# Package 'r2redux'

# February 15, 2023

**Title** R2 Statistic **Version** 1.0.13

<b>Description</b> R2 statistic for significance test. Variance and covariance of R2 values used to assess the 95% CI and p-value of the R2 difference.	
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cc\_trf

cc_trf cc_trf function
------------------------

# **Description**

This function transforms the predictive ability (R2) and its standard error (se) between the observed scale and liability scale

# Usage

```
cc_trf(R2, se, K, P)
```

## **Arguments**

R2	R2 or coefficient of determination on the observed or liability scale
se	Standard error of R2
K	Population prevalence
P	The ratio of cases in the study samples

#### Value

This function will transform the R2 and its s.e between observed scale and liability scale.Output from the command is the lists of outcomes.

R21	Transformed R2 on the liability scale
sel	Transformed se on the liability scale
R20	Transformed R2 on the observed scale
se0	Transformed se on the observed scale

# References

Lee, S. H., Goddard, M. E., Wray, N. R., and Visscher, P. M. A better coefficient of determination for genetic profile analysis. Genetic epidemiology, (2012). 36(3): p. 214-224.

```
#To get the transformed R2
output=cc_trf(0.06, 0.002, 0.05, 0.05)
output

#output$R21 (transformed R2 on the liability scale)
#0.2679337

#output$sel (transformed se on the liability scale)
#0.008931123

#output$R20 (transformed R2 on the observed scale)
#0.01343616

#output$se0 (transformed se on the observed scale)
#0.000447872
```

dat1 3

dat1

Phenotypes and 10 sets of PGSs

# **Description**

A dataset containing phenotypes and multiple PGSs estimated from 10 sets of SNPs according to GWAS p-value thresholds

# Usage

dat1

#### **Format**

A data frame with 1000 rows and 11 variables:

- V1 Phenotype, value
- V2 PGS1, for p value threshold <=1
- **V3** PGS2, for p value threshold <=0.5
- V4 PGS3, for p value threshold <=0.4
- V5 PGS4, for p value threshold <=0.3
- **V6** PGS5, for p value threshold <=0.2
- V7 PGS6, for p value threshold <=0.1
- **V8** PGS7, for p value threshold <=0.05
- **V9** PGS8, for p value threshold <=0.01
- V10 PGS9, for p value threshold <=0.001
- V11 PGS10, for p value threshold <=0.0001

dat2

Phenotypes and 2 sets of PGSs

# Description

A dataset containing phenotypes and 2 sets of PGSs estimated from 2 sets of SNPs from regulatroy and non-regulatory genomic regions

# Usage

dat2

# **Format**

A data frame with 1000 rows and 3 variables:

- V1 Phenotype
- V2 PGS1, regulatory region
- V3 PGS2, non-regulatory region

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olkin12\_1

olkin12\_1 function

# Description

olkin12\_1 function

# Usage

```
olkin12_1(omat, nv)
```

# **Arguments**

omat 3 by 3 matrix having the correlation coefficients between y, x1 and x2, i.e.

omat=cor(dat) where dat is N by 3 matrix having variables in the order of cbind

(y,x1,x2)

nv Sample size

# Value

This function will be used as source code

olkin12\_13

olkin12\_13 function

# Description

olkin12\_13 function

# Usage

```
olkin12_13(omat, nv)
```

# **Arguments**

omat 3 by 3 matrix having the correlation coefficients between y, x1 and x2, i.e.

omat=cor(dat) where dat is N by 3 matrix having variables in the order of cbind

(y,x1,x2)

nv Sample size

# Value

This function will be used as source code

olkin12\_3 5

olkin12\_3

olkin12\_3 function

# Description

olkin12\_3 function

# Usage

```
olkin12_3(omat, nv)
```

# **Arguments**

omat 3 by 3 matrix having the correlation coefficients between y, x1 and x2, i.e.

omat=cor(dat) where dat is N by 3 matrix having variables in the order of cbind

(y,x1,x2)

nv Sample size

# Value

This function will be used as source code

olkin12\_34

olkin12\_34 function

# **Description**

olkin12\_34 function

# Usage

```
olkin12_34(omat, nv)
```

# **Arguments**

omat 3 by 3 matrix having the correlation coefficients between y, x1 and x2, i.e.

