EML 6934 Verification, Validation and Uncertainty Qualification and Uncertainty Quantification

Paper helicopter project: Phase 1

Introduction

This document serves as a report on our experiments with using paper helicopters for VVUU course project.

The helicopters were made of paper and paperclips were added to simulate the weight of total helicopter. The paper helicopter autorotates when they are dropped from an altitude. The helicopters were assumed to follow one of the two models, linear or quadratic model based on the dependence of their drag on velocity. Where

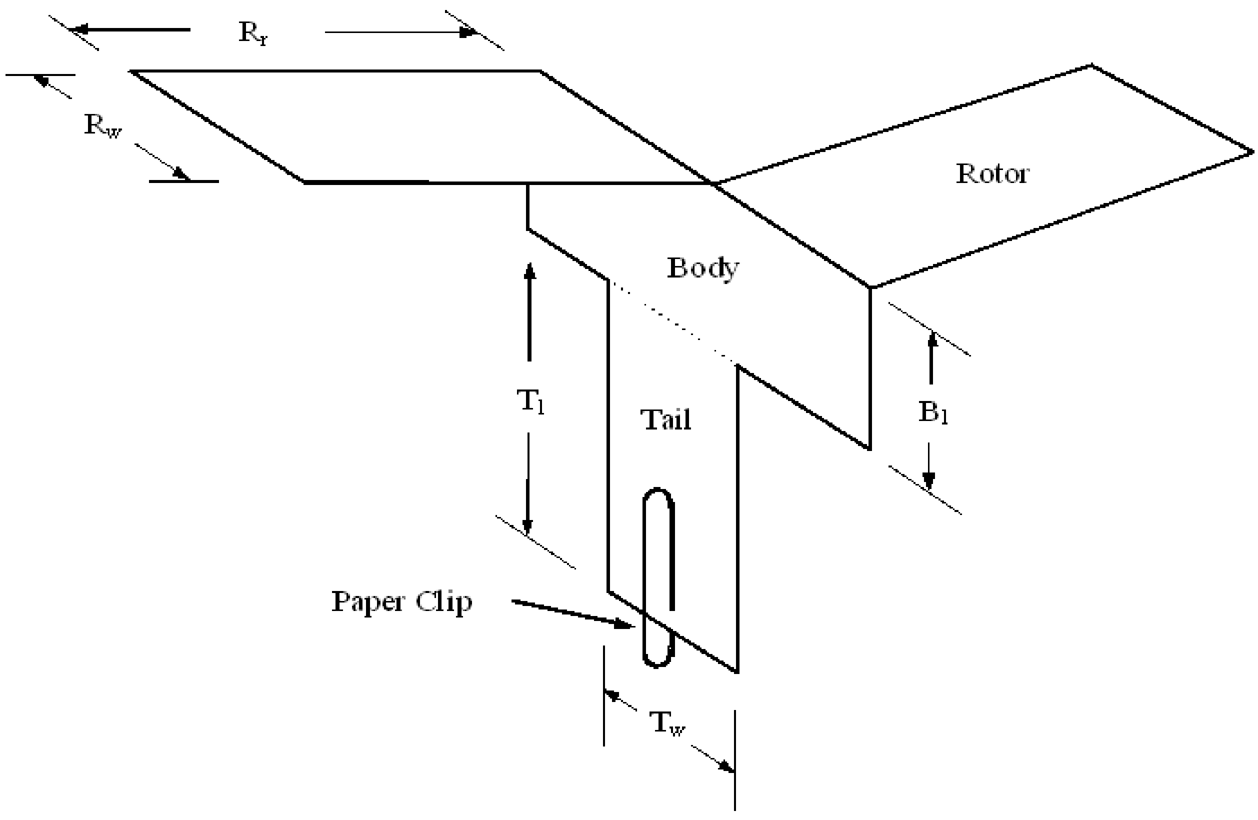


Figure 1: Paper Helicopter

the drag produced by autorotation slows their fall. A quadratic dependence of the drag on the speed is generally valid for high Reynolds numbers and a linear model appears for low Reynolds numbers.

When we release the paper helicopter, shown in the above figure, it starts to autorotate. This is a result of drag and we can express the force relation using Newton's second law, $F = ma$.

Drag can be expressed as $Drag = 0.5\rho C_D A V^2$

Hence the net Force, $ma = mg - 0.5\rho C_D A V^2$

As can be seen in the equation above, drag was assumed to be proportional to the square of velocity. However, there can be cases where we can find a linear dependence of velocity on drag.

Then, the equation of net force becomes , $ma = mg - 0.5\rho C_D A V_0 V$

V_0 is a constant term, which is assigned the value 3 ft/sec^2 for this project.

When the drag force and the mass of helicopter becomes equal, the helicopter achieves a steady state, which on solving differential equations can be expressed as:

Quadratic Model:

$$V_s = \sqrt{g/c} \text{ and } c = \frac{\rho_{\text{air}} A C_D}{2m}$$

Linear Model:

$$V_s = g/c \text{ and } c = \frac{\rho_{\text{air}} A C_D V_0}{2m}$$

To make the calculations simple, a reasonable assumption has been made, that the helicopter reaches steady state instantaneously after the drop. Then the experimental fall time and height can be used to calculate the V_S as:

$$V_S = \frac{height}{fall\ time}$$

The type of model followed by helicopter can be identified by taking helicopter with different masses and recording the fall times.

For a quadratic model the time and mass relation is :

$$\frac{t_1}{t_2} = \sqrt{\frac{m_2}{m_1}}$$

Whereas for a linear model,

$$\frac{t_1}{t_2} = \frac{m_2}{m_1}$$

For the project, we built 3 nominally identical helicopters and dropped each 10 times from same height and noted their fall times. The results of which were used for uncertainty calculations. The same experiment was performed with different masses by varying the paper pins as 1, 2 and 3. Which was used to identify the model of helicopter.

For calibration of Drag Coefficient (C_D), we used a non-informative prior for the mean and standard deviation, and used experiments to obtain the posterior distribution of the drag coefficients. We assumed that the observed drag coefficients from the tests followed a normal distribution with the mean of the true C_D and the standard deviation of test so that a likelihood function of one drop could be expressed as a conditional normal PDF for a given C_D and a standard deviation of tests, which is expressed as

$$N\left(C_{D,test} \mid C_D, \sigma_{test}\right).$$

$C_{D,test}$ is a calculated C_D from one drop

Since the prior is non-informative the likelihood function equals the posterior distribution which is a joint probability density. To compute the likelihood, we calculate the probability distribution value at each point for each C_D value and take their product. This is represented as surface and the contour plot in the 2D space of Mean of C_D and Standard deviation of C_D. These were plotted for every helicopter for all different masses.

Conditions and Tools used:

- The experiments were carried out in New Engineering Building at University of Florida.
- The Height of fall used for experiments was 226 inches.
- Fall times were measured with stop watch with resolution of 0.01 sec.
- Paper used to make helicopter were standard Xerox paper with mass of 4.5 grams each.
- Pins of average mass 0.5 grams each were used.

a.

To see if our testing skills are good enough we compared the variability of our experiments with the variability in fall times from the observations reported in the paper "Teaching a verification and validation course using simulations and experiments with paper helicopter" by Park, Choi and Haftka.

Summary measures are used to describe the amount of variability or spread in a set of data. The most common measures of variability are the range, the interquartile range (IQR), variance, and standard deviation. We here use the range and standard deviation to understand the variability.

Since the conditions in the experiments in the paper are different from our experiments, like, height of fall for helicopter in paper was 148.5 for drop with 2 pins and is 226 inches for our experiments and the dimensions of helicopter used in our experiments were 10% smaller, there will be some difference in the variability even if the conditions and observations were made with the same accuracy. The data used for the paper and in our experiments, are in the table below:

Observation	**Student A**	**Student B**	**Student C**	**Our Observations**
Height	**149 in**	**148.5 in**	**124 in**	**226 in**
1	3.52	4.63	4.63	4.52
2	3.2	3.43	3.43	4.72
3	3.12	4.05	4.05	4.51
4	3.24	3.81	3.81	4.45
5	3.33	3.36	3.36	4.67
6	3.16	3.67	3.67	4.61
7	3.25	3.9	3.9	4.49
8	3.28	3.86	3.86	4.57
9	3.15	3.59	3.59	4.5
10	3.33	3.51	3.51	4.63
Difference between Maximum and Minimum	0.4	1.27	1.27	0.27
Standard Deviation	0.117	0.371	0.14	0.181

As can be seen from above table the standard deviation in our results is comparable to the results made by the students for observation in the paper. Also, the range in our observations is also good enough, to show that the results are consistent.

b.

