Package 'r2redux'

August 2, 2022

Title R2 Statistic

Version 1.0.10	
Description R2 statistic for significance test. Variance and covariance of R2 values used to assess the 95% CI and p-value of the R2 difference.	
License GPL (>=3)	
Encoding UTF-8	
Roxygen list(markdown = TRUE)	
RoxygenNote 7.1.2	
NeedsCompilation no	
Depends R (>= 2.10)	
LazyData true	
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cc_trf

cc_trf cc_trf function

Description

This function transforms the predictive ability (R2) and its standard error (se) between the observed scale and liability scale

Usage

```
cc_trf(R2, se, K, P)
```

Arguments

R2	R2 or coefficient of determination on the observed or liability scale
se	Standard error of R2
K	Population prevalence
P	The ratio of cases in the study samples

Value

This function will transform the R2 and its s.e between observed scale and liability scale.Output from the command is the lists of outcomes.

R21	Transformed R2 on the liability scale
sel	Transformed se on the liability scale
R20	Transformed R2 on the observed scale
se0	Transformed se on the observed scale

References

Lee, S. H., Goddard, M. E., Wray, N. R., and Visscher, P. M. A better coefficient of determination for genetic profile analysis. Genetic epidemiology, (2012). 36(3): p. 214-224.

```
#To get the transformed R2
output=cc_trf(0.06, 0.002, 0.05, 0.05)
output

#output$R21 (transformed R2 on the liability scale)
#0.2679337

#output$sel (transformed se on the liability scale)
#0.008931123

#output$R20 (transformed R2 on the observed scale)
#0.01343616

#output$se0 (transformed se on the observed scale)
#0.000447872
```

dat1 3

dat1

Phenotypes and 10 sets of PGSs

Description

A dataset containing phenotypes and multiple PGSs estimated from 10 sets of SNPs according to GWAS p-value thresholds

Usage

dat1

Format

A data frame with 1000 rows and 11 variables:

- V1 Phenotype, value
- V2 PGS1, for p value threshold <=1
- **V3** PGS2, for p value threshold <=0.5
- V4 PGS3, for p value threshold <=0.4
- V5 PGS4, for p value threshold <=0.3
- **V6** PGS5, for p value threshold <=0.2
- V7 PGS6, for p value threshold <=0.1
- **V8** PGS7, for p value threshold <=0.05
- **V9** PGS8, for p value threshold <=0.01
- V10 PGS9, for p value threshold <=0.001
- V11 PGS10, for p value threshold <=0.0001

dat2

Phenotypes and 2 sets of PGSs

Description

A dataset containing phenotypes and 2 sets of PGSs estimated from 2 sets of SNPs from regulatroy and non-regulatory genomic regions

Usage

dat2

Format

A data frame with 1000 rows and 3 variables:

- V1 Phenotype
- V2 PGS1, regulatory region
- V3 PGS2, non-regulatory region

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olkin12_1

olkin12_1 function

Description

olkin12_1 function

Usage

```
olkin12_1(omat, nv)
```

Arguments

omat 3 by 3 matrix having the correlation coefficients between y, x1 and x2, i.e.

omat=cor(dat) where dat is N by 3 matrix having variables in the order of cbind

(y,x1,x2)

nv Sample size

Value

This function will be used as source code

olkin12_13

olkin12_13 function

Description

olkin12_13 function

Usage

```
olkin12_13(omat, nv)
```

Arguments

omat 3 by 3 matrix having the correlation coefficients between y, x1 and x2, i.e.

omat=cor(dat) where dat is N by 3 matrix having variables in the order of cbind

(y,x1,x2)

nv Sample size

Value

This function will be used as source code

olkin12_3 5

olkin12_3

olkin12_3 function

Description

olkin12_3 function

Usage

```
olkin12_3(omat, nv)
```

Arguments

omat 3 by 3 matrix having the correlation coefficients between y, x1 and x2, i.e.

