

## SICP Exercise 1.19

### Derivation of $p'$ and $q'$

First transformation:

$$a_{n+1} \leftarrow q(a_n + b_n) + pa_n, \quad b_{n+1} \leftarrow pb_n + qa_n, \quad (1)$$

second transformation:

$$a_{n+2} \leftarrow q(a_{n+1} + b_{n+1}) + pa_{n+1}, \quad b_{n+2} \leftarrow pb_{n+1} + qa_{n+1}, \quad (2)$$

then substituting the values from the first transformation into the  $b_{n+2}$  transformation in equation 2, we can see that:

$$b_{n+2} \leftarrow b_n(p^2 + q^2) + a_n(2pq + q^2), \quad (3)$$

and comparing with transform 1 it can be seen that

$$p' = p^2 + q^2, \quad (4)$$

and

$$q' = 2pq + q^2. \quad (5)$$