フィルタバンク理論に基づく畳込み辞書学習

～ 構造制約を利用した畳込みネットワーク構築 ～

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動作環境：MATLAB R2023a

準備

```
clear
close all

nsoltDic = ""; %"nsoltdictionary_20230529182345109";

isCodegen = false; % コード生成
setup(isCodegen)
```

```
SaivDr-4.2.2.2 exits.
Skip code generation
```

パラメータ設定

- ブロックサイズ
- 冗長度
- スパース度

```
% Block size
szBlk = [ 8 8 ];

% Redundancy ratio for RICA/K-SVD
redundancyRatio = 5/3;

% Sparsity ratio
sparsityRatio = 3/64;
```

画像の読込

学習用に正規化を適用

- $\mathbf{y} \in \mathbb{R}^N$

```
% 原画像の準備
```

```
file_yorg = "../data/yorg.png";
if ~exist(file_yorg,'file')
    unzip('http://www.ess.ic.kanagawa-it.ac.jp/std_img/monoimage2/Mono-
Image2.zip','../results')
    yfull = imread('../results/Mono-Image2/512X512/barbara512.bmp');
    ycrop = yfull(1:192,end-255:end);
    imwrite(ycrop,file_yorg)
end

% 原画像の読み込み
yorg = im2double(imread(file_yorg));
szOrg = size(yorg);
y = normalize(yorg);
```

画像表示

```
figure
imshow(yorg);
title('Original image y')
```

Original image y

離散コサイン変換（DCT）

$$[\mathbf{C}_M]_{k,n} = \sqrt{\frac{2}{M}}\alpha_k \cos\frac{k(n+1/2)\pi}{M},\ \ k,n = 0,1,\cdots,M-1$$

$$\alpha_k = \begin{cases} \frac{1}{\sqrt{2}} & k=0 \\ 1 & k = 1,2,\cdots,M-1 \end{cases}$$

基底画像

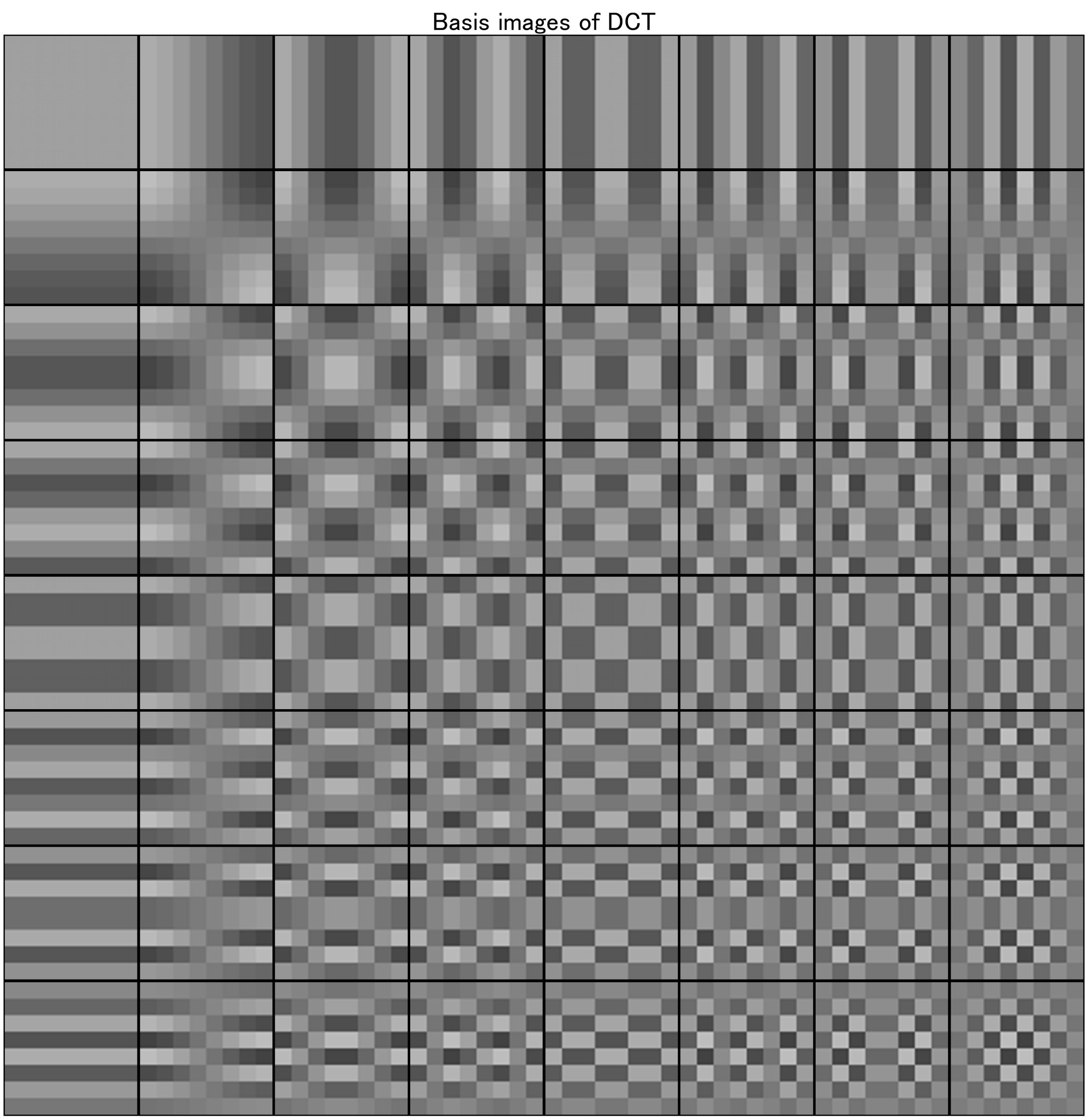
$$\mathbf{B}_{k,\ell} = \mathbf{C}_M^{-1}\mathbf{E}_{k,\ell}\mathbf{C}_M^{-T},\ \ k,\ell = 0,1,\cdots,M-1$$

$$\mathbf{E}_{k,\ell} = \mathbf{e}_k\mathbf{e}_\ell^T$$

```
basisImagesDct = zeros(szBlk(1),szBlk(2),prod(szBlk));
iBasis = 1;
for iRow=1:szBlk(1)
    for iCol=1:szBlk(2)
        E = zeros(szBlk);
        E(iRow,iCol) = 1;
        basisImagesDct(:,:,iBasis) = idct2(E,szBlk(1),szBlk(2));
        iBasis = iBasis + 1;
    end
end
```

基底画像の表示

```
figure
montage(imresize(basisImagesDct,8,'nearest')+.5,'BorderSize',[2 2])
title('Basis images of DCT')
```



ブロック DCT による合成処理とその随伴処理の定義

```
syn_blkdct = @(x) blockproc(x,szBlk,@(block_struct) idct2(block_struct.data));
adj_blkdct = @(y) blockproc(y,szBlk,@(block_struct) dct2(block_struct.data));
```

随伴関係の確認

```
x = adj_blkdct(y);
v = randn(size(x));
u = syn_blkdct(v);
assert(abs(dot(y(:),u(:))-dot(x(:),v(:)))<1e-9)
```

主成分分析（PCA）

問題設定:
直交性と次元削減

$\Phi = \mathbf{I}_M, \forall b, \forall p, \|\mathbf{x}_b\|_0 \leq p < M$

を制約条件とした最小自乗問題

$$\{\hat{\mathbf{\Phi}}, \{\hat{\mathbf{x}}_b\}_b\} = \arg \min_{\{\mathbf{\Phi}, \{\mathbf{x}_b\}_b\}} \frac{1}{2B} \sum_{b=1}^{B} \|\mathbf{y}_b - \mathbf{\Phi}\mathbf{x}_b\|_2^2$$

を解く．上式は等価的に

$$\hat{\mathbf{\Phi}} = \arg \max_{\mathbf{\Phi}} \mathrm{tr}\left(\mathbf{\Phi}_{:,0:p-1}^\mathsf{T} \hat{\mathbf{\Sigma}}_y \mathbf{\Phi}_{:,0:p-1}\right) \quad \text{s.t.} \quad \mathbf{\Phi}^\mathsf{T}\mathbf{\Phi} = \mathbf{I}_M$$

と表現できる．ただし， $\hat{\mathbf{\Sigma}}_y$は 観測ベクトル $\{\mathbf{y}_b\}_b$ （零平均を仮定）の標本分散共分散行列である.

解:
固有値分解

$$\widehat{\mathbf{\Phi}}^\mathsf{T}\widehat{\mathbf{\Sigma}_y}\widehat{\mathbf{\Phi}} = \mathbf{\Lambda}$$

ただし， $\mathbf{\Lambda} = \mathrm{diag}(\lambda_1, \lambda_2, \cdots, \lambda_M)$． $\lambda_1 \geq \lambda_2 \geq \cdots \lambda_M$ は $\hat{\mathbf{\Sigma}}_y$の固有値.

画像 $\mathbf{y}$からのデータ行列 $\mathbf{Y}$ の生成

標本平均ブロックを引く代わりに，予め正規化を適用したデータで学習

```
Y = im2col(y,szBlk,'distinct');
```

標本分散共分散行列 $\hat{\mathbf{\Sigma}}_y$の計算

```
SigmaY = cov(Y.');
```

標本分散共分散行列 $\hat{\Sigma}_y$の固有値分解

```
[Phi_pca,Lambda] = eig(SigmaY);
```

固有値 λ の大きさの降順に列ベクトルをソート (Sorting column vectors in the descending order of the eigenvalues λ)

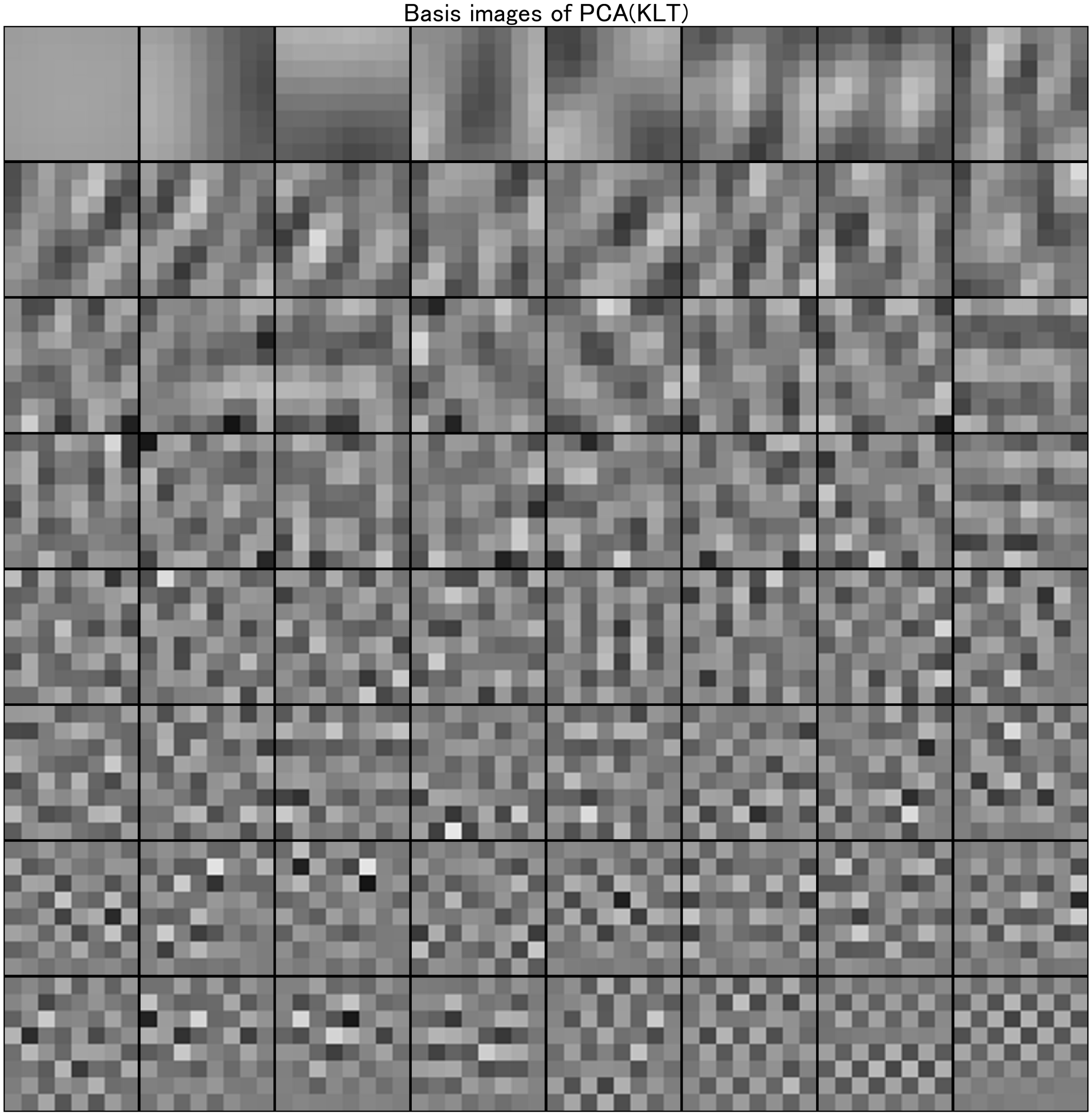
```
[~,idx] = sort(diag(Lambda),'descend');
Phi_pca = Phi_pca(:,idx);
```

固有ベクトルを基底画像に変換

```
nBases = prod(szBlk);
basisImagesPca = zeros(szBlk(1),szBlk(2),nBases);
for iBasis = 1:nBases
    basisImagesPca(:,:,iBasis) = reshape(Phi_pca(:,iBasis),szBlk(1),szBlk(2));
end
```

基底画像の表示（辞書）

```
figure
montage(imresize(basisImagesPca,8,'nearest')+.5,'BorderSize',[2 2])
title('Basis images of PCA(KLT)')
```



ブロック PCA による合成処理とその随伴処理の定義

```
syn_blkpca = @(x) col2im(Phi_pca*x,szBlk,szOrg,"distinct");
adj_blkpca = @(y) Phi_pca.'*im2col(y,szBlk,"distinct");
```

随伴関係の確認

```
x = adj_blkpca(y);
v = randn(size(x));
u = syn_blkpca(v);
assert(abs(dot(y(:),u(:))-dot(x(:),v(:)))<1e-9)
```

再構成独立成分分析（RICA）

問題設定:

$$\widehat{\mathbf{\Phi}} = \arg\min_{\mathbf{\Phi}} \frac{1}{2B}\sum_{b=1}^{B} \|\mathbf{y}_b - \mathbf{\Phi}\mathbf{\Phi}^\top\mathbf{y}_b\|_2^2 + \frac{\alpha}{B}\sum_{b=1}^{B} \rho(\mathbf{\Phi}^\top\mathbf{y}_b)$$

$$= \arg\min_{\mathbf{\Phi}} \frac{(2\alpha)^{-1}}{B}\sum_{b=1}^{B} \|\mathbf{y}_b - \mathbf{\Phi}\mathbf{\Phi}^\top\mathbf{y}_b\|_2^2 + \frac{1}{B}\sum_{b=1}^{B} \rho(\mathbf{\Phi}^\top\mathbf{y}_b)$$

ただし, $\{\mathbf{y}_n\}_n \subset \mathbb{R}^M$, $\mathbf{\Phi} = (\boldsymbol{\phi}_1, \boldsymbol{\phi}_2, \cdots, \boldsymbol{\phi}_P) \in \mathbb{R}^{M\times P}$, $M \geq P$ である.

