CS 482/628 AI: Intelligent Search

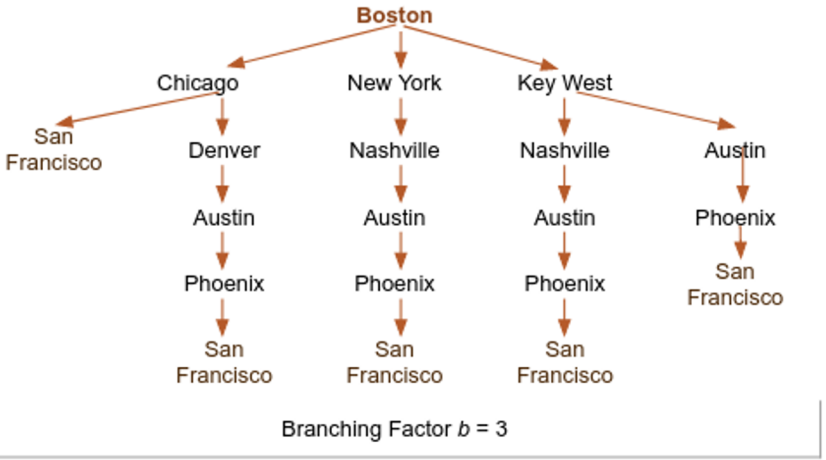
Fall 2021

previously on...

How to Search:

Generating Sequences and Data Structures

Depth
0
1
2
3
4
5



Measuring Performance

Completeness: is the strategy guaranteed to find a solution when one exists?

Time Complexity: how long does it take to find a solution?

Space Complexity: how much memory does it require to perform the search?

Optimality: Does the strategy find the best-quality solution when more than one solution exists?

Types of Uninformed Search

- Breadth-First Search: expands the shallowest nodes first
  - Complete, optimal for unit cost, exponential space complexity
- Depth-First Search: expands the deepest unexpanded node first
  - Neither complete nor optimal, but linear space complexity
- Depth Limited Search: is similar to DFS except the depth is limited
- Iterative Deepening Search: calls DFS with increasing depth limits
  - Complete, optimal for unit cost, linear space complexity
- Bidirectional Search: expands two frontiers, one around the initial state and one around the goal; stops when the two frontiers meet

More methods

Outline

- Best-first search
- A* search
- Heuristics
- Hill-climbing
- Simulated annealing

Informed (Heuristic) Search

- Uses domain-specific hints about the location of goals
- Can find solutions more efficiently than an uninformed strategy
- The hints come in the form of a heuristic function h(n)
- h(n) = estimated cost of the cheapest path from the state at node n to a goal state
    - Route-finding problem: we can estimate the distance from the current state to a goal as the straight-line distance on the map between the two points.
- A heuristic function is an inexact estimate of the evaluation function.

Review: General search

```
function GENERAL-SEARCH( problem, QUEUING-FN) returns a solution, or failure
    nodes ← MAKE-QUEUE(MAKE-NODE(INITIAL-STATE[problem]))
    loop do
        if nodes is empty then return failure
        node ← REMOVE-FRONT(nodes)
        if GOAL-TEST[problem] applied to STATE(node) succeeds then return node
        nodes ← QUEUING-FN(nodes, EXPAND(node, OPERATORS[problem]))
    end
```

A strategy is defined by picking the *order of node expansion*

Best-first search

Idea: use an *evaluation function* for each node
– estimate of "desirability"

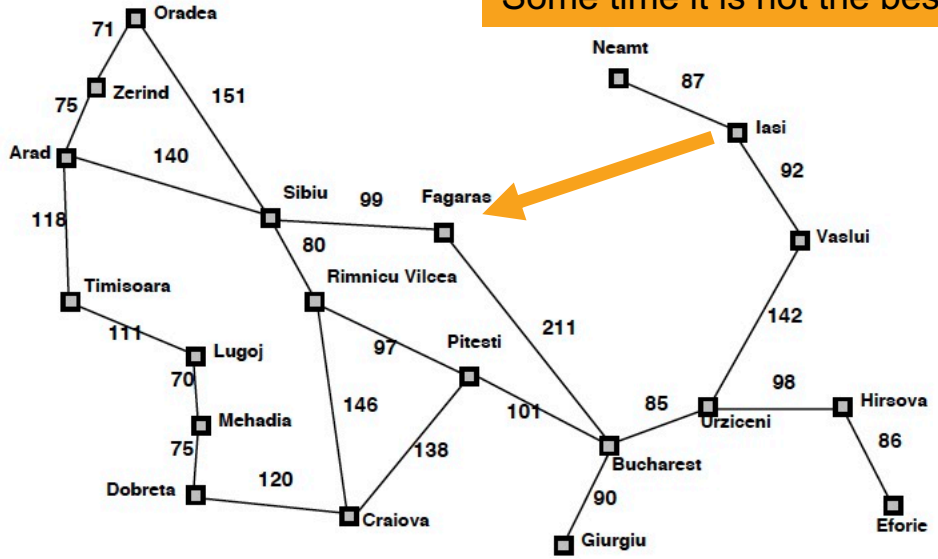
⇒ Expand most desirable unexpanded node

Implementation:
QUEUEINGFN = insert successors in decreasing order of desirability

