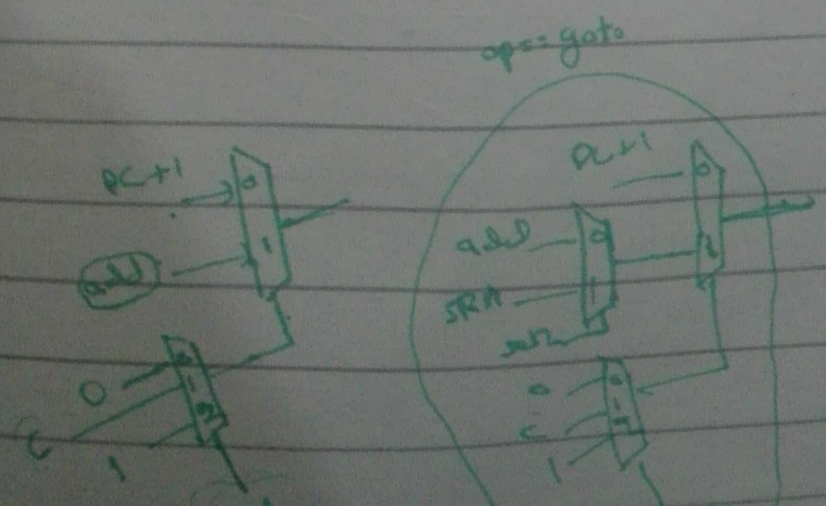
$\cos(2\pi f_c T) \leftarrow$ delay factor
effects phase

Doppler's effect: Due to relative motion of transmitter or receiver, the received signal's frequency changes and changes phase

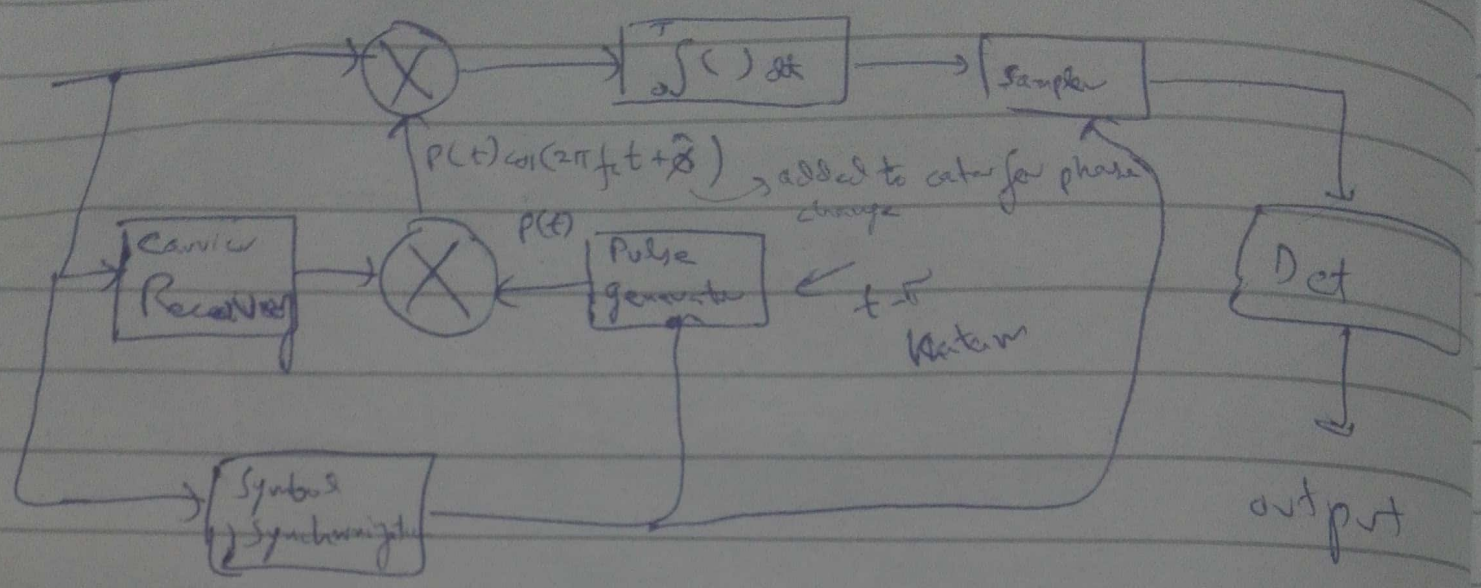
phase changes also because of ~~oscillator~~ imperfection of components hence making oscillators imperfect.