参考文献:

Le, Quoc V., Alexandre Karpenko, Jiquan Ngiam, and Andrew Y. Ng. “ICA with Reconstruction Cost for Efficient Overcomplete Feature Learning.” Advances in Neural Information Processing Systems. Vol. 24, 2011, pp. 1017–1025. https://papers.nips.cc/paper/4467-ica-with-reconstruction-cost-for-efficient-overcomplete-feature-learning.pdf.

パラメータ設定

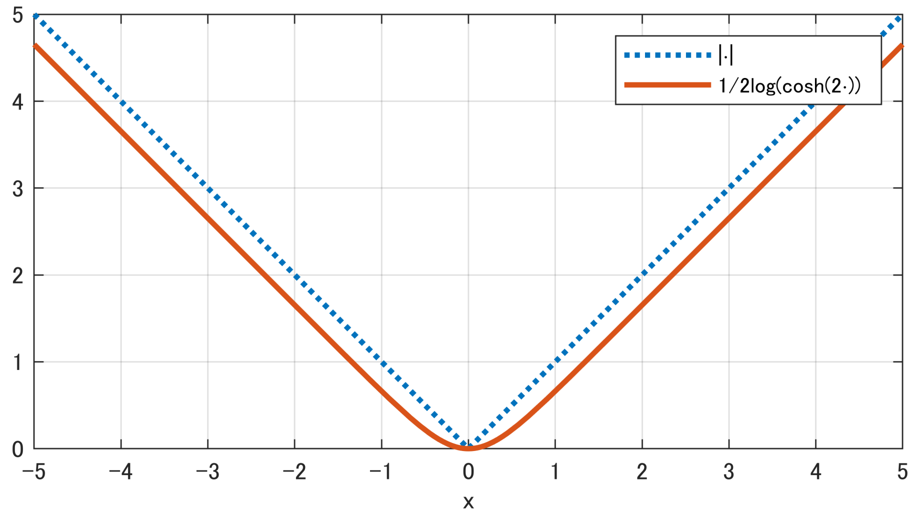
- 繰返し回数 (Number of iterations)
- 正則化パラメータ (Regularization parameter)

```
% Number of iterations
nItersRica = 1e5;
% Regularization parameter
alpha = 2e-2;
```

コントラスト関数の例

$$\rho(\mathbf{\Phi}^\top\mathbf{y}) := \frac{1}{2}\sum_{p=1}^{P} \log\circ\cosh(2\boldsymbol{\phi}_p^\top\mathbf{y})$$

```
figure
fplot(@(x) abs(x),[-5 5],':','LineWidth',2,'DisplayName','|\cdot|')
hold on
fplot(@(x) log(cosh(2*x))/2,[-5
5],'-','LineWidth',2,'DisplayName','1/2log(cosh(2\cdot))')
xlabel('x')
legend
grid on
axis equal
hold off
```



要素画像の数

```
nDims = prod(szBlk);
nAtoms = ceil(redundancyRatio*nDims);
```

辞書 $\mathbf{\Phi}$の初期化

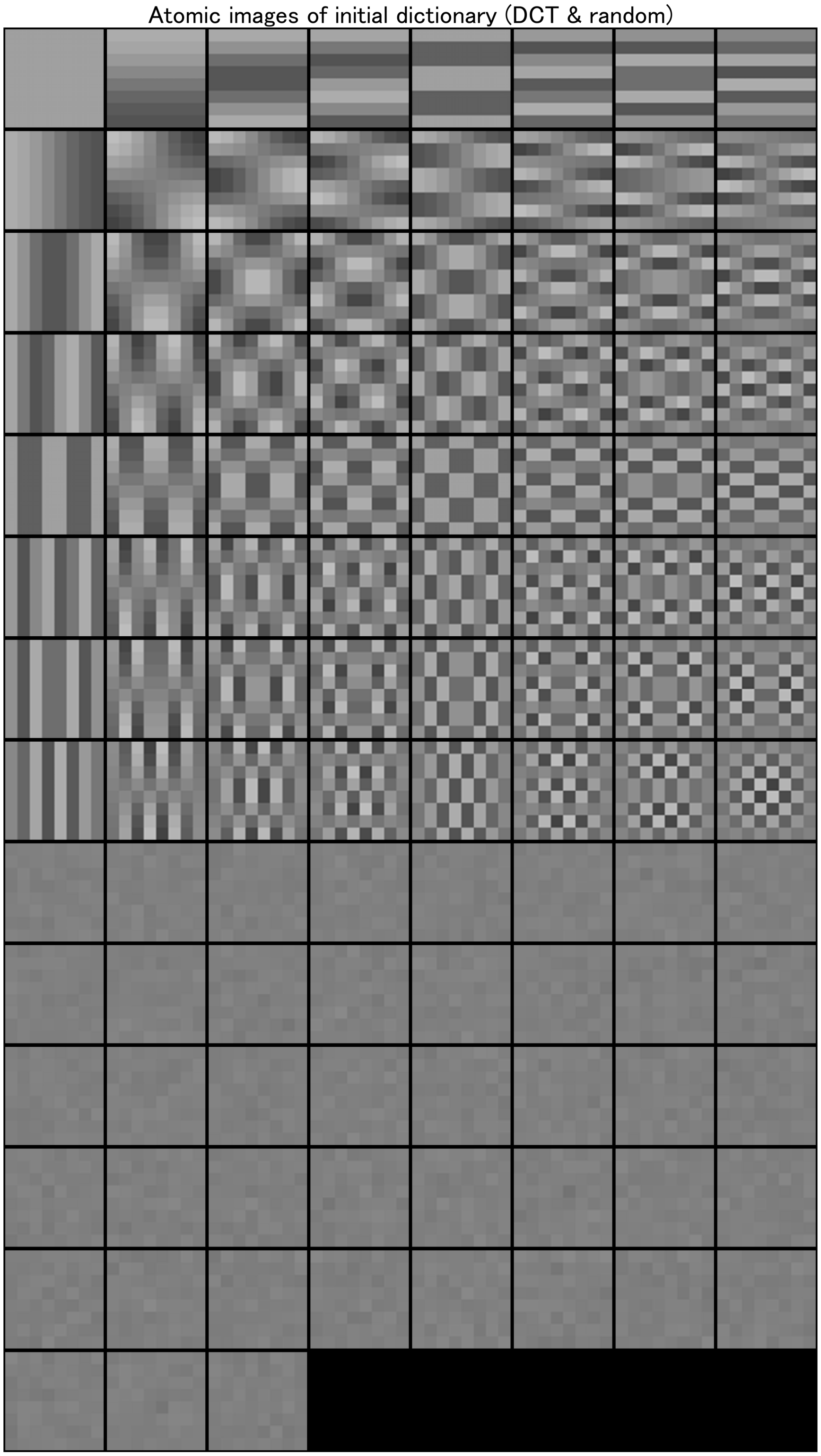
- 二次元離散コサイン変換
- ランダム

```
Phi_rica = randn(nDims,nAtoms);
Phi_rica = Phi_rica/norm(Phi_rica,'fro');
for iAtom = 1:nDims
    delta = zeros(szBlk);
    delta(iAtom) = 1;
    Phi_rica(:,iAtom) = reshape(idct2(delta),nDims,1);
end
```

要素ベクトルを要素画像に変換

```
atomicImagesRica = zeros(szBlk(1),szBlk(2),nAtoms);
for iAtom = 1:nAtoms
    atomicImagesRica(:,:,iAtom) = reshape(Phi_rica(:,iAtom),szBlk(1),szBlk(2));
end
figure
```

```
montage(imresize(atomicImagesRica,8,'nearest')+.5,'BorderSize',[2 2],'Size',
[ceil(nAtoms/8) 8])
title('Atomic images of initial dictionary (DCT & random)')
```



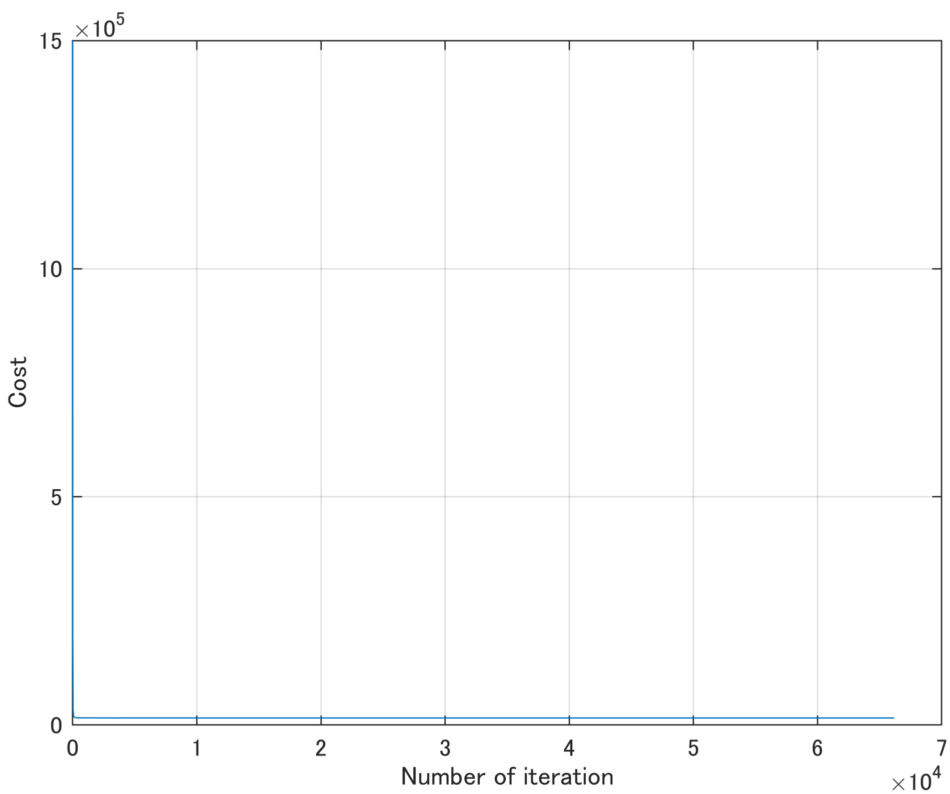
再構成 ICA オブジェクトの作成

PCA に合わせて予め正規化したデータで学習

```
model = rica(Y.',nAtoms,...
    'IterationLimit',nItersRica,...
    'ContrastFcn','logcosh',...
    'InitialTransformWeight',Phi_rica,...
    'Lambda',1/(2*alpha));
```

コスト評価のグラフ

```
info = model.FitInfo;
figure
plot(info.Iteration,info.Objective)
xlabel('Number of iteration')
ylabel('Cost')
grid on
```

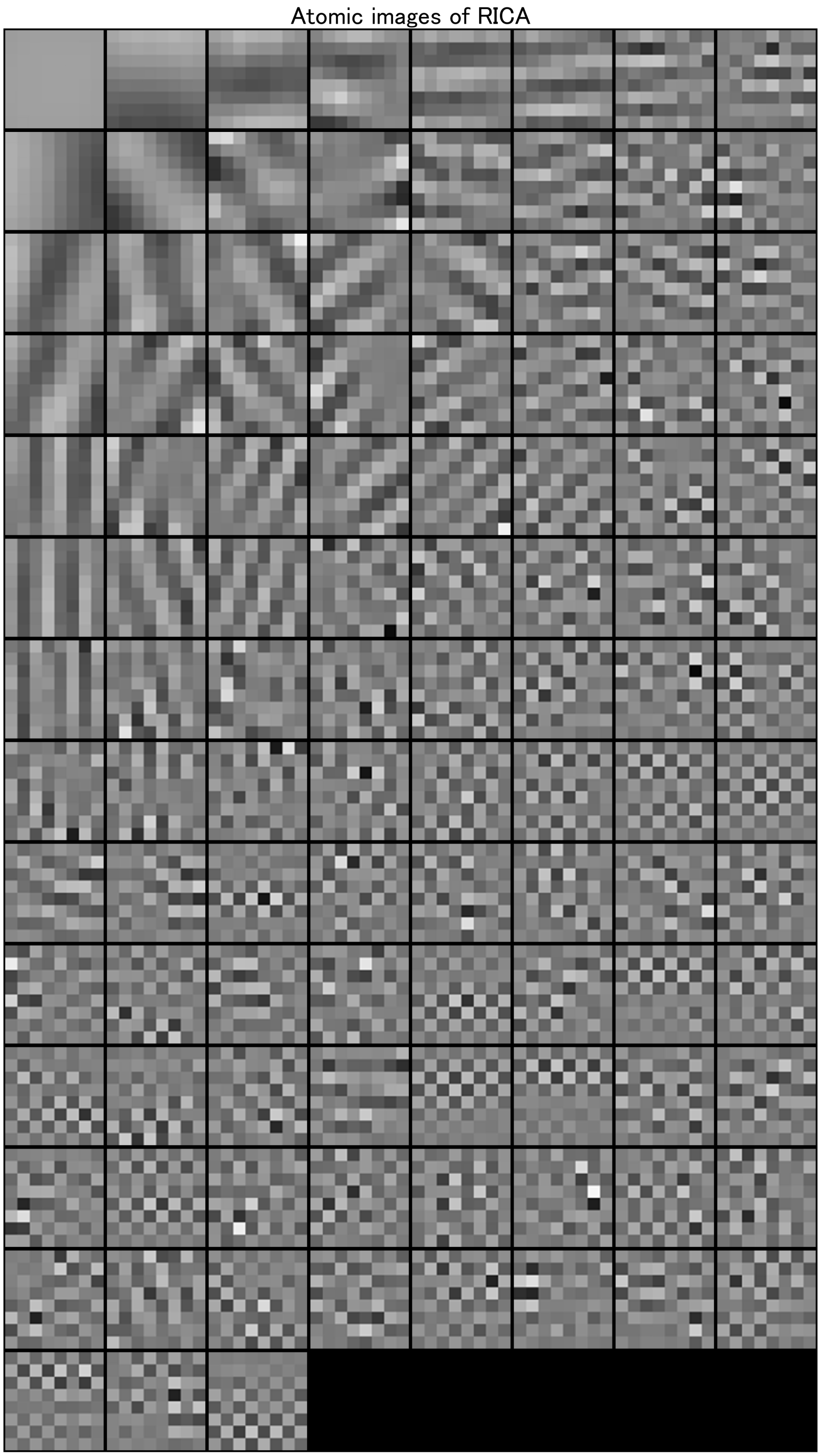


要素ベクトルを要素画像に変換

```
Phi_rica = model.TransformWeights;
atomicImagesRica = zeros(szBlk(1),szBlk(2),nAtoms);
for iAtom = 1:nAtoms
    atomicImagesRica(:,:,iAtom) = reshape(Phi_rica(:,iAtom),szBlk(1),szBlk(2));
end
```