Uncertainty in fall times of individual helicopters :

- Based on helicopter with two pins each.
- Assumption: Fall times are normally distributed.

```
fall_time_1=[4.52,4.72,4.51,4.45,4.67,4.61,4.49,4.57,4.5,4.63]';
fall_time_2=[4.35,4.24,4.65,4.77,4.31,4.48,4.34,4.43,4.22,4.27]';
fall_time_3=[4.47,4.34,4.55,4.63,4.5,4.38,4.63,4.47,4.73,4.41]';
pd1= fitdist(fall_time_1,'Normal')
pd2= fitdist(fall_time_2,'Normal')
pd3= fitdist(fall_time_3,'Normal')
mean_fall_times=[4.567,4.406,4.511]';
pd_mean= fitdist(mean_fall_times,'Normal')
```

```
pd1 =
 NormalDistribution
     mu =    4.567   [4.50418, 4.62982]
   sigma = 0.0878193   [0.0604052, 0.160324]
pd2 =
 NormalDistribution
     mu =    4.406   [4.27639, 4.53561]
   sigma = 0.181181   [0.124623, 0.330766]
pd3 =
 NormalDistribution
     mu =    4.511   [4.42245, 4.59955]
   sigma = 0.123778   [0.0851392, 0.225971]
pd_mean =
  NormalDistribution
     mu =   4.49467   [4.29163, 4.6977]
   sigma = 0.0817333   [0.0425551, 0.513672]
```

The Fall time of Helicopters with confidence level of 95% are in the range:

Helicopter 1 : [4.3914, 4.7426]

Helicopter 2 : [4.0436, 4.7684]

Helicopter 3 : [4.2634, 4.7585]

The standard uncertainty (u) of the mean is calculated from:

$$u = \frac{s}{sqrt(n)}$$

Where s is the standard deviation and n is the number of observations.

Hence the standard uncertainty in mean of fall times of helicopters are:

Helicopter 1: 0.02777 secs

Helicopter 2:0.05729 secs

Helicopter 3:0.03914 secs

Uncertainty in mean of fall times of all helicopter :

Mean fall times with 95% confidence level lie in the range [4.3312, 4.6581] secs

With a standard uncertainty of 0.047188 secs

c.

Mass of Helicopters:

Mass of A4 size paper = 450 grams

Size(area) =8.27* 11.69 = 96.6763 inches

Mass of Helicopter =

$$[2 * R_W * (R_R + B_1) + (T_1 * T_w)\ inch^2] * 0.04654\ grams/inch^2$$

$$= 0.9342\ grams$$

Average mass of a pin $= 0.5\ gram$

Assuming all the helicopters with the same number of pins weigh the same, as the uncertainty in their weights can be neglected. For determining the model, comparing the mass and fall times of all Helicopters with 1 pin and 2 pins.

i) Helicopter 1,

m2/m1 = 1.3486,

sqrt(m2/m1)= 1.1613,

<t1/t2> = $\frac{5.323}{4.567} = 1.1655$

Thus helicopter follows **quadratic model**. (t1/t2 ~ square root(m2/m1))

ii) Helicopter 2,

m2/m1 = 1.3486,

sqrt(m2/m1)= 1.1613,

<t1/t2> = $\frac{4.856}{4.406} = 1.102$

Thus helicopter follows **quadratic model**. (t1/t2 ~ square root(m2/m1))

iii) Helicopter 3,

m2/m1 = 1.3486,

sqrt(m2/m1)= 1.1613,

<t1/t2> = $\frac{5.525}{4.511} = 1.2247$

Thus helicopter follows more of a **quadratic model** than **linear model** for various observations. Will be assumed as a quadratic model for all following observations.

d.

Experimental Drag coefficients are as given below:

Since all the helicopters below follow quadratic model, the Drag coefficients can be calculated as below:

$$V_S = sqrt\left(\frac{g}{c}\right)$$

$$c = \frac{\rho_{air} A C_D}{2m}$$

$$C_D = \frac{2mg}{\rho_{air} A V_S^2} * 2402.490 \ (\,for\ unit\ conversion)$$

After C_D is obtained for all experimental values. Its mean and standard deviation is obtained assuming the drag coefficient to be normally distributed.

Copter 1			
# of Clips	**1**		Quadratic Model
Mass:	0.5 grams	1.4342	
Trial	Time(sec)	Velocity(inch/sec)	CD
1	5.05	44.75247525	1.911589672
2	5.41	41.77449168	2.19384757
3	5.2	43.46153846	2.026835985
4	5.27	42.88425047	2.081771936
5	5.41	41.77449168	2.19384757
6	5.13	44.0545809	1.972634613
7	5.48	41.24087591	2.250987255
8	5.28	42.8030303	2.089679894
9	5.35	42.24299065	2.145455362
10	5.65	40	2.392813304
Average	5.323	42.45726094	2.123854942
Copter 1			
# of Clips	**2**		Quadratic Model
Mass:	1.0 gram	1.9342	
Trial	Time(sec)	Velocity(inch/sec)	CD
1	4.52	50	2.065287181
2	4.72	47.88135593	2.252099515
3	4.51	50.11086475	2.056158851
4	4.45	50.78651685	2.001813445
5	4.67	48.39400428	2.204638265
6	4.61	49.02386117	2.148351951
7	4.49	50.33407572	2.037962845
8	4.57	49.45295405	2.111232098
9	4.5	50.22222222	2.047050739
10	4.63	48.81209503	2.167033185
Average	4.567	49.48543902	2.108461149
Copter 1			
# of Clips	**3**		Quadratic Model
Mass:	1.5 grams	2.4342	
Trial	Time(sec)	Velocity(inch/sec)	CD
1	4.88	46.31147541	3.029689369
2	4.62	48.91774892	2.715453749
3	4.59	49.23747277	2.680302615
4	4.73	47.78012685	2.846300444

5	4.8	47.08333333	2.931169505
6	4.7	48.08510638	2.810309651
7	4.82	46.8879668	2.955646806
8	4.72	47.88135593	2.834278069
9	4.63	48.81209503	2.727221683
10	4.89	46.21676892	3.042118851
Average	4.738	47.69945125	2.855936663

For Helicopter 1:

- 1 pin
  mu = 2.12595 [2.02506, 2.22683]
  sigma = 0.141027 [0.0970034, 0.25746]
- 2 pin
  mu = 2.10916 [2.05094, 2.16739]
  sigma = 0.0813915 [0.055984, 0.148589]
- 3 pin
  mu = 2.85725 [2.76483, 2.94967]
  sigma = 0.1292 [0.088868, 0.235868]

Copter 2			
# of Clips	**2**		Quadratic Model
Mass:	1.0 gram	1.932	CD
Trial	Time(sec)	Velocity(inch/sec)	
1	4.35	51.95402299	1.912855191
2	4.24	53.30188679	1.817336265
3	4.65	48.60215054	2.185795289
4	4.77	47.37945493	2.300066211
5	4.31	52.4361949	1.877837987
6	4.48	50.44642857	2.028895168
7	4.34	52.07373272	1.904070563
8	4.43	51.01580135	1.983860052
9	4.22	53.55450237	1.800232019
10	4.27	52.92740047	1.843144268
Average	4.406	51.29369042	1.96242273
Copter 2			
# of Clips	**3**		Quadratic Model
Mass:	1.5 grams	2.4342	

Trial	Time(sec)	Velocity(inch/sec)	CD
1	4.6	49.13043478	2.691994216
2	4.64	48.70689655	2.73901506
3	4.74	47.67932489	2.858348263
4	4.41	51.24716553	2.474204759
5	4.74	47.67932489	2.858348263
6	4.67	48.39400428	2.774547857
7	4.44	50.9009009	2.507981908
8	4.52	50	2.599173848
9	4.4	51.36363636	2.462996598
10	4.45	50.78651685	2.519291846
Average	4.561	49.55053716	2.64654086
Copter 2			
# of Clips	**1**		Quadratic Model
Mass:	0.5 grams	1.4342	
Trial	Time(sec)	Velocity(inch/sec)	CD
1	4.51	50.11086475	1.524631902
2	4.58	49.34497817	1.572327011
3	5.01	45.10978044	1.881426994
4	4.81	46.98544699	1.73421154
5	4.57	49.45295405	1.565468449
6	5.68	39.78873239	2.418291172
7	4.81	46.98544699	1.73421154
8	4.7	48.08510638	1.655799072
9	4.55	49.67032967	1.551796301
10	5.34	42.32209738	2.137442464
Average	4.856	46.54036244	1.767540099

For Helicopter 2:

- 1 pin
  mu = 1.77756 [1.56812, 1.987]
  sigma = 0.292778 [0.201383, 0.534498]
- 2 pin
  mu = 1.96541 [1.84779, 2.08302]
  sigma = 0.164414 [0.11309, 0.300156]
- 3 pin
  mu = 2.64859 [2.53733, 2.75985]

  sigma = 0.155531 [0.10698, 0.283939]

Copter 3			
# of Clips	**3**		Quadratic Model
Mass:	1.5 grams	2.4342	CD
Trial	Time(sec)	Velocity(inch/sec)	
1	4.88	46.31147541	3.029689369
2	4.76	47.4789916	2.882520233
3	5.01	45.10978044	3.193257278
4	4.97	45.47283702	3.142470696
5	4.81	46.98544699	2.943395434
6	4.92	45.93495935	3.079559961
7	4.85	46.59793814	2.992553589
8	4.61	49.02386117	2.70371126
9	4.7	48.08510638	2.810309651
10	4.75	47.57894737	2.870421526
Average	4.826	46.82967261	2.963009843

Copter 3			
# of Clips	**1**		Quadratic Model
Mass:	0.5 grams	1.4342	
Trial	Time(sec)	Velocity(inch/sec)	CD
1	5.22	43.29501916	2.042457014
2	5.84	38.69863014	2.556451826
3	5.81	38.89845095	2.530254372
4	5.37	42.08566108	2.161526133
5	5.68	39.78873239	2.418291172
6	5.48	41.24087591	2.250987255
7	*5.28*	42.8030303	2.089679894
8	5.63	40.14209591	2.375903016
9	5.17	43.71373308	2.003516878
10	5.77	39.16811092	2.495534311
Average	5.525	40.90497738	2.288107812

Copter 3			
# of Clips	**2**		Quadratic Model
Mass:	1.0 gram	1.9342	
Trial	Time(sec)	Velocity(inch/sec)	CD
1	4.47	50.55928412	2.019847709
2	4.34	52.07373272	1.904070563
3	4.55	49.67032967	2.092793478
4	4.63	48.81209503	2.167033185

5	4.5	50.22222222	2.047050739
6	4.38	51.59817352	1.93933038
7	4.63	48.81209503	2.167033185
8	4.47	50.55928412	2.019847709
9	4.73	47.78012685	2.261652419
10	4.41	51.24716553	1.96598753
Average	4.511	50.09975615	2.057070775

For Helicopter 3:

- 1 pin
  mu = 2.29246 [2.14236, 2.44256]
  sigma = 0.209827 [0.144326, 0.383062]
- 2 pin
  mu = 2.05846 [1.97734, 2.13959]
  sigma = 0.113399 [0.0779996, 0.207022]
- 3 pin

  mu = 2.96479 [2.8555, 3.07408]

  sigma = 0.152782 [0.105089, 0.27892]

Our objective now is to estimate unknown two parameters of normal distribution based on our observations of experimental C_D's. That is to find likelihood function, obtained as joint distribution, a function of mean and standard deviation. Since the prior given is non-informative, it will be equal to the joint posterior distribution for the Drag Coefficient.

$$p\left(\mu,\sigma^2\right) \propto \left(\sigma^2\right)^{-1}$$

- Joint posterior distribution:

$$p\left(\theta \mid y\right) \propto \prod_{i=1}^{n} p\left(y_i \mid \theta\right) \propto \sigma^{-n-2} \exp\left(-\frac{1}{2\sigma^2}\sum_{i=1}^{n}\left(y_i-\mu\right)^2\right)$$

$$\text{where} \quad s^2 = \frac{1}{n-1}\sum\left(y_i-\mu\right)^2$$

- With some algebra,

$$p\left(\mu,\sigma^2 \mid y\right) = \sigma^{-n-2} \exp\left(-\frac{1}{2\sigma^2}\left[(n-1)s^2 + n\left(\bar{y}-\mu\right)^2\right]\right)$$

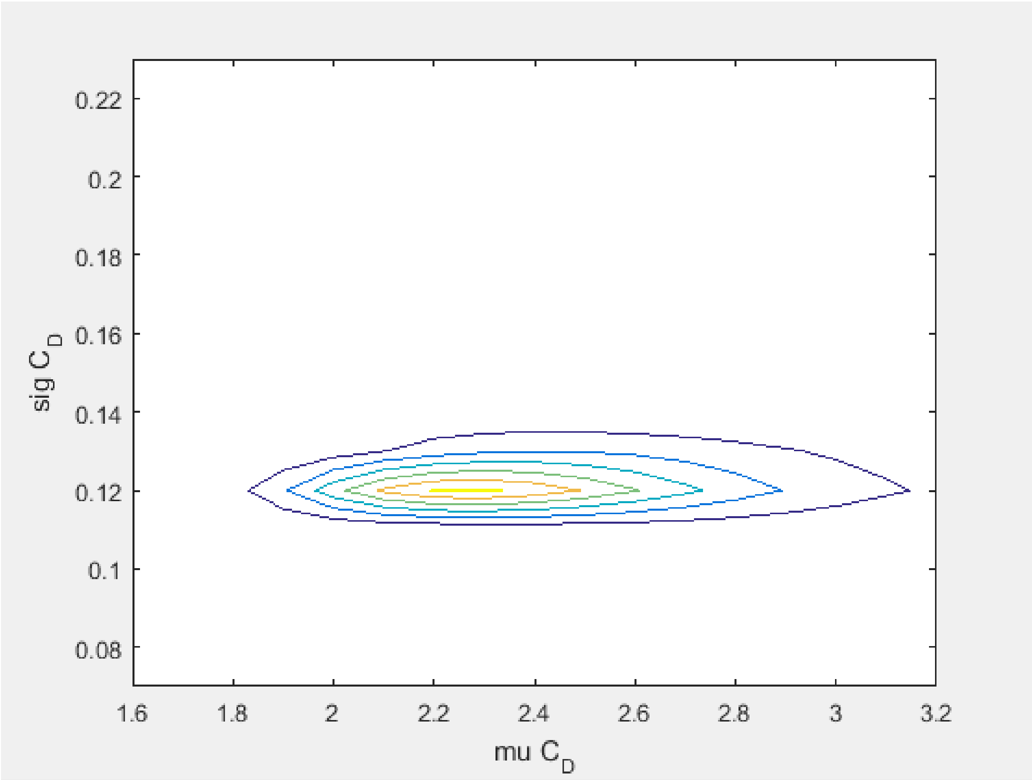
The posterior distribution of Drag coefficient is obtained separately based on all helicopters for all different cases of masses, which is plotted as a surface and a contour.

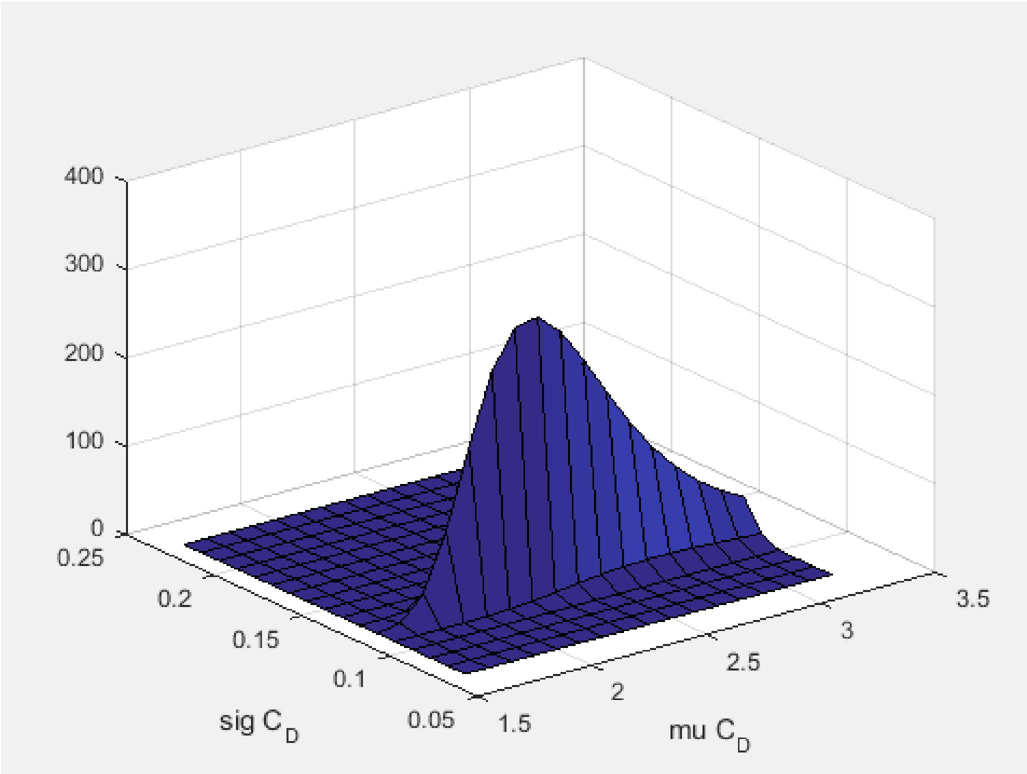
The MATLAB code to calculate Likelihood is given below:

```
%% Matlab Fuction to calculate Likelihood


H1P1=[2.019847709,1.904070563,2.092793478,2.167033185,2.047050739,1.9393303
8,2.167033185,2.019847709,2.261652419,1.96598753]';
k_=length(H1P1);
pd=fitdist(H1P1,'Normal')

mu=1.5:0.1:3.2;
x_=length(mu);
sig=0.07:0.01:0.24;
y_=length(sig);
[X,Y]=meshgrid(mu,sig);

L=meshgrid(mu,sig);
%%For Non-informative Prior
for i=1:x_
    for j=1:y_
        L(i,j)=1;
    end
end

%Likelihood Function
for i=1:x_
    for j=1:y_
        for k_=1:k_
            L(i,j)=L(i,j)*normpdf(Cd1(k_),mu(i),sig(j));
        end
    end
end

surf(X,Y,L)
%contour(X,Y,L)
xlabel('mu C_D');
ylabel('sig C_D');
```

