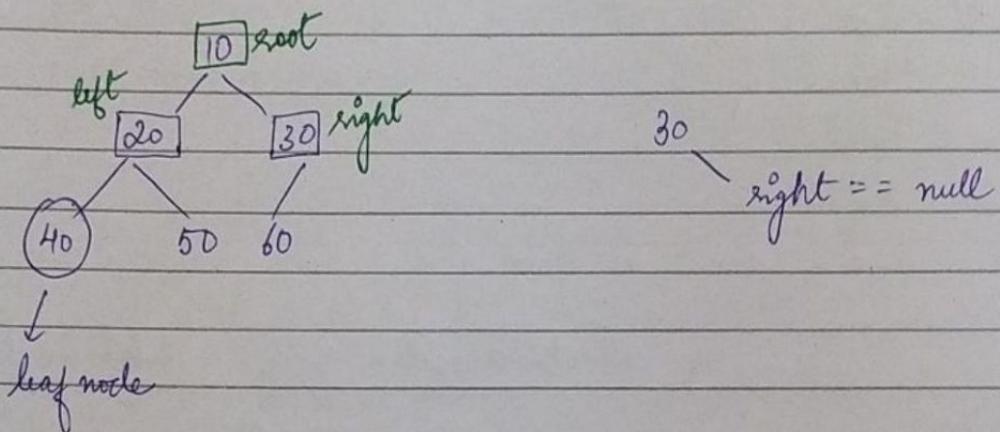


Binary Tree

- * At most 2 children

- *



- * Basic structure

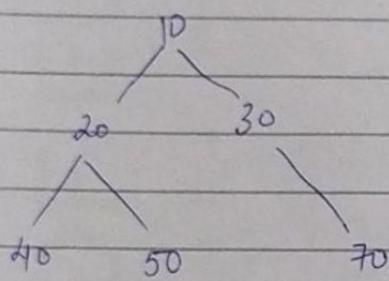
```
public class BinaryTree {  
    private class Node {  
        int data;  
        Node left;  
        Node right;  
    }  
    private Node root;
```

- * Take Input (parent, boolean)

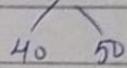
1k, true → left
3k, false → right

BT Construction → Code Github

Q1. BT → size
Max
Find
Height



⇒ $2k$ का rec. call : 20 subtree ke height



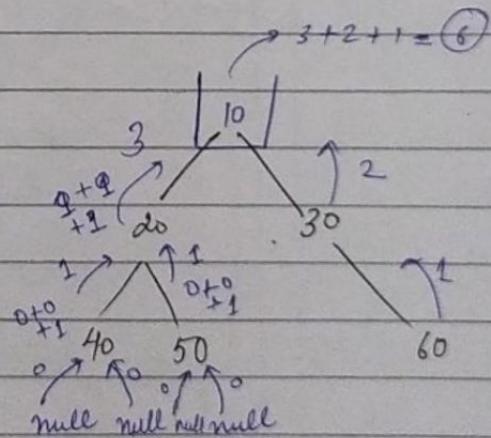
⇒ S by right subtree ka size

⇒ Done subtrees ke size ko add keke +1
kisko apne node ke naam ka.

Code :- SIZE OF BT

```

private int size (Node node) {
    if (node == null) {
        return 0;
    }
    int ls = size (node.left);
    int rs = size (node.right);
    return ls + rs + 1;
}
  
```



public int size () {
return size (root);

3

MAX OF BT

- left node se puchenge ki tere subtree me max^m value kya hai
- right node se puchenge same

Code :-

```

private int max ( Node node ) {
    if ( node == null ) {
        return Integer.MIN_VALUE;
    }
    int lm = max ( node.left );
    int rm = max ( node.right );
    return Math.max ( node.data, Math.max ( lm, rm ) );
}

```

```

public int max () {
    return max ( root );
}

```

FIND IN BT

```

private boolean find ( Node node, int item ) {
    if ( node == null ) {
        return false;
    }
}

```

```

    if ( node.data == item ) {
        return true;
    }
}

```

```

boolean ln = find ( node.left, item );
boolean rn = find ( node.right, item );
return ln || rn;
}

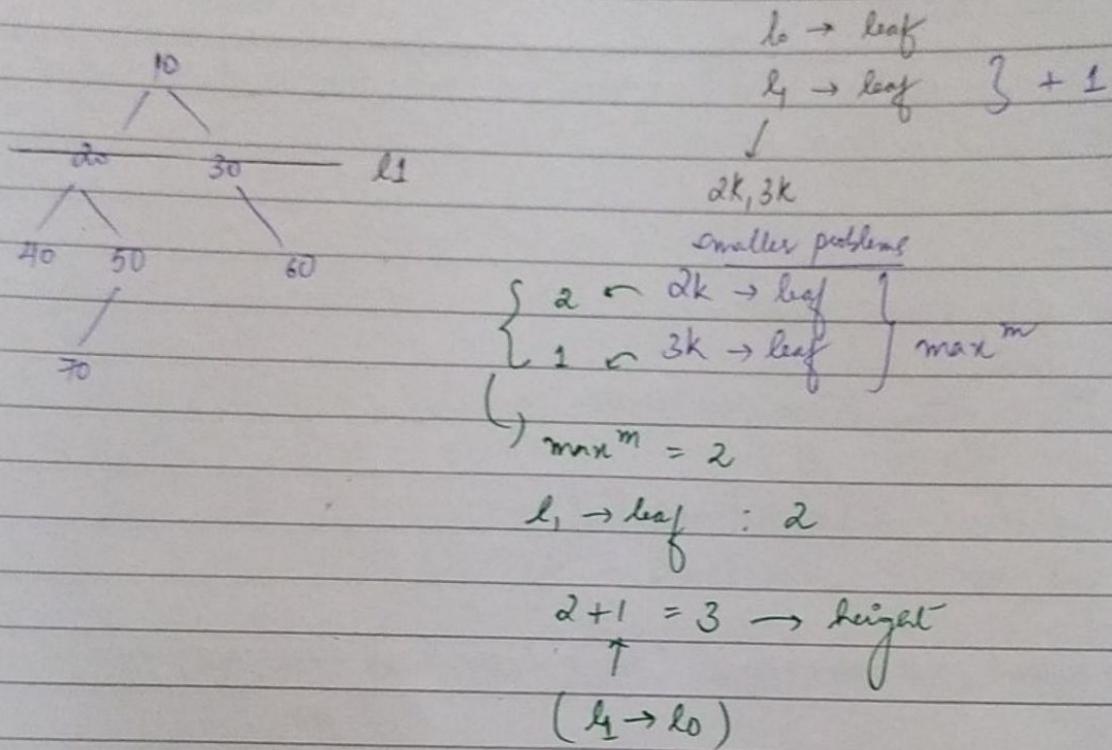
```

```

public boolean find (
    int item ) {
    return find ( root, item );
}

