Simon Fraser University

Mechatronic Systems Engineering

Final Exam for ENSC 226: Electronic Circuits

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16 April 2012

Time: 150 minutes

Name: Solutions

Student number: ______________

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Question	Mark
1	/ 10
2	/ 15
3	/ 20
4	/ 20
5	/ 20
6	/ 10
7	/ 15
Total	/110

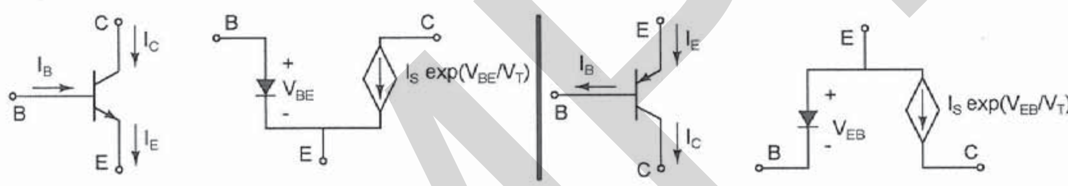
Physical model of a diode: $I_D = I_s\left(e^{\frac{V_D}{n\,V_T}} - 1\right)$

For BJTs:

	Active	Saturation	Cut-off
	BE forward biased BC reverse biased	BE forward biased BC forward biased	BE reverse biased BC reverse biased
npn	$I_C = \beta\, I_B = \alpha\, I_E\,, V_{BE} \approx 0.7V$	$I_B > \frac{I_C}{\beta}\,, V_{CE} \approx 0.2V$	$I_B \approx I_C \approx I_E \approx 0$
pnp	$I_C = \beta\, I_B = \alpha\, I_E\,, V_{EB} \approx 0.7V$	$I_B > \frac{I_C}{\beta}, V_{EC} \approx 0.2V$	$I_B \approx I_C \approx I_E \approx 0$

$$I_C(\text{active}) = I_S\, e^{\frac{|V_{BE}|}{V_T}}\left(1 + \frac{|V_{CE}|}{V_A}\right) \qquad \alpha = \frac{\beta}{\beta+1} \qquad V_T = 25mV$$

$$g_m = \frac{1}{r_m} = \frac{I_C}{V_T} \qquad r_\pi = \beta\, r_m = \frac{\beta}{g_m} \qquad r_o = \frac{V_A}{I_C}$$



For MOSFETs:

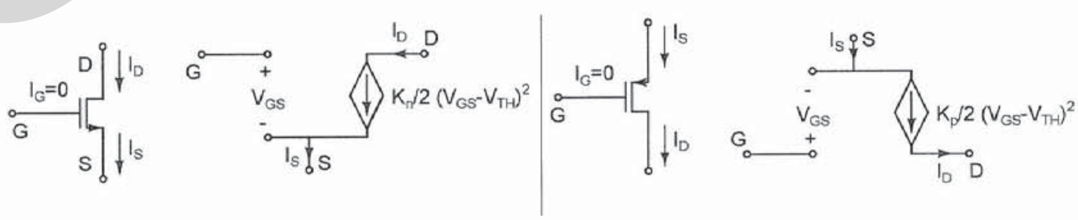
	Saturation	Triode	Cut-off
NFET (V_{TH}>0)	$V_{GS} > V_{TH}$ & $V_{GD} < V_{TH}$	$V_{GS} > V_{TH}$ & $V_{GD} > V_{TH}$	$V_{GS} < V_{TH}$ & $V_{GD} < V_{TH}$
PFET (V_{TH}<0)	$V_{GS} < V_{TH}$ & $V_{GD} > V_{TH}$	$V_{GS} < V_{TH}$ & $V_{GD} < V_{TH}$	$V_{GS} > V_{TH}$ & $V_{GD} > V_{TH}$