Special cases:
greedy search
A* search

Romania with step costs in km

Some time it is not the best



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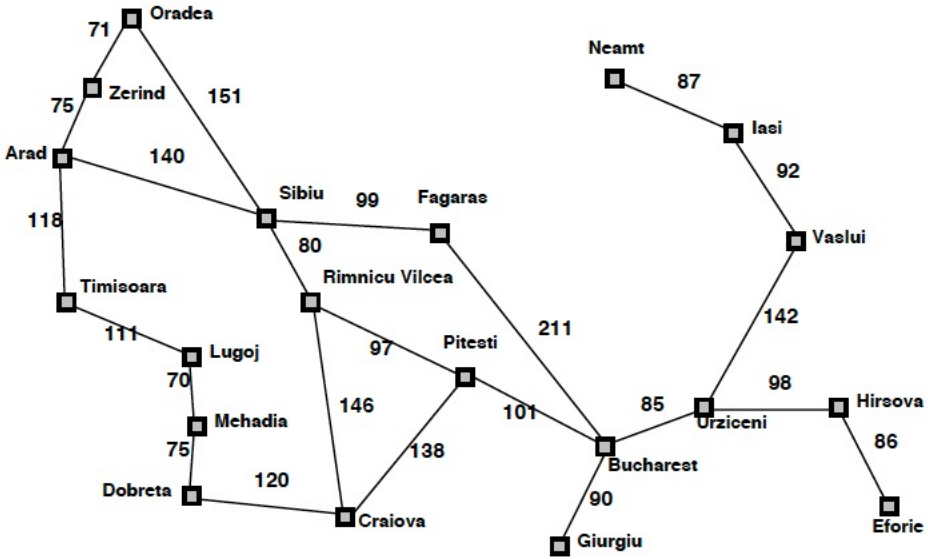
Greedy search

Evaluation function $h(n)$ (heuristic)
$=$ estimate of cost from n to $goal$

E.g., $h_{\mathrm{SLD}}(n) =$ straight-line distance from n to Bucharest

Greedy search expands the node that *appears* to be closest to goal



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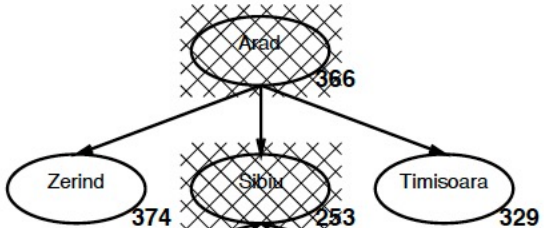

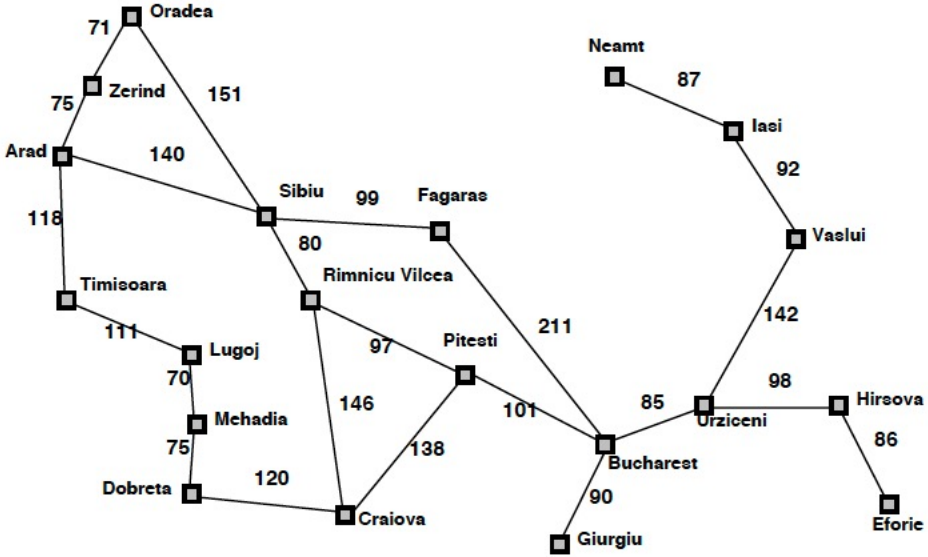

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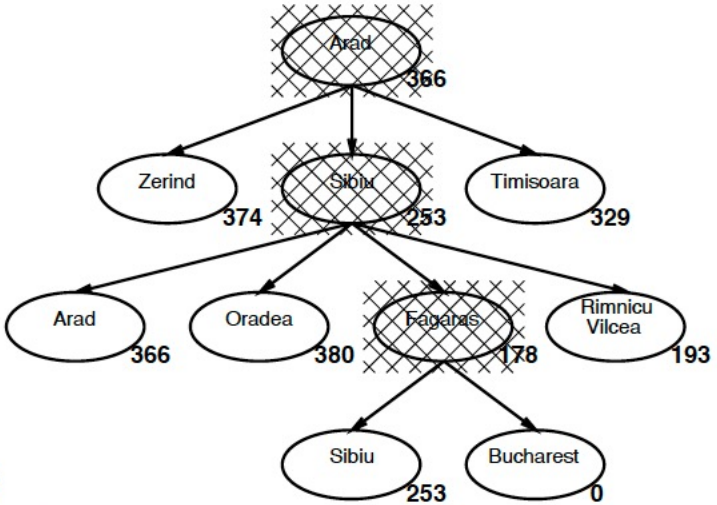