```

HEIGHT OF BT



Code :-

```
private int ht( Node node ) {
```

```
    if (node == null) {
```

```
        return -1; }
```

3

```
        int lh = ht( node.left );
```

```
        int rh = ht( node.right );
```

```
        return Math.max( lh, rh ) + 1;
```

3

```
public int ht() {
```

```
    return ht( root );
```

1

(After returning back to
 $\max(lh, rh) + 1 = 0 + 1 = 1$
height which is not size)
 \rightarrow

TC:

$$T(n) = 2T\left(\frac{n}{2}\right) + 1$$

$$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{4}\right) + 1 \quad \times 2$$

$$T\left(\frac{n}{4}\right) = 2T\left(\frac{n}{8}\right) + 1 \quad \times 2^2$$

⋮
⋮

$$T(1) = 1$$

$$\hookrightarrow nT\left(\frac{n}{n}\right) = ^n$$

GP sum formula

$$\hookrightarrow \frac{a(r^n - 1)}{r - 1}$$

$$n = 2^{\log_2 n}$$

$$2^0 + 2^1 + 2^2 + 2^3 - \dots + 2^{\log_2 n}$$

$$T(n) = \frac{2^{\log_2 n+1} - 1}{2 - 1} = 2^{\log_2 n+1} - 1$$

$= n$

$T(n) = O(n)$

DIAMETER

```
private int diameter2 (Node node) {
    if (node == null) {
        return 0;
    }
}
```

// max dist b/w 2 leaf nodes might lie in the left subtree : left ka diameter
int ld = diameter2 (node.left);
int rd = diameter2 (node.right); // right ka diameter

// self node might be the root of diameter
int sd = ht(node.left) + ht(node.right) + 2;

return Math.max (sd, Math.max (ld, rd));

}

TC:

$$T(n) = 4T\left(\frac{n}{2}\right) + 1$$

$$T(n)_2 = 4T(n_4) + 1 \times 4$$

⋮ $\times 4^2$
⋮ ⋮

$$T(n|n) = 1$$

$$\left(T\left(\frac{n}{2^{\log_2 n}}\right) \right)$$

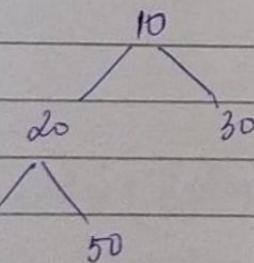
$$4^0 + 4^1 + 4^2 + \dots + 4^{\log_2 n}$$

$$\frac{4^{\log_2 n + 1} - 1}{4 - 1} = \frac{4^{\log_2 n + 1} - 1}{3} \quad (\text{constants hatada})$$

$$= 4^{\log_2 n} = \underline{\underline{n^2}} \quad : \pi$$

Diameter Better Approach

$$n^2 \rightarrow n$$



$$T(n) = 4T\left(\frac{n}{2}\right) + 1$$

$$\hookrightarrow O(N^2)$$

$$T(n) = 2T\left(\frac{n}{2}\right) + 1$$

$$\hookrightarrow O(N)$$

* Ish kisi taraf recurrence relation ham \rightarrow ye wala banadein, toh kaam ho jaayega
 (Toh hone seif 4 calls deni hain)

Is Balanced

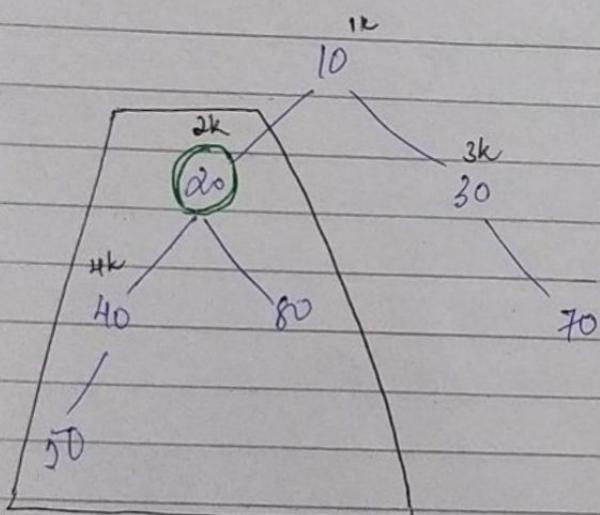
$-1, 0, 1$

Balanced : every node should've a bf of $-1, 0, 1$.

Approach 1 :-

bf : right subtree - left subtree
height height

Ek imp. chay tree me :-



* 2k memory address pe, 2k wale pure subtree ki information stored hogi.