Drift: Due to wear and tear of component.
We need generic phase estimator





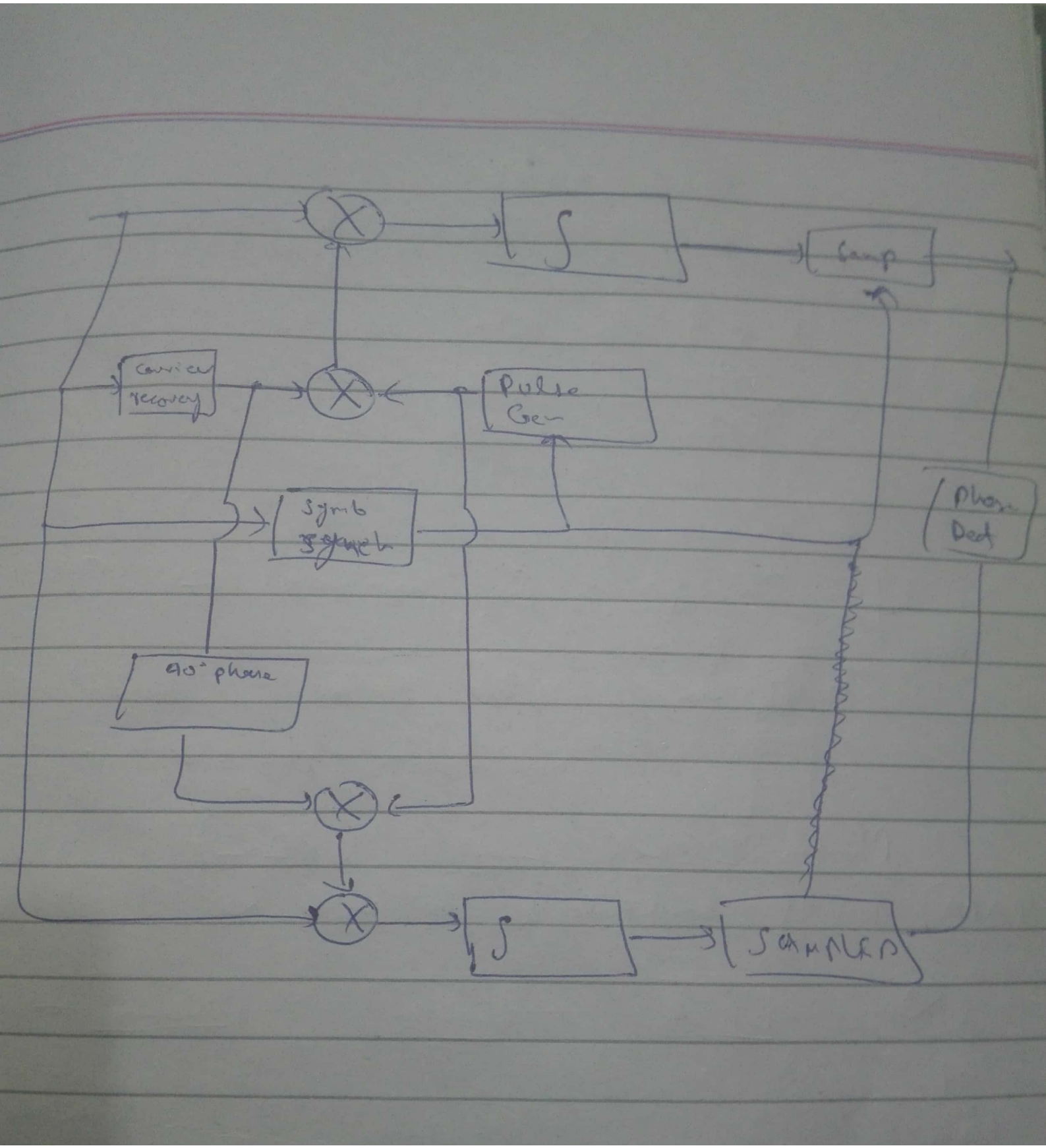
PAM



Carrier Recovery : phase estimation (
phase khatam kr rahe hain

$$r(t) = \alpha s(t-\tau)\cos(2\pi f_c(t-\tau) + \phi)$$

for one basis function (BPSK etc


Comp
Carrier recovery
Pulse Gen
Symb synch
Phase Det
90° phase
SAMPLER

$\cos(\phi - \hat{\phi})$

phase diff. reflected by cos time

ϕ less than 1 mean

$\cos^2(\phi - \hat{\phi})$

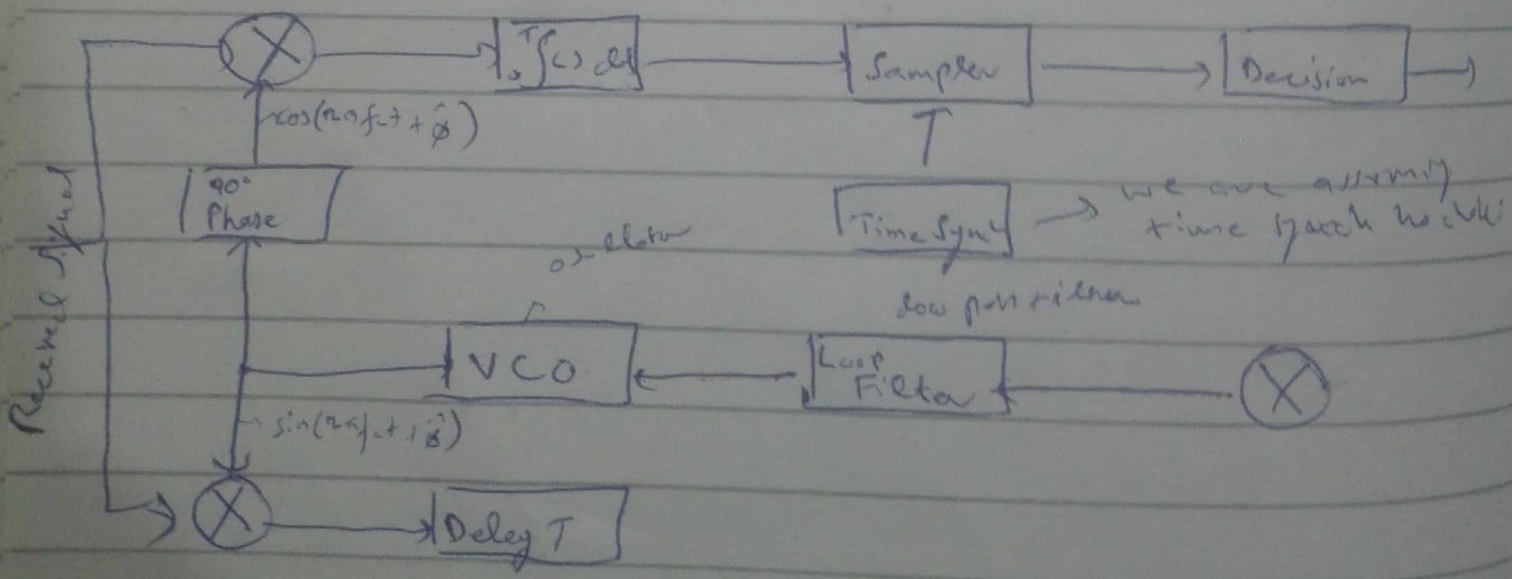
if 30° phase difference 1.25 dB loss in power.

3 dB gain/loss means double/half

Decision feedback phase locked loop

Phase Locked Loop (PLL)

PAM → sirf amplitude ki bat horaha hai



VCO = Voltage controlled oscillator

PSK should be most affected by phase difference in carrier frequency because it is entirely dependent upon phase.

$$s_i(t) = \cos\left(2\pi f_c t + \frac{i-1}{M} 2\pi\right)$$

$$+ \phi + 2\pi f_c T$$

Decision Feedback Time synchronized systems

We use ~~Time~~ Voltage Controlled clock and use differentiator instead of integrator.

18/12/17

DC (lec)

