Fluid Mechanics- MSE 223

Final Exam- Apr, 15, 2017

SFU

Student Name: **Student Number:**

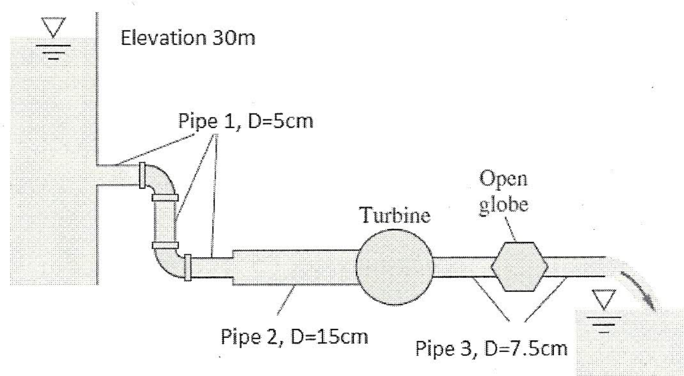
Q1- The size ***d(m)*** of droplets produced by a liquid spray nozzle is thought to depend upon the nozzle diameter ***D(m)***, jet velocity ***U(m/s)***, and the properties of the liquid; density ρ (kg/m^3), viscosity μ(kg/m.s) and surface tension **Y**(N/m). Find the dimensionless parameters by using PI theorem. Take ***D***, ρ, and ***U*** as repeating variables.

(Score: 15)

Q2- Find the total acceleration of a particle if the velocity vector field is given by: $\vec{V} = 5tx\vec{i} + 3txz\vec{j} + 2ty^2\vec{k}$ and compute the acceleration vector at the point (x,y,z)=(1,1,0).

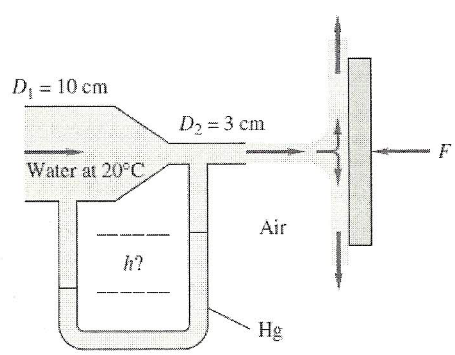
(Score: 30)

Q3- In figure below, pipe 1 is 37 m long with 5-cm diameter; pipe 2 is 23 m long with 15-cm diameter and pipe 3 is 45 m long with 7.5-cm diameter, all cast iron ($\epsilon = 0.26mm$). There are two 90° regular elbows (K=0.95), and an open globe valve (K=6.3), all screwed. If the exit elevation is zero, what horsepower is extracted by the turbine when the flow rate is 4530 cm^3/s of water at 20°C? Pipe 1 has a sharp entrance (K=0.5) and a sudden expansion K=0.79. Also exit loss of pipe 3 is K=1.0. Assume the friction factors for the pipes are: (f_1=0.0315, f_2=0.027 and f_3=0.029) ($\rho_{water} = 998\ {kg}/{m^3}$, $\mu_{water} = 1.003E-3\ kg/(m.s)$) **(Score: 30)**



Q4- Water flows through a circular nozzle, exits into the air as a jet, and strikes a plate, as shown in figure below. The force required to hold the plate steady is 70 N. Assuming frictionless one-dimensional flow, find:
(a) the velocities at sections (1) and (2); (b) the mercury manometer reading **h**.

($\rho_{water} = 998\ {kg}/{m^3}$ and $\rho_{mercury} = 13550\ {kg}/{m^3}$) **(Score: 25)**



Good Luck - Sam Behfarshad

Q1 - Final Exam, MSE 223 - SFU, Intro to Fluid Mech., Spring 2017

Given: Size of droplet = $d = f(D, U, \rho, \mu, Y)$, Repeating Var. = D, ρ, U

$Y \longrightarrow$ surface tension (N/m) $\hookrightarrow \frac{kg\,m}{s^2} \cdot \frac{1}{m}$

Find: $(P_i)_s$

Six variables, $d, D, U, \rho, \mu, Y \Rightarrow n = 6$

$d \to L$, $D \to L$, $U \to LT^{-1}$, $\rho \to ML^{-3}$, $\mu \to ML^{-1}T^{-1}$, $Y \to MT^{-2}$