* aisa nhi hoga ki, 2k memory address pe size (20) wali node ka information stored hogi.

① Yeh 2k ke rec call batayegi ki uske neeche ka tree (subtree) balanced hai k? n?

smaller problem

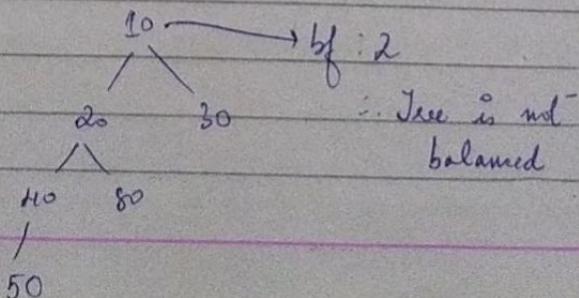
② $30 \rightarrow 3k$ wali rec call batayegi ki uske neeche ka subtree balanced hai k? n?

smaller problem

③

Then we'll also check ki 10 wali node balanced hai k? n?

Eg. suppose tree aisa hota toh :



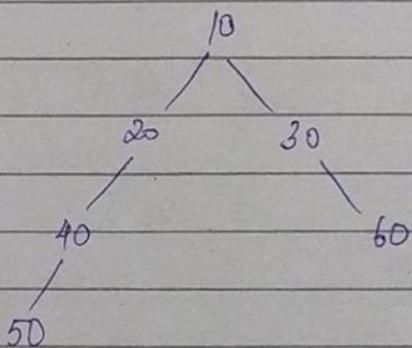
Summary :-

- 3 mandatory checks → 1. left st bal
2. right st bal
3. node bal.

Why? Let's see some eg.

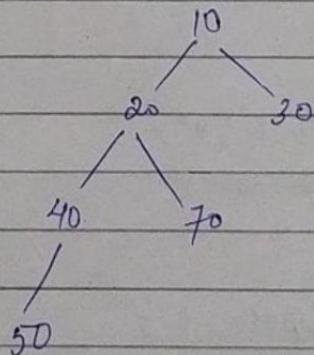
→ Suppose we don't check left st. bal

Here left st. is not balanced
left st ✓
node ✓



→ Suppose we don't check node

Here left st ✓
right st ✓
node X



Code :-

```
public boolean isBalanced () {
```

```
    return isBalanced (root);
```

```

private boolean isBalanced ( Node node ) {
    if ( node == null ) {
        return true;
    }
    boolean lb = isBalanced ( node . left );
    boolean rb = isBalanced ( node . right );
    int bf = ht ( node . left ) - ht ( node . right );
    if ( lb && rb && bf >= -1 && bf <= 1 ) {
        return true;
    } else {
        return false;
    }
}

```

3.

$$TC = \underline{\underline{O(n^2)}} \quad \left(4\pi \frac{n}{2} + 1 \right)$$

Approach 2

```

private class BallPair {
    boolean isBal = true;
    int ht = -1;
}

```

```

public boolean isBalanced () {
    return isBalanced ( root ), isBal;
}

```

private BalPair isBalanced (Node node) {

if (node == null) {

return new BalPair();

}

BalPair lbp = isBalanced (node.left);

BalPair rbp = isBalanced (node.right);

BalPair sfp = new BalPair();

boolean lf = lbp.isBal;

boolean rb = rbp.isBal;

int bf = lbp.ht - rbp.ht;

if (lf && rb && bf >= -1 && bf <= 1) {

sfp.isBal = true;

} else {

sfp.isBal = false;

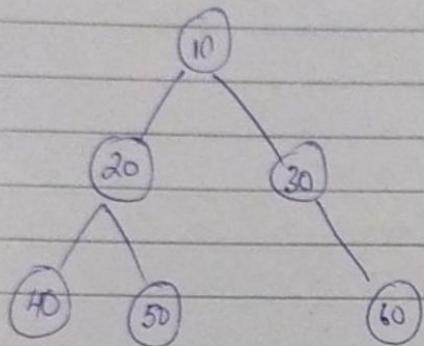
}

sfp.ht = Math.max (lbp.ht, rbp.ht) + 1;

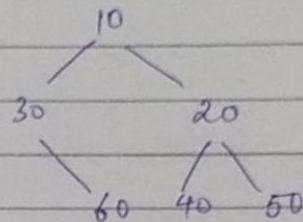
return sfp;

}

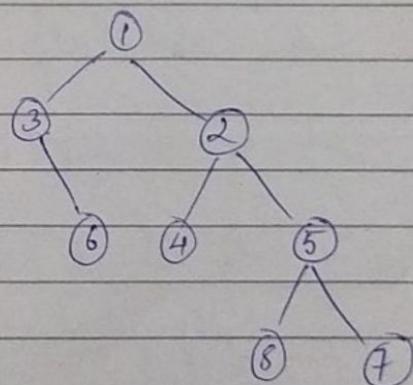
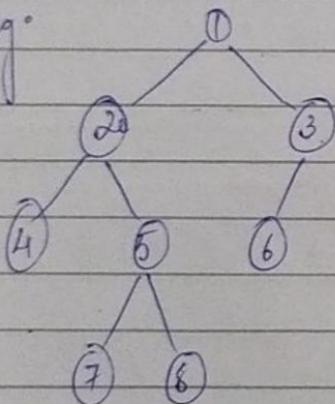
BT Flip Equivalent



10 flip :



Q.



Are they flip equivalent?

Yes

We flipped at nodes with 1, 3 and 5

* node 1. data == node 2. data

* 2 cases → Node 1 ko flip ✓
flip nahi karaya X

* flip ✓ Toh node 1 ka left subtree ko node 2 ke right subtree
match karayenge ✓

node 1 ko right subtree & node 2 ka left st match
hona chahiye .