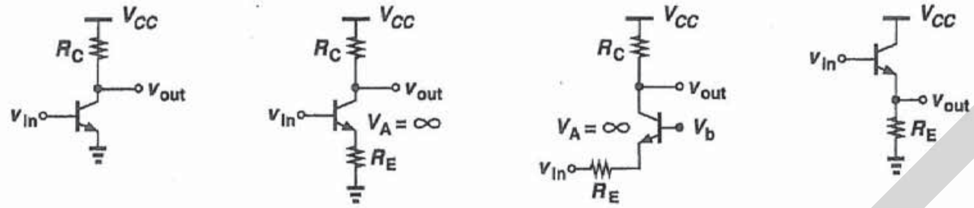
$$I_D(saturation) = \frac{K'}{2}\,(V_{GS} - V_{TH})^2\,(1 + \lambda V_{DS})$$

$K' \equiv K_n = \mu_n C_{ox}\, {}^{W}\!/_{L}$ for NFETS
$K' \equiv K_p = \mu_p C_{ox}\, {}^{W}\!/_{L}$ for PFETS

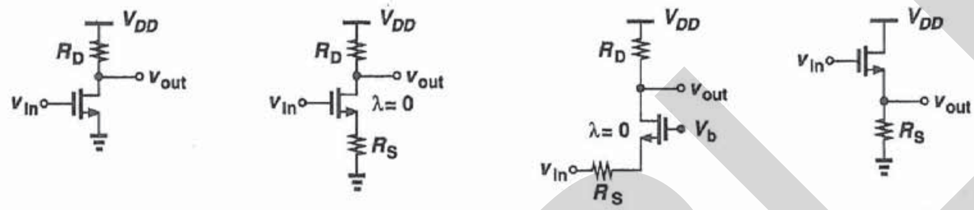
$$I_D(triode\,) = \frac{K'}{2}\,[2(V_{GS} - V_{TH})V_{DS} - V_{DS}^2] \qquad r_{DS}(deep\ triode) \approx \frac{1}{K'(V_{GS} - V_{TH})}$$

$$g_m = K'(V_{GS} - V_{TH}) = \sqrt{2\,K' I_D} = \frac{2I_D}{V_{GS} - V_{TH}} \qquad r_o = \frac{1}{\lambda I_D}$$

