Declarative Array Programming with SAC — Single Assignment C

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Informatics Institute
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Amsterdam Computer Science Summer School
on
Parallel Programming
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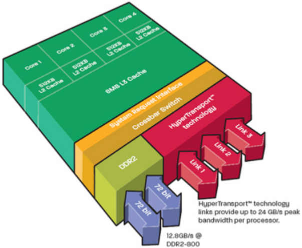
Single Assignment C: Outline of Lecture

The Free Lunch is Over: Many-Core to the Rescue

The many-core hardware zoo:



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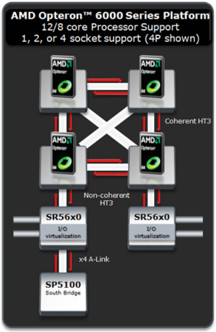

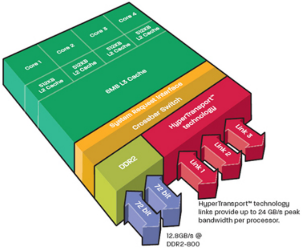

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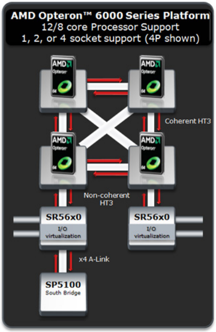

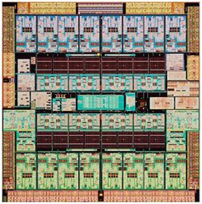

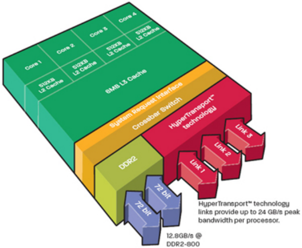

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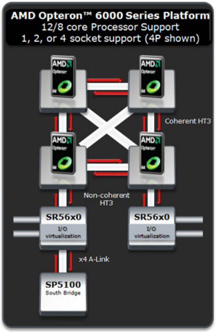

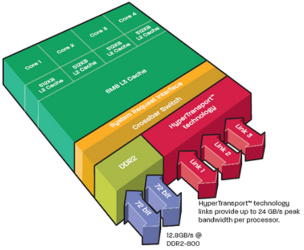

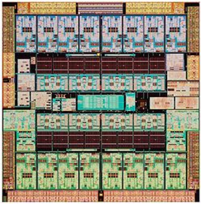

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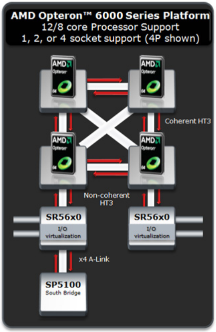

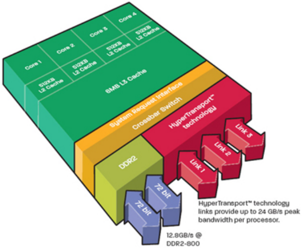

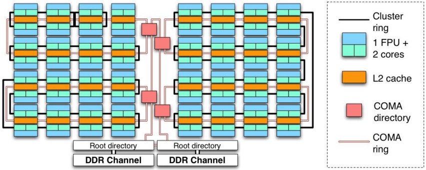

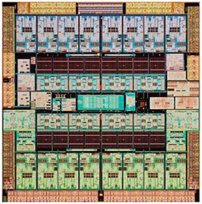

Design Rationale of SAC

Hardware in the many-core era is a zoo:

- Vastly different numbers of cores
- Vastly different core architectures: power, genericity
- Vastly different memory architectures

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Programming diverse hardware is uneconomic:

- Diverse low-level programming models
- Each requires expert knowledge
- Heterogeneous combinations of the above ?

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Genericity through abstraction:

- Program what to compute, not exactly how
- Leave execution organisation to compiler and runtime system
- Put expert knowledge into compiler, not into applications

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Genericity through abstraction:

- Program what to compute, not exactly how
- Leave execution organisation to compiler and runtime system
- Put expert knowledge into compiler, not into applications
- Let programs remain architecture-agnostic
- Compile one source to diverse target hardware
- Pursue data-parallel approach to implicitly promote concurrency

What Does Data Parallel Really Mean ?

Factorial imperative:

```
int fac( int n)
{
  f = 1;
  while (n > 1) {
    f = f * n;
    n = n - 1;
  }
  return f;
}
```

