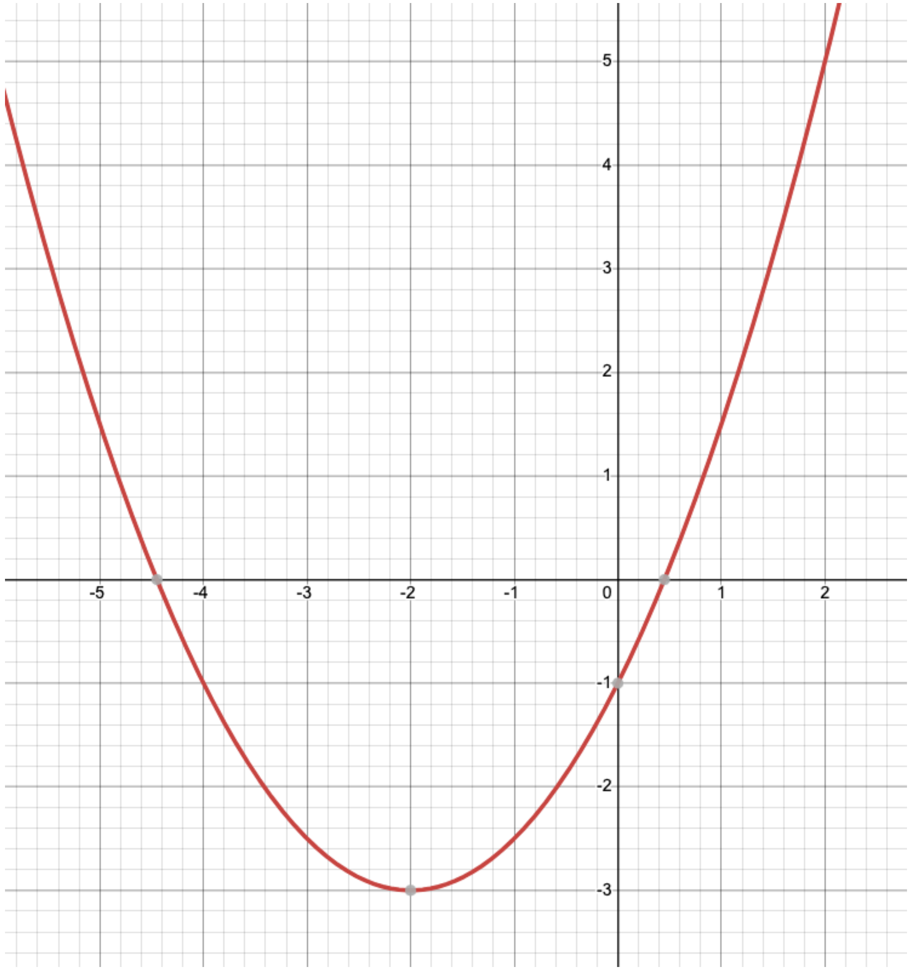
Quadratic Functions

Modeling with quadratics, Features, Important Concepts

Multiple Representations

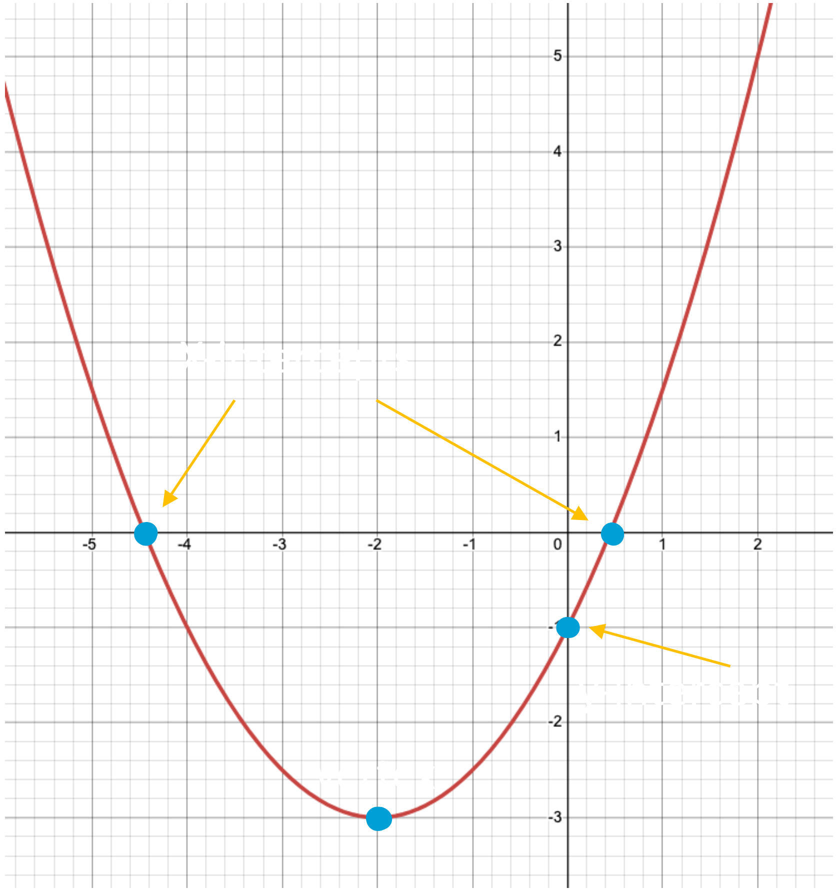


x	y
-5	
-4	
-3	
-2	
-1	
0	
1	
2	

- Equation:

$$f(x) = \frac{1}{2}x^2 + 2x - 1$$

