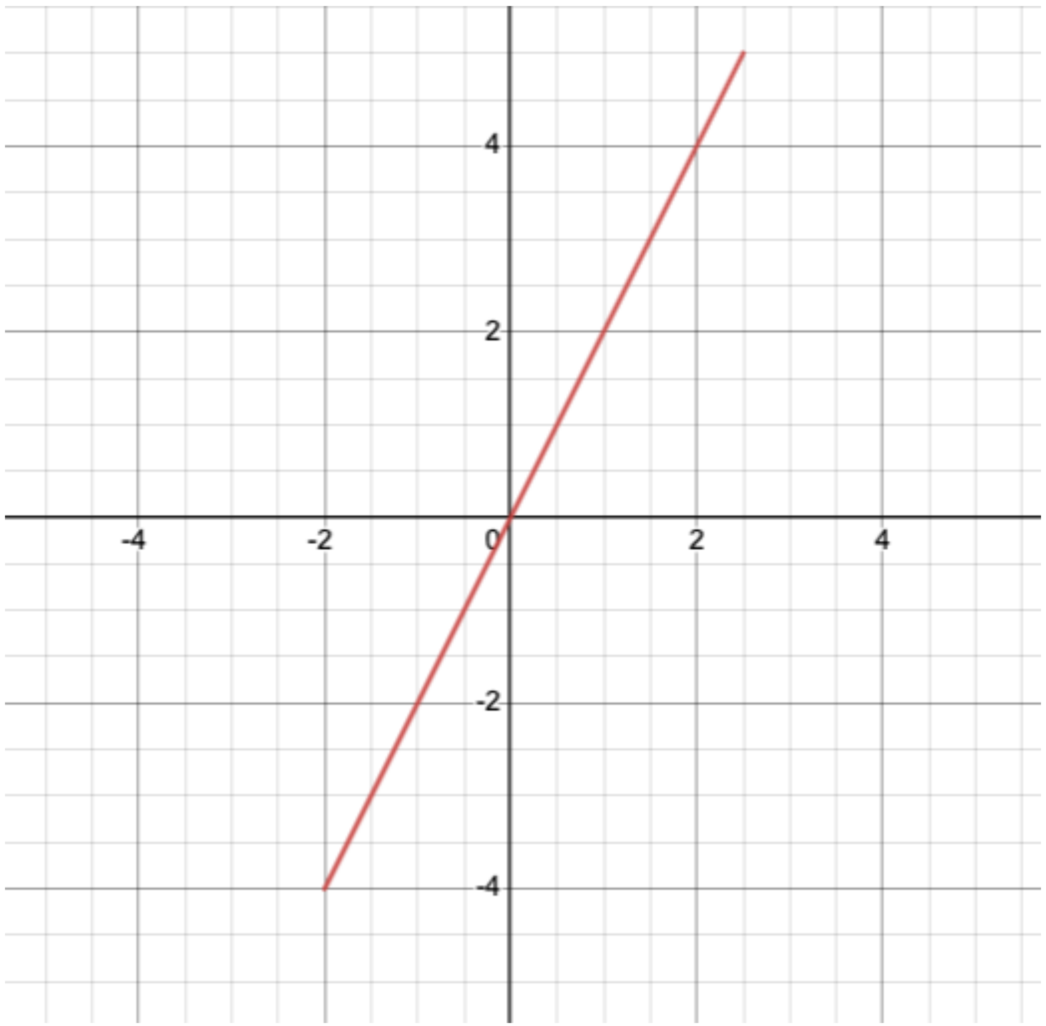
PROPERTIES, EQUATIONS, AND GRAPHS

# LINEAR FUNCTIONS AND MODELS WEEK 2

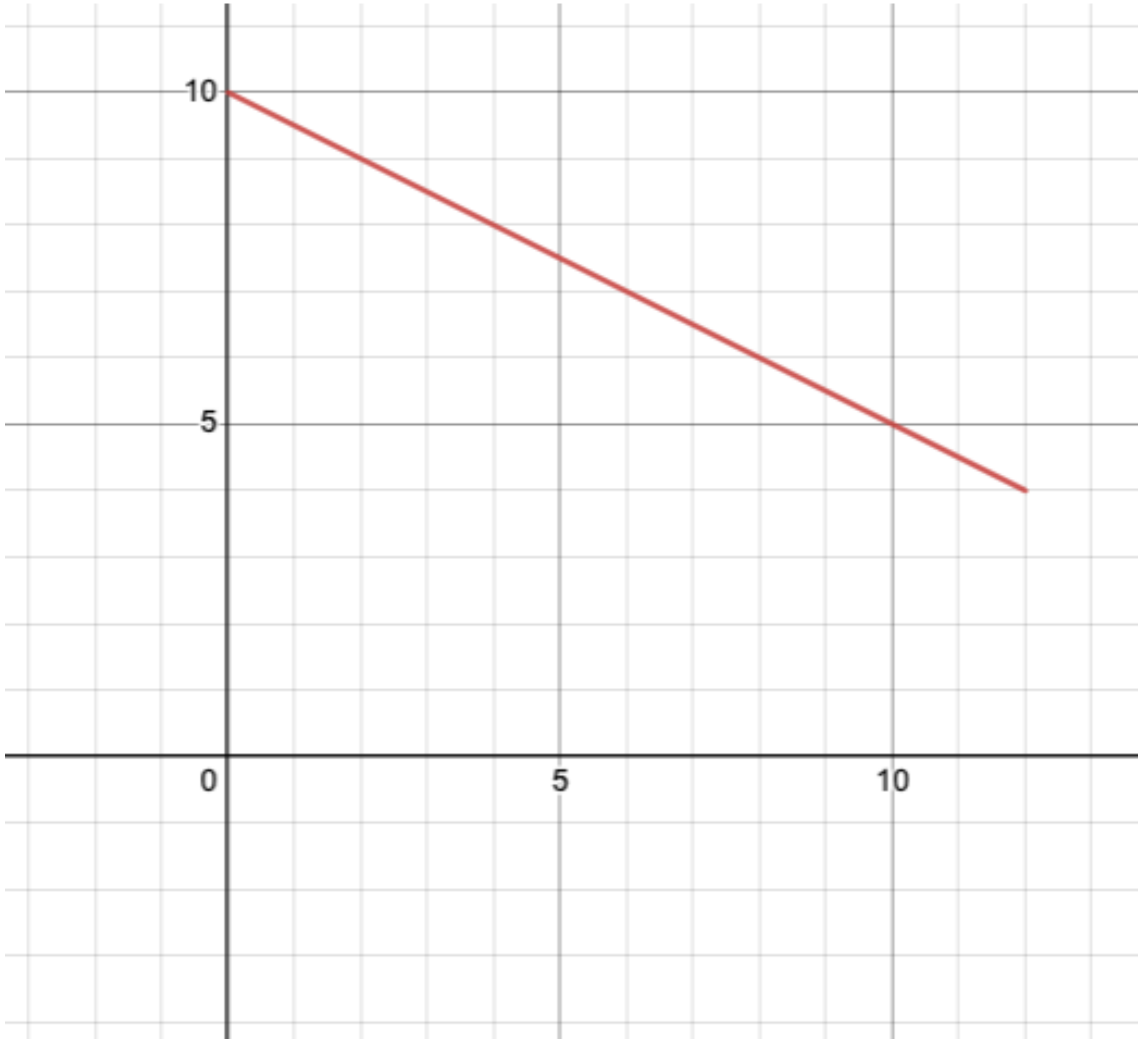
# SLOPE OF A LINE

# SLOPE IS A MEASURE OF SLANT OR ASCENT/ DESCENT




This line has POSITIVE slope. Why?

