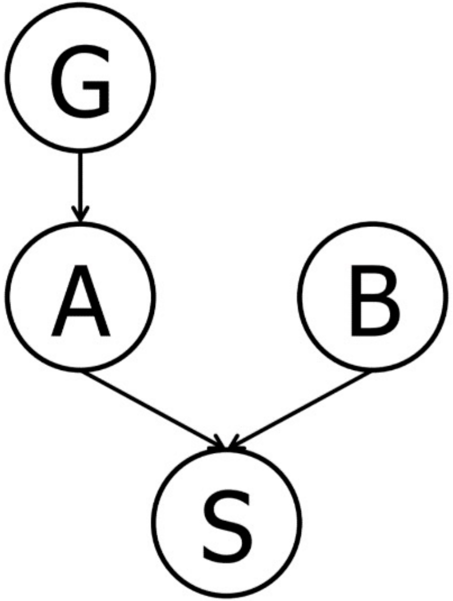
# Final Practice

# Belief net

Suppose that a patient can have a symptom (S) that can be caused by two different diseases (A and B). It is known that the variation of gene G plays a big role in the manifestation of disease A. The Bayes' Net and corresponding conditional probability tables for this situation are shown below.

| $\mathbb{P}(G)$ | |
|---|---|
| $+g$ | 0.1 |
| $-g$ | 0.9 |

| $\mathbb{P}(A\|G)$ | | |
|---|---|---|
| $+g$ | $+a$ | 1.0 |
| $+g$ | $-a$ | 0.0 |
| $-g$ | $+a$ | 0.1 |
| $-g$ | $-a$ | 0.9 |

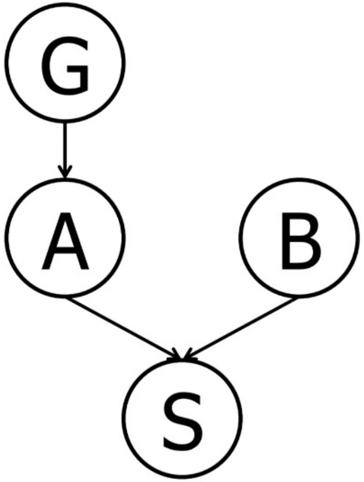



| $\mathbb{P}(B)$ | |
|---|---|
| $+b$ | 0.4 |
| $-b$ | 0.6 |

| $\mathbb{P}(S\|A,B)$ | | | |
|---|---|---|---|
| $+a$ | $+b$ | $+s$ | 1.0 |
| $+a$ | $+b$ | $-s$ | 0.0 |
| $+a$ | $-b$ | $+s$ | 0.9 |
| $+a$ | $-b$ | $-s$ | 0.1 |
| $-a$ | $+b$ | $+s$ | 0.8 |
| $-a$ | $+b$ | $-s$ | 0.2 |
| $-a$ | $-b$ | $+s$ | 0.1 |
| $-a$ | $-b$ | $-s$ | 0.9 |

| $\mathbb{P}(G)$ | |
|---|---|
| $+g$ | 0.1 |
| $-g$ | 0.9 |

| $\mathbb{P}(A\|G)$ | | |
|---|---|---|
| $+g$ | $+a$ | 1.0 |
| $+g$ | $-a$ | 0.0 |
| $-g$ | $+a$ | 0.1 |
| $-g$ | $-a$ | 0.9 |



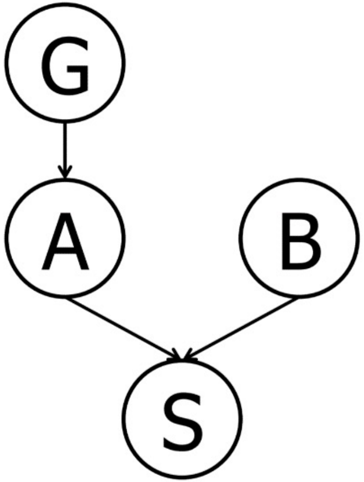

| $\mathbb{P}(B)$ | |
|---|---|
| $+b$ | 0.4 |
| $-b$ | 0.6 |

| $\mathbb{P}(S\|A,B)$ | | | |
|---|---|---|---|
| $+a$ | $+b$ | $+s$ | 1.0 |
| $+a$ | $+b$ | $-s$ | 0.0 |
| $+a$ | $-b$ | $+s$ | 0.9 |
| $+a$ | $-b$ | $-s$ | 0.1 |
| $-a$ | $+b$ | $+s$ | 0.8 |
| $-a$ | $+b$ | $-s$ | 0.2 |
| $-a$ | $-b$ | $+s$ | 0.1 |
| $-a$ | $-b$ | $-s$ | 0.9 |