Factorial functional:

```
fac n = if n <= 1
        then 1
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```

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n: 10

f: 1

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n: 10 → 9

f: 1 → 10

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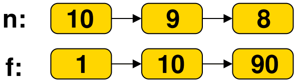

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n: 10 → 9 → 8 → ••• → 1

f: 1 → 10 → 90 → ••• → 3628800

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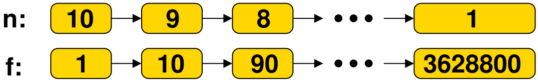

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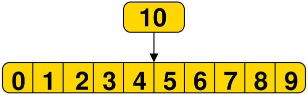

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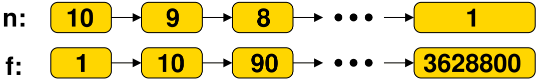

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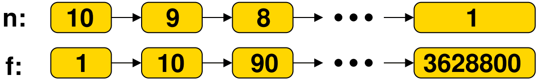

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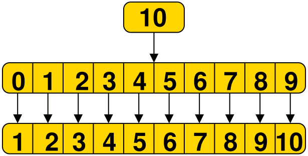

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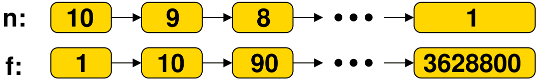

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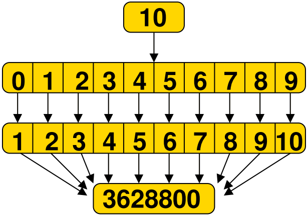

The Essence of Data Parallel Programming

prod(1+iota(n))

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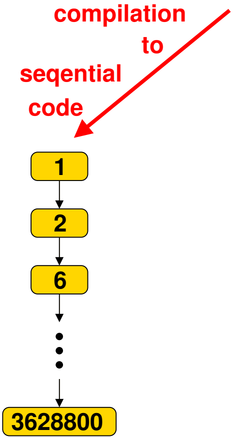

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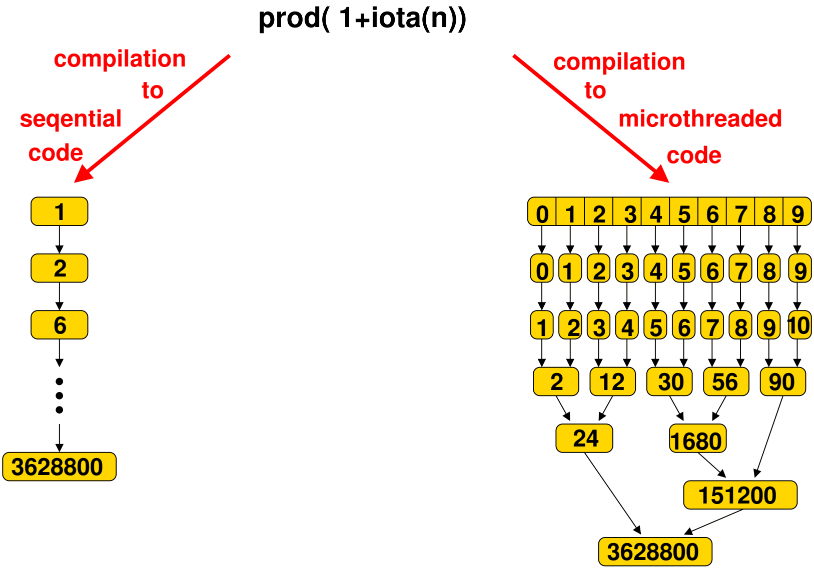

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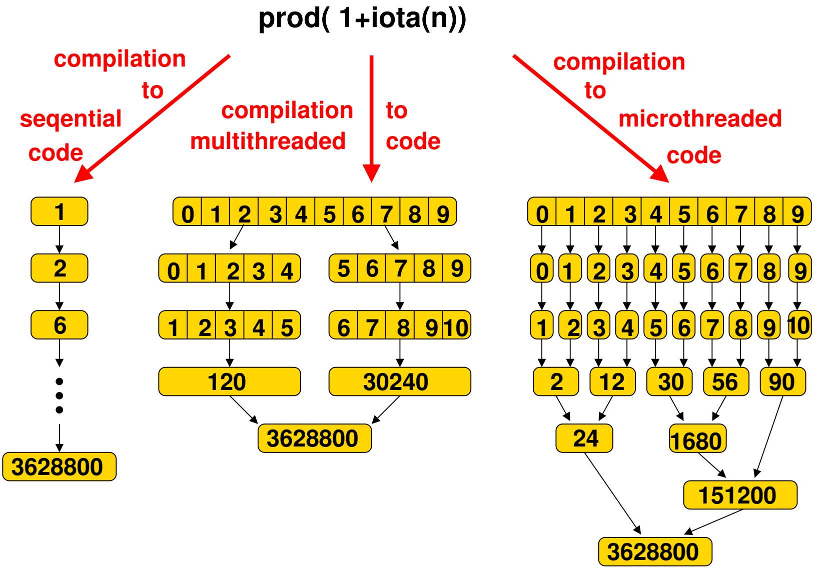

Single Assignment C: Outline of Lecture

SAC — Design Space

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High-level functional, data-parallel programming with vectors, matrices, arrays

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↓

SAC

↑

Suitability to achieve high performance
in sequential and parallel execution

SAC — Design Space

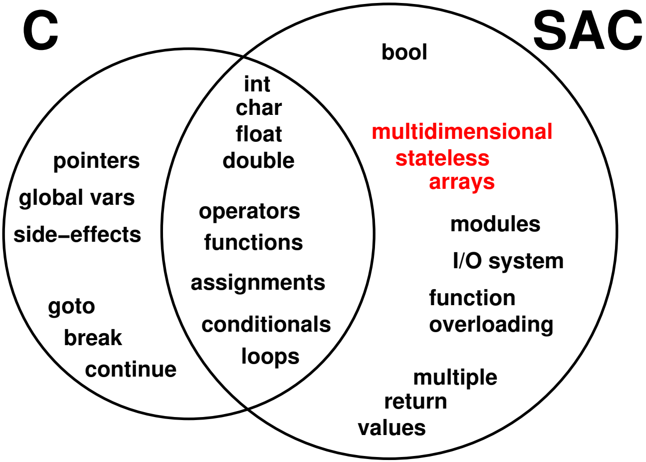
High-level functional, data-parallel programming with vectors, matrices, arrays

SAC

Easy to adopt for programmers with an imperative background

Suitability to achieve high performance in sequential and parallel execution

SAC at a glance



Introductory Example: gcd in SAC

```
int
gcd( int high, int low)
{
  if (high < low) {
    mem  = low;
    low  = high;
    high = mem;
  }
  while (low != 0) {
    quotient, remainder =
      diffmod( high, low);
    high = low;
    low  = remainder;
  }
  return( high);
}
```

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    mem  = low;
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  while (low != 0) {
    quotient, remainder =
      diffmod( high, low);
    high = low;
    low  = remainder;
  }
  return( high);
}
```

```
int, int
diffmod( int x, int y)
{
  quot   = x / y;
  remain = x % y;
  return( quot, remain);
}

int
main()
{
  return( gcd( 22, 27));
}
```

What is Functional Programming ?

