Business Math **Additional Notes Quadratic and Polynomial Functions (Applications)**

Being able to describe/write a formula for revenue, cost, or profit from given information is an important goal for this class. Consider this example:

Let the demand function for a product be given by the function $D(q) = -1.75q + 250$, where q is the quantity of items in demand and $D(q)$ is the price per item, in dollars, that can be charged when q units are sold. Suppose fixed costs of production for this item are $\$3,000$ and variable costs are $\$8$ per item produced. If 13 items are produced and sold, find the following:

A) The total revenue from selling 13 items (to the nearest penny).
Answer: $

B) The total costs to produce 13 items (to the nearest penny).
Answer: $

C) The total profits to produce 13 items (to the nearest penny. Profits may or may not be negative.).
Answer: $

Solution: First, let's get everything in terms of x and y: (some of this was in the notes last week)

Step One: Identify the givens and put in terms of x	**Step Two:** Just the facts (Revenue, Cost, Profit)	**Step 3:** Put x = 13 and solve. C(13) is total cost; R(13) is total revenue; P(13) is total profit.
$Quantity\colon\ x$	$Cost = Fixed + Variable \cdot x$	$C(13) = 3000 + 8 \cdot 13$
$Demand\colon\ -1.75x + 250$	$C(x) = 3000 + 8x$	$C(13) = 3104$
$Fixed\ Cost\colon\ 3000$	$Revenue = Demand \cdot x$	$R(13) = (-1.75 \cdot 13 + 250) \cdot 13$
$Variable\ Cost\colon\ 8$	$R(x) = (-1.75x + 250) \cdot x$	$R(13) = 2954.25$
	$P(x) = R(x) - C(x)$	$P(13) = 2954.25 - 3104$
		$P(13) = -149.75$

Next: Being able to identify key features of a curve (turning points or what we call a vertex, intercepts, is also important. **Consider the following example**:

Consider the parabola given by the equation: $f(x) = -2x^2 + 12x - 6$

Find the following for this parabola:

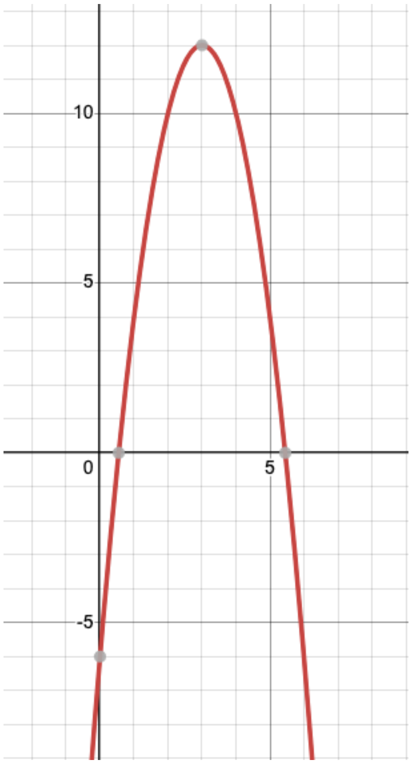
A) The vertex:

B) The vertical intercept is the point

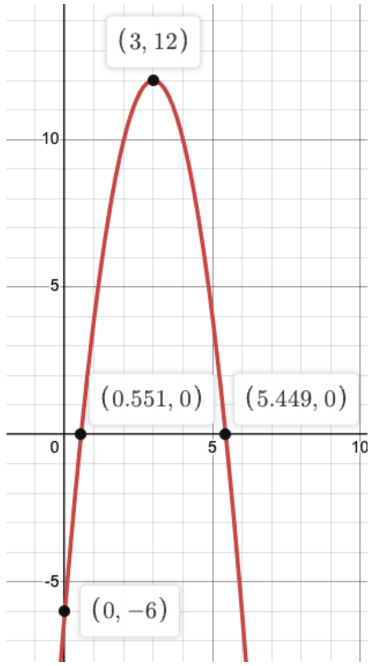
C) Find the coordinates of the two x intercepts of the parabola and write them as a list, separated by commas:

It is OK to round your value(s) to to two decimal places.

Solution: Use the Spreadsheet application, or an online graphing calculator and graph the equation. Also, **make a table** that includes **x = 0**, any **x values that appear** to be places the graph crosses the x axis, and **x value where the vertex** appears to be located.



When the equation is input into Desmos.com, the graph at the left is what we see. No points are labeled yet but if we click on the "grey dots" we see, the coordinates are revealed.



What you need to know:

The point (0, -6) is what we call the vertical intercept.
The point (3, 12) is what we call the vertex. It is the turning point of the graph.
The points (0.55, 0) and (5.45, 0) are what we call x-intercepts.

Had we chosen the spreadsheet we would need to estimate the two x-intercepts until we found one that resulted in a y value = 0.

x	$-2x^2 + 12x - 6$
0	−6
0.5	−0.5
1	4
2	10
3	12
4	10
5	4
5.5	−0.5
0.55	−0.005
5.45	−0.005

At left you can see what happened when we created a table of (x,y) values. The two numbers at the bottom, 0.55 and 5.45, were the closest we could get (with two decimal places) to a y value nearest 0.