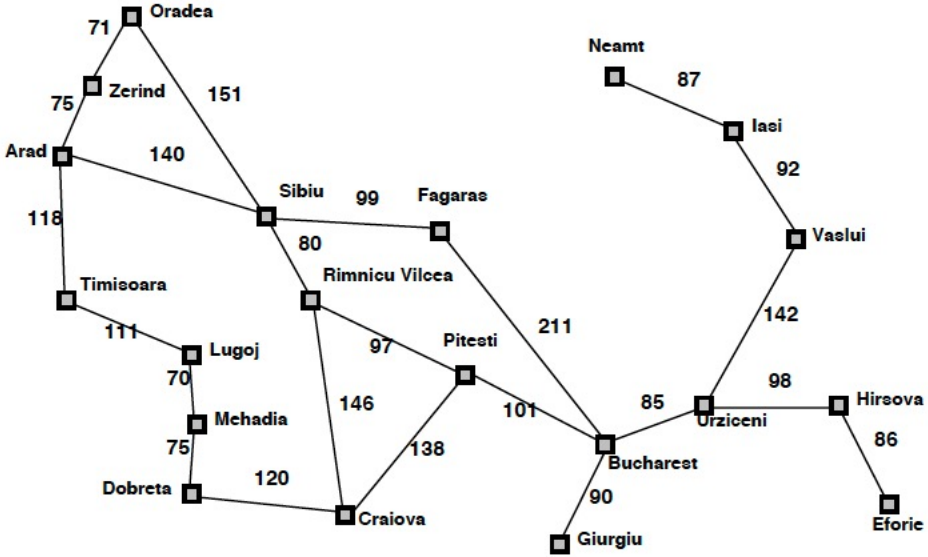

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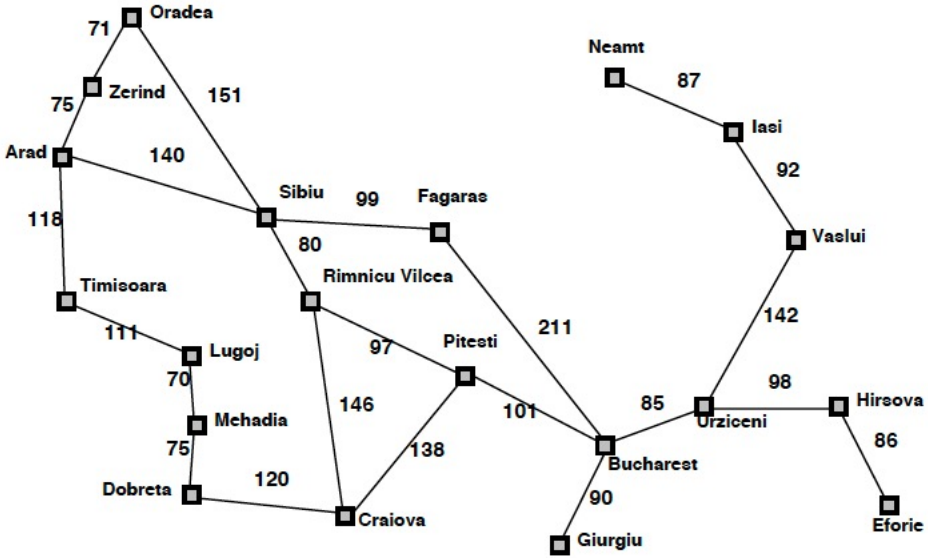
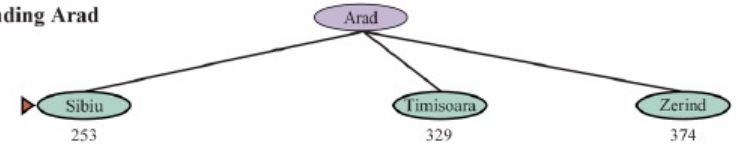


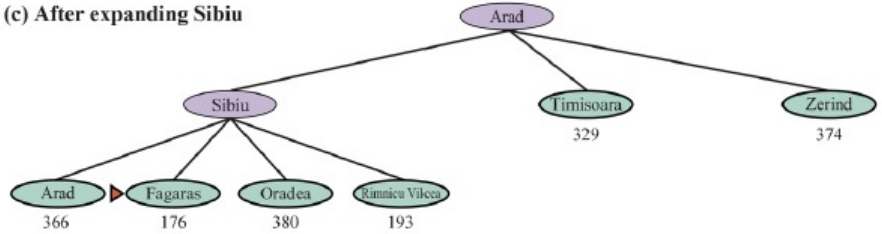
Figure 3.17

(a) The initial state

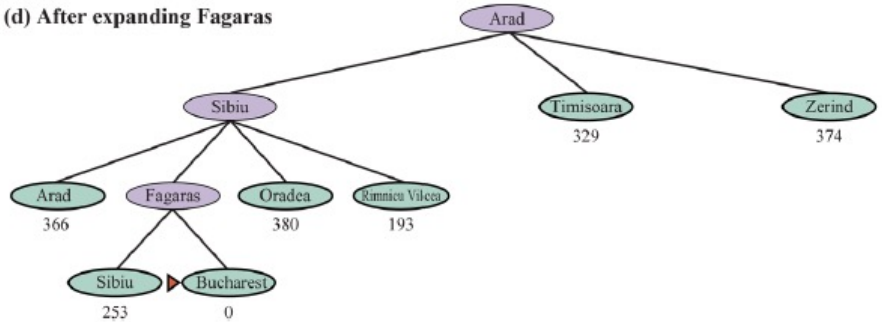
(b) After expanding Arad



(c) After expanding Sibiu



(d) After expanding Fagaras



Stages in a greedy best-first tree-like search for Bucharest with the straight-line distance heuristic h_{SLD}. Nodes are labeled with their h-values.

