# Prelims

## Linearly Seperable Data

```
GenData[n_] := Module[{X, y, pos, neg, npos, nneg},
   {npos} = Ceiling[n RandomVariate[NormalDistribution[0.5, 0.25], 1]];
   nneg = n - npos;
   X = Join[
     RandomVariate[NormalDistribution[-2, 1], {npos, 2}],
     RandomVariate[NormalDistribution[2, 1], {nneg, 2}]];
   y = Join[Table[1, {npos}], Table[-1, {nneg}]];
   pos = Take[X, npos];
   neg = Take[X, -nneg];
   {X, y, pos, neg}];

dlen = 100;
{X, y, pos, neg} = GenData[dlen];

tlen = 25;
{tpos, tneg} = Take[GenData[tlen], -2];

distro = ContourPlot[
   {PDF[NormalDistribution[-2, 1], x] PDF[NormalDistribution[-2, 1], y] == 0.001,
    PDF[NormalDistribution[2, 1], x] PDF[NormalDistribution[2, 1], y] == 0.001,
    PDF[NormalDistribution[-2, 1], x] PDF[NormalDistribution[-2, 1], y] == 0.01,
    PDF[NormalDistribution[2, 1], x] PDF[NormalDistribution[2, 1], y] == 0.01},
   {x, -5, 5}, {y, -5, 5}, ContourStyle →
    {Directive[Dashed, Blue], Directive[Dashed, Purple], Blue, Purple}];
```
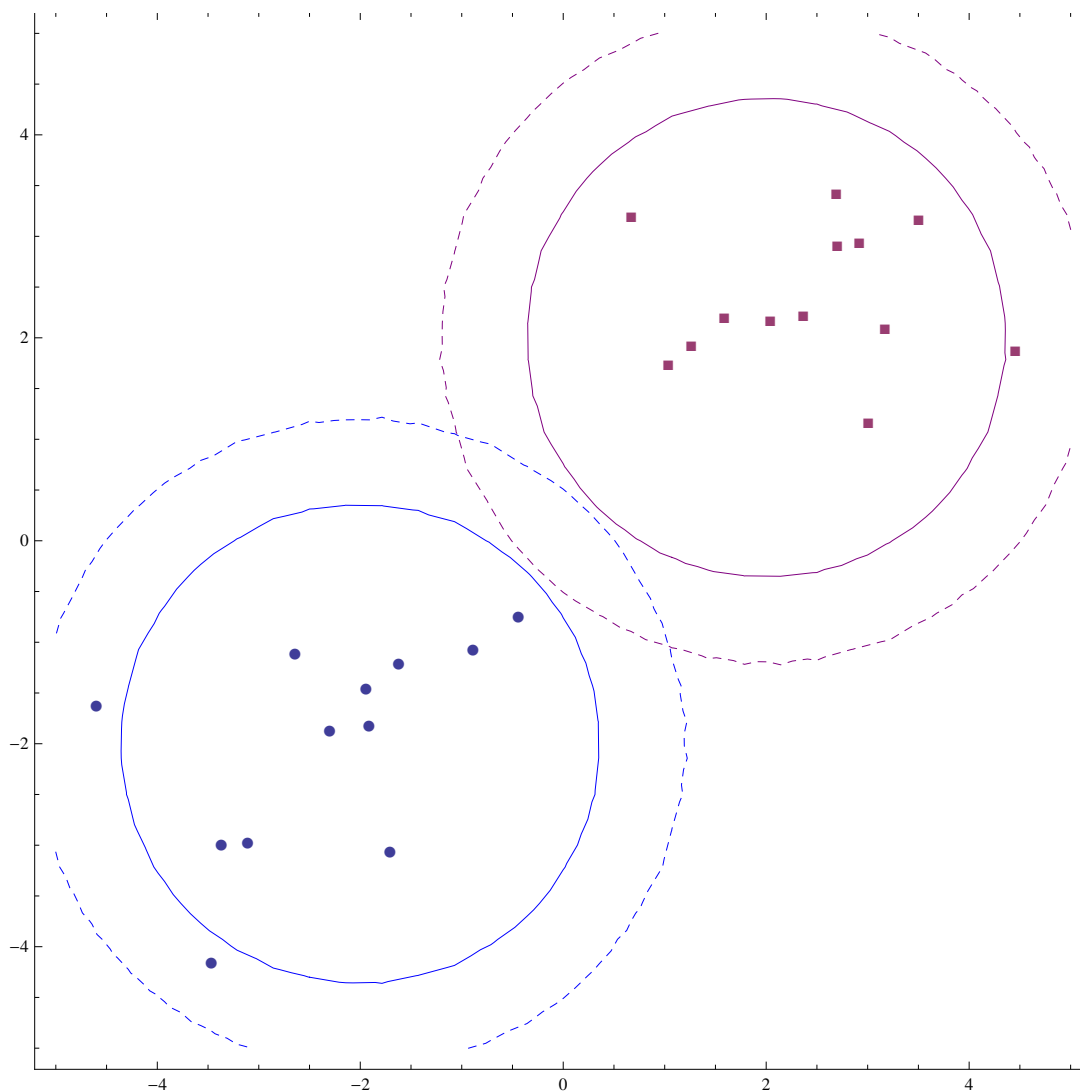
`Show[distro, dataPlot = ListPlot[{pos, neg}, PlotMarkers → Automatic]]`
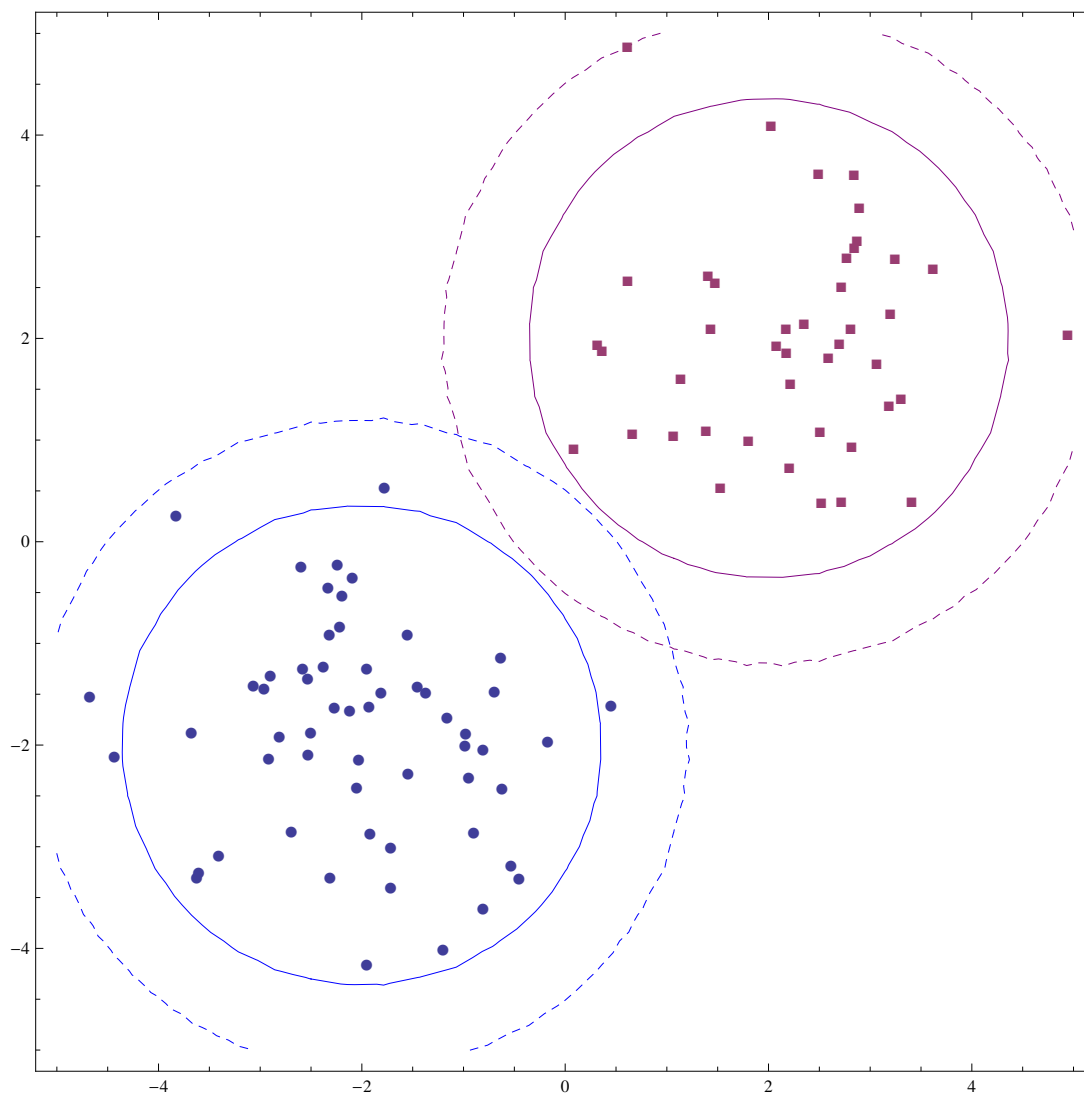
**Show[distro, testPlot = ListPlot[{tpos, tneg}, PlotMarkers → Automatic]]**