a. Compute the following entry from the joint distribution: P(+g, +a, +b, +s)
b. What is the probability that a patient has disease A? P(+a)
c. What is the probability that a patient has disease A given that they have disease B? P(+a| + b)
d. What is the probability that a patient has disease A given that they have symptom S and disease B? P(+a| + s, +b)

| $\mathbb{P}(G)$ | |
|---|---|
| $+g$ | 0.1 |
| $-g$ | 0.9 |

| $\mathbb{P}(A\|G)$ | | |
|---|---|---|
| $+g$ | $+a$ | 1.0 |
| $+g$ | $-a$ | 0.0 |
| $-g$ | $+a$ | 0.1 |
| $-g$ | $-a$ | 0.9 |




| $\mathbb{P}(B)$ | |
|---|---|
| $+b$ | 0.4 |
| $-b$ | 0.6 |

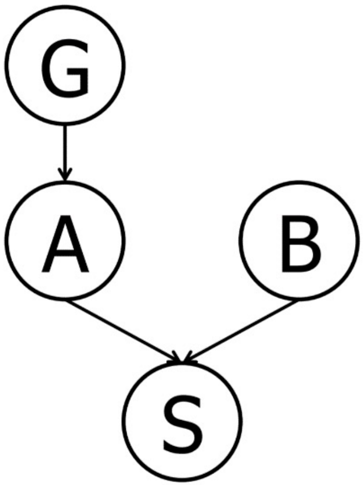
| $\mathbb{P}(S\|A,B)$ | | | |
|---|---|---|---|
| $+a$ | $+b$ | $+s$ | 1.0 |
| $+a$ | $+b$ | $-s$ | 0.0 |
| $+a$ | $-b$ | $+s$ | 0.9 |
| $+a$ | $-b$ | $-s$ | 0.1 |
| $-a$ | $+b$ | $+s$ | 0.8 |
| $-a$ | $+b$ | $-s$ | 0.2 |
| $-a$ | $-b$ | $+s$ | 0.1 |
| $-a$ | $-b$ | $-s$ | 0.9 |

a. Compute the following entry from the joint distribution: P(+g, +a, +b, +s)

- P(+g, +a, +b, +s) = P(+s | +a, +b, +g) P(+a, +b, +g)
- P(+g, +a, +b, +s) = P(+s | +a, +b) P(+a, +b, +g)
- P(+g, +a, +b, +s) = P(+s | +a, +b) P(+b) P(+a, +g)
- P(+g, +a, +b, +s) = P(+s | +a, +b) P(+b) P(+a | +g) P(+g)
- P(+g, +a, +b, +s) = 1 x 0.4 x 1 x 0.1 = 0.04

| $\mathbb{P}(G)$ | |
|---|---|
| $+g$ | 0.1 |
| $-g$ | 0.9 |

| $\mathbb{P}(A\|G)$ | | |
|---|---|---|
| $+g$ | $+a$ | 1.0 |
| $+g$ | $-a$ | 0.0 |
| $-g$ | $+a$ | 0.1 |
| $-g$ | $-a$ | 0.9 |




| $\mathbb{P}(B)$ | |
|---|---|
| $+b$ | 0.4 |
| $-b$ | 0.6 |

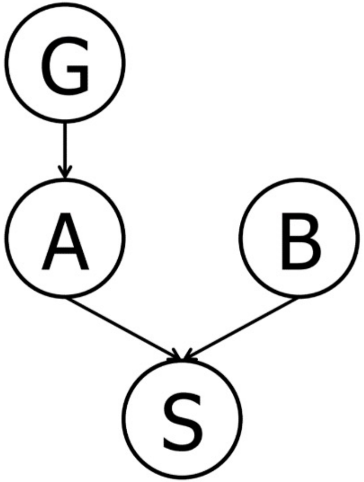
| $\mathbb{P}(S\|A,B)$ | | | |
|---|---|---|---|
| $+a$ | $+b$ | $+s$ | 1.0 |
| $+a$ | $+b$ | $-s$ | 0.0 |
| $+a$ | $-b$ | $+s$ | 0.9 |
| $+a$ | $-b$ | $-s$ | 0.1 |
| $-a$ | $+b$ | $+s$ | 0.8 |
| $-a$ | $+b$ | $-s$ | 0.2 |
| $-a$ | $-b$ | $+s$ | 0.1 |
| $-a$ | $-b$ | $-s$ | 0.9 |

b. What is the probability that a patient has disease A? P(+a)