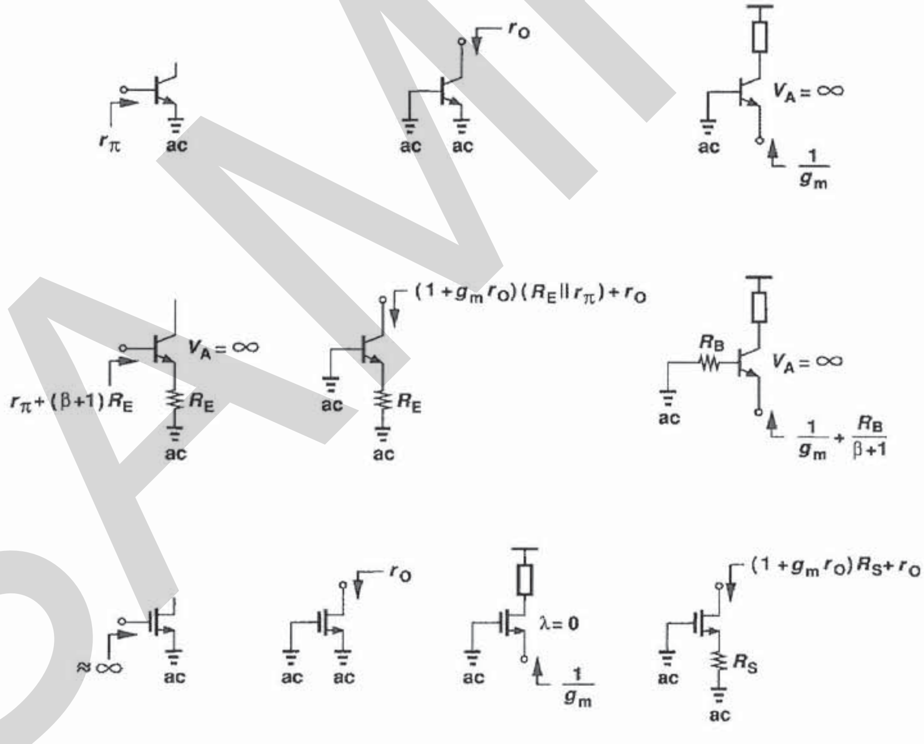


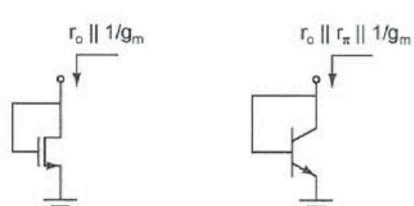


$A_v = -g_m(R_C \| r_O)$ $A_v = -\dfrac{R_C}{\frac{1}{g_m}+R_E}$ $A_v = \dfrac{R_C}{\frac{1}{g_m}+R_E}$ $A_v = \dfrac{R_E \| r_O}{\frac{1}{g_m}+R_E \| r_O}$



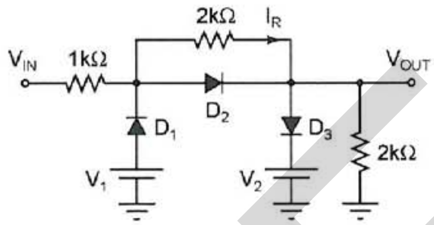
$A_v = -g_m(R_D \| r_O)$ $A_v = -\dfrac{R_D}{\frac{1}{g_m}+R_S}$ $A_v = \dfrac{R_D}{\frac{1}{g_m}+R_S}$ $A_v = \dfrac{R_S \| r_O}{\frac{1}{g_m}+R_S \| r_O}$





Question 1 [10 marks]

Consider the diode circuit to the right where the diodes are **ideal**, $V_1 = -5V$, and $V_2 = +2V$. The input voltage is slowly increased from $-10V$ to $+10V$.

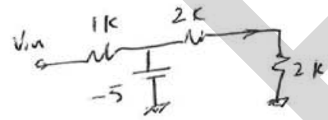


1.a. [5 marks] What is the range for I_R?

$-1.25mA < I_R <$ 0

(1)

$V_{in} \to -\infty \to D_1: on, D_2: off, D_3: off$

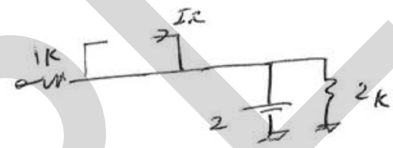


$V_{in} \Rightarrow -10 \to I_R = \frac{-5}{4K} = -1.25mA$ (2)

Diodes are OK

(1)

$V_{in} \to +\infty \to D_1: off, D_2: on, D_3: on$



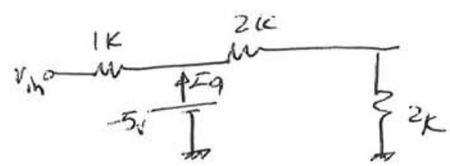
$\to I_R = 0$ (1)

Diodes are OK

1.b. [5 marks] Diode D_1 turns off as the input voltage is increased from $-10V$. What is the minimum input voltage that switches diode D_1 off?

(1)

$V_{in} \to -\infty \to D_1: on, D_2: off, D_3: on$

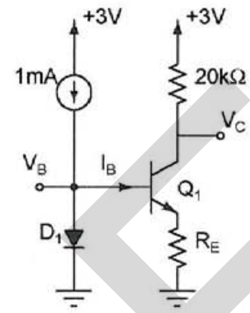


$$I_{D1} = \frac{-5 - V_{in}}{1K} - \frac{5}{4K}$$ (2)

$I_{D1} = 0 \to V_{in} = -(5 + \frac{5}{4}) = \underline{-6.25V}$

$\underline{V_{in} = -6.25V}$ (2)

Question 2 [15 marks]



Consider the circuit to the right where the base current of Q_1 can be ignored (i.e., $I_B \approx 0$ or $\beta \to \infty$). For both the transistor and the diode, assume $V_T = 25mV$, $I_S = 10^{-15}A$, and $V_A = \infty$. Provide the answers with three significant digits.