* flip X $(\text{node1}.\text{lst}, \text{node2}.\text{lst}) \&$
 $(\text{node1}.\text{rst}, \text{node2}.\text{rst})$

** Imp. note :

\Rightarrow Ham ye nhi keh she ki bilkul ditto match
 ho jayein, [ye apas me flip equivalent
 hone chaliye]

Code :

```
public boolean flipEquivalent (BinaryTree other) {
    return flipEquivalent (root, other.root);
}
```

```
private boolean flipEquivalent (Node node1, Node node2) {
    // BC
    if (node1.data != node2.data) {
        return false;
    }
}
```

```
boolean ll = flipEquivalent (node1.left, node2.left);
        right   right
        lr      left
        rl      right
                    left
```

```
return (ll && rr) || (lr && rl);
           ↘       ↙
           not flip   flip
```

// BC

if(node1 == null && node2 == null) {
 return true;

}

if(node1 == null & node2 != null) {
 return false;

}

if(node1 != null && node2 == null) {
 return false;

}

Level Order

```
public void levelOrder() {
```

```
    linkedlist<Node> queue = new linkedlist<>();
    queue.add(root);
```

```
    while (!queue.isEmpty()) {
```

```
        Node rm = queue.removeFirst();
        System.out.print(rm.data + ", ");
```

```
        if (rm.left != null) {
            queue.addLast(rm.left);
```

```
}
```

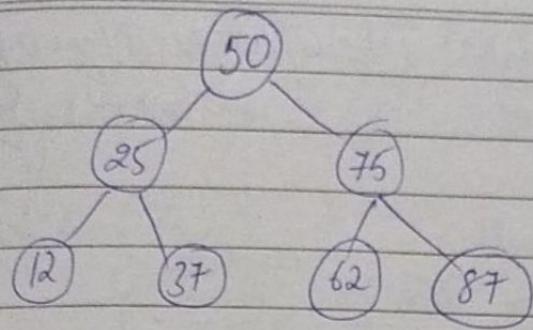
```
        if (rm.right != null) {
```

```
            queue.addLast(rm.right);
```

```
}
```

```
3
```

Is BT a BST?



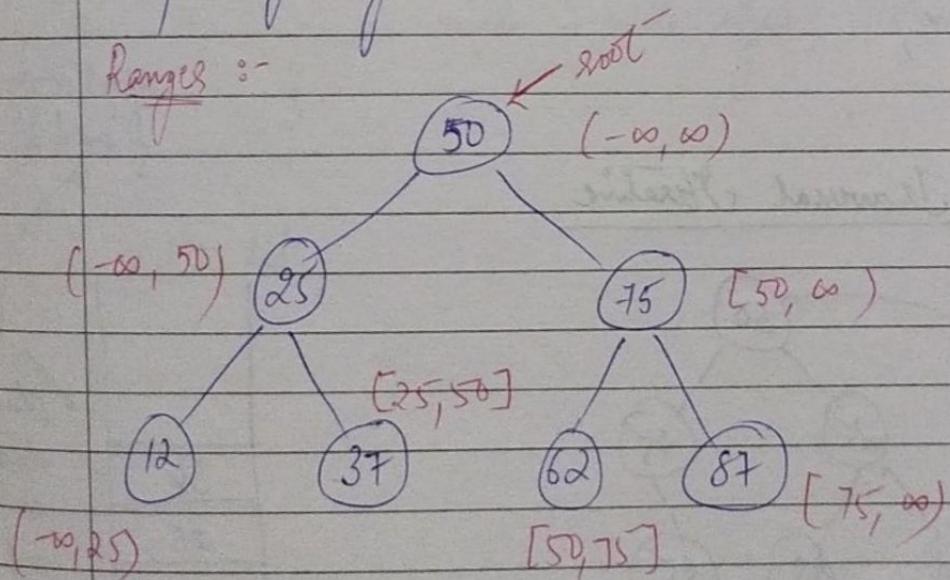
- ① Inorder traversal of the tree & store all value in an array

12, 25, 37, 50, 62, 75, 87 ← inorder

↓
sorted array will
∴ BST ✓

- ② Compare by Range

Ranges :-



Code →

public static boolean isBST (BinaryTreeNode<Integer> root, int min, int max) {

if (root == null)
return true;

if (root.data > max || root.data < min)
return false;

else if (!isBST (root.left, min, root.data))
return false;

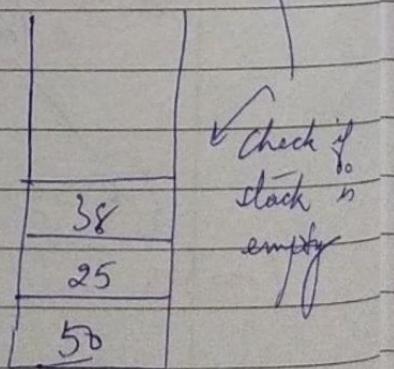
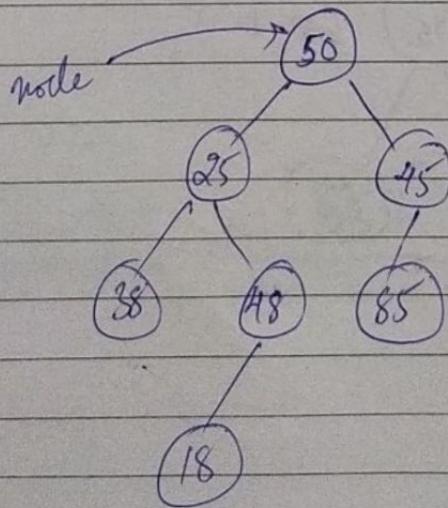
else if (!isBST (root.right, root.data, max))
return false;

return true;

