# Simon Fraser University

## Mechatronic Systems Engineering

### Final Exam for ENSC 226: Electronic Circuits

Instructor: Behraad Bahreyni

13 April 2011

Time: 150 minutes

Name: Solutions

Student number: ______________

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All writing must be submitted with this examination booklet (notes, drafts, calculations, etc.). This booklet must not be torn or mutilated in any way, and must not be taken from the examination room.

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| Question | Mark |
|---|---|
| 1 | / 40 |
| 2 | / 15 |
| 3 | / 20 |
| 4 | / 15 |
| 5 | / 10 |
| Total | /100 |

**Simon Fraser University** **ENSC226: Electronic Circuits**

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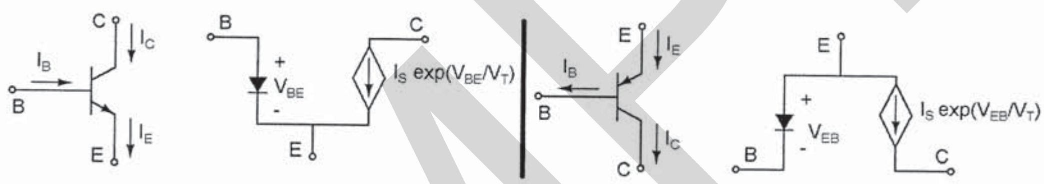
Physical model of a diode: $I_D = I_s\left(e^{\frac{V_D}{nV_T}} - 1\right)$

For BJTs:

| | Active | Saturation | Cut-off |
|---|---|---|---|
| | *BE forward biased*<br>*BC reverse biased* | *BE forward biased*<br>*BC forward biased* | *BE reverse biased*<br>*BC reverse biased* |
| npn | $I_C = \beta\, I_B = \alpha\, I_E\ , V_{BE} \approx 0.7V$ | $I_B > \frac{I_C}{\beta}\, , V_{CE} \approx 0.2V$ | $I_B \approx I_C \approx I_E \approx 0$ |
| pnp | $I_C = \beta\, I_B = \alpha\, I_E\ , V_{EB} \approx 0.7V$ | $I_B > \frac{I_C}{\beta}, V_{EC} \approx 0.2V$ | $I_B \approx I_C \approx I_E \approx 0$ |

$$I_C(\text{active}) = I_S\, e^{\frac{|V_{BE}|}{V_T}}\left(1 + \frac{|V_{CE}|}{V_A}\right) \qquad \alpha = \frac{\beta}{\beta + 1} \qquad V_T = 25mV$$

$$g_m = \frac{1}{r_m} = \frac{I_C}{V_T} \qquad r_\pi = \beta\, r_m = \frac{\beta}{g_m} \qquad r_o = \frac{V_A}{I_C}$$



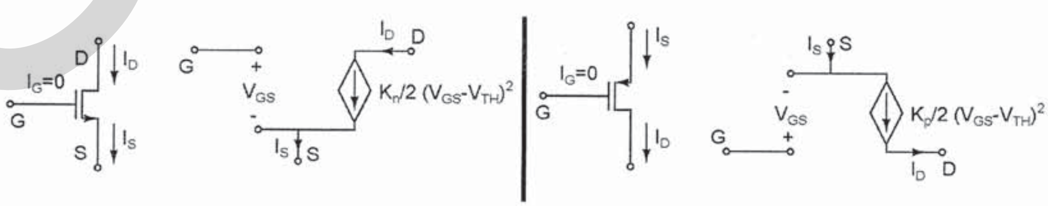

For MOSFETs:

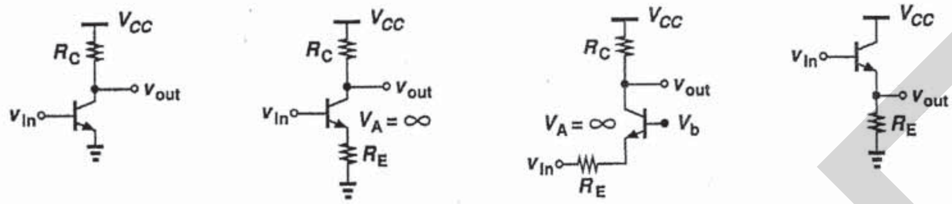
| | Saturation | Triode | Cut-off |
|---|---|---|---|
| NFET ($V_{TH}$>0) | $V_{GS} > V_{TH}\ \ \& V_{GD} < V_{TH}$ | $V_{GS} > V_{TH}\ \ \& V_{GD} > V_{TH}$ | $V_{GS} < V_{TH}\ \ \& V_{GD} < V_{TH}$ |
| PFET ($V_{TH}$<0) | $V_{GS} < V_{TH}\ \ \& V_{GD} > V_{TH}$ | $V_{GS} < V_{TH}\ \ \& V_{GD} < V_{TH}$ | $V_{GS} > V_{TH}\ \ \& V_{GD} > V_{TH}$ |