**τ = 0.01;**

**α = SeparableSVM[X, y, τ]**

{0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.530622, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0.281524, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
 0, 0, 0.812146, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}

**w = {WeightVector[α, X, y]}**
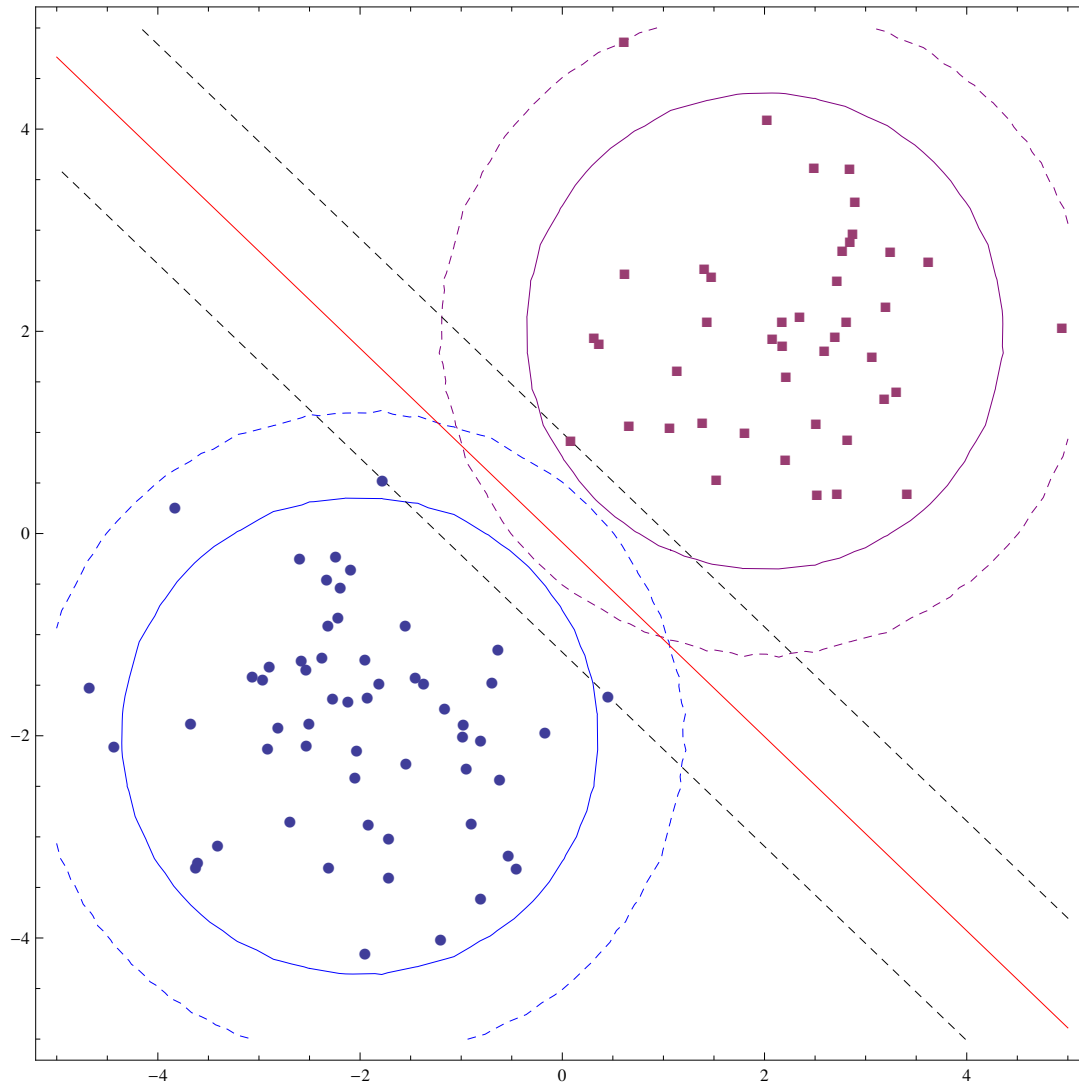
{{-0.884729, -0.921494}}

**b = Bias[α, X, y]**

-0.0816545

**g[x_, y_] := w.{{x}, {y}} + b**

```
contour = ContourPlot[{g[x, y] == -1, g[x, y] == 0, g[x, y] == 1},
    {x, -5, 5}, {y, -5, 5}, ContourStyle → {Dashed, Red, Dashed}];
```
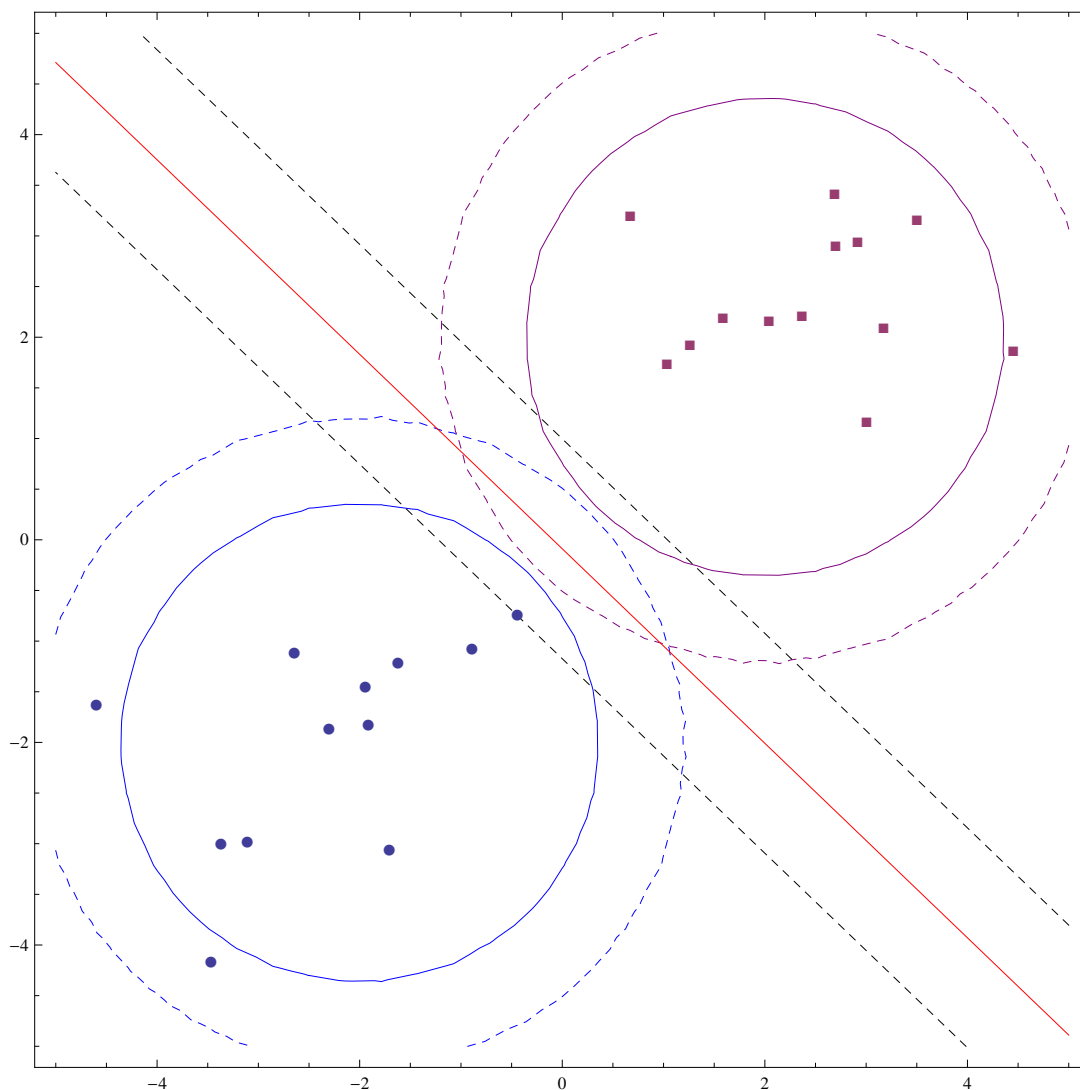
```
PDF[NormalDistribution[2, 1], x]
```

$$\frac{e^{-\frac{1}{2}(-2+x)^2}}{\sqrt{2\pi}}$$

```
Show[distro, contour, dataPlot]
```

```
Show[distro, contour, testPlot]
```



