

HEAP

Priority Queue - Elements will be removed from the queue on the basis of the element's priority.

Heap (Priority Queue)

	Sorted Array	Unsorted Array	Heap
Add	$O(n)$	$O(1)$	$O(\log n)$
Delete	$O(n)$	$O(n)$	$O(\log n)$
Get	$O(1)$	$O(n)$	$O(1)$

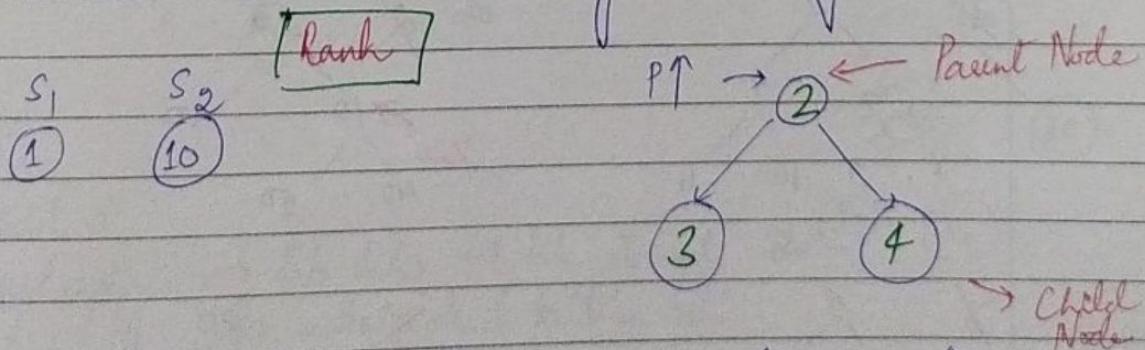
Heap

Heap is a binary tree which follows the following properties :-

1. Priority of parent is always more than children
2. It is a complete binary tree

1. PRIORITY

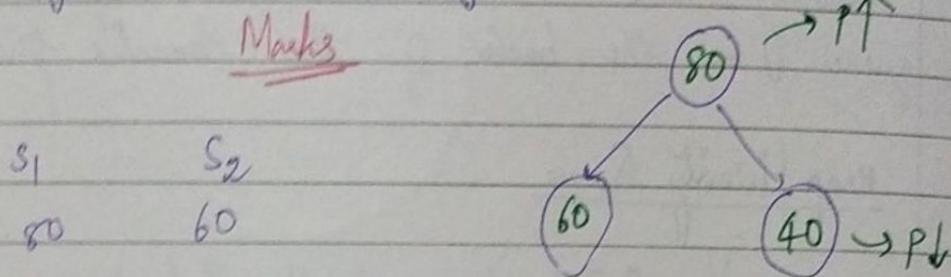
- a) Smaller Value S.V. is having ↑ Priority P.



Rank → Parent Node will have a smaller value $\rightarrow P\uparrow$
Child Node will have a greater value $\rightarrow P\downarrow$

MIN HEAP

b) Larger Value is Priority $P \uparrow$



\rightarrow Greater marks \rightarrow larger Priority $\rightarrow \therefore$ Parent Node will have greater value

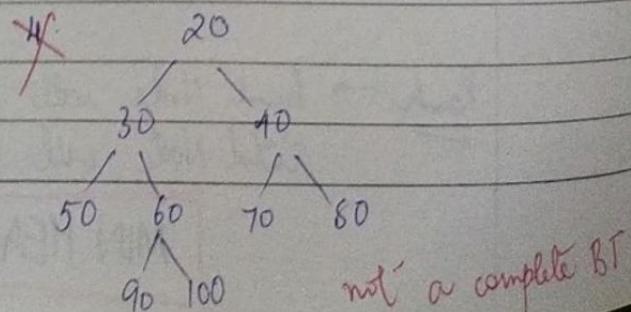
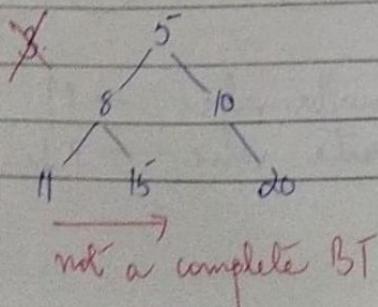
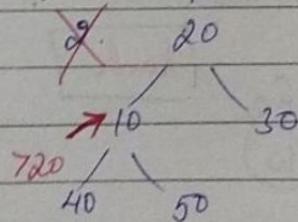
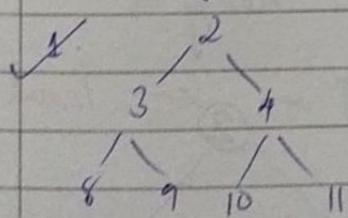
MAX HEAP

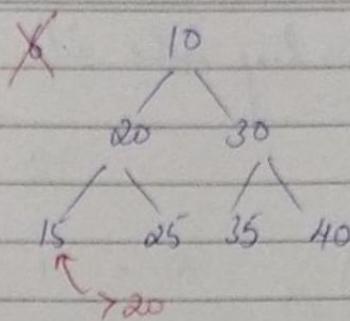
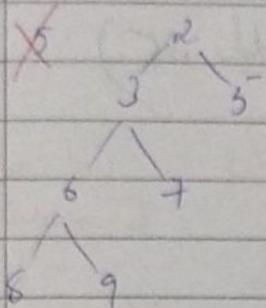
\rightarrow Parent value is more

2 COMPLETE BINARY TREE

In which every level, except the last level, is completely filled, and the last level is filled from left to right.