omat=cor(dat) where dat is N by 3 matrix having variables in the order of cbind

(y,x1,x2)

nv Sample size

# Value

This function will be used as source code

6 olkin\_beta1\_2

olkin1\_2

olkin1\_2 function

# **Description**

```
olkin1_2 function
```

# Usage

```
olkin1_2(omat, nv)
```

# Arguments

omat 3 by 3 matrix having the correlation coefficients between y, x1 and x2, i.e.

omat=cor(dat) where dat is N by 3 matrix having variables in the order of cbind

(y,x1,x2)

nv Sample size

#### Value

This function will be used as source code

# Description

This function derives Information matrix for beta  $1^2$  and beta  $2^2$  where beta 1 and 2 are regression coefficients from a multiple regression model, i.e. y = x1 \* beta 1 + x2 \* beta 2 + e, where y, x1 and x2 are column-standardised, (i.e. in the context of correlation coefficients, see Olkin and Finn 1995).

#### Usage

```
olkin_beta1_2(omat, nv)
```

# Arguments

omat 3 by 3 matrix having the correlation coefficients between y, x1 and x2, i.e.

omat=cor(dat) where dat is N by 3 matrix having variables in the order of cbind

(y,x1,x2)

nv Sample size

olkin\_beta\_inf 7

#### Value

This function will give information (variance-covariance) matrix of beta1^2 and beta2^2. To get information (variance-covariance) matrix of beta1^2 and beta2^2. Where beta1 and beta2 are regression coefficients from a multiple regression model. The outputs are listed as follows.

info	2x2 information (variance-covariance)	2x2 information (variance-covariance) matrix	
var1	Variance of beta1_2		
var2	Variance of beta2_2		
var1	2 Variance of difference between beta1	2 and beta2	2

#### References

Olkin, I. and Finn, J.D. Correlations redux. Psychological Bulletin, 1995. 118(1): p. 155.

# **Examples**

```
#To get information (variance-covariance) matrix of beta1_2 and beta2_2 where
#betal and 2 are regression coefficients from a multiple regression model.
dat=dat1
omat=cor(dat)[1:3,1:3]
#1.0000000 0.1958636 0.1970060
#0.1958636 1.0000000 0.9981003
#0.1970060 0.9981003 1.0000000
nv=length(dat$V1)
output=olkin_beta1_2(omat,nv)
output
#output$info (2x2 information (variance-covariance) matrix)
#0.04146276 0.08158261
#0.08158261 0.16111124
#output$var1 (variance of beta1_2)
#0.04146276
#output$var2 (variance of beta2_2)
#0.1611112
#output$var1_2 (variance of difference between beta1_2 and beta2_2)
#0.03940878
```

```
{\tt olkin\_beta\_inf} \qquad \textit{olkin\_beta\_inf function}
```

# **Description**

This function derives Information matrix for beta1 and beta2 where beta1 and 2 are regression coefficients from a multiple regression model, i.e. y = x1 \* beta1 + x2 \* beta2 + e, where y, x1 and x2 are column-standardised (see Olkin and Finn 1995).

8 olkin\_beta\_inf

#### Usage

```
olkin_beta_inf(omat, nv)
```

#### **Arguments**

omat 3 by 3 matrix having the correlation coefficients between y, x1 and x2, i.e.

omat=cor(dat) where dat is N by 3 matrix having variables in the order of cbind

(y,x1,x2)

nv Sample size

#### Value

This function will generate information (variance-covariance) matrix of beta1 and beta2. The outputs are listed as follows.

