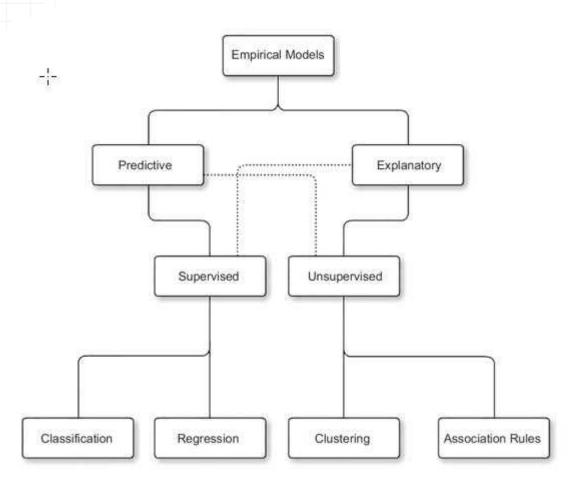


Models



Classification

Target variable is categorical. Predictors could be of any data type.

Algorithms

Decision Trees

Rule induction

kNN

Naive Bayesian

Neural Networks

Support Vector Machines

Ensemble Meta Models

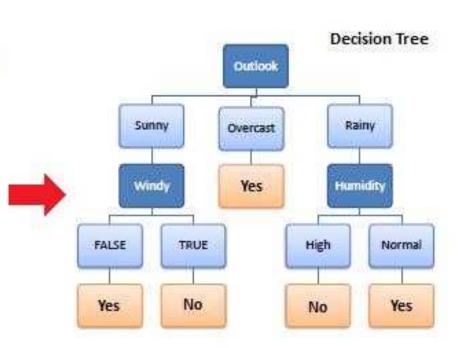


Play golf dataset

Outlook	Temperature	Humidity	Windy	Play
sunny	hot	high	FALSE	no
sunny	hot	high	TRUE	no
overcast	hot	high	FALSE	yes
rainy	mild	high	FALSE	yes
rainy	cool	normal	FALSE	yes
rainy	cool	normal	TRUE	no
overcast	cool	normal	TRUE	yes
sunny	mild	high	FALSE	no
sunny	cool	normal	FALSE	yes
rainy	mild	normal	FALSE	yes
sunny	mild	normal	TRUE	yes
overcast	mild	high	TRUE	yes
overcast	hot	normal	FALSE	yes
rainy	mild	high	TRUE	no

Decision Trees

Predictors			Target	
Outlook	Temp	Humidity	Windy	Play Golf
Rainy	Hot	High	Faice	No
Rainy	Hot	High	True	No
Overoact	Hot	High	False	Yes
Sunny	Mild	High	False	Yes
Sunny	Cool	Normal	Falce	Yes
Sunny	Cool	Normal	True	No
Overoast	Cool	Normal	True	Yes
Rainy	MRd	High	False	No
Rainy	Cool	Normal	False	Yec
Sunny	Mild	Normal	Falce	Yes
Rainy	MRd	Normal	True	Yes
Overcast	Mild	High	True	Yes
Overoast	Hot	Normal	Falce	Yes
Sunny	MRd	High	True	No



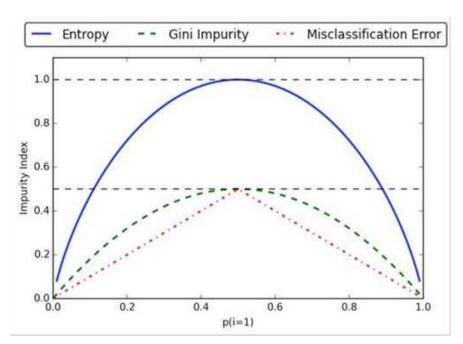
Measure of impurity

Every split ties to make child node more pure.

Gini impurity

Information Gain (Entropy)

Misclassification Error



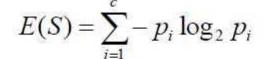
		Row No.	Play	Outlook
		1	no	sunny
		2	no	sunny
		3	yes	overcast
		4	yes	rain
Table 4.2 Computing the Info Attributes	ormation Gain for All	5	yes	rain
Attribute	Information Gain	6	no	rain
Temperature	0.029	7	yes	overcast
Humidity	0.102	8	no	sunny
Wind Outlook	0.048 0.247	9	yes	sunny
Commonweal Annabara	U-9-20-0-0-0 993	10	yes	rain
		11	yes	sunny
		12	yes	overcast
		13	yes	overcast
		14	no	rain

Entropy

$$E(T,X) = \sum_{c \in X} P(c)E(c)$$

		Play Golf		44
		Yes	No	5
Outlook	Sunny	3	2	5
	Overcast	4	0	4
	Rainy	2	3	5
				14





Entropy(PlayGolf) = Entropy (5,9) = Entropy (0.36, 0.64) = - (0.36 log₂ 0.36) - (0.64 log₂ 0.64) = 0.94



Yes 2	No 2
2	2
~	
4	2
3	1
029	
Di-	C-14
	3 029 Play

Play Golf

2

		Play Golf	
<u></u> \		Yes	No
Humidity	High	3	4
	Normal	6	1
	Gain = 0	.152	

Sunny

Rainy

Overcast

Outlook

	Gain = 0.048				
(T)	T	(T	V		

False

$$\underline{Gain(T, \underline{X})} = \underline{Entropy(T)} - \underline{Entropy(\underline{T}, \underline{X})}$$

G(PlayGolf, Outlook) = E(PlayGolf) - E(PlayGolf, Outlook)