3

if not empty, pop
add to list

Inorder Traversal Iterative



Stack of node

list : [38, 25, 18, 48, 50, 85, 45]

Leetcode sol :-

Stack <TreeNode> st = new Stack<>();

ArrayList <Integer> list = new ArrayList<>();

while (true) {

if (~~node~~ ^{node} == null) {

st.push(~~node~~);

node = node.left;

}

else {

if (st.isEmpty()) {

break;

}

node = st.pop();

list.add(node.val);

node = node.right;

}

}

return list;

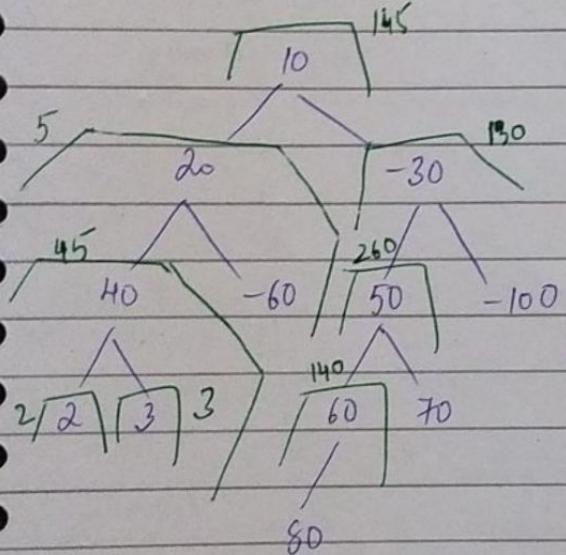
SUM OF ALL NODES

```
if (node == null) {
    return 0;
}
```

```
int ls = sum (node.left);
int rs = sum (node.right);
```

```
return ls + rs + node.data;
```

MAX SUBTREE SUM



max subtree sum = 260

Logic yehi
wala lagega

Code :-

113c

```
int ls = sum (node.left);
int rs = sum (node.right);
```

```
return ls + rs + node.data
```

→ sum of all nodes

Code :-

Max subtree sum

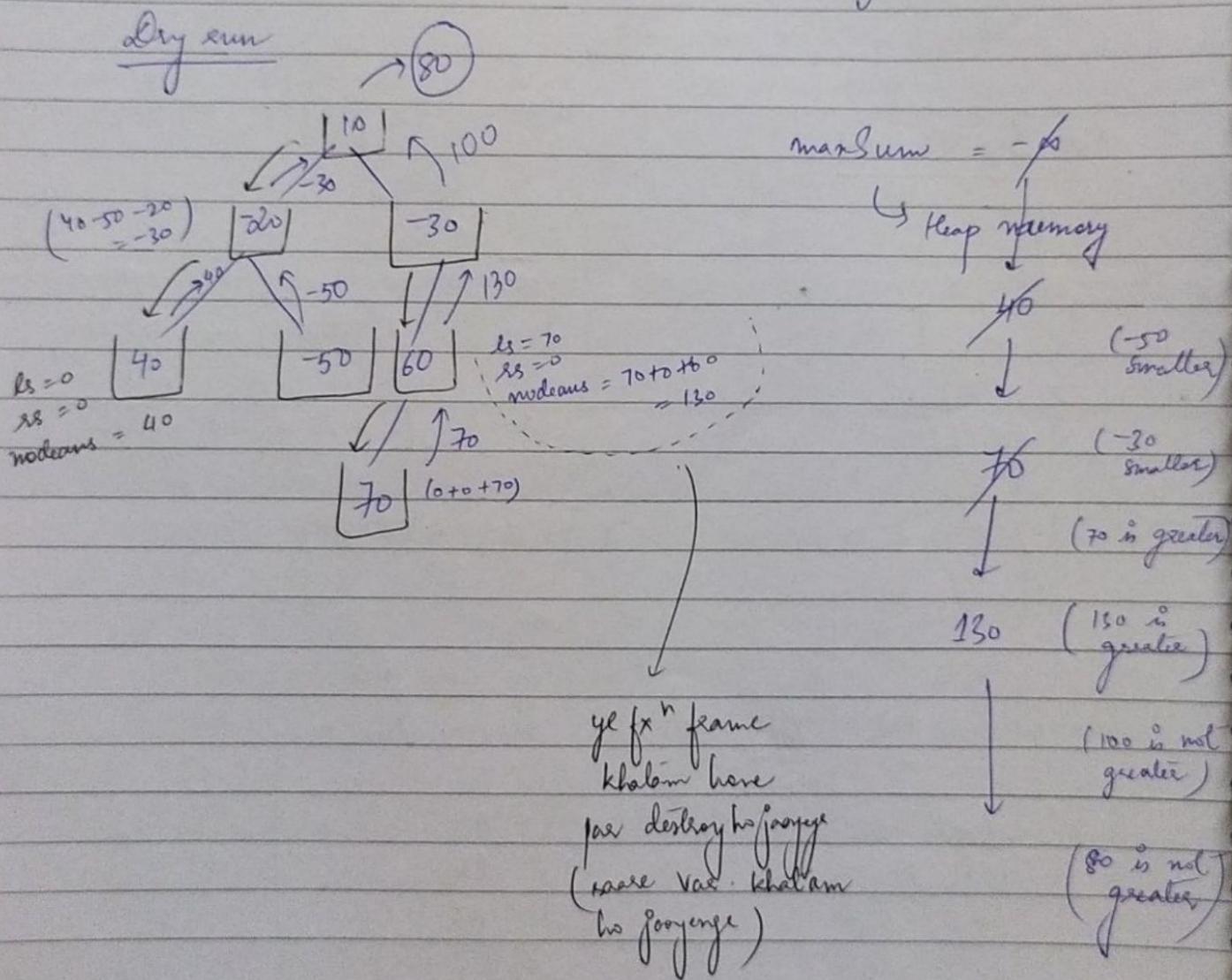
A1

A2

A3

Approach 1: Using global Variable

- * Use the mansum global variable rakhlega:
initialized with $-\infty$
- * Have subtree sum ko mansum se compare karoge



