

History of Integers

We look at integers from a historical point of view.

You might have noticed that there is one common class of numbers that we haven't much discussed, yet: negative numbers. One of the choices that the author of your textbook has made is to discuss the various kinds of operations (addition, subtraction, multiplication, division) as a whole, including many of the types of numbers in the discussion. This helps to highlight the fact that multiplication, for instance, is the same thing no matter what kinds of numbers you use. Other textbooks take a different approach: looking at whole numbers and their operations, then fractions and their operations, and so on. We happen to think that our textbook has made the better choice (and you can agree or disagree), but we have deviated a little in this instance.

Now that you have noticed these facts, you might be wondering: why?

There are several reasons, but the ones we would like to discuss here are the pattern of history and the difficulty of finding a good representation.

These two reasons are very much related. If we look over how numbers as a concept were developed throughout history, we definitely find evidence of whole numbers by 3000BC in Ancient Egypt. Beginning around the same period of time in Ancient Babylon, people were starting to use a place-value system which included what we would call decimals, except the Ancient Babylonians used base 60 instead of base 10 as we do. The Ancient Egyptians also had notation for unit fractions, and elaborate methods of combining these unit fractions to express any other fraction they would like. If you're interested in this subject, you can find a nice timeline of very early mathematics at MacTutor .

So, we have seen evidence of whole numbers, fractions, and even decimals from the beginning of mathematics. Negative numbers, however, are a different story. We see the first evidence of negative numbers appearing sometime between 200BC and 200AD in Chinese mathematics, and around the 7th century AD in Indian mathematics. Western European mathematicians didn't start really using negatives in their calculations until the 17th century AD, and didn't allow them as answers to problems or give them equal status amongst the positive numbers, decimals, and fractions until the 19th century!

To emphasize this point a little bit, here are some things that happened in history *before* most Western mathematicians were working with negative numbers.

- The Hindu-Arabic numerals are developed in India and the Middle East around the 6th and 7th centuries. They are brought to Western Europe in the early 1200s.
- The bubonic plague hit Europe and Asia in the mid 1300s.
- Gutenberg invents the printing press around 1440.
- Christopher Columbus sails to the New World around 1492.
- Michelangelo begins painting the Sistine Chapel around 1508.
- Martin Luther publishes his 95 theses, and the Reformation begins around 1517.
- Galileo finds the Earth revolves around the sun around 1613.
- Descartes and Fermat invent coordinate geometry around 1637.
- Isaac Newton and Wilhelm Leibniz invent calculus around 1670.

Learning outcomes:

Author(s):

See MacTutor at http://www-history.mcs.st-and.ac.uk/Chronology/3000BC_500BC.html

See Hindu-Arabic numerals at https://en.wikipedia.org/wiki/Arabic_numerals

(Of course, any such list leaves out an incredible number of important events! Our list here is focused on Western history, as these events are usually more familiar to people who have studied history in the United States. Also, in this time period, mathematics in Western Europe tended to be advancing more rapidly than in other parts of the world. You can find other world history timelines online if you are interested, such as World History AD Timeline .)

Much of mathematicians' hesitation to treat negative numbers as actual numbers came from the idea that mathematics was a subject that should make sense in the real world, and model real-world phenomena. People didn't have a very good representation for negative numbers that made sense all the time. For example, let's consider a few problems.

Question 1 For each of the problems below, write an expression that would solve the problem, i.e. $3 + 4$.

- (a) Jaci has 8 apples, and Joseph has 12 apples. How many apples do the two children have together?

$$\boxed{8 + 12}$$

- (b) Jaci has $\frac{1}{2}$ of an apple, and Joseph has $\frac{5}{7}$ of an apple. How many apples do the two children have together?

$$\boxed{\frac{1}{2} + \frac{5}{7}}$$

- (c) Jaci has -8 apples, and Joseph has -12 apples. How many apples do the two children have together?

$$\boxed{-8 + -12}$$

Now, you might say that negative numbers weren't too bad in that situation: after all, you could likely answer the question because you've recognized the structure of addition is the same in all three cases. But what does the problem actually *mean*? What does it mean to have -8 apples? Maybe you can reconcile this, as Indian mathematicians did in their earliest interpretations, to mean that Jaci owes someone 8 apples, and Joseph owes someone 12 apples, and perhaps this interpretation of negative numbers as debts makes sense in some cases.

Let's try another example.

Question 2 For each of the problems below, write an expression that would solve the problem, i.e. $3 + 4$.

- (a) Sasha has 8 bags, and each bag contains 12 sheets of stickers. How many sheets of stickers does Sasha have in total? $\boxed{(8)(12)}$

- (b) Sasha has $\frac{1}{2}$ of a box of stickers, and the box originally contained $\frac{5}{7}$ of a sheet of stickers. How many sheets of stickers does Sasha have?

$$\boxed{\frac{1}{2} \frac{5}{7}}$$

- (c) Sasha has -8 bags, and each bag contains -12 sheets of stickers. How many sheets of stickers does Sasha have in total? $\boxed{(-8)(-12)}$

Again, you could likely answer the third question, but the meaning here is likely even less clear. The question itself doesn't even seem to make sense – we have just asked what looks like the right question based on the patterns in the previous two parts. If we understand the objects in the groups to be debts, what does it mean to have a negative group? Why should the overall answer to this question be positive?

See World History AD Timeline at <https://www.finchier.org/History/WorldAD.shtml>

We will look at several kinds of models for these problems in order to try to make some sense of negative numbers, and hopefully be able to write story problems that make sense. We expect this to be pretty challenging: after all, mathematicians struggled to make sense of these ideas for centuries!