info 2x2 information (variance-covariance) matrix

var1 Variance of beta1 var2 Variance of beta2

var1 2 Variance of difference between beta1 and beta2

#### References

Olkin, I. and Finn, J.D. Correlations redux. Psychological Bulletin, 1995. 118(1): p. 155.

```
#To get information (variance-covariance) matrix of beta1 and beta2 where
#betal and 2 are regression coefficients from a multiple regression model.
dat=dat1
omat=cor(dat)[1:3,1:3]
#omat
#1.0000000 0.1958636 0.1970060
#0.1958636 1.0000000 0.9981003
#0.1970060 0.9981003 1.0000000
nv=length(dat$V1)
output=olkin_beta_inf(omat,nv)
output
#output$info (2x2 information (variance-covariance) matrix)
#0.2531406 -0.2526212
#-0.2526212 0.2530269
#output$var1 (variance of beta1)
#0.2531406
#output$var2 (variance of beta2)
#0.2530269
#output$var1_2 (variance of difference between beta1 and beta2)
#1.01141
```

olkin\_beta\_ratio

```
olkin_beta_ratio olkin_beta_ratio function
```

# **Description**

This function derives variance of beta  $1^2 / R^2$  where beta 1 and 2 are regression coefficients from a multiple regression model, i.e. y = x1 \* beta 1 + x2 \* beta 2 + e, where y, x1 and x2 are column-standardised (see Olkin and Finn 1995).

#### Usage

```
olkin_beta_ratio(omat, nv)
```

#### **Arguments**

omat 3 by 3 matrix having the correlation coefficients between y, x1 and x2, i.e.

omat=cor(dat) where dat is N by 3 matrix having variables in the order of cbind

(y,x1,x2)

nv sampel size

#### Value

This function will generate the variance of the proportion, i.e. beta1\_2/R^2. The outputs are listed as follows.

```
ratio_var Variance of ratio
```

#### References

Olkin, I. and Finn, J.D. Correlations redux. Psychological Bulletin, 1995. 118(1): p. 155.

```
#To get information (variance-covariance) matrix of beta1 and beta2 where
#beta1 and 2 are regression coefficients from a multiple regression model.
dat=dat2
omat=cor(dat)[1:3,1:3]
#omat
#1.0000000 0.1497007 0.136431
#0.1497007 1.0000000 0.622790
#0.1364310 0.6227900 1.000000

nv=length(dat$V1)
output=olkin_beta_ratio(omat,nv)
output
#r2redux output

#output$ratio_var (Variance of ratio)
#0.08042288
```

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r2_beta_var

# **Description**

This function estimates var(beta1^2) and (beta2^2), and beta1 and 2 are regression coefficients from a multiple regression model, i.e. y = x1 \* beta1 + x2 \* beta2 + e, y, x1 and x2 are column-standardised (see Olkin and Finn 1995). y is y is y 1 matrix having the dependent variable, y 1 is y 1 matrix having the ith explanatory variable. y 2 in y 3 matrix having the jth explanatory variable. y 3 and y 2 indicates the ith and jth column in the data (y 1 or y 2 should be a single interger between 1 - y 3.

# Usage

```
r2_beta_var(dat, v1, v2, nv)
```

# **Arguments**

dat	N by $(M+1)$ matrix having variables in the order of $cbind(y,x)$	
v1	This can be set as $v1=1$ , $v1=2$ , $v1=3$ or any value between 1 - M based on combination	
v2	This can be set as $v2=1$ , $v2=2$ , $v2=3$ , or any value between 1 - M based on combination	
nv	Sample size	

#### Value

This function will estiamte the variance of beta  $1^2$  and beta  $2^2$ , and the covariance between beta  $1^2$  and beta  $2^2$ , i.e. the information matrix of squared regression coefficients. beta 1 and beta 2 are regression coefficients from a multiple regression model, i.e. y = x1 \* beta 1 + x2 \* beta 2 + e, where y, x1 and x2 are column-standardised. The outputs are listed as follows.

```
beta1_sq
                 beta1_sq
                 beta2_sq
beta2_sq
                 Variance of beta1_sq
var1
                 Variance of beta2_sq
var2
                 Variance of difference between beta1 sq and beta2 sq
var1 2
                 Covariance between beta1_sq and beta2_sq
COV
upper_beta1_sq
                 upper limit of 95% CI for beta1_sq
lower_beta1_sq
                 lower limit of 95% CI for beta1_sq
upper_beta2_sq
                 upper limit of 95% CI for beta2_sq
lower_beta2_sq
                 lower limit of 95% CI for beta2_sq
```

 $r2\_diff$  11

#### References

Olkin, I. and Finn, J.D. Correlations redux. Psychological Bulletin, 1995. 118(1): p. 155.