Important Features of Quadratic Functions



X-intercepts can be determined by looking at the graph; they can also be calculated by solving the equation $\frac{1}{2}x^2 + 2x - 1 = 0$

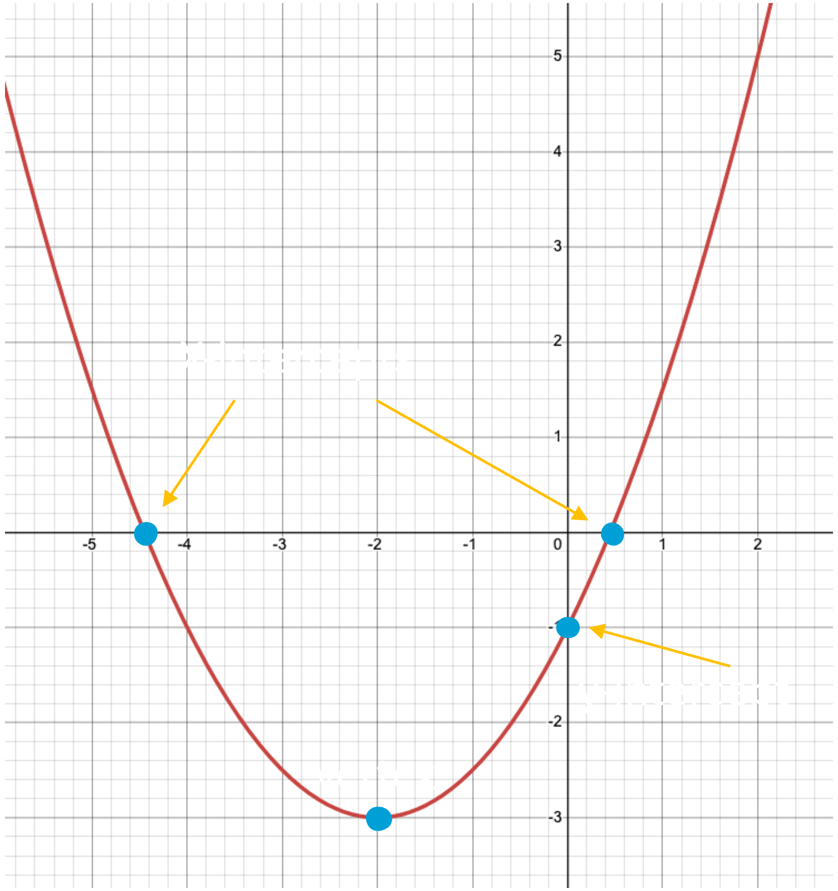
Y-intercept is just f(0)