$j = 3$, $(L, M, T) \Rightarrow$

No of $\pi = n - j = 6 - 3 = 3$, π_1, π_2, π_3, Repeating Var. = D, ρ, U

$\pi_1 = D^a \rho^b U^c d = (L)^a (ML^{-3})^b (LT^{-1})^c (L) = L^0 M^0 T^0$

For L $\Rightarrow$ $a - 3b + c + 1 = 0 \Rightarrow \boxed{a = -1}$

" M $\Rightarrow$ $b = 0 \Rightarrow \boxed{b = 0}$

" T $\Rightarrow$ $-c = 0 \Rightarrow \boxed{c = 0}$

$\Rightarrow \pi_1 = D^{-1} d \Rightarrow \boxed{\pi_1 = \frac{d}{D}}$

5/15

$\pi_2 = D^a \rho^b U^c \mu = (L)^a (ML^{-3})^b (LT^{-1})^c (ML^{-1}T^{-1}) = M^0 L^0 T^0$

for L $\Rightarrow$ $a - 3b + c - 1 = 0 \Rightarrow a - 3(-1) + (-1) - 1 = 0 \Rightarrow \boxed{a = -1}$

" M $\Rightarrow$ $b + 1 = 0 \Rightarrow \boxed{b = -1}$

" T $\Rightarrow$ $-c - 1 = 0 \Rightarrow \boxed{c = -1}$

$\Rightarrow \pi_2 = D^{-1} \rho^{-1} U^{-1} \mu = \frac{\mu}{D \rho U} \Rightarrow \boxed{\pi_2 = \frac{D \rho U}{\mu}}$

Re. No.

5/15

$\pi_3 = D^a \rho^b U^c Y = (L)^a (ML^{-3})^b (LT^{-1})^c (MT^{-2}) = M^0 L^0 T^0$

for L $\Rightarrow$ $a - 3b + c = 0 \Rightarrow a - 3(-1) + (-2) = 0 \Rightarrow \boxed{a = -1}$

" M $\Rightarrow$ $b + 1 = 0 \Rightarrow \boxed{b = -1}$

" T $\Rightarrow$ $-c - 2 = 0 \Rightarrow \boxed{c = -2}$

$\Rightarrow \pi_3 = D^{-1} \rho^{-1} U^{-2} Y \Rightarrow \boxed{\pi_3 = \frac{Y}{\rho D U^2}}$

5/15

So $\frac{d}{D} = f\left(\frac{\rho D U}{\mu}, \frac{\rho U^2 D}{Y}\right)$ 3π

Ans.

Q2, Final Exam, MSE-223, Fluid Mechanics, SFU, Spring 2017

Given: $\vec{V} = 5tx\,\vec{i} + 3txz\,\vec{j} + 2ty^2\,\vec{k}$

Find: $\vec{a} = ?$

$u = 5tx$, $V = 3txz$, $w = 2ty^2$

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}$$ ← Total derivative

Step ① $$\frac{\partial \vec{V}}{\partial t} = \frac{\partial u}{\partial t}\vec{i} + \frac{\partial v}{\partial t}\vec{j} + \frac{\partial w}{\partial t}\vec{k} = (5x)\vec{i} + (3xz)\vec{j} + (2y^2)\vec{k}$$ 5/30

Step ②

$$u\frac{\partial V}{\partial x} = (5tx)\frac{\partial}{\partial x}\left(5tx\vec{i} + 3txz\vec{j} + 2ty^2\vec{k}\right) = (5tx)(5t\vec{i} + 3tz\vec{j} + 0\vec{j})$$
$$= 25t^2x\vec{i} + 15t^2xz\vec{j}$$ 5/30

$$v\frac{\partial V}{\partial y} = (3txz)\frac{\partial}{\partial y}(\quad " \quad) = 3txz(0\vec{i} + 0\vec{j} + 4ty\vec{k})$$
$$= 12t^2xyz\,\vec{k}$$ 5/30

$$w\frac{\partial V}{\partial z} = (2ty^2)\frac{\partial}{\partial z}(\quad " \quad) = (2ty^2)(0\vec{i} + 3tx\vec{j} + 0\vec{k})$$
$$= 6t^2xy^2\vec{j}$$ 5/30

Step 3: Combine all 4 items above.