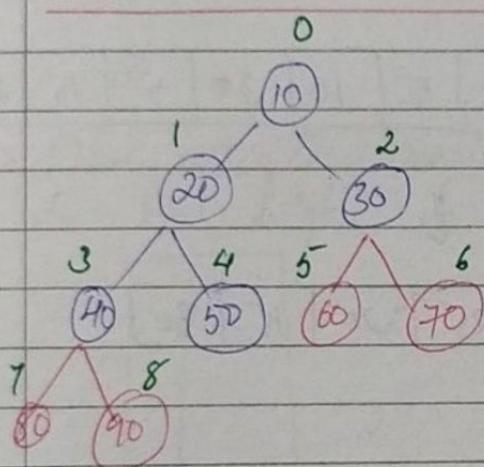
Q Which of the foll. is a min heap?





Not a complete BT

ADDITION & DELETION IN HEAP



$$p_i = 1$$

$$l_i = \boxed{2p_i + 1} = 3$$

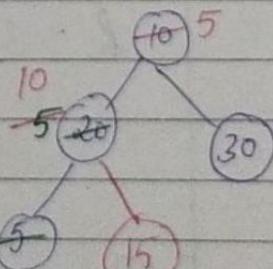
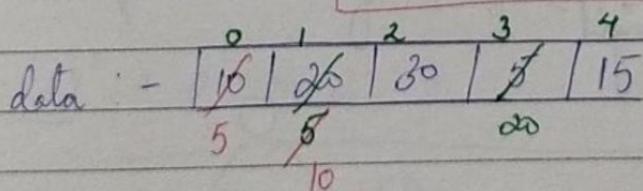
$$r_i = \boxed{2p_i + 2} = 4$$

$$p_i^* = 3$$

$$l_i^* = 7$$

$$r_i^* = 8$$

$l_{ci} = 2p_i + 1$	$r_{ci} = 2p_i + 2$
$p_i = \frac{l_{ci} - 1}{2}$	$p_i^* = \frac{r_{ci} - 2}{2}$

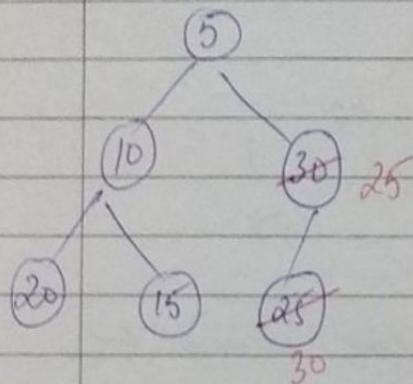


1. add 5

2. upheapify \rightarrow tab tak jab tak element 20 ke uska correct position nahi mil jaata

3. add 15

Now, the heap looks like - 4. (Add 25)



($25 < 30$: we need
to swap)

ArrayList :- data \Rightarrow

5	10	30	5	15	25
---	----	----	---	----	----

 \downarrow

5	10	25	20	15	30
---	----	----	----	----	----

data :-

5	10	25	20	15	30
---	----	----	----	----	----

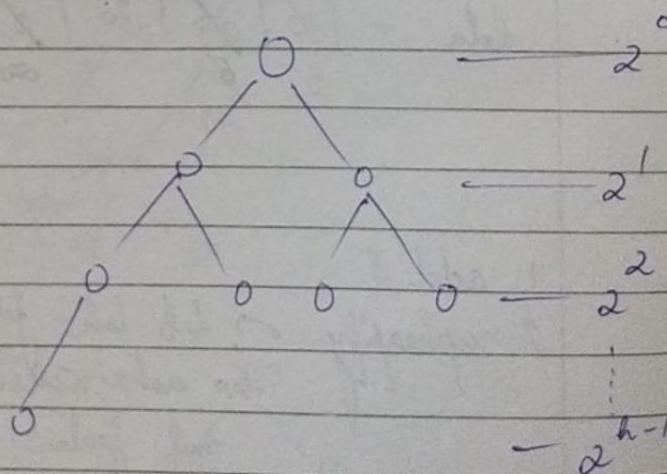
(Code \rightarrow Min Heap (add, deletion etc.) see Github)

COMPLEXITY ANALYSIS -

Q1. What are the minimum no. of nodes required in CBT to have height h ?

