# [illegible]

## Contents



## 1. [illegible]

[illegible]

$$\frac{\partial \mathcal{L}}{\partial q_i} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_i} = 0 \quad (1)$$

[illegible] $\mathcal{L}$ [illegible](Lagrangean)[illegible] $q_i$ [illegible] $\dot{q}_i$ [illegible]([illegible]) [illegible]

[illegible]

$$L = T - V \quad (2)$$

[illegible] $T$ [illegible] $V$ [illegible]

[illegible] [illegible]

### 1.1. [illegible]

[illegible] $x$ [illegible] [illegible] $x(t)$ [illegible] [illegible] $t = t_0$ [illegible] $x_0$[illegible] $t = t_1$ [illegible] $x_1$ [illegible] $t_0 < t < t_1$ [illegible]

[illegible] $f(x, \dot{x})$ [illegible] [illegible] $x(t_0) = x_0$ [illegible] $x(t_1) = x_1$ [illegible](Action)[illegible] [illegible]

$$A = \int_{t_0}^{t_1} f(x(t), \dot{x}(t))\, \mathrm{d}t \quad (3)$$

[illegible]

### 1.2. [illegible]

[illegible] [illegible] [illegible]([illegible])[illegible]

[illegible] [illegible] [illegible] $x$ [illegible] [illegible]

[illegible] [illegible] [illegible]

[illegible] $f(x, \dot{x})$ [illegible] $A$ [illegible] $\delta$ [illegible]

$$\delta A = \int_{t_0}^{t_1} \delta f(x, \dot{x})\, \mathrm{d}t = 0 \tag{4}$$

[illegible]

[illegible] $\delta f(x, \dot{x})$ [illegible] $f$ [illegible] $x$ [illegible] $x_i$ [illegible] $t$ [illegible] $\varepsilon$ [illegible]

$$f\left(x_i, \frac{x_i - x_{i-1}}{\varepsilon}\right). \tag{5}$$

[illegible]

[illegible] $i$ [illegible] $x_i$ [illegible] $f$ [illegible] $f$ [illegible] $i$ [illegible] $f_i$ [illegible] $x_i$ [illegible]

$$\begin{aligned} \frac{\partial}{\partial x_i} &= \frac{\partial}{\partial x_i} f\left(x_i, \frac{x_i - x_{i-1}}{\varepsilon}\right) \\ &= \frac{\partial f_i}{\partial x_i} + \frac{\partial v_i}{\partial x_i} \frac{\partial f_i}{\partial v_i} \\ &= \frac{\partial f_i}{\partial x_i} + \frac{1}{\varepsilon} \frac{\partial f_i}{\partial v_i} \end{aligned} \tag{6}$$

[illegible] $v_i \equiv \frac{x_i - x_{i-1}}{\varepsilon}$ [illegible]

[illegible] $\frac{\partial f}{\partial x_i}$ [illegible] $f_{i+1}$ [illegible]

$$\begin{aligned} \frac{\partial}{\partial x_i} f_{i+1} &= \frac{\partial}{\partial x_i} f\left(x_{i+1}, \frac{x_{i+1} - x_i}{\varepsilon}\right) \\ &= \frac{\partial v_{i+1}}{\partial x_i} \frac{\partial f_{i+1}}{\partial v_{i+1}} \\ &= -\frac{1}{\varepsilon} \frac{\partial f_{i+1}}{\partial v_{i+1}} \end{aligned} \tag{7}$$

[illegible]

$$\frac{\partial f_i}{\partial x_i} + \frac{1}{\varepsilon} \frac{\partial f_i}{\partial v_i} - \frac{1}{\varepsilon} \frac{\partial f_{i+1}}{\partial v_{i+1}}. \tag{8}$$

[illegible]

[illegible] $\frac{f_{i+1} - f_i}{\varepsilon}$ [illegible] $\varepsilon \to 0$ [illegible] $v_i$ [illegible]

[illegible]

$$\frac{\partial f_i}{\partial x_i} - \frac{d}{dt} \frac{\partial f_i}{\partial \dot{x}_i} = 0 \tag{9}$$

[illegible] $i$ [illegible] $t$ [illegible]

$$\begin{aligned} x &= X \cos(\omega t) + Y \sin(\omega t) \\ y &= X \sin(\omega t) - Y \cos(\omega t). \end{aligned} \tag{10}$$