$$I_D(saturation) = \frac{K'}{2}\,(V_{GS} - V_{TH})^2\,(1 + \lambda V_{DS}) \qquad \begin{array}{ll} K' \equiv \mu_n C_{ox} {}^{W}\!/_{L} & \text{for NFETS} \\ K' \equiv \mu_p C_{ox} {}^{W}\!/_{L} & \text{for PFETS} \end{array}$$

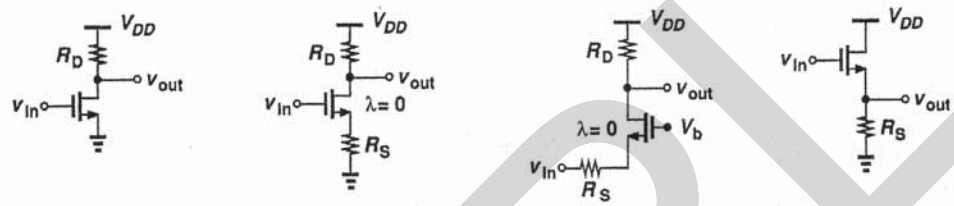
$$I_D(triode\,) = \frac{K'}{2}\,[2(V_{GS} - V_{TH})V_{DS} - V_{DS}^2] \qquad r_{DS}(triode) \approx \frac{1}{K'(V_{GS} - V_{TH})}$$

$$g_m = K'(V_{GS} - V_{TH}) = \sqrt{2\,K' I_D} = \frac{2I_D}{V_{GS} - V_{TH}} \qquad r_o = \frac{1}{\lambda I_D}$$