omat=cor(dat) where dat is N by 3 matrix having variables in the order of cbind

(y,x1,x2)

nv Sample size

Value

This function will be used as source code

olkin12_34

olkin12_34 function

Description

olkin12_34 function

Usage

```
olkin12_34(omat, nv)
```

Arguments

omat 3 by 3 matrix having the correlation coefficients between y, x1 and x2, i.e.

omat=cor(dat) where dat is N by 3 matrix having variables in the order of cbind

(y,x1,x2)

nv Sample size

Value

This function will be used as source code

6 olkin_beta1_2

olkin1_2	olkin1_2 function
----------	-------------------

Description

olkin1_2 function

Usage

```
olkin1_2(omat, nv)
```

Arguments

omat 3 by 3 matrix having the correlation coefficients between y, x1 and x2, i.e.

omat=cor(dat) where dat is N by 3 matrix having variables in the order of cbind

(y,x1,x2)

nv Sample size

Value

This function will be used as source code

```
olkin_beta1_2 olkin_beta1_2 function
```

Description

This function derives Information matrix for beta 1^2 and beta 2^2 where beta 1 and 2 are regression coefficients from a multiple regression model, i.e. y = x1 beta 1 + x2 beta 2 + e, where y, x1 and x2 are column-standardised, (i.e. in the context of correlation coefficients, see Olkin and Finn 1995).

Usage

```
olkin_beta1_2(omat, nv)
```

Arguments

omat 3 by 3 matrix having the correlation coefficients between y, x1 and x2, i.e.

omat=cor(dat) where dat is N by 3 matrix having variables in the order of cbind

(y,x1,x2)

nv Sample size

Value

var1_2

This function will give information (variance-covariance) matrix of beta1^2 and beta2^2. To get information (variance-covariance) matrix of beta1^2 and beta2^2. Where beta1 and beta2 are regression coefficients from a multiple regression model. The outputs are listed as follows.

C	1 6	
info	2x2 information (variance-covariance) matrix	
var1	Variance of beta1^2	
var2	Variance of beta2^2	

Variance of difference between beta1^2 and beta2^2

olkin_beta_inf 7

References

Olkin, I. and Finn, J.D. Correlations redux. Psychological Bulletin, 1995. 118(1): p. 155.

Examples

```
#To get information (variance-covariance) matrix of beta1_2 and beta2_2 where
#betal and 2 are regression coefficients from a multiple regression model.
omat=cor(dat)[1:3,1:3]
#omat
#1.0000000 0.1958636 0.1970060
#0.1958636 1.0000000 0.9981003
#0.1970060 0.9981003 1.0000000
nv=length(dat$V1)
output=olkin_beta1_2(omat,nv)
output
#output$info (2x2 information (variance-covariance) matrix)
#0.04146276 0.08158261
#0.08158261 0.16111124
#output$var1 (variance of beta1^2)
#0.04146276
#output$var2 (variance of beta2^2)
#0.1611112
#output$var1_2 (variance of difference between beta1^2 and beta2^2)
#0.03940878
```

olkin_beta_inf olk

olkin_beta_inf function

Description

This function derives Information matrix for beta1 and beta2 where beta1 and 2 are regression coefficients from a multiple regression model, i.e. y = x1 * beta1 + x2 * beta2 + e, where y, x1 and x2 are column-standardised (see Olkin and Finn 1995).

Usage

```
olkin_beta_inf(omat, nv)
```

Arguments

omat 3 by 3 matrix having the correlation coefficients between y, x1 and x2, i.e.

omat=cor(dat) where dat is N by 3 matrix having variables in the order of cbind

(y,x1,x2)

nv Sample size

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Value

This function will generate information (variance-covariance) matrix of beta1 and beta2. The outputs are listed as follows.

info
 var1
 var2
 var2
 var1
 var2
 var2
 var1
 var2
 var2
 var2
 var2
 var3
 var4
 var4
 var5
 var6
 <li

References

Olkin, I. and Finn, J.D. Correlations redux. Psychological Bulletin, 1995. 118(1): p. 155.