= 0.940 - 0.693 = 0.247Play Golf Yes No 3 Sunny 2

4 Outlook Overcast 0 Rainy 2 3 Gain = 0.247

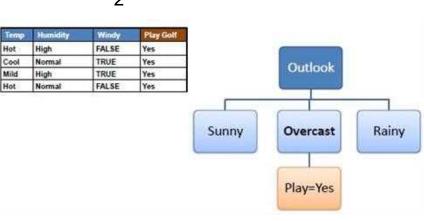
Sub table / sub tree

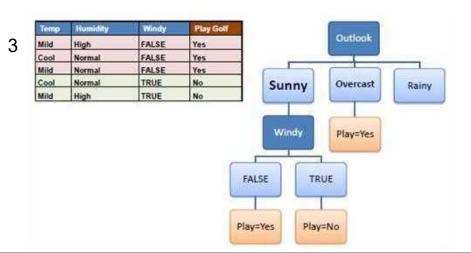


Overcast

Rainy

Collinor.	remp	Humaday	willing	Play Gon	
Sunny	Mild	High	FALSE	Yes	
Sunny	Cool	Normal	FALSE	Yes	
Sunny	Cool	Normal	TRUE	No	
Sunny	Mild	Normal	FALSE	Yes	
Sunny	Mild	High	TRUE	No	
Overcast	Hot	High	FALSE	Yes	
Overcast	Cool	Normal	TRUE	Yes	
Overcast	Mild	High	TRUE	Yes	
Overcast	Hot	Normal	FALSE	Yes	
VIII.	operation	1	100000	F01	
Rainy	Hat	High	FALSE	No	0
Rainy	Hat	High	TRUE	No	
Rainy	Mild	High	FALSE	No	
Rainy	Cool	Normal	FALSE	Yes	
Rainy	Mild	Normal	TRUE	Yes	





Decision tree – rule extraction

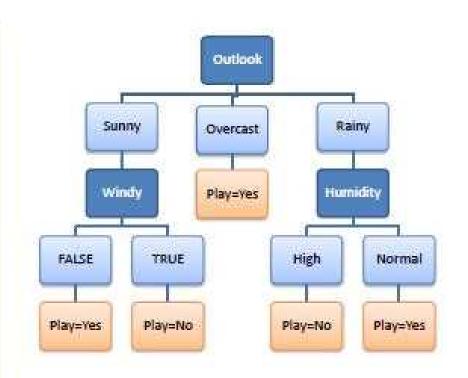
R₁: IF (Outlook=Sunny) AND (Windy=FALSE) THEN Play=Yes

R₂: IF (Outlook=Sunny) AND (Windy=TRUE) THEN Play=No

R₃: IF (Outlook=Overcast) THEN Play=Yes

R₄: IF (Outlook=Rainy) AND (Humidity=High) THEN Play=No

R_s: IF (Outlook=Rain) AND (Humidity=Normal) THEN Play=Yes



Decision Tree Algorithm

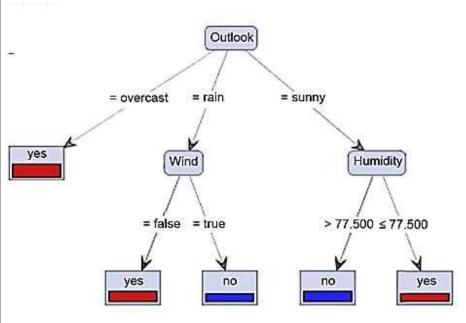
- A general algorithm for a decision tree can be described as follows:
 - 1. Pick the best attribute (highest IG)
 - 2. Make branches,
 - 3. for each branch **repeat** the process for the subset / sub table until one of stop conditions is satisfied.

DT: When to Stop Splitting Data?

- No attribute satisfies a minimum information gain threshold
- A maximal depth is reached
- There are less than a certain number of examples in the current subtree



Tree to Rules



Rule 1: if (Outlook = overcast) then yes

Rule 2: if (Outlook = rain) and (Wind = false) then yes

Rule 3: if (Outlook = rain) and (Wind = true) then no

Rule 4: if (Outlook = sunny and (Humidity > 77.5) then no

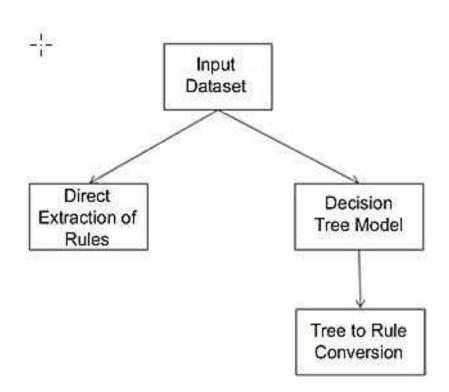
Rule 5: if (Outlook = sunny and (Humidity ≤ 77.5) then yes

Rules

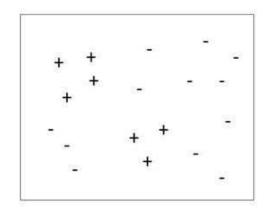
 $R = \{ r_1 \cap r_2 \cap r_3 \cap ... r_k \}$ Where k is the number of disjuncts in a rule set. Individual disjuncts can be represented as

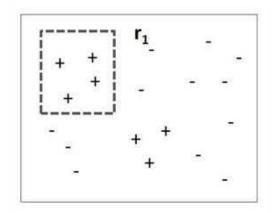
 r_i = (antecedent or condition) then (consequent)

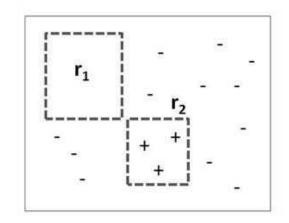
Approaches



Sequential covering







Rule accuracy A
$$(r_i) = \frac{\text{Correct records covered by rule}}{\text{All records covered by the rule}}$$

Rule-based Classifier (Example)