[illegible] [illegible] [illegible] [illegible] [illegible]

[illegible]

$$\mathcal{L} = \frac{m}{2}(\dot{x}^2 + \dot{y}^2) \tag{11}$$

[illegible]

[illegible] (2) [illegible]

$$\mathcal{L} = \frac{m}{2}\left(\dot{X}^2 + \dot{Y}^2\right) + \frac{\omega^2 m}{2}(X^2 + Y^2) + \frac{\omega m}{2}\left(X\dot{Y} - Y\dot{X}\right) \tag{12}$$

[illegible]

[illegible] [illegible] $r = X^2 + Y^2$ [illegible] $r$ [illegible] [illegible]

[illegible] $X$ [illegible]

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial X} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{X}} &= \omega^2 mX + \frac{\omega m}{2}\dot{Y} - m\ddot{X} + \frac{\omega m}{2}\dot{Y} = 0 \\ \therefore m\ddot{X} &= \omega^2 mX + \omega m\dot{Y} \end{aligned} \tag{13}$$

[illegible] $ma = F$ [illegible] $X$ [illegible] [illegible] $Y$ [illegible] $\dot{X}$ [illegible] [illegible] [illegible] [illegible] [illegible]

## 1.3. [illegible]

[illegible] [illegible] $r, \theta$ [illegible] [illegible]

$$\begin{aligned} x &= r\cos(\theta) \\ y &= r\sin(\theta). \end{aligned} \tag{14}$$

[illegible]

$$\begin{aligned} \mathcal{L} &= \frac{m}{2}(\dot{x}^2 + \dot{y}^2) \\ &= \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\theta}^2\right) \end{aligned} \tag{15}$$

[illegible]

[illegible] $r$ [illegible]

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial r} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{r}} &= mr\dot{\theta}^2 - \frac{d}{dt}m\dot{r} \\ &= mr\dot{\theta}^2 - m\ddot{r} = 0 \\ \therefore mr\dot{\theta}^2 &= m\ddot{r}. \end{aligned} \tag{16}$$

[illegible] $F = ma$ [illegible] $\theta^2$ [illegible] [illegible] $r$ [illegible] 2 [illegible]

[illegible] $\theta$ [illegible]

$$\frac{\partial \mathcal{L}}{\partial \theta} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = -mr^2\dot{\theta} = 0. \tag{17}$$

[illegible]

[illegible]

## 1.4. [illegible]

[illegible] [illegible]

[illegible]

$$\delta f(\vec{q}) = \sum_i \frac{\partial f}{\partial q_i}\delta q_i \tag{18}$$

[illegible] $f(\dot{q})$ [illegible] $q_{i(i=1,2,\ldots,n)}$ [illegible]

[illegible] [illegible] [illegible] [illegible] [illegible]

[illegible] $q_i$ [illegible] $\delta q_i$ [illegible] $f_{i(\dot{q})}$ [illegible]

$$\delta q_i = f_i(\vec{q})\delta \tag{19}$$

[illegible] [illegible] $\delta$ [illegible] [illegible] $\delta$ [illegible]

[illegible] $q_i$ [illegible] $\dot{q}_i$ [illegible]

$$\delta \mathcal{L}\left(\vec{q}, \vec{\dot{q}}\right) = \sum_i \left(\frac{\partial \mathcal{L}}{\partial q_i}\delta q_i + \frac{\partial \mathcal{L}}{\partial \dot{q}_i}\delta \dot{q}_i\right). \tag{20}$$

[illegible]

[illegible] Equation 1 [illegible] [illegible]

$$\frac{\partial \mathcal{L}}{\partial \dot{q}_i} \equiv p_i. \tag{21}$$

[illegible] [illegible]

[illegible]

$$\frac{\partial \mathcal{L}}{\partial q_i} = \dot{p}_i \tag{22}$$

[illegible]

[illegible] Equation 20 [illegible]

$$\delta \mathcal{L}\left(\vec{q}, \vec{\dot{q}}\right) = \sum_i \left(\dot{p}_i \delta q_i + p_i \delta \dot{q}_i\right) \tag{23}$$

[illegible] [illegible]

$$\delta \mathcal{L}\left(\vec{q}, \vec{\dot{q}}\right) = \frac{d}{dt}\sum_i p_i \delta q_i \tag{24}$$

[illegible]