- P(+a) = P(+a, +g) + P(+a, -g)
- P(+a) = P(+a | +g) P(+g) + P(+a | -g) P(-g)
- P(+a) = 1x0.1 + 0.1x0.9 = 0.19

| $\mathbb{P}(G)$ | |
|---|---|
| $+g$ | 0.1 |
| $-g$ | 0.9 |

| $\mathbb{P}(A \mid G)$ | | |
|---|---|---|
| $+g$ | $+a$ | 1.0 |
| $+g$ | $-a$ | 0.0 |
| $-g$ | $+a$ | 0.1 |
| $-g$ | $-a$ | 0.9 |




| $\mathbb{P}(B)$ | |
|---|---|
| $+b$ | 0.4 |
| $-b$ | 0.6 |

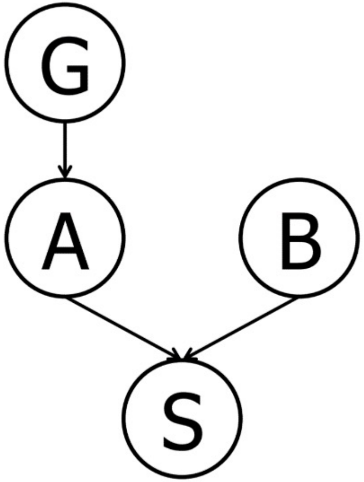
| $\mathbb{P}(S \mid A, B)$ | | | |
|---|---|---|---|
| $+a$ | $+b$ | $+s$ | 1.0 |
| $+a$ | $+b$ | $-s$ | 0.0 |
| $+a$ | $-b$ | $+s$ | 0.9 |
| $+a$ | $-b$ | $-s$ | 0.1 |
| $-a$ | $+b$ | $+s$ | 0.8 |
| $-a$ | $+b$ | $-s$ | 0.2 |
| $-a$ | $-b$ | $+s$ | 0.1 |
| $-a$ | $-b$ | $-s$ | 0.9 |

c. What is the probability that a patient has disease A given that they have disease B? $P(+a \mid +b)$

- No dependency between a and b
- $P(+a \mid +b) = P(+a) = 0.19$

| $\mathbb{P}(G)$ | |
|---|---|
| $+g$ | 0.1 |
| $-g$ | 0.9 |

| $\mathbb{P}(A \mid G)$ | | |
|---|---|---|
| $+g$ | $+a$ | 1.0 |
| $+g$ | $-a$ | 0.0 |
| $-g$ | $+a$ | 0.1 |
| $-g$ | $-a$ | 0.9 |



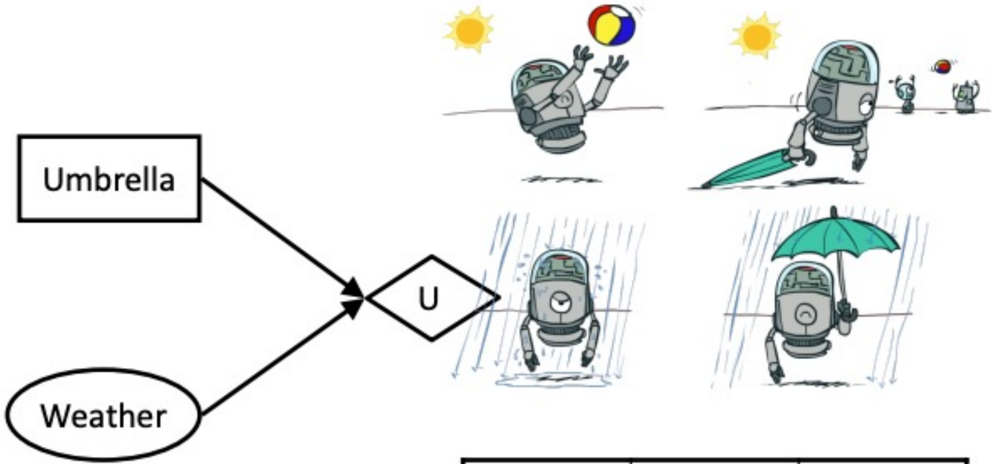

| $\mathbb{P}(B)$ | |
|---|---|
| $+b$ | 0.4 |
| $-b$ | 0.6 |

| $\mathbb{P}(S \mid A, B)$ | | | |
|---|---|---|---|
| $+a$ | $+b$ | $+s$ | 1.0 |
| $+a$ | $+b$ | $-s$ | 0.0 |
| $+a$ | $-b$ | $+s$ | 0.9 |
| $+a$ | $-b$ | $-s$ | 0.1 |
| $-a$ | $+b$ | $+s$ | 0.8 |
| $-a$ | $+b$ | $-s$ | 0.2 |
| $-a$ | $-b$ | $+s$ | 0.1 |
| $-a$ | $-b$ | $-s$ | 0.9 |