Execution Model:

Imperative programming:
Sequence of instructions
that step-wise manipulate the program state

Functional programming:
Context-free substitution of expressions
until fixed point is reached

Functional Semantics of SAC

SAC: → **Functional pseudo code:**

```
{
   ...
   a = 5;
   b = 7;
   a = a + b;
   return( a);
}
```

```
...
let a = 5
in let b = 7
in let a = a + b
in a
```

Functional Semantics of SAC

SAC: → **Functional pseudo code:**

```
int fac( int n)
{
  if (n>1) {
    r = fac( n-1);
    f = n * r;
  }
  else {
    f = 1;
  }
  return( f);
}
```

```
fun fac n =
  if n>1
  then let r = fac (n-1)
       in let f = n * r
       in f
  else let val f = 1
       in f
```

Functional Semantics of SAC

SAC: → **Functional pseudo code:**

```
int fac( int n)
{
  f = 1;

  while (n>1) {
    f = f * n;
    n = n - 1;
  }

  return( f);
}
```

```
fun fac n =
  let rec fac_while f n =
    if n>1
    then let f = f * n
         in let n = n - 1
         in fac_while f n
    else f
  in
    fac_while 1 n
```

The Role of Functions

Mathematics:
context-free mapping of argument values to result values

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Imperative programming:

subroutine with side-effects on global state

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subroutine with side-effects on global state

Functional programming in SAC:
context-free mapping of argument values to result values

The Role of Variables

Mathematics:
name/placeholder of a value

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Imperative programming:
name of a memory location

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Functional programming in SAC:
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The Role of Arrays

Mathematics:
functions from indices to values

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$\Updownarrow$

Imperative programming:

contiguous fragments of addressable memory

The Role of Arrays

Mathematics:
functions from indices to values

Imperative programming:
contiguous fragments of addressable memory

Functional programming in SAC:
stateless multidimensional indexable collections of values

Think Data Parallel !

Think Arrays !

or: everything is an array

- Vectors are arrays.
- Matrices are arrays.
- Tensors are arrays.
- .... are arrays.

Think Data Parallel !

Think Arrays !

or: everything is an array

- Vectors are arrays.
- Matrices are arrays.
- Tensors are arrays.
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- Operations map arrays to arrays.
- Even scalars are arrays.
- Even iteration spaces are arrays.
- And arrays are values.

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- Vectors are arrays.
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- Operations map arrays to arrays.
- Even scalars are arrays.
- Even iteration spaces are arrays.
- And arrays are values.

Arrays are structured collections of data, not projections into memory.

Single Assignment C: Outline of Lecture

Multidimensional Arrays in SAC

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$$

dim: 2
shape: [3,3]
data: [1,2,3,4,5,6,7,8,9]

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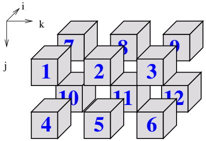

dim: 3
shape: [2,2,3]
data: [1,2,3,4,5,6,7,8,9,10,11,12]

Multidimensional Arrays in SAC

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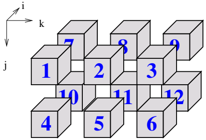

dim: 3
shape: [2,2,3]
data: [1,2,3,4,5,6,7,8,9,10,11,12]

[1, 2, 3, 4, 5, 6]

dim: 1
shape: [6]
data: [1,2,3,4,5,6]

Multidimensional Arrays in SAC

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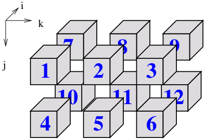

dim: 3
shape: [2,2,3]
data: [1,2,3,4,5,6,7,8,9,10,11,12]

[1, 2, 3, 4, 5, 6]

dim: 1
shape: [6]
data: [1,2,3,4,5,6]

42

dim: 0
shape: [ ]
data: [42]

Built-in Array Operations

- Defining a vector:

```
vec = [1,2,3,4,5,6];
```

Built-in Array Operations

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- Defining a higher-dimensional array:

```
mat = [vec,vec];
mat = reshape( [3,2], vec);
```

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- Defining a vector:
  `vec = [1,2,3,4,5,6];`
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- Querying for the shape of an array:
  `shp = shape( mat);` ➡ [3,2]

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```
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```
→ [3,2]

- Querying for the rank of an array:

```
rank = dim( mat);
```
→ 2

Built-in Array Operations

- Defining a vector:
  `vec = [1,2,3,4,5,6];`
- Defining a higher-dimensional array:
  `mat = [vec,vec];`
  `mat = reshape( [3,2], vec);`
- Querying for the shape of an array:
  `shp = shape( mat);` ➡ [3,2]
- Querying for the rank of an array:
  `rank = dim( mat);` ➡ 2
- Selecting elements:
  `x = sel( [4], vec);` ➡ 5
  `y = sel( [2,1], mat);` ➡ 6
  `x = vec[[4]];` ➡ 5
  `y = mat[[2,1]];` ➡ 6

With-Loops: Versatile Array Comprehensions

```
A  =  with {
        ([1,1] <= iv < [4,4]) :  e(iv);
      }:  genarray( [5,4],  def  );
```

- Multidimensional array comprehensions
- Mapping from index domain into value domain