$r(t) = s(t) + n(t)$

⇒ → Delay.
→ Attenuation.

⇒ Interference
↳ Constructive → Signal Amplifies
↳ Destructive → Attenuation.

⇒ Fading Effect:-
→ Interference due to delay
→ Attenuation (Power loss) due to delay.

⇒ **MULTIPATH FADING CHANNED**

PAM/ASK:-

pulse to deal with ISI.

$s_i(t) = A_i\, p(t) \cdot \cos(2\pi f_c t)$

for now → ignoring delay and attenuation factor

$r(t) = s_i(t) + \alpha_1 s_i(t - \tau_1) + n(t)$

α_1 → attenuation factor; τ_1 → delay

$r(t) = A_i\, p(t) \cos(2\pi f_c t) + \alpha_1 p(t-\tau_1) \cos(2\pi f_c (t-\tau_1)) + n(t)$

⇒ removing carrier frequency

↳ $y(t) = A_i p(t) + \alpha_1 p(t-\tau_1) \cos(2\pi f_c \tau_1) + n(t)$

A_i → attenuation; $\cos(2\pi f_c \tau_1)$ → phase factor due to delay

Q- 2 come [illegible] straight path and 1 comes with delay and adds up with 2

QAM :-

$$s(t) = \sum_k \left[Re\{S_k\} p(t-kT) \right] \cos(2\pi f_c t)$$
$$+ \sum_k \left[Im\{S_k\} p(t-kT) \right] \sin(2\pi f_c t)$$

$$\text{Channel} = \underbrace{\delta(t)}_{\text{direct path}} + \underbrace{\sum_{i=1}^{K} \alpha_i \delta(t-\tau_i)}_{K \text{ reflections paths}}$$