- Equation in standard form:

$$f(x) = \frac{1}{2}x^2 + 2x - 1$$

Where $a = \frac{1}{2}, b = 2, and\ c = -1$

Important Features of Quadratic Functions



The vertex will be labeled (h, k) and can be calculated using the values of $a, b, and\ c$ from the equation:

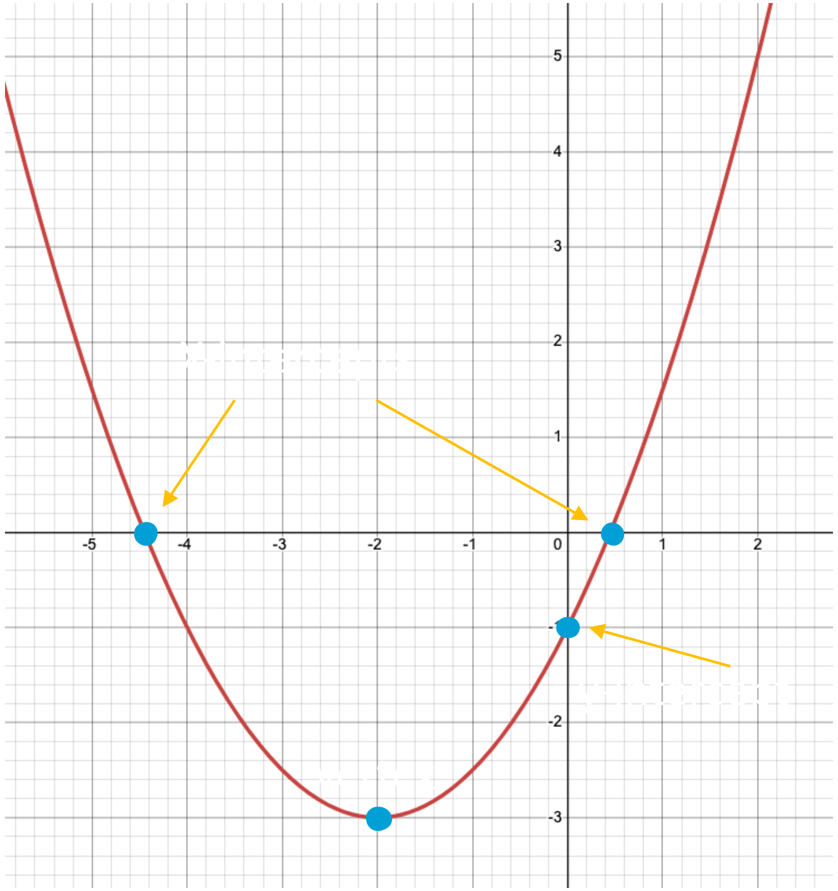
$$h = -\frac{b}{2a}, k = f(h)$$

- Equation in standard form:

$$f(x) = \frac{1}{2}x^2 + 2x - 1$$

Where $a = \frac{1}{2}, b = 2, and\ c = -1$

Important Features of Quadratic Functions



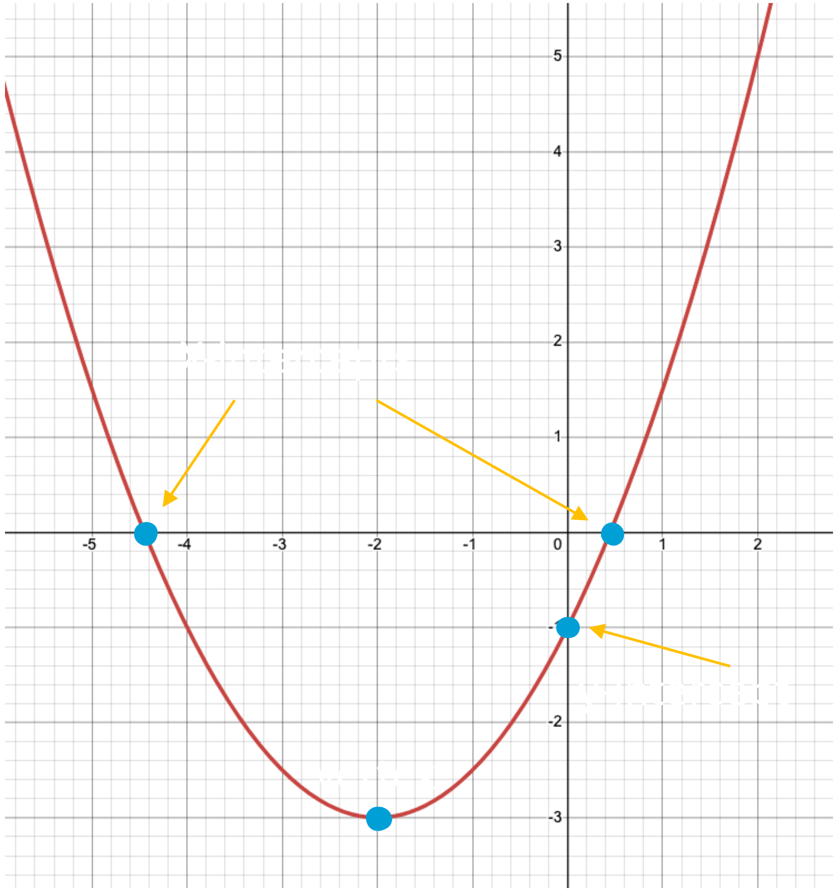
The vertex of this function is (-2,-3) which always is either the *minimum* or *maximum* of the function. Which do you think it is?

- Equation in standard form:

$$f(x) = \frac{1}{2}x^2 + 2x - 1$$

Where $a = \frac{1}{2}, b = 2, and\ c = -1$

Important Features of Quadratic Functions



The vertex (-2,-3) also allows us to rewrite the function in *vertex form*:

$$f(x) = a(x-h)^2 + k$$

Plugging in our numbers we get:

$$f(x) = \frac{1}{2}(x-(-2))^2 + (-3)$$

$$f(x) = \frac{1}{2}(x+2)^2 - 3$$

- Equation in standard form:

$f(x) = \frac{1}{2}x^2 + 2x - 1$

Where $a = \frac{1}{2}, b = 2, and\ c = -1$

Why quadratic functions matter

When we multiply two linear functions, we get a quadratic function.

Example: Revenue = Unit cost * Quantity and Unit Cost is not constant but linear

Example: Revenue = Demand * Quantity and Demand is linear

Application #1: Maximize Revenue in a Subscription

Let	Let x = the unit charge per subscription, S = the number of subscribers, and revenue $R(x) = S * x$
Assume	Assume $S = -2500x + 159{,}000$ (which means it acts like a demand function)
Show	Show that the maximum revenue will occur when x = \$31.80 and revenue amount will be \$2,528,100

Application #2: Maximize profit in sales of a new wearable fitness device

Let x = the quantity of devices sold, $p(x)$ = the price as a function of x, and revenue $R(x) = p(x) * x$. Cost must also be given.

Profit = Revenue - Cost

Assume $p(x) = -0.002x + 160$ and $C(x) = 700,000 + 30x$

What does 30 represent? What is the meaning behind -0.002?

Write an equation for Profit. Is it a quadratic?

Application #2: Maximize profit in sales of a new wearable fitness device

- So Profit = $y = -0.002x^2 + 130x - 700{,}000$
- Show that the maximum profit occurs when the selling price is $95
- Show that the profit will be $1,412,500