[0,0]	[0,1]	[0,2]	[0,3]
[1,0]	[1,1]	[1,2]	[1,3]
[2,0]	[2,1]	[2,2]	[2,3]
[3,0]	[3,1]	[3,2]	[3,3]
[4,0]	[4,1]	[4,2]	[4,3]

→

def	def	def	def
def	e([1,1])	e([1,2])	e([1,3])
def	e([2,1])	e([2,2])	e([2,3])
def	e([3,1])	e([3,2])	e([3,3])
def	def	def	def

index domain **value domain**

With-Loops: Modarray Variant

```
A  =  with {
        ([1,1] <= iv < [3,4]) :  e(iv);
      }:  modarray( B );
```

$$\Downarrow$$

$$A = \begin{pmatrix} B[[0,0]] & B[[0,1]] & B[[0,2]] & B[[0,3]] & B[[0,4]] \\ B[[1,0]] & e([1,1]) & e([1,2]) & e([1,3]) & B[[1,4]] \\ B[[2,0]] & e([2,1]) & e([2,2]) & e([2,3]) & B[[2,4]] \\ B[[3,0]] & B[[3,1]] & B[[3,2]] & B[[3,3]] & B[[3,4]] \end{pmatrix}$$

With-Loops: Fold Variant

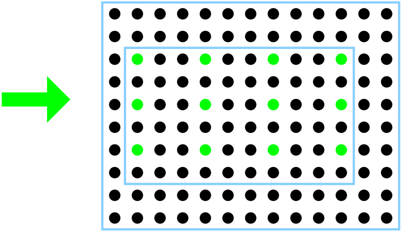
```
A  =  with {
        ([1,1] <= iv < [3,4]) :  e(iv);
      }:  fold( ⊕, neutr );
```

$$A = neutr \oplus e([1,1]) \oplus e([1,2]) \oplus e([1,3]) \oplus e([2,1]) \oplus e([2,2]) \oplus e([2,3])$$

($\oplus$ denotes associative, commutative binary function.)

Grid Generators

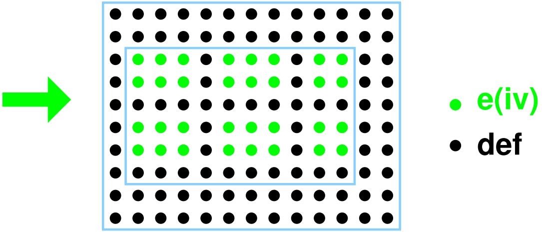
```
A  =  with {
        ([2,1] <= iv < [8,11] step [2,3]):
             e(iv);
      }:  genarray( [10,13],  def  );
```



- e(iv)
- def

Grid Width Generators

```
A  =  with {
        ([2,1] <= iv < [8,11] step [3,4] width [2,3]):
            e(iv);
      }:  genarray( [10,13],  def  );
```



Multi-Generator/Multi-Operator With-Loops

Example

```
A, B = with {
          ([1,1] <=      iv    < [3,4]) : A[iv], B[iv] ;
          ([3,1] <=    [i,j]   < [6,4]) : i, j ;
          ([0,5] <= iv=[i,j]   < [3,7]) : B[iv], i+j ;
       }: (genarray( [10,10], 0), modarray( C) ) ;
```

Properties:

- Multiple generators, sequence matters
- Multiple operators, can be different
- Index can be vector, sequence of scalars or both

Single Assignment C: Outline of Lecture

Principle of Abstraction

Element-wise subtraction of arrays:

```
int[20,20] (-) (int[20,20] A, int[20,20] B)
{
  res = with {
          ([0,0] <= iv < [20,20]) : A[iv] - B[iv];
        }: genarray( [20,20], 0);
  return( res);
}
```

Principle of Abstraction

```
int[20,20] (-) (int[20,20] A, int[20,20] B)
{
  res = with {
          ([0,0] <= iv < [20,20]) : A[iv] - B[iv];
        }: genarray( [20,20], 0);
  return( res);
}
```

Shape-generic code

```
int[.,.] (-) (int[.,.] A, int[.,.] B)
{
  shp = min( shape(A), shape(B));
  res = with {
          ([0,0] <= iv < shp) : A[iv] - B[iv];
        }: genarray( shp, 0);
  return( res);
}
```

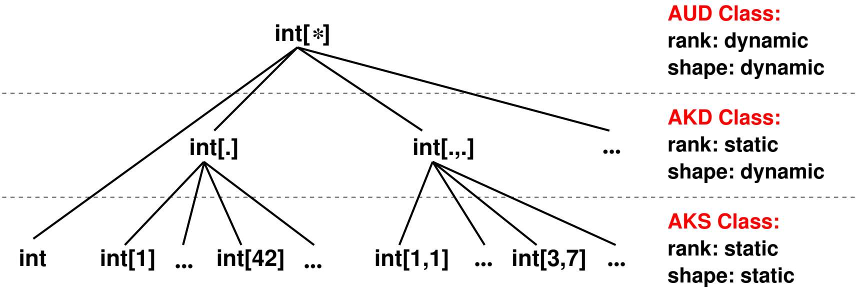
Principle of Abstraction

```
int[.,.] (-) (int[.,.] A, int[.,.] B)
{
  shp = min( shape(A), shape(B));
  res = with {
          ([0,0] <= iv < shp) : A[iv] - B[iv];
        }: genarray( shp, 0);
  return( res);
}
```

Rank-generic code

```
int[*] (-) (int[*] A, int[*] B)
{
  shp = min( shape(A), shape(B));
  res = with {
          (0*shp <= iv < shp) : A[iv] - B[iv];
        }: genarray( shp, 0);
  return( res);
}
```