# SLOPE IS A MEASURE OF SLANT OR ASCENT/ DESCENT




This line has NEGATIVE slope. Why?

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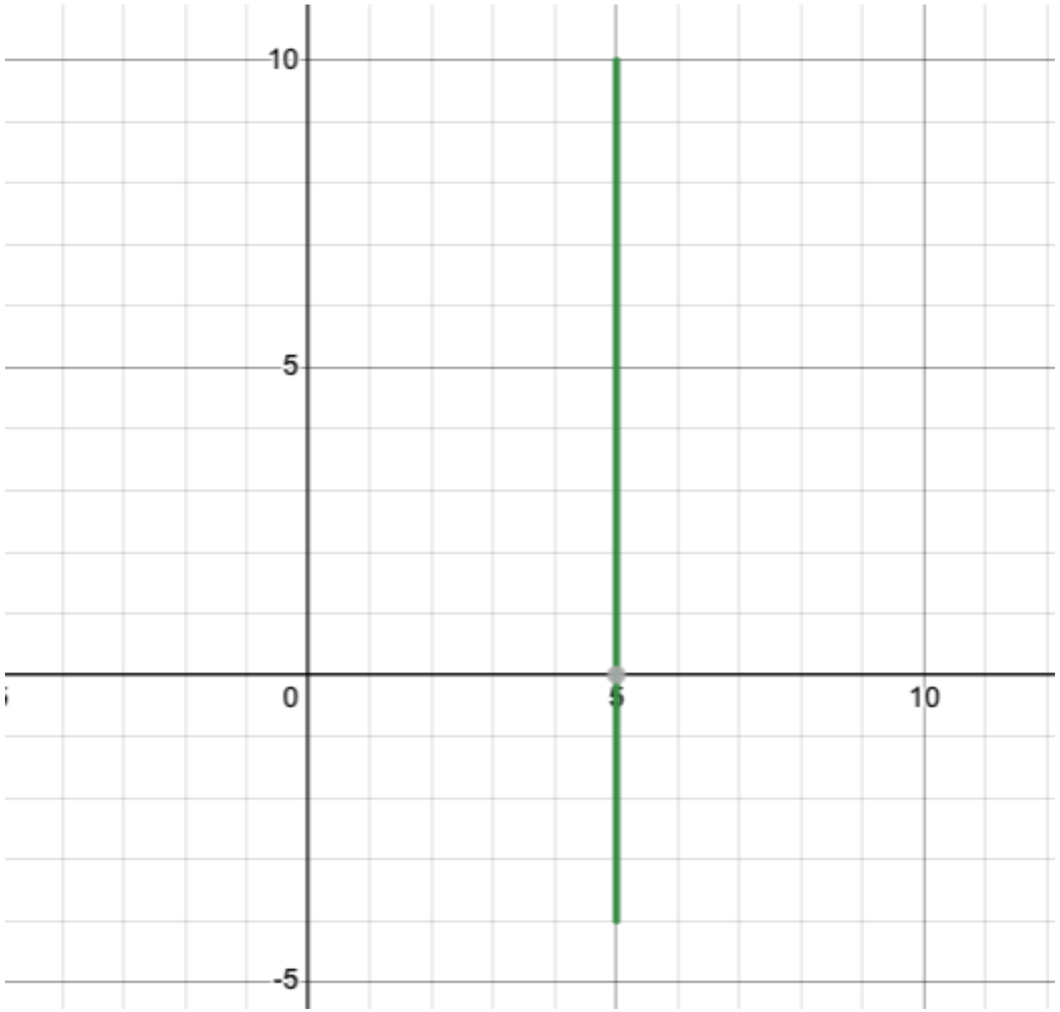
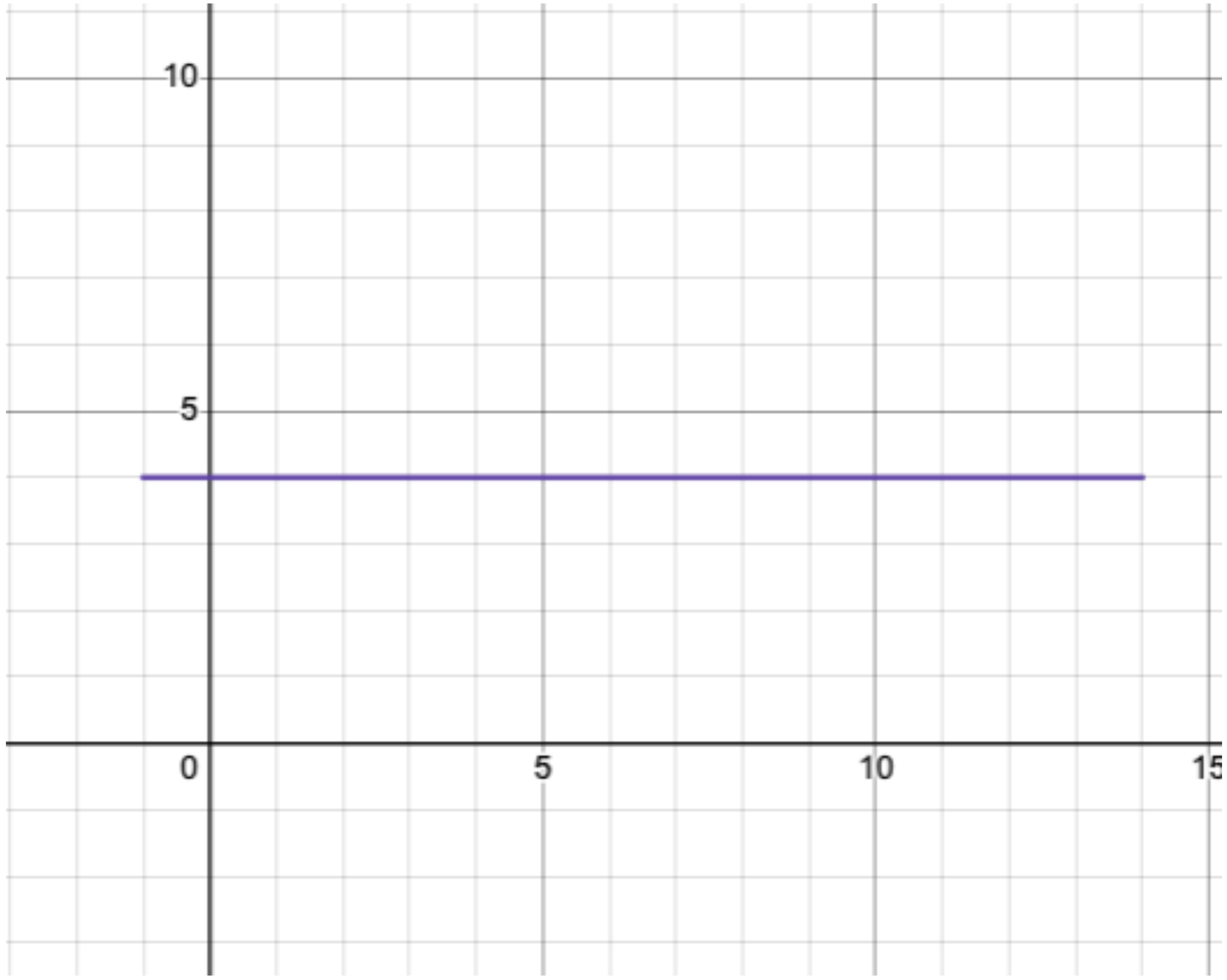



