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SIMON FRASER UNIVERSITY
School of Mechatronic Systems Engineering
Quiz I: Monday, May 27^{th} - Summer 2019

SFU

Last name, First name:

Name: Student #: Mark: /10

1- Consider the equation of motion for a single DOF mass-spring-damper system

$$m\ddot{x}(t) = f(t) - b\dot{x}(t) - kx(t),$$

where the constants $m(kg)$, $b(Ns/m)$ and $k(N/m)$ are respectively mass, viscous damping constant and spring constant, $x(m)$ is the displacement of the mass from the static equilibrium, and $f(N)$ is the applied force.

a) Using the Laplace transform, obtain the trajectory $x(t)$ of the mass when we apply a force $f(t) = 0.1u(t)$ ($u(t)$ is the unit step function.)i Assume that $m = 0.1$, $b = 0.2$, $k = 0.2$, and the mass is initially still at the position $x(0) = 0.01$. [**3 marks**]

$\dot{x}(0) = 0, \mathcal{L}\left[\frac{dx}{dt}\right] = SX(S) - x(0)$

b) Check if the final value $\lim_{t\to\infty} x(t)$ obtained from the calculated trajectory $x(t)$ and the final value obtained by the final value theorem are the same. [**1 marks**]

$\mathcal{L}\left[\frac{d^2x}{dt^2}\right] = S\left[SX(S) - x(0)\right] - \dot{x}(0)$

$= S^2X(S) - Sx(0) - \dot{x}(0)$

a)

$0.1\ddot{x}(t) = 0.1U(t) - 0.2\dot{x}(t) - 0.2x(t)$

$\Rightarrow \ddot{x}(t) = U(t) - 2\dot{x}(t) - 2x(t)$

$\xrightarrow{\mathcal{L}} S^2X(S) - Sx(0) - \dot{x}(0) = 1/S - 2(SX(S) - x(0)) - 2X(S)$

$\Rightarrow (S^2+2S+2)X(S) = 1/S + (S+2)x(0) + \dot{x}(0) \Rightarrow$

$X(S) = \frac{1}{S(S^2+2S+2)} + \frac{0.01(S+2)}{S^2+2S+2}$

$\Rightarrow \frac{A}{S} + \frac{BS+C}{S^2+2S+2} = \frac{1}{S(S^2+2S+2)} \Rightarrow 1 = A(S^2+2S+2) + S(BS+C)$

$\Rightarrow \begin{cases} A+B=0 \\ 2A+C=0 \\ 2A=1 \end{cases} \Rightarrow \begin{matrix} A=1/2 \\ B=-1/2 \\ C=-1 \end{matrix}$

$X(S) = \frac{A}{S} + \frac{BS+C}{S^2+2S+2} + 0.01\frac{(S+1)+1}{(S+1)^2+1} = \frac{A}{S} + \frac{B(S+1)+C-B}{(S+1)^2+1} + 0.01\frac{(S+1)+1}{(S+1)^2+1}$

$\xrightarrow{\mathcal{L}^{-1}} x(t) = AU(t) + e^{-t}\{B\cos t + (C-B)\sin t\} + 0.01e^{-t}\{\cos t + \sin t\}$

b)

$SX(S) = \frac{1}{S^2+2S+2} + \frac{0.01S(S+2)}{S^2+2S+2} \xrightarrow{\text{Poles are}} -1 \pm j$

Since all poles are in the LHP then Final Value theorem can be applied

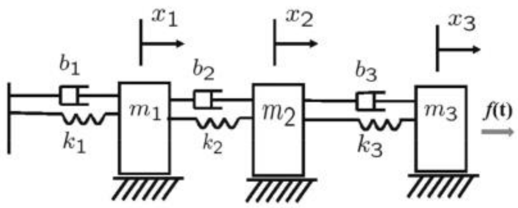
$\lim_{t\to\infty} x(t) = \lim_{S\to 0} SX(S) = \lim_{S\to 0}\left[\frac{1}{S^2+2S+2} + \frac{0.01S(S+2)}{S^2+2S+2}\right] = 1/2$

2- Consider a 3-DOFs mass spring damper system below, where x, f, m, k and b denote respectively the displacement of masses, the force input, the mass, the spring and damper coefficients. Assume no friction between the mass and the ground.

a) Using the Newton's second law for each mass and taking Laplace transform, draw a block diagram of the system. [**3 marks**]

b) Using the closed loop block diagram formula, obtain the following transfer function. [**3 marks**]

- $T(s) := X_2(s)/F(s)$: Transfer function from input $f(t)$ to output x_2.



