Thesis notes

1st February

Measuring polarization

Modularity can be used to ascertain polarization, not to find it.

- New and more reliable measures are defined on an undirected and weighted graph.

From "A Measure of Polarization on Social Media Networks Based on Community Boundaries", P Guerra, W Meira Jr, C Cardie

A new measure (1)

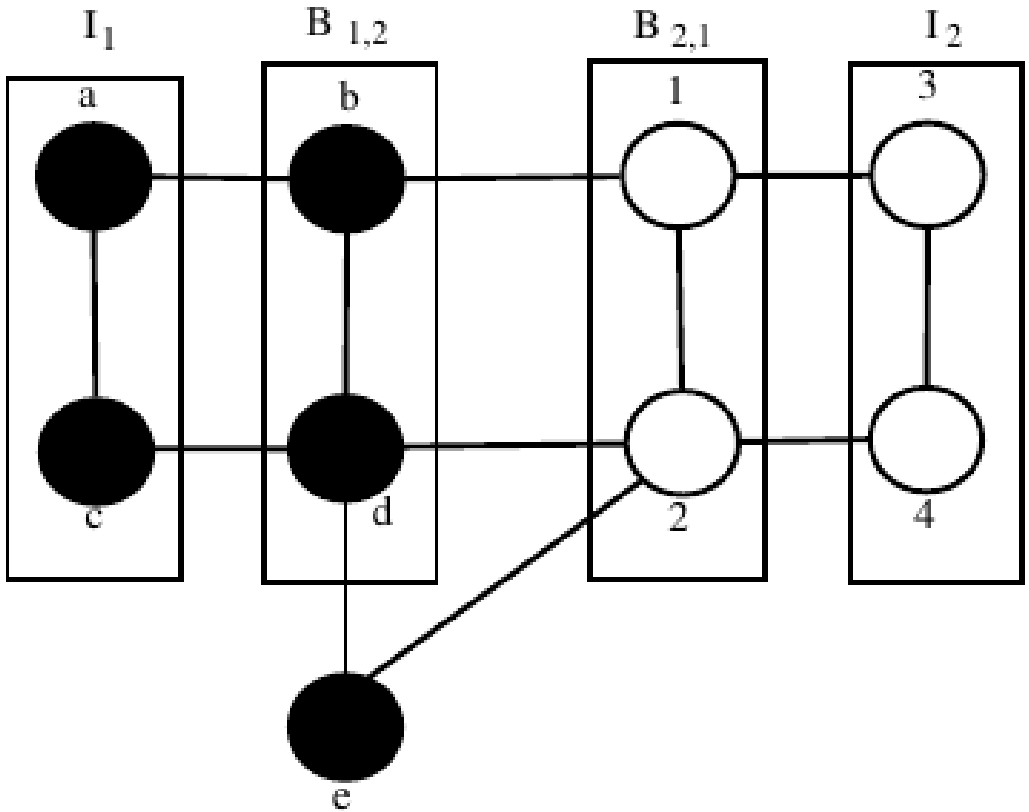
Graph is partitioned and, by considering nodes on the **boundary** (i.e. being connected to a node in the other community and to an *inner* node of its community) the following measure is defined

$$P = \frac{1}{|B|} \sum_{v \in B} \left[\frac{d_i(v)}{d_i(v) + d_b(v)} - 0.5 \right] \tag{1}$$

$P \in [-0.5, 0.5]$

- P close to 0.5 means there is polarization
- P close to 0 or < 0 means there is no polarization

A new measure (2)



Adapting the measure (1)

- Unweighted and undirected graph (defined in the paper)

$$P = \frac{1}{|B|} \sum_{v \in B} \left[\frac{d_i(v)}{d_i(v) + d_b(v)} - 0.5 \right] \tag{2}$$

- Weighted and directed graph (adapted)

$$P' = \frac{1}{|B|} \sum_{v \in B} \frac{1}{2} \cdot \frac{s_{out,i}(v) - s_{out,b}(v)}{\sum_{w \in OUT(v)} |w|} \tag{3}$$

$P' \in [-0.5, 0.5]$, polarization is present if P' close to 0.5

Adapting the measure (2)

$$P' = \frac{1}{|B|} \sum_{v \in B} \frac{1}{2} \cdot \frac{s_{out,i}(v) - s_{out,b}(v)}{\sum_{w \in OUT(v)} |w|}$$

- A term of the sum, relative to a single vertex $v \in B$, reaches the maximum if
  - the out-strength towards nodes in the other community are negative and
  - all edges towards nodes in the same community are positive
- A term of the sum is 0 if $s_{out,i}(v) = s_{out,b}(v)$

Another measure of polarization

Non-polarized communities should promote hubs in the boundary.

The correlation ρ between the ascending rank of the degree among all nodes r and the rank of the degree among the nodes in the boundary r_b is computed.

It can be adapted to weighed and directed graphs by ranking nodes according to $(s_{out} + s_{in})$: if there is no polarization users in the boundary interact positevily in both directions and with both communities.