$$\vec{a} = \frac{d\vec{V}}{dt} = \left[(5x)\vec{i} + (3xz)\vec{j} + (2y^2)\vec{k}\right] + \left[(25t^2x)\vec{i} + (15t^2xz)\vec{j}\right] + \left[(12t^2xyz\vec{k})\right] + (6t^2xy^2)\vec{j}$$

$$\vec{a} = \left[(5x) + (25t^2x)\right]\vec{i} + \left[(3xz) + (15t^2xz) + (6t^2xy^2)\right]\vec{j} + \left[(2y^2) + (12t^2xyz)\right]\vec{k}$$ 5/30

Ans. a

at $(x, y, z) = (1, 1, 0)$

$$\Rightarrow \vec{a} = \left[(5\times1) + (25\times1\times t^2)\right]\vec{i} + \left[0 + 0 + (6\times1\times1\times t^2)\right]\vec{j} + \left[(2\times1) + (0)\right]\vec{k}$$

$$\vec{a} = (5 + 25t^2)\vec{i} + 6t^2\vec{j} + 2k$$ 5/30

Ans. b

Another way:

$\vec{a} = \vec{a_x} + \vec{a_y} + \vec{a_z}$,

$$\begin{cases} a_x = \frac{du}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} \\ a_y = \frac{dv}{dt} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} \\ a_z = \frac{dw}{dt} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} \end{cases}$$

Q3-Final Exam, MSE-223, SFU, SPRING 2017.

Given: $\varepsilon = 0.26\,mm$, $\Delta Z = 30\,m$, $Q = 4530\ cm^3/s = 0.00453\ m^3/s$

Pipe 1 $\Rightarrow$ $L_1 = 37\,m \Rightarrow \frac{L}{D}\Big|_1 = 740$; $D_1 = 5\,cm = 0.05\,m \Rightarrow \frac{\varepsilon}{D}\Big|_1 = 0.0052$

Pipe 2 $\Rightarrow$ $L_2 = 23\,m \Rightarrow \frac{L}{D}\Big|_2 = 153.3$; $D_2 = 15\,cm = 0.15\,m$; $\frac{\varepsilon}{D}\Big|_2 = 0.00173$

Pipe 3 $\Rightarrow$ $L_3 = 45\,m$, $\frac{L}{D}\Big|_3 = 600$; $D_3 = 7.5\,cm = 0.075\,m$; $\frac{\varepsilon}{D}\Big|_3 = 0.00347$

2 Elbow $\Rightarrow 2\times K = 0.95$
1 Open Glove Valve $\Rightarrow K = 6.3$
Sharp Enterance $\Rightarrow K = 0.5$
Exit $\Rightarrow K = 1.0$

$\rho_w = 998\ kg/m^3$

$\mu_w = 1.003E{-3}\ \frac{N.S}{m^2}$

$$Q_1 = A_1 V_1 \Rightarrow V_1 = \frac{Q_1}{A_1} = \frac{0.00453}{\frac{\pi}{4}(0.05)^2} = 2.3\ m/s \quad , \quad V_2 = \frac{Q}{A_2} = \frac{0.00453}{\frac{\pi}{4}(0.15)^2} = 0.256\ m/s$$

$$V_3 = \frac{Q}{A_3} = \frac{0.00453}{\frac{\pi}{4}(0.075)^2} = 1.0254\ m/s$$

6/30 $\Rightarrow V_1, V_2, V_3$

$$Re_1 = \frac{\rho_1 V_1 D_1}{\mu} = \frac{998\times 2.3\times 0.05}{1.003E{-3}} = 114427 \quad , \quad Re_2 = \frac{998\times 0.256\times 0.15}{1.003E{-3}} = 38208$$

$$Re_3 = \frac{998\times 1.0254\times 0.075}{1.003E{-3}} = 76521$$

Exit vel. is zero

Eq 3.75 $\Rightarrow$ Ber. Eq. BTW A & B $\rightarrow$ Reservior top surf, Reservior Down stream

$$\left(\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1\right) = \left(\frac{p_2}{\rho g} + \frac{V_2^2}{2g} + Z_2\right) + h_{Turb.} - h_{Pump} + h_{fric} + h_{min\ loss}$$

Head loss $\Rightarrow$ 12/30

$$\Rightarrow \Delta Z = Z_1 - Z_2 = h_T + h_{fric} + h_{min} \Rightarrow h_T = \Delta Z - \sum h_f - \sum h_{min}.$$

$\Delta Z = 30\,m$,

We now have to find friction factor for each pipe from Moody diag.