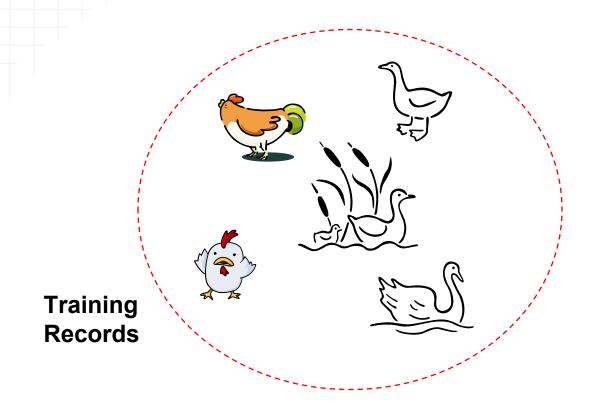
Name	Blood Type	Give Birth	Can Fly	Live in Water	Class
human	warm	yes	no	no	mammals
python	cold	no	no	no	reptiles
salmon	cold	no	no	yes	fishes
whale	warm	yes	no	yes	mammals
frog	cold	no	no	sometimes	amphibians
komodo	cold	no	no	no	reptiles
bat	warm	yes	yes	no	mammals
pigeon	warm	no	yes	no	birds
cat	warm	yes	no	no	mammals
leopard shark	cold	yes	no	yes	fishes
turtle	cold	no	no	sometimes	reptiles
penguin	warm	no	no	sometimes	birds
porcupine	warm	yes	no	no	mammals
eel	cold	no	no	yes	fishes
salamander	cold	no	no	sometimes	amphibians
gila monster	cold	no	no	no	reptiles
platypus	warm	no	no	no	mammals
owl	warm	no	yes	no	birds
dolphin	warm	yes	no	yes	mammals
eagle	warm	no	yes	no	birds
	· · · · · · · · · · · · · · · · · · ·	•	· ———	· · · · · · · · · · · · · · · · · · ·	•

R1: (Give Birth = no) ∧ (Can Fly = yes) → Birds
R2: (Give Birth = no) ∧ (Live in Water = yes) → Fishes
R3: (Give Birth = yes) ∧ (Blood Type = warm) →
Mammals

R4: (Give Birth = no) \land (Can Fly = no) \rightarrow Reptiles R5: (Live in Water = sometimes) \rightarrow Amphibians



KNN

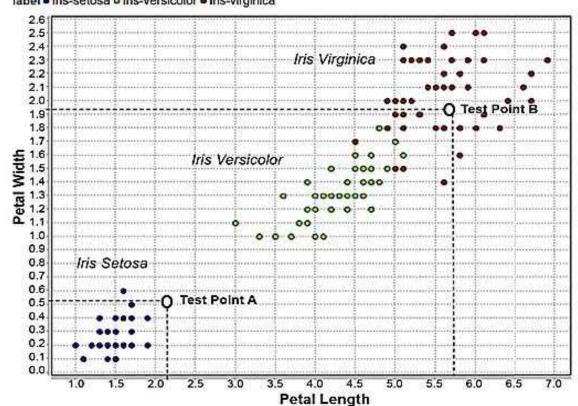


Test Record

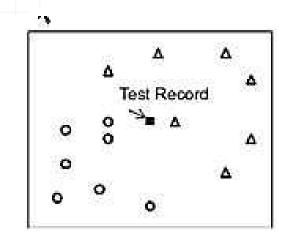


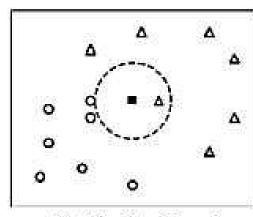
Guess the species for A and B

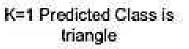


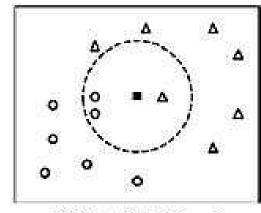


KNN



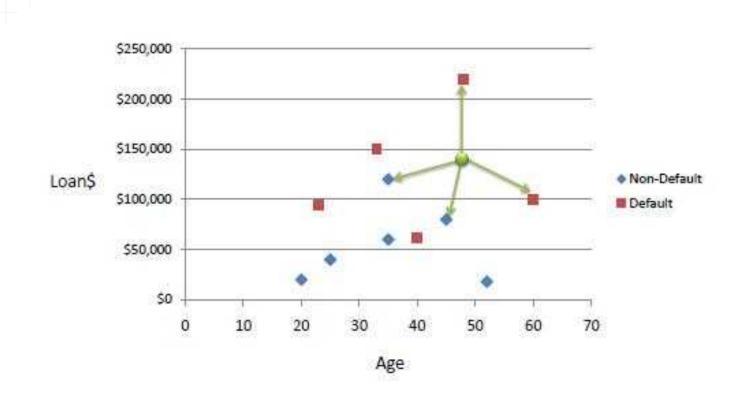




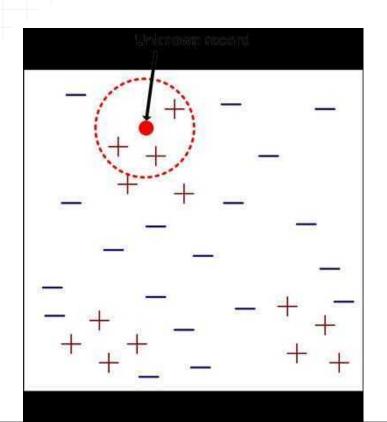


K=3 Predicted Class is circle

kNN, k=3



Nearest-Neighbor Classifiers



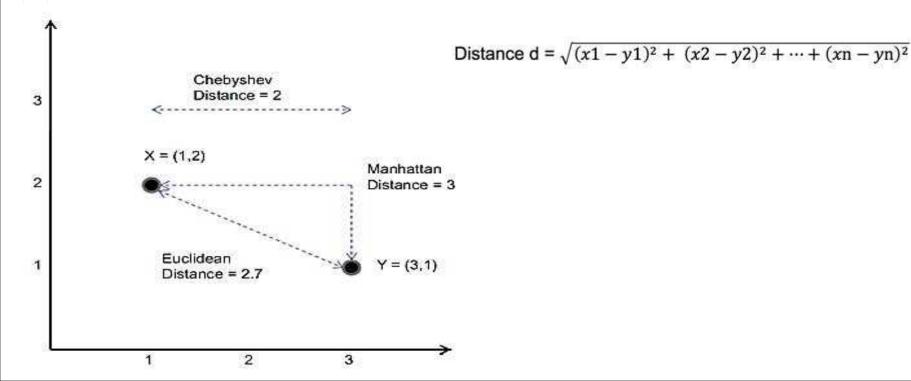
Requires the following:

- A set of labeled records
- Proximity metric to compute distance/similarity between a pair of records e.g., Euclidean distance
- The value of k, the number of nearest neighbors to retrieve
- A method for using class labels of K nearest neighbors to determine the class label of unknown record (e.g., by taking majority vote)
- Take the majority vote of class labels among the k-nearest neighbors

Measure of Proximity

Distance

Distance d =
$$\sqrt{(x1-y1)^2 + (x2-y2)^2}$$



Measure of Proximity

Correlation similarity

Correlation (X, Y) =
$$\frac{s_{xy}}{s_x + s_y}$$

Simple matching coefficient

Simple matching coefficient (SMC) = $\frac{matching\ occurences}{total\ occurences}$

Jaccard Similarity

 $Jaccard\ coefficient = \frac{common\ occurences}{total\ occurences}$

Cosine similarity

Cosine similarity (|X,Y|) =
$$\frac{x \cdot y}{\|x\| \|y\|}$$

Hamming distance

Hamming Distance

$$D_{H} = \sum_{i=1}^{k} |x_{i} - y_{i}|$$

$$x = y \Rightarrow D = 0$$

$$x \neq y \Rightarrow D = 1$$

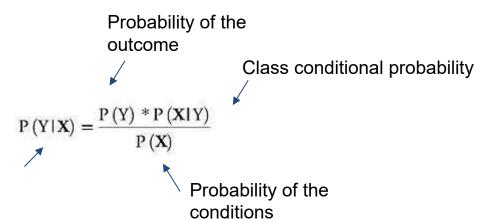
Х	Y	Distance
Male	Male	0
Male	Female	1

KNN algorithm steps

- 1. Choose the number of nearest neighbors (K) and the distance metric to use.
- 2. Train the model by storing the data in memory. Since KNN is a lazy learner, it does not build a model during the training phase. Instead, it stores the training data in memory and uses it to make predictions during the testing phase.
- 3. Test the model by making predictions on new data.
 - To make a prediction for a new data point:
 - The KNN algorithm calculates the distance between the new data point and all the points in the training dataset.
 - Then selects the K nearest neighbors based on the chosen distance metric and uses them to make a prediction.
 - For classification tasks, the prediction is the most common class among the K nearest neighbors. For regression tasks, the prediction is the average of the values of the K nearest neighbors.
- 4. Evaluate the model by measuring its performance on the test data. This



Bayes' theorem



Posterior probability

Bayes Classifier

- A probabilistic framework for solving classification problems
- Conditional Probability:

$$P(Y \mid X) = \frac{P(X,Y)}{P(X)}$$

Bayes theorem:

$$P(Y \mid X) = \frac{P(X \mid Y)P(Y)}{P(X)}$$

 $P(X \mid Y) = \frac{P(X,Y)}{P(Y)}$

Data set

No.	Tamamanatura V	Urumi ditu V	Outlook X ₃	WindV	Dlaw (Class Label) V
NO.	Temperature X ₁	Humidity X ₂	Ontlook v3	Wind X ₄	Play (Class Label) Y
1	high	med	sunny	false	no
2	high	high	sunny	true	no
3	low	low	rain	true	no
4	med	high	sunny	false	no
5	low	med	rain	true	no
6	high	med	overcast	false	yes
7	low	high	rain	false	yes
8	low	med	rain	false	yes
9	low	low	overcast	true	yes
10	low	low	sunny	false	yes
11	med	med	rain	false	yes
12	med	low	sunny	true	yes
13	med	high	overcast	true	yes
14	high	low	overcast	false	yes

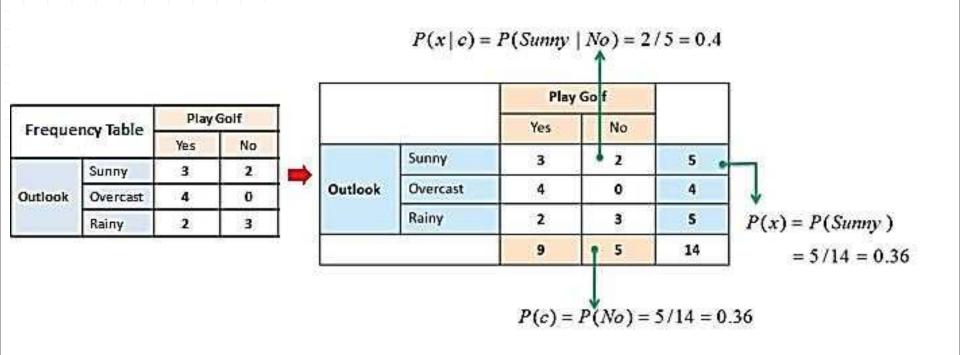
Likelihood Table Frequency Table Play Golf Play Golf Yes No Yes No Sunny 3 2 Sunny 3/9 2/5 Outlook 0 Outlook 4/9 0/5 Overcast 4 Overcast 3 2/9 3/5 Rainy 2 Rainy Play Golf Play Golf Yes No Yes No High 3 4 High 3/9 4/5 Humidity Humidity Normal 1 6/9 1/5 6 Normal Play Golf Play Golf Yes No Yes No Hot Hot 2/9 2 2 2/5 Mild Mild 4 2 Temp. 4/9 2/5 Temp. Cool 1/5 Cool 3 1 3/9 Play Golf Play Golf Yes No Yes No False 2 False 6/9 6 2/5 Windy Windy True 3 3 True 3/9 3/5

$$P(x \mid c) = P(Sunny \mid Yes) = 3/9 = 0.33$$

P(c) = P(Yes) = 9/14 = 0.64

			Delia Carlo	-	Likelihood Table		Play Golf			
Frequency Table		Play Golf			Likelinood lable		Yes	No		
		Yes	No			Sunny	3/9	2/5	5/14	
Outlook	Sunny	3	2		Outlook	and the second	5434530	FS STOCKS	20000000	P(x) = P(Sunny) = 5/14 = 0.36
	Overcast	4	0			Overcast	4/9	0/5	4/14	
	Rainy	2	3			Rainy	2/9	3/5	5/14	
							9/14	5/14		