$$h = 3$$

n: Total no. of nodes
in BT



$$2^0 + 2^1 + 2^2 + \dots + 2^{h-1} + 1 = n$$

GP

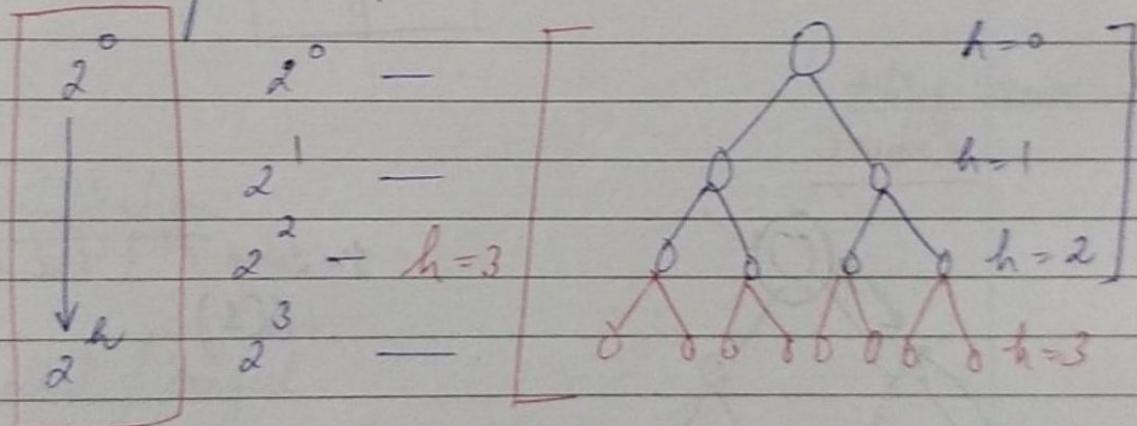
$$\frac{2^{h-1}}{2-1} + 1 = n$$

$$2^h = n$$

$$h = \log_2 n$$

$$[h = O(\log n)]$$

Q2. What are the minimum no. of nodes required in CBT to have height h?



$$2^0 + 2^1 + 2^2 + \dots + 2^h = n$$

GP

$$2^{h+1} - 1 = n$$

$$2^{h+1} = n + 1$$

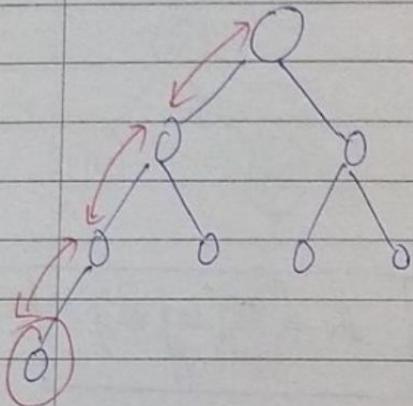
$$\log_2 2^{h+1} = \log_2 (n+1)$$

$$h+1 = \log_2 (n+1) \Rightarrow$$

$$h = \log_2 (n+1) - 1$$

⇒ $h = O(\log n)$

Add



$$\underline{O(1)} + \underline{O(h)}$$

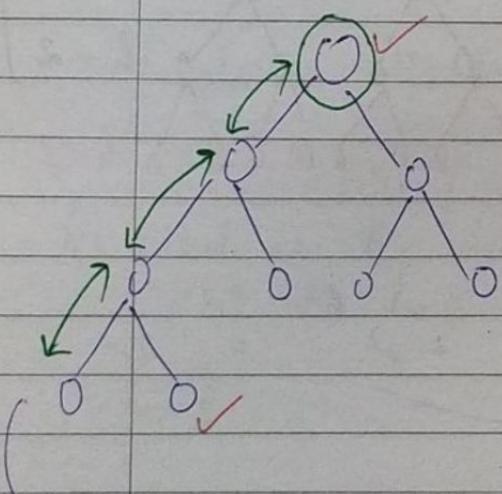
to add the
element at
the end of
the array

upheapify

worst case we
swap kte kte we
reach the root element

$$O(h) = \boxed{O(\log n)}$$

Remove



1. Swap 1st & last element
 $O(1)$

2. remove last one element
hai (after swapping)

$$\underline{O(1)} + \underline{O(h)} = O(h)$$

swap downheapify

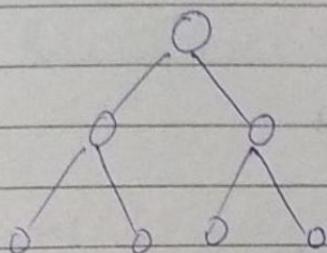
In worst case →
downheapify kte kte
we reach the last

Yahan 3 swaps hne
hai, 3 → dependent
on h

(BT : $h = O(\log n)$)

Remove : $O(\log n)$

Get



data.get(0)

$O(1)$

Summarize :-

1. Add : $O(\log n)$
2. Remove : $O(\log n)$
3. Get : $O(1)$

Aditya Verma

Identification

1. K
2. Smallest / Largest

$K + \text{smallest} \Rightarrow \text{Max heap}$

$K + \text{largest} \Rightarrow \text{Min heap}$

Sorting

$n \log n$ (merge sort)