要素画像の表示（辞書）

```
figure
montage(imresize(atomicImagesRica,8,'nearest')+.5,'BorderSize',[2 2],'Size',
[ceil(nAtoms/8) 8])
title('Atomic images of RICA')
```



ブロック RICA による合成処理とその随伴処理の定義

```
syn_blkrica = @(x) col2im(Phi_rica*x,szBlk,szOrg,"distinct");
adj_blkrica = @(y) Phi_rica.'*im2col(y,szBlk,"distinct");
```

随伴関係の確認

```
x = adj_blkrica(y);
v = randn(size(x));
u = syn_blkrica(v);
assert(abs(dot(y(:),u(:))-dot(x(:),v(:)))<1e-9)
```

K-特異値分解

パラメータ設定

- 繰返し回数 (Number of iterations)

```
% Number of iterations
nItersKsvd = 2000;
```

問題設定 (Problem setting):

$$\{\hat{\mathbf{\Phi}}, \{\hat{\mathbf{x}}_n\}\} = \arg\min_{\{\mathbf{\Phi},\{\mathbf{x}_n\}\}} \frac{1}{2S}\sum_{n=1}^{S}\|\mathbf{y}_n - \mathbf{\Phi}\hat{\mathbf{x}}_n\|_2^2, \quad \text{s.t. } \forall n, \|\mathbf{x}_n\|_0 \leq K$$

アルゴリズム (Algorithm):

スパース近似ステップと辞書更新ステップを繰返す.

- スパース近似ステップ (Sparse approximation step)

$$\hat{\mathbf{x}}_n = \arg\min_{\mathbf{x}_n} \frac{1}{2}\|\mathbf{y}_n - \hat{\mathbf{\Phi}}\mathbf{x}_n\|_2^2 \quad \text{s.t. } \|\mathbf{x}_n\|_0 \leq K$$

- 辞書更新ステップ (Dictionary update step)

$$\hat{\mathbf{\Phi}} = \arg\min_{\mathbf{\Phi}} \frac{1}{2S}\sum_{n=1}^{S}\|\mathbf{y}_n - \mathbf{\Phi}\hat{\mathbf{x}}_n\|_2^2 = \arg\min_{\mathbf{\Phi}} \frac{1}{2S}\left\|\left(\mathbf{Y} - \sum_{p\neq k}\boldsymbol{\phi}_p\hat{\mathbf{X}}_{p,:}\right) - \boldsymbol{\phi}_k\hat{\mathbf{X}}_{k,:}\right\|_F^2$$

係数の数 (Number of coefficients)

```
nCoefsKsvd = max(floor(sparsityRatio*nDims),1);
```

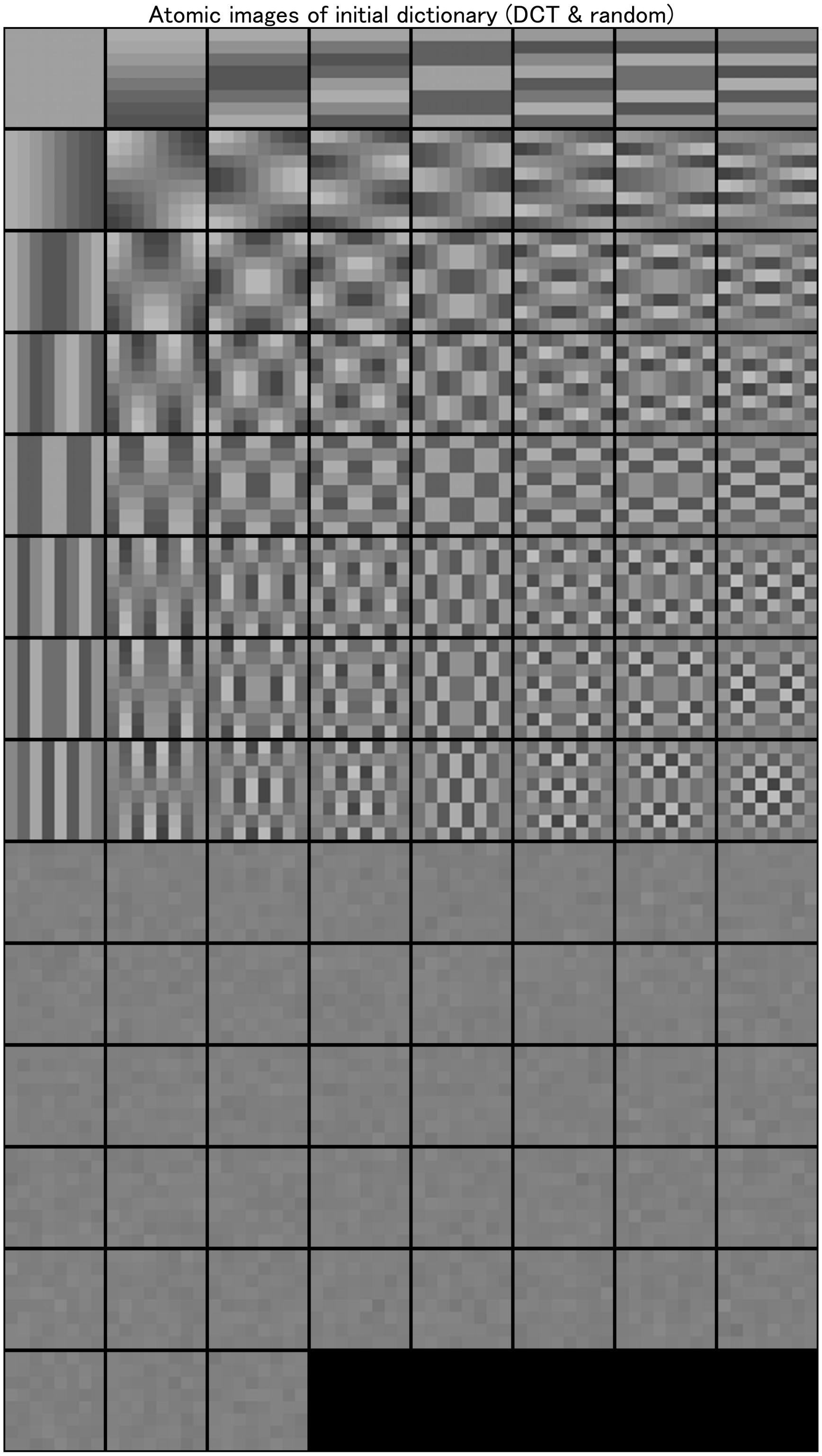
辞書 $\mathbf{\Phi}$の初期化 (Initializatio of dictionary $\mathbf{\Phi}$)

- 二変量離散コサイン変換(Bivariate DCT)
- ランダム (random)

```
Phi_ksvd = randn(nDims,nAtoms);
Phi_ksvd = Phi_ksvd/norm(Phi_ksvd,'fro');
for iAtom = 1:nDims
    delta = zeros(szBlk);
    delta(iAtom) = 1;
    Phi_ksvd(:,iAtom) = reshape(idct2(delta),nDims,1);
end
```

要素ベクトルを要素画像に変換 (Reshape the atoms into atomic images)

```
atomicImagesKsvd = zeros(szBlk(1),szBlk(2),nAtoms);
for iAtom = 1:nAtoms
    atomicImagesKsvd(:,:,iAtom) = reshape(Phi_ksvd(:,iAtom),szBlk(1),szBlk(2));
end
figure
montage(imresize(atomicImagesKsvd,8,'nearest')+.5,'BorderSize',[2 2],'Size',
[ceil(nAtoms/8) 8])
title('Atomic images of initial dictionary (DCT & random)')
```



スパース近似ステップと辞書更新ステップの繰り返し

- スパース近似： 直交マッチング追跡 (OMP)
- 辞書更新： 特異値分解と 1-ランク近似 (SVD and 1-rank approximation)

辞書更新の内容

1. $k \leftarrow 1$
2. 誤差行列 $\mathbf{E}_k$ を定義： $\mathbf{E}_k := \mathbf{Y} - \sum_{p \neq k} \boldsymbol{\phi}_p \hat{\mathbf{X}}_{p,:}$
3. データ行 $\hat{\mathbf{X}}_{k,:}$ の非零値を抽出する行列 $\boldsymbol{\Omega}_k$ を定義： $\hat{\mathbf{X}}_{k,:}^R = \hat{\mathbf{X}}_{k,:}\boldsymbol{\Omega}_k \Leftrightarrow \hat{\mathbf{X}}_{k,:}^R \boldsymbol{\Omega}_k^T = \hat{\mathbf{X}}_{k,:}$
4. 誤差行列 $\mathbf{E}_k$ を行列 $\boldsymbol{\Omega}_k$ で縮退： $\mathbf{E}_k^R := \mathbf{E}_k \boldsymbol{\Omega}_k$
5. 縮退した誤差行列 $\mathbf{E}_k^R$ を特異値分解： $\mathbf{E}_k^R = \mathbf{USV}^T = (\mathbf{u}_1, \mathbf{u}_2, \cdots, \mathbf{u}_r)\mathrm{diag}(\sigma_1, \sigma_2, \cdots, \sigma_r)(\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_r)^T$
6. 要素ベクトル $\boldsymbol{\phi}_k$ を更新： $\mathbf{k} \leftarrow \mathbf{u}_1$
7. データ行 $\hat{\mathbf{X}}_{k,:}$ を更新： $\hat{\mathbf{X}}_{k,:} \leftarrow \sigma_1 \mathbf{v}_1^T$
8. $k \leftarrow k+1$
9. $k \leq N$ ならば 2. へ $k > N$ ならば終了

ただし， σ_1 を最大特異値とする.

交互ステップの繰返し計算

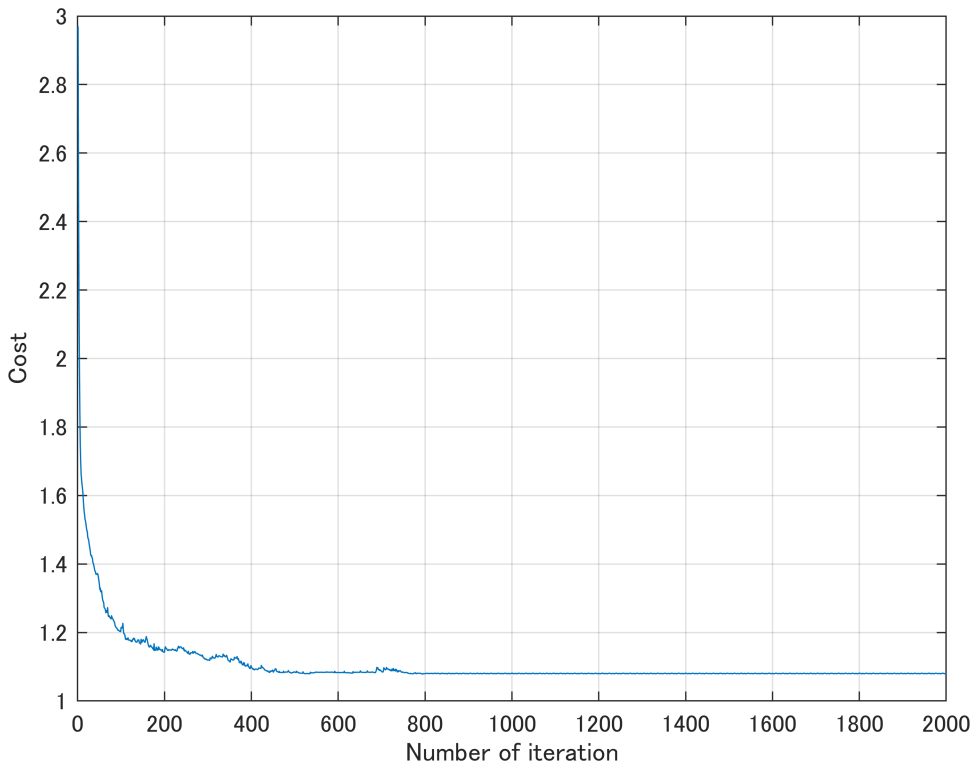
PCA に合わせて予め正規化したデータで学習

```
cost = zeros(1,nItersKsvd);
nSamples = size(Y,2);
for iIter = 1:nItersKsvd
    X = zeros(nAtoms,nSamples);
    % Sparse approximation
    for iSample = 1:nSamples
        y_ = Y(:,iSample);
        x = omp(y_,Phi_ksvd,nCoefsKsvd);
        X(:,iSample) = x;
    end
    % Dictionary update
    for iAtom = 1:nAtoms
        idxset = setdiff(1:nAtoms,iAtom);
        xk = X(iAtom,:);
        suppk = find(xk);
        %
        Ekred = Y(:,suppk)-Phi_ksvd(:,idxset)*X(idxset,suppk);
        %
        if ~isempty(suppk)
            [U,S,V] = svd(Ekred,'econ');
            ak = U(:,1);
            xkred = S(1,1)*V(:,1)';
            %
            Phi_ksvd(:,iAtom) = ak;
            X(iAtom,suppk) = xkred;
        end
    end
    cost(iIter) = (norm(Y-Phi_ksvd*X,'fro')^2)/(2*nSamples);
```

```
end
```