2.a. [5 marks] What is V_B?

$V_B =$ 0.691 V

$$\underbrace{V_B = V_T \ln \frac{I_D}{I_S}}_{3} = 0.691 \quad 2$$

2.b. [7 marks] If $R_E = 1k\Omega$, what is the collector current of Q_1 (Hint: use iterations)?

$I_C =$ 0.0672 mA

$$V_B = V_{BE} + I_C R_E \quad 2$$

$$\rightarrow 0.691 = V_T \ln \frac{I_C}{I_S} + I_C R_E$$

at least 3 iterations 3

$I_C = 67.4 \mu A$ either 2

$V_{BE} = 0.623V$

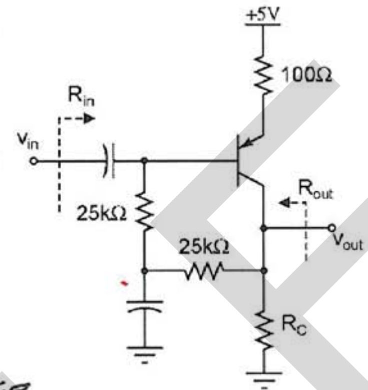
2.c. [3 marks] What is V_C for the collector current calculated in the above?

$V_C =$ 1.652 V

$$\underbrace{V_C = V_{CC} - I_C R_C}_{2} = 1.652V \quad 1$$

Question 3 [20 marks]

Consider the circuit shown to the right where the capacitors are assumed to be AC short circuits. Assume $\beta = 100$, $V_T = 25mV$, $V_{EB}(on) = 0.6V$, $V_{EC}(sat) = 0.2V$, and $|V_A| = \infty$.



3.a. [4 marks] If $R_C = 10k\Omega$, what is the collector current of the transistor? Verify the operating region of the transistor.

$I_C =$ 0.411 mA Operating region: Active

2

$$V_{CC} = I_E(100 + R_C) + I_B(50k) + V_{EB}$$

$\rightarrow I_C = 411\mu A$ 1

$V_C = 4.15V$, $V_E = 4.96V$

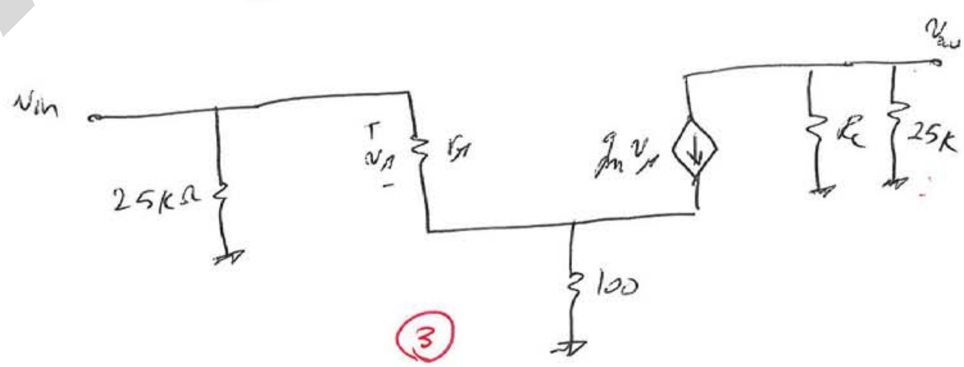
$\rightarrow V_{EC} \simeq 0.8V$, $V_{EB} \simeq 0.6V$ $\Big\}$ $\rightarrow$ Active 1

3.b. [4 marks] Calculate the small signal parameters and draw the small signal equivalent circuit.

$g_m =$ 16.4 mS $r_\pi =$ 6.08 $k\Omega$

$g_m = \frac{I_C}{V_T} = 16.4\ mS$ $r_\pi = \beta / g_m = 6.08k\Omega$

1



3

3.c. [4 marks] What is the voltage gain of the circuit?