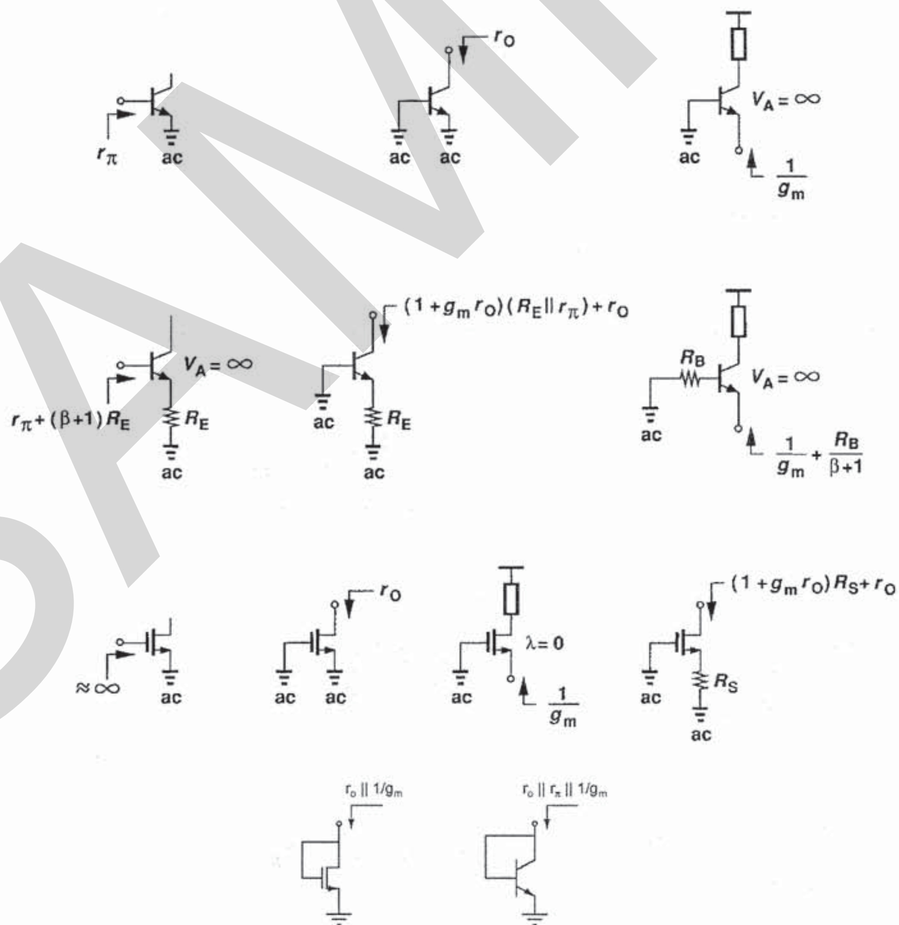





$A_v = -g_m(R_C \| r_O)$ $A_v = -\dfrac{R_C}{\frac{1}{g_m}+R_E}$ $A_v = \dfrac{R_C}{\frac{1}{g_m}+R_E}$ $A_v = \dfrac{R_E \| r_O}{\frac{1}{g_m}+R_E \| r_O}$



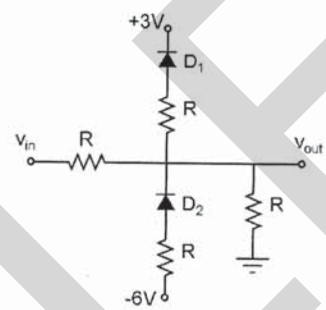

$A_v = -g_m(R_D \| r_O)$ $A_v = -\dfrac{R_D}{\frac{1}{g_m}+R_S}$ $A_v = \dfrac{R_D}{\frac{1}{g_m}+R_S}$ $A_v = \dfrac{R_S \| r_O}{\frac{1}{g_m}+R_S \| r_O}$



**Question 1.** [40 marks]

Circle the correct answer for each of the following problems.

***1.a.*** [5 marks] What is the range for the output voltage if $-36V < v_{in} < +36V$? Assume the diodes are ideal.




(a) $-12V < v_{out} < +12V$ (b) $-10V < v_{out} < +9V$

(c) $-14V < v_{out} < +13V$ (d) $-18V < v_{out} < +18V$

(e) None of the above

$v_{out} = \frac{v_{in}}{3} + 1$ for large $v_{in}$

$v_{out} = \frac{v_{in}}{3} - 2$ for small $v_{in}$

***1.b.*** [5 marks] A student wants to measure the input resistance of an amplifier using the method you learned in Lab 4. However, he only has a fixed 10kΩ resistor instead of a proper variable resistor. When he puts this resistor in series with the input of the amplifier, the output signal amplitude changes from 1V to 0.80V. What is the input resistance of this amplifier?

(a) 10kΩ (b) 20kΩ

(c) 30kΩ (d) 40kΩ +5

(e) None of the above


+2 for the model

$\frac{R_{in}}{10k + R_{in}} = 0.8 \rightarrow 0.2 R_{in} = 8k\Omega$

$\rightarrow R_{in} = 40k\Omega$

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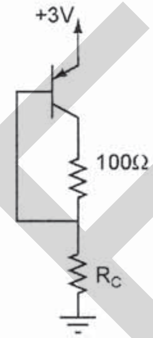
***1.c.*** [5 marks] Assume $V_A = \infty$, $\beta = \infty$ (i.e., ignore the base current) and $R_C = 2.4k\Omega$. What should the value of $I_s$ be such that the base-collector junction is forward biased by 100mV?




(a) $4.83 \times 10^{-15} A$ (b) $1.27 \times 10^{-17} A$

(c) $3.76 \times 10^{-14} A$ (+5) (d) $8.03 \times 10^{-12} A$

(e) None of the above

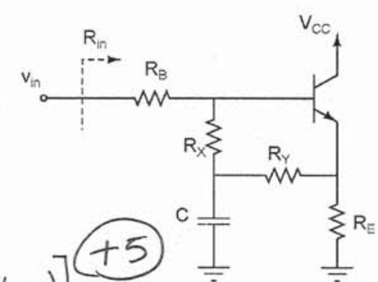
$I_C = 1mA$ (+2)

$3 = 0.025 Ln \frac{1mA}{I_s} + 2.4k \times 1mA$

$\Rightarrow I_s = 3.775 \times 10^{-14} A$

***1.d.*** [5 marks] Assume the circuit is properly biased to keep the transistor in active region and $V_A < \infty$ (i.e., take Early effect into account). Using formulas provided on page 3, find out the input resistance, $R_{in}$, of this circuit.