Examples

```
#To get information (variance-covariance) matrix of betal and beta2 where
#beta1 and 2 are regression coefficients from a multiple regression model.
dat=dat1
omat=cor(dat)[1:3,1:3]
#omat
#1.0000000 0.1958636 0.1970060
#0.1958636 1.0000000 0.9981003
#0.1970060 0.9981003 1.0000000
nv=length(dat$V1)
output=olkin_beta_inf(omat,nv)
output
#output$info (2x2 information (variance-covariance) matrix)
#0.2531406 -0.2526212
#-0.2526212 0.2530269
#output$var1 (variance of beta1)
#0.2531406
#output$var2 (variance of beta2)
#0.2530269
#output$var1_2 (variance of difference between beta1 and beta2)
#1.01141
```

olkin_beta_ratio olkin_beta_ratio function

Description

This function derives variance of beta $1^2 / R^2$ where beta 1 and 2 are regression coefficients from a multiple regression model, i.e. y = x1beta + x2beta + e, where y, x1 and x2 are column-standardised (see Olkin and Finn 1995).

Usage

```
olkin_beta_ratio(omat, nv)
```

r2_beta_var

Arguments

omat 3 by 3 matrix having the correlation coefficients between y, x1 and x2, i.e.

omat=cor(dat) where dat is N by 3 matrix having variables in the order of cbind

(y,x1,x2)

nv sampel size

Value

This function will generate the variance of the proportion, i.e. beta1^2/R^2. The outputs are listed as follows.

ratio_var Variance of ratio

References

Olkin, I. and Finn, J.D. Correlations redux. Psychological Bulletin, 1995. 118(1): p. 155.

Examples

```
#To get information (variance-covariance) matrix of beta1 and beta2 where
#beta1 and 2 are regression coefficients from a multiple regression model.
dat=dat1
omat=cor(dat)[1:3,1:3]
#omat
#1.0000000 0.1958636 0.1970060
#0.1958636 1.0000000 0.9981003
#0.1970060 0.9981003 1.0000000

nv=length(dat$V1)
output=olkin_beta_ratio(omat,nv)
output
#r2redux output

#output$ratio_var (Variance of ratio)
#27.20206
```

r2_beta_var

r2_beta_var

Description

This function estimates var(beta1^2) and (beta2^2), and beta1 and 2 are regression coefficients from a multiple regression model, i.e. y = x1beta1 + x2beta2 + e, y, x1 and x2 are column-standardised (see Olkin and Finn 1995). y is N by 1 matrix having the dependent variable, x1 is N by 1 matrix having the jth explanatory variable. x1 is N by 1 matrix having the jth explanatory variable. x1 and x2 indicates the ith and jth column in the data (x1 or x2 should be a single interger between 1 - M, see Arguments below).

Usage

```
r2_beta_var(dat, v1, v2, nv)
```

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Arguments

dat	N by $(M+1)$ matrix having variables in the order of cbind (y,x)	
v1	This can be set as $v1=1$, $v1=2$, $v1=3$ or any value between 1 - M based on combination	
v2	This can be set as $v2=1$, $v2=2$, $v2=3$, or any value between 1 - M based on combination	
nv	Sample size	

Value

This function will estiamte the variance of beta1 2 and beta2 2 and, and the covariance between beta1 2 and beta2 2 , i.e. the information matrix of squared regression coefficients. beta1 and beta2 are regression coefficients from a multiple regression model, i.e. y = x1beta1 + x2beta2 + e, where y, x1 and x2 are column-standardised. The outputs are listed as follows.

```
beta1_sq
                 beta1_sq
beta2_sq
                 beta2_sq
                 Variance of beta1_sq
var1
                 Variance of beta 2 sq
var2
                 Variance of difference between t1 and t2
var1_2
                 Covariance between t1 and t2
upper_beta1_sq
                 upper limit of 95% CI for beta1_sq
lower_beta1_sq
                 lower limit of 95% CI for beta1_sq
upper_beta2_sq
                 upper limit of 95% CI for beta2_sq
lower_beta2_sq
                 lower limit of 95% CI for beta2_sq
```