Properties of greedy search

Complete??

Time??

Space??

Optimal??

Properties of greedy search

Complete?? No–can get stuck in loops, e.g.,
Iasi $\rightarrow$ Neamt $\rightarrow$ Iasi $\rightarrow$ Neamt $\rightarrow$
Complete in finite space with repeated-state checking

Time?? $O(b^m)$, but a good heuristic can give dramatic improvement

Space?? $O(b^m)$—keeps all nodes in memory

Optimal?? No

A* search

Idea: avoid expanding paths that are already expensive

Evaluation function $f(n) = g(n) + h(n)$

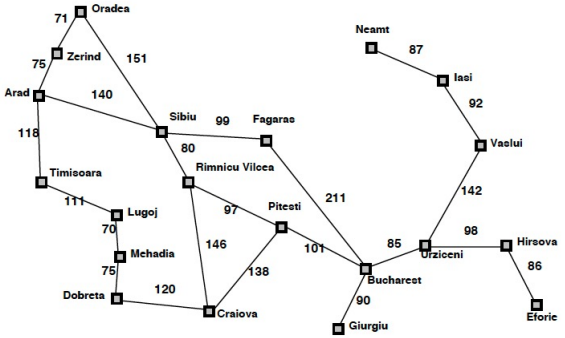
$g(n)$ = cost so far to reach n
$h(n)$ = estimated cost to goal from n
$f(n)$ = estimated total cost of path through n to goal

A* search uses an $admissible$ heuristic
i.e., $h(n) \leq h^*(n)$ where $h^*(n)$ is the $true$ cost from n.

E.g., $h_{\mathrm{SLD}}(n)$ never overestimates the actual road distance

Theorem: A* search is optimal

A* search example



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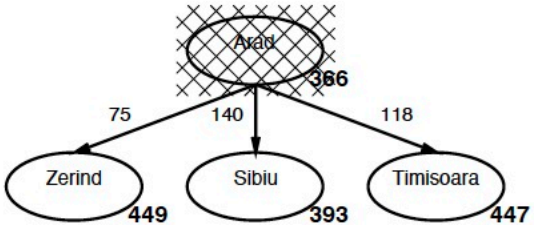

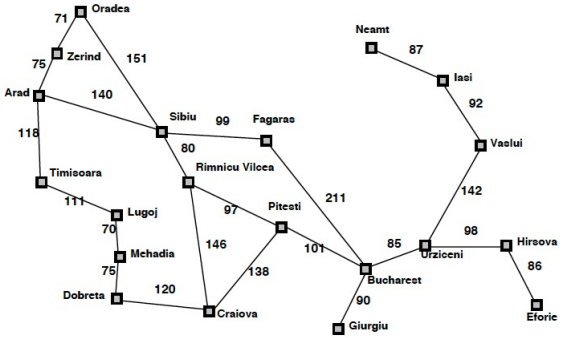

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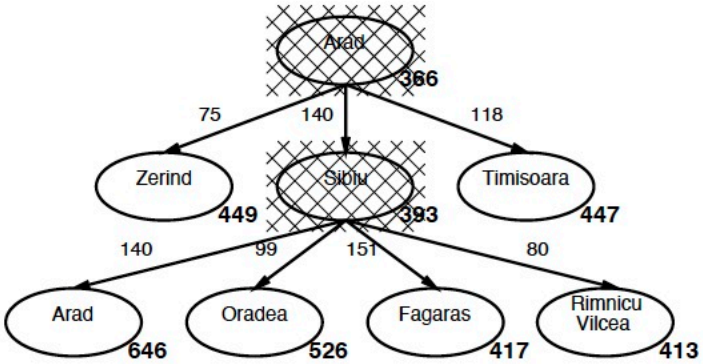

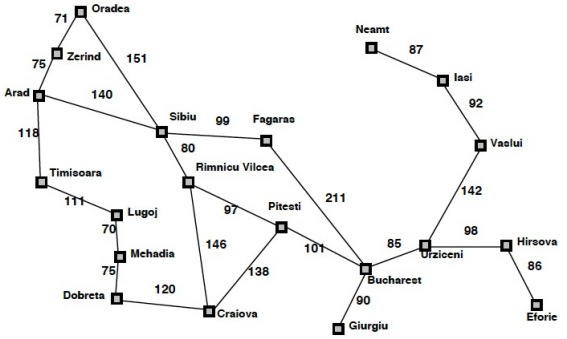

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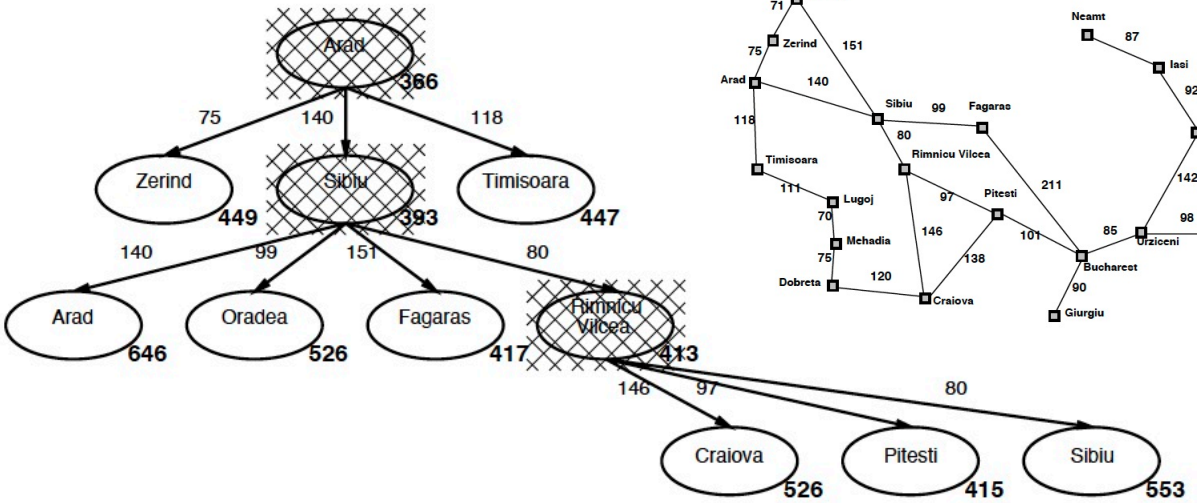