[illegible] $\delta q_i$ [illegible] [illegible] Equation 19 [illegible]

$$\delta\mathcal{L}\left(\vec{q},\dot{\vec{q}}\right) = \frac{d}{dt}\sum_i p_i \delta q_i \tag{25}$$

[illegible]

$\delta$ [illegible]

$$\frac{d}{dt}\sum_i p_i f_i(\vec{q}) = 0 \tag{26}$$

[illegible]

[illegible]

$$Q \equiv \sum_i p_i f_i(\vec{q}) \tag{27}$$

[illegible]

[illegible]

[illegible]

## 2. [illegible]

[illegible]

[illegible] $q_i, \dot{q}_i$ [illegible] [illegible] $\mathcal{L}(\dot{q},\ddot{q})$ [illegible]

[illegible]

$$\frac{d\mathcal{L}\left(\vec{q},\dot{\vec{q}}\right)}{dt} = \sum_i \left\{ \frac{\partial\mathcal{L}}{\partial q_i}\dot{q}_i + \frac{\partial\mathcal{L}}{\partial\dot{q}_i}\ddot{q}_i \right\} \tag{28}$$

[illegible] Equation 21, Equation 22 [illegible]

$$\frac{d\mathcal{L}}{dt} = \sum_i \left(p_i\dot{q} + \dot{p}_i\ddot{q}_i\right). \tag{29}$$

[illegible]

$$\frac{d\mathcal{L}}{dt} = \sum_i \frac{d}{dt} p_i \dot{q}_i \tag{30}$$

[illegible]

$$\frac{d}{dt}\left\{ \mathcal{L} - \sum_i p_i \dot{q}_i \right\} = 0 \tag{31}$$

[illegible]

[illegible]

$$\mathcal{H} \equiv \sum_i p_i \dot{q}_i - \mathcal{L} \tag{32}$$

[illegible] [illegible] [illegible]

[illegible] 1 [illegible] [illegible] $p_i = m_i \dot{x}_i$ [illegible]

$$\begin{aligned}
\mathcal{H} &= \sum_i p_i \dot{q}_i - \mathcal{L} \\
&= \sum_i \left[ m_i \dot{x}_i^2 - \left\{ \frac{m}{2} \dot{x}_i - V(x_i) \right\} \right] \\
&= \sum_i \left\{ \frac{m \dot{x}_i^2}{2} + V(x_i) \right\}.
\end{aligned} \tag{33}$$

[illegible] [illegible] [illegible] [illegible]

[illegible] Equation 28 [illegible] Equation 31 [illegible] $\mathcal{L}$ [illegible] $\mathcal{L}(\dot{q}, \ddot{q}, t)$ [illegible]

$$\begin{aligned}
\frac{d\mathcal{L}}{dt} &= \sum_i \frac{d}{dt} p_i \dot{q}_i + \frac{\partial \mathcal{L}}{\partial t} \\
\frac{d\mathcal{H}}{dt} &= -\frac{\partial \mathcal{L}}{\partial t}.
\end{aligned} \tag{34}$$

[illegible] [illegible]

## 3. [illegible]

[illegible]

$$d\mathcal{H} = \sum_i \left[ \dot{q}_i dp_i + p_i d\dot{q} - \frac{\partial \mathcal{L}}{\partial \dot{q}_i} d\dot{q}_i - \frac{\partial \mathcal{L}}{\partial q_i} dq_i \right] \tag{35}$$

[illegible] $\frac{\partial \mathcal{L}}{\partial \dot{q}_i} = p_i$, $\frac{\partial \mathcal{L}}{\partial p_i} = \dot{q}_i$ [illegible]

$$d\mathcal{H} = \sum_i \left( \dot{q}_i dp_i - \dot{p}_i dq_i \right) \tag{36}$$

[illegible] $p_i$ [illegible] $q_i$ [illegible]

$$\begin{aligned}
\frac{\partial \mathcal{H}}{\partial p_i} &= \dot{q}_i \\
\frac{\partial \mathcal{H}}{\partial q_i} &= -\dot{p}_i
\end{aligned} \tag{37}$$

[illegible]

[illegible]

## 4. [illegible]


1. Leonard Susskind, Classical Mechanics | Lecture 3
2. Leonard Susskind, Classical Mechanics | Lecture 4

3. Leonard Susskind, Classical Mechanics | Lecture 5
4. Wikipedia’s Noether’s theorem article