# **Examples**

```
#To get the 95% CI of beta1_sq and beta2_sq
#betal and beta2 are regression coefficients from a multiple regression model,
#i.e. y = x1 * beta1 + x2 * beta2 + e, where y, x1 and x2 are column-standardised.
dat=dat2
nv=length(dat$V1)
v1=c(1)
v2=c(2)
output=r2_beta_var(dat,v1,v2,nv)
output
#r2redux output
#output$beta1_sq (beta1_sq)
#0.01118301
#output$beta2_sq (beta2_sq)
#0.004980285
#output$var1 (variance of beta1_sq)
#7.072931e-05
#output$var2 (variance of beta2_sq)
#3.161929e-05
#output$var1_2 (variance of difference between beta1_sq and beta2_sq)
#0.000162113
#output$cov (covariance between beta1_sq and beta2_sq)
#-2.988221e-05
#output$upper_beta1_sq (upper limit of 95% CI for beta1_sq)
#0.03037793
#output$lower_beta1_sq (lower limit of 95% CI for beta1_sq)
#-0.00123582
#output$upper_beta2_sq (upper limit of 95% CI for beta2_sq)
#0.02490076
#output$lower_beta2_sq (lower limit of 95% CI for beta2_sq)
#-0.005127546
```

r2\_diff

r2\_diff function

# **Description**

This function estimates  $var(R2(y\sim x[,v1]) - R2(y\sim x[,v2]))$  where R2 is the R squared value of the model, y is N by 1 matrix having the dependent variable, and x is N by M matrix having M explanatory variables. v1 or v2 indicates the ith column in the x matrix (v1 or v2 can be multiple values between 1 - M, see Arguments below)

r2\_diff

#### Usage

```
r2_diff(dat, v1, v2, nv)
```

#### **Arguments**

dat	N by (M+1) matrix having variables in the order of cbind(y,x)
v1	This can be set as $v1=c(1)$ or $v1=c(1,2)$
v2	This can be set as $v2=c(2)$ , $v2=c(3)$ , $v2=c(1,3)$ or $v2=c(3,4)$
nv	Sample size

#### Value

This function will estimate significant difference between two PGS (either dependent or independent and joint or single). To get the test statistics for the difference between  $R2(y\sim x[,v1])$  and  $R2(y\sim x[,v2])$ . (here we define  $R2_1=R2(y\sim x[,v1])$ ) and  $R2_2=R2(y\sim x[,v2])$ ). The outputs are listed as follows.

```
R2_1
rsq1
                 R2_2
rsq2
                 Variance of R2_1
var1
var2
                 variance of R2 2
                 Variance of difference between R2_1 and R2_2
var_diff
                 two tailed P-value for significant difference between R2_1 and R2_2
r2_based_p
r2_based_p_one_tail
                 one tailed P-value for significant difference
                 Differences between R2_1 and R2_2
mean_diff
upper_diff
                 Upper limit of 95% CI for the difference
lower_diff
                 Lower limit of 95% CI for the difference
```

```
#To get the test statistics for the difference between R2(y \sim x[,v1]) and #R2(y \sim x[,v2]). (here we define R2_1=R2(y \sim x[,v1])) and R2_2=R2(y \sim x[,v2]))) dat=dat1 nv=length(dat$V1) v1=c(1) v2=c(2) output=r2_diff(dat,v1,v2,nv) output #r2redux output #r2redux output #output$rsq1 (R2_1) #0.03836254 #output$rsq2 (R2_2) #0.03881135 #output$var1 (variance of R2_1) #0.0001436128
```

```
#output$var2 (variance of R2_2)
#0.0001451358
#output$var_diff (variance of difference between R2_1 and R2_2)
#5.678517e-07
#output$r2_based_p (two tailed p-value for significant difference)
#0.5514562
#output$r2_based_p_one_tail(one tailed p-value for significant difference)
#0.2757281
#output$mean_diff (differences between R2_1 and R2_2)
#-0.0004488044
#output$upper_diff (upper limit of 95% CI for the difference)
#0.001028172
#output$lower_diff (lower limit of 95% CI for the difference)
#-0.001925781
#To get the test statistics for the difference between R2(y\sim x[,v1]+x[,v2]) and
\#R2(y\sim x[,v2]). (here R2_1=R2(y\sim x[,v1]+x[,v2]) and R2_2=R2(y\sim x[,v1]))
dat=dat1
nv=length(dat$V1)
v1=c(1,2)
v2=c(1)
output=r2_diff(dat,v1,v2,nv)
#r2redux output
#output$rsq1 (R2_1)
#0.03896678
#output$rsq2 (R2_2)
#0.03836254
#output$var1 (variance of R2_1)
#0.0001473686
#output$var2 (variance of R2_2)
#0.0001436128
#output$var_diff (variance of difference between R2_1 and R2_2)
#2.321425e-06
#output$r2_based_p (p-value for significant difference between R2_1 and R2_2)
#0.4366883
#output$mean_diff (differences between R2_1 and R2_2)
#0.0006042383
#output$upper_diff (upper limit of 95% CI for the difference)
#0.00488788
```