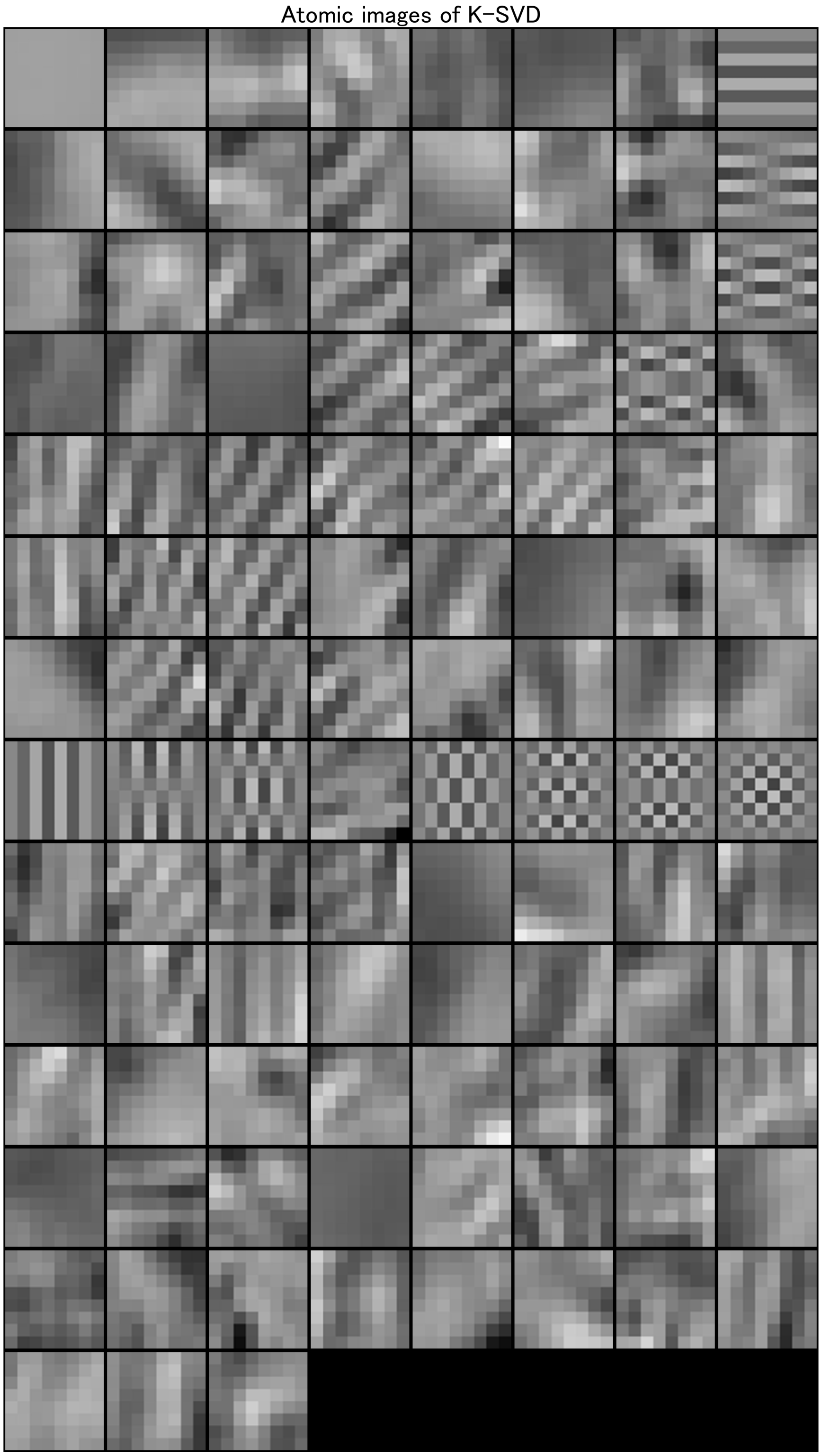
コスト評価のグラフ

```
figure
plot(cost)
xlabel('Number of iteration')
ylabel('Cost')
grid on
```



要素ベクトルを要素画像に変換

```
atomicImagesKsvd = zeros(szBlk(1),szBlk(2),nAtoms);
for iAtom = 1:nAtoms
    atomicImagesKsvd(:,:,iAtom) = reshape(Phi_ksvd(:,iAtom),szBlk(1),szBlk(2));
end
figure
montage(imresize(atomicImagesKsvd,8,'nearest')+.5,'BorderSize',[2 2],'Size',
[ceil(nAtoms/8) 8])
title('Atomic images of K-SVD')
```



ブロック K-特異値分解による合成処理とその随伴処理の定義

```
syn_blkksvd = @(x) col2im(Phi_ksvd*x,szBlk,szOrg,"distinct");
adj_blkksvd = @(y) Phi_ksvd.'*im2col(y,szBlk,"distinct");
```

随伴関係の確認

```
x = adj_blkksvd(y);
v = randn(size(x));
u = syn_blkksvd(v);
assert(abs(dot(y(:),u(:))-dot(x(:),v(:)))<1e-9)
```

2 変量ラティス構造冗長フィルタバンク

例として，（偶対称チャネルと奇対称チャネルが等しい）偶数チャネル、偶数のポリフェーズ次数をもつタイプ I 非分離冗長重複変換(NSOLT)

$$\mathbf{E}(z_\mathrm{v},z_\mathbf{h})=\left(\prod_{n_\mathrm{h}=1}^{\nu_\mathrm{h}/2}\mathbf{V}_{2n_\mathrm{h}}^{\{\mathrm{h}\}}\bar{\mathbf{Q}}(z_\mathrm{h})\mathbf{V}_{2k_\mathrm{h}-1}^{\{\mathrm{h}\}}\mathbf{Q}(z_\mathrm{h})\right)\left(\prod_{n_\mathrm{v}=1}^{\nu_\mathrm{v}/2}\mathbf{V}_{2n_\mathrm{v}}^{\{\mathrm{v}\}}\bar{\mathbf{Q}}(z_\mathrm{v})\mathbf{V}_{2n_\mathrm{v}-1}^{\{\mathrm{v}\}}\mathbf{Q}(z_\mathrm{v})\right)\mathbf{V}_0\mathbf{E}_0,$$

$$\mathbf{R}(z_\mathrm{v},z_\mathbf{h})=\mathbf{E}^\mathsf{T}(z_\mathrm{v}^{-1},z_\mathrm{h}^{-1}),$$

を採用する．ただし，

- $\mathbf{E}(z_\mathrm{v},z_\mathrm{h})$: 分析フィルタバンクの Type-I ポリフェーズ行列
- $\mathbf{R}(z_\mathrm{v},z_\mathrm{h})$: 合成フィルタバンクの Type-II ポリフェーズ行列
- $z_d\in\mathbb{C},d\in\{\mathrm{v},\mathrm{h}\}$: Z-変換の変数
- $\nu_d\in\mathbb{N},d\in\{\mathrm{v},\mathrm{h}\}$:方向 d のポリフェーズ次数(重複ブロック数)
- $\mathbf{V}_0=\begin{pmatrix}\mathbf{W}_0 & \mathbf{O}\\ \mathbf{O} & \mathbf{U}_0\end{pmatrix}\begin{pmatrix}\mathbf{I}_{M/2}\\ \mathbf{O}\\ \mathbf{I}_{M/2}\\ \mathbf{O}\end{pmatrix}\in\mathbb{R}^{P\times M}$, $\mathbf{V}_n^{\{d\}}=\begin{pmatrix}\mathbf{I}_{P/2} & \mathbf{O}\\ \mathbf{O} & \mathbf{U}_n^{\{d\}}\end{pmatrix}\in\mathbb{R}^{P\times P},d\in\{\mathrm{v},\mathrm{h}\}$, $\mathbf{W}_0,\mathbf{U}_0,\mathbf{U}_n^{\{d\}}\in\mathbb{R}^{P/2\times P/2}$は直交行列
- $\mathbf{Q}(z)=\mathbf{B}_P\begin{pmatrix}\mathbf{I}_{P/2} & \mathbf{O}\\ \mathbf{O} & z^{-1}\mathbf{I}_{P/2}\end{pmatrix}\mathbf{B}_P$, $\bar{\mathbf{Q}}(z)=\mathbf{B}_P\begin{pmatrix}z\mathbf{I}_{P/2} & \mathbf{O}\\ \mathbf{O} & \mathbf{I}_{P/2}\end{pmatrix}\mathbf{B}_P$, $\mathbf{B}_P=\frac{1}{\sqrt{2}}\begin{pmatrix}\mathbf{I}_{P/2} & \mathbf{I}_{P/2}\\ \mathbf{I}_{P/2} & -\mathbf{I}_{P/2}\end{pmatrix}$

【References】

- MATLAB SaivDr Package: https://github.com/msiplab/SaivDr
- S. Muramatsu, K. Furuya and N. Yuki, "Multidimensional Nonseparable Oversampled Lapped Transforms: Theory and Design," in IEEE Transactions on Signal Processing, vol. 65, no. 5, pp. 1251-1264, 1 March1, 2017, doi: 10.1109/TSP.2016.2633240.
- S. Muramatsu, T. Kobayashi, M. Hiki and H. Kikuchi, "Boundary Operation of 2-D Nonseparable Linear-Phase Paraunitary Filter Banks," in IEEE Transactions on Image Processing, vol. 21, no. 4, pp. 2314-2318, April 2012, doi: 10.1109/TIP.2011.2181527.
- S. Muramatsu, M. Ishii and Z. Chen, "Efficient parameter optimization for example-based design of nonseparable oversampled lapped transform," 2016 IEEE International Conference on Image Processing (ICIP), Phoenix, AZ, 2016, pp. 3618-3622, doi: 10.1109/ICIP.2016.7533034.
- Furuya, K., Hara, S., Seino, K., & Muramatsu, S. (2016). Boundary operation of 2D non-separable oversampled lapped transforms. *APSIPA Transactions on Signal and Information Processing, 5*, E9. doi:10.1017/ATSIP.2016.3.

2 次元画像の階層的分析

$R_M^P(\tau)$ をツリーレベル τの階層構造フィルタバンクの冗長度とすると、

$$R_M^P(\tau)=\begin{cases}(P-1)\tau+1, & M=1,\\ \dfrac{P-1}{M-1}-\dfrac{P-M}{(M-1)M^{\tau}}, & M\geq 2.\end{cases}$$

となる.

パッケージダウンロード

```
% SaivDr パッケージバージョン
SAIVDR_VER = "4.2.2.2";
SAIVDR_DIR = "SaivDr-"+SAIVDR_VER;
if ~exist(SAIVDR_DIR,"dir")
    unzip("https://github.com/msiplab/SaivDr/archive/refs/tags/"+SAIVDR_VER+".zip")
else
    disp(SAIVDR_DIR+" exits.")
end
```

```
SaivDr-4.2.2.2 exits.
```

```
ccd = cd(SAIVDR_DIR);
setpath
if isempty(dir("./mexcodes/fcn_*"))
    mybuild
else
    disp("MEX files exist.")
end
```

```
MEX files exist.
```

```
cd(ccd)
```

構成パラメータ設定

```
%{
% Decimation factor (Strides)
decFactor = [2 2]; % [μv μh]

% Number of channels ( sum(nChannels) >= prod(decFactors) )
nChannels = [3 3]; % [Ps Pa] (Ps=Pa)

% Number of tree levels
nLevels = 4;

% Polyphase Order
ppOrder = [4 4];
%}

% Decimation factor (Strides)
```

```
decFactor =  [4 4]; % [μv μh]

% Number of channels ( sum(nChannels) >= prod(decFactors) )
nChannels = [13 13]; % [Ps Pa] (Ps=Pa)

% Number of tree levels
nLevels = 2;

% Polyphase Order
ppOrder = [2 2];

% Redundancy
P = sum(nChannels);
M = prod(decFactor);
redundancyNsolt = ...
    (prod(decFactor)==1)*((P-1)*nLevels+1) + ...
    (prod(decFactor)>1)*((P-1)/(M-1)-(P-M)/((M-1)*M^nLevels))
```

```
redundancyNsolt = 1.6641
```

```
assert(redundancyNsolt<redundancyRatio)

% Sparsity ratio
% Number of patchs per image
nSubImgs = 30;

% No DC-leakage
noDcLeakage = true
```

```
noDcLeakage = logical
   1
```

辞書の設定

```
if exist("../data/"+nsoltDic+".mat","file")
    S = load("../data/"+nsoltDic);
    analysisnet = S.analysisnet;
    synthesisnet = S.synthesisnet;
    nLevels_ = extractnumlevels(analysisnet);
    decFactor_ = extractdecfactor(analysisnet);
    nChannels_ = extractnumchannels(analysisnet);

    assert(nLevels==nLevels_)
    assert(all(decFactor==decFactor_))
    assert(all(nChannels==nChannels_))
else
    % Number of iterations
    nItersNsolt = 10;

    % Standard deviation of initial angles
    stdInitAng = 1e-1; %pi/6;
```

```
    % Patch size for training
    szPatchTrn = [128 128]; % > [ (Ny+1)My (Nx+1)Mx ]

    % Mini batch size
    miniBatchSize = 10;

    % Number of Epochs (1 Epoch = nSubImgs/miniBachSize iterlations)
    maxEpochs = 20;

    % Number of iterations
    maxIters = nSubImgs/miniBatchSize * maxEpochs

    % Training options
    opts = trainingOptions('sgdm', ... % Stochastic gradient descent w/ momentum
        ...'Momentum', 0.9000,...
        'InitialLearnRate',1.0e-04,...
        ...'LearnRateScheduleSettings','none',...
        ...'L2Regularization',1.0e-04,...
        ...'GradientThresholdMethod','l2norm',...
        ...'GradientThreshold',Inf,...
        'MaxEpochs',maxEpochs,...30,...
        'MiniBatchSize',miniBatchSize,...128,...
        'Verbose',1,...
        'VerboseFrequency',10,...50,...
        ...'ValidationData',[],...
        ...'ValidationFrequency',50,...
        ...'ValidationPatience',Inf,...
        ...'Shuffle','once',...
        ...'CheckpointPath','',...
        ...'ExecutionEnvironment','auto',...
        ...'WorkerLoad',[],...
        ...'OutputFcn',[],...
        'Plots','none',...'training-progress',...
        ...'SequenceLength','longest',...
        ...'SequencePaddingValue',0,...
        ...'SequencePaddingDirection','right',...
        ...'DispatchInBackground',0,...
        'ResetInputNormalization',0);...1
```