## Spiral Data

$$\text{func}[t\_, y\_] := \frac{2}{17\,\text{Pi}}\,\{y\,t\,\text{Sin}[t], y\,t\,\text{Cos}[t]\}$$

```
Clear[next];
```

$$\text{next}[a\_, n\_] := \text{next}[a, n] = a + \text{NArgMin}\Big[\Big\{\frac{1}{n}\,\text{Sum}\Big[\text{Norm}\Big[D\Big[\text{func}\Big[a + i\,\frac{b}{n}, 1\Big]\Big]\Big], \{i, 0, n\}\Big],$$
$$\frac{1}{n}\,\text{Sum}\Big[\text{Norm}\Big[D\Big[\text{func}\Big[a + i\,\frac{b}{n}, 1\Big]\Big]\Big], \{i, 0, n\}\Big] > \frac{1}{20}, b > 0\Big\}, \{b\}\Big][[1]];$$

```
Clear[next];
```

$$\text{next}[a\_, n\_] := \text{next}[a, n] = a + \text{NArgMin}[\{\text{Norm}[\text{func}[a, 1] - \text{func}[a + b, 1]],$$
$$\text{Norm}[\text{func}[a, 1] - \text{func}[a + b, 1]] > 1/20, b > 0\}, \{b\}][[1]];$$

```
indices = Table[Nest[next[#, 10] &, 13 / 25 Pi, i], {i, 1, 200}]
```

```
{2.25815, 2.75919, 3.18869, 3.57022, 3.91676, 4.23637, 4.53442, 4.81472, 5.08008,
 5.33265, 5.57409, 5.80576, 6.02873, 6.24392, 6.45208, 6.65386, 6.8498, 7.04039,
 7.22603, 7.40709, 7.5839, 7.75673, 7.92584, 8.09147, 8.25382, 8.41307, 8.56939,
 8.72295, 8.87388, 9.02231, 9.16836, 9.31214, 9.45376, 9.59331, 9.73087, 9.86653,
 10.0004, 10.1325, 10.2629, 10.3916, 10.5188, 10.6445, 10.7688, 10.8916, 11.0131,
 11.1333, 11.2522, 11.3698, 11.4863, 11.6016, 11.7157, 11.8288, 11.9408, 12.0518,
 12.1617, 12.2707, 12.3788, 12.4859, 12.5921, 12.6974, 12.8018, 12.9054, 13.0082,
 13.1102, 13.2114, 13.3118, 13.4115, 13.5105, 13.6087, 13.7063, 13.8031, 13.8993,
 13.9948, 14.0897, 14.184, 14.2776, 14.3706, 14.463, 14.5549, 14.6461, 14.7368, 14.827,
 14.9166, 15.0057, 15.0942, 15.1822, 15.2698, 15.3568, 15.4434, 15.5294, 15.615,
 15.7001, 15.7848, 15.869, 15.9528, 16.0361, 16.119, 16.2015, 16.2836, 16.3653,
 16.4465, 16.5274, 16.6078, 16.6879, 16.7676, 16.8469, 16.9259, 17.0045, 17.0827,
 17.1606, 17.2381, 17.3153, 17.3921, 17.4686, 17.5447, 17.6206, 17.6961, 17.7713,
 17.8461, 17.9207, 17.995, 18.0689, 18.1426, 18.2159, 18.289, 18.3617, 18.4342,
 18.5064, 18.5783, 18.65, 18.7213, 18.7924, 18.8633, 18.9338, 19.0041, 19.0742,
