UNHM MATH 422 Exam #2 NAME Key
Date

PLEASE REMEMBER: Show your reasoning and/or math work in every problem. Some questions have more than one part. Make sure you answer every part of the question that was asked. You are being asked to calculate some things and interpret others.

Section 1: These questions all involve mathematical reasoning with quadratic functions (at least in part) or polynomial functions. Recall that a quadratic function in standard form is $y = a \cdot x^2 + bx + c$ whose vertex is (h, k) and $h = \frac{-b}{2a}$, $k = a \cdot h^2 + b \cdot h + c$. A quadratic in "vertex form" is given by $y = a \cdot (x - h)^2 + k$.

1. (4 pts)

Let the **unit price** for a certain sneaker be given by: $price = -0.85x + 250$, where x is the quantity of sneakers in demand and $price$ is in dollars, that can be charged when x units are sold. Recall that $Revenue = price * quantity$

A) The total revenue R(x) = $x \cdot (-0.85x + 250)$

B) The total revenue when producing/selling 106 items (to the nearest penny).

Answer: $ 16,949.40 $= 106(-0.85(106)+250)$

2. (4 pts)

Consider the parabola (quadratic) given by the equation: $f(x) = x^2 - 2x - 8$
Find the following for this parabola:

A) The vertex: $(1,-9)$ Is it a maximum? Or minimum? ~~ma~~ minimum
b/c $a=1$ (positive)

B) The vertical intercept (y-intercept) is the point $(0,-8)$

$a=1,\ b=-2,\ c=-8$
vertex $= (h,k)$ $h = -(-2) \div (2 \times 1) = 1$
$k = 1^2 - 2(1) - 8 = -9$

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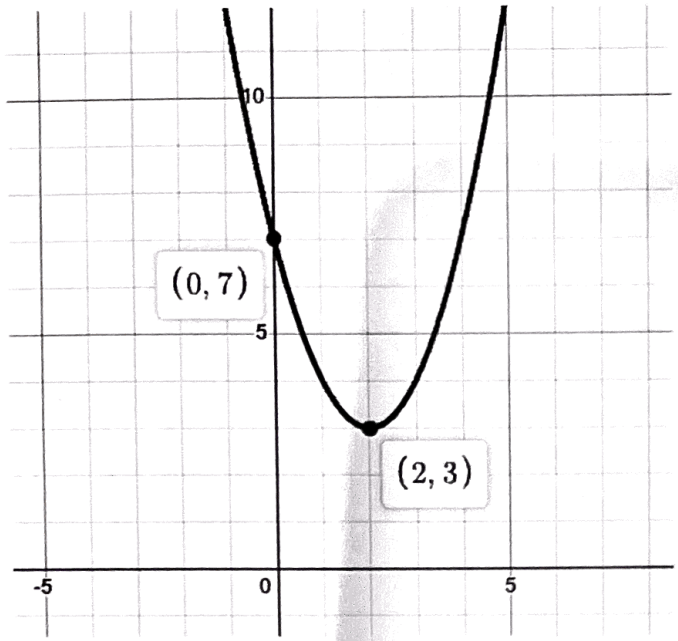
3. (4 pts)

Write an equation (any form) for the function graphed below:

f(x) = $(x-2)^2+3$



$y = a(x-h)^2+k$

$(2,3) = (h,k)$

$y = a(x-2)^2+3$

Find a by "plugging in" (0,7)

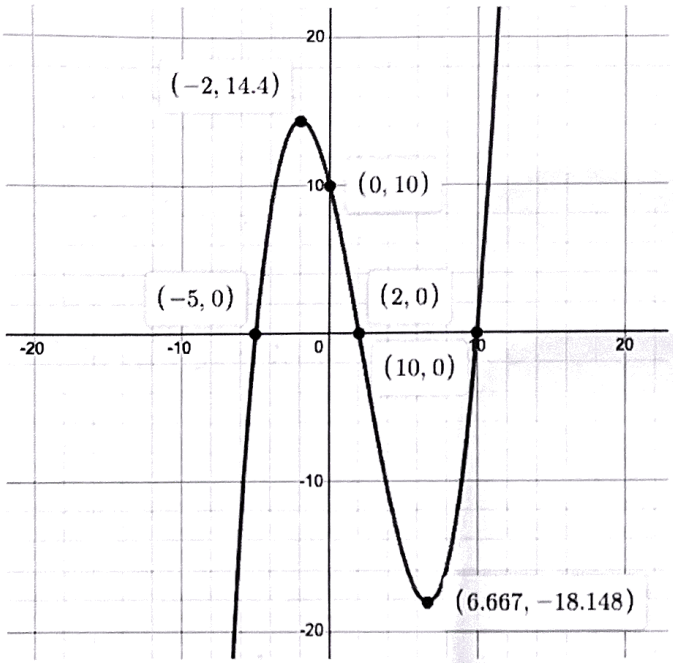
$x=0, y=7$:

$7 = a(-2)^2+3$

$4 = 4a$

$1 = a$

4. (4 pts)



A polynomial function is shown on this graph.