Heap K

$n \log k$

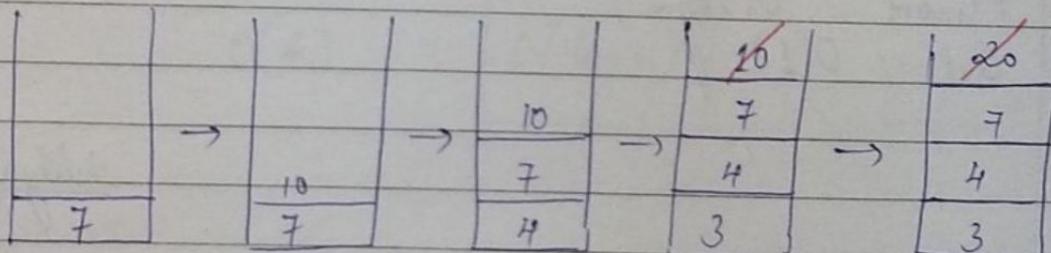
(Improved complexity)

k^{th} smallest element

k_{smallest}] \rightarrow Qualify for hap

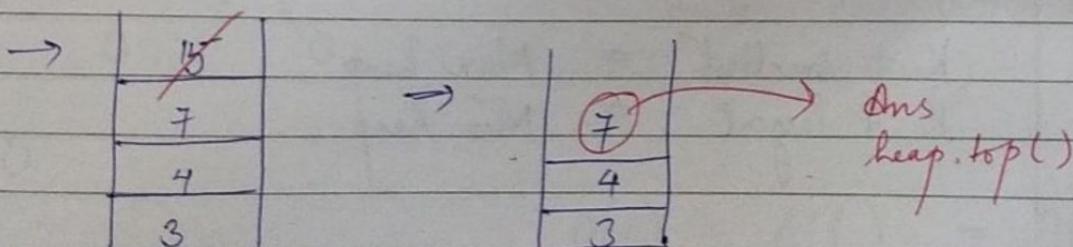
Which heap? Max Heap

arr [] : 7 10 4 3 20 15
k = 3



$$\text{size} = 1 \quad \text{size} = 2 \quad \text{size} = 3 \quad \text{size} > k \quad \text{size} > k$$

||
K ∴ pop ∴ pop



$\text{size} > k$
∴ f_{top}

Note: - Max Heap makes sure the largest element is always at the top (it does not guarantee that all elements are pushed in sorted order in the heap)

Code : using max heap

PriorityQueue<Integer> pq = new PriorityQueue<>(
 Comparator.reverseOrder());

```
for(int i = 0 ; i < size ; i++) {  
    pq.add(a[i]);  
    if (pq.size() > k) {  
        pq.poll();  
    }  
}
```

*→ to build
max heap*

return pq.peek();

3

Q:

Return K largest elements in array

arr[] : 7 10 4 3 20 15

k = 3

[20 | 15 | 10] → op.

①

Approach 1 : Sorting

[3 | 4 | 7 | 10 | 15 | 20]

| |
return

But, sorting 3, 4 & 7 is extra work that we're doing

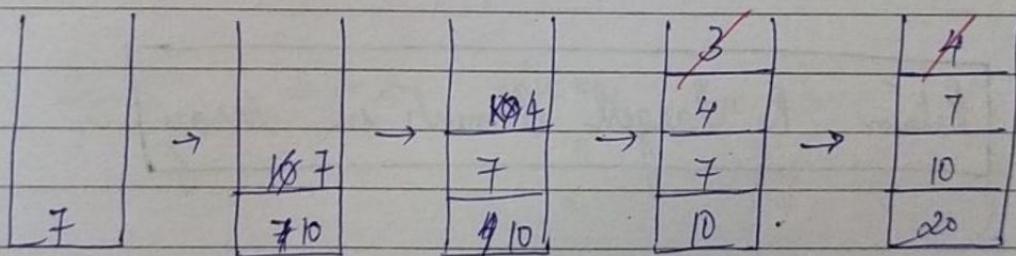
Hence, we need to optimize it

That we can do with help of heap

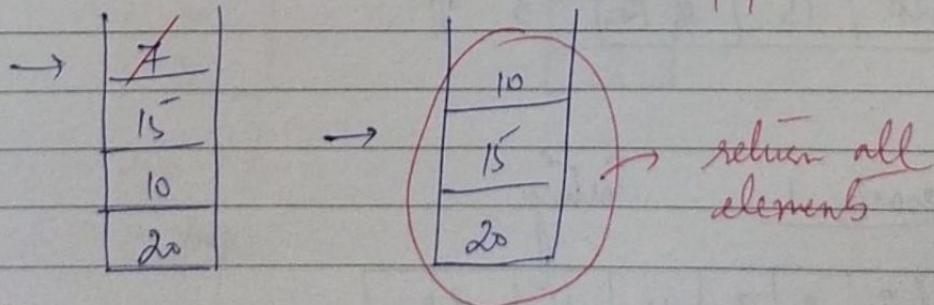
Identification

1. k
 2. Largest } \Rightarrow Min Heap

Approach 2 :