 19.144, 19.2135, 19.2828, 19.3518, 19.4206, 19.4892, 19.5575, 19.6256, 19.6934,
 19.761, 19.8284, 19.8955, 19.9625, 20.0292, 20.0957, 20.1619, 20.228, 20.2938,
 20.3594, 20.4248, 20.49, 20.555, 20.6198, 20.6844, 20.7488, 20.813, 20.877, 20.9407,
 21.0044, 21.0678, 21.131, 21.194, 21.2569, 21.3195, 21.382, 21.4443, 21.5064,
 21.5683, 21.6301, 21.6917, 21.7531, 21.8143, 21.8754, 21.9363, 21.997, 22.0576,
 22.118, 22.1782, 22.2383, 22.2982, 22.3579, 22.4175, 22.477, 22.5362, 22.5953,
 22.6543, 22.7131, 22.7718, 22.8303, 22.8887, 22.9469, 23.0049, 23.0629, 23.1206}
```

```
funcPlot = ParametricPlot[{func[t, 1], func[t, -1]}, {t, 0, (8 + 1 / 2) Pi}];
```

```
Nd = 200;
(*data=Table[Join[func[(i+12)/25Pi,(-1)^i+RandomReal[]/25]
      (*+{RandomReal[],RandomReal[]}/15*),{(-1)^i}],{i,1,Nd}]//N;*)
(*data=Import["spiral-data.csv"];*)
dist = NormalDistribution[0, 0.02];
RandomIndices[n_, k_] := Intersection[Round /@ Table[RandomReal[] * n, {k}]];
genPoint[t_, c_] :=
  Join[func[t + RandomReal[] / 25, c] + RandomVariate[dist] {1, 1}, {c}];
data = Join[Table[genPoint[indices[[i]], 1], {i, 1, Length[indices]}],
    Table[genPoint[indices[[i]], -1], {i, 1, Length[indices]}]];
test = Join[Table[genPoint[indices[[i]], 1],
     {i, RandomIndices[Length[indices], Ceiling[0.2 Length[indices]]]}],
    Table[genPoint[indices[[i]], -1],
     {i, RandomIndices[Length[indices], Ceiling[0.2 Length[indices]]]}]];
Nd = Length[data];
xdata = Table[{data[[i, 1]], data[[i, 2]]}, {i, 1, Nd}];
ydata = Table[data[[i, 3]], {i, 1, Nd}];
pos = Table[{item[[1]], item[[2]]}, {item, Select[data, #[[3]] > 0 &]}];
neg = Table[{item[[1]], item[[2]]}, {item, Select[data, #[[3]] < 0 &]}];
tpos = Table[{item[[1]], item[[2]]}, {item, Select[test, #[[3]] > 0 &]}];
tneg = Table[{item[[1]], item[[2]]}, {item, Select[test, #[[3]] < 0 &]}];
```
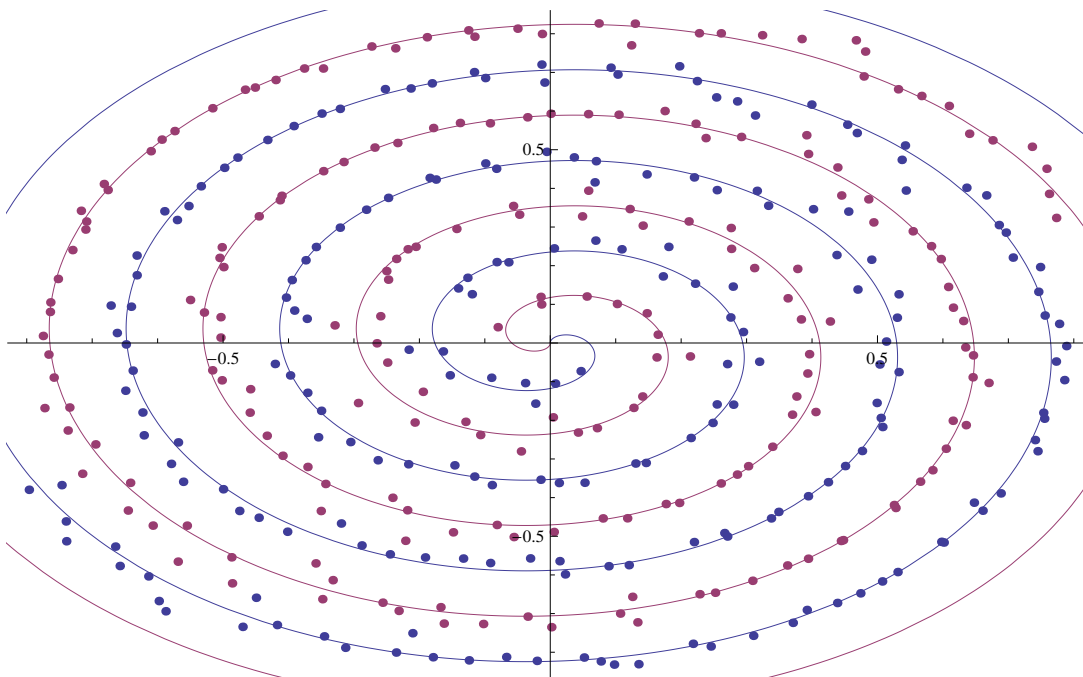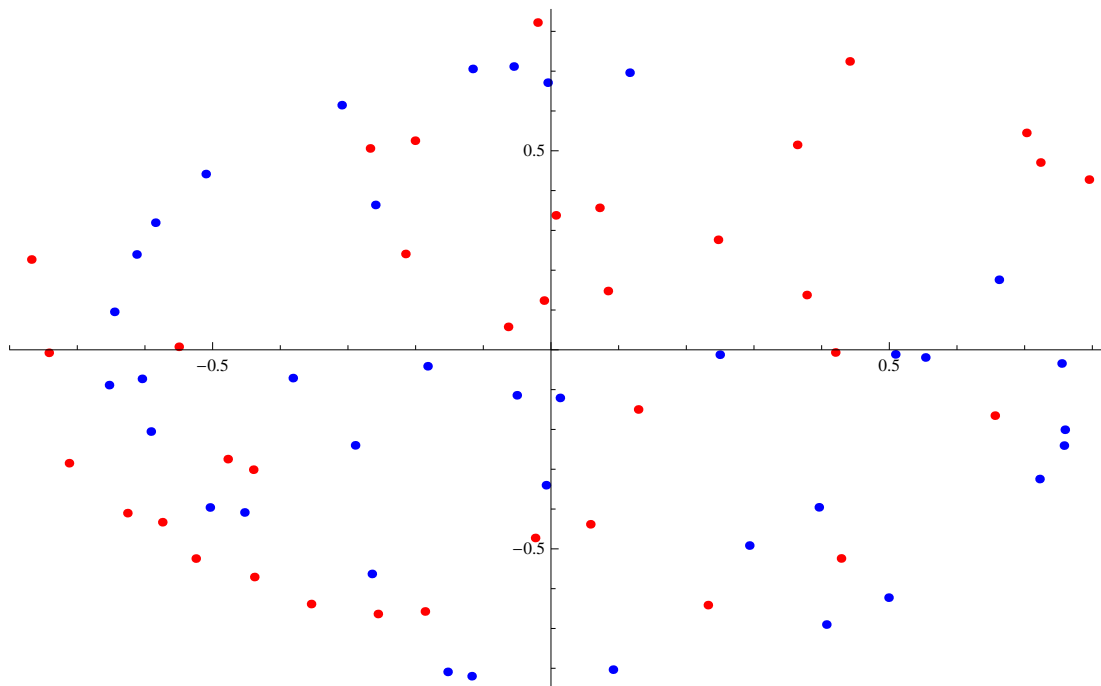
**dPlot = ListPlot[{pos, neg}, PlotStyle → PointSize[Medium]]**
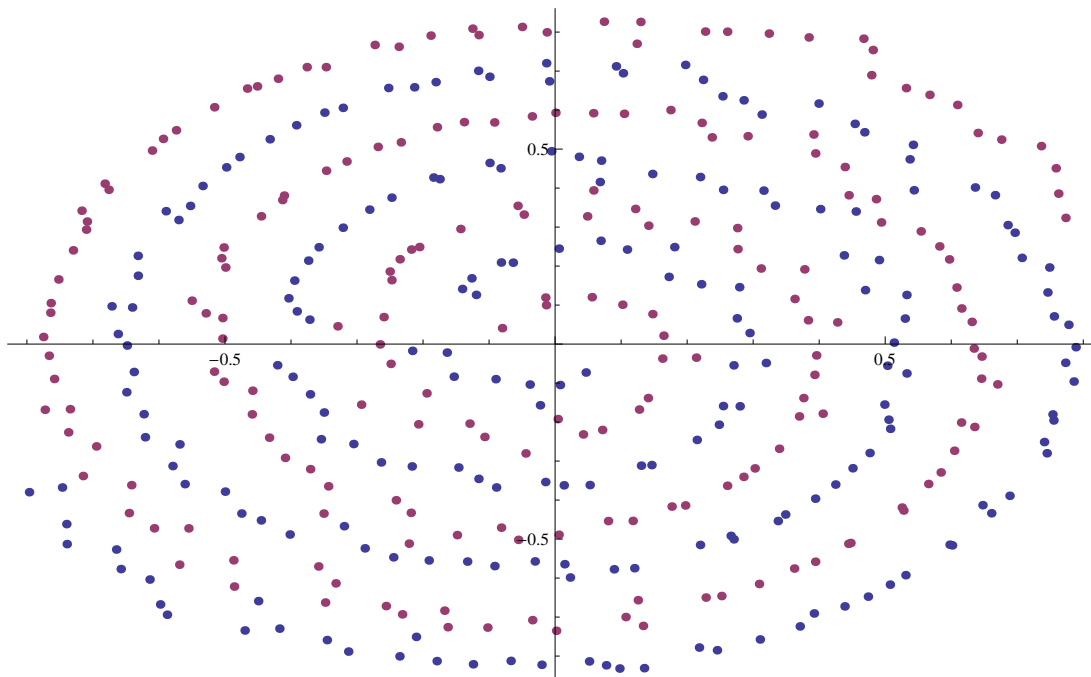