Ans
 $= 130$

Code :

```
int manSum = Integer.MIN_VALUE;
```

```
public int maxSubtreeSum () {  
    maxSubtreeSum (root);  
    return manSum;
```

}

```
private int maxSubtreeSum (Node node) {
```

```
    if (node == null) {  
        return 0;
```

}

```
    int ls = maxSubtreeSum (node.left);
```

```
    int rs = maxSubtreeSum (node.right);
```

```
    int nodeans = ls + rs + node.data;
```

```
    if (nodeans > manSum) {  
        manSum = nodeans;
```

}

```
    return nodeans;
```

}

Approach 2 : Recursion is returning something (main subtree sum)

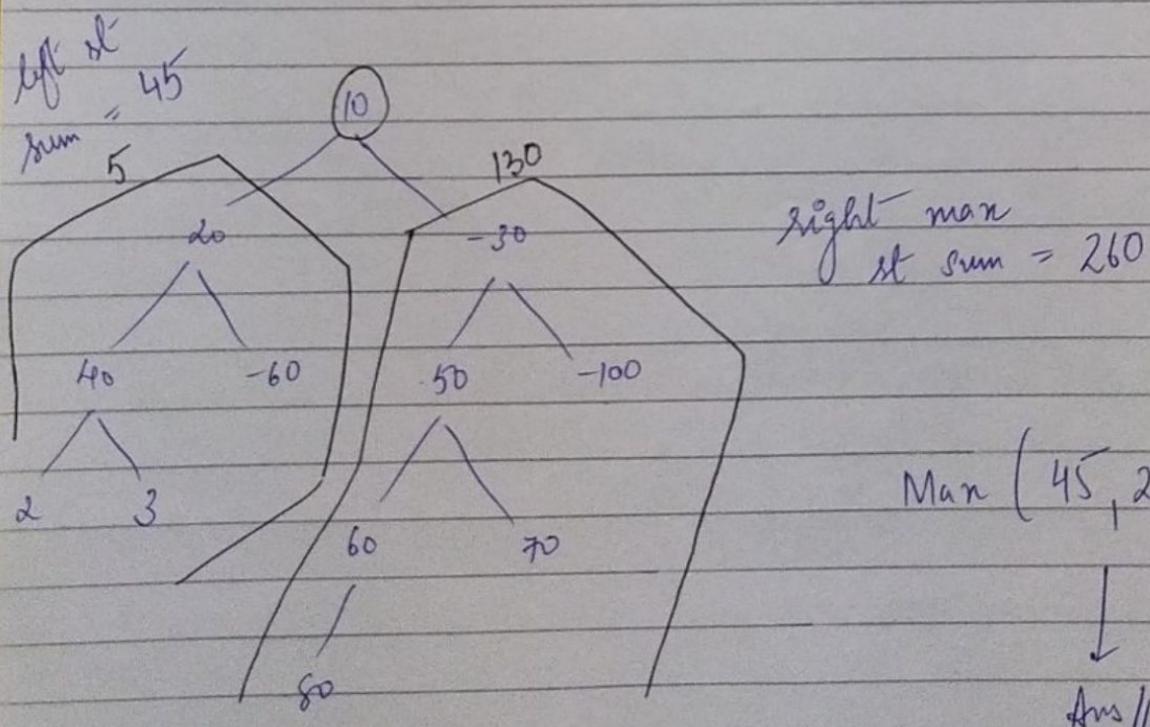
- * Rec call ka meaning clear hona chahiye mind me

* $\begin{cases} 45 \\ 260 \end{cases}$ } left : main st sum
 $\begin{cases} 260 \\ 45 \end{cases}$ } right : main st sum
 (recursive calls) They are different

$$\text{node. data} + \frac{\text{left}}{\text{entire sum}} + \frac{\text{right}}{\text{entire sum}}$$

$$10 + \frac{5}{145} + \frac{130}{145}$$

$$10 + 5 + 130 = 145$$



Code :

```
if ( node == null ) {  
    return Integer.MIN_VALUE;  
}
```

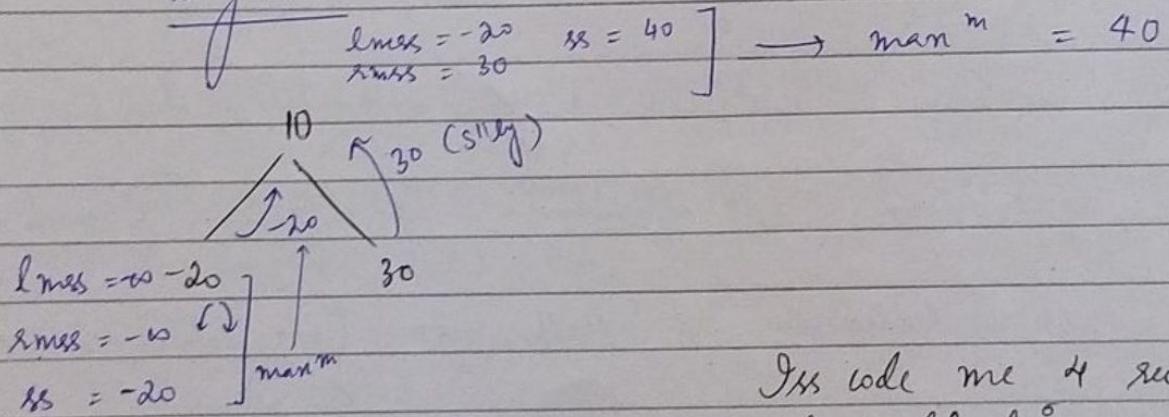
```
int lManSubtreeSum = manSubtreeSum ( node.left );
```

```
int rManSubtreeSum = manSubtreeSum ( node.right );
```

```
int selfSum = sum ( node );
```

```
return Math.max ( selfSum, Math.max ( lManSubtreeSum,  
                                         rManSubtreeSum ) );
```

