Review for Unit Exam - Sets and Probability

10 of the 11 questions on the test next week will be similar to these. Instead of the customary 12 questions, there will only be 11 questions. As such, each question on the exam will be worth 5 points. Your "base score" will be 45 instead of 52.

1.

Let S be the universal set, where:

$$S = \{1,2,3, \ldots ,18,19,20\}$$

Let sets A and B be subsets of S , where:

Set $A = \{2,5,13,16,17,20\}$

Set $B = \{2,5,17\}$

Find the following:

LIST the elements in the set ($A \cup B$): {2,5,13,16,17,20} (combine the two sets)

LIST the elements in the set ($A \cap B$): {2,5,17} (list those things the two sets have in common)

LIST the elements in the set A^c : {1,3,4,6,7,8,9,10,11,12,14,15,18,19} (everything in S that is NOT in A)

2.

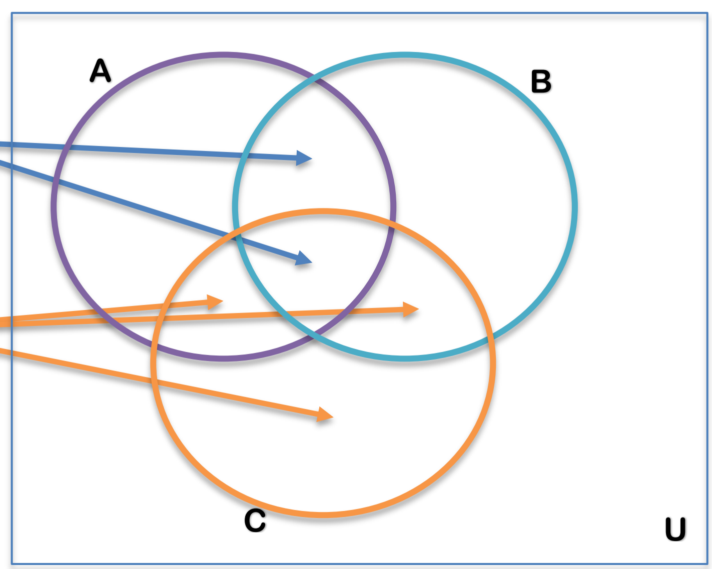
Let the Universal set U = {letters of the alphabet a through j}.

Let A = {c, g, h, j}, B = {d, f, g, h}, and C = {a, d, h, j}

List the elements of the set $(A \cap B) \cup C$:

$A \cap B$

$\cup\, C$



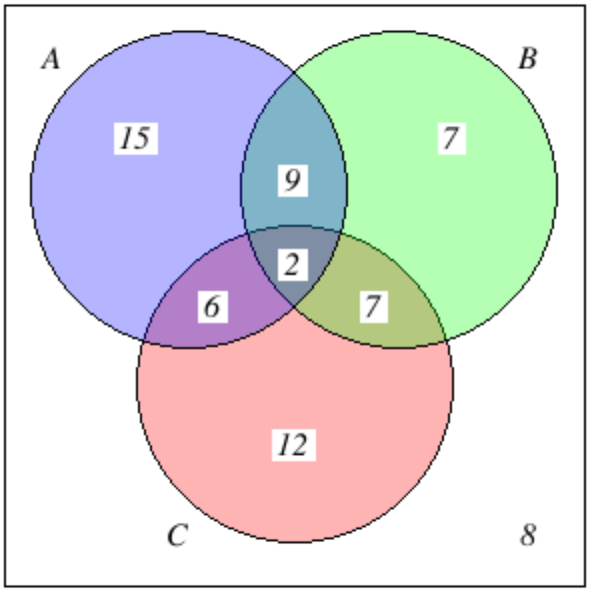
List things in C first, and then also

include things in BOTH A and B:

{g,h} U {a,d,h,j} = {a,d,g,h,j}

3.

The Venn diagram here shows the **cardinality** of each set. Use this to find the cardinality of each given set.



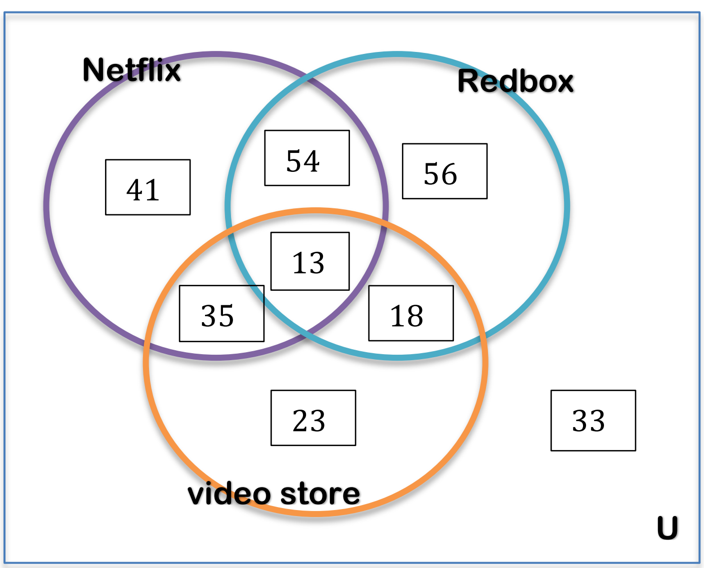
$n(A)$ = 15+9+2+6 = 32

$n(A \cap C)$ = 6+2 = 8

$n(A \cap B \cap C^c)$ = 9

4. A survey was given asking whether they watch movies at home from Netflix, Redbox, or a video store. Use the results to determine how many people use Redbox.