**Show[dPlot, funcPlot]**

```
tPlot = ListPlot[{tpos, tneg},
  PlotStyle → {Directive[PointSize[Medium], Blue], Directive[PointSize[Medium], Red]}]
```



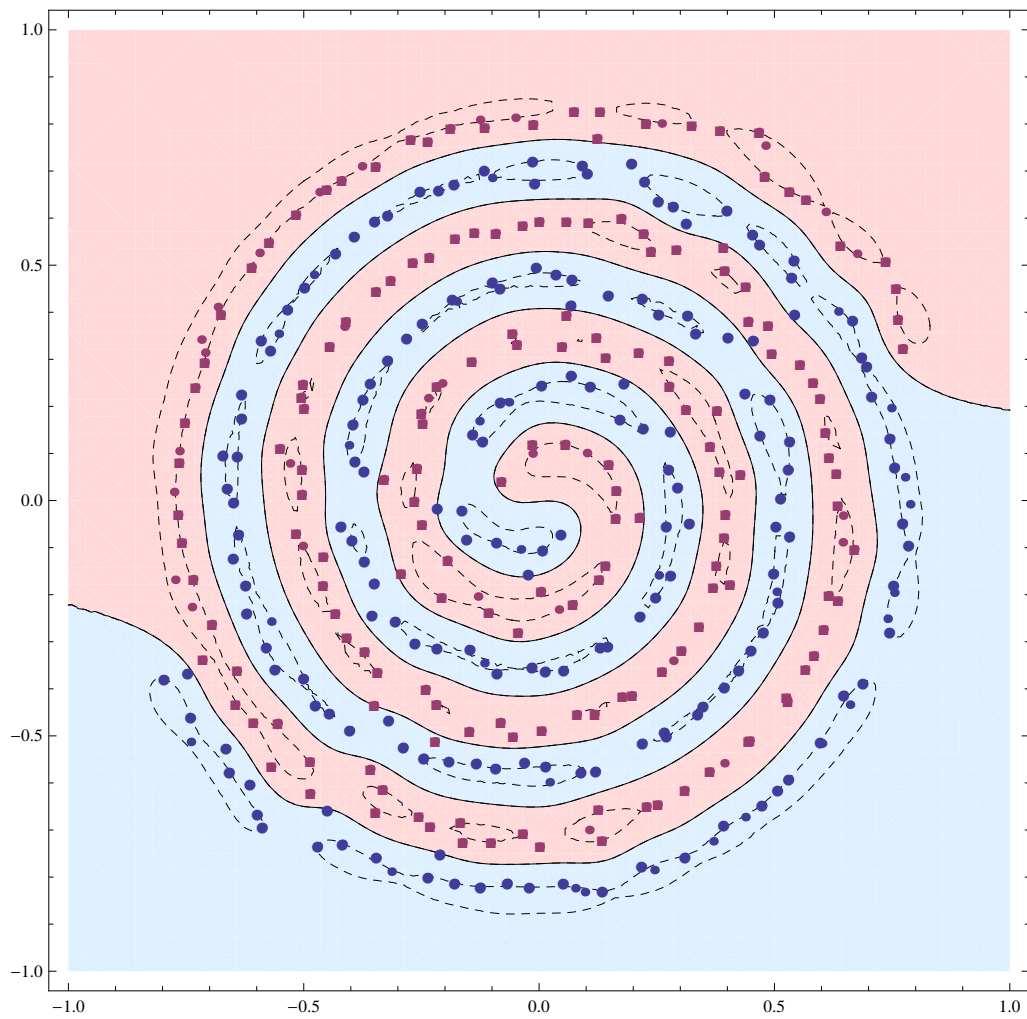## ■ As a separable dataset

```
? SeparableSVM
```