d. What is the probability that a patient has disease A given that they have symptom S and disease B? P(+a| + s, +b)

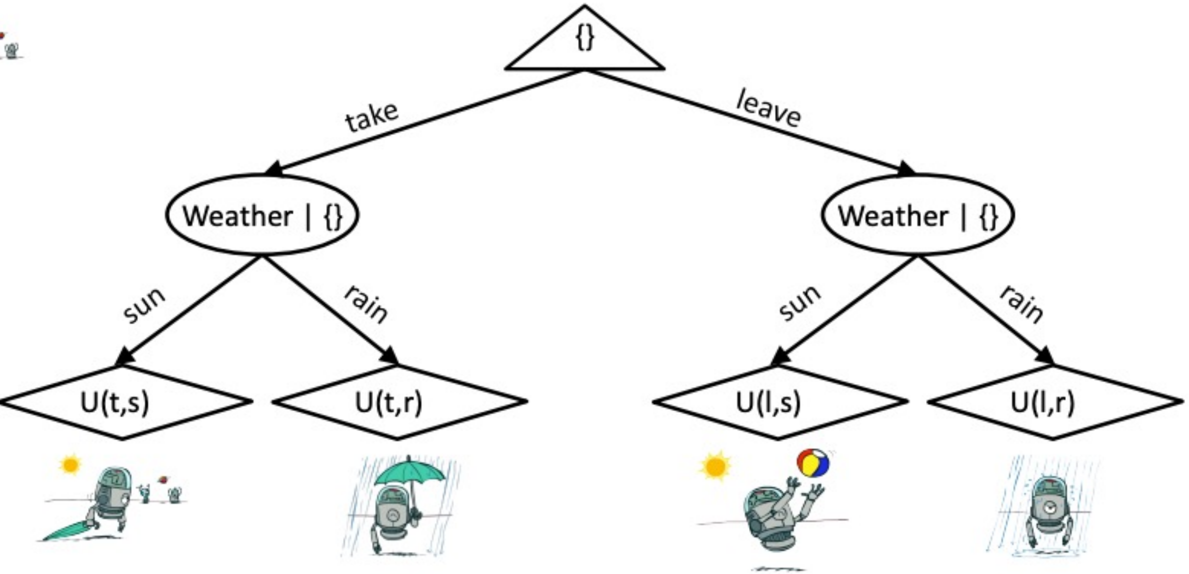
- $P(+a|+s,+b) = \frac{P(+a,+s,+b)}{P(+s,+b)}$
- P(+s, +b, +a) = P(+s | +a, +b) P(+a, + b) = P(+s | +a, +b) P(+a) P(+b) = 1 x 0.19 x 0.4
- P(+s, +b) = P(+s, +b, +a) + P(+s, +b, -a) =
- P(+s, +b) = P(+s | +a, +b) P(+a, +b) + P(+s | -a, +b) P(-a, +b) =
- P(+s, +b) = P(+s | +a, +b) P(+a) P(+b) + P(+s | -a, +b) P(-a) (+b) = 1 x 0.19 x 0.4 + 0.8 x 0.81 x 0.4
- P(+a | +s, +b) = 0.2267

# Making simple decision




| W | P(W) |
|---|---|
| sun | 0.7 |
| rain | 0.3 |

| A | W | U(A,W) |
|---|---|---|
| leave | sun | 100 |
| leave | rain | 0 |
| take | sun | 20 |
| take | rain | 70 |




| W | P(W) |
|---|---|
| sun | 0.7 |
| rain | 0.3 |

| A | W | U(A,W) |
|---|---|---|
| leave | sun | 100 |
| leave | rain | 0 |
| take | sun | 20 |
| take | rain | 70 |