General Polynomials

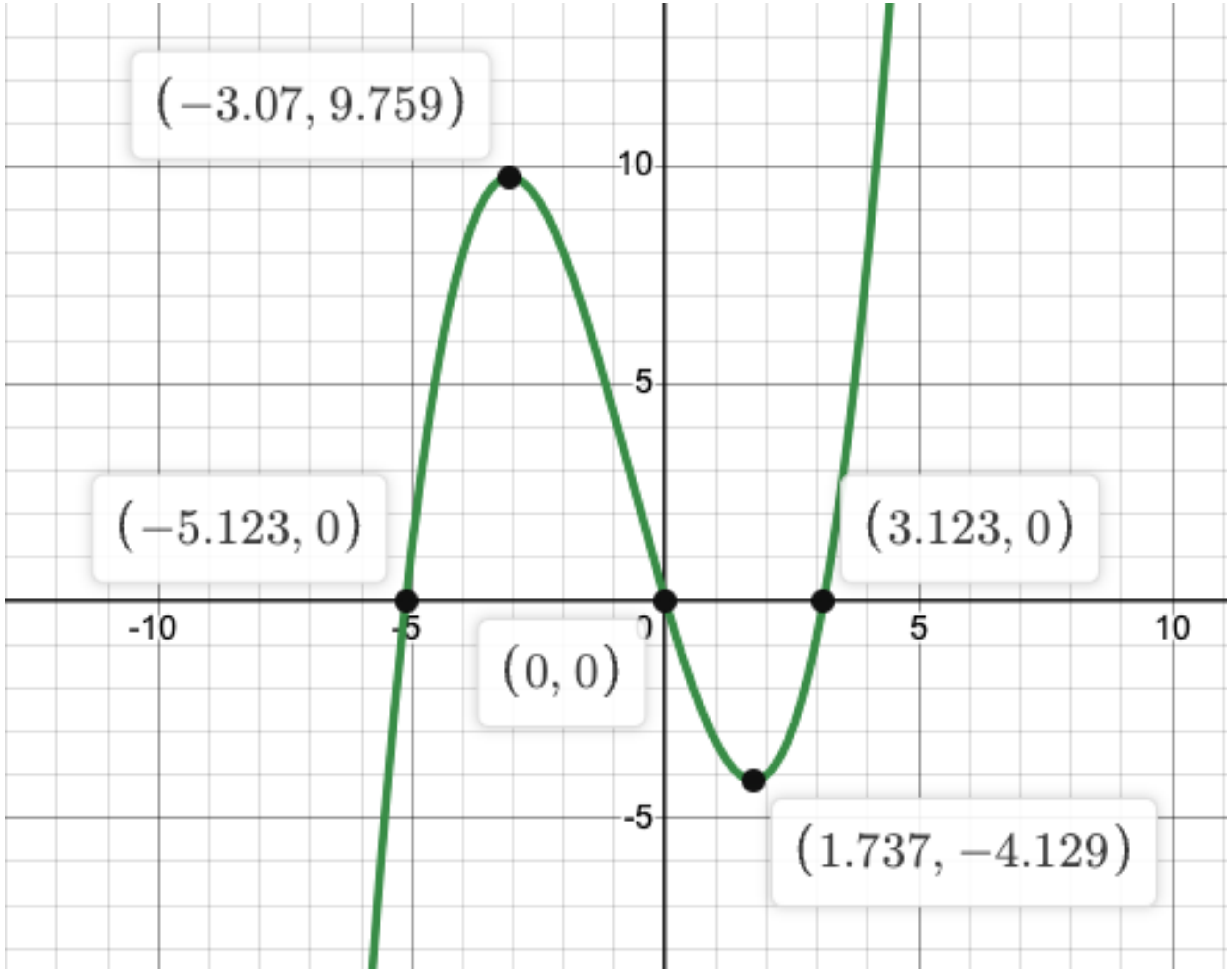
- $y = \frac{1}{4}x^3 + \frac{1}{2}x^2 - 4x$
- $y = -0.1x^4 + 3x^2$
- $y = 0.2x^4 - 3x^3 + 6x^2 + 30x$

Using a graphing tool of your choice, can you identify primary features of these three polynomials? Note: There will be more than one vertex for each of these