SeparableSVM[X, y, τ] trains a separable SVM on data X, labels y, and solution tolerance τ (see QPSolve). Returns the multiplier vector α. Option KernelFunction determines the kernel to use; defaults to IdentityKernel (i.e. no kernel)

```
krbf[x_, y_] := RBFKernel[x, y, 100];

α = SeparableSVM[xdata, ydata, 0.01, KernelFunction → krbf];

ynew[α_, X_, y_, k_] := Sum[α[[i]] y[[i]] k[#, X[[i]]], {i, 1, Length[X]}] &

rbfY = ynew[α, xdata, ydata, krbf];

rbfPlot = ContourPlot[rbfY[{x, y}], {x, -1, 1}, {y, -1, 1}, Contours → {-1, 0, 1},
  ContourStyle → {Dashed, Black, Dashed}, ContourShading → None];

rbfPlotShading = ContourPlot[rbfY[{x, y}], {x, -1, 1}, {y, -1, 1}, Contours → {0},
  ContourStyle → {Dashed, Black, Dashed}, ContourShading → {LightRed, LightBlue}];

SVs = ListPlot[{({#[[1]], #[[2]]}) & /@ #1[[1]], ({#[[1]], #[[2]]}) & /@ #2[[1]]} & @@
  SplitBy[Table[data[[i]], {i, SupportVectors[α, ydata]}], #[[3]] &],
  PlotMarkers → Automatic];
```

**SupportVectors[α, ydata]**

{{1, 2, 3, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29,
    30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52,
    53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74,
    75, 76, 77, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 97,
    98, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 112, 113, 114, 115,
    116, 117, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 130, 132, 133, 134, 135,
    136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 148, 149, 150, 151, 153, 154,
    157, 158, 159, 162, 163, 164, 166, 168, 169, 170, 173, 174, 176, 177, 180, 181, 182,
    183, 184, 185, 186, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200},
   {201, 202, 204, 206, 207, 208, 209, 210, 211, 212, 214, 215, 216, 218, 219, 220, 221,
    222, 223, 224, 225, 226, 228, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240,
    241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255, 256, 257, 258,
    259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 273, 274, 275, 277,
    278, 279, 280, 281, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295,
    296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 314,
    315, 316, 317, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 331, 332, 333,
    334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350,
    352, 354, 355, 357, 359, 360, 361, 364, 366, 368, 369, 371, 372, 374, 375, 376, 377,
    379, 381, 382, 383, 384, 385, 387, 388, 389, 391, 392, 393, 395, 397, 398, 399, 400}}

**Show[rbfPlotShading, rbfPlot, dPlot, SVs]**

**Show[rbfPlotShading, rbfPlot, tPlot]**
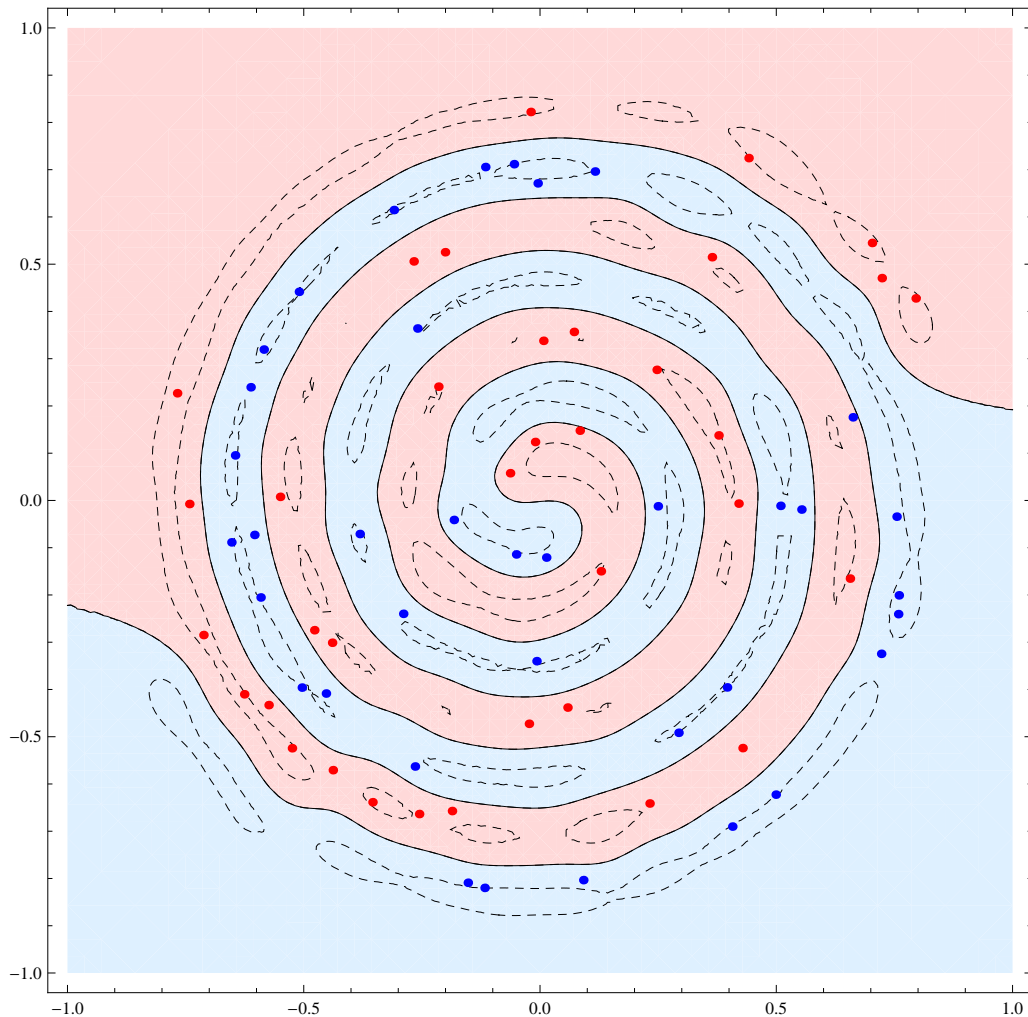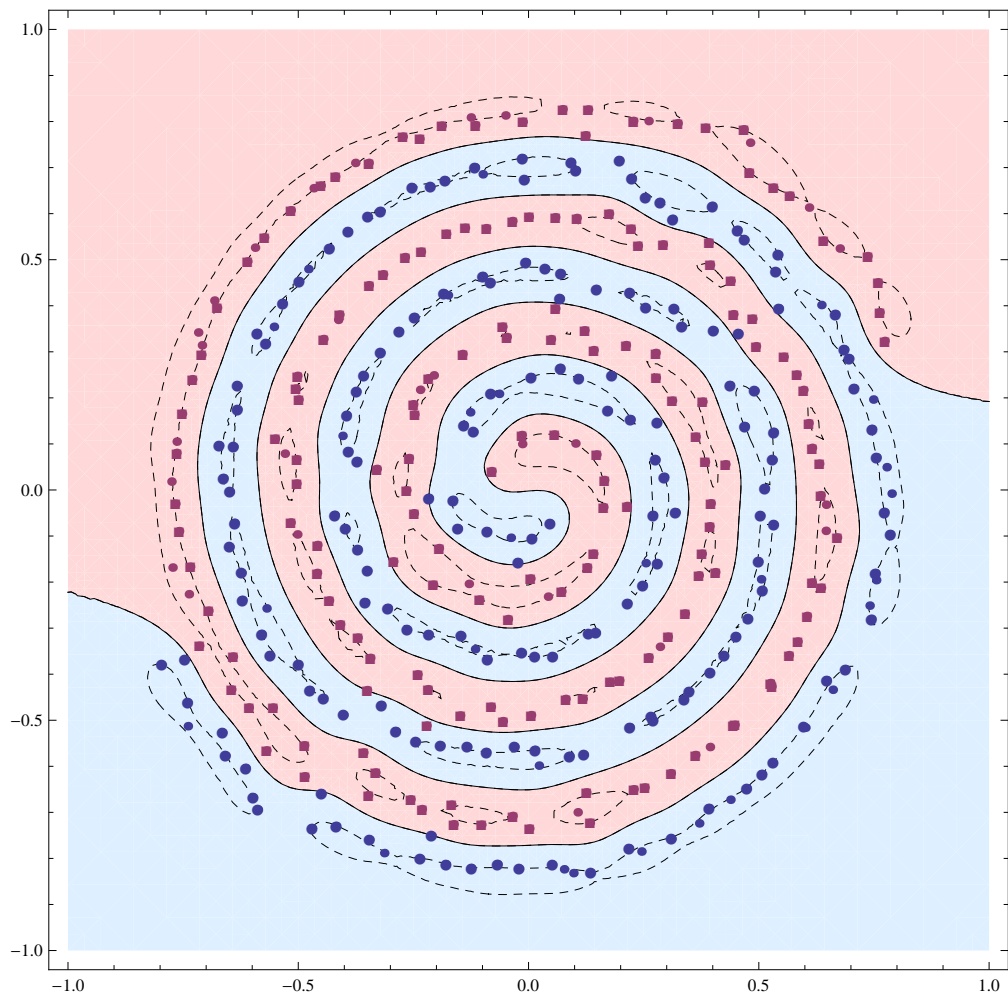


