**3.18.** The result of a single pulse (impulse) transmission is a received sequence of samples (impulse response), with values 0.1, 0.3, –0.2, 1.0, 0.4, –0.1, 0.1, where the leftmost sample is the earliest. The value 1.0 corresponds to the mainlobe of the pulse, and the other entries correspond to adjacent samples. Design a 3-tap transversal equalizer that forces the ISI to be zero at one sampling point on each side of the mainlobe. Calculate the values of the equalized output pulses at times $k = 0, \pm 1, \dots, \pm 3$. After equalization, what is the largest magnitude sample contributing to ISI, and what is the sum of all the ISI magnitudes?

**3.18 Solution** $\{x_{-3} \dots x_0, \dots, x_3\} =$

$$0.1,\ 0.3,\ -0.2,\ 1.0,\ 0.4,\ -0.1,\ 0.1$$

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} x_0 & x_{-1} & x_{-2} \\ x_1 & x_0 & x_{-1} \\ x_2 & x_1 & x_0 \end{bmatrix} \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix}$$

Solution for the weights $c_n$ can best be performed using matrix inversion as expressed by Equation (3.89b). It is helpful to use computer assistance. The result is

$$\begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} 0.8752 & 0.2593 & -0.2107 \\ -0.3079 & 0.8347 & 0.2593 \\ 0.2107 & -0.3079 & 0.8752 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Thus, the equalizer weights are

$c_{-1} = 0.2593$, $c_0 = 0.8347$, $c_1 = -0.3079$

The output samples $\{z(k)\}$ are found by convolving the input samples and the filter tap weights using Equation (3.86). For the times $k = 0, \pm 1, \ldots, \pm 3$, we obtain the equalized sample points

$$\{z(k)\} = 0.1613,\ 0.1678,\ 0.0000,$$

$$1.0000,\ 0.0000,\ -0.1807,\ 0.1143$$

Largest sample magnitude contributing to ISI $= 0.1807$

Sum of ISI magnitudes $= 0.6241$

**3.17.** A desired impulse response of a communication system is the ideal $h(t) = \delta(t)$, where $\delta(t)$ is the impulse function. Assume that the channel introduces ISI so that the overall impulse response becomes $h(t) = \delta(t) + \alpha\delta(t - T)$, where $\alpha < 1$, and $T$ is the symbol time. Derive an expression for the impulse response of a zero-forcing filter that will equalize the effects of ISI. Demonstrate that this filter suppresses the ISI. If the resulting suppression is deemed inadequate, how can the filter design be modified to increase the ISI suppression further?

## 3.17

The overall (channel and system) impulse response is $h(t) = \delta(t) + \alpha\delta(t-T)$. We need a compensating (equalizing) filter with impulse response $c(t)$ that forces $h(t) * c(t) = \delta(t)$ and zero everywhere else (zero-forcing filter). The impulse response of the equalizing filter can have the following form:

$$c(t) = c_0\delta(t) + c_1\delta(t-T) + c_2\delta(t-2T) + c_3\delta(t-3T) + \cdots$$

where $\{c_k\}$ are the weights or filter values at times k = 0, 1, 2, 3, ... After equalizing, the system output is obtained by convolving the overall impulse response with the filter impulse response, as follows:

$$\begin{aligned} h(t) * c(t) = {} & c_0\delta(t) + c_1\delta(t-T) + c_2\delta(t-2T) + c_3\delta(t-3T) + \cdots \\ & + \alpha c_0\delta(t-T) + \alpha c_1\delta(t-2T) + \alpha c_2\delta(t-3T) + \cdots \end{aligned}$$

We solve for the $\{c_k\}$ weights recursively, forcing the output to be equal to 1 at time $t = 0$, and to be 0 elsewhere.

| At $t =$ | Contribution to output | Let $c_0 = 1$ | Output |
|---|---|---|---|
| 0 | $c_0$ | $c_0 = 1$ | 1 |
| $T$ | $c_1 + \alpha c_0$ | $c_1 + \alpha c_0 = 0$<br>$c_1 = -\alpha c_0$<br>$c_1 = -\alpha$ | 0 |
| $2T$ | $c_2 + \alpha c_1$ | $c_2 + \alpha c_1 = 0$<br>$c_2 = -\alpha c_1$<br>$c_2 = +\alpha^2$ | 0 |
| $3T$ | $c_3 + \alpha c_2$ | $c_3 + \alpha c_2 = 0$<br>$c_3 = -\alpha c_2$<br>$c_3 = -\alpha^3$ | 0 |
| $4T$ | $\alpha c_3$ | | $-\alpha^4$ |