This line has NO slope (UNDEFINED). Why?

# SLOPE IS A MEASURE OF SLANT OR ASCENT/ DESCENT




This line has slope = 0 (FLAT LINE). Why?

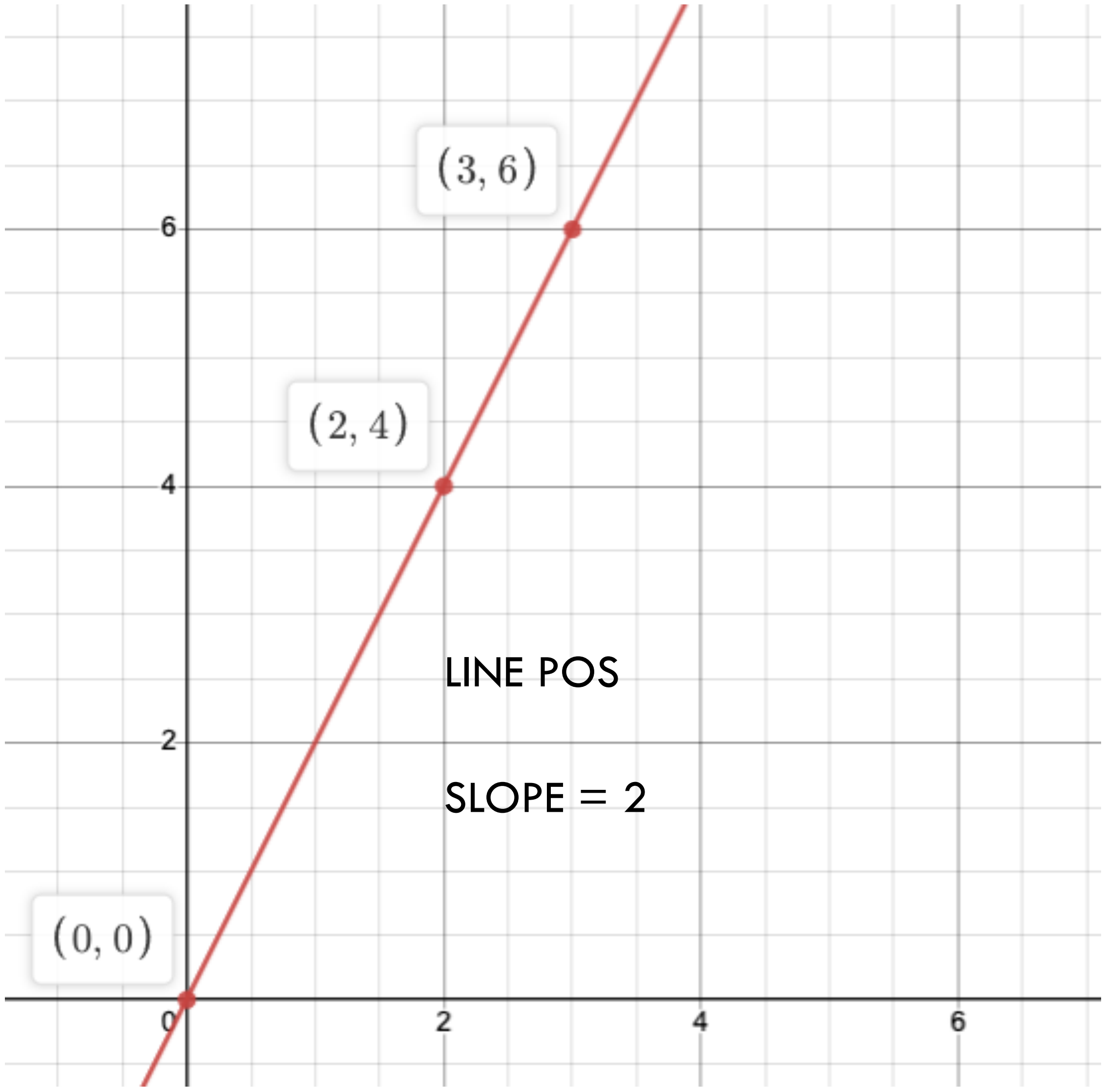
# HOW DO WE CALCULATE SLOPE?

- We need the coordinates of TWO points which we will call $(x_0, y_0)$ and $(x_1, y_1)$.
  - Why the subscripts? We use subscripts to identify, or label, something specific which is to be treated as a number (not a variable)
- Now we subtract in columns:

$$\begin{array}{rcc} & (x_1 & , y_1) \\ - & (x_0 & , y_0) \\ \hline \end{array}$$

- The result is two numbers: $\Delta x = x_1 - x_0 \quad y_1 - y_0 = \Delta y$ which measure the change in x and the corresponding change in y.
- Divide for the slope (ORDER matters!) slope $= \boldsymbol{m} = \dfrac{\Delta y}{\Delta x} = \Delta y \div \Delta x$
  - New symbols: $m$ = slope of a line, $\Delta y$ means change in y, $\Delta x$ means change in x