-0.005 is "very near" 0.

Our answers need to be entered as (x,y) ordered pairs.

Vertex: (3, 12)

Vertical intercept: (0, -6)

X-intercepts: (0.55,0), (5.45,0)

What if we are asked to interpret and answer questions when we only know the profit equation? Here is an example:

A company's profit when it sells *x* thousand items is predicted to be
$P(x) = -3x^2 + 1212x - 18000.$

a) What is the company's startup costs?

$ [ ]

b) How many items does the company need to sell to break even? (to the nearest thousand items)

[ ] thousand items

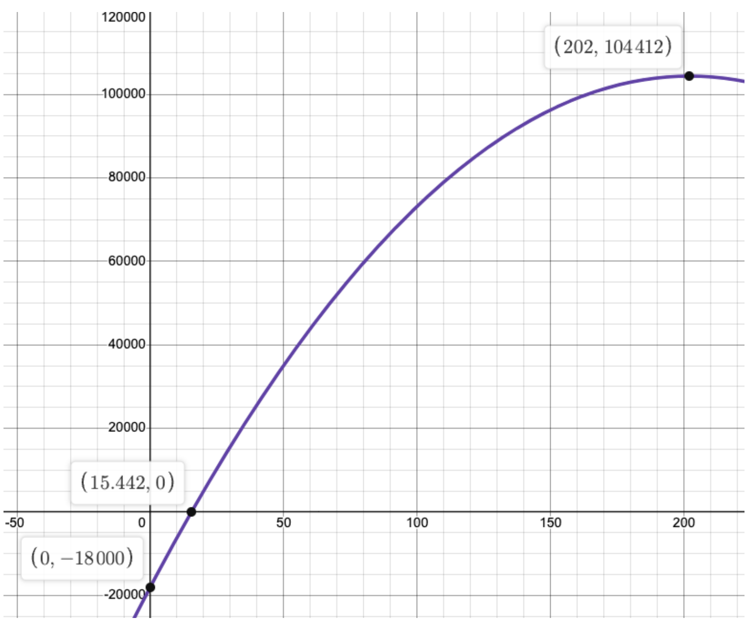
c) How many items should the company sell to maximize profit? (to the nearest thousand items)

[ ] thousand items

The startup cost is the cost when nothing is produced. In other words x=0.

Look for the constant term. In this problem the startup cost is 18000. It's also the **vertical intercept** of the graph but made into a **positive** number.

The "break even" point is where profit = 0. That would be the x-intercept(s). To maximize profit we are looking for the turning point, the vertex. The only tricky part to this was getting everything visible on desmos.com. Using a spreadsheet, we would need to expand our choice of x values to get to where we see the vertex appear.



Here the startup cost was $18000. The "break even" point occurred when the number of items (quantity) was 15.442

The maximum profit occurred at x = 202, which answers the question about quantity.

"How many items" will always be x!

Our answers: 18000 is startup cost, the break-even point is rounded to 15. And the number of items to maximize profit is 202.

Finally, suppose we are asked a question about revenue, but we are not given the demand equation. Instead, we are given information that would help us find the demand equation. Here is an example:

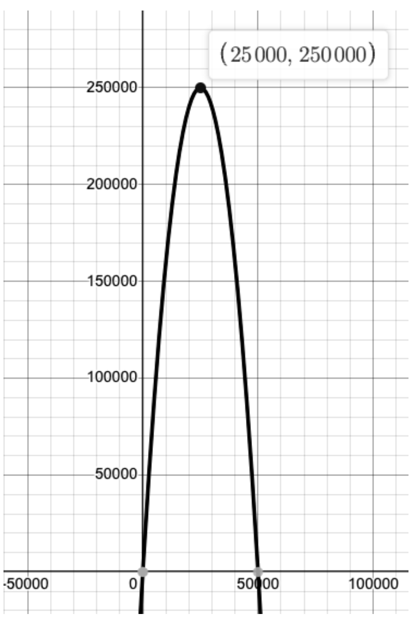
A baseball team plays in a stadium that holds 70000 spectators. With the ticket price at $8 the average attendance has been 28000. When the price dropped to $5, the average attendance rose to 35000. Assume that attendance is linearly related to ticket price.

What ticket price would maximize revenue? $

Since our goal is to find a maximum (probably a vertex) of the revenue equation, we need to build it.

Step One:	**Step Two:**	**Step Three:**
demand is going to be the line that passes through the points (28000,8) and (35000,5)	Find the equation y = mx + b using the two points	R(x) = (demand) * x
Why? x = quantity sold and y=unit price	$y = -0.0004x + 20$	$R(x) = (-0.0004x + 20) \cdot x$

The reason why the decimal for the slope is so small is because our x values are really big but y is really small in comparison.



When we graph the equation for R(x) in desmos.com or another graphing tool, we must zoom way out to locate the vertex. This is where the maximum revenue will occur.

Remember that x = quantity sold and y = revenue on this graph. Maximum revenue will occur when x = 25000.

The ***question asks about the ticket price***.

$$price = y = -0.0004x + 20$$
$$price = -0.0004 \cdot 25000 + 20$$

So the answer is: price = 10