References

Olkin, I. and Finn, J.D. Correlations redux. Psychological Bulletin, 1995. 118(1): p. 155.

```
#To get the 95% CI of beta1_sq and beta2_sq
#beta1 and beta2 are regression coefficients from a multiple regression model,
#i.e. y = x1.beta1 + x2.beta2 +e, where y, x1 and x2 are column-standardised.

dat=dat2
nv=length(dat$V1)
v1=c(1)
v2=c(2)
output=r2_beta_var(dat,v1,v2,nv)
output
#r2redux output
#output$beta1_sq (beta1_sq)
#0.01118301

#output$beta2_sq (beta2_sq)
```

 $r2_diff$

```
#0.004980285
#output$var1 (variance of beta1_sq)
#7.072931e-05
#output$var2 (variance of beta2_sq)
#3.161929e-05
#output$var1_2 (variance of difference between beta1_sq and beta2_sq)
#0.000162113
#output$cov (covariance between t1 and t2)
#-2.988221e-05
#output$upper_beta1_sq (upper limit of 95% CI for beta1_sq)
#0.03037793
#output$lower_beta1_sq (lower limit of 95% CI for beta1_sq)
#-0.00123582
#output$upper_beta2_sq (upper limit of 95% CI for beta2_sq)
#0.02490076
#output$lower_beta2_sq (lower limit of 95% CI for beta2_sq)
#-0.005127546
```

r2_diff

r2_diff function

Description

This function estimates $var(R2(y\sim x[,v1]) - R2(y\sim x[,v2]))$ where R2 is the R squared value of the model, y is N by 1 matrix having the dependent variable, and x is N by M matrix having M explanatory variables. v1 or v2 indicates the ith column in the x matrix (v1 or v2 can be multiple values between 1 - M, see Arguments below)

Usage

```
r2_diff(dat, v1, v2, nv)
```

Arguments

dat	N by (M+1) matrix having variables in the order of cbind(y,x)
v1	This can be set as $v1=c(1)$ or $v1=c(1,2)$
v2	This can be set as $v2=c(2)$, $v2=c(3)$, $v2=c(1,3)$ or $v2=c(3,4)$
nv	Sample size

Value

This function will estimate significant difference between two PGS (either dependent or independent and joint or single). To get the test statistics for the difference between $R2(y\sim x[,v1])$ and $R2(y\sim x[,v2])$. (here we define $R2_1=R2(y\sim x[,v1])$) and $R2_2=R2(y\sim x[,v2])$). The outputs are listed as follows.

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```
R2 1
rsq1
                 R2_2
rsq2
                 Variance of R2_1
var1
                 variance of R2 2
var2
var diff
                 Variance of difference between R2 1 and R2 2
                 two tailed P-value for significant difference between R2_1 and R2_2
r2_based_p
r2_based_p_one_tail
                 one tailed P-value for significant difference
                 Differences between R2_1 and R2_2
mean_diff
upper_diff
                 Upper limit of 95% CI for the difference
lower diff
                 Lower limit of 95% CI for the difference
```

Examples

#0.001028172

```
\#To get the test statistics for the difference between R2(y\sim x[,v1]) and
\#R2(y\sim x[,v2]). (here we define R2_1=R2(y\sim x[,v1])) and R2_2=R2(y\sim x[,v2])))
dat=dat1
nv=length(dat$V1)
v1=c(1)
v2=c(2)
output=r2_diff(dat,v1,v2,nv)
output
#r2redux output
#output$rsq1 (R2_1)
#0.03836254
#output$rsq2 (R2_2)
#0.03881135
#output$var1 (variance of R2_1)
#0.0001436128
#output$var2 (variance of R2_2)
#0.0001451358
#output$var_diff (variance of difference between R2_1 and R2_2)
#5.678517e-07
#output$r2_based_p (two tailed p-value for significant difference between R2_1 and R2_2)
#0.5514562
#output$r2_based_p_one_tail(one tailed p-value for significant difference)
#0.2757281
#output$mean_diff (differences between R2_1 and R2_2)
#-0.0004488044
#output$upper_diff (upper limit of 95% CI for the difference)
```

r2_enrich_beta