Shapely Array Type Hierarchy With Subtyping



AUD	:	Array of Unknown Dimension
AKD	:	Array of Known Dimension
AKS	:	Array of Known Shape

Function Overloading

Example:

```
int[20,20] (-) (int[20,20] A, int[20,20] B) {...}

int[.,.]   (-) (int[.,.] A,   int[.,.] B) {...}

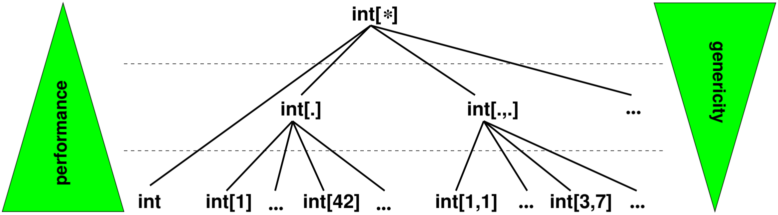
int[*]     (-) (int[*] A,     int[*] B) {...}
```

Features:

- Multiple function definitions with same name, but
  - different numbers of arguments
  - different base types
  - different shapely types
- No restriction on function semantics
- Argument subtyping must be monotonous
- Dynamic function dispatch

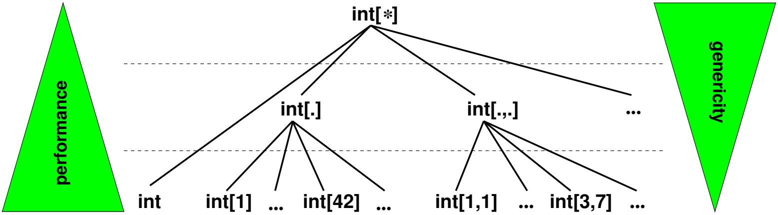
Genericity vs Performance

Dilemma:



Genericity vs Performance

Dilemma:



Solution: Specialisation:

- Compiler infers argument shapes
- Compiler creates specialised instances of generic functions

Genericity vs Performance

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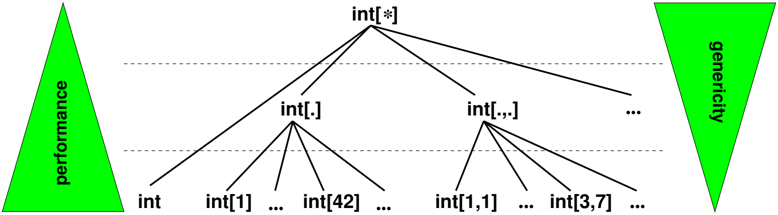


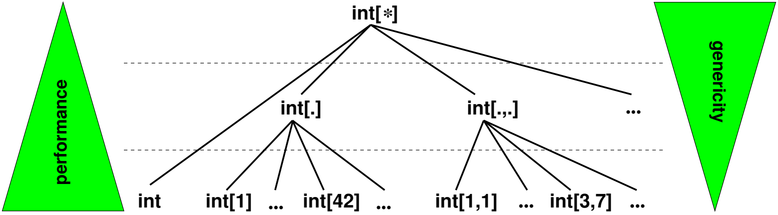
Solution: Specialisation:

- Compiler infers argument shapes
- Compiler creates specialised instances of generic functions

But what if the compiler cannot infer argument shapes ?

Specialisation Directives

Dilemma:



Specialisation directives:

```
int[*] (-) (int[*] A, int[*] B) {...}

specialize int[.,.]   (-) (int[.,.]   A, int[.,.]   B);
specialize int[20,20] (-) (int[20,20] A, int[20,20] B);
specialize int[50,50] (-) (int[50,50] A, int[50,50] B);
```

Principle of Composition

Characteristics:

- Step-wise composition of functions
- from previously defined functions
- or basic building blocks (with-loop defined)

Example: convergence test

```
bool
is_convergent (double[*] new, double[*] old, double eps)
{
  return( all( abs( new - old) < eps));
}
```

Principle of Composition

Example: convergence test

```
bool
is_convergent (double[*] new, double[*] old, double eps)
{
  return( all( abs( new - old) < eps));
}
```

Advantages:

- Rapid prototyping
- High confidence in correctness
- Good readability of code

Execution through Context-Free Substitution

Convergence Test:

```
is_convergent( [1,2,3,8], [3,2,1,4], 3 )
```

Execution through Context-Free Substitution

Convergence Test:

```
is_convergent( [1,2,3,8], [3,2,1,4], 3 )
```

```
all( abs( [1,2,3,8] - [3,2,1,4]) < 3 )
```

Execution through Context-Free Substitution

Convergence Test:

```
is_convergent( [1,2,3,8], [3,2,1,4], 3 )
```

$\Downarrow$

```
all( abs( [1,2,3,8] - [3,2,1,4]) < 3 )
```

$\Downarrow$

```
all( abs( [-2,0,2,4]) < 3 )
```

Execution through Context-Free Substitution

Convergence Test:

```
is_convergent( [1,2,3,8], [3,2,1,4], 3 )

all( abs( [1,2,3,8] - [3,2,1,4]) < 3 )

all( abs( [-2,0,2,4]) < 3 )

all( [2,0,2,4] < 3 )
```