Dry run



This code makes 4 rec calls

log shr^o bin^o

sum (node)

sum (node.left)

sum (node.right)

Approach 3 :

Pai se keke

hoga

(Reduce TC)

TC ↑

Code :

```
private class manSubtreeSumPair {  
    int entreeSum = 0;  
    int manSubtreeSum f = Integer.MIN_VALUE;  
}
```

```
public int manSubtreeSum () {  
    return manSubtreeSum (root).manSubtreeSum;  
}
```

```
private manSubtreeSumPair manSubtreeSum (Node node) {  
    // BC  
    manSubtreeSumPair lp = manSubtreeSum (node.left);  
    rp = manSubtreeSum (node.right);  
    sp = new manSubtreeSumPair ();  
}
```

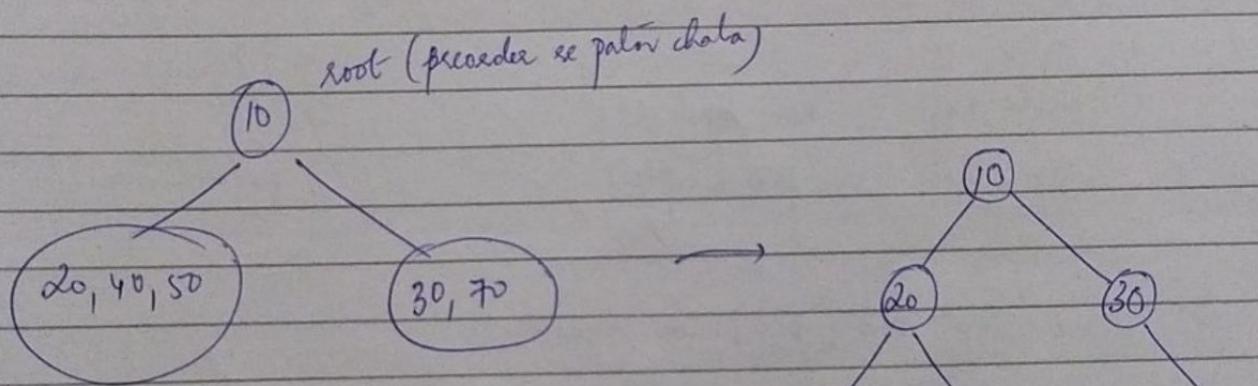
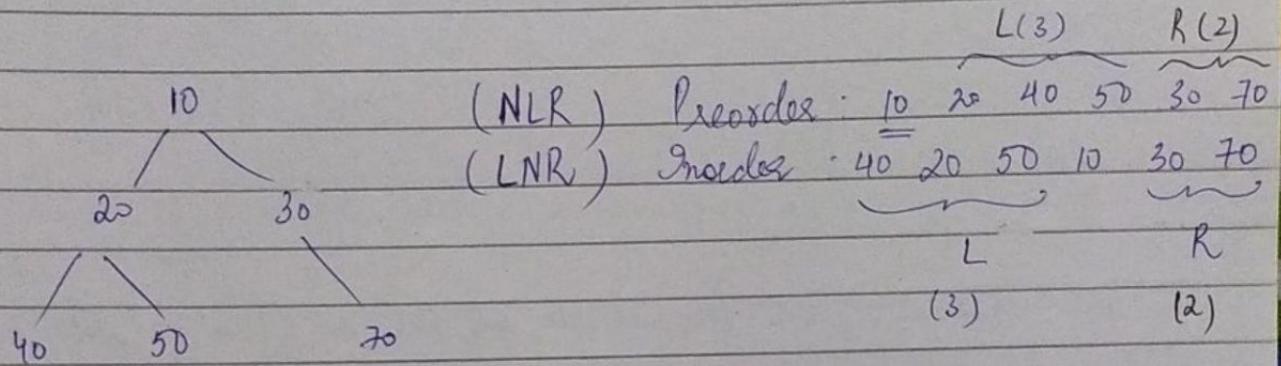
$$sp.\text{entreeSum} = lp.\text{entreeSum} + rp.\text{entreeSum} + \text{node.data}$$

```
sp.manSubtreeSum = Math.min (sp.entreeSum,  
    Math.max (lp.manSubtreeSum, rp.manSubtreeSum));
```

return sp;

```
// BC  
if (node == null) {  
    return new manSubtreeSumPair ();  
}
```

Imp Construct Tree from Preorder & Inorder traversal
Postorder & Inorder



↓ L R (Inorder ko dekhe path
20, 40, 50 (NLR) data)

40, 20, 50 (LNRL)
L R

left subtree ka root

→ root
30, 70 (NLR)

30, 70 (LNRL)

→ right

Preorder : [0 1 2 3 4 5 6
10, 20, 40, 30, 50, 60, 70] NLR

Inorder : [0 1 2 3 4 5 6
40, 20, 10, 50, 30, 60, 70] LNR

$$plo = 0 \quad phi = 6$$

$$ilo = 0 \quad ihi = 6$$

private Node construct(int[] pre, int plo, int phi, int[] in, int ilo, int ihi) {