```
#output$lower_diff (lower limit of 95% CI for the difference)
#-0.001925781
#To get the test statistics for the difference between R2(y\sim x[,v1]+x[,v2]) and
\#R2(y\sim x[,v2]). (here R2_1=R2(y\sim x[,v1]+x[,v2]) and R2_2=R2(y\sim x[,v1]))
dat = dat 1
nv=length(dat$V1)
v1=c(1,2)
v2=c(1)
output=r2_diff(dat,v1,v2,nv)
#r2redux output
#output$rsq1 (R2_1)
#0.03896678
#output$rsq2 (R2_2)
#0.03836254
#output$var1 (variance of R2_1)
#0.0001473686
#output$var2 (variance of R2_2)
#0.0001436128
#output$var_diff (variance of difference between R2_1 and R2_2)
#2.321425e-06
#output$r2_based_p (p-value for significant difference between R2_1 and R2_2)
#0.4366883
#output$mean_diff (differences between R2_1 and R2_2)
#0.0006042383
#output$upper_diff (upper limit of 95% CI for the difference)
#0.00488788
#output$lower_diff (lower limit of 95% CI for the difference)
#-0.0005576171
```

r2_enrich_beta r2_enrich_beta

Description

This function estimates $var(beta1^2/R^2)$, beta1 and R^2 are regression coefficient and the coefficient of determination from a multiple regression model, i.e. y = x1beta1 + x2beta2 + e, where y, x1 and x2 are column-standardised (see Olkin and Finn 1995). y is y by 1 matrix having the dependent variable, and y is y by 1 matrix having the ith explanatory variables. y is y by 1 matrix having the y indicates the ith and y indicates the ith and y indicates the data (y1 or y2 should be a single interger between 1 - y4, see Arguments below).

r2_enrich_beta

Usage

```
r2_enrich_beta(dat, v1, v2, nv, exp1)
```

Arguments

dat	N by $(M+1)$ matrix having variables in the order of $cbind(y,x)$	
v1	These can be set as $v1=1$, $v1=2$, $v1=3$ or any value between 1 - M based on combination	
v2	These can be set as $v2=1$, $v2=2$, $v2=3$, or any value between 1 - M based on combination	
nv	Sample size	
exp1	The expectation of the ratio (e.g. ratio of # SNPs in genomic partitioning)	

Value

This function will test the ratio which is significantly different from the expectation. To get the test statistic for the ratio which is significantly different from the expectation. var[(t1/exp)-(t2/(1-exp))], where $t1 = beta1^2$ and $t2 = beta2^2$. beta1 and beta2 are regression coefficients from a multiple regression model, i.e. y = x1.beta1 + x2.beta2 + e, where y, x1 and x2 are column-standardised. The outputs are listed as follows.

```
beta1_sq
                 beta1_sq
beta2_sq
                 beta2_sq
ratio1
                 beta1_sq/R^2
ratio2
                 t1/R^2
                 variance of ratio 1
ratio_var1
                 variance of ratio 2
ratio var2
upper_ratio1 upper limit of 95% CI for ratio 1
lower_ratio1 lower limit of 95% CI for ratio 1
upper_ratio2 upper limit of 95% CI for ratio 2
lower_ratio2 lower limit of 95% CI for ratio 2
                 two tailed P-value for beta1_sq/R^2 is significantly different from exp1
enrich_p1
enrich p1 one tail
                 one tailed P-value for beta1_sq/R^2 is significantly different from exp1
enrich_p2
                 P-value for beta2_sq/R2 is significantly different from (1-exp1)
enrich_p2_one_tail
                 one tailed P-value for beta2_sq/R2 is significantly different from (1-exp1)
```

References

Olkin, I. and Finn, J.D. Correlations redux. Psychological Bulletin, 1995. 118(1): p. 155.

r2_enrich_beta