$\text{size} > k$ $\text{size} > k$
 $\therefore \text{pop}$ $\therefore \text{pop}$



pop

emptying the heap

`while ($pq \cdot \text{size}() > 0$) {`

`System.out.println($pq \cdot \text{peek}()$);`
 `$pq \cdot \text{pop}()$;`

}

Q:

Sort a K Sorted array | Sort Nearly Sorted array

Ans:

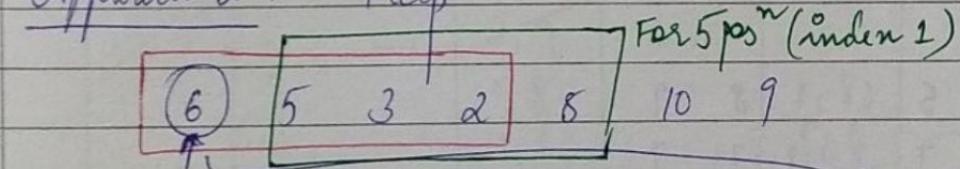
`arr: [6 5 3 2 8 10 9]`

`k = 3`

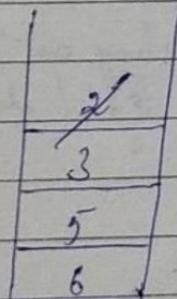
Approach 1: Sorting
 Merge Sort $O(n \log n)$

But we didn't use (K)

Approach 2: Heap



Min Heap

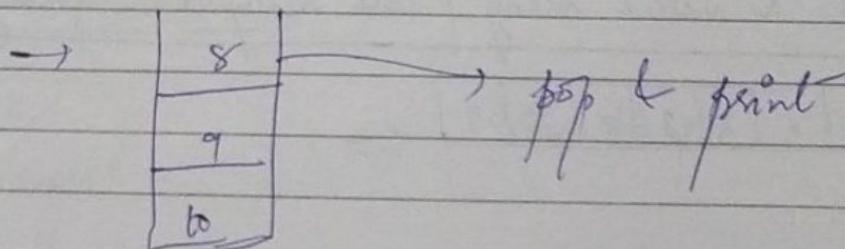
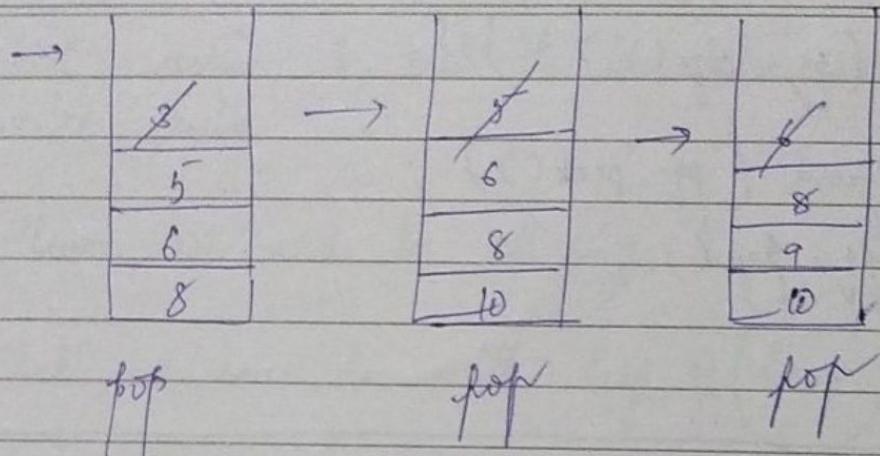


ye toh pata hai ki
 value tal index 0
 would be kill 6532
 we don't need to see
 aage

pop

$\lceil n \log k \rceil$, kyunhi 'k' se upper w. of elements
 ke liye sort nhi kar sake