# CALCULATE SLOPE FROM A GRAPH AND TWO POINTS



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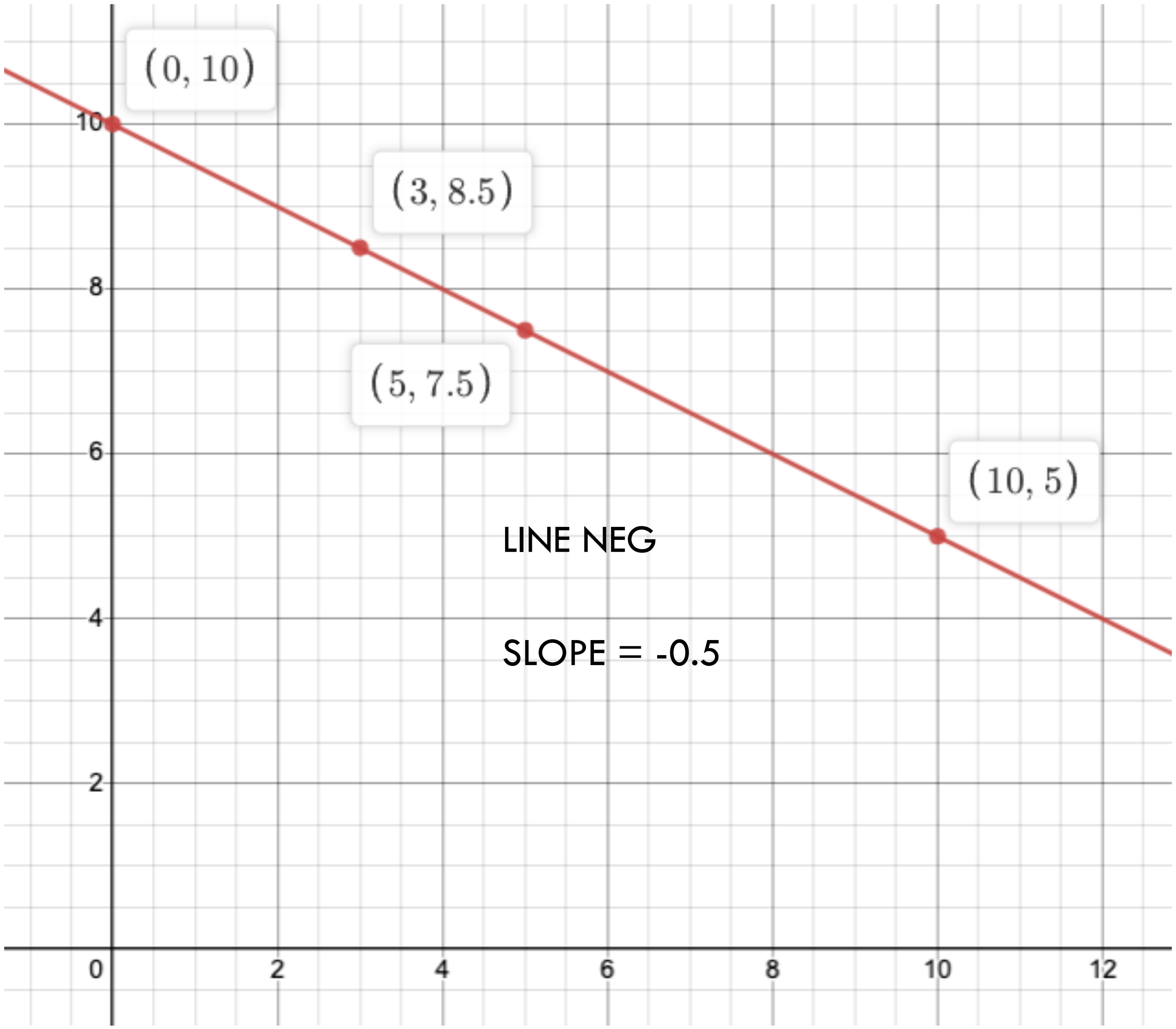

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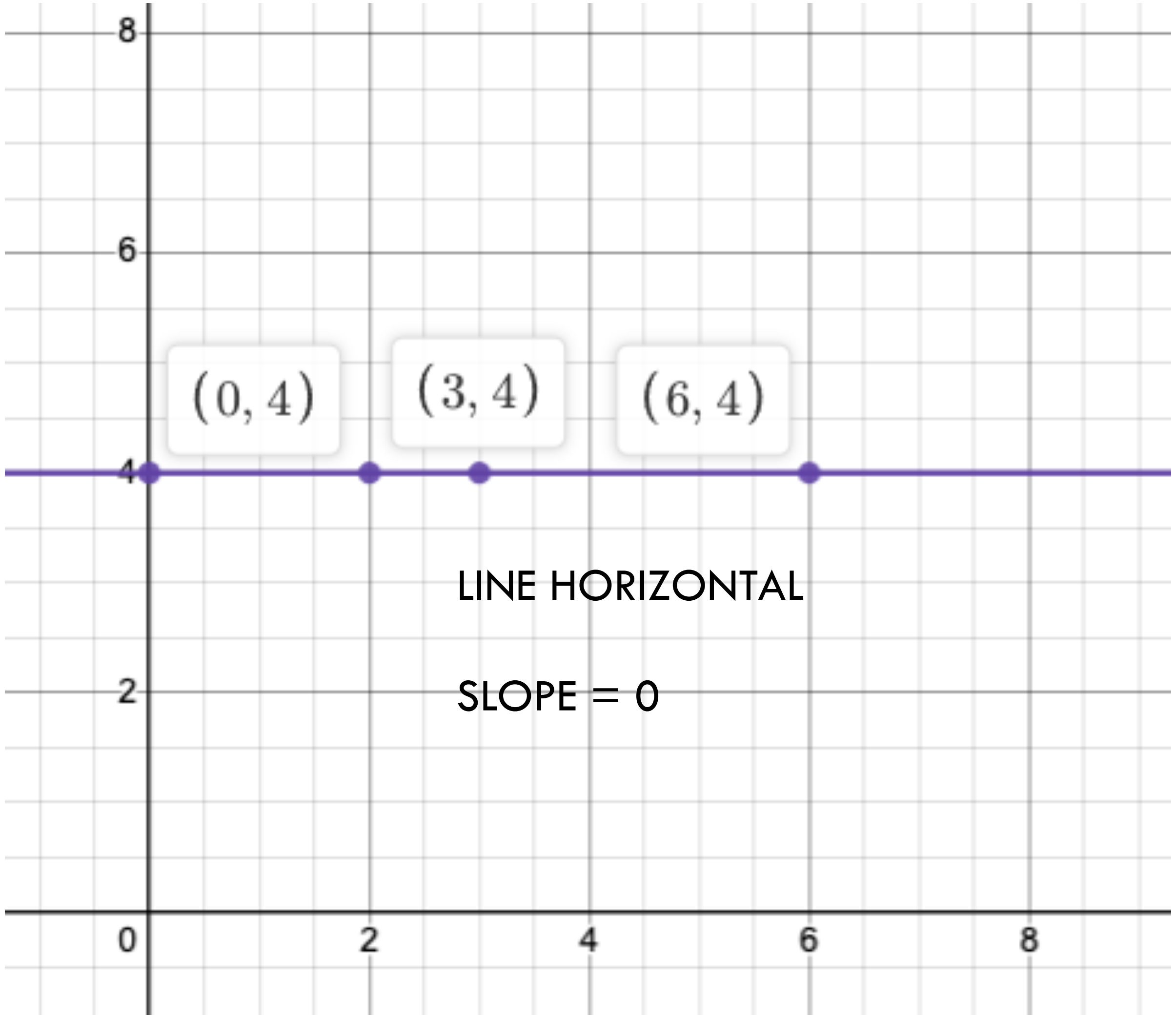