For m_3: $m_3 \dfrac{d^2 x_3}{dt^2} = f(t) - K_3(-x_2 - (-x_3)) - b_3(-\dot{x}_2 - (-\dot{x}_3))$ ①

For m_2: $m_2 \dfrac{d^2 x_2}{dt^2} = K_3(x_3 - x_2) + b_3(\dot{x}_3 - \dot{x}_2) - K_2(-x_1 - (-x_2)) - b_3(-\dot{x}_1 - (-\dot{x}_2))$ ②

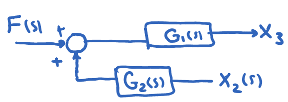
For m_1: $m_1 \dfrac{d^2 x_1}{dt^2} = K_2(x_2 - x_1) + b_2(\dot{x}_2 - \dot{x}_1) - K_1(0 - (-x_1)) - b_1(0 - (-\dot{x}_1))$ ③

① $\Longrightarrow m_3 s^2 X_3(s) = F(s) - K_3(X_3(s) - X_2(s)) - b_3 s(X_3(s) - X_2(s))$

$\Longrightarrow F(s) + (b_3 s + K_3) X_2(s) = (m_3 s^2 + K_3 + b_3 s) X_3(s)$

$\Longrightarrow X_3(s) = \underbrace{\dfrac{1}{m_3 s^2 + b_3 s + K_3}}_{G_1} \left[F(s) + \underbrace{(b_3 s + K_3)}_{G_2} X_2(s)\right]$

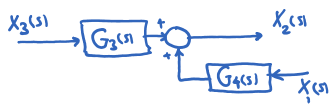
$\Longrightarrow X_3(s) = G_1(s)\left[F(s) + G_2(s) X_2(s)\right]$



② $\Longrightarrow (m_2 s^2 + (b_2 + b_3)s + (K_2 + K_3)) X_2(s) = (b_3 s + K_3) X_3(s) + (b_2 s + K_2) X_1(s)$

$\Longrightarrow X_2(s) = \underbrace{\dfrac{b_3 s + K_3}{m_2 s^2 + (b_2 + b_3)s + (K_2 + K_3)}}_{G_3} X_3(s) + \underbrace{\dfrac{b_2 s + K_2}{m_2 s^2 + (b_2 + b_3)s + (K_2 + K_3)}}_{G_4} X_1(s)$

$\Longrightarrow X_2(s) = G_3(s) X_3(s) + G_4(s) X_1(s)$

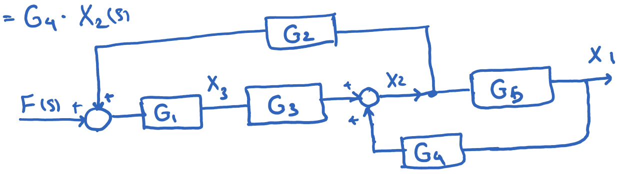


③ $\Longrightarrow [m_1 s^2 + (b_2 + b_1)s + (K_2 + K_1)] X_1(s) = (b_2 s + K_2) X_2(s)$

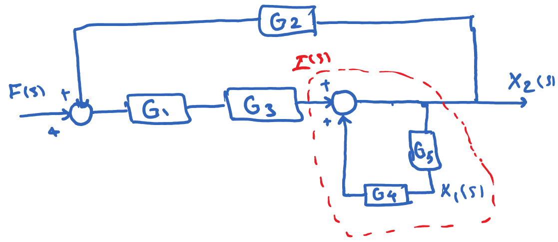
$\Longrightarrow X_1(s) = \dfrac{b_2 s + K_2}{m_1 s^2 + (b_2 + b_1)s + (K_2 + K_1)} X_2(s)$

$$\Rightarrow X_1(s) = \underbrace{\frac{b_2 s + K_2}{m_1 s^2 + (b_2+b_1)s + (K_2+K_1)}}_{G_5} X_2(s)$$

$$\Rightarrow X_1(s) = G_4 \cdot X_2(s)$$

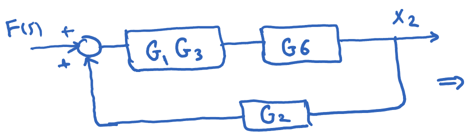


$$T = \frac{X_2(s)}{F(s)} = ?$$



$$G_6 = \frac{X_2(s)}{I(s)} = \frac{1}{1 - G_4 G_5}$$

Then the above block diagram can be re-drawn as:



$\Rightarrow$

$$T(s) = \frac{X_2(s)}{F(s)} = \frac{G_1 G_3 G_6}{1 - G_1 G_2 G_3 G_6} = \frac{G_1 G_3 \cdot \frac{1}{1 - G_4 G_5}}{1 - G_1 G_2 G_3 \cdot \frac{1}{1 - G_4 G_5}}$$

$$= \frac{G_1 G_3}{1 - G_4 G_5 - G_1 G_2 G_3}$$