op: 2 3 5 6 8 9 10



Q. K Closest Numbers

arr[] : 5 6 7 8 9

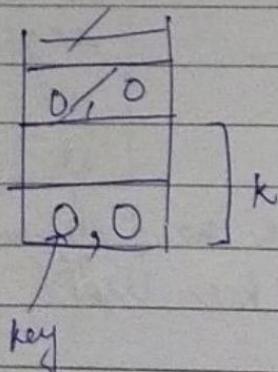
k = 3

X = 7

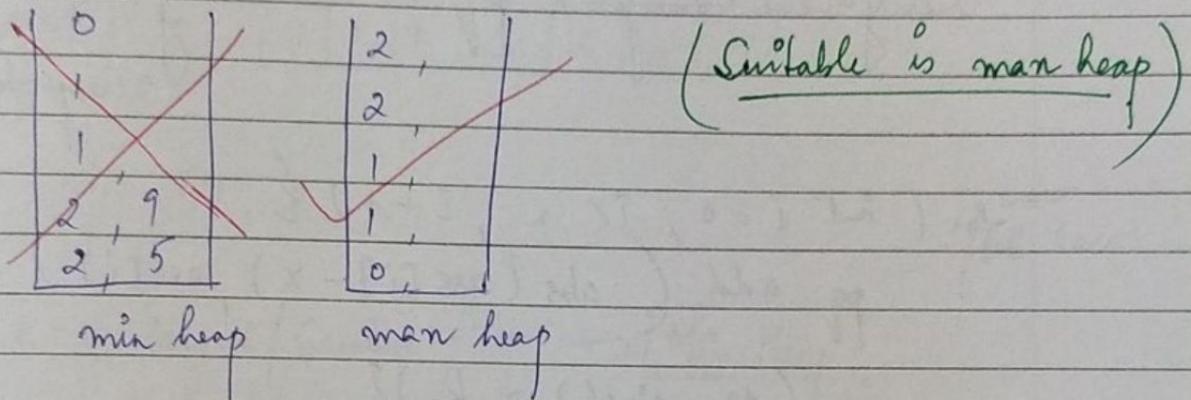
5 6 7 8 9

7 7 7 7 7

diff \Rightarrow 2 1 0 1 2 \rightarrow sorting on the basis of that
(difference)

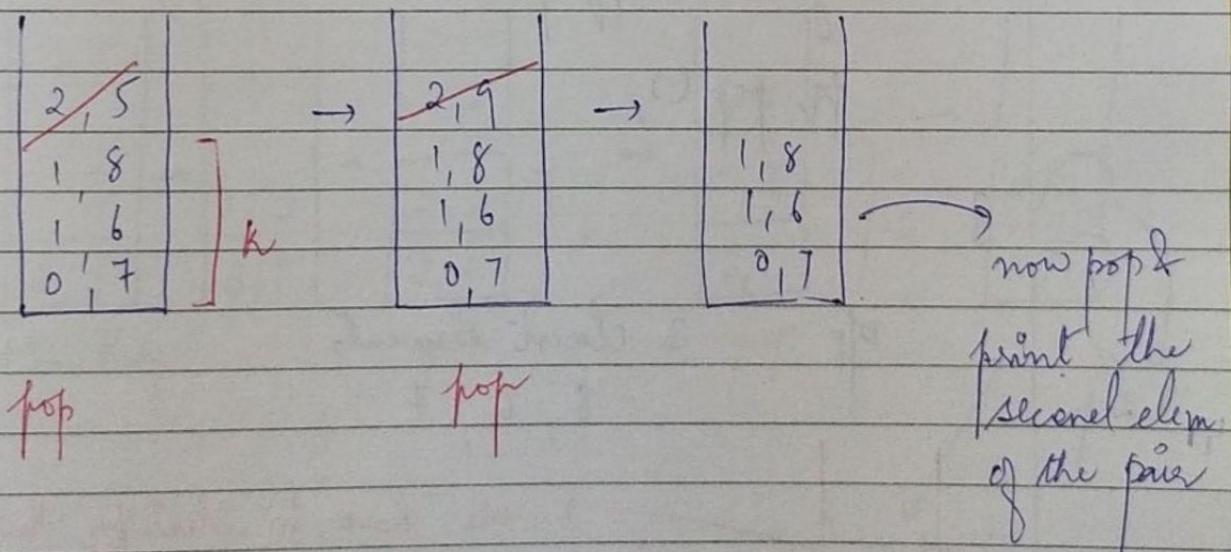


which heap??



we always pop the max element once \rightarrow Here max is 2, 2,
 \therefore we cannot make
 min heap.

Pair class \Rightarrow $\boxed{\text{abs}(\text{arr}[i] - x), \text{arr}[i]}$



Code:

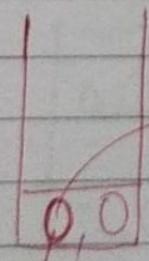
Priority Queue (^{Pair}Integers) pq = new Priority Queue<X>;
Comparator<X> compare

```
for (int i = 0; i < n; i++) {
    pq.add (abs (arr[i] - x), arr[i])
    if (pq.size() > k) {
        pq.pop();
    }
}
```

```
while (pq.size() > 0) {
    System.out.println (pq.peek().second);
    pq.pop();
}
```