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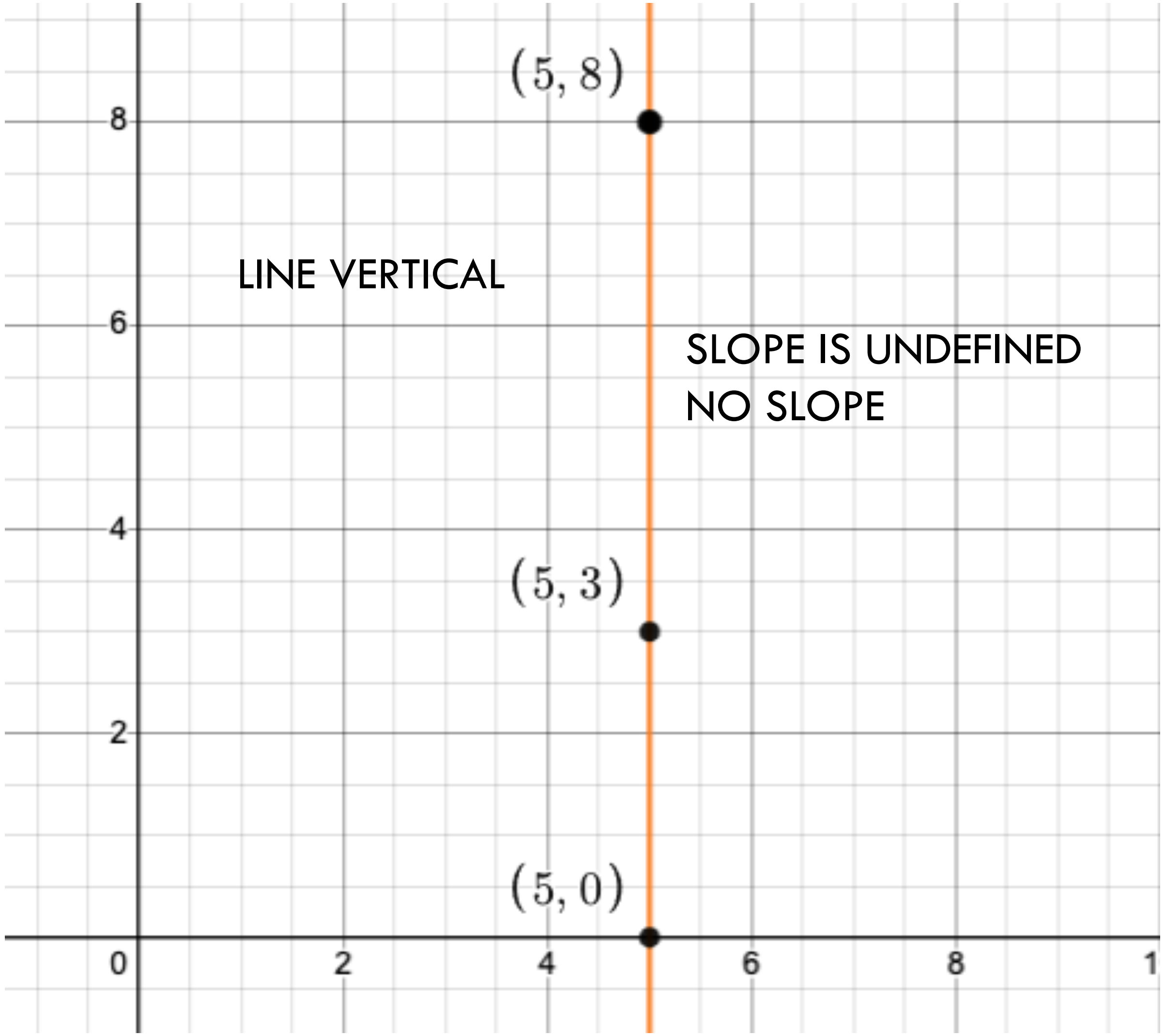
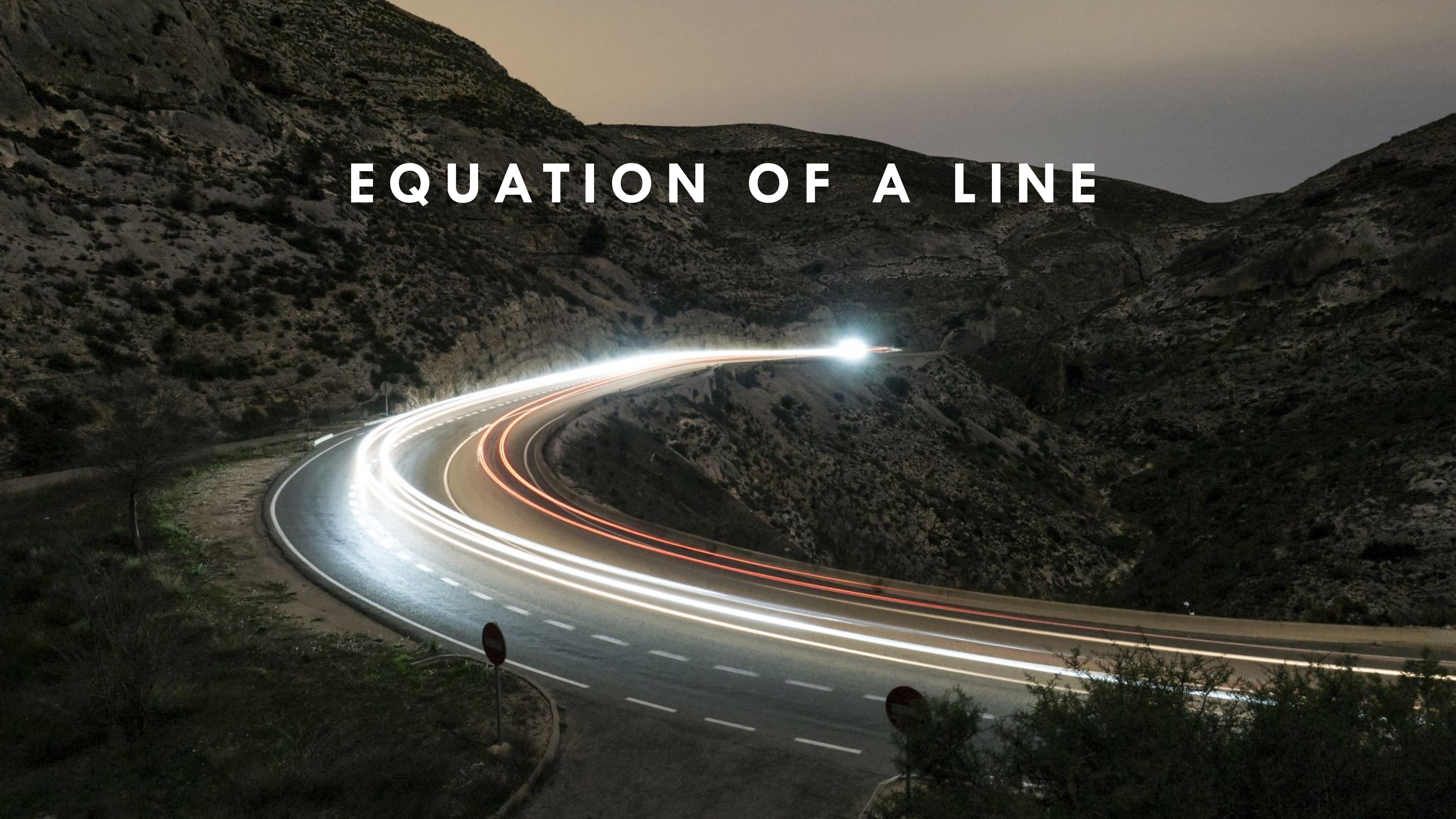