$A_v = \frac{v_{out}}{v_{in}} =$ -44.2 V/V

$$A_V = \underbrace{\frac{-g_m R_C}{1 + g_m R_E}}_{2} = \frac{-16.4m \times (25k \parallel 10k)}{1 + 16.4m \times 100} = -44.2 V/V \quad 2$$

3.d. [4 marks] What is the input resistance of the circuit?

$R_{in} =$ 9.822 $k\Omega$

$$R_{in} = \overbrace{25k \parallel (r_\pi + (\beta+1)R_E)}^{2}$$

$$= 9.822 k\Omega \quad 2$$

3.e. [4 marks] What is the output resistance of the circuit?

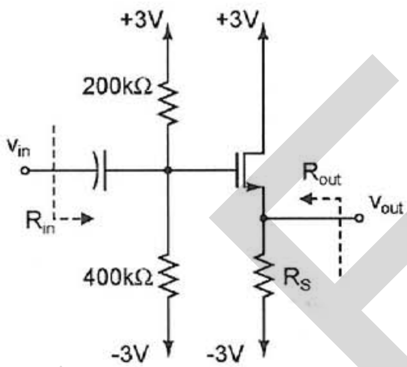
$R_{out} =$ 7.142 $k\Omega$

$$R_{out} = R_C \parallel 25k\Omega \quad 2$$

$$= 7.142 k\Omega \quad 2$$

Question 4 [20 marks]

Consider the circuit shown to the right where the capacitor is AC short circuit. Assume $R_S = 15k\Omega$, $V_{TH} = 2V$, $K_n = 1mA/V^2$, and $\lambda = 0$.



4.a. [6 marks] What is the drain current of the transistor? Verify the operating region of the transistor.

$I_D =$ 0.103 mA

Operating region: Active

$$V_G = -\frac{2}{6}\times 3 + \frac{4}{6}\times 3 = +1V \quad 1$$

$$V_G = V_{GS} + I_D R_S - 3 \rightarrow 1 = V_{GS} + \frac{15k\Omega \times 1mA/V^2}{2}(V_{GS}-2)^2 - 3$$

$$\underbrace{7.5V_{GS}^2 + 26 - 29V_{GS} = 0}_{1} \rightarrow V_{GS} = \begin{cases} 2.45V > V_{TH} \checkmark \; 1 \\ 1.41V < V_{TH} \end{cases}$$

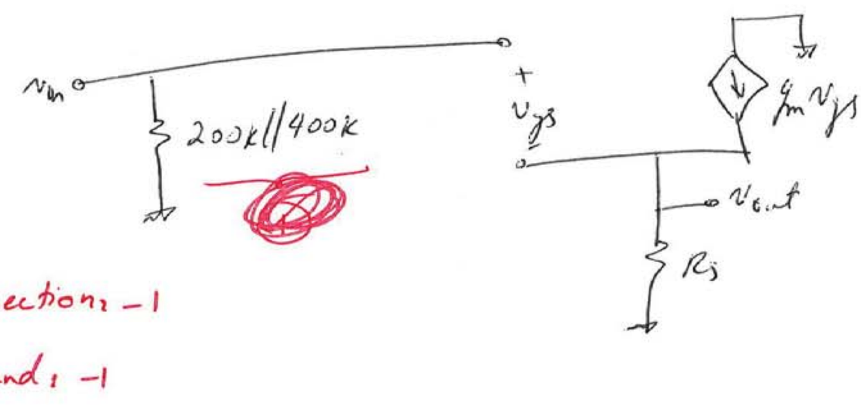
$$\rightarrow V_{GS} = 2.45V \rightarrow I_D = 103\mu A \quad 1$$

$$\left.\begin{matrix} V_D = 3V > V_G - V_{TH} = 1 - 2 \\ V_{GS} > V_{TH} \end{matrix}\right\} \rightarrow \text{Active} \quad 2$$

4.b. [3 marks] Calculate g_m and draw the small signal equivalent circuit.