Helicopter 1

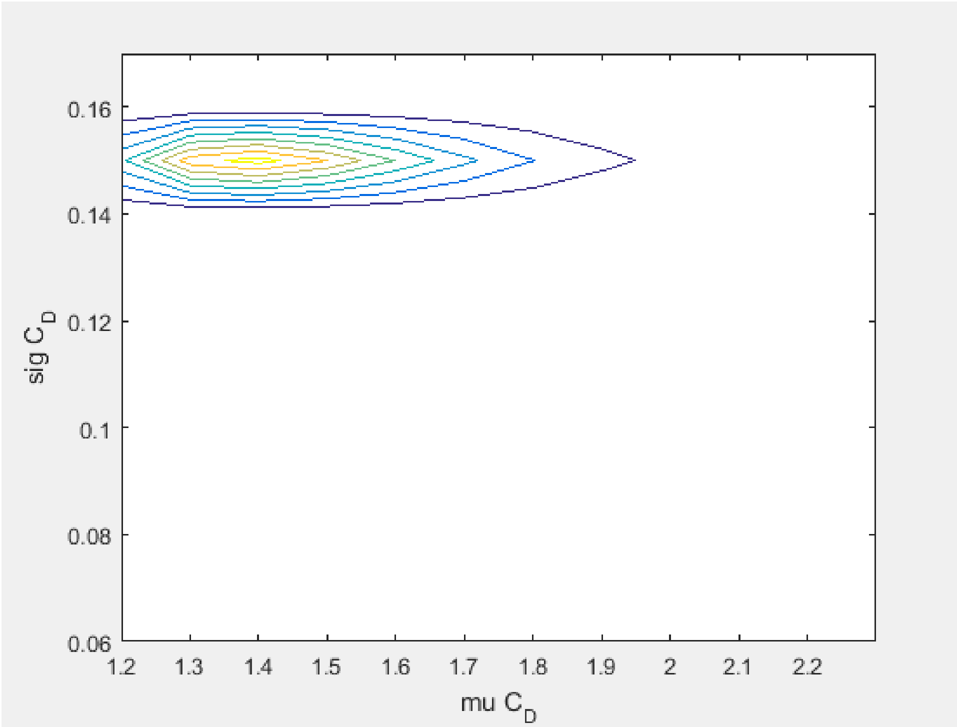
#Pins 1:

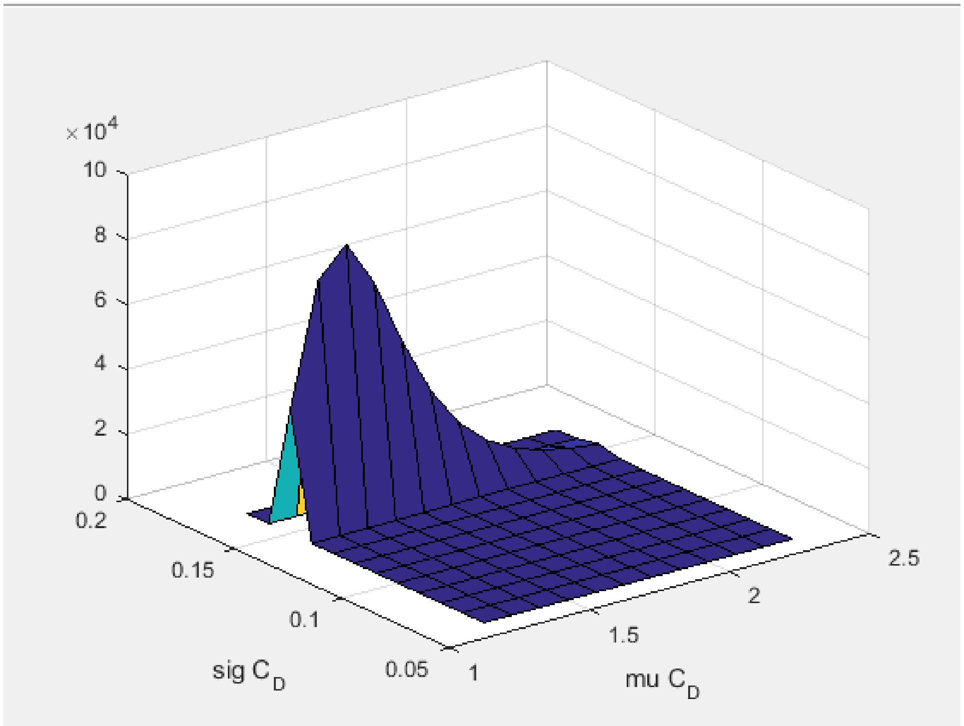




Helicopter 1

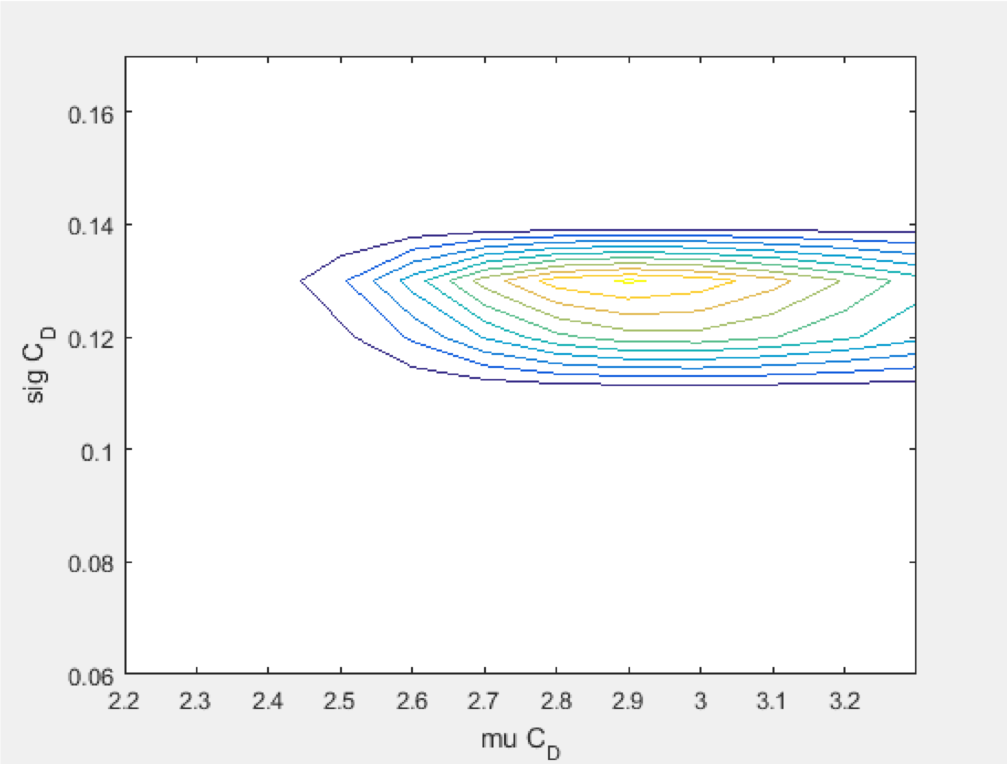
#Pins 2:

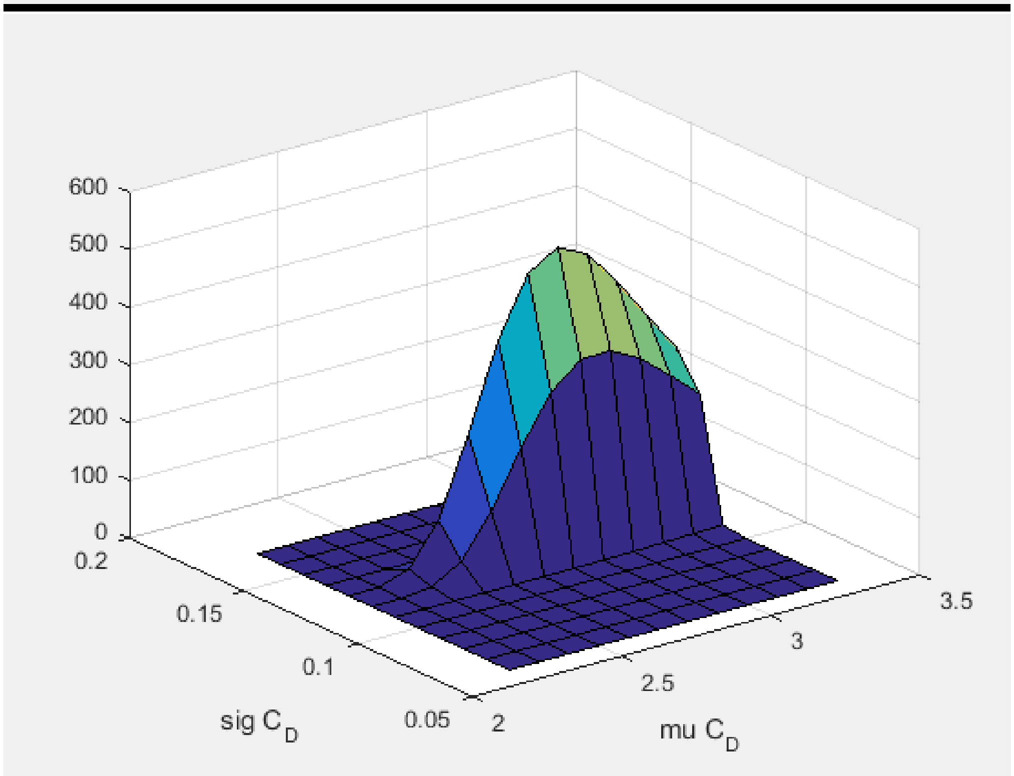




Helicopter 1

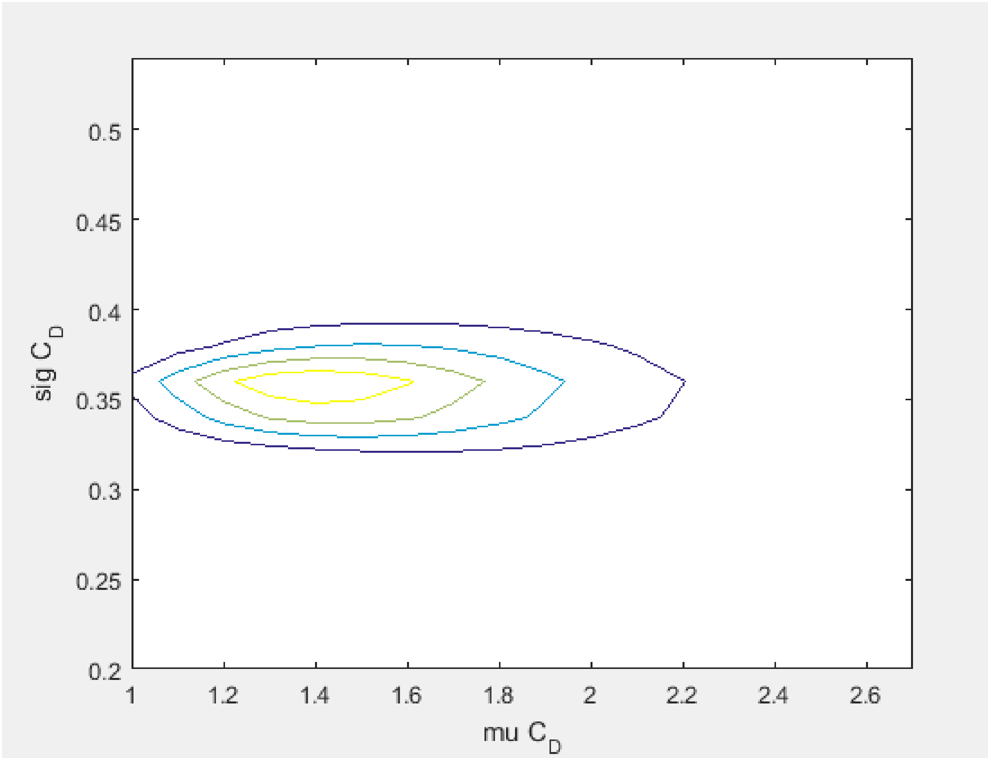
#Pins 3:

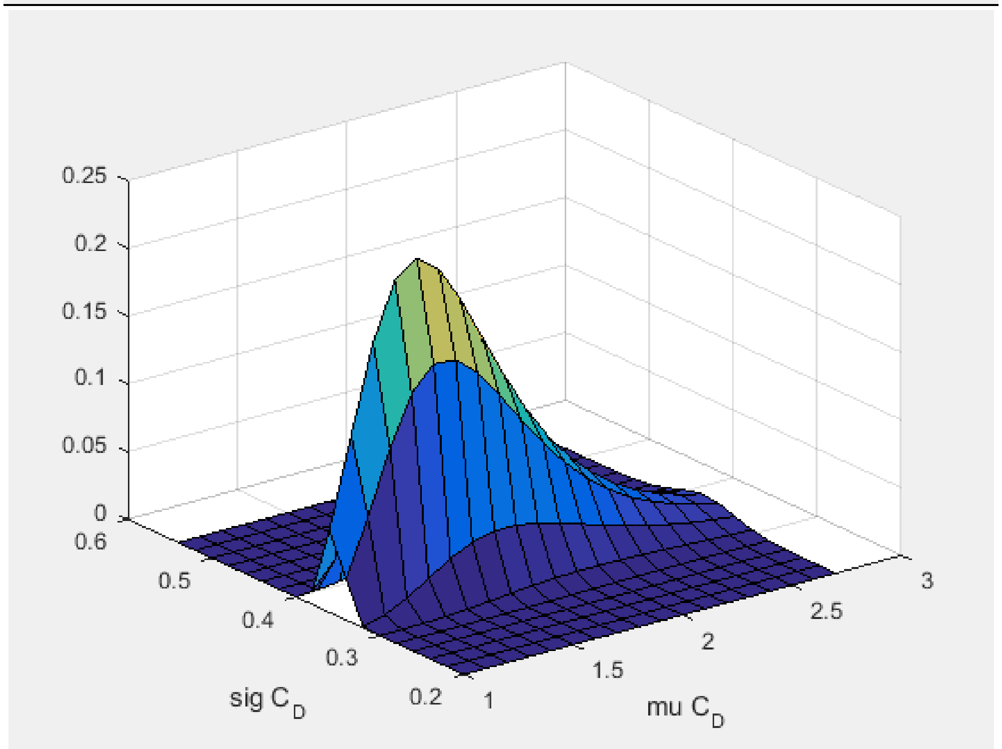




Helicopter 2

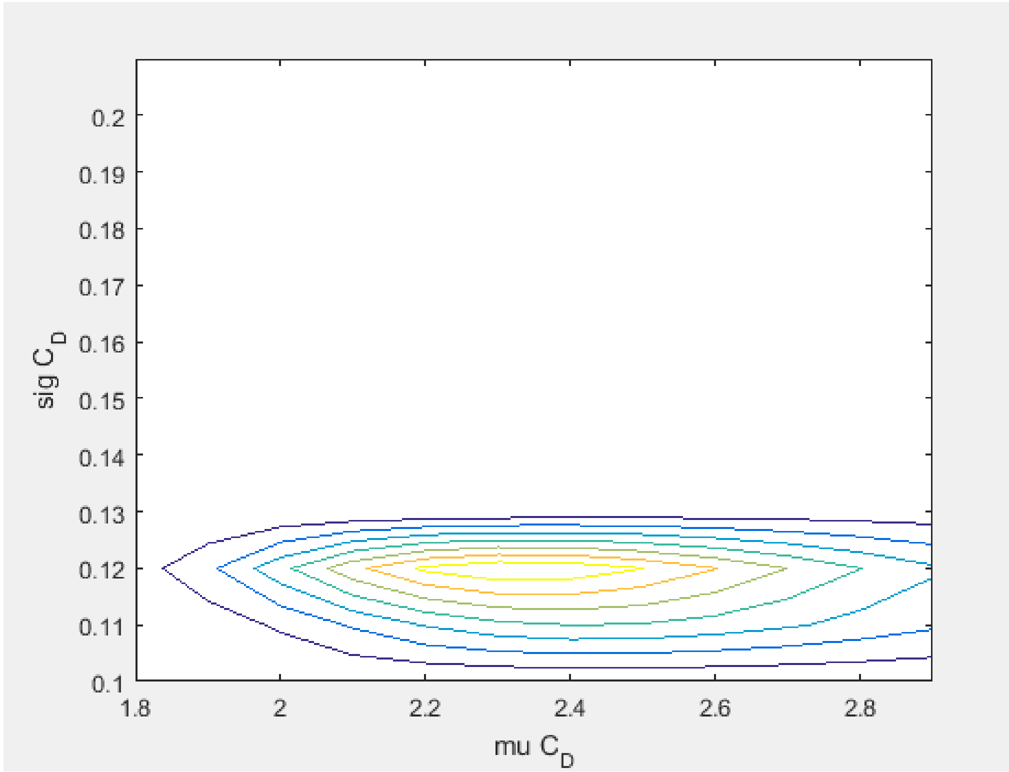
#Pins 1:

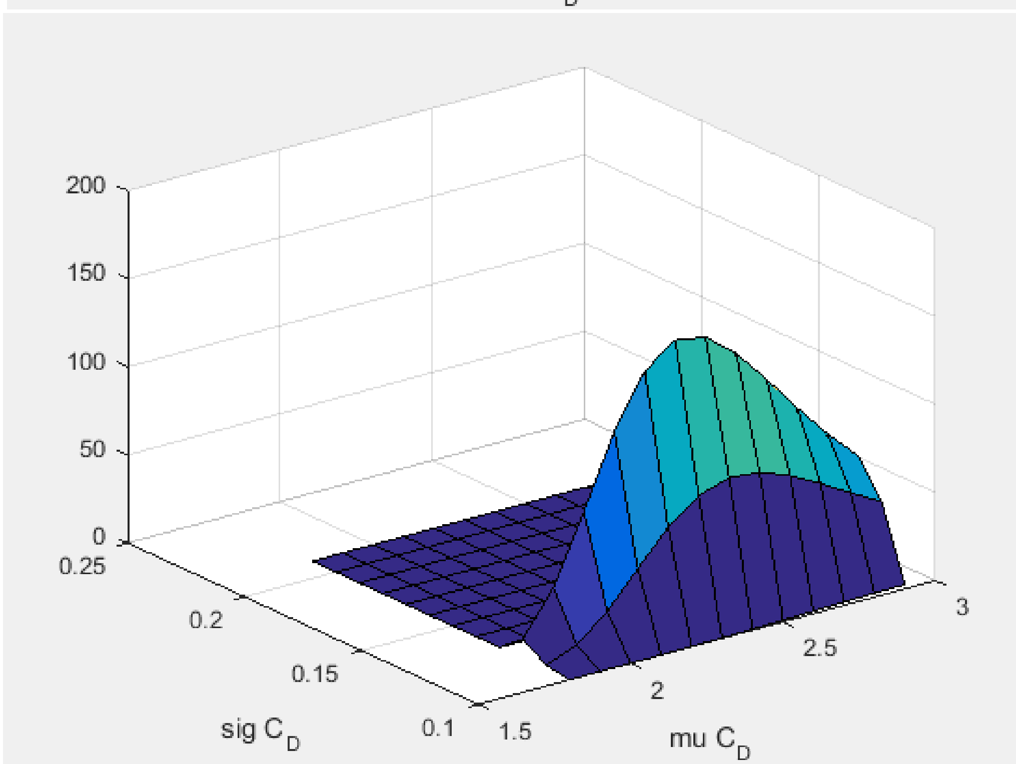




Helicopter 2

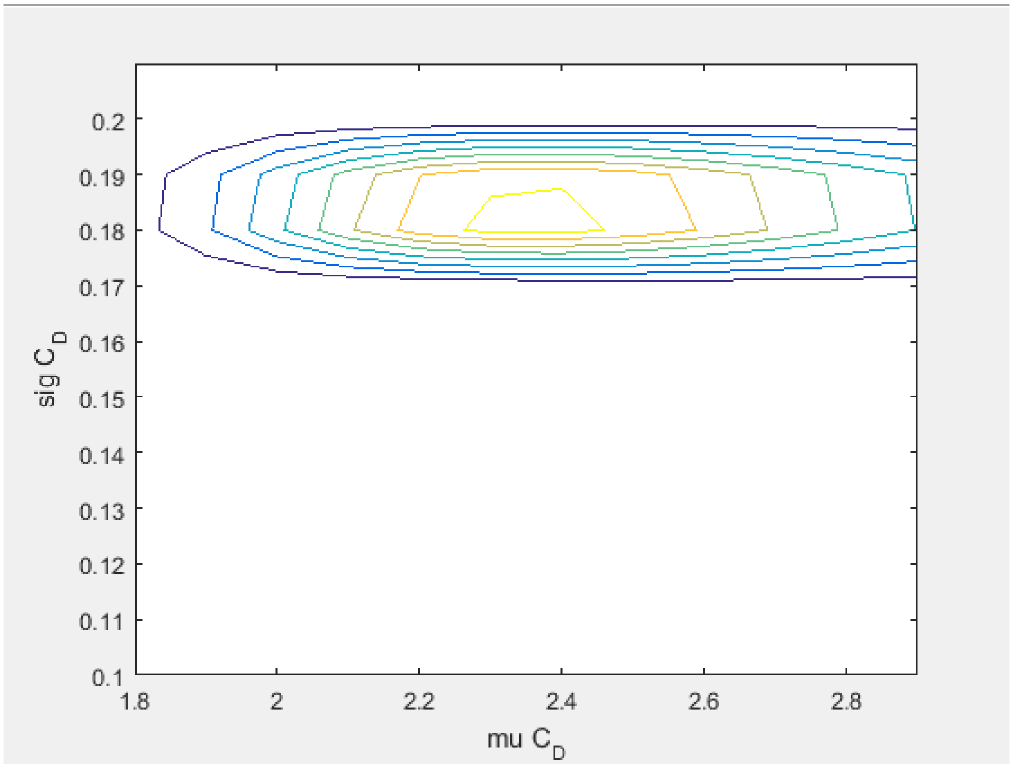
#Pins 2:

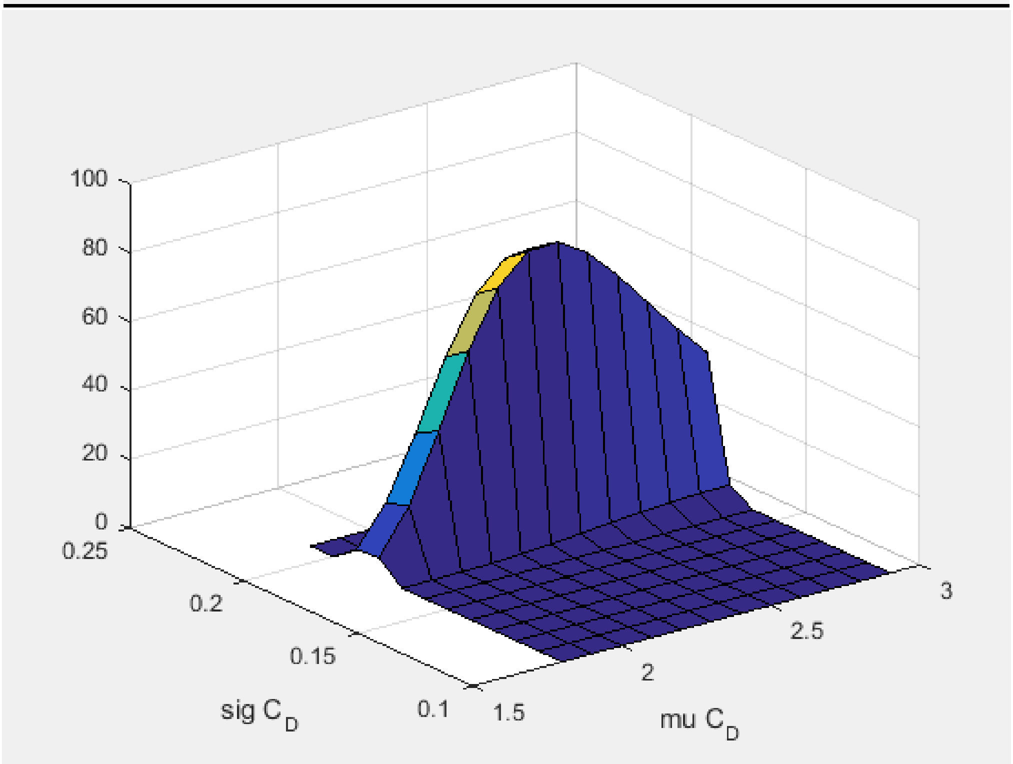

0.2
0.19
0.18
0.17
0.16
0.15
0.14
0.13
0.12
0.11
0.1
sig C_D
1.8
2
2.2
2.4
2.6
2.8
mu C_D