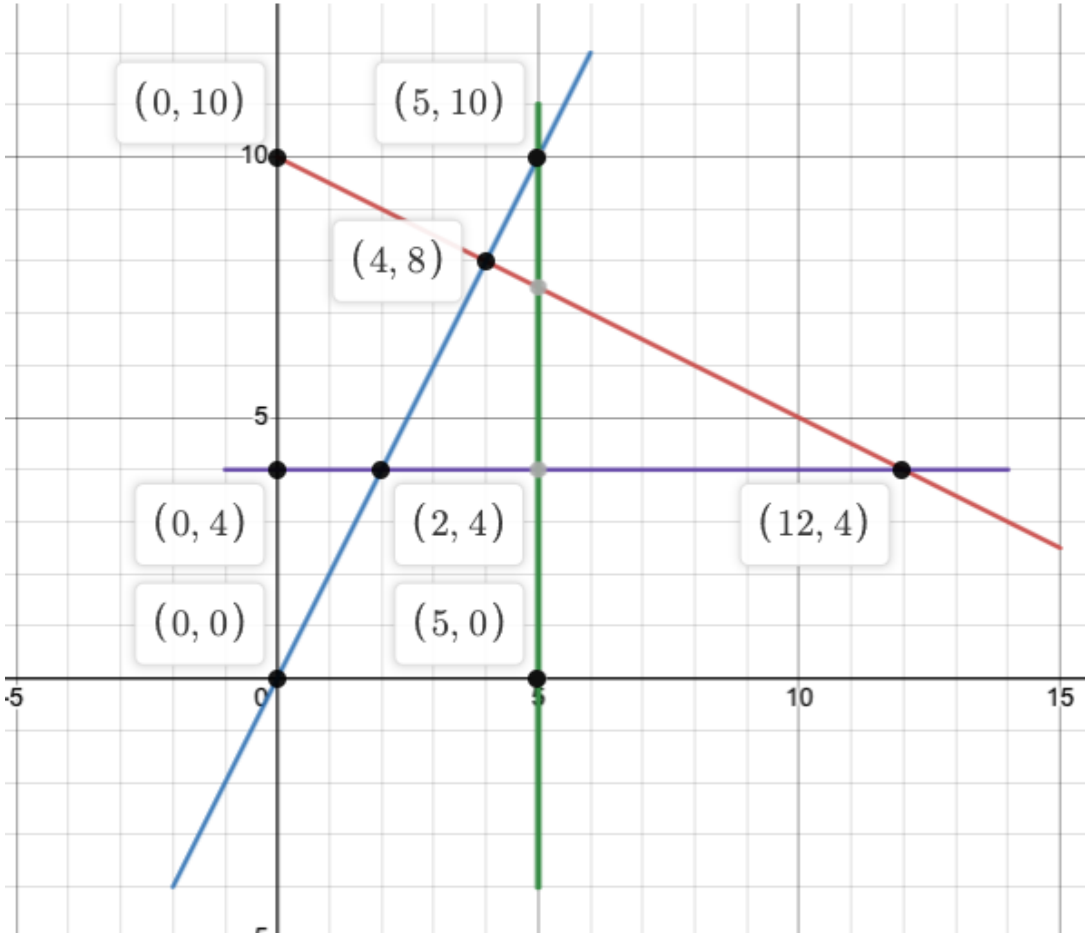
# EQUATION OF A LINE



# Y INTERCEPT OF A LINE

- What is an intercept?
- It is the location where the line crosses the y axis
- It is the y value when x is 0
- Notice vertical lines never touch the y axis
- Every line has one! (except vertical lines)

- What are the y-intercepts shown here?



# HOW DO WE WRITE THE EQUATION?

The equation of any line is in the form: $f(x) = y = mx + b$

- m is the slope (which is calculated given any two points)
- The number b is the y-intercept. We may need to solve for b. If so, we use $\boldsymbol{b} = y - m \cdot x$ with (x,y) a point on the line and m = slope
- Example: If $m = 3$ and $\boldsymbol{b} = -8$ the equation will be:

$$y = 3x - 8$$

- Can you? Find the slope and y-intercept of a line that passes through the points (0, 6) and (5, 46)?
- Then use those numbers to write the equation

Answers: slope (m) = $40 \div 5 = 8$ and y-intercept (b) = 6 … gotcha! The point (0,6) is the y-intercept !!

Equation: $y = 8x + 6$

# HOW DO WE WRITE THE EQUATION?

In business, we often use linear equations for supply and demand. The *y variable represents price,* and the x variable represents quantity.

So $y = mx + b$ means something specific.

1) The value of b will be the price then quantity = 0

2) The value of m will be measuring change in price over change in quantity

Use caution when looking at supply/demand examples on Youtube. Equations are given quantity as a function of price.

Suppose quantity demanded is 0 when price is $18. Consumer demand goes up to 6 when the price is at $12. What is the demand equation in this situation?

Answer: The two (x, y) points are (0, 18) and (6, 12). Slope is -1 and b = 18. Therefore, the equation for price over demand = $y = -x + 18$

# HOW DO WE WRITE THE EQUATION?

In business, we often use linear equations for supply and demand. The ***y variable represents price,*** and the x variable represents quantity.

So $y = mx + b$ means something specific.

1) The value of b will be the price then quantity = 0

2) The value of m will be measuring change in price over change in quantity

Use caution when looking at supply/demand examples on Youtube. Equations are given as quantity as a function of price.

Suppose quantity supplied is 0 when price is $4. Supply goes up to 12 when the price is at $16. What is the supply equation in this situation?