Execution through Context-Free Substitution

Convergence Test:

```
is_convergent( [1,2,3,8], [3,2,1,4], 3 )
```

⇓

```
all( abs( [1,2,3,8] - [3,2,1,4]) < 3 )
```

⇓

```
all( abs( [-2,0,2,4]) < 3 )
```

⇓

```
all( [2,0,2,4] < 3 )
```

⇓

```
all( [true, true, true, false])
```

Execution through Context-Free Substitution

Convergence Test:

```
is_convergent( [1,2,3,8], [3,2,1,4], 3 )
```

$\Downarrow$

```
all( abs( [1,2,3,8] - [3,2,1,4]) < 3 )
```

$\Downarrow$

```
all( abs( [-2,0,2,4]) < 3 )
```

$\Downarrow$

```
all( [2,0,2,4] < 3 )
```

$\Downarrow$

```
all( [true, true, true, false])
```

$\Downarrow$

```
false
```

Shape-Generic Programming

2-dimensional convergence test:

$$\texttt{is\_convergent(}\ \begin{pmatrix} 1 & 2 \\ 3 & 8 \end{pmatrix},\ \begin{pmatrix} 3 & 2 \\ 1 & 7 \end{pmatrix},\ 3\ \texttt{)}$$

Shape-Generic Programming

2-dimensional convergence test:

$$\texttt{is\_convergent(}\ \begin{pmatrix} 1 & 2 \\ 3 & 8 \end{pmatrix},\ \begin{pmatrix} 3 & 2 \\ 1 & 7 \end{pmatrix},\ 3\ \texttt{)}$$

3-dimensional convergence test:

$$\texttt{is\_convergent(}\ \begin{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 8 \end{pmatrix} \\ \begin{pmatrix} 6 & 7 \\ 2 & 8 \end{pmatrix} \end{pmatrix},\ \begin{pmatrix} \begin{pmatrix} 2 & 1 \\ 0 & 8 \end{pmatrix} \\ \begin{pmatrix} 1 & 1 \\ 3 & 7 \end{pmatrix} \end{pmatrix},\ 3\ \texttt{)}$$

The Power of With-Loops

- NO large collection of built-in operations
  - Simplified compiler design

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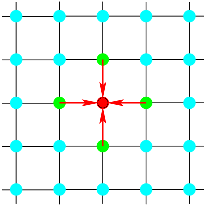
  - Improved maintainability
  - Improved extensibility
- Composition of building blocks
  - Rapid prototyping
  - High confidence in correctness
  - Good readability of code
- General intermediate representation for array operations
  - Basis for code optimization
  - Basis for implicit parallelization

Single Assignment C: Outline of Lecture

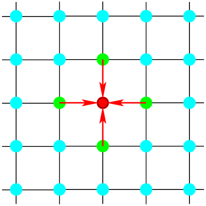
Case Study: Convolution



Algorithmic principle:

Compute weighted sums of neighbouring elements

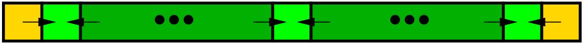
Case Study: Convolution



Algorithmic principle:

Compute weighted sums of neighbouring elements

Fixed boundary conditions (1-dimensional):



Case Study: Convolution

Algorithmic principle:

Compute weighted sums
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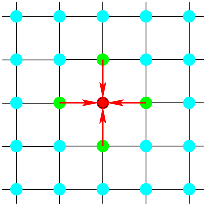
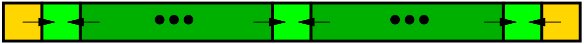
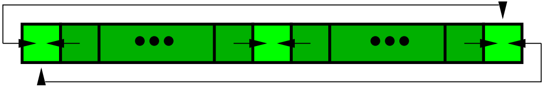


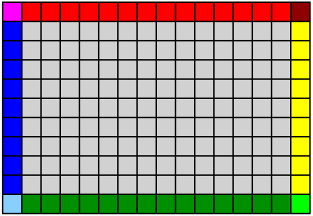
Fixed boundary conditions (1-dimensional):



Periodic boundary conditions (1-dimensional):



Case Study: Convolution



Problem:

- 9 different situations in 2-dimensional grids
- 27 different situations in 3-dimensional grids
- ...

Convolution Step in SaC

1-dimensional:

```
double[.] convolution_step (double[.] A)
{
  R = A + rotate( 1, A) + rotate( -1, A);
  return( R / 3.0);
}
```

Convolution Step in SaC

1-dimensional:

```
double[.] convolution_step (double[.] A)
{
  R = A + rotate( 1, A) + rotate( -1, A);
  return( R / 3.0);
}
```

N-dimensional:

```
double[*] convolution_step (double[*] A)
{
  R = A;
  for (i=0; i<dim(A); i++) {
    R = R + rotate( i, 1, A) + rotate( i, -1, A);
  }
  return( R / tod( 2 * dim(A) + 1));
}
```

Convolution in SaC

Fixed number of iterations:

```
double[*] convolution (double[*] A, int iter)
{
  for (i=0; i<iter; i++) {
    A = convolution_step( A);
  }

  return( A);
}
```

Convolution in SaC

Variable number of iterations with convergence check:

```
double[*] convolution (double[*] A, double eps)
{
  do {
    A_old = A;
    A = convolution_step( A_old);
  }
  while (!is_convergent( A, A_old, eps));

  return( A);
}
```

Convolution in SaC

Variable number of iterations with convergence test:

```
double[*] convolution (double[*] A, double eps)
{
  do {
    A_old = A;
    A = convolution_step( A_old);
  }
  while (!is_convergent( A, A_old, eps));

  return( A);
}
```

Convergence criterion:

```
bool
is_convergent (double[*] new, double[*] old, double eps)
{
  return( all( abs( new - old) < eps));
}
```

Convolution with Static Boundary Conditions

1-dimensional:

```
double[.] convolution_step (double[.] A)
{
  conv  = (A + shift( 1, A) + shift( -1, A)) / 3.0;

  inner = tile( shape( conv) - 2, [1], conv);
  res   = embed( inner, [1], A);

  return( res);
}
```

Library functions:

tile (shp, offset, array) takes a subarray of shape *shp* at *offset* from *array*.

embed (inner, offset, outer) creates an array like *outer* except for elements from index *offset* which are taken from *inner* for the whole shape of *inner*.

