MSE 211: Computational Methods for Engineers

SIMON FRASER UNIVERSITY

MECHATRONIC SYSTEMS ENGINEERING

Midterm Examination (Type 1) – February 26, 2018

Instructor: Kambiz Hajikolaei Time: 120 minutes

Non-programmable calculators may be used

No smartphones or other electronic devices may be used

One page (8.5" x 11", one side) of handwritten notes is permitted

Answer all the questions in the booklet and bubble sheet (not the question sheet)

Part I: True-False (use bubble sheet for this part, True: A and False: B) [25 marks]

1) Newton-Raphson method converges quadratically
2) If a matrix has a very small determinant, then the matrix is nearly singular
3) If a matrix has a small condition number, then the matrix is nearly singular
4) If A is a nonsingular matrix, then the condition number of A and A^{-1} are the same
5) The idea in structured programming is that any numerical algorithm can be composed of using the three fundamental structures: Sequence, Selection, Repetition
6) For finding a simple root of a nonlinear equation, Secant method usually converges faster than bisection method
7) For finding a simple root of a nonlinear equation, Secant method usually converges faster than Newton-Raphson method
8) The steepest descent algorithm requires the calculation of partial derivatives
9) Decreasing the step size leads to decreasing the round-off error
10) In well-conditioned systems, a small change in the coefficients result in large changes in the solution
11) Gauss-Jordan elimination method does not require back substitution
12) All the norms satisfy $\|x\| \geq 0$
13) All norms satisfy $\|x+y\| \geq \|x\| + \|y\|$
14) Brent's method is one of bracketing methods
15) Bisection method guarantees to converge
16) Decreasing the step size leads to increasing the truncation error
17) Bracketing methods are usually faster than open methods
18) In the false position method, the size of the interval decreased by half at each step
19) In the false position method, one endpoint might stay fixed while the other converges to the root
20) In Newton-Raphson method, the derivative of the function has to be found analytically
21) Both the Secant and Muller's methods are using quadratic approximations
22) Powell's method is a type of pattern search optimization
23) If $|H| < 0 \quad and \quad \frac{\partial^2 f}{\partial x^2} < 0$, then $f(x,y)$ has a local maximum
24) In general, there is no guarantee that the global minimum/maximum has been found
25) A unimodal function has a single maximum or a minimum

Part II: Short answer questions:

a b c

1) [3 marks] Consider the quadratic $f(x) = x^2 - 0.8x + 0.15$
   a) Why is the interval [0.1, 0.6] not a satisfactory starting interval for bisection?
   b) If you start with [0, 0.32], which root is reached with bisection?
   c) Using the interval in (b), what is the maximum number of iterations required to attain an absolute error of 0.001?
2) [3 marks] Carry out one iteration of Newton's method applied to the system of nonlinear equations with starting point [1 2]

$$x + 2y - 2 = 0$$
$$x^2 + 4y^2 - 4 = 0$$

3) [4 marks] While searching for the minimum of

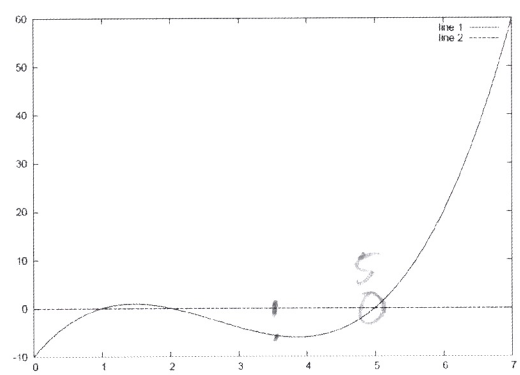
$$f(x) = x_1^4 + x_2^4 + 2x_1^2x_2^2 + 2x_1^2 - 2x_2^2 + 1$$

We terminate at the following points:

a) $x = (0,0)$ b) $x = (0,1)$ c) $x = (0,-1)$ d) $x = (1,1)$

Classify each point as
- Local maximum
- Local minimum
- Neither of above

4) [1 mark] The graph of the function $f(x) = x^3 - 8x^2 + 17x - 10$ is given by:



If the search starts with the bracket [0 7], which root will the bisection method converge to?

5) [1 mark] Show that $f(x) = 3x^4 - e^x - 1$ has a root on the interval [0 2]
6) [1 mark] How many significant figure does 0.001267 have?
7) [1 mark] Convert 1101.0011 to base 10
8) [1 marks] Use centered difference to estimate the first derivative of
   $f(x) = -0.2x^3 - 0.5x^2 + 0.25x + 1.3$
   At $x = 1.3$ using step size of 0.2

Part III: Problems

P1 [15 marks] Solve the following system of equations by LU decomposition without pivoting.

$$4x_1 - x_2 + x_3 = 6$$
$$8x_1 + 3x_2 - x_3 = 10$$
$$3x_1 + x_2 + x_3 = 9$$

Confirm x_1 using Cramer's rule.

P2 [15 marks] Use the Secant method with $x_0 = -1$ and $x_1 = 0$ to approximate one of the roots of

$$f(x) = 4\sin x - x - 1$$

Continue the iterations until the absolute value of the approximate error estimate falls below an error criterion conforming to one significant figure. In each iteration, use nine significant digits when calculating x and $f(x)$.

P3 [10 marks] Apply the method of steepest descent to find the minimum of function

$$f(x, y) = 4x^2 - 4xy + 2y^2$$

With initial point $x_0 = (2, 3)$.