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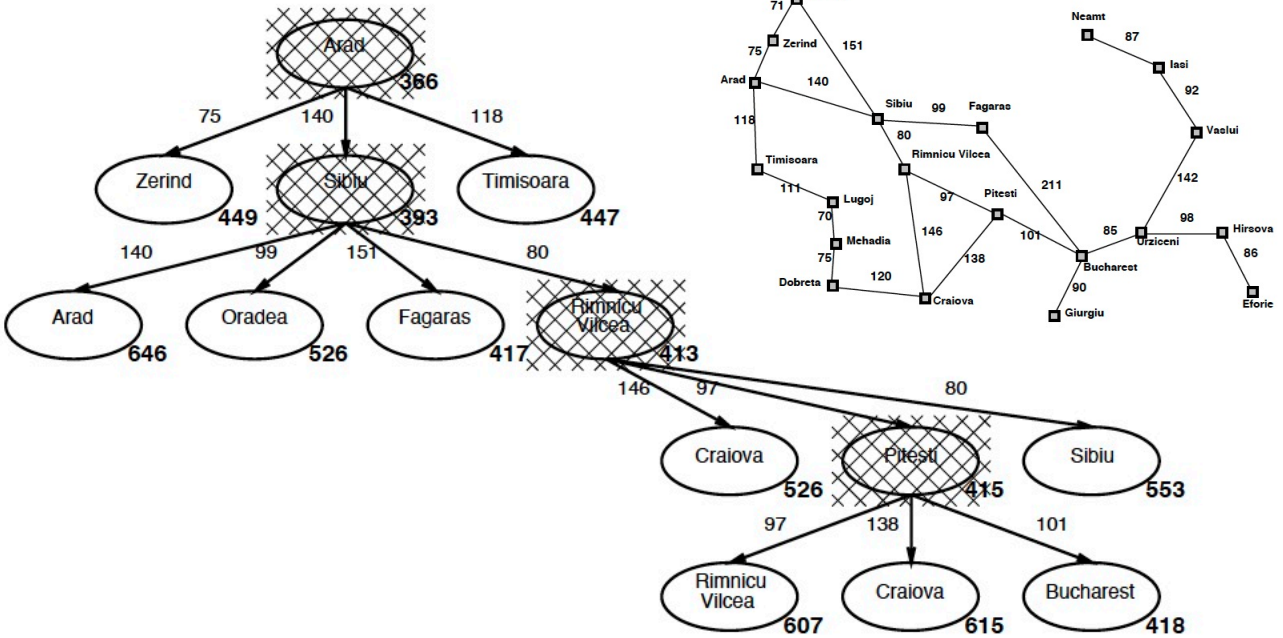

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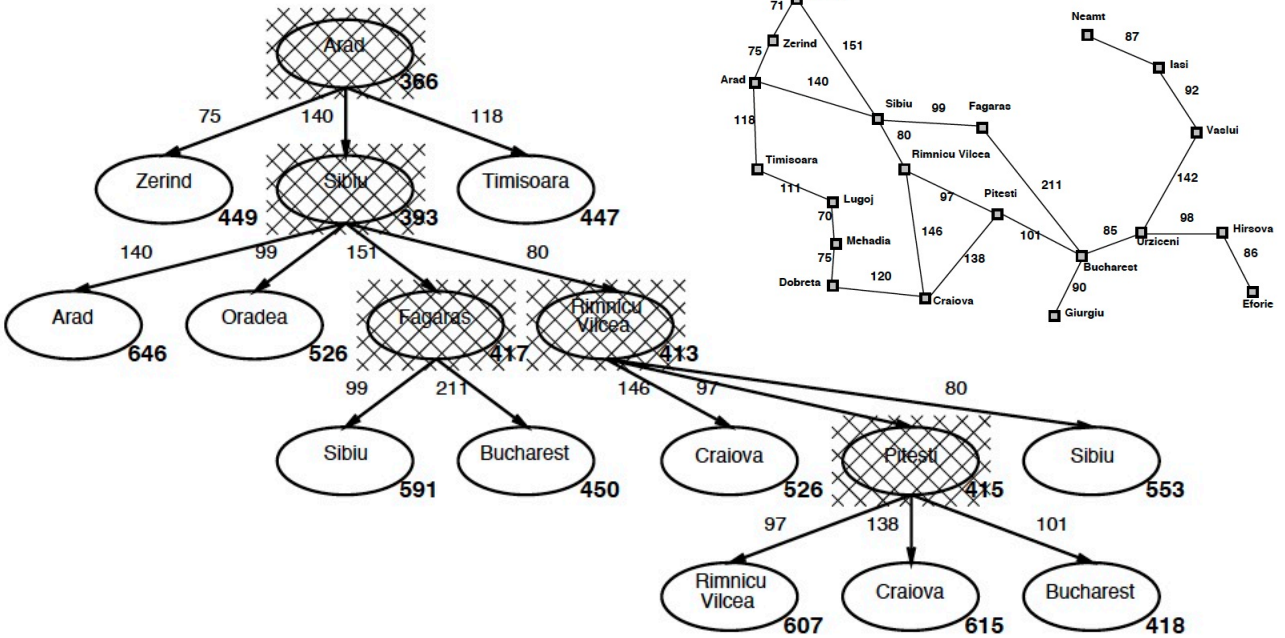

