Paper helicopter project: Phase 1

Helicopters rely on a phenomenon called autorotation to slow their descent to the ground when they lose power. The air-flow past the rotors generated by the downward speed causes the rotor to spin and generate drag that slows down the fall. The paper helicopter is a simple construction that shares this property of autorotation when falling to the ground, and in the Approximation and Optimization in Engineering Design class, the objective was to build a paper helicopter (see Figure 1) that takes the longest to fall to the ground from a given height. In this project the objective is to validate the analytical model used to predict the fall time.

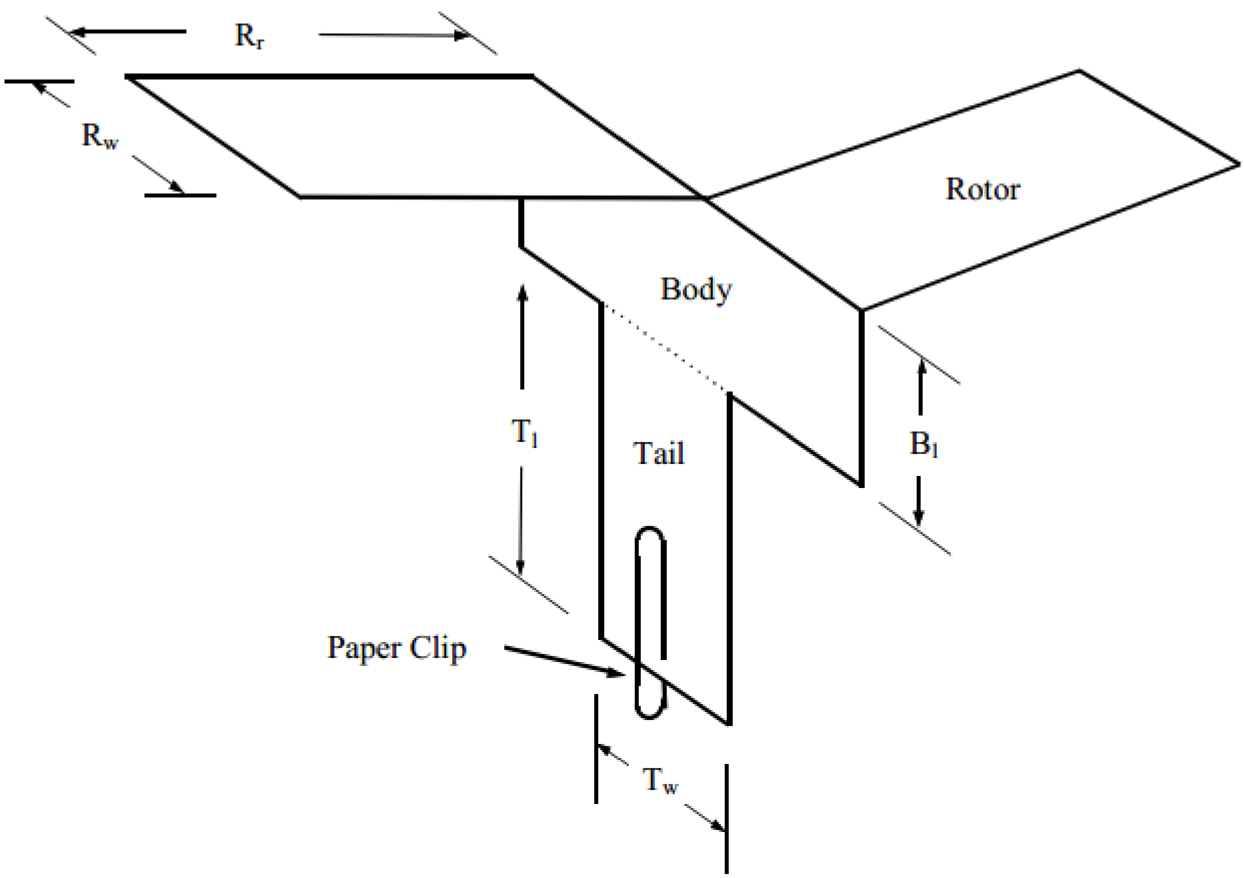

Figure 1: Paper helicopter: Dimensions in inches used for this homework: Rr = 4.0, Rw = 1.8, Tw = 0.9, Bl = 1.3, Tl = 1.8.

As the paper helicopter rotates and falls to the ground, the upward force (drag D) on it is traditionally approximately as $D = \frac{1}{2}\rho_{air}V^2 AC_D$ $\qquad A = \pi R_r^2$

However, in previous year's projects, students have found that some of the helicopters appeared to have linear dependence on the speed, so that

$$D = \frac{1}{2} \rho_{air} V V_0 A C_D$$

For some reference speed V0 (For our problem, a reasonable value is V0=3ft/sec).. The objective of the project is to validate one of the two models for the drag force for your helicopters. For that you will need to build three nominally identical helicopters. More details about construction are given in Siorek and Haftka, 98 (can be found in the files section).

For this project use the dimensions given in Figure 1 using regular copy machine paper ("Xerox paper") and one, two or three paper clips. In order to distinguish between the quadratic and linear models, you will have to test the same helicopters with different weights. In order to explore the difference between linear and quadratic helicopters, you need to photograph the first second of the fall of at least two helicopters. If some of your helicopters behave linearly and some quadratically, you need to photograph at least one of each.

Project Phase 1: (due 4/10)

1. **Comparison with results reported in paper by Park, Choi and Haftka:** For this step use two small (#1) paper clips and compare the variability in your results with those reported in the paper in order to see if your construction and testing skills are good enough. Note that this year's helicopters are about 10% smaller in dimension, in order to reduce their flexibility. Do not use a printer to print the helicopter outline on the construction paper, because this will induce curvature in the paper.
2. **Uncertainty quantification**; Build 3 nominally identical helicopter and drop each 10 times from the same height. It is recommended that you use a height of at least 10 feet, so that the free-fall part of the motion will have only a small effect. The purpose of this step is to estimate the uncertainty in the fall time. Assume that the measured fall times are normally distributed.
3. **Determination of model:** Repeat the experiments with at least three different weights (e.g. different number of paper clips) to determine whether your helicopters follow a linear or a quadratic model. Note that it is possible that some of your helicopters will have a linear model and some quadratic. Caution: For some weights the helicopter will not behave well (late start of autorotation or excessive swaying). Such weights are not useful for determining model.
4. **Calibration:** Use the experiments to calibrate the drag coefficient, CD. Start with a non-informative prior for the mean and standard deviation, and use the experiments to obtain the posterior distribution of the drag coefficient. Assume that observed drag coefficients from tests follow a normal distribution with the mean of the true CD and the standard deviation of tests so that a likelihood function of one drop is expressed as a conditional normal PDF for a given CD and a standard deviation of tests, which is expressed as $N\left(C_{D,test} \mid C_D, \sigma_{test}\right)$. $C_{D,test}$ is an calculated CD from one drop.

Report: The report will need to provide enough information for you to be able to understand it without any additional documentation five years from now, and be able to repeat all calculations from the data given in the report.