Umbrella = leave

$$\mathrm{EU}(\mathrm{leave}) = \sum_w P(w)U(\mathrm{leave}, w)$$

$$= 0.7 \cdot 100 + 0.3 \cdot 0 = 70$$

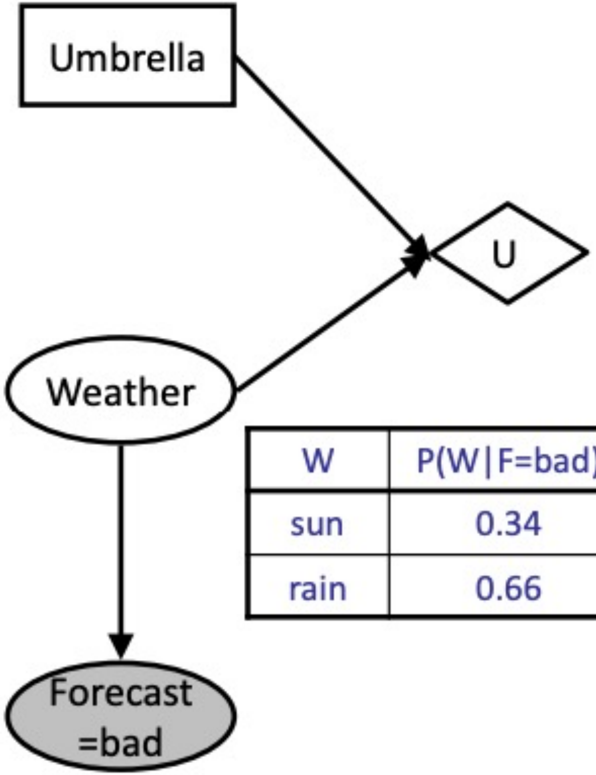
Umbrella = take

$$\mathrm{EU}(\mathrm{take}) = \sum_w P(w)U(\mathrm{take}, w)$$

$$= 0.7 \cdot 20 + 0.3 \cdot 70 = 35$$

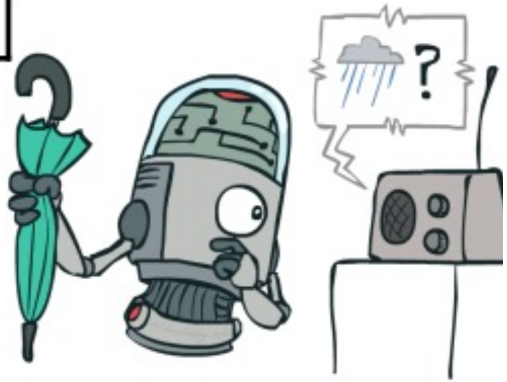
Optimal decision = leave

$$\mathrm{MEU}(\varnothing) = \max_a \mathrm{EU}(a) = 70$$




| A | W | U(A,W) |
|---|---|---|
| leave | sun | 100 |
| leave | rain | 0 |
| take | sun | 20 |
| take | rain | 70 |

| W | P(W\|F=bad) |
|---|---|
| sun | 0.34 |
| rain | 0.66 |




Umbrella = leave

$$\mathrm{EU}(\mathrm{leave}|\mathrm{bad}) = \sum_w P(w|\mathrm{bad})U(\mathrm{leave}, w)$$

$$= 0.34 \cdot 100 + 0.66 \cdot 0 = 34$$

Umbrella = take

$$\mathrm{EU}(\mathrm{take}|\mathrm{bad}) = \sum_w P(w|\mathrm{bad})U(\mathrm{take}, w)$$

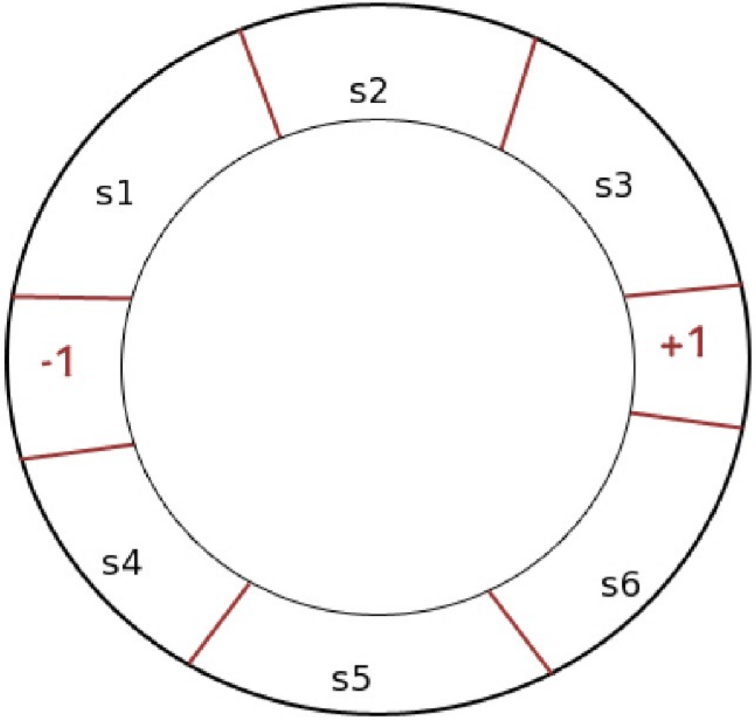
$$= 0.34 \cdot 20 + 0.66 \cdot 70 = 53$$