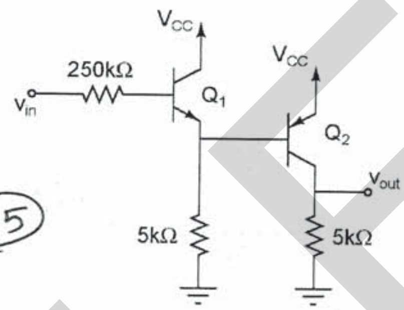
The capacitor is short-circuit for small-signal analysis.




$R_{in} = R_B + R_X || [r_\pi + (\beta+1)(R_E || R_Y || r_o)]$ (+5)

(+3) if $r_o$ is omitted

***1.e.*** [5 marks] Assume the circuit is properly biased to keep the transistors in active region and $\beta = 200$, $V_A = \infty$, and $I_{C1} = I_{C2} = 1mA$. Use formulas on page 3 to find the voltage gain, $A_v = \frac{v_{out}}{v_{in}}$, of this circuit.




CC CE

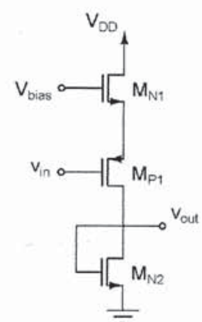
(a) $A_v = 132\ V/V$ (b) $A_v = -132\ V/V$ +5

(c) $A_v = 198\ V/V$ (d) $A_v = -198\ V/V$

(e) None of the above

+2 if (a) is seleted

***1.f.*** [5 marks] Assume the circuit is properly biased to keep the transistors in active region and that the transcunductance of the PMOSFETs is twice as big as that of the NMOSFETs; i.e., $g_{mn1} = g_{mn2} = 0.5\ g_{mp}$. Using formulas on page 3, find the voltage gain, $A_v = \frac{v_{out}}{v_{in}}$, of the circuit.




+5 (a) $A_v = -{}^{2}/_{3}$ (b) $A_v = +{}^{2}/_{3}$ +2

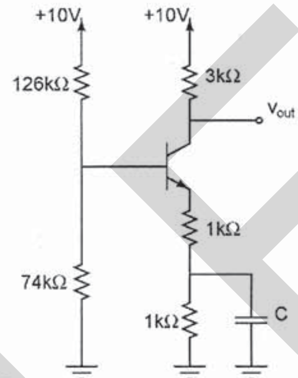
(c) $A_v = -{}^{1}/_{3}$ (d) $A_v = +{}^{1}/_{3}$

(e) None of the above

$$A_v = \frac{-1/g_{mn}}{1/g_{mp} + 1/g_{mp}} = \frac{-1}{+1+0.5}$$

+2 for right formula & negative answer

***1.g.*** [5 marks] Assume $V_A = \infty$, $V_{BE,on} = 0.7V$, $V_{CE,sat} = 0.2V$, and $\beta = \infty$ (i.e., ignore the base current). What is the maximum symmetric swing at the output? In other words, what is the peak-to-peak value of the largest output signal without distortion?




Maximum symmetric swing = 4.6✓ V +1

What limits the swing, cut-off or saturation? Saturation +2

+1 for 4.12

$I_{CQ} = 1.5mA$

$V_{CQ} = 5.5$

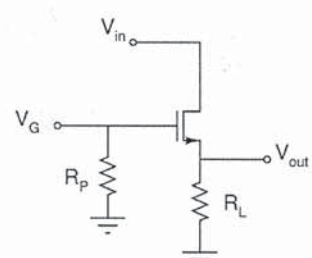
$\min V_C = 3.2$

$\max V_C = 10$

$\rightarrow \Delta V_{C\max} = 2.3V$ +2 $\rightarrow$ Pk-Pk swing $= 4.6V$

Saturation

***1.h.*** [5 marks] We know that the threshold voltage of the transistor shown here is less than $2.5V$. Assume that $R_L = 100\Omega$ and $V_{in} = 100mV$. For $V_G = 3V$, $v_{out} = 80mV$ and for $V_G = 4V$, $v_{out} = 90mV$. What is the threshold voltage of this transistor?




