MSE381-Quiz2

Tuesday, June 18, 2019 2:03 PM

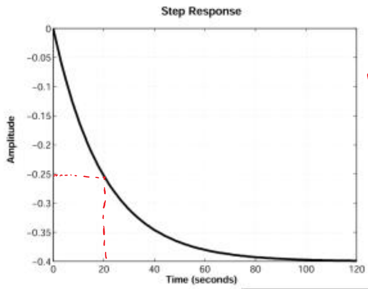
SIMON FRASER UNIVERSITY
School of Mechatronic Systems Engineering
Quiz II: Monday, June 17^{th} - Summer 2019

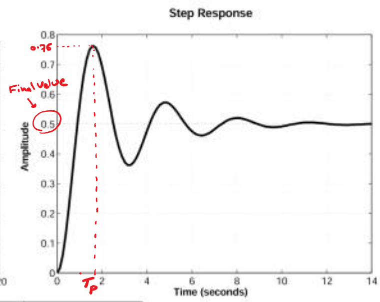
SFU

Last name, First name:

Name: Student #: Mark: /10

1- For the following two step responses, obtain (approximately) the transfer functions for the first and second-order systems. (You may need to use a calculator.)





Settling time	$\approx \frac{3}{\zeta\omega_n}$ or $\frac{4}{\zeta\omega_n}$
Peak time	$\frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$
Peak value	$1 + e^{-\zeta\pi/\sqrt{1-\zeta^2}}$
Percent overshoot	$100e^{-\zeta\pi/\sqrt{1-\zeta^2}}$

τ : 63% of final value

$0.63 \times 0.4 = 0.252 \Rightarrow$

$\boxed{\tau = 20}$ 1 mark

$T(s) = \frac{-0.4}{20s+1}$ 1 mark

$P.O. = \frac{0.76-0.5}{0.5} = 52\%$ 0.5 mark

$\Rightarrow \zeta = \frac{|\ln \frac{P.O.}{100}|}{\sqrt{\pi^2 + (\ln \frac{P.O.}{100})^2}}$

$\Rightarrow \zeta = \frac{|-0.654|}{\sqrt{3.14^2 + (-0.654)^2}}$

$\Rightarrow \boxed{\zeta = 0.2}$ 1 mark

$T_p \simeq 1.6 \Rightarrow \frac{\pi}{\omega_d} = 1.6 \Rightarrow \omega_d = 1.964$ $\Rightarrow \omega_n = \frac{1.964}{\sqrt{1-\zeta^2}}$

$\omega_d = \omega_n\sqrt{1-\zeta^2}$

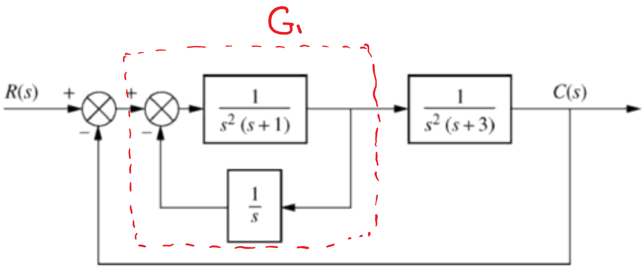
$\Rightarrow \boxed{\omega_n = 2}$ 1 mark

Final value = 0.5

$\Rightarrow T(s) = 0.5 \times \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \Rightarrow$

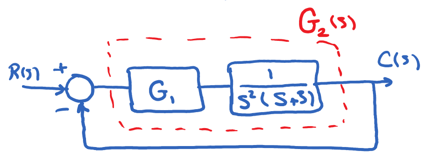
$T(s) = 0.5 \cdot \frac{4}{s^2 + 0.8s + 4}$ 0.5 mark

2- Given the system shown below, find the following:



$$G_1 = \frac{\frac{1}{s^2(s+1)}}{1 + \frac{1}{s^2(s+1)} \times \frac{1}{s}} = \frac{s}{s^4 + s^3 + 1}$$ 0.5 mark

(a) The closed-loop transfer function
(b) The system type
(c) The steady-state error for an input of $5u(t)$
(d) The steady-state error for an input of $5tu(t)$



For $r(t) = u(t), e_{ss} = \dfrac{R}{1+K_p}$ and $K_p = \lim_{s \to 0} G(s)$

For $r(t) = tu(t), e_{ss} = \dfrac{R}{K_v}$ and $K_v = \lim_{s \to 0} sG(s)$

$$G_2(s) = G_1 \cdot \frac{1}{s^2(s+3)} = \frac{1}{s(s^5 + 4s^4 + 3s^3 + s + 3)}$$ 0.5 mark

a) $$T(s) = \frac{G_2(s)}{1 + G_2(s)} = \frac{1}{s^6 + 4s^5 + 3s^4 + s^2 + 3s + 1}$$ 1 mark

b) system is type 1. 1 mark
7

c) $K_p = \infty \Rightarrow e_{ss} \to 0$ 1 mark

d) $K_v = \lim s \cdot G_2(s) = 1/3 \Rightarrow e_{ss} = 5/K_v = 15$ 1 mark
0

Hint: Since the system is unstable (the coefficient of s^3 is zero) the steady-state error is meaningless. But it is supposed that the system's stability in not our concern.