Optimal decision = take

$$\mathrm{MEU}(F = \mathrm{bad}) = \max_a \mathrm{EU}(a|\mathrm{bad}) = 53$$

# Utility Iteration Algorithm

Consider the above MDP, representing a robot on a circular wheel. The wheel is divided into eight states and the available actions are to move clockwise or counterclockwise. The robot has a poor sense of direction and will move with probability 0.6 in the intended direction and with probability 0.4 in the opposite direction. All states have reward zero, except the terminal states which have rewards −1 and +1 as shown. The discount factor is $\gamma = 0.9$.

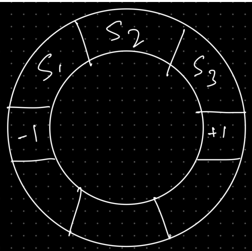



Compute the numeric values of the state-value function U(s) for states s1 through s6: compute U(s1), U(s2), ...

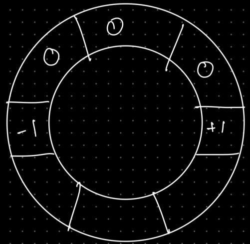
- By symmetry, we know that
  - U(s1) = U(s4)
  - U(s2) = U(s5)
  - U(s3) = U(s6)

- To update U (s1), U (s2), and U (s3), we use Bellman equation
- There is no cost in moving
- Note that since $\gamma$ is not 1, remember to multiply the updated utility with $\gamma$
  - U(s1) = 0.9 [ 0.4∗(−1) + 0.6 ∗U(s2)]
  - U(s2) = 0.9 [ 0.4*U(s1) + 0.6∗U(s3)]
  - U(s3) = 0.9 [ 0.4∗U(s2) + 0.6∗(+1)]

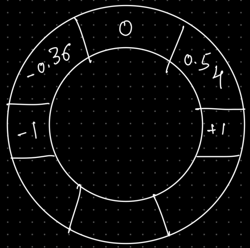
- [ 0,0,0 ] → [ -0.36, 0, 0.54 ] → ...




$V_0$




$V_1$

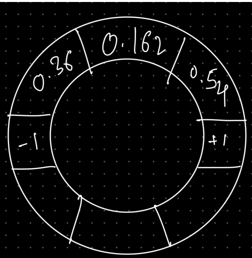



$S_1 = 0 + 0.9(0.4) - 1 = \underline{-0.36}$

$S_2 = 0 + 0.9(0.6)\ 0 = 0$

$S_3 = 0 + 0.9(0.6) + 1 = \underline{0.54}$

$V_2 =$



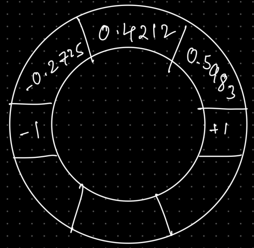

$S_1 = 0 + 0.9(0.4) - 1 = -0.36$

$S_2 = 0 + 0.9(0.6(0.54) + 0.4(-0.36))$

$= 0.162$

$S_3 = 0 + 0.9(0.6)(1) = 0.54$

$V_3$




$S_1 = 0 + 0.9(0.6(0.162) + 0.4(-1))$

$= \underline{-0.2725}$

$S_2 = 0.9(0.6(0.54) + 0.4(0.36))$

$= 0.4212$

$S_3 = 0.9(0.6 \times 1 + 0.4 \times 0.162)$

$= 0.5983$

# Hidden Markov Models

Every day, your pet cat Markov is either in a good or bad mood, with one mood transition between days. Your cat's behavior on a day depends on its mood, but the true mood is always a bit of a mystery. On day one, Markov is in a good mood. The cat mood process can be modeled using an HMM with the following parameters:

$P(M_1)$

| | |
|---|---|
| *good* | 1 |
| *bad* | 0 |

$P(M_t \mid M_{t-1} = good)$

| | |
|---|---|
| *good* | 3/4 |
| *bad* | 1/4 |

$P(M_t \mid M_{t-1} = bad)$

| | |
|---|---|
| *good* | 1/4 |
| *bad* | 3/4 |

Transitions

$P(B_t \mid M_t = good)$

| | |
|---|---|
| *hiss* | 0/5 |
| *meow* | 3/5 |
| *purr* | 2/5 |

$P(B_t \mid M_t = bad)$

| | |
|---|---|
| *hiss* | 3/5 |
| *meow* | 1/5 |
| *purr* | 1/5 |

Emissions

$P(M_1)$

| | |
|---|---|
| *good* | 1 |
| *bad* | 0 |

$P(M_t \mid M_{t-1} = good)$

| | |
|---|---|
| *good* | 3/4 |
| *bad* | 1/4 |

$P(M_t \mid M_{t-1} = bad)$

| | |
|---|---|
| *good* | 1/4 |
| *bad* | 3/4 |

Transitions

$P(B_t \mid M_t = good)$

| | |
|---|---|
| *hiss* | 0/5 |
| *meow* | 3/5 |
| *purr* | 2/5 |

$P(B_t \mid M_t = bad)$

| | |
|---|---|
| *hiss* | 3/5 |
| *meow* | 1/5 |
| *purr* | 1/5 |

Emissions

- What is P(M2 = good)?
- What is P(B2 = meow)?
- What is P(M2 = good|B2 = meow)?
- What does P($B_T$ = meow) approach for very large T?

$P(M_1)$

| | |
|---|---|
| *good* | 1 |
| *bad* | 0 |

$P(M_t \mid M_{t-1} = good)$

| | |
|---|---|
| *good* | 3/4 |
| *bad* | 1/4 |

$P(M_t \mid M_{t-1} = bad)$

| | |
|---|---|
| *good* | 1/4 |
| *bad* | 3/4 |

Transitions

$P(B_t \mid M_t = good)$

| | |
|---|---|
| *hiss* | 0/5 |
| *meow* | 3/5 |
| *purr* | 2/5 |

$P(B_t \mid M_t = bad)$

| | |
|---|---|
| *hiss* | 3/5 |
| *meow* | 1/5 |
| *purr* | 1/5 |

Emissions

- What is P(M2 = good)?
    - P(M2 = good) = P(M2 = good|M1 = good) = ¾

$P(M_1)$

| | |
|---|---|
| *good* | 1 |
| *bad* | 0 |

$P(M_t \mid M_{t-1} = good)$

| | |
|---|---|
| *good* | 3/4 |
| *bad* | 1/4 |

$P(M_t \mid M_{t-1} = bad)$

| | |
|---|---|
| *good* | 1/4 |
| *bad* | 3/4 |

Transitions

$P(B_t \mid M_t = good)$

| | |
|---|---|
| *hiss* | 0/5 |
| *meow* | 3/5 |
| *purr* | 2/5 |

$P(B_t \mid M_t = bad)$

| | |
|---|---|
| *hiss* | 3/5 |
| *meow* | 1/5 |
| *purr* | 1/5 |

Emissions

- What is P(B2 = meow)?
  - P(B2 = meow) = P(B2 = meow, M2 = good) + P(B2 = meow, M2 = bad)
  - P(B2 = meow) = P(B2 = meow | M2 = good)P(M2 = good) + P(B2 = meow | M2 = bad)P(M2 = bad)
  - P(B2 = meow) = (3/5 )(3/4 ) + (1/5 )(1/4 ) = ½

| $P(M_1)$ | |
|---|---|
| *good* | 1 |
| *bad* | 0 |

| $P(M_t \mid M_{t-1} = good)$ | |
|---|---|
| *good* | 3/4 |
| *bad* | 1/4 |

| $P(M_t \mid M_{t-1} = bad)$ | |
|---|---|
| *good* | 1/4 |
| *bad* | 3/4 |

Transitions

| $P(B_t \mid M_t = good)$ | |
|---|---|
| *hiss* | 0/5 |
| *meow* | 3/5 |
| *purr* | 2/5 |

| $P(B_t \mid M_t = bad)$ | |
|---|---|
| *hiss* | 3/5 |
| *meow* | 1/5 |
| *purr* | 1/5 |