(a) $2.0V$

(b) $2.2V$ +5

(c) $2.1V$

(d) $2.3V$

(e) None of the above

$r_{DS1} = 100\left(\frac{100}{80} - 1\right) = 25\Omega$

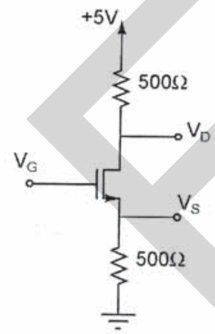
$r_{DS2} = 100\left(\frac{100}{90} - 1\right) = \frac{100}{9}\Omega$

$\rightarrow \frac{r_{ds1}}{r_{DS2}} = \frac{25 \times 9}{100} = \frac{4 - V_{TH}}{3 - V_{TH}}$

$\rightarrow 27 - 9V_{TH} = 16 - 4V_{TH} \Rightarrow V_{TH} = \frac{11}{5} = 2.2V$

**Question 2.** [15 marks]

For the transistor shown in the circuit to the right, $K_n = 50\ mA/V^2$, $\lambda = 0$, and $V_{TH} = 1V$. We slowly change $V_G$ from 0 to +5V.
At what gate voltages does the operating region of the transistor change from one to the other?




Cut-off: $0 < V_G < 1$ (+2)

Saturation: $1 < V_G < 3.7$

Triode: $3.7 < V_G < 5$ (+1)

$V_D = V_G - V_{TH} = V_{DD} - I_D R_D$ (+3)

$V_G = V_{GS} + I_D R_D$ (+3)

$5 - 500\, I_D = V_{GS} + 500\, I_D - 1$ (+3)

$\Rightarrow \begin{cases} V_{GS} = 1.43 \checkmark \\ V_{GS} = 0.53 \end{cases}$

$I_D = 4.63\,mA \rightarrow V_G = 3.714\,V$ (+3)

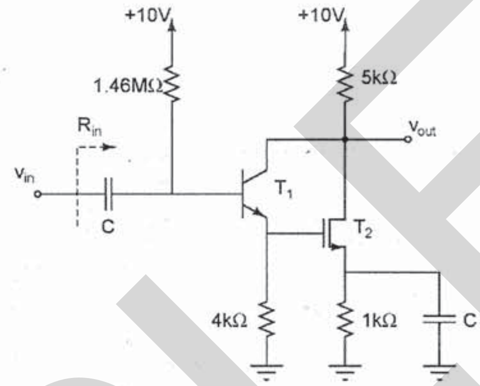
**Question 3.** [20 marks]

Consider the circuit shown to the right.

For the BJT, $\beta = 100$, $V_A = \infty$, $V_{BE,on} = 0.7V$, $V_{CE,sat} = 0.2V$, and $V_A = \infty$.

For the MOSFET, $K_n = 100\ mA/V^2$, $\lambda = 0$, and $V_{TH} = 1V$.

Capacitors are assumed AC short circuits.




