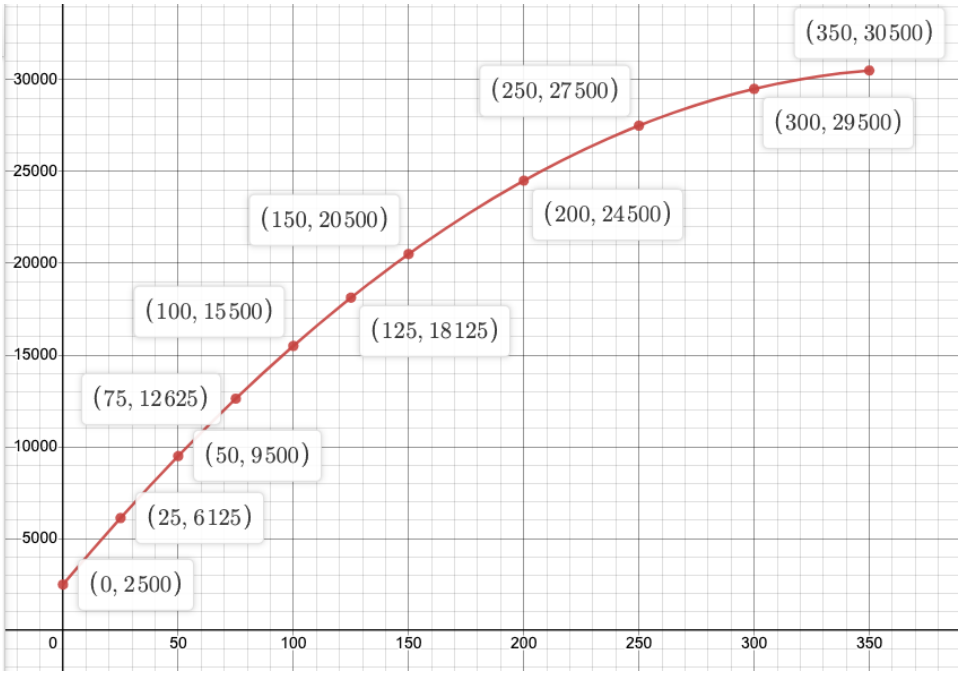
UNHM MATH 422 FINAL EXAM NAME_____

PLEASE REMEMBER: Show your reasoning and/or math work in every problem. ALL questions have more than one part. Make sure you answer every part of the question that was asked. You are being asked to calculate some things and interpret others.

1. (8 pts)

The total cost *C(x)*, in dollars, to produce *x* items is given by the function graphed below.