Answer: The two (x, y) points are (0, 4) and (12, 16). Slope is +1 and b = 4. Therefore, the equation for price over supply = $y = x + 4$

# MAKE A GRAPH

- Your turn: Make a graph of the supply and demand equations that we just found.
- Set up a table of values. Choose 3 or 4 values of x and calculate the y values that correspond to the chosen values of x.
- Plot the points. Label the supply and demand.
- Equilibrium is accomplished when supply equals demand at a certain price. How would we go about finding the equilibrium in this example?

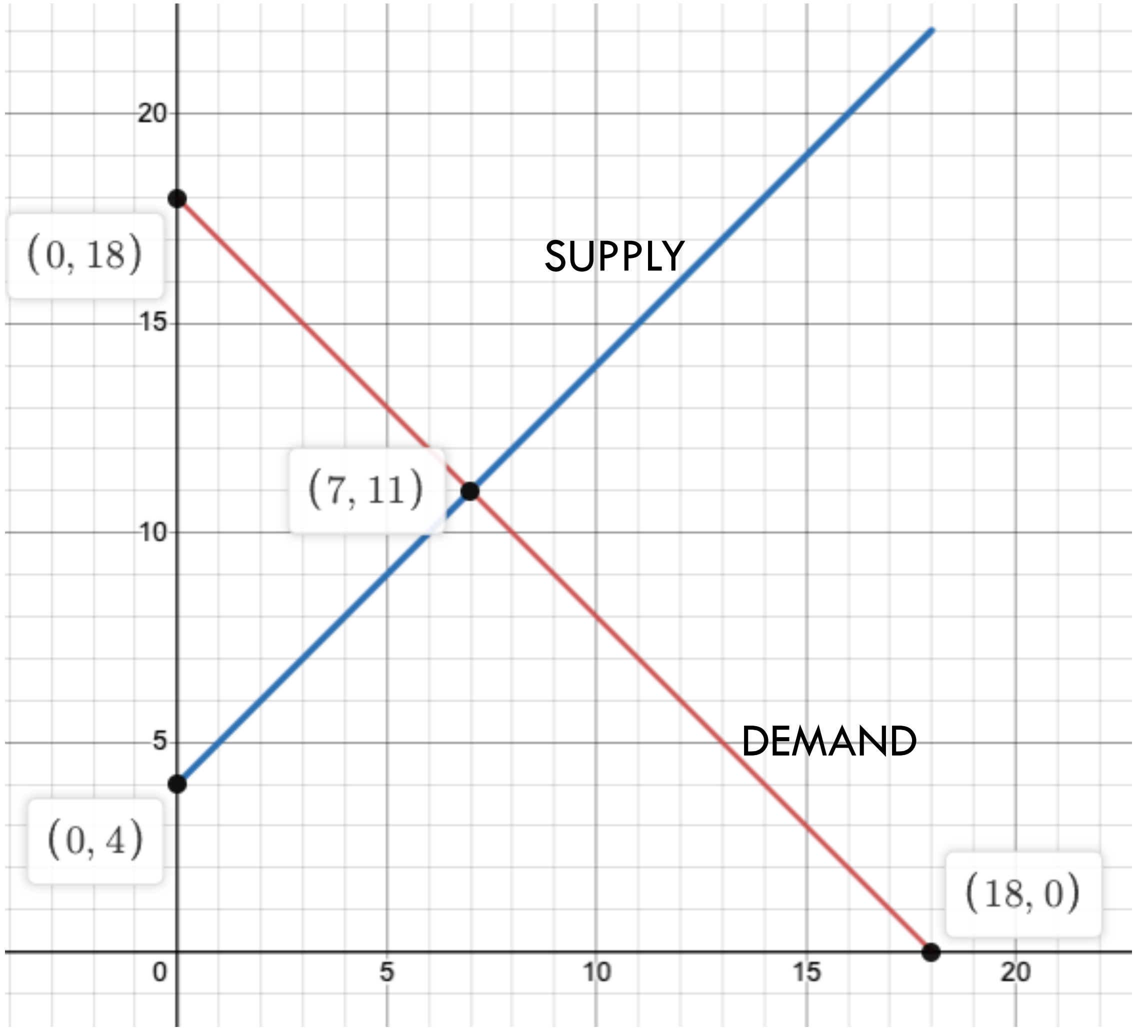
# SUPPLY AND DEMAND LINES

The slope of demand is always negative (quantity demanded goes UP as price goes DOWN)

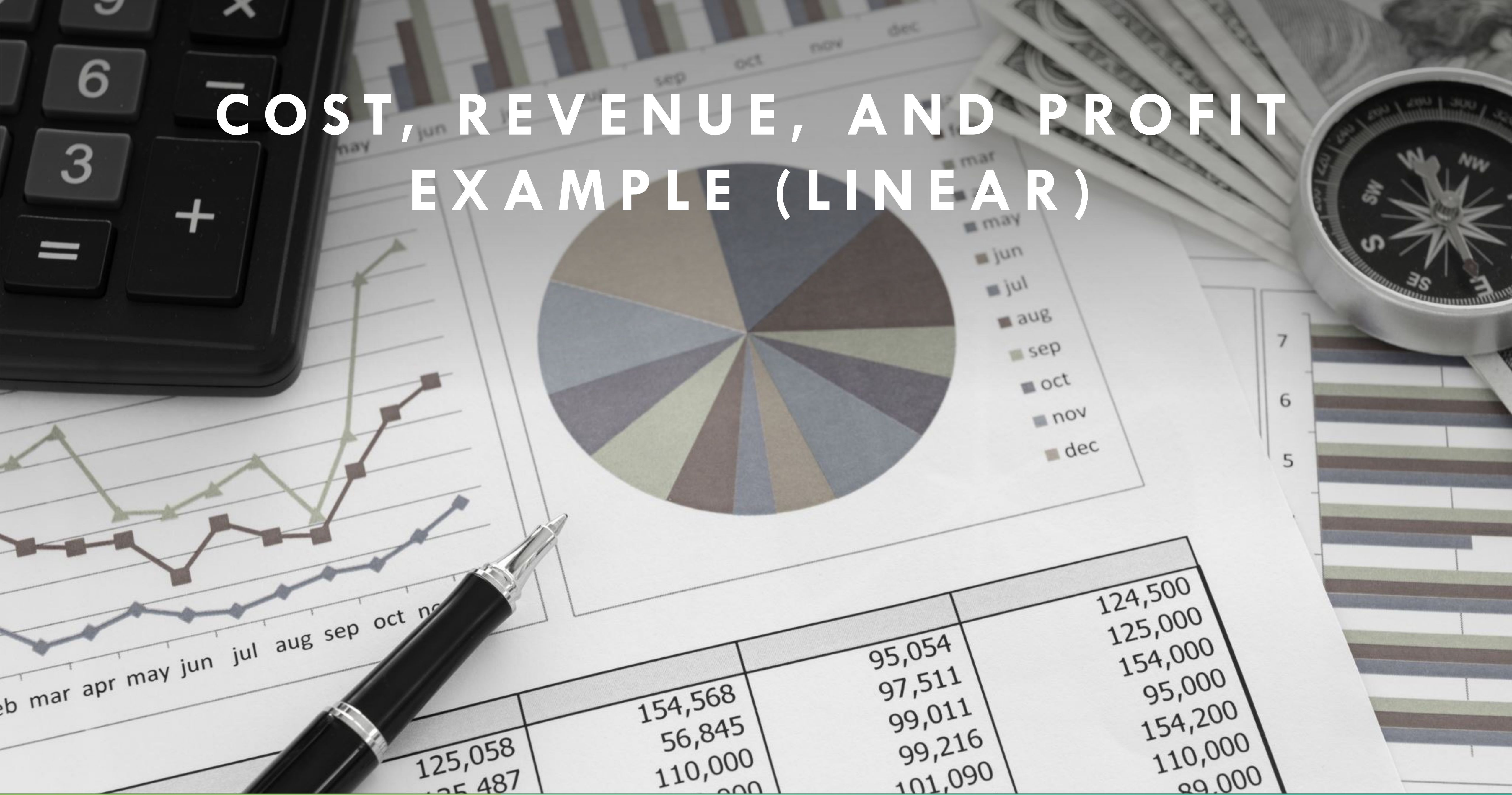
Supply slope is always positive (quantity in supply goes UP as price goes UP)