層構造の構築

```
    import saivdr.dcnn.*
    analysislgraph = fcn_creatensoltlgraph2d([],...
        'InputSize',szPatchTrn,...
        'NumberOfChannels',nChannels,...
        'DecimationFactor',decFactor,...
        'PolyPhaseOrder',ppOrder,...
        'NumberOfLevels',nLevels,...
        'NumberOfVanishingMoments',noDcLeakage,...
```

```
        'Mode','Analyzer');
    synthesislgraph = fcn_creatensoltlgraph2d([],...
        'InputSize',szPatchTrn,...
        'NumberOfChannels',nChannels,...
        'DecimationFactor',decFactor,...
        'PolyPhaseOrder',ppOrder,...
        'NumberOfLevels',nLevels,...
        'NumberOfVanishingMoments',noDcLeakage,...
        'Mode','Synthesizer');

    figure
    subplot(1,2,1)
    plot(analysislgraph)
    title('Analysis NSOLT')
    subplot(1,2,2)
    plot(synthesislgraph)
    title('Synthesis NSOLT')

    % Construction of deep learning network.
    synthesisnet = dlnetwork(synthesislgraph);

    % Initialize
    nLearnables = height(synthesisnet.Learnables);
    for iLearnable = 1:nLearnables
        if synthesisnet.Learnables.Parameter(iLearnable)=="Angles"
            layerName = synthesisnet.Learnables.Layer(iLearnable);
            synthesisnet.Learnables.Value(iLearnable) = ...
                cellfun(@(x) x+stdInitAng*randn(size(x)), ...
                synthesisnet.Learnables.Value(iLearnable),'UniformOutput',false);
        end
    end

    % Copy the synthesizer's parameters to the analyzer
    synthesislgraph = layerGraph(synthesisnet);
    analysislgraph = fcn_cpparamssyn2ana(analysislgraph,synthesislgraph);
    analysisnet = dlnetwork(analysislgraph);
```

随伴関係（完全再構成）の確認

NSOLT はパーセバルタイト性を満たす.

```
    nOutputs = nLevels+1;
    x = rand(szPatchTrn,'single');
    s = cell(1,nOutputs);
    dlx = dlarray(x,'SSCB'); % Deep learning array (SSCB:
 Spatial,Spatial,Channel,Batch)
    [s{1:nOutputs}] = analysisnet.predict(dlx);
    dly = synthesisnet.predict(s{:});
    display("MSE: " + num2str(mse(dlx,dly)))
```

要素画像の初期状態

```
    import saivdr.dcnn.*
    figure
    atomicimshow(synthesisnet,[],2^(nLevels-1))
    title('Atomic images of initial NSOLT')
```

訓練画像の準備

画像データストアからランダムにパッチを抽出

PCA に合わせて予め正規化したデータで学習

```
    imds = imageDatastore(file_yorg,"ReadFcn",@(x) normalize(im2single(imread(x))));
    patchds = 
randomPatchExtractionDatastore(imds,imds,szPatchTrn,'PatchesPerImage',nSubImgs);
    figure
    minibatch = preview(patchds);
    responses = minibatch.ResponseImage;
    responses = cellfun(@(x) x/3 + 0.5,responses,'UniformOutput',false);
    montage(responses,'Size',[2 4]);
    drawnow
```

畳み込み辞書学習

問題設定:

$$\{\hat{\boldsymbol{\theta}},\{\hat{\mathbf{x}}_n\}\}=\arg\min_{\{\boldsymbol{\theta},\{\mathbf{x}_n\}\}}\frac{1}{2S}\sum_{n=1}^{S}\|\mathbf{y}_n-\mathbf{D}_{\boldsymbol{\theta}}\hat{\mathbf{x}}_n\|_2^2,\quad \mathrm{s.t.}\ \forall n,\|\mathbf{x}_n\|_0\leq K,$$

ただし，$\mathbf{D}_{\boldsymbol{\theta}}$は設計パラメータベクトル $\boldsymbol{\theta}$ をもつ畳み込み辞書.

アルゴリズム:

スパース近似ステップと辞書更新ステップを繰返す.

- Sparse approximation step

$$\hat{\mathbf{x}}_n=\arg\min_{\mathbf{x}_n}\frac{1}{2}\|\mathbf{y}_n-\hat{\mathbf{D}}\mathbf{x}_n\|_2^2\quad \mathrm{s.t.}\ \|\mathbf{x}_n\|_0\leq K$$

- Dictionary update step

$$\hat{\boldsymbol{\theta}}=\arg\min_{\boldsymbol{\theta}}\frac{1}{2S}\sum_{n=1}^{S}\|\mathbf{y}_n-\mathbf{D}_{\boldsymbol{\theta}}\hat{\mathbf{x}}_n\|_2^2$$

$$\hat{\mathbf{D}}=\mathbf{D}_{\hat{\boldsymbol{\theta}}}$$

採用するスパース近似と辞書更新の手法:

- Sparse approximation：Iterative hard thresholding
- Dictionary update： Stochastic gradient descent w/ momentum

```
    % Check if IHT works for dlarray
    %x = dlarray(randn(szPatchTrn,'single'),'SSCB');
    %[y,coefs{1:nOutputs}] = iht(x,analysisnet,synthesisnet,sparsityRatio);
```

辞書学習の繰返し計算

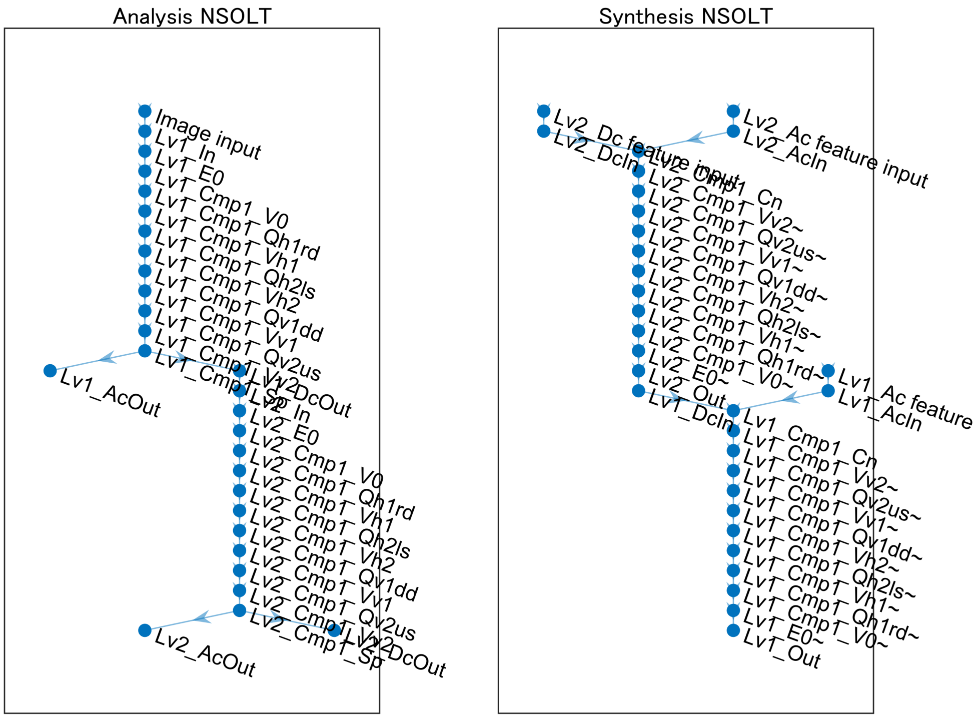
```
    import saivdr.dcnn.*
    %profile on
    for iIter = 1:nItersNsolt

        % Sparse approximation (Applied to produce an object of 
TransformedDatastore)
        coefimgds = transform(patchds, @(x) 
iht4patchds(x,analysisnet,synthesisnet,sparsityRatio));

        % Synthesis dictionary update
        trainlgraph = synthesislgraph.replaceLayer('Lv1_Out',...
            regressionLayer('Name','Lv1_Out'));
        trainednet = trainNetwork(coefimgds,trainlgraph,opts);

        % Analysis dictionary update (Copy parameters from synthesizer to analyzer)
        trainedlgraph = layerGraph(trainednet);
        analysislgraph = fcn_cpparamssyn2ana(analysislgraph,trainedlgraph);
        analysisnet = dlnetwork(analysislgraph);

        % Check the adjoint relation (perfect reconstruction)
        checkadjointrelation(analysislgraph,trainedlgraph,nLevels,szPatchTrn);

        % Replace layer
        synthesislgraph = trainedlgraph.replaceLayer('Lv1_Out',...
            nsoltIdentityLayer('Name','Lv1_Out'));
        synthesisnet = dlnetwork(synthesislgraph);

    end
    %profile off
    %profile viewer
```

訓練ネットワークの保存

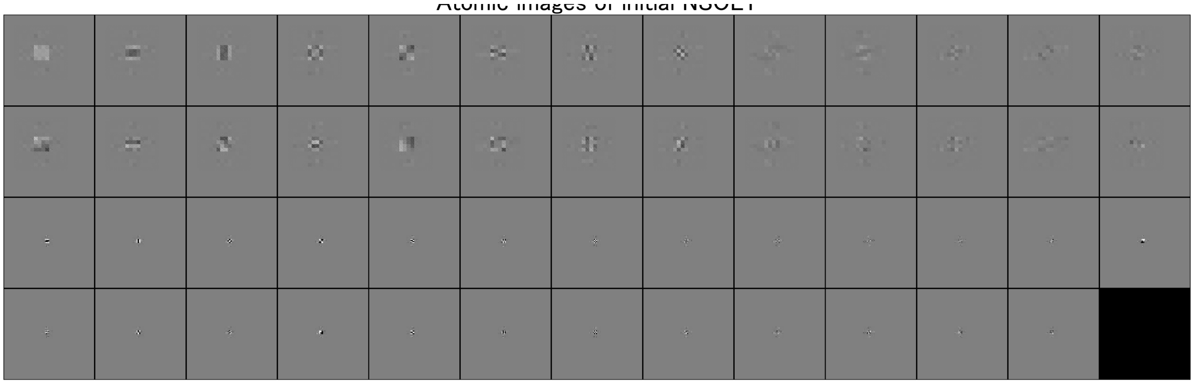
```
    import saivdr.dcnn.*
    synthesislgraph = layerGraph(synthesisnet);
    analysislgraph = fcn_cpparamssyn2ana(analysislgraph,synthesislgraph);
    analysisnet = dlnetwork(analysislgraph);
    save(sprintf('../results/
nsoltdictionary_%s',datetime('now','Format','yyyyMMddHHmmssSSS')),'analysisnet','syn
thesisnet','nLevels')
```

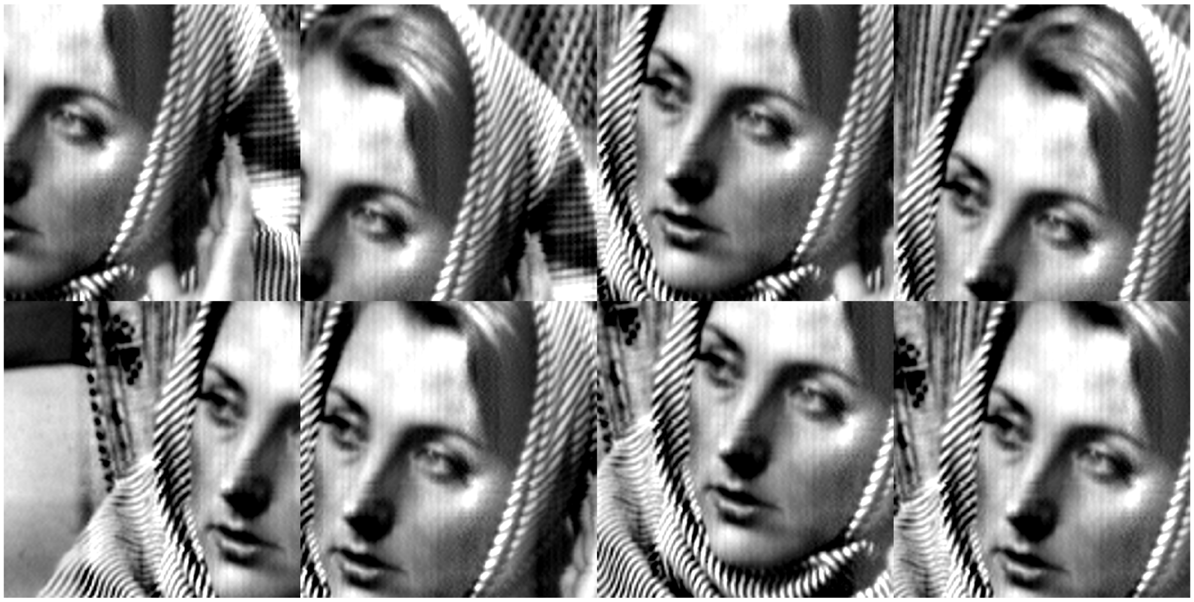
```
end
```

```
maxIters = 60
```




```
Copy angles from Lv1_Cmp1_V0~ to Lv1_Cmp1_V0
Copy angles from Lv1_Cmp1_Vh1~ to Lv1_Cmp1_Vh1
Copy angles from Lv1_Cmp1_Vh2~ to Lv1_Cmp1_Vh2
Copy angles from Lv1_Cmp1_Vv1~ to Lv1_Cmp1_Vv1
Copy angles from Lv1_Cmp1_Vv2~ to Lv1_Cmp1_Vv2
Copy angles from Lv2_Cmp1_V0~ to Lv2_Cmp1_V0
Copy angles from Lv2_Cmp1_Vh1~ to Lv2_Cmp1_Vh1
Copy angles from Lv2_Cmp1_Vh2~ to Lv2_Cmp1_Vh2
Copy angles from Lv2_Cmp1_Vv1~ to Lv2_Cmp1_Vv1
Copy angles from Lv2_Cmp1_Vv2~ to Lv2_Cmp1_Vv2
    "MSE: 2.1942e-10"
```





単一の GPU で学習中。

エポック	反復	経過時間 （ｈｈ：ｍｍ：ｓｓ）	ミニバッチ　ＲＭＳＥ	ミニバッチ損失	基本学
１	１	００：００：１３	４６．３８	１０７５．４	１．０
４	１０	００：０１：２３	４１．０２	８４１．２	１．０
７	２０	００：０２：４０	４０．７１	８２８．６	１．０
１０	３０	００：０３：５６	３８．６６	７４７．４	１．０
１４	４０	００：０５：１５	３９．０２	７６１．３	１．０
１７	５０	００：０６：３１	３９．８４	７９３．６	１．０
２０	６０	００：０７：４７	４０．５１	８２０．６	１．０