Posterior Probability:
$$P(c \mid x) = P(Yes \mid Sunny) = 0.33 \times 0.64 \div 0.36 = 0.60$$



Posterior Probability:
$$P(c \mid x) = P(No \mid Sunny) = 0.40 \times 0.36 \div 0.36 = 0.40$$

Class conditional probability

Class Cond	itional Probability of Ten	nperature	
Temperature (X ₁)	$P(X_1 Y = no)$	P(X ₁ Y = yes)	
high	2/5	2/9	
med	1/5	3/9	
low	2/5	4/9	

Condition	al Probability of Humidity,	Outlook, and Wind	
Humidity (X ₂)	$P(X_1 Y = no)$	P(X ₁ Y = yes)	
high low med	2/5 1/5 2/5	2/9 4/9 3/9	
Outlook (X ₃)	$P(X_1 Y = no)$	P(X ₁ Y = yes)	
overcast Rain sunny	0/5 2/5 3/5	4/9 3/9 2/9	
Wind (X ₄)	$P(X_1 Y = no)$	$P(X_1 Y = yes)$	
false true	2/5 3/5	6/9 3/9	

Test record

Table 4.7 Test Record					
No.	Temperature X ₁	Humidity X ₂	Outlook X ₃	Wind X ₄	Play (Class Label) Y
Unlabeled Test	high	low	sunny	false	?

Calculation of posterior probability P(Y/X)

$$P(Y = yes|X) = \frac{P(Y) * \prod_{i=1}^{n} P(Xi|Y)}{P(X)}$$

$$= P(Y = yes) * \{P(Temp = high|Y = yes) * P(Humidity = low|Y = yes) * P(Outlook = sunny|Y = yes) * P(Wind = false|Y = yes)\}/P(X)$$

$$= 9/14 * \{2/9 * 4/9 * 2/9 * 6/9\}/P(X)$$

$$= 0.0094/P(X)$$

$$P(Y = no|X) = 5/14 * \{2/5 * 4/5 * 3/5 * 2/5\}$$

We normalize both the estimates by dividing both by (0.0094 + 0.027) to get

 0.0274 ± 0.0094

Likelihood of (Play = yes) =
$$\frac{0.0094}{0.0274 + 0.0094}$$
 = 26%
Likelihood of (Play = no) = $\frac{0.0274}{0.0274}$ = 74%

= 0.0274/P(X)

Issues

- Incomplete training set -> Use laplace correction
- Continuous numeric attributes -> Use Probability density function

Numerical

 $\sigma = \left[\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \mu)^2 \right]$

yes

no

Play Golf

$$\frac{1}{-1}\sum_{i=1}^{\infty} (x_i)^{-1}$$

85 90 70 95 91

 $P(\text{humidity} = 74 \mid \text{play} = \text{yes}) = \frac{1}{\sqrt{2\pi} (10.2)} e^{-\frac{(74-79.1)^2}{2(10.2)^2}} = 0.0344$

 $P(\text{humidity} = 74 \mid \text{play} = \text{no}) = \frac{1}{\sqrt{2\pi}(9.7)} e^{-\frac{(74.862)^2}{2(9.7)^2}} = 0.0187$

Mean

79.1

86.2

Standard deviation

Normal distribution

StDev

10.2

9.7

More example NB

Outlook	Temp	Humidity	Windy	Play
Rainy Cool		High	True	?

Rainy Cool High True ?
$$P(Yes \mid X) = P(Rainy \mid Yes) \times P(Cool \mid Yes) \times P(High \mid Yes) \times P(True \mid Yes) \times P(Yes)$$

 $P(Yes \mid X) = 2/9 \times 3/9 \times 3/9 \times 3/9 \times 9/14 = (0.00529)$

$$P(No \mid X) = P(Rainy \mid No) \times P(Cool \mid No) \times P(High \mid No) \times P(True \mid No) \times P(No)$$

$$P(No \mid X) = 3/5 \times 1/5 \times 4/5 \times 3/5 \times 5/14 = 0.02057$$

$$0.8 = \frac{0.02057}{0.02057 + 0.00529}$$

Example of Naïve Bayes Classifier

Name	Give Birth	Can Fly	Live in Water	Have Legs	Class
human	yes	no	no	yes	mammals
python	no	no	no	no	non-mammals
salmon	no	no	yes	no	non-mammals
whale	yes	no	yes	no	mammals
frog	no	no	sometimes	yes	non-mammals
komodo	no	no	no	yes	non-mammals
bat	yes	yes	no	yes	mammals
pigeon	no	yes	no	yes	non-mammals
cat	yes	no	no	yes	mammals
leopard shark	yes	no	yes	no	non-mammals
turtle	no	no	sometimes	yes	non-mammals
penguin	no	no	sometimes	yes	non-mammals
porcupine	yes	no	no	yes	mammals
eel	no	no	yes	no	non-mammals
salamander	no	no	sometimes	yes	non-mammals
gila monster	no	no	no	yes	non-mammals
platypus	no	no	no	yes	mammals
owl	no	yes	no	yes	non-mammals
dolphin	yes	no	yes	no	mammals
eagle	no	yes	no	yes	non-mammals

A: attributes

M: mammals

N: non-mammals

$$P(A|M) = \frac{6}{7} \times \frac{6}{7} \times \frac{2}{7} \times \frac{2}{7} = 0.06$$

$$P(A|N) = \frac{1}{13} \times \frac{10}{13} \times \frac{3}{13} \times \frac{4}{13} = 0.0042$$

$$P(A|M)P(M) = 0.06 \times \frac{7}{20} = 0.021$$

$$P(A|N)P(N) = 0.004 \times \frac{13}{20} = 0.0027$$

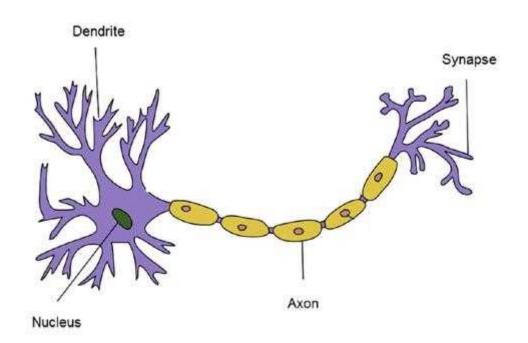
P(A|M)P(M) > P(A|N)P(N)

=> Mammals

Give Birth	Can Fly	Live in Water	Have Legs	Class
yes	no	yes	no	?