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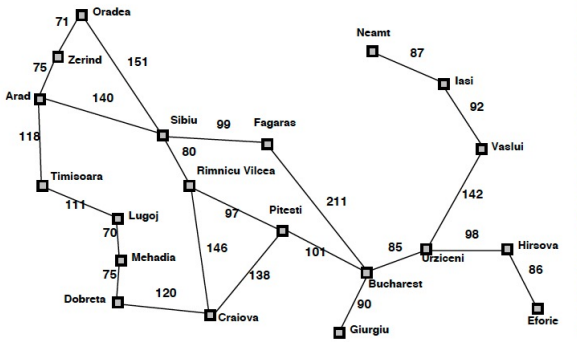

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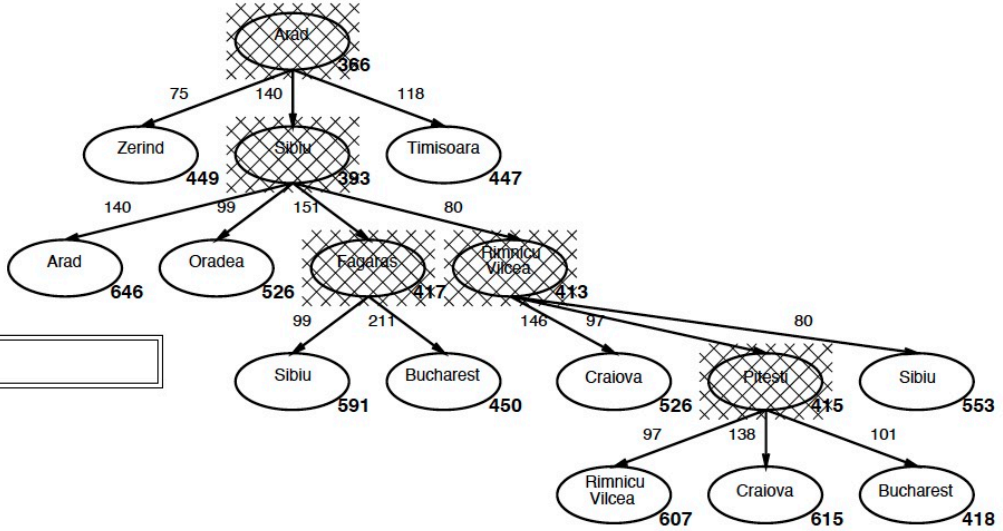
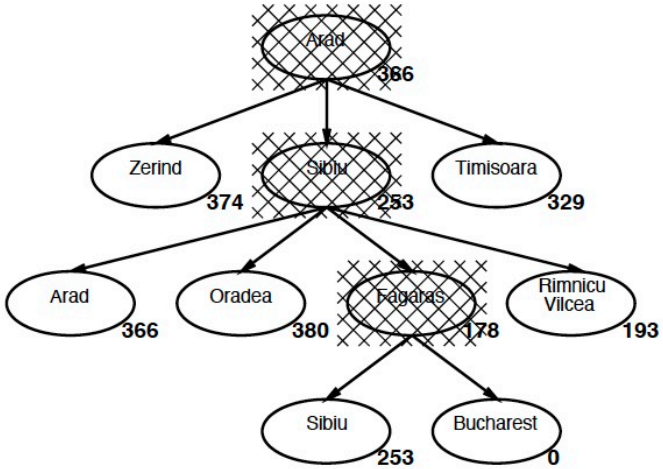


Greedy search example



Optimality of A* (proof)

- Assume that G_2 has been generate and is in the queue
- Let n is an unexpanded node on a shortest path to an optimal goal G_1
- $f(G_2) = g(G_2)$ because $h(G_2) = 0$
- $f(G_2) = g(G_2) > g(G_1)$ since G_2 is suboptimal
- $g(G_1) = g(n) + h^*(n) \geq g(n) + h(n)$ since h is admissible
- Therefore, $g(G_1) \geq f(n)$
- $f(G_2) > g(G_1) \geq f(n)$ since h is admissible
- Since $f(G_2) > f(n)$, A* will never select G_2 for expansion

Evaluation function $f(n) = g(n) + h(n)$

$g(n)$ = cost so far to reach n
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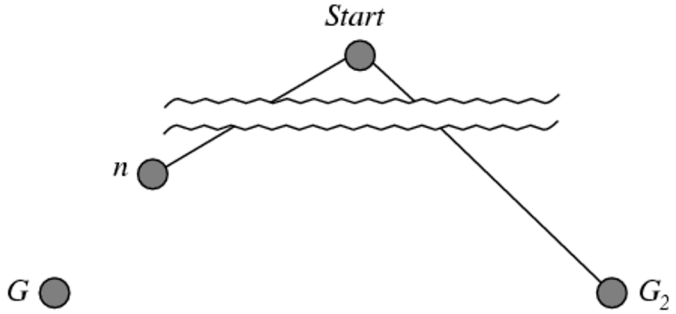
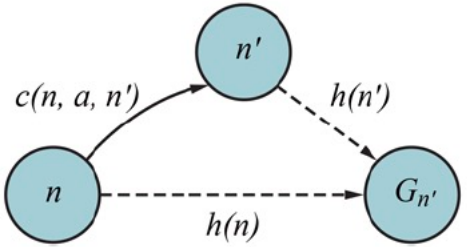
A* search uses an *admissible* heuristic
i.e., $h(n) \leq h^*(n)$ where $h^*(n)$ is the *true* cost from n.