//BC ① Create a new node with plo

Node nn = new Node();

nn.data = pre[plo]; // 10 wali node

② search for pre[phi] in inorder

int si = -1, nel = 0;

for (int i = ilo ; i <= ihi ; i++) {
if (pre[phi] == in[i]) {

si = i;

break;

} nel++;

index of 10

|| si = 2

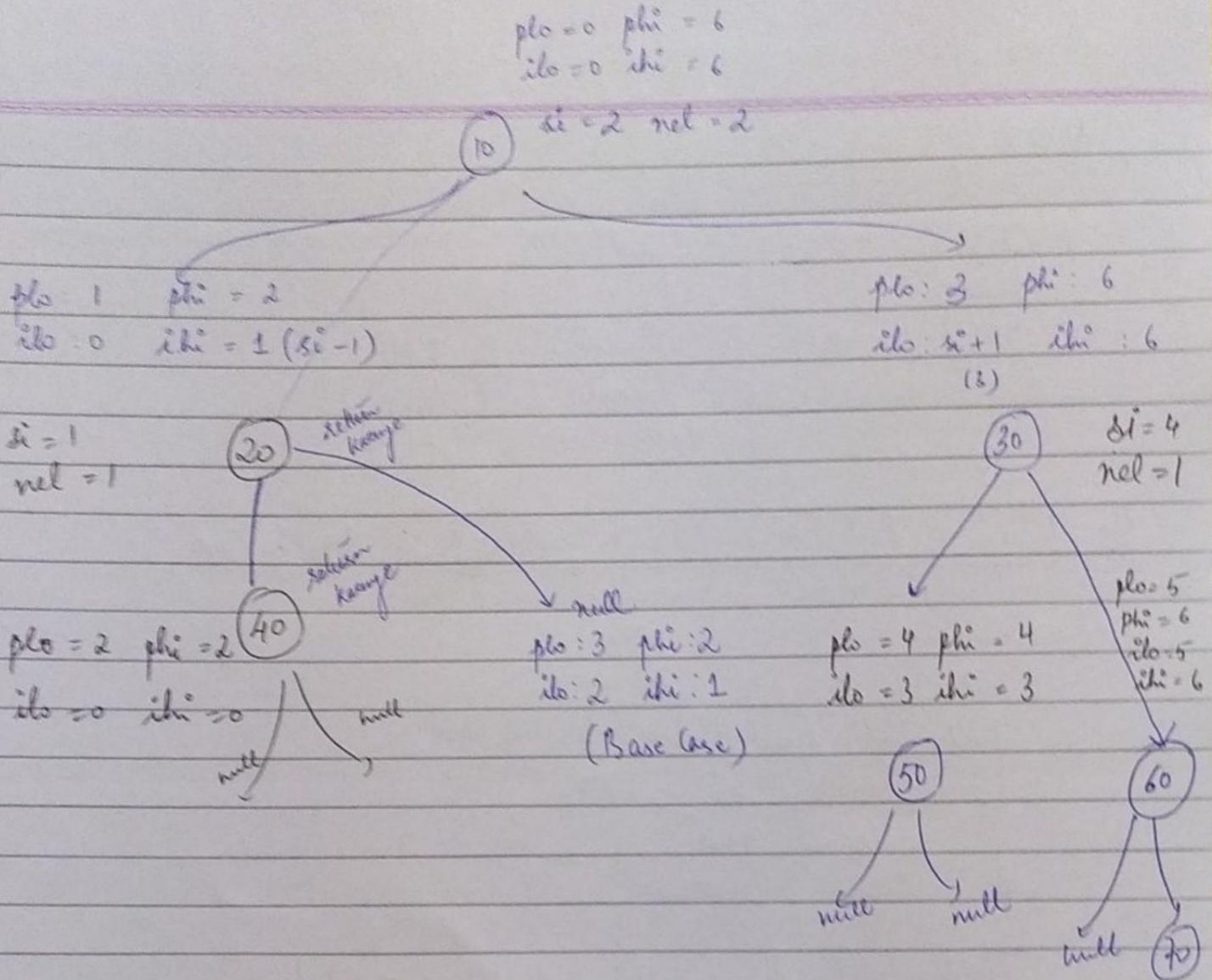
|| nel = 2 ^{10 se}
^{of elem}
(40, 20)

③ left and right child call

nn.left = construct(pre, plo + 1, plo + nel, in, ilo, si - 1);

nn.right = construct(pre, plo + nel + 1, phi, in, si + 1, ihi);

return nn;



// Base Case

```

if ( ilo > ihu || plo > phi ) {
  return null;
}
  
```

BFS & DFS in Tree

(BFS)

→ Level Order

(DFS)

- 1. Inorder (LNR)
- 2. Preorder (NLR)
- 3. Postorder (LRN)

Q. Sum of Root to Leaf Binary Numbers

Leetcode

class Solution {

 int sum = 0;

 public int sumRootToLeaf(TreeNode root) {

 dfs (root, "");

 return sum;

}

 public void dfs (TreeNode root, String cur) {

 if (root == null) {

 return;

}

 if (root.left == null && root.right == null) {

 sum += Integer.parseInt (cur + root.val, 2);

}

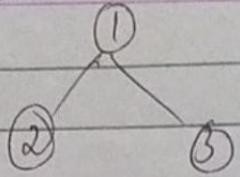
 dfs (root.left, cur + root.val);

 dfs (root.right, cur + root.val);

}

)

Q. Similar Ques



$$12 + 13 = \underline{\underline{25}}$$

class Solution {

int sum = 0;

public int sumNumbers (TreeNode root) {

dfs (root, 0);

return sum;

}

public void dfs (TreeNode root, int cur) {

if (root == null)

return;

cur = cur * 10 + root.val;

if (root.left == null && root.right == null) {

sum += cur;

}

dfs (root.left, cur);

dfs (root.right, cur);

}

J