学習終了: 最大数のエポックが完了しました。

```
Copy angles from Lv1_Cmp1_V0~ to Lv1_Cmp1_V0
Copy angles from Lv1_Cmp1_Vh1~ to Lv1_Cmp1_Vh1
Copy angles from Lv1_Cmp1_Vh2~ to Lv1_Cmp1_Vh2
Copy angles from Lv1_Cmp1_Vv1~ to Lv1_Cmp1_Vv1
Copy angles from Lv1_Cmp1_Vv2~ to Lv1_Cmp1_Vv2
Copy angles from Lv2_Cmp1_V0~ to Lv2_Cmp1_V0
Copy angles from Lv2_Cmp1_Vh1~ to Lv2_Cmp1_Vh1
Copy angles from Lv2_Cmp1_Vh2~ to Lv2_Cmp1_Vh2
Copy angles from Lv2_Cmp1_Vv1~ to Lv2_Cmp1_Vv1
Copy angles from Lv2_Cmp1_Vv2~ to Lv2_Cmp1_Vv2
    "MSE: 2.9328e-14"
```

単一の GPU で学習中。

エポック	反復	経過時間 （ｈｈ：ｍｍ：ｓｓ）	ミニバッチ　ＲＭＳＥ	ミニバッチ損失	基本学
１	１	００：００：１２	３６．１１	６５１．９	１．０
４	１０	００：０１：２２	３５．０１	６１２．９	１．０
７	２０	００：０２：３９	３５．７８	６４０．０	１．０
１０	３０	００：０３：５６	３５．６７	６３６．１	１．０
１４	４０	００：０５：１６	３６．６３	６７０．８	１．０
１７	５０	００：０６：３３	３３．７６	５６９．９	１．０
２０	６０	００：０７：４９	３４．５３	５９６．０	１．０

学習終了: 最大数のエポックが完了しました。

```
Copy angles from Lv1_Cmp1_V0~ to Lv1_Cmp1_V0
Copy angles from Lv1_Cmp1_Vh1~ to Lv1_Cmp1_Vh1
Copy angles from Lv1_Cmp1_Vh2~ to Lv1_Cmp1_Vh2
```

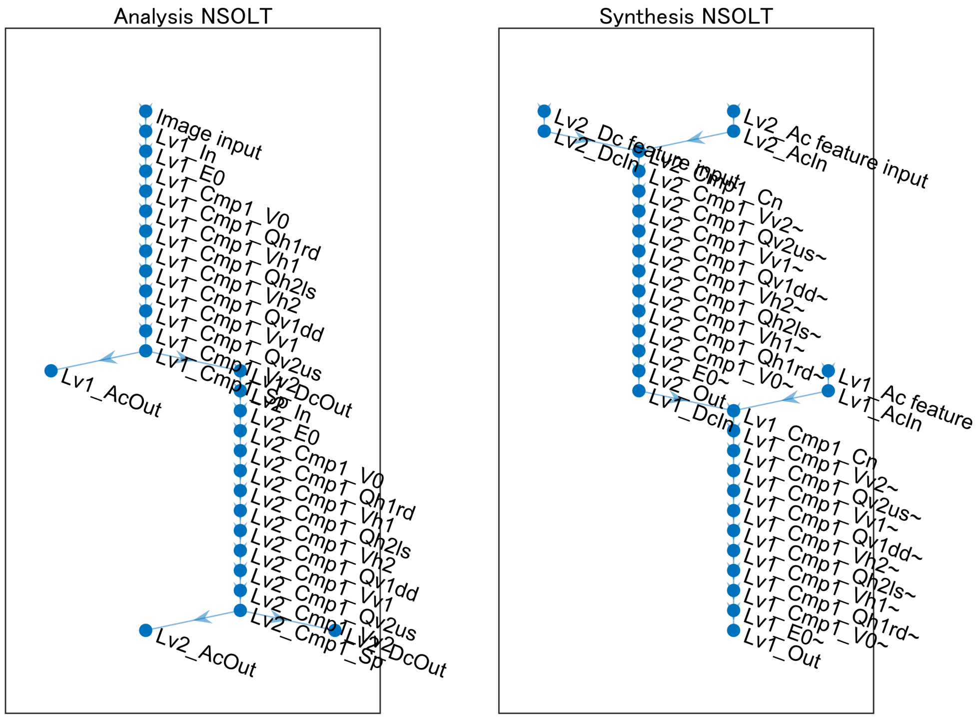
```
Copy angles from Lv1_Cmp1_Vv1~ to Lv1_Cmp1_Vv1
Copy angles from Lv1_Cmp1_Vv2~ to Lv1_Cmp1_Vv2
Copy angles from Lv2_Cmp1_V0~ to Lv2_Cmp1_V0
Copy angles from Lv2_Cmp1_Vh1~ to Lv2_Cmp1_Vh1
Copy angles from Lv2_Cmp1_Vh2~ to Lv2_Cmp1_Vh2
Copy angles from Lv2_Cmp1_Vv1~ to Lv2_Cmp1_Vv1
Copy angles from Lv2_Cmp1_Vv2~ to Lv2_Cmp1_Vv2
    "MSE: 2.8555e-14"
単一の GPU で学習中。
|==============================================================================
|  エポック  |  反復  |     経過時間     |  ミニバッチ　ＲＭＳＥ  |  ミニバッチ損失  |  基本
|            |        |  （ｈｈ：ｍｍ：ｓｓ）  |                        |                  |
|==============================================================================
|          １ |      １ |    ００：００：１０ |              ３３．３０ |        ５５４．５ | １．０
|          ４ |    １０ |    ００：０１：１３ |              ３１．５４ |        ４９７．４ | １．０
|          ７ |    ２０ |    ００：０２：２９ |              ３１．９８ |        ５１１．４ | １．０
|        １０ |    ３０ |    ００：０３：４７ |              ３３．６９ |        ５６７．５ | １．０
|        １４ |    ４０ |    ００：０５：０９ |              ３３．１５ |        ５４９．５ | １．０
|        １７ |    ５０ |    ００：０６：２６ |              ３１．９９ |        ５１１．７ | １．０
|        ２０ |    ６０ |    ００：０７：４３ |              ３３．２２ |        ５５１．７ | １．０
|==============================================================================
学習終了: 最大数のエポックが完了しました。
Copy angles from Lv1_Cmp1_V0~ to Lv1_Cmp1_V0
Copy angles from Lv1_Cmp1_Vh1~ to Lv1_Cmp1_Vh1
Copy angles from Lv1_Cmp1_Vh2~ to Lv1_Cmp1_Vh2
Copy angles from Lv1_Cmp1_Vv1~ to Lv1_Cmp1_Vv1
Copy angles from Lv1_Cmp1_Vv2~ to Lv1_Cmp1_Vv2
Copy angles from Lv2_Cmp1_V0~ to Lv2_Cmp1_V0
Copy angles from Lv2_Cmp1_Vh1~ to Lv2_Cmp1_Vh1
Copy angles from Lv2_Cmp1_Vh2~ to Lv2_Cmp1_Vh2
Copy angles from Lv2_Cmp1_Vv1~ to Lv2_Cmp1_Vv1
Copy angles from Lv2_Cmp1_Vv2~ to Lv2_Cmp1_Vv2
    "MSE: 3.3966e-14"
単一の GPU で学習中。
|==============================================================================
|  エポック  |  反復  |     経過時間     |  ミニバッチ　ＲＭＳＥ  |  ミニバッチ損失  |  基本
|            |        |  （ｈｈ：ｍｍ：ｓｓ）  |                        |                  |
|==============================================================================
|          １ |      １ |    ００：００：１３ |              ３２．６１ |        ５３１．６ | １．０
|          ４ |    １０ |    ００：０１：２２ |              ３２．５６ |        ５３０．０ | １．０
|          ７ |    ２０ |    ００：０２：３８ |              ３１．２３ |        ４８７．６ | １．０
|        １０ |    ３０ |    ００：０３：５３ |              ３３．０３ |        ５４５．３ | １．０
|        １４ |    ４０ |    ００：０５：１２ |              ３２．７０ |        ５３４．６ | １．０
|        １７ |    ５０ |    ００：０６：２７ |              ３１．３８ |        ４９２．４ | １．０
|        ２０ |    ６０ |    ００：０７：４３ |              ３１．１２ |        ４８４．４ | １．０
|==============================================================================
学習終了: 最大数のエポックが完了しました。
Copy angles from Lv1_Cmp1_V0~ to Lv1_Cmp1_V0
Copy angles from Lv1_Cmp1_Vh1~ to Lv1_Cmp1_Vh1
Copy angles from Lv1_Cmp1_Vh2~ to Lv1_Cmp1_Vh2
Copy angles from Lv1_Cmp1_Vv1~ to Lv1_Cmp1_Vv1
Copy angles from Lv1_Cmp1_Vv2~ to Lv1_Cmp1_Vv2
Copy angles from Lv2_Cmp1_V0~ to Lv2_Cmp1_V0
Copy angles from Lv2_Cmp1_Vh1~ to Lv2_Cmp1_Vh1
Copy angles from Lv2_Cmp1_Vh2~ to Lv2_Cmp1_Vh2
Copy angles from Lv2_Cmp1_Vv1~ to Lv2_Cmp1_Vv1
Copy angles from Lv2_Cmp1_Vv2~ to Lv2_Cmp1_Vv2
    "MSE: 3.1136e-14"
単一の GPU で学習中。
|==============================================================================
|  エポック  |  反復  |     経過時間     |  ミニバッチ　ＲＭＳＥ  |  ミニバッチ損失  |  基本
|            |        |  （ｈｈ：ｍｍ：ｓｓ）  |                        |                  |
|==============================================================================
|          １ |      １ |    ００：００：１３ |              ３２．９７ |        ５４３．６ | １．０
```

```
|         ４ |      １０ |        ００：０１：２３ |              ３０．３４ |              ４６０．４ | １．０
|         ７ |      ２０ |        ００：０２：３９ |              ３１．４８ |              ４９５．６ | １．０
|       １０ |      ３０ |        ００：０３：５５ |              ３１．３６ |              ４９１．８ | １．０
|       １４ |      ４０ |        ００：０５：１４ |              ３１．９５ |              ５１０．３ | １．０
|       １７ |      ５０ |        ００：０６：３０ |              ３１．９９ |              ５１１．７ | １．０
|       ２０ |      ６０ |        ００：０７：４６ |              ３０．８６ |              ４７６．１ | １．０
|=========================================================================================================
学習終了: 最大数のエポックが完了しました。
Copy angles from Lv1_Cmp1_V0~ to Lv1_Cmp1_V0
Copy angles from Lv1_Cmp1_Vh1~ to Lv1_Cmp1_Vh1
Copy angles from Lv1_Cmp1_Vh2~ to Lv1_Cmp1_Vh2
Copy angles from Lv1_Cmp1_Vv1~ to Lv1_Cmp1_Vv1
Copy angles from Lv1_Cmp1_Vv2~ to Lv1_Cmp1_Vv2
Copy angles from Lv2_Cmp1_V0~ to Lv2_Cmp1_V0
Copy angles from Lv2_Cmp1_Vh1~ to Lv2_Cmp1_Vh1
Copy angles from Lv2_Cmp1_Vh2~ to Lv2_Cmp1_Vh2
Copy angles from Lv2_Cmp1_Vv1~ to Lv2_Cmp1_Vv1
Copy angles from Lv2_Cmp1_Vv2~ to Lv2_Cmp1_Vv2
    "MSE: 2.949e-14"
単一の GPU で学習中。
|=========================================================================================================
|   エポック   |    反復    |         経過時間          |     ミニバッチ　ＲＭＳＥ     |      ミニバッチ損失      |    基本
|              |            |     （ｈｈ：ｍｍ：ｓｓ）    |                          |                          |
|=========================================================================================================
|         １ |        １ |        ００：００：１４ |              ３２．７５ |              ５３６．３ | １．０
|         ４ |      １０ |        ００：０１：２４ |              ３０．８７ |              ４７６．６ | １．０
|         ７ |      ２０ |        ００：０２：４１ |              ３１．８２ |              ５０６．２ | １．０
|       １０ |      ３０ |        ００：０３：５７ |              ３１．６８ |              ５０１．８ | １．０
|       １４ |      ４０ |        ００：０５：１６ |              ３２．０８ |              ５１４．５ | １．０
|       １７ |      ５０ |        ００：０６：３２ |              ３０．６２ |              ４６８．９ | １．０
|       ２０ |      ６０ |        ００：０７：４７ |              ２９．７２ |              ４４１．５ | １．０
|=========================================================================================================
学習終了: 最大数のエポックが完了しました。
Copy angles from Lv1_Cmp1_V0~ to Lv1_Cmp1_V0
Copy angles from Lv1_Cmp1_Vh1~ to Lv1_Cmp1_Vh1
Copy angles from Lv1_Cmp1_Vh2~ to Lv1_Cmp1_Vh2
Copy angles from Lv1_Cmp1_Vv1~ to Lv1_Cmp1_Vv1
Copy angles from Lv1_Cmp1_Vv2~ to Lv1_Cmp1_Vv2
Copy angles from Lv2_Cmp1_V0~ to Lv2_Cmp1_V0
Copy angles from Lv2_Cmp1_Vh1~ to Lv2_Cmp1_Vh1
Copy angles from Lv2_Cmp1_Vh2~ to Lv2_Cmp1_Vh2
Copy angles from Lv2_Cmp1_Vv1~ to Lv2_Cmp1_Vv1
Copy angles from Lv2_Cmp1_Vv2~ to Lv2_Cmp1_Vv2
    "MSE: 2.7673e-14"
単一の GPU で学習中。
|=========================================================================================================
|   エポック   |    反復    |         経過時間          |     ミニバッチ　ＲＭＳＥ     |      ミニバッチ損失      |    基本
|              |            |     （ｈｈ：ｍｍ：ｓｓ）    |                          |                          |
|=========================================================================================================
|         １ |        １ |        ００：００：１４ |              ３３．２４ |              ５５２．３ | １．０
|         ４ |      １０ |        ００：０１：２４ |              ３０．８７ |              ４７６．４ | １．０
|         ７ |      ２０ |        ００：０２：４０ |              ３０．７８ |              ４７３．８ | １．０
|       １０ |      ３０ |        ００：０３：５６ |              ３１．４６ |              ４９４．９ | １．０
|       １４ |      ４０ |        ００：０５：１６ |              ３１．４８ |              ４９５．６ | １．０
|       １７ |      ５０ |        ００：０６：３３ |              ３２．７０ |              ５３４．６ | １．０
|       ２０ |      ６０ |        ００：０７：５０ |              ３２．５３ |              ５２９．１ | １．０
|=========================================================================================================
学習終了: 最大数のエポックが完了しました。
Copy angles from Lv1_Cmp1_V0~ to Lv1_Cmp1_V0
Copy angles from Lv1_Cmp1_Vh1~ to Lv1_Cmp1_Vh1
Copy angles from Lv1_Cmp1_Vh2~ to Lv1_Cmp1_Vh2
Copy angles from Lv1_Cmp1_Vv1~ to Lv1_Cmp1_Vv1
Copy angles from Lv1_Cmp1_Vv2~ to Lv1_Cmp1_Vv2
Copy angles from Lv2_Cmp1_V0~ to Lv2_Cmp1_V0
```

```
Copy angles from Lv2_Cmp1_Vh1~ to Lv2_Cmp1_Vh1
Copy angles from Lv2_Cmp1_Vh2~ to Lv2_Cmp1_Vh2
Copy angles from Lv2_Cmp1_Vv1~ to Lv2_Cmp1_Vv1
Copy angles from Lv2_Cmp1_Vv2~ to Lv2_Cmp1_Vv2
    "MSE: 3.1389e-14"
単一の GPU で学習中。
|==================================================================================
|   エポック   |   反復   |      経過時間      |   ミニバッチ ＲＭＳＥ   |   ミニバッチ損失   |   基本学
|              |          |   （ｈｈ：ｍｍ：ｓｓ）   |                         |                    |
|==================================================================================
|          １ |       １ |     ００：００：１４ |            ３１．８１ |          ５０５．８ | １．０
|          ４ |     １０ |     ００：０１：２５ |            ３０．８６ |          ４７６．１ | １．０
|          ７ |     ２０ |     ００：０２：３８ |            ３０．６７ |          ４７０．３ | １．０
|        １０ |     ３０ |     ００：０３：５４ |            ３０．１７ |          ４５５．２ | １．０
|        １４ |     ４０ |     ００：０５：１３ |            ３１．７４ |          ５０３．８ | １．０
|        １７ |     ５０ |     ００：０６：２９ |            ３１．８２ |          ５０６．２ | １．０
|        ２０ |     ６０ |     ００：０７：４４ |            ２９．６７ |          ４４０．０ | １．０
|==================================================================================
学習終了: 最大数のエポックが完了しました。
Copy angles from Lv1_Cmp1_V0~ to Lv1_Cmp1_V0
Copy angles from Lv1_Cmp1_Vh1~ to Lv1_Cmp1_Vh1
Copy angles from Lv1_Cmp1_Vh2~ to Lv1_Cmp1_Vh2
Copy angles from Lv1_Cmp1_Vv1~ to Lv1_Cmp1_Vv1
Copy angles from Lv1_Cmp1_Vv2~ to Lv1_Cmp1_Vv2
Copy angles from Lv2_Cmp1_V0~ to Lv2_Cmp1_V0
Copy angles from Lv2_Cmp1_Vh1~ to Lv2_Cmp1_Vh1
Copy angles from Lv2_Cmp1_Vh2~ to Lv2_Cmp1_Vh2
Copy angles from Lv2_Cmp1_Vv1~ to Lv2_Cmp1_Vv1
Copy angles from Lv2_Cmp1_Vv2~ to Lv2_Cmp1_Vv2
    "MSE: 2.9605e-14"
単一の GPU で学習中。
|==================================================================================
|   エポック   |   反復   |      経過時間      |   ミニバッチ ＲＭＳＥ   |   ミニバッチ損失   |   基本学
|              |          |   （ｈｈ：ｍｍ：ｓｓ）   |                         |                    |
|==================================================================================
|          １ |       １ |     ００：００：１２ |            ３０．１７ |          ４５５．１ | １．０
|          ４ |     １０ |     ００：０１：２２ |            ３０．８０ |          ４７４．３ | １．０
|          ７ |     ２０ |     ００：０２：３８ |            ３１．６１ |          ４９９．６ | １．０
|        １０ |     ３０ |     ００：０３：５４ |            ３０．４３ |          ４６２．９ | １．０
|        １４ |     ４０ |     ００：０５：１４ |            ２９．８３ |          ４４５．０ | １．０
|        １７ |     ５０ |     ００：０６：３１ |            ３０．００ |          ４５０．０ | １．０
|        ２０ |     ６０ |     ００：０７：４７ |            ３０．４１ |          ４６２．４ | １．０
|==================================================================================
学習終了: 最大数のエポックが完了しました。
Copy angles from Lv1_Cmp1_V0~ to Lv1_Cmp1_V0
Copy angles from Lv1_Cmp1_Vh1~ to Lv1_Cmp1_Vh1
Copy angles from Lv1_Cmp1_Vh2~ to Lv1_Cmp1_Vh2
Copy angles from Lv1_Cmp1_Vv1~ to Lv1_Cmp1_Vv1
Copy angles from Lv1_Cmp1_Vv2~ to Lv1_Cmp1_Vv2
Copy angles from Lv2_Cmp1_V0~ to Lv2_Cmp1_V0
Copy angles from Lv2_Cmp1_Vh1~ to Lv2_Cmp1_Vh1
Copy angles from Lv2_Cmp1_Vh2~ to Lv2_Cmp1_Vh2
Copy angles from Lv2_Cmp1_Vv1~ to Lv2_Cmp1_Vv1
Copy angles from Lv2_Cmp1_Vv2~ to Lv2_Cmp1_Vv2
    "MSE: 2.7443e-14"
単一の GPU で学習中。
|==================================================================================
|   エポック   |   反復   |      経過時間      |   ミニバッチ ＲＭＳＥ   |   ミニバッチ損失   |   基本学
|              |          |   （ｈｈ：ｍｍ：ｓｓ）   |                         |                    |
|==================================================================================
|          １ |       １ |     ００：００：１３ |            ２９．５６ |          ４３７．０ | １．０
|          ４ |     １０ |     ００：０１：２３ |            ３０．９８ |          ４７９．８ | １．０
|          ７ |     ２０ |     ００：０２：４０ |            ３２．２０ |          ５１８．５ | １．０
|        １０ |     ３０ |     ００：０３：５７ |            ３１．５０ |          ４９６．２ | １．０
```