$\alpha_0 = 1$
$\tau_0 = R$

$$r(t) = \underbrace{s(t) + \sum_{i=1}^{K} \alpha_i s(t-\tau_i)}_{\text{total } (K+1) \text{ paths}} + n_c(t)$$

$$r(t) = \sum_{i=0}^{K} \alpha_i s(t-\tau_i) + n_c(t)$$

$$r(t) = \sum_i \alpha_i \left\{ \sum_k \left[Re\{S_k\} p(t-kT-\tau_i) \right] \cos(2\pi f_c (t-\tau_i)) \right.$$
$$\left. + \sum_k \left[Im\{S_k\} p(t-kT-\tau_i) \right] \sin(2\pi f_c (t-\tau_i)) \right\}$$
$$+ n_c(t).$$

$$\boxed{Y(t) = \sum_k S_k \left[\sum_i \alpha_i e^{-j2\pi f_c \tau_i} p(t-kT-\tau_i) \right] + n(t)}$$

$\rightarrow$ Final signal.

2/12/17

DC (lec)

QAM (continued) :-

$$r(t) = \sum_{i=0}^{K} \alpha_i \, s(t-\tau_i) + n_c(t)$$

$$s(t) = \left[\sum_k Re\{s_k\} \, p(t-kT) \right] \cos(2\pi f_c t)$$

$$+ \left[\sum_k Im\{s_k\} \, p(t-kT) \right] \sin(2\pi f_c t)$$

so

$$r(t) = \sum_{i=0}^{K} \alpha_i \left(\sum_k \left[Re\{s_k\} \, p(t-\tau_i-kT) \right] \cos(2\pi f_c (t-\tau_i)) \right.$$

$$\left. + \left[\sum_k \left[Im\{s_k\} \, p(t-kT-\tau_i) \right] \sin(2\pi f_c (t-\tau_i)) \right] \right)$$

$$+ n_c(t)$$

$$= \sum_{i=0}^{K} \alpha_i \left(\left[\sum_k \left[Re\{s_k\} \, p(t-\tau_i-kT) \right] \right] \left[\cos(2\pi f_c t) \cos(2\pi f_c \tau_i) \right. \right.$$

$$\left. + \sin(2\pi f_c t) \sin(2\pi f_c \tau_i) \right]$$

$$+ \left[\sum_k \left[Im\{s_k\} \, p(t-\tau_i-kT) \right] \right]$$

$$\left[\sin(2\pi f_c t) \cos(2\pi f_c \tau_i) - \sin(2\pi f_c \tau_i) \cos(2\pi f_c t) \right] \Big)$$

Doplas Effect =?

No direct Path ← Rayleigh (most Basic channel)

Fading channel =?

Removing $(f_c t)$ → carrier frequency.

$$= \sum_{i=0}^{k} \alpha_i \left[\sum_k \left[Re\{s_k\} p(t-\tau_i-kT) \right] \left[\cos(2\pi f_c \tau_i) + j\sin(2\pi f_c \tau_i) \right] \right.$$

$$\left. + \left[\sum_k \left[Im\{s_k\} p(t-\tau_i-kT) \right] \left[j\cos(2\pi f_c \tau_i) - \sin(2\pi f_c \tau_i) \right] \right] \right]$$

$$= \sum_{i=0}^{k} \alpha_i \left[\sum_k \left[Re\{s_k\} p(t-\tau_i-kT) \right] \left[e^{j2\pi f_c \tau_i} \right] \right.$$

$$\left. + \sum_k \left[Im\{s_k\} p(t-\tau_i-kT) \right] j \left[\cos(2\pi f_c \tau_i) + j\sin(2\pi f_c \tau_i) \right] \right]$$

$$= \sum_{i=0}^{k} \alpha_i \left[\sum_k \left[Re\{s_k\} p(t-\tau_i-kT) \right] \left[e^{j2\pi f_c \tau_i} \right] \right.$$

$$\left. + \sum_k \left[Im\{s_k\} p(t-\tau_i-kT) \right] j \left[e^{j2\pi f_c \tau_i} \right] \right] + n_c(t)$$

after calculations :-

$$Y(t) = \sum_k s_k \left[\sum_i \alpha_i \exp\left[-j\, 2\pi f_c \tau_i \right] p(t-kT-\tau_i) \right] + n(t)$$

$$j\left[\cos(2\pi f_c \tau_i) - \frac{1}{j} \sin(2\pi f_c \tau_i) \right]$$

$$j\left[\cos(2\pi f_c \tau_i) - (-j)\sin(2\pi f_c \tau_i) \right]$$