```
#To get the test statistic for the ratio which is significantly
#different from the expectation, this function estiamtes
\#var (beta1^2/R^2), where
#beta1^2 and R^2 are regression coefficients and the
#coefficient of dterminationfrom a multiple regression model,
\#i.e. y = x1*beta1 + x2*beta2 +e, where y, x1 and x2 are
#column-standardised.
dat=dat2
nv=length(dat$V1)
v1=c(1)
v2=c(2)
expected_ratio=0.04
output=r2_enrich_beta(dat,v1,v2,nv,expected_ratio)
output
#r2redux output
#output$beta1_sq (beta1_sq)
#0.01118301
#output$beta2_sq (beta2_sq)
#0.004980285
#output$ratio1 (beta1_sq/R^2)
#0.4392572
#output$ratio2 (beta2_sq/R^2)
#0.1956205
#output$ratio_var1 (variance of ratio 1)
#0.08042288
#output$ratio_var2 (variance of ratio 2)
#0.0431134
#output$upper_ratio1 (upper limit of 95% CI for ratio 1)
#0.9950922
#output$lower_ratio1 (lower limit of 95% CI for ratio 1)
#-0.1165778
#output$upper_ratio2 upper limit of 95% CI for ratio 2)
#0.6025904
#output$lower_ratio2 (lower limit of 95% CI for ratio 2)
#-0.2113493
\#output\$enrich_p1 (two tailed P-value for beta1_sq/R^2 is
#significantly different from exp1)
#0.1591692
\#output\$enrich\_p1\_one\_tail (one tailed P-value for beta1_sq/R^2
#is significantly different from exp1)
#0.07958459
```

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```
#output$enrich_p2 (P-value for beta2_sq/R2 is significantly
#different from (1-exp1))
#0.000232035

#output$enrich_p2_one_tail (one tailed P-value for beta2_sq/R2 is
#significantly different from (1-exp1))
#0.0001160175
```

r2_var

r2_var function

Description

This function estimates $var(R2(y\sim x[,v1]))$ where R2 is the R squared value of the model, where R2 is the R squared value of the model, y is N by 1 matrix having the dependent variable, and x is N by M matrix having M explanatory variables. v1 indicates the ith column in the x matrix (v1 can be multiple values between 1 - M, see Arguments below)

Usage

```
r2_var(dat, v1, nv)
```

Arguments

dat	N by $(M+1)$ matrix having variables in the order of $cbind(y,x)$
v1	This can be set as $v1=c(1)$, $v1=c(1,2)$ or possibly with more values
nv	Sample size

Value

This function will test the null hypothesis for R2. To get the test statistics for $R2(y\sim x[,v1])$. The outputs are listed as follows.

```
rsq R2
var Variance of R2
r2_based_p P-value under the null hypothesis, i.e. R2=0
upper_r2 Upper limit of 95% CI for R2
lower_r2 Lower limit of 95% CI for R2
```

```
#To get the test statistics for R2(y~x[,v1])
dat=dat1
nv=length(dat$V1)
v1=c(1)
output=r2_var(dat,v1,nv)
output
#r2redux output
```

r_diff

```
#output$rsq (R2)
#0.03836254
#output$var (variance of R2)
#0.0001436128
#output$r2_based_p (P-value under the null hypothesis, i.e. R2=0)
#1.188162e-10
#output$upper_r2 (upper limit of 95% CI for R2)
#0.06433782
#output$lower_r2 (lower limit of 95% CI for R2)
#0.01764252
#To get the test statistic for R2(y \sim x[,v1]+x[,v2]+x[,v3])
dat=dat1
nv=length(dat$V1)
v1=c(1,2,3)
r2_var(dat,v1,nv)
#r2redux output
#output$rsq (R2)
#0.03836254
#output$var (variance of R2)
#0.0001436128
#output$r2_based_p (R2 based P-value)
#1.188162e-10
#output$upper_r2 (upper limit of 95% CI for R2)
#0.06433782
#output$lower_r2 (lower limit of 95% CI for R2)
#0.01764252
```