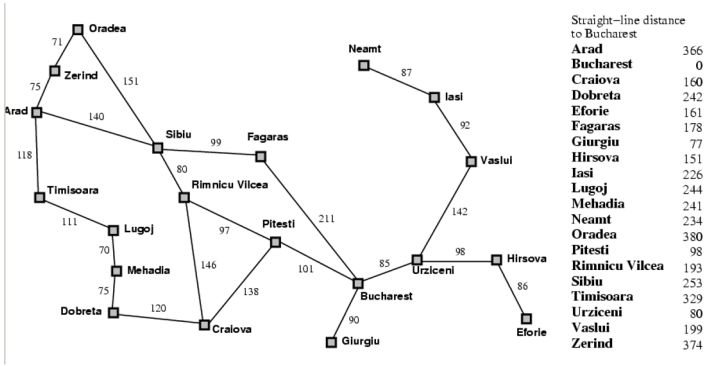


Consistency



- A slightly stronger property is consistency
- h(n) is consistent if $h(n) \leq c(n,a,n') + h(n')$
- It is called “triangle inequality”
- Every consistent heuristic is admissible (but not vice versa)
- Therefore, with a consistent heuristic, A* is cost-optimal
- With a consistent heuristic, we never re-add a state
- Note that the straight-line distance above is consistent!
- Many implementations avoid inconsistent heuristics, but Felner et al. (2011) argues that the worst effects rarely happen in practice

How does A* Search Work?



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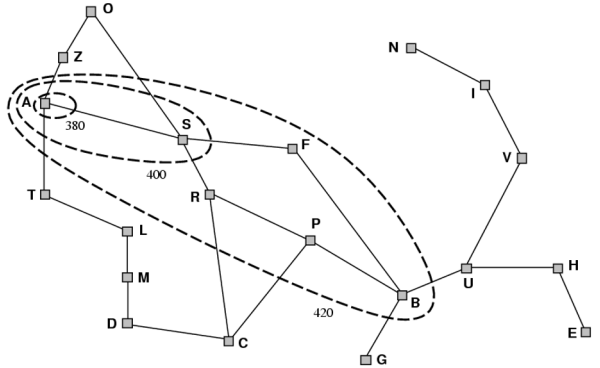


- A useful way to visualize is to draw contours in the state space
- Inside the contour labeled 400, all nodes have $f(n) = g(n) + h(n) \leq 400$
- Because A* expands the frontier node of the lowest f-cost, A* search gradually adds “f-contours” of nodes.
- Contour I has all nodes with $f = f_i$ where $f_i < f_{i+1}$

Completeness of A*

- A* expands nodes in order of increasing *f*
- When would a solution not be found?
  - Node with an infinite branching factor
  - A path with a finite path cost but an infinite number of nodes
- A* is complete when
  - There is a finite branching factor
  - Every operator costs at least some positive ε

Complexity of A*

- Computation time is limited by the quality of the heuristic function (but is still exponential)
  - Issue #1 : Choosing the right heuristic function can have a large impact

- More serious problem is that all generated nodes need to be kept in memory
  - Issue #2 : Can we limit the memory requirements?

Properties of A*

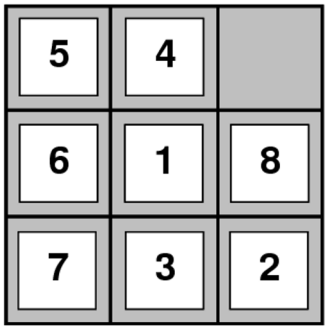
Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time?? Exponential in [relative error in h $\times$ length of soln.]

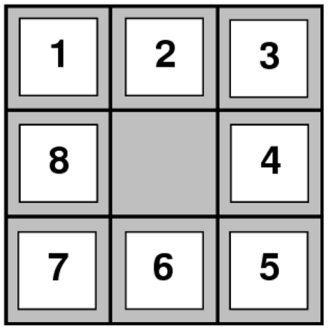
Space?? Keeps all nodes in memory

Optimal?? Yes—cannot expand f_{i+1} until f_i is finished

Issue #1: Choosing a Heuristic Function



Start State



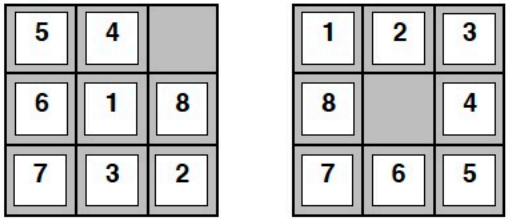
Goal State

- Must be admissible (never overestimate)
- Heuristics for the 8-Puzzle
  - *h1* = number of tiles in the wrong position
  - *h2* = sum of the distances of the tiles from their goal positions (city block distance)

Admissible heuristics

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles
$h_2(n)$ = total Manhattan distance
(i.e., no. of squares from desired location of each tile)



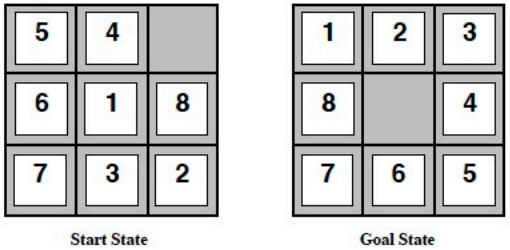
Start State Goal State

$h_1(S)$ =??
$h_2(S)$ =??