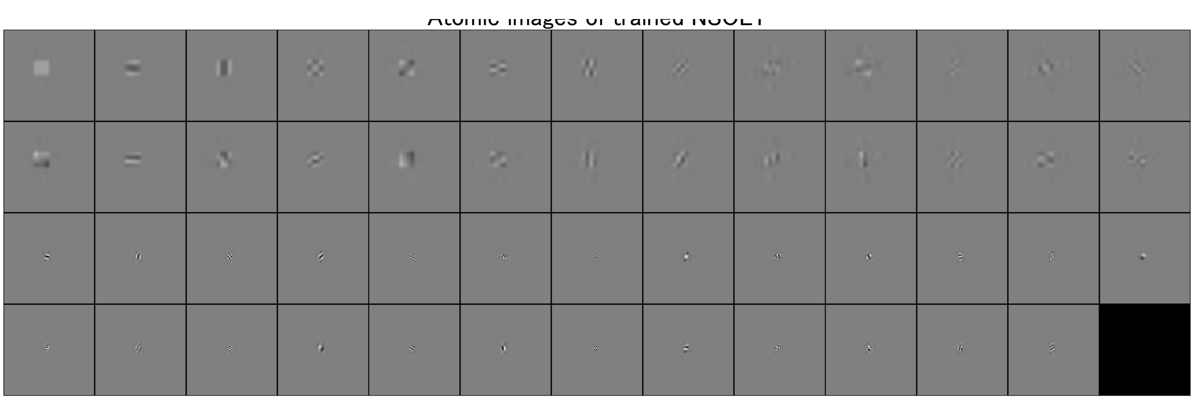
```
|        １４  |     ４０  |        ００：０５：１４  |            ２９．６７  |         ４４０．１  |  １．００
|        １７  |     ５０  |        ００：０６：２４  |            ３０．９７  |         ４７９．７  |  １．００
|        ２０  |     ６０  |        ００：０７：４０  |            ２９．９９  |         ４４９．７  |  １．００
|======================================================================================
学習終了: 最大数のエポックが完了しました。
Copy angles from Lv1_Cmp1_V0~ to Lv1_Cmp1_V0
Copy angles from Lv1_Cmp1_Vh1~ to Lv1_Cmp1_Vh1
Copy angles from Lv1_Cmp1_Vh2~ to Lv1_Cmp1_Vh2
Copy angles from Lv1_Cmp1_Vv1~ to Lv1_Cmp1_Vv1
Copy angles from Lv1_Cmp1_Vv2~ to Lv1_Cmp1_Vv2
Copy angles from Lv2_Cmp1_V0~ to Lv2_Cmp1_V0
Copy angles from Lv2_Cmp1_Vh1~ to Lv2_Cmp1_Vh1
Copy angles from Lv2_Cmp1_Vh2~ to Lv2_Cmp1_Vh2
Copy angles from Lv2_Cmp1_Vv1~ to Lv2_Cmp1_Vv1
Copy angles from Lv2_Cmp1_Vv2~ to Lv2_Cmp1_Vv2
    "MSE: 3.1444e-14"
Copy angles from Lv1_Cmp1_V0~ to Lv1_Cmp1_V0
Copy angles from Lv1_Cmp1_Vh1~ to Lv1_Cmp1_Vh1
Copy angles from Lv1_Cmp1_Vh2~ to Lv1_Cmp1_Vh2
Copy angles from Lv1_Cmp1_Vv1~ to Lv1_Cmp1_Vv1
Copy angles from Lv1_Cmp1_Vv2~ to Lv1_Cmp1_Vv2
Copy angles from Lv2_Cmp1_V0~ to Lv2_Cmp1_V0
Copy angles from Lv2_Cmp1_Vh1~ to Lv2_Cmp1_Vh1
Copy angles from Lv2_Cmp1_Vh2~ to Lv2_Cmp1_Vh2
Copy angles from Lv2_Cmp1_Vv1~ to Lv2_Cmp1_Vv1
Copy angles from Lv2_Cmp1_Vv2~ to Lv2_Cmp1_Vv2
```

```
analysislgraph = layerGraph(analysisnet);
synthesislgraph = layerGraph(synthesisnet);

figure
subplot(1,2,1)
plot(analysislgraph)
title('Analysis NSOLT')
subplot(1,2,2)
plot(synthesislgraph)
title('Synthesis NSOLT')
```



要素画像の表示

```
import saivdr.dcnn.*

figure
atomicimshow(synthesisnet,[],2^(nLevels-1))
title('Atomic images of trained NSOLT')
```



推論用 NSOLT ネットワークの構築

```
% Assemble analyzer
analysislgraph4predict = analysislgraph;
analysislgraph4predict = analysislgraph4predict.replaceLayer('Image input',...
    imageInputLayer(szOrg,'Name','Image input','Normalization','none'));
```

```
for iLayer = 1:height(analysislgraph4predict.Layers)
    layer = analysislgraph4predict.Layers(iLayer);
    if contains(layer.Name,"Lv"+nLevels+"_DcOut") || ...
            ~isempty(regexp(layer.Name,'^Lv\d+_AcOut','once'))
        analysislgraph4predict = analysislgraph4predict.replaceLayer(layer.Name,...
            regressionLayer('Name',layer.Name));
    end
end
analysisnet4predict = assembleNetwork(analysislgraph4predict);

% Assemble synthesizer
synthesislgraph4predict = synthesislgraph;
synthesislgraph4predict = synthesislgraph4predict.replaceLayer('Lv1_Out',...
    regressionLayer('Name','Lv1_Out'));
for iLayer = 1:height(synthesislgraph4predict.Layers)
    layer = synthesislgraph4predict.Layers(iLayer);
    if contains(layer.Name,'Ac feature input')
        iLv = str2double(layer.Name(3));
        sbSize = szOrg.*(decFactor.^(-iLv));
        newlayer = ...
            imageInputLayer([sbSize
(sum(nChannels)-1)],'Name',layer.Name,'Normalization','none');
        synthesislgraph4predict = synthesislgraph4predict.replaceLayer(...
            layer.Name,newlayer);
    elseif contains(layer.Name,sprintf('Lv%0d_Dc feature input',nLevels))
        iLv = str2double(layer.Name(3));
        sbSize = szOrg.*(decFactor.^(-iLv));
        newlayer = ...
            imageInputLayer([sbSize 1],'Name',layer.Name,'Normalization','none');
        synthesislgraph4predict = synthesislgraph4predict.replaceLayer(...
            layer.Name,newlayer);
    end
end
synthesisnet4predict = assembleNetwork(synthesislgraph4predict);
```

随伴関係（完全再構成）の確認

NSOLT はパーセバルタイト性を満たす.

```
u = rand(szOrg,'single');
[s{1:nLevels+1}] = analysisnet4predict.predict(u);
v = synthesisnet4predict.predict(s{1:nLevels+1});
assert(mse(u,v)<1e-9)
```

NSOLT による合成処理とその随伴処理の定義

```
nsoltconfig.nLevels = nLevels;
szCoefs = zeros(nLevels+1,3);
for iLevel = 1:nLevels+1
    s_iLevel = s{iLevel};
    szCoefs(iLevel,1) = size(s_iLevel,1);
```

```
    szCoefs(iLevel,2) = size(s_iLevel,2);
    szCoefs(iLevel,3) = size(s_iLevel,3);
end
nsoltconfig.szCoefs = szCoefs;
syn_nsolt = @(x) synthesisnsolt(x,synthesisnet4predict,nsoltconfig);
adj_nsolt = @(y) analysisnsolt(y,analysisnet4predict,nsoltconfig);
```

随伴関係の確認

```
x = adj_nsolt(y);
v = randn(size(x));
u = syn_nsolt(v);
assert(abs(dot(y(:),u(:))-dot(x(:),v(:)))<1e-3)
```

繰返しハード閾値処理(IHT)によるスパース近似の比較

辞書の準備

```
blkdct  = { syn_blkdct,  adj_blkdct,  "Block DCT", true };
blkpca  = { syn_blkpca,  adj_blkpca,  "Block PCA", true };
blkrica = { syn_blkrica, adj_blkrica, "Block RICA", false };
blkksvd = { syn_blkksvd, adj_blkksvd, "Block K-SVD", false };
nsolt   = { syn_nsolt,   adj_nsolt,   "NSOLT", true };
dicset  = { blkdct, blkpca, blkrica, blkksvd, nsolt };
nDics   = length(dicset);
```