with $Re_1 = 114427$ & $\frac{\varepsilon_1}{D_1} = 0.0052 \Rightarrow f_1 = 0.0315$

" $Re_2 = 38208$ & $\frac{\varepsilon_2}{D_2} = 0.00173 \Rightarrow f_2 = 0.027$

" $Re_3 = 76521$ & $\frac{\varepsilon_3}{D_3} = 0.00347 \Rightarrow f_3 = 0.029$

$h_{f_1} = f_1 \frac{L_1 V_1^2}{D_1\, 2g} = 6.285\ m$

$h_{f_2} = f_2 \frac{L_2}{D_2}\frac{V_2^2}{2g} = 0.01383\ m$

$h_{f_3} = f_3 \frac{L_3}{D_3}\frac{V_3^2}{2g} = 0.9325\ m$

$$h_T = 30 - \frac{(2.3)^2}{2(9.81)}\Big[\overbrace{0.0315(740)}^{23.31} + 0.5 + 2(0.95) + \underbrace{0.79}_{\text{exp. loss}}\Big] - \frac{(0.256)^2}{2(9.81)}\Big[\overbrace{(0.027)(153.3)}^{4.139}\Big]$$

3.19

$$- \frac{(1.0254)^2}{2(9.81)}\Big[(0.029)(600) + \overbrace{6.3 + 1}^{7.3}\Big]$$

$h_T \rightarrow$ 6/30

$$h_T = 30 - 7.145 - 0.0138 - 1.324 = 21.518\ m$$

$$\text{Turbine Power} = \rho g Q h_T = (998)(9.81)(0.00453)(21.518) = 954.33\ \text{Watt}$$

$$P_{Turbine} = \frac{954.33\ W}{745.70\ \frac{W}{hp}} = 1.2798\,hp \simeq 1.28\ hp.$$

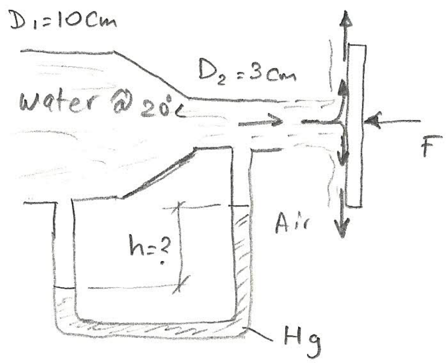
Ans.

$\Rightarrow$ 6/30

Q4, Final Exam, MSE-223, Fluid Mechanics, SFU, Spring 2017

(a) V_1 & V_2 = ?

(b) h = ?



First from momentum equ. at point 2:

$$\sum F = F = -\dot{m}_{in} u_{in} = -\rho A_2 V_2^2$$

$$\Rightarrow -70 = -(998)\left(\frac{\pi}{4}\right)(0.03)^2(V_2^2)$$

$$\Rightarrow V_2^2 = 99.23 \Rightarrow \boxed{V_2 = 9.96\ m/s}$$ Ans. 6/30

$$Q_1 = Q_2 \Rightarrow V_1 A_1 = V_2 A_2 \Rightarrow V_1 = \frac{V_2 A_2}{A_1} = \frac{9.96 \times \frac{\pi}{4}(0.03)^2}{\frac{\pi}{4}(0.1)^2} = 0.8965 \approx 0.9\ m/s$$

$$\Rightarrow \boxed{V_1 = 0.9\ m/s}$$ 6/30

Ans.

Applying Bernoulli's equ bet. 1 & 2

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + \cancel{z_1}^{0} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + \cancel{z_2}^{0}$$

$$\frac{P_1 - P_2}{\rho \cancel{g}} = \frac{V_2^2 - V_1^2}{2\cancel{g}} \Rightarrow \Delta P = \rho\left(\frac{V_2^2 - V_1^2}{2}\right) = \frac{1}{2}(998)(9.96^2 - 0.9^2) = 49097.4\ Pa$$ 6/30

$$P_1 + \gamma_w h - \gamma_m h = P_2 \Rightarrow P_1 - P_2 = (\gamma_m - \gamma_w) h = \Delta P$$

$$\Rightarrow h = \frac{\Delta P}{(\gamma_m - \gamma_w)} = \frac{49097}{(13550 - 998) \times (9.81)} = 0.3987\ m$$

$$\boxed{h = 0.4\ m} = 40\ cm$$ 7/30