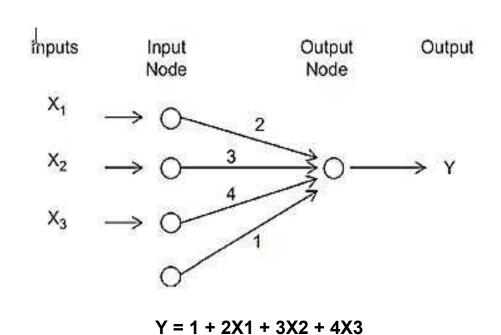
Neurons – biological neuron



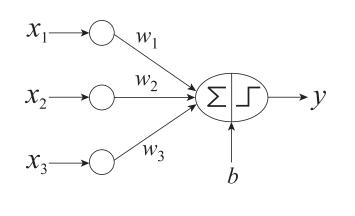
Artificial Neural Networks (ANN)

- Basic Idea: A complex non-linear function can be learned as a composition of simple processing units
- ANN is a collection of simple processing units (nodes) that are connected by directed links (edges)
 - Every node receives signals from incoming edges, performs computations, and transmits signals to outgoing edges
 - Analogous to human brain where nodes are neurons and signals are electrical impulses
 - Weight of an edge determines the strength of connection between the nodes
- Simplest ANN: Perceptron (single neuron)

Model



Basic Architecture of Perceptron

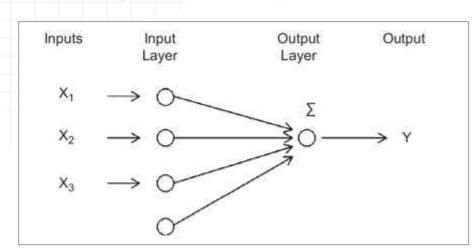


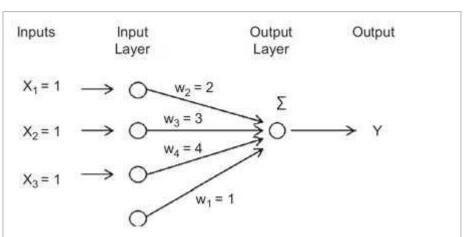
$$y = \begin{cases} 1, & \text{if } \mathbf{w}^T \mathbf{x} + b > 0. \\ -1, & \text{otherwise.} \end{cases}$$

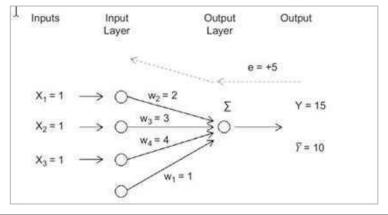
$$\begin{split} \tilde{\mathbf{w}} &= (\mathbf{w}^T \ b)^T \qquad \tilde{\mathbf{x}} = (\mathbf{x}^T \ 1)^T \\ & \qquad \hat{y} = sign(\tilde{\mathbf{w}}^T \tilde{\mathbf{x}}) \\ & \qquad \uparrow \\ & \qquad \text{Activation Function} \end{split}$$

- Learns linear decision boundaries
- Related to logistic regression (activation function is sign instead of sigmoid)

Compute output

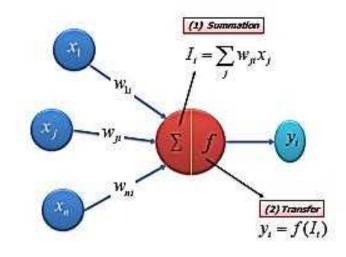




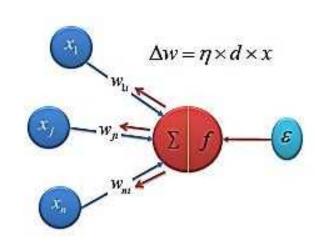


NN training

Feedforward Input Data



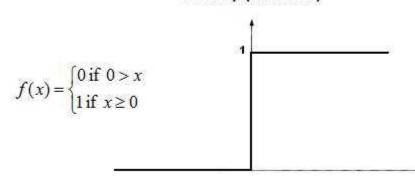
Backward Error Propagation



 $W_{ij} = W_{ij} + delta$

Activation functions

Unit step (threshold)

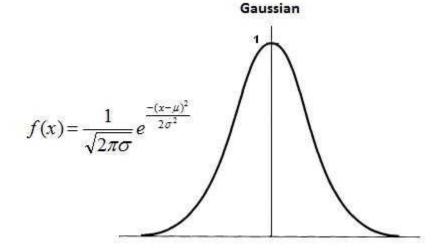


$f(x) = \frac{1}{1 + e^{-\beta x}}$

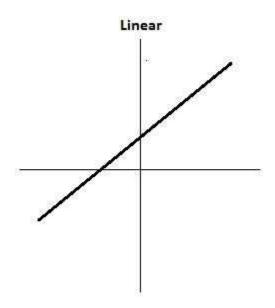
Sigmoid

Piecewise Linear

$$f(x) = \begin{cases} 0 & \text{if } x \le x_{\min} \\ mx + b & \text{if } x_{\max} > x > x_{\min} \\ 1 & \text{if } x \ge x_{\max} \end{cases}$$