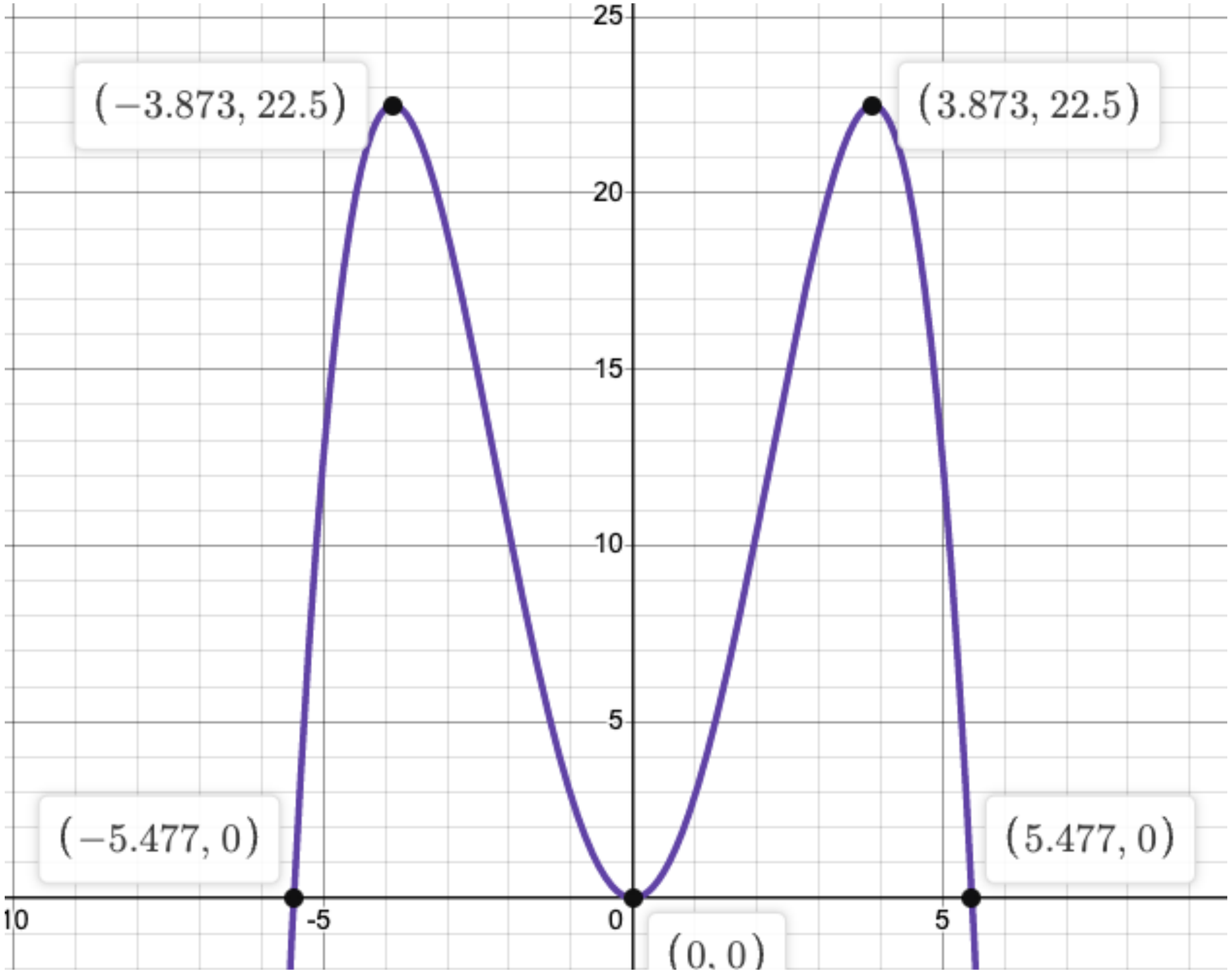
$$y = \frac{1}{4}x^3 + \frac{1}{2}x^2 - 4x$$

We have two vertices (a maximum and minimum point)

We have 3 x-intercepts
One of the x-intercepts is also the y-intercept



$$y = -0.1x^4 + 3x^2$$



We have three vertices (two maximum and one minimum point)