41 only use Netflix	56 only use Redbox
23 only use a video store	18 use only a video store and Redbox
54 use only Netflix and Redbox	35 use only a video store and Netflix
13 use all three	33 use none of these



Redbox users: 56 + 54 + 13 + 18

141

5. A group of people were asked if they had run a red light in the last year. 178 responded "yes", and 309 responded "no".

Find the probability that if a person is chosen at random, they have run a red light in the last year. You must get the total of the survey: 178 + 309 = 487

Probability of running a red light = $\frac{178}{487} = 0.366$

6. Giving a test to a group of students, the grades and sex are summarized below

	A	B	C	Total
Male	3	4	16	23
Female	11	14	5	30
Total	14	18	21	53

If one student was chosen at random, find the probability that the student was male.

Answer: Probability = $\frac{Number\ of\ males}{Total\ number\ of\ students} = \frac{23}{53} = 0.434$

7. Giving a test to a group of students, the grades and sex are summarized below

	A	B	C	Total
Male	7	17	14	38
Female	9	10	11	30
Total	16	27	25	68

If one student is chosen at random, find the probability that the student was male AND got a "A".

Answer: Probability = $\frac{Number\ of\ male\ students\ who\ got\ an\ A}{Total\ number\ of\ students} = \frac{7}{68} = 0.103$

Using "Bayes" Formula: Probability of A and B = $\frac{16}{68} \cdot \frac{7}{16}$ OR also $\frac{38}{68} \cdot \frac{7}{38}$

8. Kenneth buys a bag of cookies that contains 9 chocolate chip cookies, 4 peanut butter cookies, 6 sugar cookies and 5 oatmeal cookies.

What is the probability that Kenneth reaches in the bag and randomly selects a chocolate chip cookie from the bag, eats it, then reaches back in the bag and randomly selects a peanut butter cookie? NOTE: There are 9 + 4 + 6 + 5 = 24 cookies in the bag

Let A = selecting a chocolate chip cookie, B = selecting a peanut butter cookie

Pr(A and B) = Pr(A) * Pr(B|A) = $\frac{9}{24} \cdot \frac{4}{23} = \frac{36}{552} = 0.065$

9. Suppose a jar contains 18 red marbles and 21 blue marbles. If you reach in the jar and pull out 2 marbles at random at the same time, find the probability that both are red.

Use the same logic as with problem #8. Probability of choosing a red marble, and then another red marble: Let A = the event of selecting a red marble.

Solution: To start, there are 39 marbles in the jar.

Then Pr(A and A) = Pr(A) * Pr(A|A) = $\frac{18}{39} \cdot \frac{17}{38} = \frac{306}{1482} = 0.206$

10. A certain disease occurs in **4% of the population**. The *false negative rate is 30%* and the ***false positive rate is 1%***. (***On the test you will need to make the following table yourself***)

Part A: Make a contingency table describing the situation:

	Tests positive	Tests negative	Row Totals
Has disease	= 70% of row total 2.8	= 30% of row total 1.2	**4**
Does not have disease	*= 1% of row total* *0.96*	= 99% of row total 95.04	**96**
Column Totals	2.8 + 0.96 = 3.76	1.2 + 95.04 = 96.24	**100**

What is the probability that a person has the disease ***given that*** they test positive?

Solution: When we see the phrase "given that" we will use only numbers in that row or column. Here, we are "given that" the person tests positive, so we take the numbers from the "tests positive" column ONLY:

Pr(has disease | tests positive) = $2.8 \div 3.76 = 0.745$ which means 74.5% likelihood

What is the probability that a person has the disease ***given that*** they test negative?

Solution: We take numbers from the "Tests negative" column only:

Pr (has disease | tests negative) = $1.2 \div 96.24 = 0.0125$

11. A company estimates that 15% of their products will fail after the original warranty period but within 2 years of the purchase, with a replacement cost of $500.

If they offer a 2-year extended warranty for $72, what is the company's expected value of each warranty sold?
Note: the statement "15% of products fail within the extended warranty period" is a statement about probability of failure, hence probability that the company will need to replace the product for the customer

x	Explanation:	Pr(x)	$x \cdot \Pr(x)$
72 – 500 = - 428	Company loses $428 due to replacement during extended warranty period	0.15	- $64.20
+ 72	Company makes $72 because the product does not fail	0.85	$61.20

Expected value = - $64.20 + $61.20 = - $3.00 (in other words, a loss of $3.00) per customer on average

Follow-up question: By what amount would the expected value change if the warranty price was $82 instead of $72?

Answer: (-418)*(0.15) + (82)*(0.85) = $7.00 (company would make $7.00 in income) for a net increase of $10.00 per customer/per extended warranty sold