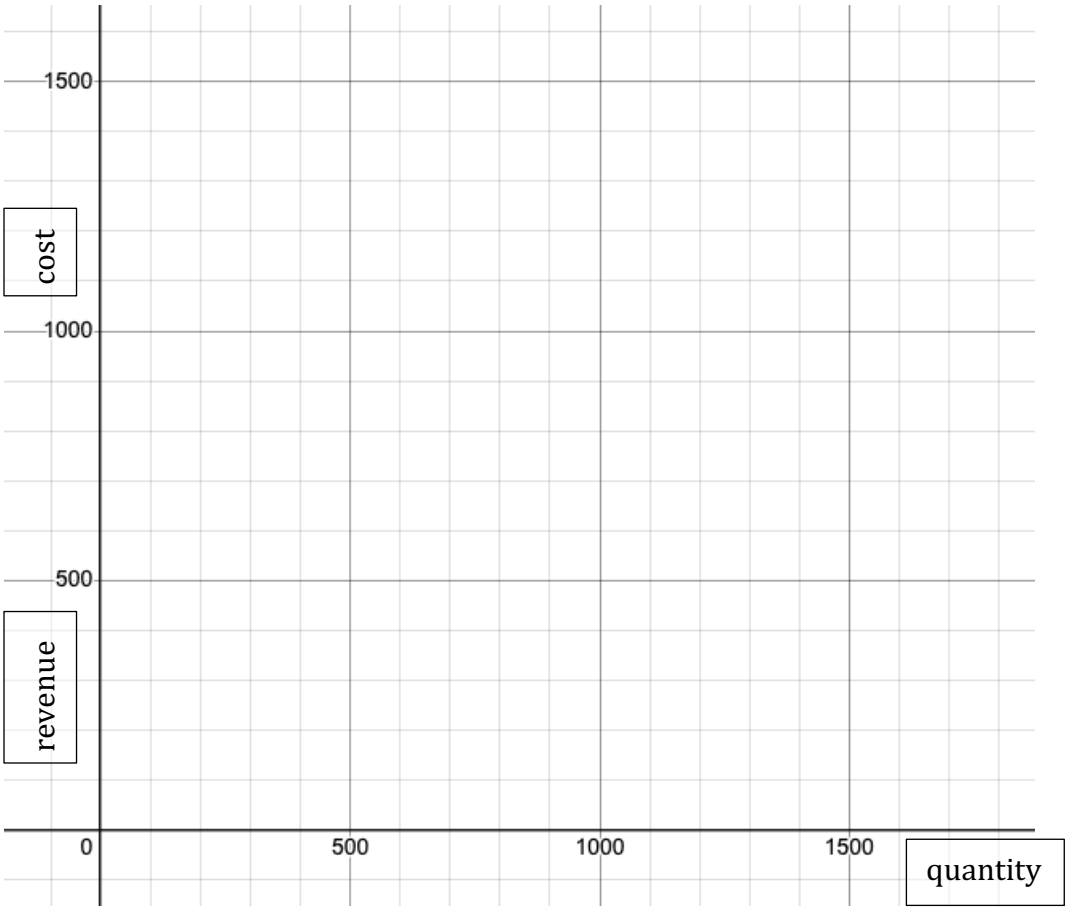


a. Find the average increase in cost per item (the average rate of change) when increasing production from 50 units to 150 units.

Show your work here:

b. The cost function is $C(x) = -0.2(x - 375)^2 + 30625$ What is the ***meaning*** of 375, and of 30625 in the context of cost over quantity of goods?

2. (8 pts) It costs $C(x) = 500 + 0.5x$ to make x cans of SPAM. The revenue from sales of SPAM is $R(x) = 1.5x$. Make a graph of the cost and revenue functions. They are both what type of function? _____ (exponential, quadratic, or linear)



What is the break-even point between cost and revenue?

3. (8 pts) The value of business furniture is assumed to depreciate (decrease) in value at a **rate** of 9% per year. The value was $37,500 when it was purchased in 2020.

a) ***Find an equation*** for the value of the furniture. Let y = the value, x = years after 2020

b) In what year will the value drop to half its purchase value?

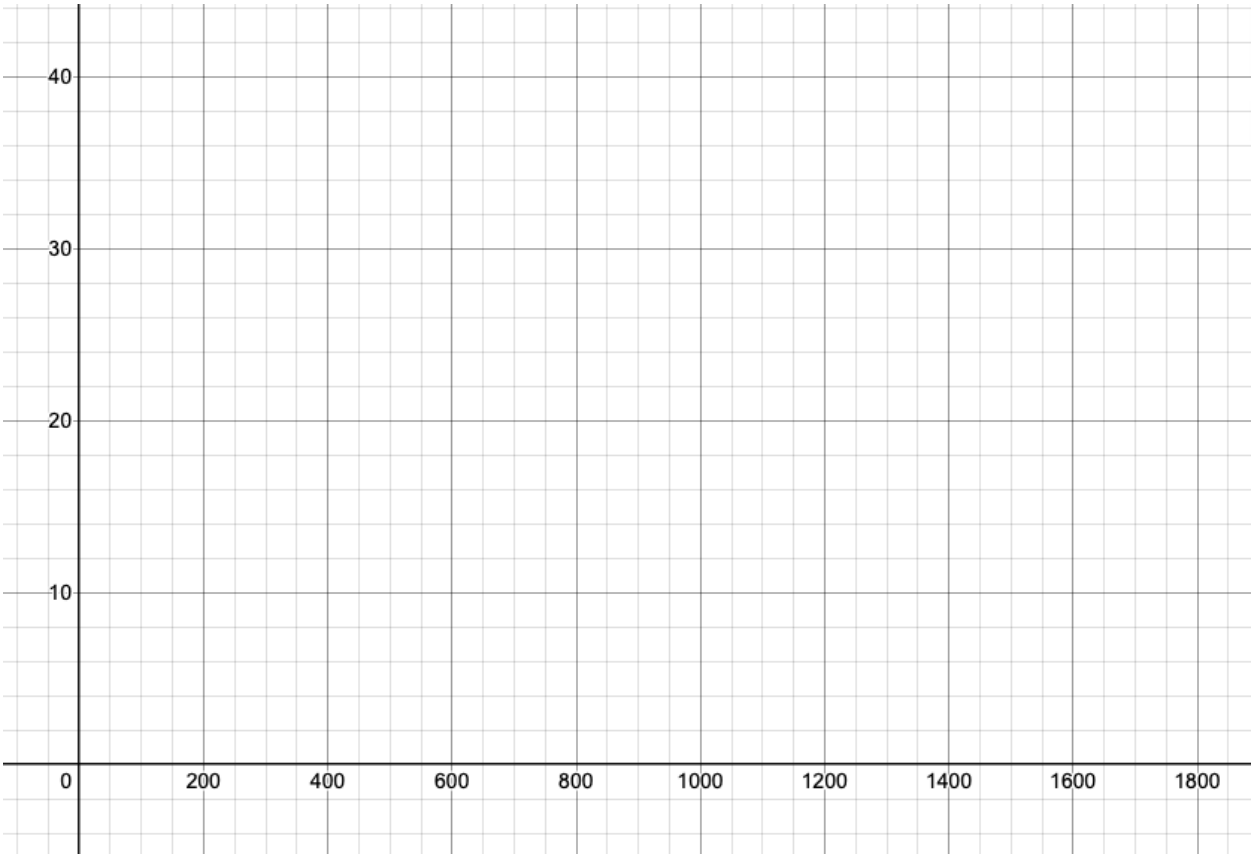
4. (8 pts) Shania buys a bag of cookies that contains 8 chocolate chip cookies, 5 peanut butter cookies, 2 sugar cookies and 7 oatmeal cookies.