Convolution with Static Boundary Conditions

N-dimensional:

```
double[*] convolution_step (double[*] A)
{
  conv = A;
  for (i=0; i<dim(A); i++) {
    conv = conv + shift( i, 1, A) + shift( i, -1, A);
  }
  conv = conv / tod( 2 * dim(A) + 1));

  inner = tile(shape(conv) - 2, shape(conv) * 0 + 1, conv);
  res   = embed(inner, shape(conv) * 0 + 1, A);

  return( res);
}
```

Single Assignment C: Outline of Lecture

At last: Hello World

As simple as pie (for C hackers):

```
import StdIO: all;

int main()
{
  printf( "Hello World !!");
  return( 0);
}
```

Input and Output in General

Example:

```
import StdIO: all;
import ArrayIO: all;

int main()
{
  a = 42;
  b = [1,2,3,4,5];

  errcode, outfile = fopen( "filename", "w");

  fprintf( outfile, "a = %d\n", a);
  fprint( outfile, b);

  fclose( outfile);

  return(0);
}
```

Input and Output in General

Example:

```
import StdIO: all;
import ArrayIO: all;

int main()
{
  a = 42;
  b = [1,2,3,4,5];

  errcode, outfile = fopen( "filename", "w");

  fprintf( outfile, "a = %d\n", a);
  fprint( outfile, b);

  fclose( outfile);

  return(0);
}
```

But is this functional code ?

Input and Output in General

Example functionalised by compiler:

```
FileSystem, int main( FileSystem theFileSystem)
{
  a = 42;
  b = [1,2,3,4,5];

  theFileSystem, errcode, outfile
    = fopen( theFileSystem, "filename", "w");

  outfile = fprintf( outfile, "a = %d\n", a);
  outfile = fprint( outfile, b);

  theFileSystem = fclose( theFileSystem, outfile);

  return( theFileSystem, 0);
}
```

FileSystem and File are uniqueness types.

Input and Output in General

Example completed:

```
import StdIO: all;
import ArrayIO: all;
import SysErr: all;

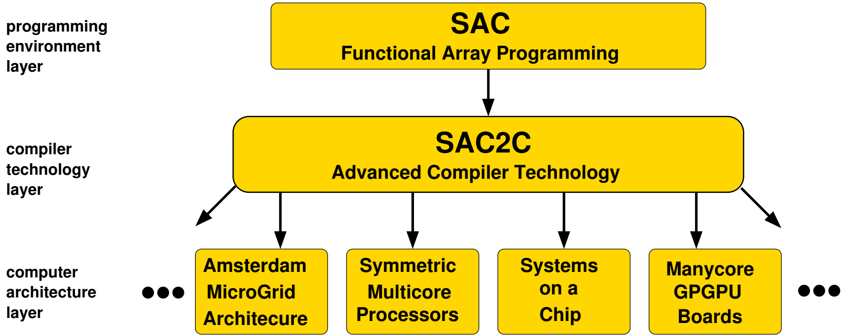
int main()
{
  a = 42;
  b = [1,2,3,4,5];
  errcode, outfile = fopen( "somefile", "w");
  if (fail(errcode)) {
    fprintf( stderr, "%s\n", strerror( errcode));
  }
  else {
    fprintf( outfile, "a = %d\n", a);
    fprint( outfile, b);

    fclose( outfile);
  }
  return(0);
}
```

Single Assignment C: Outline of Lecture

- Design Rationale of SAC
- Language Design of SAC
- SAC Arrays
- Abstraction and Composition
- Case Study: Convolution
- Modules and Input/Output
- Compilation Challenge

Compilation Challenge



Some Compilation Challenges

- **Challenge 1: Stateless Arrays**
  - How to avoid copying?
  - How to avoid boxing small arrays?
  - How to do memory management efficiently?

Some Compilation Challenges

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  - How to avoid multiple array traversals?

Some Compilation Challenges

- **Challenge 1: Stateless Arrays**
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- **Challenge 3: Shape-Invariant Specifications**
  - How to generate efficient loop nestings?
  - How to represent arrays with different static knowledge?

Some Compilation Challenges

- **Challenge 1: Stateless Arrays**
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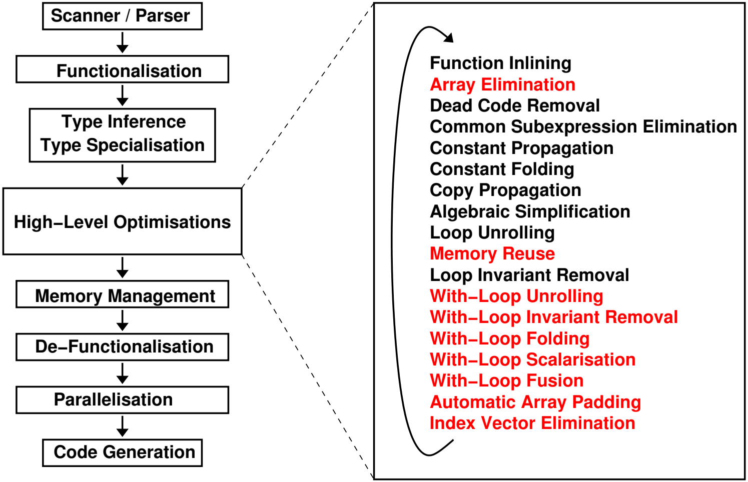
  - How to avoid multiple array traversals?
- **Challenge 3: Shape-Invariant Specifications**
  - How to generate efficient loop nestings?
  - How to represent arrays with different static knowledge?
- **Challenge 4: Organisation of Concurrent Execution**
  - How to schedule index spaces to threads ?
  - When to synchronise (and when not) ?