Admissible heuristics

E.g., for the 8-puzzle:

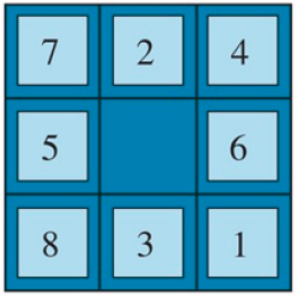
$h_1(n)$ = number of misplaced tiles
$h_2(n)$ = total Manhattan distance
(i.e., no. of squares from desired location of each tile)



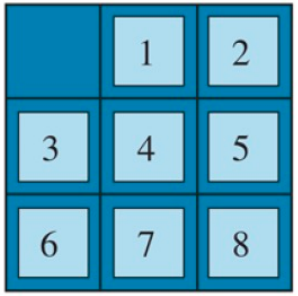
Start State Goal State

$h_1(S)$ =?? 7
$h_2(S)$ =?? 2+3+3+2+4+2+0+2 = 18

Dominance



Start State



Goal State

- How many reachable states in an 8-puzzle?
- 9!/2 = 181,400 states → easily stored in memory
- But what for 15-puzzle?
- Over 10 trillion → need a good admissible heuristic function
  - *h1* = number of tiles in the wrong position
  - *h2* = sum of the distances of the tiles from their goal positions (city block distance)
- $h_2(n) > h_1(n)$ for all n (both admissible) then h_2 dominates h_1 and is better for search

Effect of Heuristic Accuracy on Performance in the 8-puzzle

	Search Cost			Effective Branching Factor		
d	IDS	A*(h_1)	A*(h_2)	IDS	A*(h_1)	A*(h_2)
2	10	6	6	2.45	1.79	1.79
4	112	13	12	2.87	1.48	1.45
6	680	20	18	2.73	1.34	1.30
8	6384	39	25	2.80	1.33	1.24
10	47127	93	39	2.79	1.38	1.22
12	364404	227	73	2.78	1.42	1.24
14	3473941	539	113	2.83	1.44	1.23
16	–	1301	211	–	1.45	1.25
18	–	3056	363	–	1.46	1.26
20	–	7276	676	–	1.47	1.27
22	–	18094	1219	–	1.48	1.28
24	–	39135	1641	–	1.48	1.26

- Compare iterative-deepening with A* using h1 (# misplaced tiles) and h2 (city block distance)
- Effective branching factor b*
  - Number of expanded nodes = 1 + b* + (b*) 2 + … + (b*) depth
  - b* remains relatively constant across many measurements
- Always better to use a heuristic with higher values, so long as it does not overestimate

Issue #2: Limiting Memory Utilization

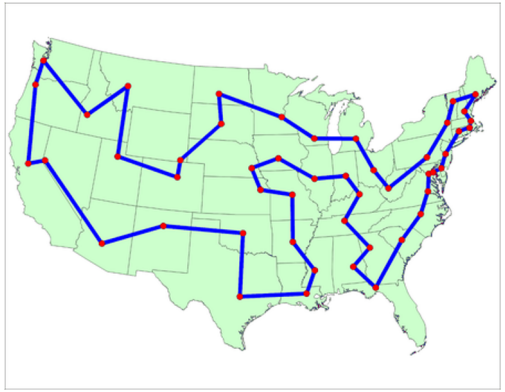
- If we can maintain a bound on the memory, we might be willing to wait for a solution

- Two techniques for Memory Bounded Search:
  - Iterative deepening A* (IDA*)
  - Recursive Best-First-Search (RBFS)

Iterative Deepening A* Search (IDA*)

- Each iteration is a depth-first search with a limit based on f rather than on depth
- Complete and optimal (with same caveats as A*)
- Requires space proportional to the longest path that it explores
- Can have competitive time complexity, since the overhead of maintaining the nodes in memory is greatly reduced

Problems with IDA*



- In the TSP, different heuristic function value for each state
- Each contour contains only one additional node
- If A* expands N nodes, the IDA* will expand

  $1+2+3+4+\ldots+N = O(N^2)$ nodes
- If N is too large for memory, N^2 is too long to wait
- Runs into problems because it recalculates every node

Recursive Best-First Search (RBFS)

- total path cost (f) =
  actual path so far (g) +
  heuristic estimate of future path to goal (h)

- uses f_limit variable to keep track of best *alternative* path available from any ancestor of the current node

Properties of Recursive Best-First Search (RBFS)

- RBFS will
  - be complete given sufficient memory to store the shallowest solution path
  - be optimal if the heuristic function is admissible (and you have enough memory to store the solution)

- Both RBFS and IDA* use not enough memory
  - Require at most linear space with the depth of the tree

Next Week

- Can you search without building a tree?
- What happens when you don't get to make the choice at each level of the search space?
- Game Playing
  - Minimax
  - Alpha-beta pruning