The point where the two graphs intersect is called the equilibrium point. Where supply equals demand.

Equilibrium occurs when price is $11



# COST, REVENUE, AND PROFIT EXAMPLE (LINEAR)



# WRITING COST AND REVENUE EQUATIONS

- An enterprising young person decides to sell baked goods at a local playground. The town permits its residents to rent a booth for this purpose.
- It cost her \$0.50 each for the baked goods. This is what we call ***unit cost***, because it is given as "cost per item". $Variable\ cost\ = \boldsymbol{unit\ cost \cdot x}$
- She pays the town \$120 to rent the booth for a week. This is what we call a ***fixed cost*** because she has to pay it no matter how much she sells.
- Each baked good will sell for \$1.50. This is what we will call ***unit price***.

# WRITING COST EQUATION

- Cost is C(x)
- The independent variable is x. Throughout this course x will represent “quantity” of goods made/bought/sold
- $C(x) = \ Fixed\ Cost\ \ + Unit\ Cost \cdot x$
- $C(x) = Fixed\ Cost + Variable\ Cost$

$$C(x) = 120 + 0.50x$$

# WRITING REVENUE EQUATION

- Revenue is R(x)
- $R(x) = Unit\ price \cdot x$

$$R(x) = 1.5x$$

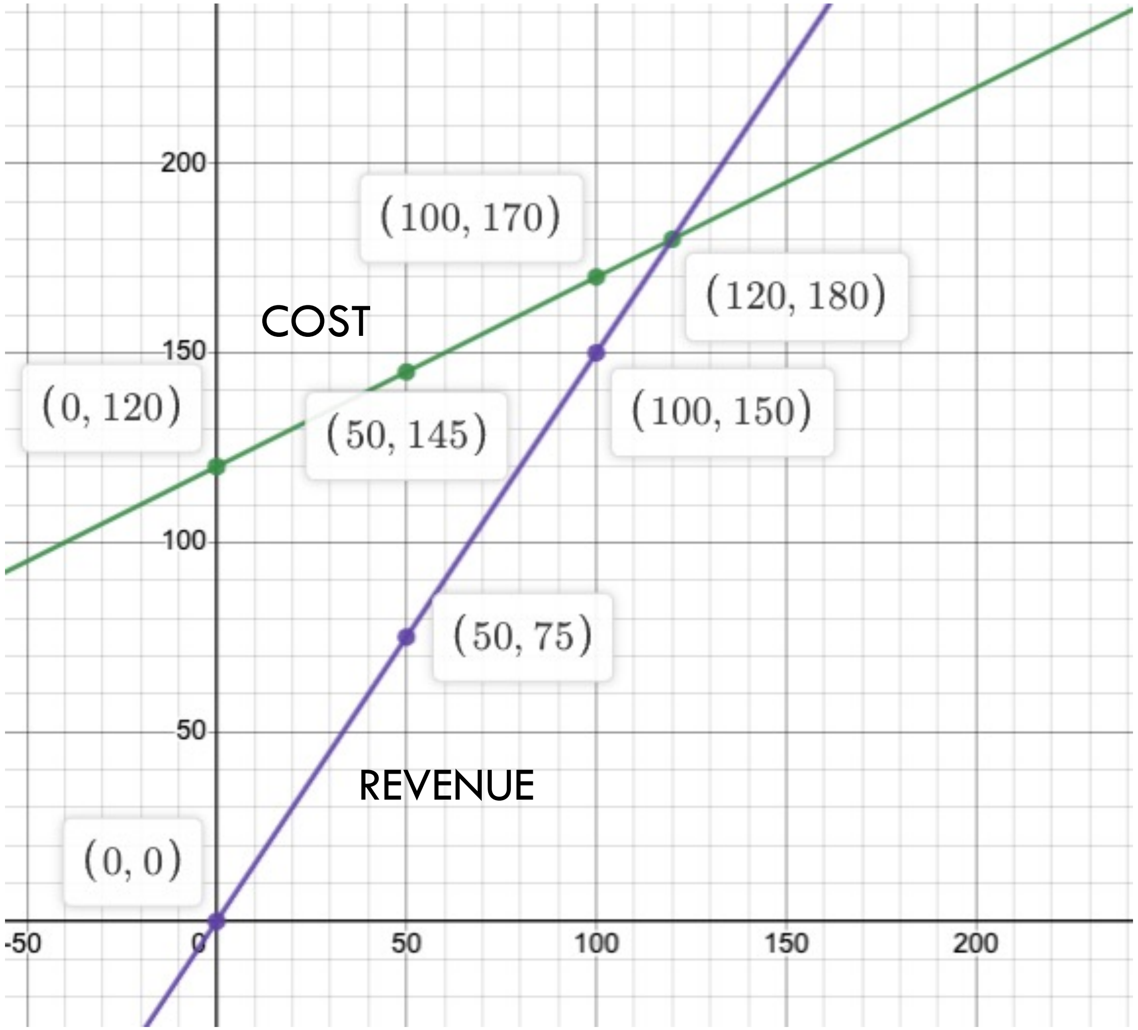
# GRAPH OF COST AND REVENUE

Make a graph of cost and revenue. Make sure to label COST and REVENUE.

Use x values of 0, 50, 100, and 120 in a table of values.

What is the meaning of the point where cost and revenue intersect?

Answer: (120,180) is called the break-even point. She needs to sell more than 120 baked goods before starting to make $.



# PROFIT EQUATION

- Profit is given the name P(x)
- $P(x) = R(x) - C(x)$
- $P(x) = 1.5x - (120 + 0.5x)$
- $P(x) = 1.5x - 120 - 0.5x$
- $P(x) = 1.0x - 120$
- $P(x) = x - 120$