$g_m =$ 0.454 mS

$$g_m = \sqrt{2K_n I_D} = 0.454mS$$



Wrong connection: -1

Missing ground: -1

4.c. [4 marks] What is the voltage gain of the circuit?

$A_v = \frac{v_{out}}{v_{in}} =$ 0.87 V/V

$$A_v = \underbrace{\frac{g_m R}{1 + g_m R_S}}_{2} = \frac{15k}{15k + 1/0.454mS} = 0.87 \, V/V \quad 2$$

4.d. [3 marks] What is the input resistance of the circuit?

$R_{in} =$ 133 $k\Omega$

$$R_{in} = \underbrace{200k \| 400k}_{2} = 133k\Omega \quad 1$$

4.e. [4 marks] What is the output resistance of the circuit?

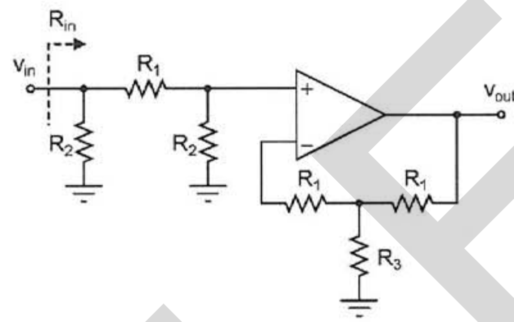
$R_{out} =$ 1.92 $k\Omega$

$$\underbrace{R_{out} = R_S \| 1/g_m}_{2} = 1920\Omega \quad 2$$

Question 5 [20 marks]

The OpAmps are ideal in circuits shown below.

5.a. [5 marks] What is the input resistance of the circuit as a function of resistor values?



$R_{in} =$ $R_2 // (R_1 + R_2)$

$R_{in} = R_2 // (R_1 + R_2)$

5

5.b. [5 marks] What is the voltage gain of the circuit assuming $R_1 = 2k\Omega$, $R_2 = 10k\Omega$, and $R_3 = 100\Omega$?

1

$A_v = \frac{v_{out}}{v_{in}} =$ $+21$ V/V

2

$v^+ = v_{in}\left(\frac{R_2}{R_1 + R_2}\right) = v^-$

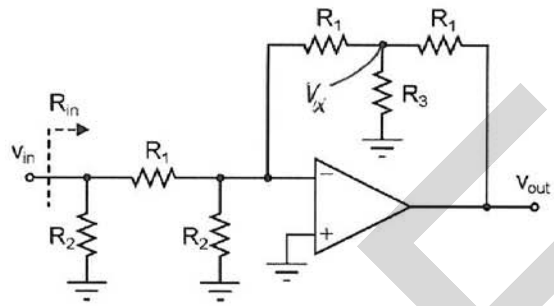
$I^- = 0 \rightarrow V_{out} = \left(1 + \frac{R_1}{R_3}\right) v^+$ 2

$\rightarrow V_{out} = \left(1 + \frac{R_1}{R_3}\right)\left(\frac{R_2}{R_1 + R_2}\right) v_{in}$

8

5.c. [4 marks] What is the input resistance of the circuit as a function of resistor values?

$R_{in} = \underline{R_1 \| R_2}$



$R_{in} = R_2 \| R_1$ 4

5.d. [6 marks] What is the voltage gain of the circuit assuming $R_1 = 2k\Omega$, $R_2 = 10k\Omega$, and $R_3 = 100\Omega$?