Output: 3 closest elements

8, 6, 7



→ we have to identify this key
ki kispe sort karna hai

key → # itself
→ frequency
→ difference

Q. Top k frequent numbers

$arr[] = [1 | 1 | 1 | 3 | 2 | 2 | 4]$

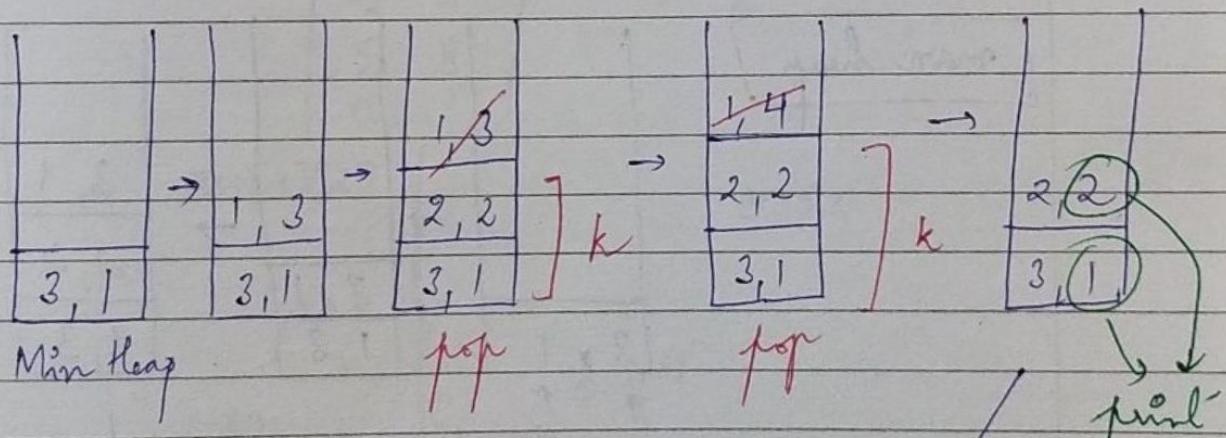
$k = 2$

O/P: 1, 2

feel how

$k + \text{Largest/Greatest}/\text{Top} \rightarrow \underline{\text{Min Heap}}$

Hash Map	1	3
	3	1
	2	2
	4	1
↑	↑	
$arr[i]$		freq



(For the symbol of priority queue → check github repo) Output

Q. [Frequency Sort]

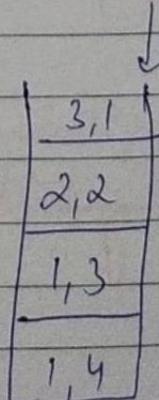
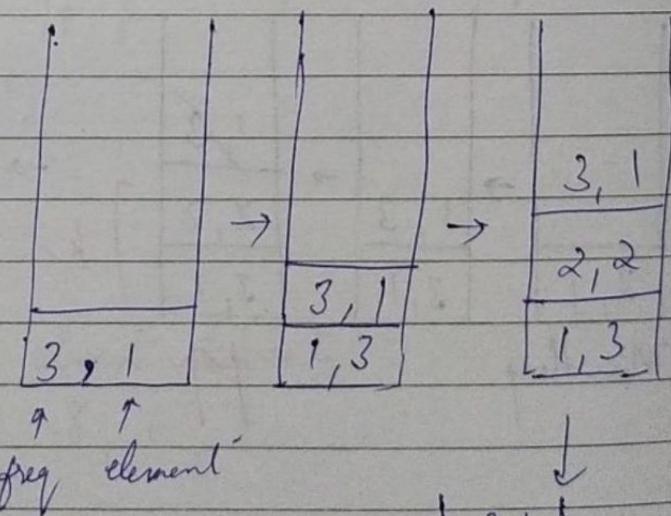
arr[] : 1 1 1 1 3 2 2 4

ofp : 1 1 1 2 2 3 4
 ↓ ↓ ↓ ↓
 3 2 1 1

Sort based on frequency

Map	key	value
1	3	
3	1	
2	2	
4	1	

[max heap]



while ($pq.size() > 0$) {

 int freq = pq.top().first;

 int elem = pq.top().second;

 for (int i = 0; i <= freq; i++) {
 cout (elem);

}

 pq.pop();

}

Q. k-Closest Points to Origin

$$arr[] = \begin{bmatrix} 1 & 3 \\ -2 & 2 \\ 5 & 8 \\ 0 & 1 \end{bmatrix} \quad k = 2$$

arr[4][2]

○ (5, 8)

(-2, 2) ○

○ (1, 3)

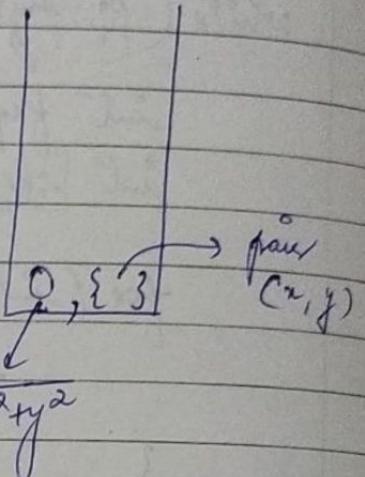
○ (0, 1)

(0, 0) origin

minimize $\rightarrow \sqrt{x^2 + y^2}$

$k=2$

2 closest points from origin.



pair<int < pair<int, int>> p;