r_diff

r_diff function

Description

This function estimates $var(R(y\sim x[,v1]) - R(y\sim x[,v2]))$ where R is the correlation between y and x, y is N by 1 matrix having the dependent variable, and x is N by M matrix having M explanatory variables. v1 or v2 indicates the ith column in the x matrix (v1 or v2 can be multiple values between 1 - M, see Arguments below)

Usage

```
r_diff(dat, v1, v2, nv)
```

 r_diff

Arguments

dat	N by (M+1) matrix having variables in the order of cbind(y,x)
v1	This can be set as $v1=c(1)$ or $v1=c(1,2)$
v2	This can be set as $v2=c(2)$, $v2=c(3)$, $v2=c(1,3)$ or $v2=c(3,4)$
nv	Sample size

Value

This function will estimate significant difference between two PGS (either dependent or independent and joint or single). To get the test statistics for the difference between $R2(y\sim x[,v1])$ and $R2(y\sim x[,v2])$. (here we define $R2_1=R2(y\sim x[,v1])$) and $R2_2=R2(y\sim x[,v2])$). The outputs are listed as follows.

```
R_1
r1
                 R 2
r2
                 Variance of R_1
var1
var2
                 variance of R_2
                 Variance of difference between R_1 and R_2
var_diff
                 P-value for significant difference between R_1 and R_2 for two tailed test
r2_based_p
r_based_p_one_tail
                 P-value for significant difference between R_1 and R_2 for one tailed test
                 Differences between R_1 and R_2
mean_diff
                 Upper limit of 95% CI for the difference
upper_diff
                 Lower limit of 95% CI for the difference
lower diff
```

```
#To get the test statistics for the difference between R(y\sim x[,v1]) and
\#R(y\sim x[,v2]). (here we define R_1=R(y\sim x[,v1])) and R_2=R(y\sim x[,v2])))
dat=dat1
nv=length(dat$V1)
v1=c(1)
v2=c(2)
output=r_diff(dat,v1,v2,nv)
output
#r2redux output
#output$r1 (R_1)
#0.1958636
#output$r2 (R_2)
#0.197006
#output$var1 (variance of R_1)
#0.0009247466
#output$var2 (variance of R_1)
#0.0001451358
```

```
#output$var_diff (variance of difference between R_1 and R_2)
#3.65286e-06
#output$r_based_p (two tailed p-value for significant difference between R_1 and R_2)
#0.5500319
#output$r_based_p_one_tail (one tailed p-value for significant difference between R_1 and
#0.2750159
#output$mean diff
\#-0.001142375 (differences between R2_1 and R2_2)
#output$upper_diff (upper limit of 95% CI for the difference)
#0.002603666
#output$lower_diff (lower limit of 95% CI for the difference)
#-0.004888417
#To get the test statistics for the difference between R(y\sim x[,v1]+[,v2]) and
\#R(y \sim x[,v2]). (here R_1=R(y \sim x[,v1]+x[,v2]) and R_2=R(y \sim x[,v1]))
nv=length(dat$V1)
v1=c(1,2)
v2=c(2)
output=r_diff(dat,v1,v2,nv)
output
#output$r1
#0.1974001
#output$r2
#0.197006
#output$var1
#0.0009235848
#output$var2
#0.0009238836
#output$var_diff
#3.837451e-06
#output$r2_based_p
#0.8405593
#output$mean_diff
#0.0003940961
#output$upper_diff
#0.004233621
#output$lower_diff
#-0.003445429
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