Emissions

- What is P(M2 = good|B2 = meow) ?
  - P(M2 = good|B2 = meow) = P(B2 = meow|M2 = good)P(M2 = good)/ P(B2 = meow) = (3/5 )(3/4 )/( 1/2 ) = 9/10

$P(M_1)$

| | |
|---|---|
| *good* | 1 |
| *bad* | 0 |

$P(M_t \mid M_{t-1} = good)$

| | |
|---|---|
| *good* | 3/4 |
| *bad* | 1/4 |

$P(M_t \mid M_{t-1} = bad)$

| | |
|---|---|
| *good* | 1/4 |
| *bad* | 3/4 |

Transitions

$P(B_t \mid M_t = good)$

| | |
|---|---|
| *hiss* | 0/5 |
| *meow* | 3/5 |
| *purr* | 2/5 |

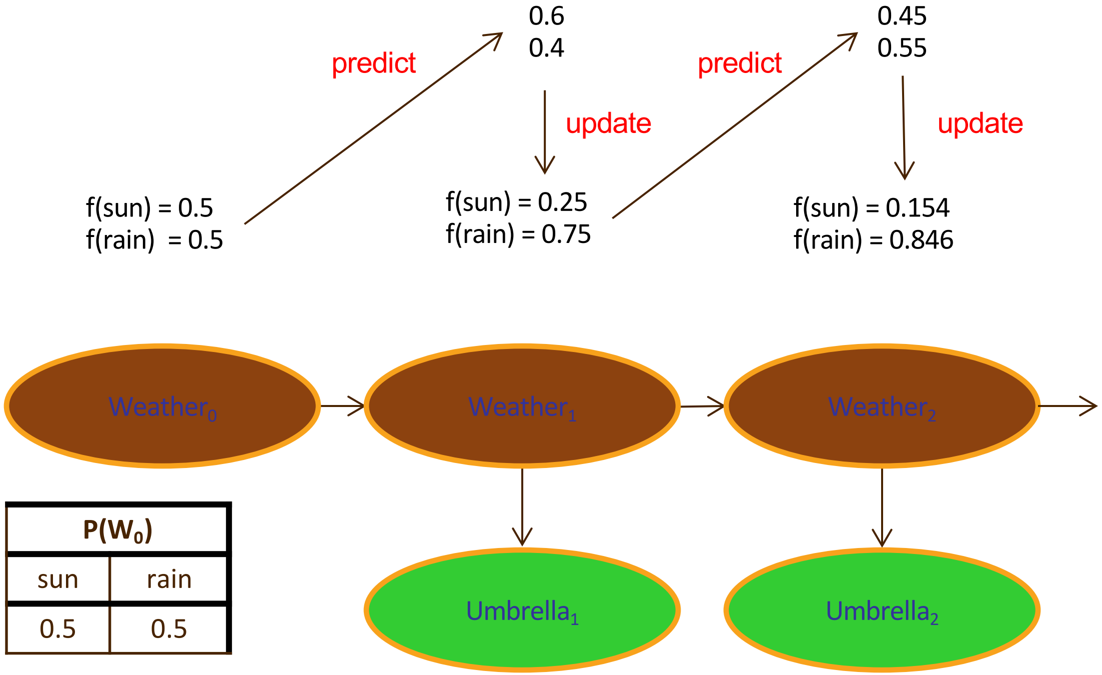
$P(B_t \mid M_t = bad)$

| | |
|---|---|
| *hiss* | 3/5 |
| *meow* | 1/5 |
| *purr* | 1/5 |

Emissions

- What does $P(B_T = \text{meow})$ approach for very large T?
  - $P(B_T = \text{meow}) = P(B_T = \text{meow} | M_T = \text{good})P(M_T = \text{good}) + P(B_T = \text{meow} | M_T = \text{bad})P(M_T = \text{bad})$
  - $P(B_T = \text{meow}) = (3/5)(1/2) + (1/5)(1/2) = 2/5$

$\alpha\, P(e_{t+1}|X_{t+1})\, P(X_{t+1}|\, e_{1:t})$




| P($W_0$) | |
|---|---|
| sun | rain |
| 0.5 | 0.5 |

| $W_{t-1}$ | P($W_t$\|$W_{t-1}$) | |
|---|---|---|
| | sun | rain |
| sun | 0.9 | 0.1 |
| rain | 0.3 | 0.7 |

| $W_t$ | P($U_t$\|$W_t$) | |
|---|---|---|
| | true | false |
| sun | 0.2 | 0.8 |
| rain | 0.9 | 0.1 |