200
150
100
50
0
0.25
0.2
0.15
0.1
sig C_D
1.5
2
2.5
3
mu C_D

Helicopter 2

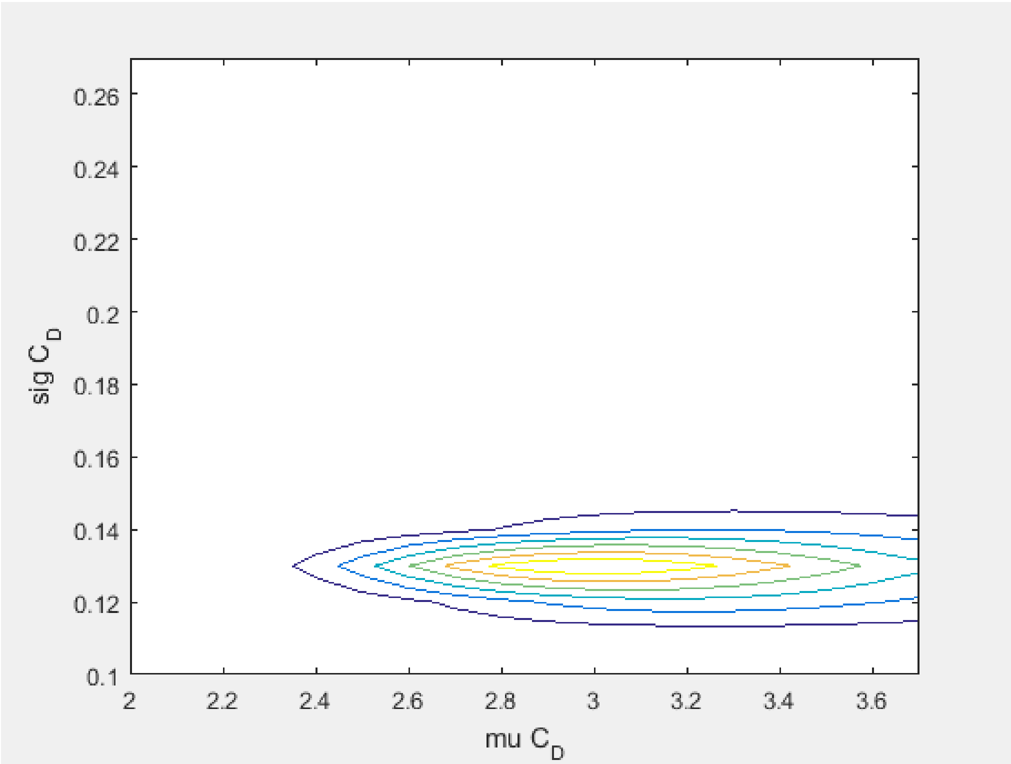
#Pins 3:

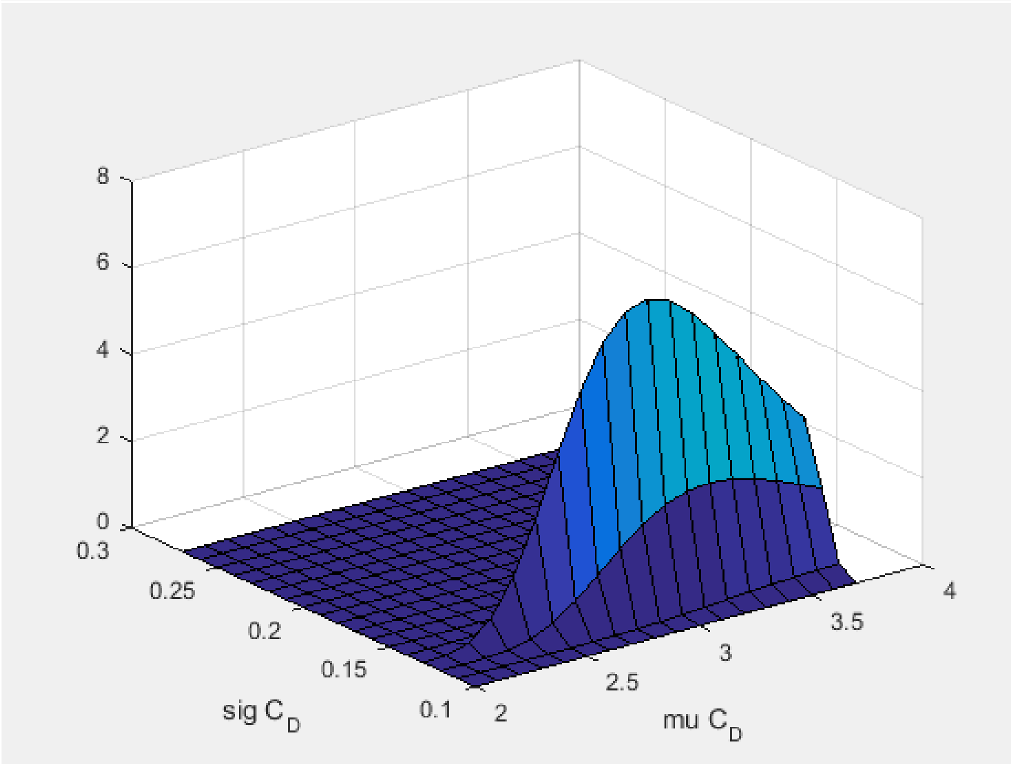




Helicopter 3

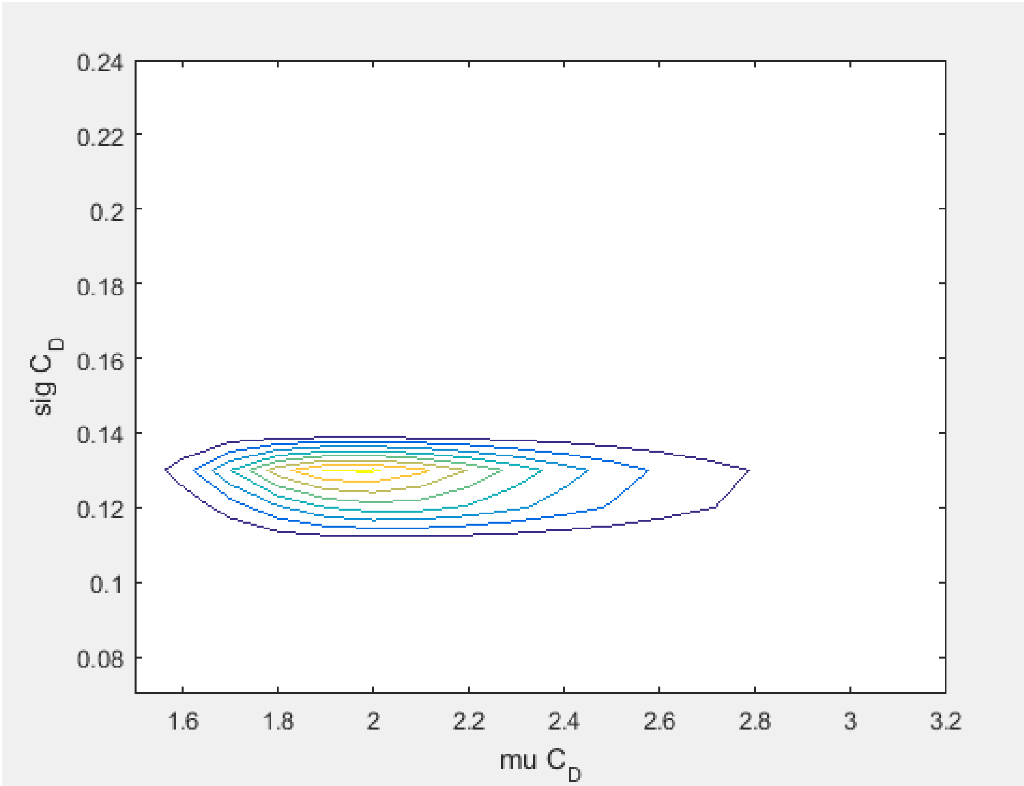
#Pins 1:

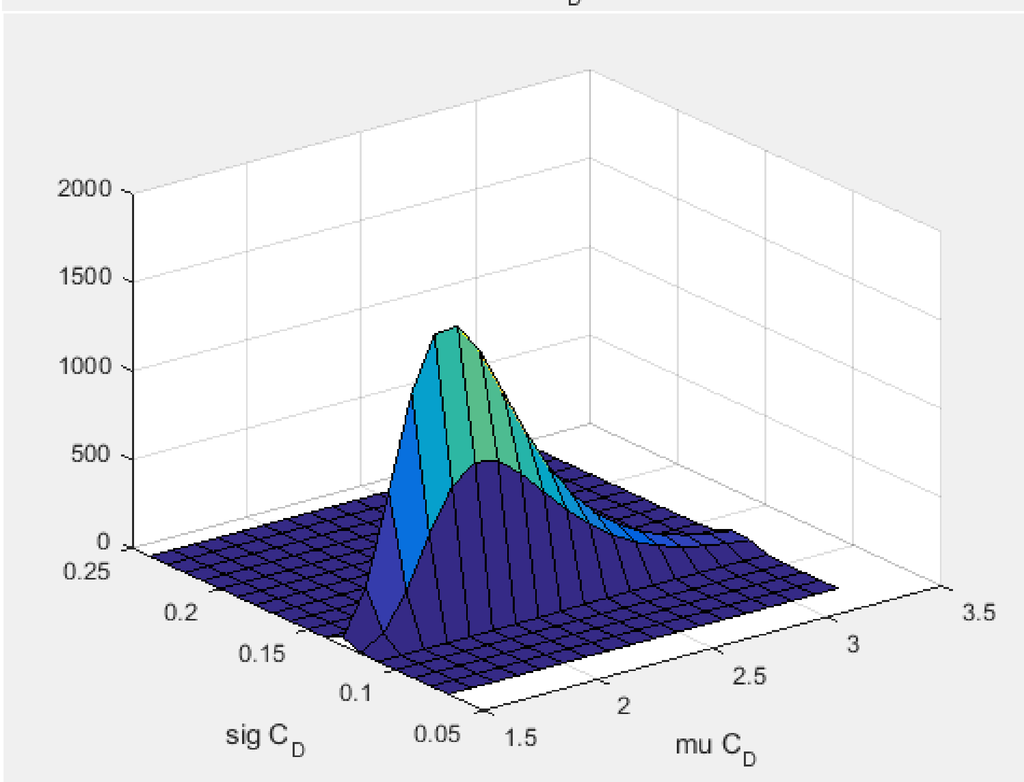




Helicopter 3

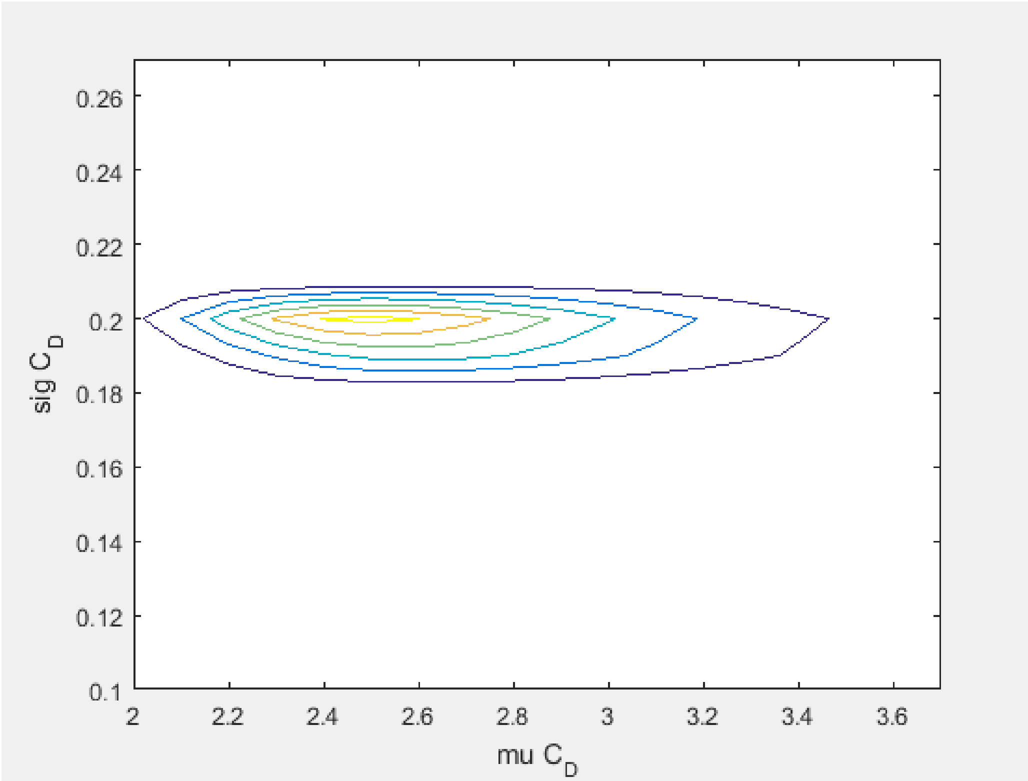
#Pins 2:

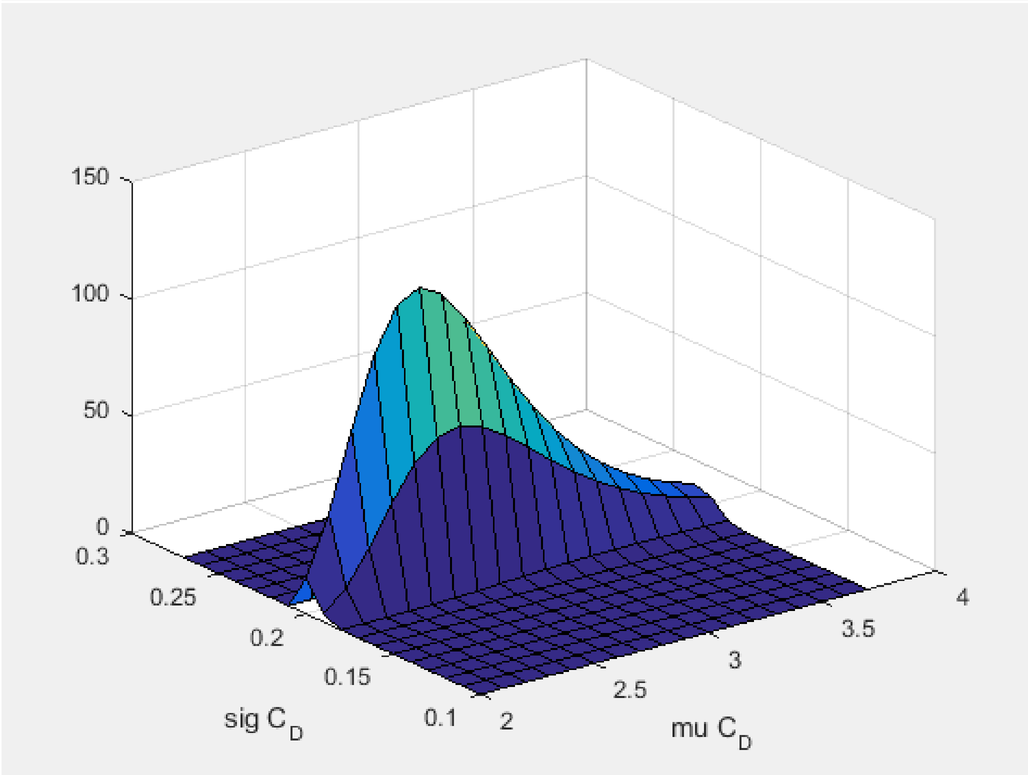

sig C_D
mu C_D
0.24
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
1.6
1.8
2
2.2
2.4
2.6
2.8
3
3.2


2000
1500
1000
500
0
0.25
0.2
0.15
0.1
0.05
sig C_D
1.5
2
2.5
3
3.5
mu C_D

Helicopter 3

#Pins 3:





References:

1. Park, C., Choi, J., & Haftka, R. T. (2016). Teaching a Verification and Validation Course Using Simulations and Experiments With Paper Helicopters. *Journal of Verification, Validation and Uncertainty Quantification, 1*(3), 031002. doi:10.1115/1.4033889
2. Oberkamph, W., & Roy, C. (n.d.). *VERIFICATION AND VALIDATION IN SCIENTIFIC COMPUTING*. New York: Cambridge University Press, .
3. Measures of Variability. (n.d.). Retrieved April 10, 2017, from http://onlinestatbook.com/2/summarizing_distributions/variability.html