a) What is the probability that Shania's friend reaches in the bag to get 2 cookies and randomly selects a chocolate chip cookie ***and*** a peanut butter cookie from the bag?

b) What is the probability that Shania randomly chooses a sugar cookie ***given that*** the friend already chose a chocolate chip cookie and peanut butter cookie?

5. (8 pts)

a) For the function $f(x) = \sqrt{x}$ calculate y values for these values of x: x = 100, 225, 400, 900, and 1600. Use this information to graph the function (as a curve). Label these 5 points.



b) Solve for x from a given y: For y = 24, what is x?

6. (8 pts) Given a ball thrown vertically upward according to the formula

$h(x) = 30 + 21.5x - 4.9x^2$ with height measured in meters and x is time in seconds. The ground is at h(x)=0.

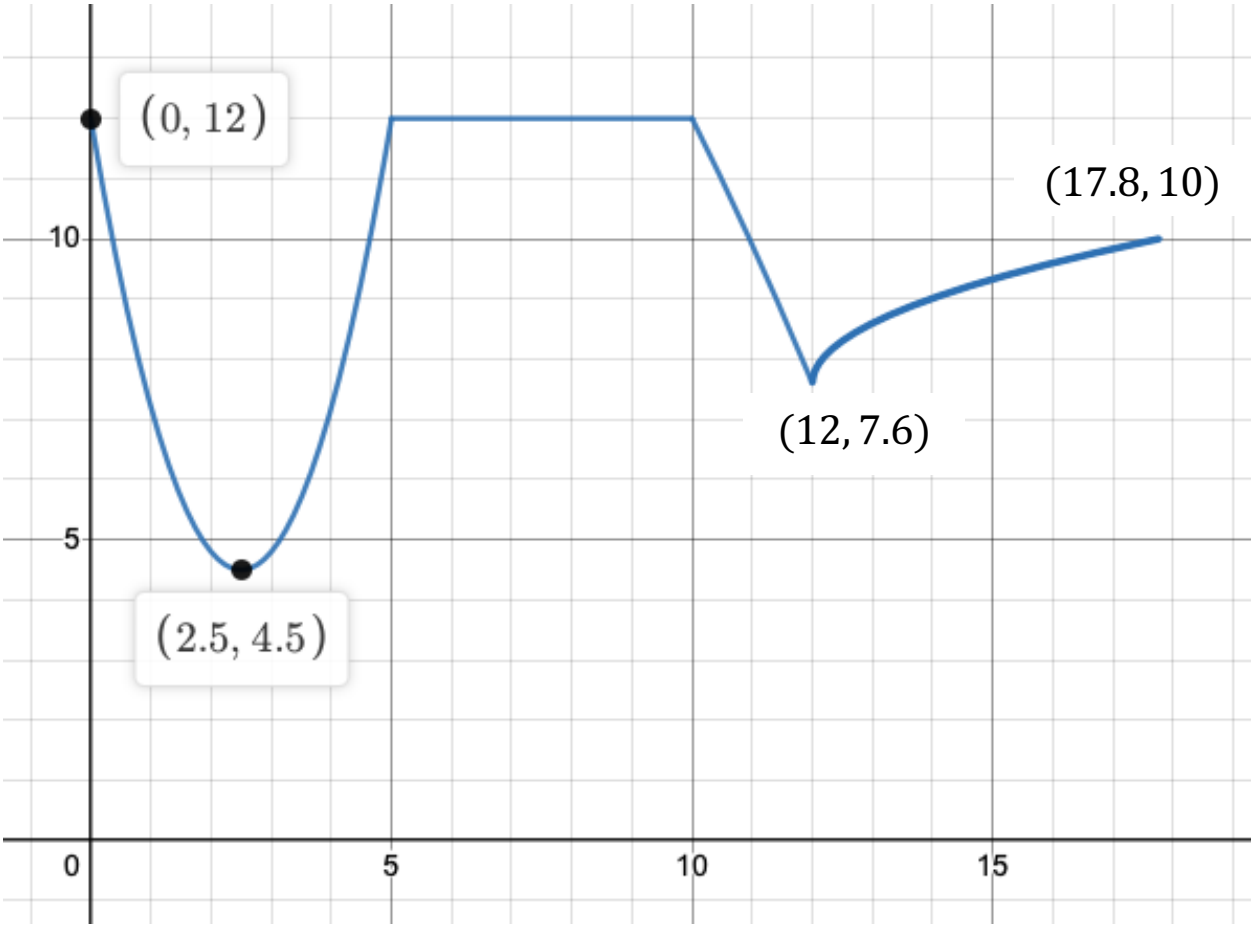
a. Find the slope of the secant line that passes through the curve at x = 1 and x = 2.

