Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with $m = 45$ minutes and $s = 8$ minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

   A. 0.3875
   B. 0.2676
   C. 0.5
   D. 0.6987

   Ans

Standard Normal Variable $Z=(x-\mu)/\sigma = (x-45)/8$

Thus the question can be answered by using the normal table to find

$Pr(X \leq 50) = Pr(Z \leq (50 -45)/8.0) = Pr(Z \leq 0.625)=73.4\%$

Probability that the service manager will not meet his demand will be = 100-73.4 = 26.6% or 0.2676

B = 0.2676

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean $m = 38$ and Standard deviation $s = 6$. For each statement below, please specify True/False. If false, briefly explain why.

   A. More employees at the processing center are older than 44 than between 38 and 44.

   Ans

   False because prob(38<x<44) > prob(x>44)

   prob(x>44) = 0.1586

   prob(38<x<44) = 0.3414

   B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans

True because prob(x<30) = 9.12% , and 9.125 0f 400 = 36.48 which is around 36.

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between $2X_1$ and $X_1 + X_2$? Discuss both their distributions and parameters.

   Ans

   For 2X1,

   = N((2mu), (2sigma)^2) = N(2(mu), 4(sigma)^2)

   For X1+X2,

   = N((mu), (sigma)^2) + N((mu), (sigma)^2) = N(2(mu), 2(sigma)^2)

   For (2X1 - (X1+X2)),

   =~N(4(mu), 6(sigma)^2)

4. Let $X \sim N(100, 20^2)$. Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

    A. 90.5, 105.9
    B. 80.2, 119.8
    C. 22, 78
    D. 48.5, 151.5
    E. 90.1, 109.9

Ans

So since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.
By finding the Standard Normal Variable Z (Z Value), we can calculate the X values

$$Z = (X - m)/ s$$

For Probability 0.005 the Z Value is -2.57 (from Z Table).

$Z * \sigma + \mu = X$

$Z(-0.005)*20+100 = -(-2.57)*20+100 = 151.4$

$Z(+0.005)*20+100 = (-2.57)*20+100 = 48.6$

D = 48.5, 151.5

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
    A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
    B. Specify the 5^{th} percentile of profit (in Rupees) for the company
    C. Which of the two divisions has a larger probability of making a loss in a given year?