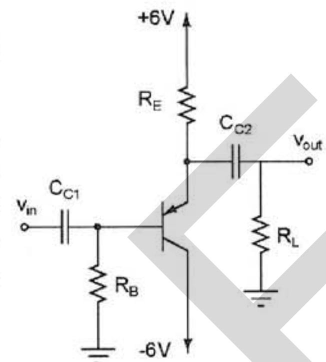
$A_v = \frac{v_{out}}{v_{in}} = \underline{-22}$ V/V 2

$$\left.\begin{aligned} &\frac{V_x}{R_3} + \frac{V_x}{R_1} + \frac{V_x - V_{out}}{R_1} = 0 \\ &V_x = -\frac{R_1}{R_1} v_{in} \end{aligned}\right\} \rightarrow v_{out} = -R_1 \underbrace{\left(\frac{2}{R_1} + \frac{1}{R_3}\right)}_{2} v_{in}$$

$$\rightarrow A_v = -\left(2 + \frac{R_1}{R_3}\right)$$ 2

Question 6 [10 marks]

Shown to the right is a common-collector circuit similar to what you studied in Lab 3.



6.a. [4 marks] We monitor the output signal while applying a signal with a constant amplitude to the input of the circuit. We notice that the amplifier gain at 1kHz is smaller than the gain at 100kHz. The gain remains relatively constant for frequencies higher than 100kHz. Explain what may be the most possible reason for this observation.

Impedance of coupling capacitors is high @ LF.

4

6.b. [6 marks] We measure the gain of the amplifier at 1MHz with $R_L = 100\Omega$ to be 0.9V/V. When we change the load resistance to $R_L = 200\Omega$, the gain increases to 0.95V/V. What is the output resistance of the circuit? (Hint: you do not need device parameters or component values.)

$$A_v = A_{vo} \frac{R_L}{R_L + r_o}$$

4

$$\left.\begin{aligned} &\to 0.9 = A_{vo} \frac{100}{100 + r_o} \\ &0.95 = A_{vo} \frac{200}{200 + r_o} \end{aligned}\right\} \to \underline{\underline{r_o = 11.76\,\Omega}}$$

2

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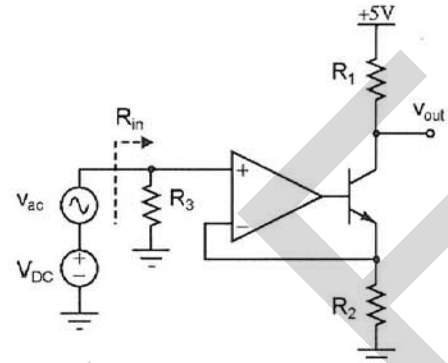
Question 7 [15 marks]



The OpAmp is ideal in circuit shown to the right.

The transistor parameters are $\beta = 100$, $V_T = 25mV$, $V_{BE}(on) = 0.6V$, and $V_{CE}(sat) = 0.2V$. Assume $R_1 = 35k\Omega$, $R_2 = 1k\Omega$, and $R_3 = 1M\Omega$.

V_{DC} is a $100mV$ DC voltage source and v_{ac} is a small signal AC voltage source.

7.a. [4 marks] What are the DC collector current and voltage of the transistor?

$I_C =$ 0.0991 mA $V_C =$ 1.53 V

$$I_C = \frac{V_{DC}}{R_2} \times \frac{\beta}{\beta+1} = \frac{0.1V}{1k} \times \frac{100}{101} = 99.1\mu A \quad 3$$

$$\rightarrow V_C = 5 - I_C R_C = 5 - 99.1\mu A \times 35k\Omega = 1.531 \quad 1$$

7.b. [7 marks] What is the small signal voltage gain of the circuit?

$A_v = \frac{v_{out}}{v_{ac}} =$ -34.65 V/V

$$\overbrace{v_{out} = -i_c R_1 = -\frac{v_{ac}}{R_2} R_1 \times \frac{\beta}{\beta+1}}^{4} \Rightarrow \overbrace{A_v = -\frac{R_1}{R_2} \times \frac{\beta}{\beta+1}}^{2}$$

$$\rightarrow A_v = -34.65 V/V \quad 1$$

7.c. [4 marks] What is the input resistance of the circuit?

$R_{in} =$ $1M\Omega$

$$R_{in} = R_3 = 1M\Omega \quad 4$$

You may use this space to answer any of the questions with proper reference to the question.