### ■ As a nonseparable dataset

**? NonseparableSVM**

NonSeparableSVM[X,y,C,τ] trains an non−separable SVM on data X, labels
    y, penalty term C and solution tolerance τ (see QPSolve). Returns the multiplier vector α.
    Option KernelFunction determines the kernel to use; defaults to IdentityKernel (ie, no kernel)

**α2 = NonseparableSVM[xdata, ydata, 1.0, 0.01, KernelFunction → krbf];**

**rbfY2 = ynew[α, xdata, ydata, krbf];**

**rbfPlot2 = ContourPlot[rbfY2[{x, y}], {x, -1, 1}, {y, -1, 1}, Contours → {-1, 0, 1},**
    **ContourStyle → {Dashed, Black, Dashed}, ContourShading → None];**

**rbfPlotShading2 = ContourPlot[rbfY2[{x, y}], {x, -1, 1}, {y, -1, 1}, Contours → {0},**
    **ContourStyle → {Dashed, Black, Dashed}, ContourShading → {LightRed, LightBlue}];**

```
SVs2 = ListPlot[{({#[[1]], #[[2]]}) & /@ #1[[1]], ({#[[1]], #[[2]]}) & /@ #2[[1]]} & @@
     SplitBy[Table[data[[i]], {i, SupportVectors[α2, ydata]}], #[[3]] &],
    PlotMarkers → Automatic];
```

```
SupportVectors[α2, ydata]
```

```
{{1, 2, 3, 5, 6, 7, 8, 9, 10, 12, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29,
   30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52,
   53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74,
   75, 76, 77, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 97,
   98, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 112, 113, 114, 115,
   116, 117, 119, 120, 121, 123, 124, 125, 126, 127, 128, 129, 130, 132, 133, 134, 135,
   136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 148, 149, 150, 151, 153, 154,
   157, 158, 159, 162, 163, 164, 166, 168, 169, 170, 173, 174, 176, 177, 180, 181, 182,
   183, 184, 185, 186, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200},
  {201, 202, 204, 206, 207, 208, 209, 210, 211, 212, 214, 215, 216, 218, 219, 220, 221,
   222, 223, 224, 225, 226, 228, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240,
   241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 253, 254, 255, 256, 257, 258,
   259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 273, 274, 275, 277,
   278, 279, 280, 281, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295,
   296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 314,
   315, 316, 317, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 331, 332, 333,
   334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350,
   352, 354, 355, 357, 359, 360, 361, 364, 366, 368, 369, 371, 372, 374, 375, 376, 377,
   379, 381, 382, 383, 384, 385, 387, 388, 389, 391, 392, 393, 395, 397, 398, 399, 400}}
```

**Show[rbfPlotShading2, rbfPlot2, dPlot, SVs2]**

**Show[rbfPlotShading2, rbfPlot2, tPlot]**