Linear activation function



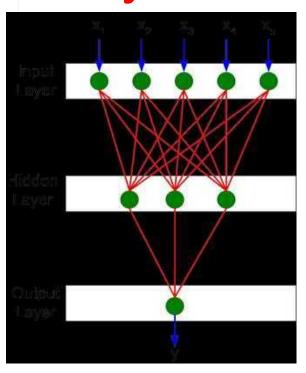
NN training algorithm

- Initialize the weights (w₀, w₁, ..., w_d) randomly
- Repeat
 - For each training example (x_i, y_i)
 - Compute \widehat{y}_i
 - Update the weights:

$$w_i^{(k+1)} = w_i^{(k)} + \lambda (y_i - \hat{y}_i^{(k)}) x_{ij}$$

- Until stopping condition is met
- k: iteration number; λ : learning rate

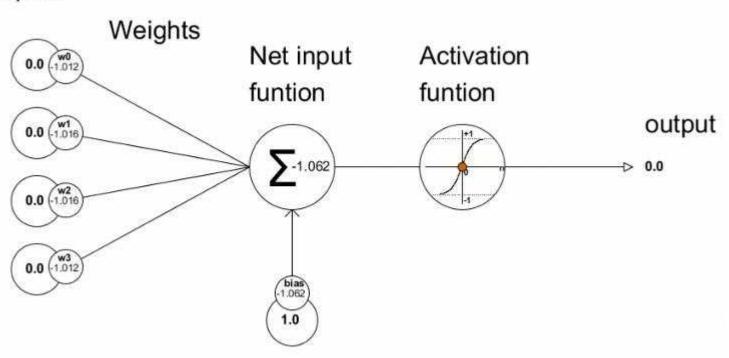
Multi-layer Neural Network



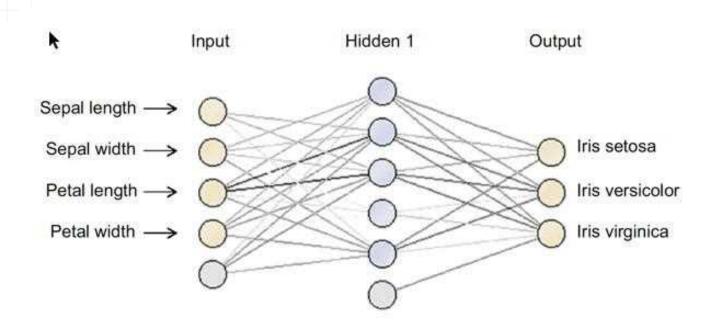
- More than one hidden layer of computing nodes
- Every node in a hidden layer operates on activations from preceding layer and transmits activations forward to nodes of next layer
- Also referred to as "feedforward neural networks"

NN training

Inputs

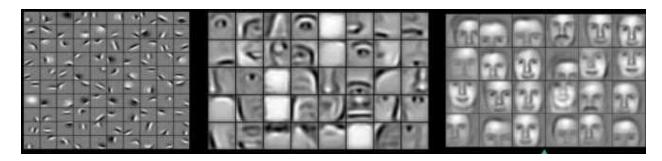


Multilayer NN for Iris dataset



Why Multiple Hidden Layers?

- Activations at hidden layers can be viewed as features extracted as functions of inputs
- Every hidden layer represents a level of abstraction
 - Complex features are compositions of simpler features



- Number of layers is known as depth of ANN
 - Deeper networks express complex hierarchy of features

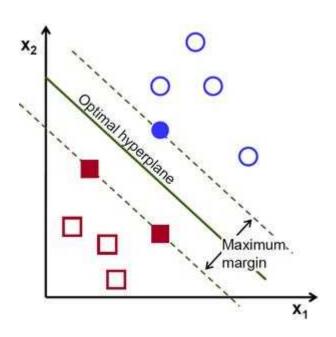
Design Issues in ANN

- Number of nodes in input layer
 - One input node per binary/continuous attribute
 - k or log₂ k nodes for each categorical attribute with k values
- Number of nodes in output layer
 - One output for binary class problem
 - k or log₂ k nodes for k-class problem
- Number of hidden layers and nodes per layer
- Initial weights and biases
- Learning rate, max. number of epochs, mini-batch size for mini-batch SGD, ...

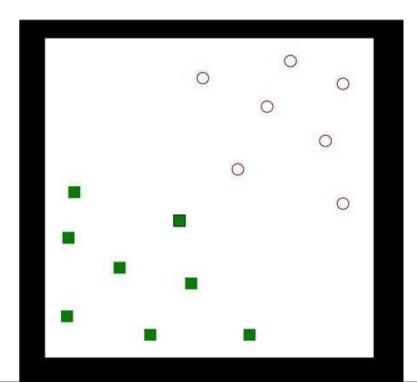
Deep Learning Trends

- Training **deep** neural networks (more than 5-10 layers) could only be possible in recent times with:
 - Faster computing resources (GPU)
 - Larger labeled training sets
- Algorithmic Improvements in Deep Learning
 - Responsive activation functions (e.g., RELU)
 - Regularization (e.g., Dropout)
 - Supervised pre-training
 - Unsupervised pre-training (auto-encoders)
- Specialized ANN Architectures:
 - Convolutional Neural Networks (for image data)
 - Recurrent Neural Networks (for sequence data)
- Generative Models: Generative Adversarial Networks GANs

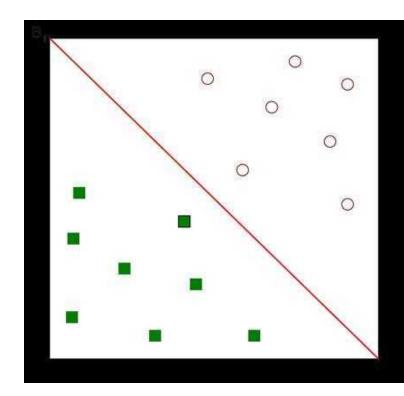
SUPPORT VECTOR MACHINES



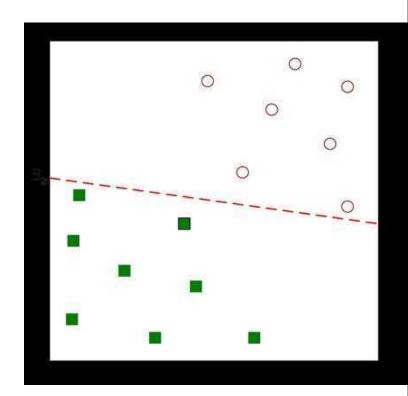
Find a linear hyperplane (decision boundary) that will separate the data