Challenge 5: Implementing a Fully-Fledged Compiler



Compiler Engineering

sac2c is a large-scale compilation technology project

- **SAC** compiler + runtime library:
  - 300,000 lines of code
  - about 1000 files
  - about 250 compiler passes
  - + standard prelude
  - + standard library
- More than 10 years of research and development
- Approaching one hundred man years of investment
- Complete compiler construction infrastructure

The SAC Project

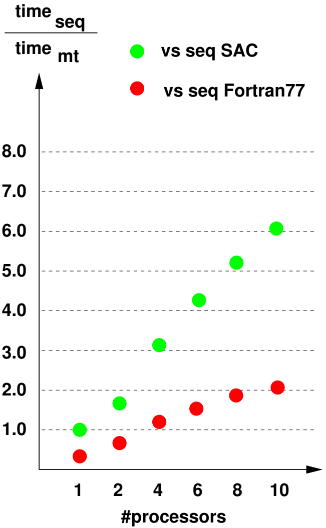
International partners:

- University of Kiel, Germany (1994–2005)
- University of Toronto, Canada (since 2000)
- University of Lübeck, Germany (2001–2008)
- University of Hertfordshire, England (2003–2012)
- University of Amsterdam, Netherlands (since 2008)
- Heriot-Watt University, Scotland (since 2011)

Always Looking for New Faces !!

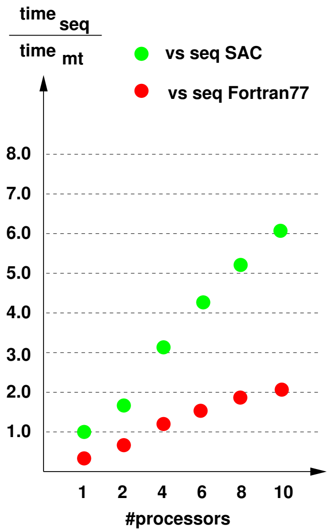
Single Assignment C: Outline of Lecture

Runtime Performance: Standard Multiprocessor

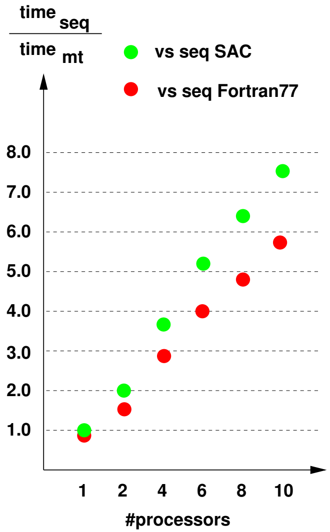


NAS benchmark FT

Runtime Performance: Standard Multiprocessor



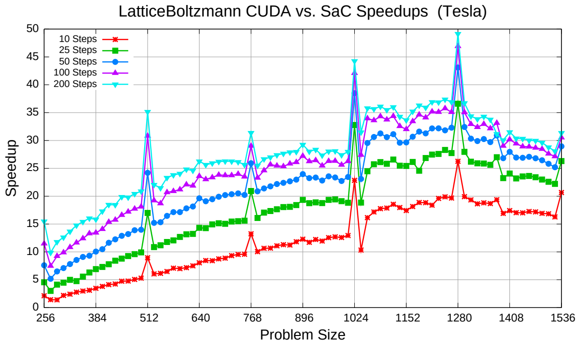
NAS benchmark FT



NAS benchmark MG

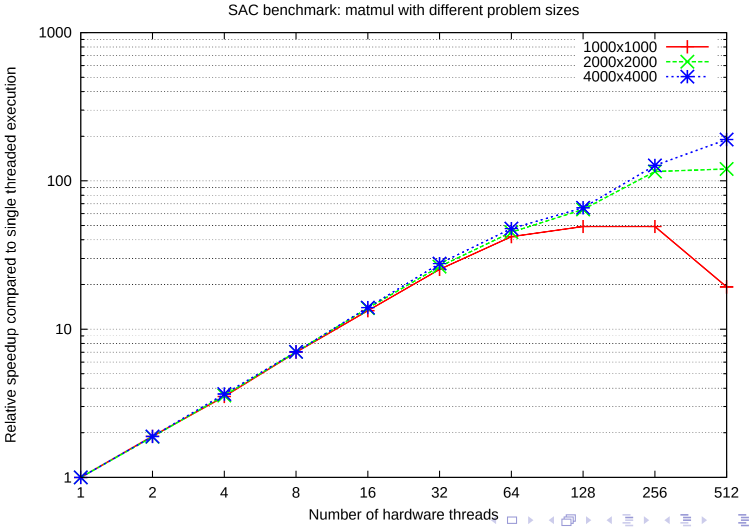
Runtime Performance: NVidia Tesla

Lattice-Boltzmann:



Runtime Performance: Ultra Sparc T3-4 Server

Matrix Multiplication:



Summary

Language design:

- High-level array processing
- Functional state-less semantics but C-like syntax
- Abstraction and composition
- Shape-generic programming
- (Almost) index-free programming

Summary

Language design:

- High-level array processing
- Functional state-less semantics but C-like syntax
- Abstraction and composition
- Shape-generic programming
- (Almost) index-free programming

Language implementation:

- Fully-fledged compiler
- Automatic parallelisation
- Automatic memory management
- High-level program transformation
- Large-scale machine-independent optimisation

The End

Questions ?

Check out www.sac-home.org !!