Are there any extrema (max/min)?

Yes ✓ No ___

If so, *name a point* that represents a **maximum** (if any): (-2, 14.4)

Name a point that represents a minimum (if any): (6.667, -18.148)

What is the y-intercept? 10 or (0, 10)

Are there any x-intercepts? If so, name it/them: (-5, 0), (2, 0), (10, 0)

Section 2: These questions all involve exponential or logarithmic growth or decay. Recall that an exponential function has the form $y = a \cdot b^x$ where $b = 1 + r$ or $b = 1 - r$

5. (4 pts)

A vehicle purchased for $29,500 **depreciates** at a rate of 12% per year. Determine the approximate value of the vehicle 13 years after purchase. Round to the nearest whole dollar. If $\boldsymbol{P}$ is the price (value) of the vehicle in $\boldsymbol{x}$ years, use the equation $\boldsymbol{P = P_0 * b^x}$

$ 5,599

Depreciation means $b = 1 - r$

$b = 1 - 0.12 = 0.88$

$P_0 = 29,500$ & $x = 13$ years

$P = 29,500 \times 0.88^{13}$

$= \$5,598.82$ ↑ round up to $5,599

6. (4 pts)

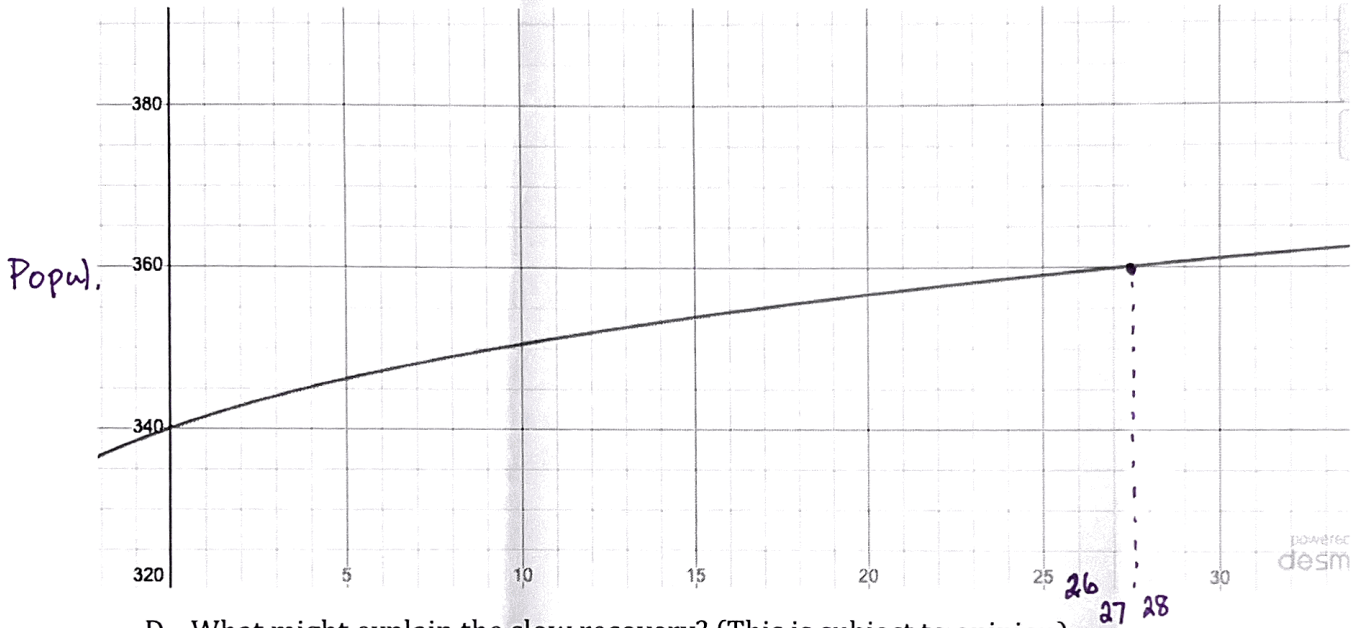
Efforts to save the North Atlantic right whale have been underway since 2008 but the population **continues to decline**. Cc: us.whales.org Suppose that increased protections happen so that the right whale population starts to make a comeback according to the function (x in years from now):

$W = 35 \cdot \log(x + 10) + 305$. (That is log base 10). Please answer the following questions:

A. In how many years would the population reach 360? 27-28 years

B. What is the population now? 340

C. In 100 years' time, what will the population be? 376 $35 \times \log(110) + 305$



D. What might explain the slow recovery? (This is subject to opinion)

Possible answers are:

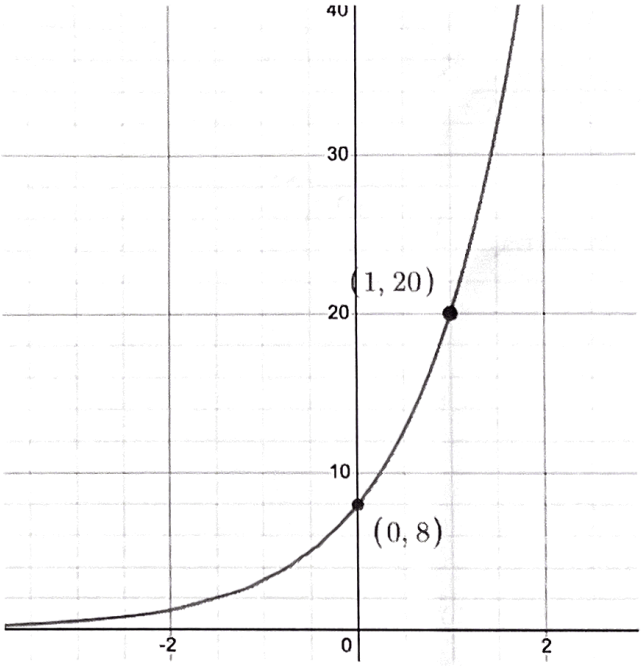
a) few fertile females (breeding actively)

b) whales have long lifespans & breed infrequently

c) at such low number (population) it will take a long time to recover

d) other risk factors may result in few calves living long enough to reproduce

7. (4 pts)



Write the equation that represents the graph of an exponential function shown here.

$y = a \cdot b^x$ ① (0,8)

$8 = a \cdot b^0$

$8 = a \cdot 1$

$8 = a$

② (1,20)

$y = 8 \cdot b^x$

$20 = 8 \cdot b^1$

$2.5 = b$

$y = 8(2.5^x)$

8. (4 pts) Please show your work!

An investment of 5,000 dollars is placed in an account for 25 years at an interest rate of 3.7% per year, **compounded continuously**. How much interest will have been earned in 25 years? (Interest earned = Future value – Initial Value)

$ 7,609.34

$A = P \cdot e^{rt}$

$A = 5000 \cdot e^{25(0.037)}$

$= 12,609.34$

$I = A - P$

$= 12609.34 - 5000$

Section 3: These questions all concern financial situations that involve simple or compound interest. Please refer to your formula sheet, select the appropriate equation for each question, and calculate your answer. For partial credit, please write (show) here the exact calculation that you made.

9. (4 pts)

You can afford a $250 per month car payment. You found a 4-year loan at 3% simple interest. How big of a loan can you afford? Note that the accumulated value (A) will be the total of all payments made. You are looking for the present value (P)

$ 10,714.28

① Step 1:

$A = 250 \times 4 \times 12$

$= 12,000$

$I = P \times r \times t$

② $A = P(1+rt)$

$A \div (1+rt) = P$

$12,000 \div (1 + .03 \times 4)$

10. (4 pts)

You deposit $2000 in an account earning 4.2% interest compounded monthly. How much will you have in the account in 18 years?

$ 4253.86

$A = P(1 + \frac{r}{k})^{kt}$ $k = 12$

$P = 2000$
$r = 0.042$
$k = 12$
$t = 18$

$A = 2000(1 + \frac{.042}{12})^{12 \cdot 18}$

$= 4253.86$

11. (4 pts)

A young executive is going to purchase a vacation property for investment purposes. The sale price is $750,000. She will make a down-payment of $221,000. The market value is likely to increase by a rate of 2.8% compounded monthly.

What is the amount she needed to mortgage? $ 529,000

What will the value of the property be in 30 years? $ 1,735,576.18

$750{,}000(1 + \frac{.028}{12})^{12 \times 30} = \$1735576.177\ldots$

12. (4 pts)

If a person wanted to save $50,000 as a down-payment on a house, and they had $10,000 to invest now, how long would it take to reach their goal at 12% interest compounded quarterly?

$k = 4$

$time = Log(\frac{A}{P}) \div [k \cdot Log(1 + \frac{r}{k})]$

$Log(\frac{50000}{10000}) \div [4 \cdot Log(1 + \frac{.12}{4})]$

$Log(5) \div [4 \cdot Log(1.03)]$

$13.612\ldots$

13.6 years