We have 3 x-intercepts

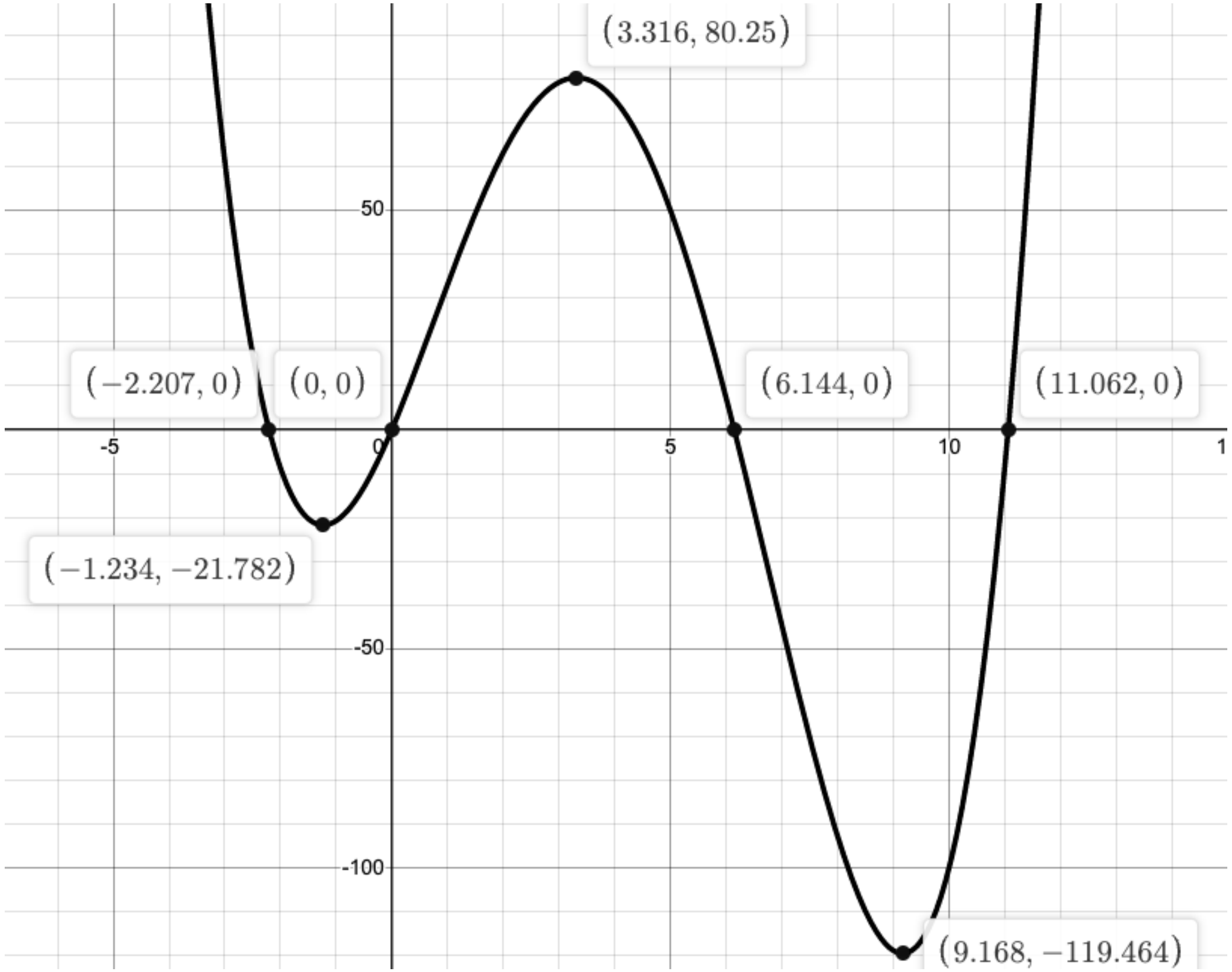
One of the x-intercepts is also the y-intercept as well as being a minimum (0,0). This polynomial also has symmetry.

$y = 0.2x^4 - 3x^3 + 6x^2 + 30x$

We have three vertices (one maximum and two minimum points)

We have 4 x-intercepts

One of the x-intercepts is also the y-intercept.



A polynomial in factored form

$$y = 0.25x(x-3)(x+5)$$

For each factor we see an x-intercept. Factors here are: $x, x-3, and\ x+5$