b. Find the slope of another secant line that passes through the curve at x = 1.5 and x = 2.

c. Which of these is a better estimate of the derivative of h(x) at x = 1.5 seconds? Explain your choice. (You may calculate $h'(x) and\ evaluate\ it\ for\ x = 1.5$ but it is not necessary.

d. How fast is the ball moving at 1.5 seconds after being thrown?

7. (8 pts) Here is a graph of a function, f(x). Some points are labeled.



a. Assume that the endpoints are on the function. State the domain and range of f(x).

b. List the points where the derivative (slope) does not exist.

c. At x = 13, approximate the slope of the curve.

d. Where is the slope = 0 ?

8. (8 pts)

When $f(x) = 2x^2 + 3x + 5$, we are interested in how slope changes on the graph.

a) What is the vertex of this function?

b) For the line that connects the points (0.5, 7) and (0, 5) calculate the slope.

c) For the line that connects the points (0.5, 7) and the point (1, 10) calculate the slope.

d) What do these answers tell us about the slope at the point (0.5, 7)?

9. (8 pts)

a) What is the derivative of the function $f(x) = -1x^3 + x^2 + 8x$?

b) Name the point where $f'(x) = 0$.

10. (8 pts) The cost to produce x items is given by the formula $C(x) = -0.5x^2 + 70x + 250$ for x in [0, 60].

a) Write the unit cost function. Calculate the Unit cost for production (x) = 40

b) Write the marginal cost function. Calculate C'(40). Write a sentence that summarizes what this number means.

11. (8 pts) Given the cost function as in question #10, the demand for the product is defined by $price = 160 - 2x$.

a) Write the equation for revenue, R(x). Use this to calculate the revenue for x = 40.

b) What is the production level that maximizes revenue?

c) What is the marginal revenue function? Calculate R'(x) and write a sentence describing what R'(40) means.

12. (8 points) Given the cost and revenue as in questions 10 and 11.

a) Write the formula for profit, $P(x) = R(x) - C(x)$. Calculate the profit when $x = 40$.

b) Where is the break-even point? (the smallest production (x) when profit = 0) Round up to the nearest whole number.

c) We are interested in profit between $x = 10$ and $x = 60$. At what production level do we see profit maximized?

13. (4 points) Consider the fact that all three functions in questions 10-12 are quadratic functions. Name the three vertices, (h, k) for cost, revenue, and profit. Write a sentence or two about why maximum profit doesn't occur at the same place as maximum revenue.

Some useful Formulas:

Linear functions: y = mx + b, where b = (0,b) and $m = \frac{\Delta y}{\Delta x} = (y_1 - y_0) \div (x_1 - x_0)$

Exponential growth or decay. Recall that an exponential function has the form $y = a \cdot b^x$ where $b = 1 + r$ or $b = 1 - r$

You may use your notes for other formulas.