MSE 426/726 Introduction to Optimal Engineering Design
Term Test 2

March 27, 2017

Name: ______________________ Student Number: ______________________

Note:
- This is a closed-book exam.
- Each student is permitted to bring one page of US letter size with hand-written notes on one side to the exam.
- Only non-communicative calculators are allowed.

Problem 1 True (T) or False (F); Fill "A" for True and "B" for False on the bubble sheet. (28 marks)

T 1. The Newton method converges in just one iteration from any starting point for a strictly convex quadratic function.
T 2. The BFGS method always gives a direction of descent for the cost function, while the Newton's method does not guarantee that.
F 3. In Genetic algorithms (GA), the value of the mutation rate should be at least 0.5.
T 4. In GA, the value of the crossover rate should be at least 0.5.
F 5. In certain constrained optimization search strategies, the step size could be a vector having two or more numbers.
F 6. GA is a deterministic global optimization method that doesn't need any information about the gradients.
F 7. If the obtained solution satisfies the KKT condition, we know for sure this point is a local optimum.
T 8. The interior penalty method may cause discontinuity to the cost function and result in divergence.
F 9. The feasible direction method for constrained optimization involves solving an optimization problem to find the minimum K in order to find an appropriate search direction.
T 10. One of the disadvantages of GA is its often high number of function calls.
F 11. When there is no explicit function expressions for objective functions, fmincon(.) cannot be used for optimization.
F 12. In an algorithm using the exterior penalty method, the penalty factor usually starts with a large number and gradually reduces to zero for convergence.
F 13. A regular point is a point at which the first derivative of the objective function is equal to zero.
T 14. An optimum solution for a convex programming problem (both objective and constraints are convex) is always unique.

Problem 2 Multiple-choice questions for Problems 15-27 (Fill the corresponding oval on the bubble sheet) (4 marks each for a total of 52 marks).

15. An engineer is going to use the optimization toolbox of Matlab to optimize the design of a heat exchanger to have the highest heat transfer rate (expressed by the function $f(\vec{x})$) as well as the lowest manufacturing costs (expressed by the function $g(\vec{x})$). Which of the following functions could suit their purpose?
   a) gamultiobj(fitnessfcn,nvars,A,b,Aeq,beq,LB,UB,nonlcon,options)
   b) ga(fitnessfcn,nvars,A,b,Aeq,beq,LB,UB,nonlcon,options)
   c) fmincon(fun,x0,A,b,Aeq,beq,lb,ub,nonlcon,options)
   d) fminunc(fun,x0,options)

16. In order to test the optimality condition, for a Hessian matrix of an objective function, which one shows the type of $[H_0]=\begin{pmatrix} -2 & 2 & 0 \\ 2 & -2 & 0 \\ 0 & 0 & -2 \end{pmatrix}$?
   a) Positive definite
   b) Negative definite
   C a) Negative semidefinite
   d c) Indefinite

17. For $f(\vec{x})=2x_1^2+x_2^2+3x_1x_2$ subject to $h(x_1,x_2)=x_1^2+2x_1x_2-1=0$ and $g(x_1,x_2)=x_1^2+x_2^2-1\le 0$, which one shows the correct Lagrangian function for finding the optimum point by using the Lagrange multipliers?
   b) $L=2x_1^2+x_2^2+3x_1x_2+v\left(x_1^2+2x_1x_2-1\right)+u\left(x_1^2+x_2^2-1\right)$
   c) $L=2x_1^2+x_2^2+3x_1x_2+v\left(x_1^2+2x_1x_2-1\right)+u\left(x_1^2+x_2^2-1+s^2\right)$
   d) $L=2x_1^2+x_2^2+3x_1x_2$
   e) $L=2x_1^2+x_2^2+3x_1x_2+v\left(x_1^2+2x_1x_2-1+s_1^2\right)+u\left(-x_1^2-x_2^2+1+s_2^2\right)$

18. Which of the following equations doesn't belong to the system of equations that should be solved to obtain the optimum of Problem 17?
   a) $4x_1+3x_2+2vx_1+2vx_2+2ux_1=0$
   b) $2us=0$
   c) $3x_1+2x_2+vx_1+2ux_2=0$
   d) $x_1^2+2x_1x_2-1=0$