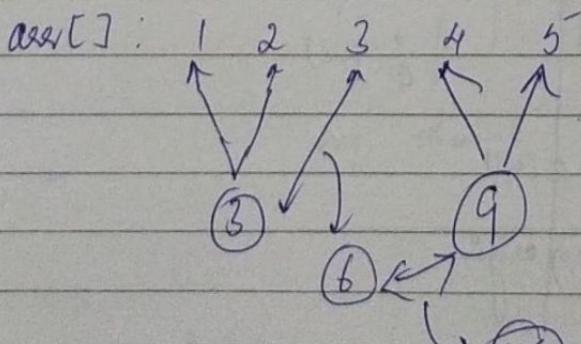
To ease out, we'll sort acc to
 $(\sqrt{x^2 + y^2})^2 = x^2 + y^2$

Max heap Banana length

{ jab samajh na sakte hō konsa heap banana hō ljh
make both & check konsa suitable hō & then
implement that heap)

Q.

Connect ropes to minimize the cost

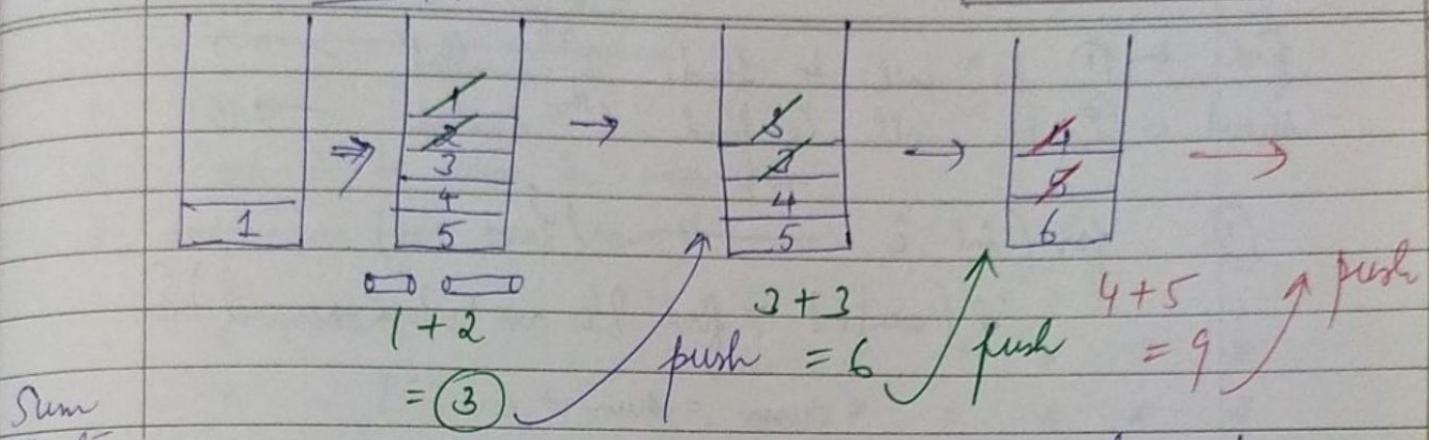


Cost =

$$3 + 6 + 9 + 15$$

$$9 + 9 + 15$$

$$18 + 15 \\ = 33$$

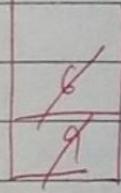
Min Heap

Sum
Tcost

$$= 3 + 6$$

$$+ 9 + 15$$

$$= 33$$

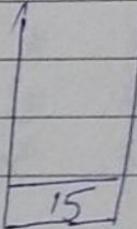


pop

$$\Rightarrow 6 + 9$$

$$= 15$$

push



≥ 2 nth
hai

pop kth & add
to Tcost

return Tcost

Ans

Q.

Sum of Elements

arr[] : [1 | 3 | 12 | 5 | 15 | 11]

$$k_1 = 3$$

$$k_2 = 6$$

concept

Variation

[1 | 3 | 12 | 5 | 15 | 11]

↑
3rd
smallest

$$\rightarrow 11 + 12 = 23 \text{ Ans}$$

↑
6th
smallest

first \leftarrow ① Fn call to find 3rd smallest $\rightarrow 5$
 second \leftarrow ② Fn call to find 6th smallest $\rightarrow 15$

```
③ for (int i = 0; i < n; i++) {
    if (arr[i] > first && arr[i] < second) {
        sum = sum + arr[i];
    }
}
return sum;
```

Variations	$k_1 \rightarrow$ Largest	$k_1 \rightarrow$ smallest
	$k_2 \rightarrow$ Largest	$k_2 \rightarrow$ largest
$k_1 \rightarrow$ Largest	$k_1 \rightarrow$ smallest	
	$k_2 \rightarrow$ smallest	$k_2 \rightarrow$ smallest

Are variations bar sahi hain gus ke

But **CONCEPT** wahi rahaega !!!

ALL CODES : GITHUB.