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```
\#output$lower_diff (lower limit of 95% CI for the difference) \#-0.0005576171
```

#### **Description**

This function estimates  $var(beta1^2/R^2)$ , beta1 and  $R^2$  are regression coefficient and the coefficient of determination from a multiple regression model, i.e. y = x1 \* beta1 + x2 \* beta2 + e, where y, x1 and x2 are column-standardised (see Olkin and Finn 1995). y is y by 1 matrix having the dependent variable, and y is y by 1 matrix having the ith explanatory variables. y is y by 1 matrix having the jth explanatory variables. y indicates the ith and jth column in the data (y1 or y2 should be a single interger between 1 - y4, see Arguments below).

# Usage

```
r2_enrich_beta(dat, v1, v2, nv, exp1)
```

# Arguments

combination	dat	N by $(M+1)$ matrix having variables in the order of cbind $(y,x)$	
nv Sample size	v1	These can be set as $v1=1$ , $v1=2$ , $v1=3$ or any value between 1 - M based on combination	
1	v2	These can be set as v2=1, v2=2, v2=3, or any value between 1 - M based on combination	
exp1 The expectation of the ratio (e.g. ratio of # SNPs in genomic partitioning)	nv	Sample size	
	exp1	The expectation of the ratio (e.g. ratio of # SNPs in genomic partitioning)	

# Value

This function will estimate var(beta1 $^2$ /R $^2$ ), beta1 and R $^2$  are regression coefficient and the coefficient of determination from a multiple regression model, i.e. y = x1 \* beta1 + x2 \* beta2 + e, where y, x1 and x2 are column-standardised. The outputs are listed as follows.

```
beta1_sq
                 beta1_sq
beta2_sq
                 beta2_sq
                 beta1 sq/R^2
ratio1
                 beta2_sq/R^2
ratio2
ratio_var1
                 variance of ratio 1
                 variance of ratio 2
ratio_var2
upper_ratio1 upper limit of 95% CI for ratio 1
lower_ratio1 lower limit of 95% CI for ratio 1
upper_ratio2 upper limit of 95% CI for ratio 2
lower ratio2 lower limit of 95% CI for ratio 2
                 two tailed P-value for beta1_sq/R^2 is significantly different from exp1
enrich_p1
```

r2\_enrich\_beta

```
enrich_p1_one_tail
one tailed P-value for beta1_sq/R^2 is significantly different from exp1
enrich_p2 P-value for beta2_sq/R2 is significantly different from (1-exp1)
enrich_p2_one_tail
one tailed P-value for beta2_sq/R2 is significantly different from (1-exp1)
```

#### References

Olkin, I. and Finn, J.D. Correlations redux. Psychological Bulletin, 1995. 118(1): p. 155.