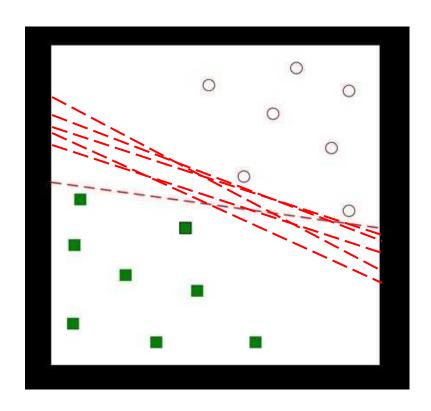
One Possible Solution



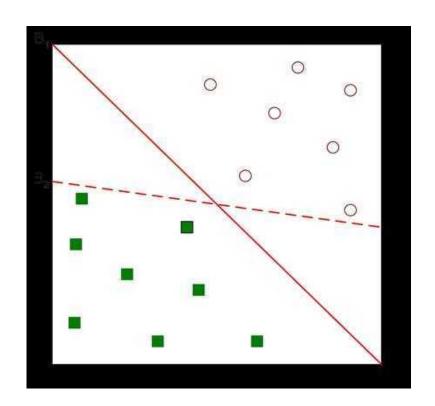
Another possible solution



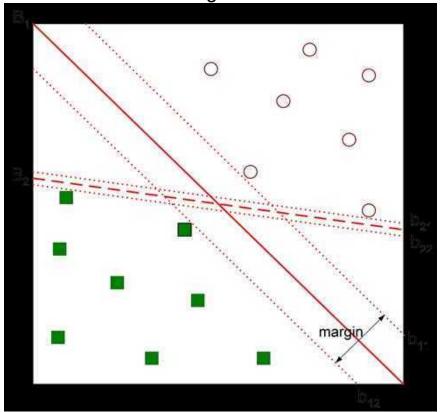
Other possible solutions

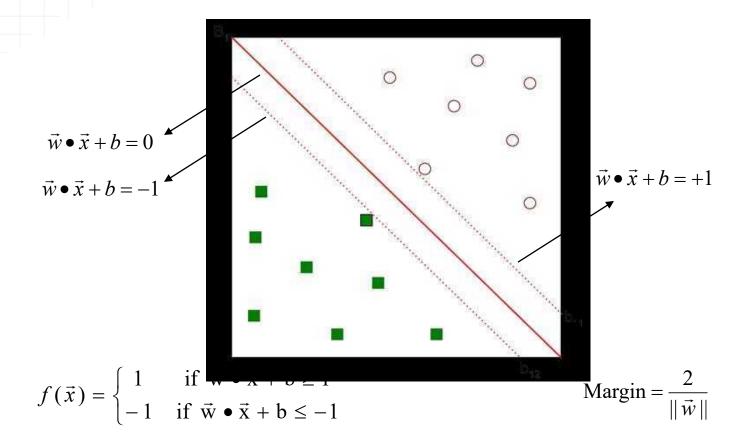


- Which one is better? B1 or B2?
- How do you define better?



Find hyperplane maximizes the margin => B1 is better than B2





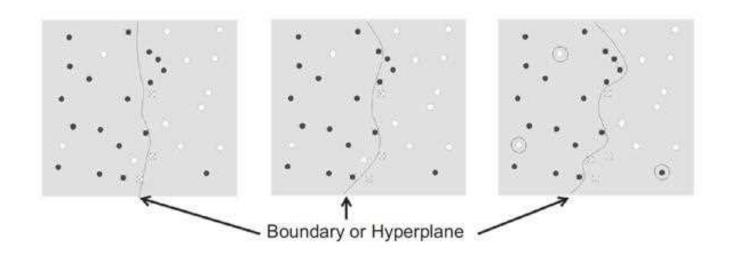
Linear SVM

Linear model:

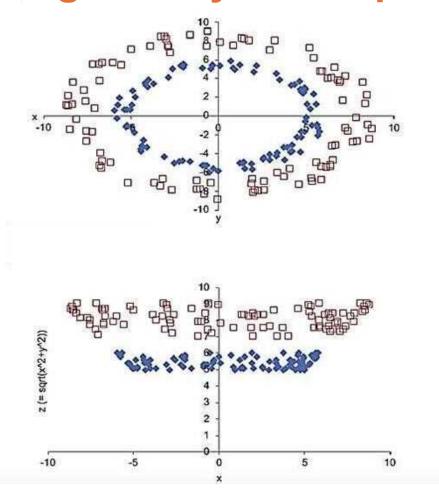
$$f(\vec{x}) = \begin{cases} 1 & \text{if } \vec{w} \cdot \vec{x} + b \ge 1 \\ -1 & \text{if } \vec{w} \cdot \vec{x} + b \le -1 \end{cases}$$

- Learning the model is equivalent to determining the values of \vec{w} and b
 - Find from training data

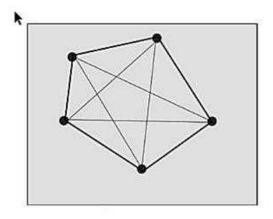
Boundary

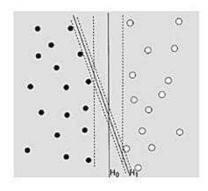


Transforming linearly non-separable data

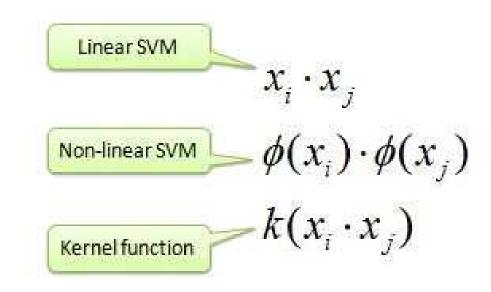


Optimal hyperplane





SVM kernels





Ensemble model

Wisdom of the Crowd

Meta learners = sum of several base models

Reduces the model generalization error

Ensemble models