19. Genetic algorithm is used to find the optimum of $f(x)=4x^3-4x^2+x$ in the range $x\in[3,10]$. Which one of the following functions cannot be used as a fitness function?
   a) $f(x)$ b) $f(\sin x)$ c) $f(e^x)$ d) $f(x)+1000$

20. A function is to be optimized by the genetic algorithm, and the fitness function is $f(x) = -x^2 + 2x + 100$. A random population is generated as follows to be used in the selection process: 01011, 01001, 01000, and 00010. Which string is the fittest?
   a) 00010
   b) 01011
   c) 01001
   d) 01000

21. If the following table is obtained in the selection process of solving a problem by genetic algorithm and the Roulette wheel numbers are 0.34, 0.15, 0.87, and 0.45, which string is not selected at all?

String	Cumulative probability (CP)
00011	0.2
01011	0.27
10001	0.50
11111	1.00

   a) 00011
   b) 01011
   c) 10001
   d) 11111

22. The pairs shown in the following table are selected for the crossover in the solution of a problem by genetic algorithm. Which one shows the offspring for the cross points shown in the following table?

Crossover point	Parents	Offspring
r=2	01110	
	11101	
r=4	10011	
	10001	

   a) 01101, 10110, 10011, 11001
   b) 00101, 10010, 10011, 10001
   c) 01101, 11110, 10011, 10001
   d) 01110, 11101, 10011, 10001

23. The crossover step in solving a problem by genetic algorithm has resulted in the following offspring: 00101, 10010, 10011, and 10001. The following random numbers are generated for mutation of the offspring: 0.45, 0.0012, 0.63, 0.91, 0.22, 0.56, 0.0004, 0.81, 0. 03, 0.32, 0.77, 0.42, 0.0009, 0.37, 0.97, 0.21, 0.65, 0.00101, 0.11, 0.12. Which one shows the mutated offspring if P_m=0.001?
   a) 01101, 10010, 10111, 10001
   b) 01101, 11000, 10111, 10101
   c) 00101, 10010, 10011, 10001
   d) 00101, 11010, 10111, 10001

24. For the following table of experimental data, determine degrees of freedom for the factors, their interaction, and error for ANOVA analysis of the data as well as the total degrees of freedom.

		Factor B		
		0	**25**	**107**
Factor A	**2%**	10 7 9	5 4 7	3 4 2
	45%	5 4 6	3 4 5	1 2 2
	78%	4 8 6	5 5 9	1 2 3

a) $d_A = 3,\ d_B = 3,\ d_{AB} = 9,\ d_E = 32, d_{tot} = 47$
b) $d_A = 2,\ d_B = 2,\ d_{AB} = 4,\ d_E = 18, d_{tot} = 26$
c) $d_A = 2,\ d_B = 2,\ d_{AB} = 9,\ d_E = 26, d_{tot} = 39$
d) $d_A = 3,\ d_B = 3,\ d_{AB} = 6,\ d_E = 18, d_{tot} = 30$

25. For the following ANOVA table, which statement is correct if $F_{0.05,3,32} = 2.92$ and $F_{0.05,9,32} = 2.21$?

Source of variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
A	60	3	20	64
B	45	3	15	48
Interaction	22	9	2.4444	7.82
Error	10	32	0.3125	
Total	137	47		

a) Factors A and B have insignificant effects.
b) Factor A is more significant than B.
c) Factors A and B have significant effects on the data, but their interaction is insignificant.
d) The effects of factors A and B as well as their interaction are significant.

26. Which of the following equations shows the equivalent linear fitting problem for fitting the curve defined by $y = a(x_1 + x_1x_2)^b e^{cx_1}$ through a set of data?

a) $y = ab\log(x_1 + x_1x_2) + cx_1 \log e$

b) $\log y = ab\log(x_1 + x_1x_2) + cx_1 \log e$

c) $\log y = \log a + b\log(x_1 + x_1x_2) + cx_1 \log e$

d) $y = \log ab + \log(x_1 + x_1x_2) + cx_1 \log e$

27. For fitting $y = b_0 + b_1e^x + b_2x^2$ through three points of $y(0) = 3$, $y(1) = 7$, and $y(2) = 16$ by the least square method, what is the least square estimate of $\begin{bmatrix} b_0 \\ b_1 \\ b_2 \end{bmatrix}$?