```
#To get the test statistic for the ratio which is significantly
#different from the expectation, this function estiamtes
\#var (beta1^2/R^2), where
\#beta1^2 and R^2 are regression coefficients and the
#coefficient of dterminationfrom a multiple regression model,
\#i.e. y = x1 * beta1 + x2 * beta2 +e, where y, x1 and x2 are
\#column-standardised.
dat=dat2
nv=length(dat$V1)
v1=c(1)
v2=c(2)
expected_ratio=0.04
output=r2_enrich_beta(dat,v1,v2,nv,expected_ratio)
output
#r2redux output
#output$beta1_sq (beta1_sq)
#0.01118301
#output$beta2_sq (beta2_sq)
#0.004980285
#output$ratio1 (beta1_sq/R^2)
#0.4392572
#output$ratio2 (beta2_sq/R^2)
#0.1956205
#output$ratio_var1 (variance of ratio 1)
#0.08042288
#output$ratio_var2 (variance of ratio 2)
#0.0431134
#output$upper_ratio1 (upper limit of 95% CI for ratio 1)
#0.9950922
#output$lower_ratio1 (lower limit of 95% CI for ratio 1)
#-0.1165778
#output$upper_ratio2 upper limit of 95% CI for ratio 2)
#0.6025904
```

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```
#output$lower_ratio2 (lower limit of 95% CI for ratio 2)
#-0.2113493

#output$enrich_p1 (two tailed P-value for beta1_sq/R^2 is
#significantly different from exp1)
#0.1591692

#output$enrich_p1_one_tail (one tailed P-value for beta1_sq/R^2
#is significantly different from exp1)
#0.07958459

#output$enrich_p2 (two tailed P-value for beta2_sq/R2 is
#significantly different from (1-exp1))
#0.000232035

#output$enrich_p2_one_tail (one tailed P-value for beta2_sq/R2
#is significantly different from (1-exp1))
#0.0001160175
```

r2\_var

r2\_var function

#### **Description**

This function estimates  $var(R2(y\sim x[,v1]))$  where R2 is the R squared value of the model, where R2 is the R squared value of the model, y is N by 1 matrix having the dependent variable, and x is N by M matrix having M explanatory variables. v1 indicates the ith column in the x matrix (v1 can be multiple values between 1 - M, see Arguments below)

#### Usage

```
r2_var(dat, v1, nv)
```

# **Arguments**

dat	N by $(M+1)$ matrix having variables in the order of $cbind(y,x)$	
v1	This can be set as $v1=c(1)$ , $v1=c(1,2)$ or possibly with more values	
nv	Sample size	

#### Value

This function will test the null hypothesis for R2. To get the test statistics for  $R2(y\sim x[,v1])$ . The outputs are listed as follows.

rsq R2
var Variance of R2
r2\_based\_p P-value under the null hypothesis, i.e. R2=0
upper\_r2 Upper limit of 95% CI for R2
lower\_r2 Lower limit of 95% CI for R2

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```
#To get the test statistics for R2(y\sim x[,v1])
dat=dat1
nv=length(dat$V1)
v1=c(1)
output=r2_var(dat,v1,nv)
output
#r2redux output
#output$rsq (R2)
#0.03836254
#output$var (variance of R2)
#0.0001436128
#output$r2_based_p (P-value under the null hypothesis, i.e. R2=0)
#1.188162e-10
#output$upper_r2 (upper limit of 95% CI for R2)
#0.06433782
#output$lower_r2 (lower limit of 95% CI for R2)
#0.01764252
#To get the test statistic for R2(y \sim x[,v1]+x[,v2]+x[,v3])
dat=dat1
nv=length(dat$V1)
v1=c(1,2,3)
r2_var(dat,v1,nv)
#r2redux output
#output$rsq (R2)
#0.03836254
#output$var (variance of R2)
#0.0001436128
#output$r2_based_p (R2 based P-value)
#1.188162e-10
#output$upper_r2 (upper limit of 95% CI for R2)
#0.06433782
#output$lower_r2 (lower limit of 95% CI for R2)
#0.0176425
#When comparing two independent sets of PGSs
#Let's assume dat1$V1 and dat2$V2 are independent for this example
#(e.g. male PGS vs. female PGS)
```

 $r_diff$ 