***3.a.*** [8 marks] Calculate the operating point parameters of both transistors.

$I_C =$ 0.5 $mA$ $I_D =$ 0.87 $mA$

$V_{CE} =$ 1.15 $V$ $V_{DS} =$ 2.26 $V$

$I_C = 0.5mA$ +3

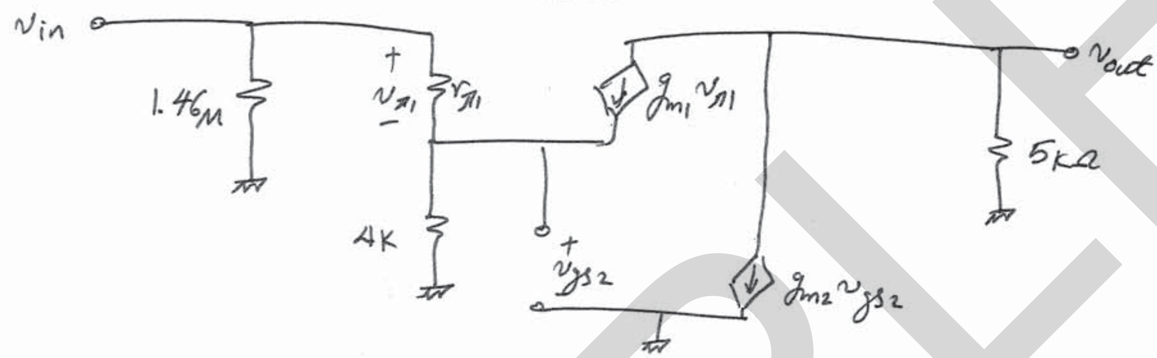
$I_D = 0.87\ mA$, $V_{GS} = 1.132$ +3

$V_{CE1} = 1.15_V$ +1

$V_{DS2} = 2.26_V$ +1

***3.b.*** [4 marks] Draw the small signal model of the circuit.

−1 for Floating nodes, missing parts




***3.c.*** [8 marks] Use the small signal model to calculate the voltage gain of the circuit.

$A_v = \frac{v_{out}}{v_{in}} =$ −66.3 V/V

$v_{in} = v_{\pi 1} + v_{gs2}$

$v_{out} = -(g_{m1} v_{\pi 1} + g_{m2} v_{gs2}) R_D$ +2

$v_{gs2} = \left(\frac{v_{\pi 1}}{r_{\pi 1}} + g_{m1} v_{\pi 1}\right) \times 4k\Omega$

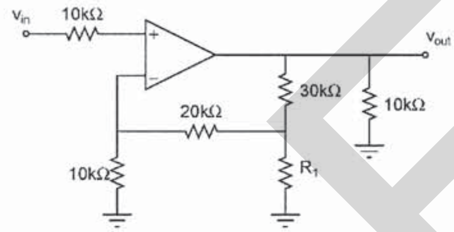
$v_{in} = 81.8\, v_{\pi 1} = \frac{8\cancel{0}.8}{81.8} v_{gs2}$

$\cancel{v_{gs}}$

$\rightarrow A_v = -66.3$ +4

**Question 4.** [15 marks]

Assume the OpAmp in circuit shown here is ideal.




***3.a.*** [10 marks] What is the voltage gain of the circuit if $R_1 = 2k\Omega$.

$A_v = \frac{v_{out}}{v_{in}} =$ +51 V/V

$$\frac{V_x}{R_1} + \frac{V_x - V^-}{20k} + \frac{V_x - V_o}{30k} = 0 \quad +2$$

$$\frac{V_x - V^-}{20k} = \frac{V^-}{110k} \quad +2$$

Ignoring 10K at $v^+$ & $v_o$ → +4

$R_1 = 2K\Omega = +51$

+2

***3.b.*** [5 marks] What is the voltage gain of the circuit if $R_1 \to \infty$.

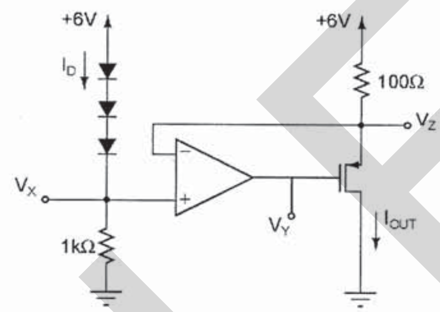
$A_v = \frac{v_{out}}{v_{in}} =$ +6 V/V

$R_1 \to \infty \to A_v = +6\,V/V$

+5

**Question 5.** [10 marks]

Assume the OpAmp in circuit shown here is ideal. For the MOSFET, $K_p = 200\ mA/V^2$, $\lambda = 0$, and $V_{TH} = -2V$. The diodes are identical.




***5.a.*** [3 marks] For each diode, $I_D = I_S\, e^{\frac{V_D}{nV_T}}$ where $I_S = 10^{-9}A$ and $n = 2$. What is $V_X$?

$V_X =$ 3.8 $V$

$6 = 2 \times 3 \times 0.025 \ln \frac{I_D}{10^{-9}} + I_D \times 1k$ ★

$I_D = 3.9mA \rightarrow$ ★ $= 6.17$

$I_D = 3.8mA \rightarrow V_X = 3.8V$

***5.b.*** [4 marks] What is $I_{OUT}$?

$I_{OUT} =$ 22 $mA$

$I_{out} = \frac{2.2}{100} = 22\,mA$ +2

+2

***5.c.*** [3 marks] What is $V_Y$?

$V_Y =$ 1.33 $V$

$I_D = 0.1(V_{GS}+2)^2 \rightarrow V_{GS} = \begin{cases} -2.47V & ✓ \\ -1.53V & X \end{cases}$ +2

$V_y - V_z = -2.47V \rightarrow V_y = 3.8 - 2.47 = 1.33V$

+2

**You may use this space to answer any of the questions with proper reference to the question.**

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