Therefore, the filter impulse response is:

$$c(t) = \delta(t) - \alpha\delta(t-T) + \alpha^2\delta(t-2T) - \alpha^3\delta(t-3T)$$

And the output is:

$$\begin{aligned} r(t) = h(t) * c(t) &= 1\times\delta(t) + 0\times\delta(t-2T) + 0\times\delta(t-3T) - \alpha^4\times\delta(t-4T) \\ &= \delta(t) - \alpha^4\delta(t-4T) \end{aligned}$$

The filter can be designed as a tapped delay line. The longer it is (more taps), the more ISI terms can be forced to zero. If $\alpha = ½$, then the 4-tap filter described above has an impulse response represented by a 1 plus three 0s, and the resulting ISI has a magnitude of $(1/2)^4 = 1/256$. Further ISI suppression can be accomplished with a longer filter.

**11-1** An equivalent discrete-time channel with white gaussian noise is shown in Fig. P11-1.

**a** Suppose we use a linear equalizer to equalize the channel. Determine the tap coefficients $c_{-1}$, $c_0$, $c_1$ of a three-tap equalizer. To simplify the computation, let the AWGN be zero.

**b** The tap coefficients of the linear equalizer in (a) are determined recursively via the algorithm

$$\mathbf{C}_{k+1} = \mathbf{C}_k - \Delta \mathbf{g}_k, \qquad \mathbf{C}_k = [c_{-1k} \quad c_{0k} \quad c_{1k}]'$$

where $\mathbf{g}_k = \mathbf{\Gamma C}_k - \mathbf{b}$ is the gradient vector and $\Delta$ is the step size. Determine the range of values of $\Delta$ to ensure convergence of the recursive algorithm. To simplify the computation, let the AWGN be zero.

## Problem 11.1 :

(a)

$$F(z) = \frac{4}{5} + \frac{3}{5}z^{-1} \Rightarrow X(z) = F(z)F^*(z^{-1}) = 1 + \frac{12}{25}\left(z + z^{-1}\right)$$

Hence :

$$\mathbf{\Gamma} = \begin{bmatrix} 1 & \frac{12}{25} & 0 \\ \frac{12}{25} & 1 & \frac{12}{25} \\ 0 & \frac{12}{25} & 1 \end{bmatrix} \quad \xi = \begin{bmatrix} 3/5 \\ 4/5 \\ 0 \end{bmatrix}$$

and :

$$\mathbf{C}_{opt} = \begin{bmatrix} c_{-1} \\ c_0 \\ c_1 \end{bmatrix} = \mathbf{\Gamma}^{-1}\xi = \frac{1}{\beta}\begin{bmatrix} 1-a^2 & -a & a^2 \\ -a & 1 & -a \\ a^2 & -a & 1-a^2 \end{bmatrix}\begin{bmatrix} 3/5 \\ 4/5 \\ 0 \end{bmatrix}$$

where $a = 0.48$ and $\beta = 1 - 2a^2 = 0.539$. Hence :

$$\mathbf{C}_{opt} = \begin{bmatrix} 0.145 \\ 0.95 \\ -0.456 \end{bmatrix}$$

(b) The eigenvalues of the matrix $\mathbf{\Gamma}$ are given by :

$$|\mathbf{\Gamma} - \lambda \mathbf{I}| = 0 \Rightarrow \begin{vmatrix} 1-\lambda & 0.48 & 0 \\ 0.48 & 1-\lambda & 0.48 \\ 0 & 0.48 & 1-\lambda \end{vmatrix} = 0 \Rightarrow \lambda = 1,\ 0.3232,\ 1.6768$$

The step size $\Delta$ should range between :

$$0 \leq \Delta \leq 2/\lambda_{\max} = 1.19$$

You already have the two papers for Fading Channels and the last two topics will only be covered theoretically.