```
nv=length(dat1$V1)
v1=c(1)
output1=r2_var(dat1,v1,nv)
nv=length(dat2$V1)
v1=c(1)
output2=r2_var(dat2,v1,nv)
#To get the difference between two independent sets of PGSs
r2_diff_independent=abs(output1$rsq-output2$rsq)
#To get the variance of the difference between two independent sets of PGSs
var_r2_diff_independent= output1$var+output2$var
sd_r2_diff_independent=sqrt(var_r2_diff_independent)
#To get p-value (following eq. 15 in the paper)
chi=r2_diff_independent^2/var_r2_diff_independent
p_value=pchisq(chi,1,lower.tail=FALSE)
#to get 95% CI (following eq. 15 in the paper)
uci=r2_diff_independent+1.96*sd_r2_diff_independent
lci=r2_diff_independent-1.96*sd_r2_diff_independent
```

r\_diff

r\_diff function

#### **Description**

This function estimates  $var(R(y\sim x[,v1]) - R(y\sim x[,v2]))$  where R is the correlation between y and x, y is N by 1 matrix having the dependent variable, and x is N by M matrix having M explanatory variables. v1 or v2 indicates the ith column in the x matrix (v1 or v2 can be multiple values between 1 - M, see Arguments below)

# Usage

```
r_diff(dat, v1, v2, nv)
```

# Arguments

dat	N by $(M+1)$ matrix having variables in the order of $cbind(y,x)$
v1	This can be set as $v1=c(1)$ or $v1=c(1,2)$
v2	This can be set as $v2=c(2)$ , $v2=c(3)$ , $v2=c(1,3)$ or $v2=c(3,4)$
nv	Sample size

#### Value

This function will estimate significant difference between two PGS (either dependent or independent and joint or single). To get the test statistics for the difference between  $R(y\sim x[,v1])$  and  $R(y\sim x[,v2])$ . (here we define  $R_1=R(y\sim x[,v1])$ ) and  $R_2=R(y\sim x[,v2])$ ). The outputs are listed as follows.

 $r_{-}$ diff

```
var2 variance of R_2

var_diff Variance of difference between R_1 and R_2

r2_based_p P-value for significant difference between R_1 and R_2 for two tailed test
r_based_p_one_tail
P-value for significant difference between R_1 and R_2 for one tailed test

mean_diff Differences between R_1 and R_2

upper_diff Upper limit of 95% CI for the difference

lower_diff Lower limit of 95% CI for the difference
```

```
#To get the test statistics for the difference between R(y\sim x[,v1]) and
\#R(y \sim x[,v2]). (here we define R_1=R(y \sim x[,v1])) and R_2=R(y \sim x[,v2])))
dat=dat1
nv=length(dat$V1)
v1=c(1)
v2=c(2)
output=r_diff(dat,v1,v2,nv)
output
#r2redux output
#output$r1 (R_1)
#0.1958636
#output$r2 (R_2)
#0.197006
#output$var1 (variance of R_1)
#0.0009247466
#output$var2 (variance of R_1)
#0.0001451358
#output$var_diff (variance of difference between R_1 and R_2)
#3.65286e-06
#output$r_based_p (two tailed p-value for significant difference between R_1 and R_2)
#0.5500319
#output$r_based_p_one_tail (one tailed p-value for significant difference between R_1 and
#0.2750159
#output$mean_diff
\#-0.001142375 (differences between R2_1 and R2_2)
#output$upper_diff (upper limit of 95% CI for the difference)
#0.002603666
#output$lower_diff (lower limit of 95% CI for the difference)
#-0.004888417
#To get the test statistics for the difference between R(y\sim x[,v1]+[,v2]) and
```

 $r_diff$ 

```
\#R(y \sim x[,v2]). (here R_1=R(y \sim x[,v1]+x[,v2]) and R_2=R(y \sim x[,v1]))
nv=length(dat$V1)
v1=c(1,2)
v2=c(2)
output=r_diff(dat,v1,v2,nv)
output
#output$r1
#0.1974001
#output$r2
#0.197006
#output$var1
#0.0009235848
#output$var2
#0.0009238836
#output$var_diff
#3.837451e-06
#output$r2_based_p
#0.8405593
#output$mean_diff
#0.0003940961
#output$upper_diff
#0.004233621
#output$lower_diff
#-0.003445429
```

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