a) $\left(\begin{bmatrix} 1 & 1 & 1 \\ 0 & e & e^2 \\ 0 & 1 & 4 \end{bmatrix}^T \begin{bmatrix} 1 & 0 & 0 \\ 1 & e & 1 \\ 1 & e^2 & 4 \end{bmatrix}\right)^{-1} \begin{bmatrix} 1 & 1 & 1 \\ 0 & e & e^2 \\ 0 & 1 & 4 \end{bmatrix}^T \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$

b) $\left(\begin{bmatrix} 1 & 1 & 1 \\ 0 & e & e^2 \\ 0 & 1 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & e & 1 \\ 1 & e^2 & 4 \end{bmatrix}\right)^{-1} \begin{bmatrix} 1 & 1 & 1 \\ 0 & e & e^2 \\ 0 & 1 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$

c) $\left(\begin{bmatrix} 1 & 1 & 1 \\ 1 & e & e^2 \\ 0 & 1 & 4 \end{bmatrix}^T \begin{bmatrix} 1 & 1 & 0 \\ 1 & e & 1 \\ 1 & e^2 & 4 \end{bmatrix}\right)^{-1} \begin{bmatrix} 1 & 1 & 1 \\ 1 & e & e^2 \\ 0 & 1 & 4 \end{bmatrix}^T \begin{bmatrix} 3 \\ 7 \\ 16 \end{bmatrix}$

d) $\left(\begin{bmatrix} 1 & 1 & 1 \\ 1 & e & e^2 \\ 0 & 1 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 1 & e & 1 \\ 1 & e^2 & 4 \end{bmatrix}\right)^{-1} \begin{bmatrix} 1 & 1 & 1 \\ 1 & e & e^2 \\ 0 & 1 & 4 \end{bmatrix} \begin{bmatrix} 3 \\ 7 \\ 16 \end{bmatrix}$

Problem 3. Multiple-choice questions (Fill the corresponding oval on the bubble sheet) (5 marks each; 20 marks in total).

28. For the function $f(\vec{x}) = x_1^2 + 2x_2^2 + 2x_3^2 + 2x_1x_2 + 2x_2x_3$, one has found two consecutive points at $\vec{x}_0 = (2, 4, 10)^T$ and $\vec{x}_1 = (0.0956, -2.348,\ 2.381)^T$. The conjugate gradient direction at $\vec{x}_1$ is which one of the following?

a) $\vec{d} = [-7.5 \quad -35.562 \quad -52.828]^T$

b) $\vec{d} = [-12 \quad 40 \quad 18]^T$

c) $\vec{d} = [-4.5 \quad -4.438 \quad 4.828]^T$

d) $\vec{d} = [4.312 \quad 3.813 \quad -5.578]^T$

e) None of the above

29. For the following optimization problem defined as

$$\min f = x^2 + y^2 - 2x - 2y + 2$$
$$\textit{subject to}$$
$$-2x - y + 4 \leq 0$$
$$-x - 2y + 4 \leq 0$$

By applying the Lagrange multiplier method, which one of the following is the solution point?

a) $(4/3, 4/3)^T$
b) $(1,1)^T$
c) $(1.2, 1.4)^T$
d) $(1.4, 1.2)^T$
e) None of the above

30. For the problem above, applying the KKT condition at point $(4,0)^T$, which of the following is true?

a) λ=(4.667, -3.333)^T and $(4,0)^T$ is not a constrained optimum.
b) $\Delta\vec{f} + \vec{G}\lambda=\{0\}$ at this point and $(4,0)^T$ is a constrained optimum.
c) $\lambda = 0.4$ and $(4,0)^T$ is a constrained optimum
d) $\lambda = 0.4$ and $(4,0)^T$ is not a constrained optimum

31. Considering $[H_0] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $\vec{x}_0 = (0,1)$ for $f(\vec{x}) = 2x_1^2 + x_2^2 + 3x_1x_2$, which one shows $[H_1]$ from the BFGS method?

a) $\begin{pmatrix} 4.36 & 2.46 \\ 2.46 & 2.80 \end{pmatrix}$

b) $\begin{pmatrix} 2.46 & 4.36 \\ 4.36 & 2.80 \end{pmatrix}$

c) $\begin{pmatrix} 2.80 & 2.46 \\ 2.46 & 4.36 \end{pmatrix}$

d) $\begin{pmatrix} 2.80 & 4.36 \\ 4.36 & 2.46 \end{pmatrix}$