IHT

$$\mathbf{x}^{(t+1)} \leftarrow \mathscr{H}_{BK}\left(\mathbf{x}^{(t)} + \mu^{(t)}\hat{\mathbf{D}}^{\mathsf{T}}\left(\mathbf{y} - \hat{\mathbf{D}}\mathbf{x}^{(t)}\right)\right)$$

$$t \leftarrow t + 1$$

- T. Blumensath and M. E. Davies, "Normalized Iterative Hard Thresholding: Guaranteed Stability and Performance," in IEEE Journal of Selected Topics in Signal Processing, vol. 4, no. 2, pp. 298-309, April 2010, doi: 10.1109/JSTSP.2010.2042411.

```
% 平均値を引いた画像を用意（近似後に平均値を加算）
ymean = mean(y,"all");
y = yorg - ymean;
% 準備
c = 1e-3;
kappa = 1/(1-c)+1e-1;
nItersIht = 3000;
nCoefs = floor(sparsityRatio*prod(szOrg));
psnrs = zeros(nItersIht,nDics);
ssims = zeros(nItersIht,nDics);
yaprxs = cell(1,nDics);
% 繰り返し処理
for iDic = 1:nDics
    dic_ = dicset{iDic};
    synproc = dic_{1};
```

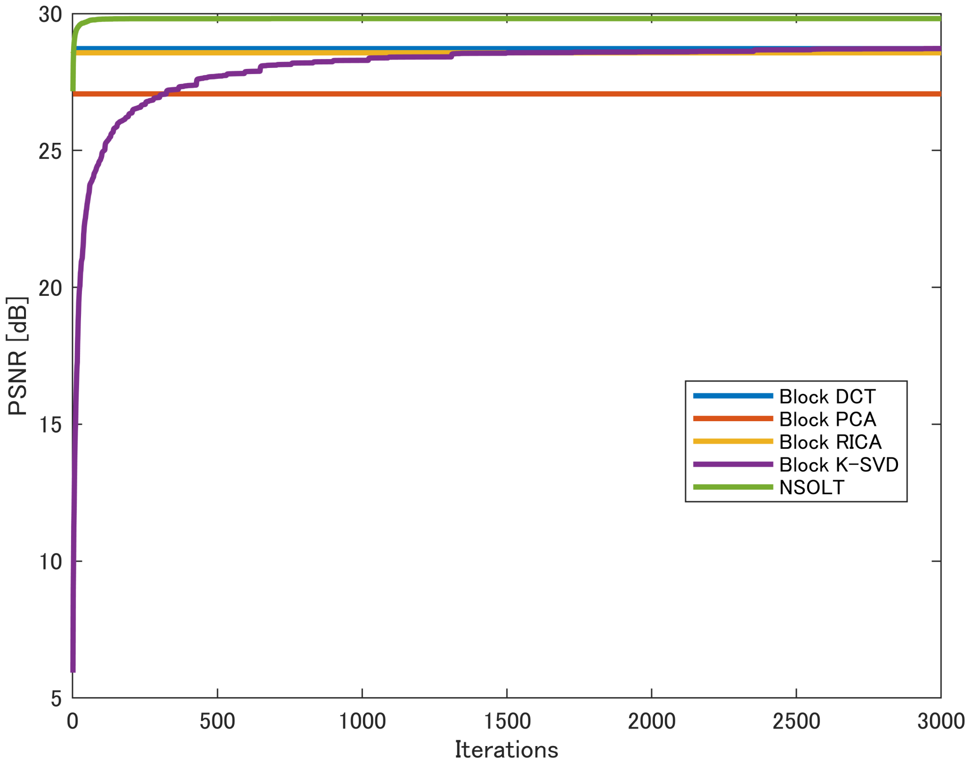
```
    adjproc = dic_{2};
    dicname = dic_{3};
    isStepSizeNormalized = dic_{4};
    % IHT
    display(dicname)
    s = adjproc(y); % D^Ty
    xt = zeros(size(s),'like',s); % x1 = 0;
    if ~isStepSizeNormalized
        suppt = find(hardthresh(s,nCoefs)); % Γ1 = supp(H_K(D^Ty))
        maskt = (abs(s)~=0);
    end
    for iIter=1:nItersIht
        % Gradient descent
        gt = adjproc(y-synproc(xt)); % g = D^T(y-Dxn)
        if isStepSizeNormalized % 正規化なし
            mu = (1-c);
            xtp1 = hardthresh(xt+mu*gt,nCoefs);
        else % 正規化を適用
            ggt = gt(suppt); % g_Γn
            ugt = synproc(maskt.*gt); % D_Γn^T g_Γn
            mu = (ggt.'*ggt)/(ugt(:).'*ugt(:));
            ttp1 = hardthresh(xt+mu*gt,nCoefs); % ~xn+1 = H_K(xn + μn gn)
            supptp1 = find(ttp1); % Γn+1 = supp(~xn+1)
            if length(supptp1)==length(suppt) && all(supptp1==suppt)
                xtp1 = ttp1; % xn+1 = ~xn+1
            else
                dxt = ttp1-xt; % ~xn+1 - xn
                omega = (1-c)*(norm(dxt,'fro')/norm(synproc(dxt),'fro'))^2;
                if mu <= omega
                    xtp1 = ttp1; % xn+1 = ~xn+1
                else
                    while mu > omega
                        mu = mu/(kappa*(1-c));
                        ttp1 = hardthresh(xt+mu*gt,nCoefs); % ~xn+1 = H_K(xn + μn
gn)

                        dxt = ttp1-xt; % ~xn+1 - xn
                        omega = (1-c)*(norm(dxt,'fro')/norm(synproc(dxt),'fro'))^2;
                    end
                    supptp1 = find(ttp1);  % Γn+1 = supp(~xn+1)
                    xtp1 = ttp1; % xn+1 = ~xn+1
                end
            end
            % Update
            suppt = supptp1;
            maskt = 0*maskt;
            maskt(suppt) = 1;
        end
        xt = xtp1;
        % Monitoring
        checkSparsity = nnz(xt)/prod(szOrg)<=sparsityRatio;
```

```
        assert(checkSparsity)
        yaprx_ = synproc(xt);
        psnr_ = psnr(cast(yaprx_,'like',y),y);
        ssim_ = ssim(cast(yaprx_,'like',y),y);
        psnrs(iIter,iDic) = psnr_;
        ssims(iIter,iDic) = ssim_;
        %fprintf("IHT(%d) PSNR: %6.4f\n",iIter,psnr_);
    end
    yaprxs{iDic} = yaprx_ + ymean;
end
```

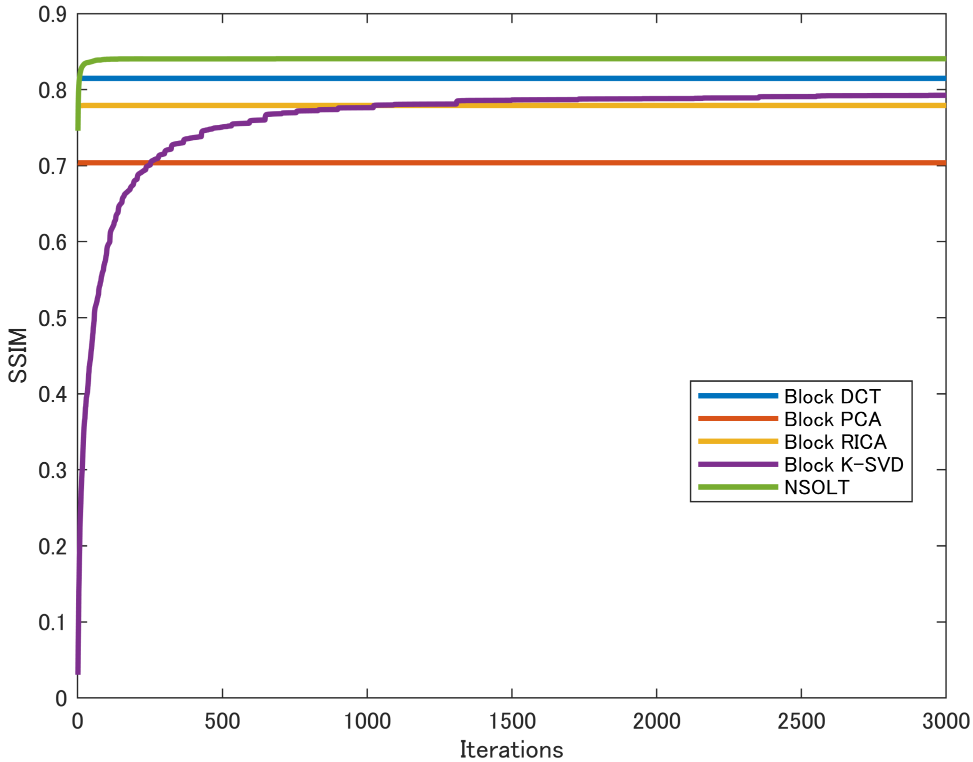
```
dicname = 
"Block DCT"
dicname = 
"Block PCA"
dicname = 
"Block RICA"
dicname = 
"Block K-SVD"
dicname = 
"NSOLT"
```

近似結果の表示

```
dicnames = [blkdct{3},blkpca{3},blkrica{3},blkksvd{3},nsolt{3}];
psnrtbl = array2table(psnrs,'VariableNames',
[blkdct{3},blkpca{3},blkrica{3},blkksvd{3},nsolt{3}]);
psnrtbl = horzcat(table((1:nItersIht).','VariableNames',"Iterations"),psnrtbl);
ssimtbl = array2table(ssims,'VariableNames',
[blkdct{3},blkpca{3},blkrica{3},blkksvd{3},nsolt{3}]);
ssimtbl = horzcat(table((1:nItersIht).','VariableNames',"Iterations"),ssimtbl);

% PSNR のグラフ
figure
plot(psnrtbl,"Iterations",dicnames,'LineWidth',2)
ylabel('PSNR [dB]')
legend('Location','best')
```



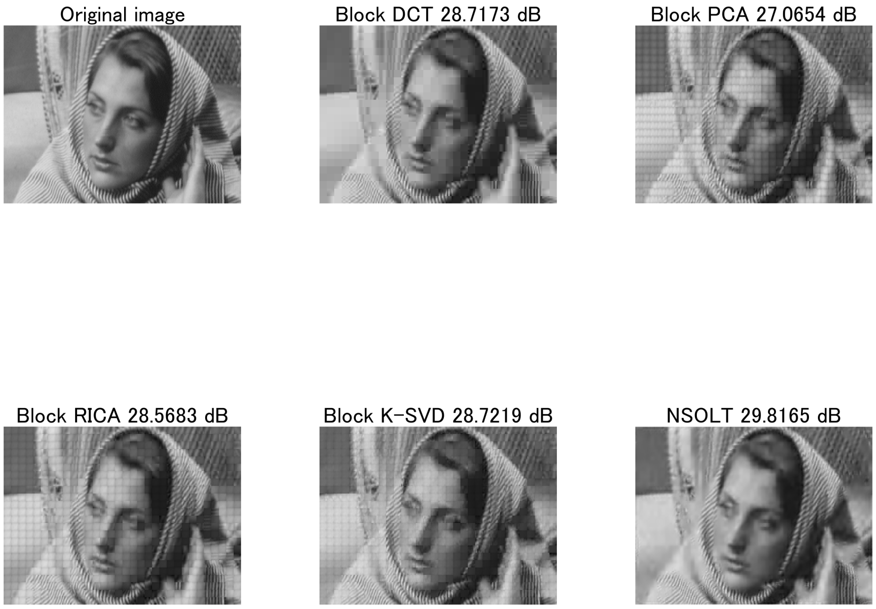

```
% SSIM のグラフ
figure
plot(ssimtbl,"Iterations",dicnames,'LineWidth',2)
ylabel('SSIM')
legend('Location','best')
```




```
% 原画像の表示
figure
tiledlayout(2,3)
nexttile
imshow(yorg)
title("Original image")
% 近似画像の表示
for idx = 1:nDics
    yaprx = yaprxs{idx};
    dicname = dicnames(idx)
    file_yaprx = "../results/yaprx_" + replace(lower(dicname),' ','_') +".png";
    imwrite(yaprx,file_yaprx)
    %
    nexttile
    imshow(yaprxs{idx})
    title(dicname+" "+num2str(psnrs(end,idx))+" dB")
end
```

```
dicname = 
"Block DCT"
dicname = 
"Block PCA"
dicname = 
"Block RICA"
dicname = 
```

```
"Block K-SVD"
dicname = 
"NSOLT"
```



【関数定義】

NSOLT 合成処理関数

```
function y = synthesisnsolt(x,synthesisnet4predict,config)
nLevels = config.nLevels;
szCoefs = config.szCoefs;
s = cell(1,nLevels+1);
sidx = 1;
for iLevel = 1:nLevels+1
    sz_iLevel = szCoefs(iLevel,:);
    eidx = sidx+prod(sz_iLevel)-1;
    x_iLevel = x(sidx:eidx);
    s{iLevel} = reshape(x_iLevel,sz_iLevel);
    sidx = eidx+1;
end
y = synthesisnet4predict.predict(s{1:nLevels+1});
end
```

NSOLT 分析処理関数

```
function x = analysisnsolt(y,analysisnet4predict,config)
nLevels = config.nLevels;
```

```
[s{1:nLevels+1}] = analysisnet4predict.predict(y);
x = [];
for iLevel = 1:nLevels+1
    x = [x; s{iLevel}(:)];
end
end
```

ハード閾値処理

```
function y = hardthresh(x,K)
v = sort(abs(x(:)),'descend');
thk = v(K);
y = (abs(x)>thk).*x;
end
```

深層学習配列に対する繰返しハード閾値処理(IHT)のバッチ処理

```
function newdata = iht4patchds(oldtbl,analyzer,synthesizer,sparsityRatio)
% IHT for InputImage in randomPatchExtractionDatastore
%
nInputs = length(synthesizer.InputNames);

% Apply IHT process for every input patch
restbl = removevars(oldtbl,'InputImage');
dlv = dlarray(cat(4,oldtbl.InputImage{:}),'SSCB');
[~,dlcoefs{1:nInputs}] = iht4dlarray(dlv,analyzer,synthesizer,sparsityRatio);
coefs = cellfun(@(x) permute(num2cell(extractdata(x),1:3),[4 1 2
3]),dlcoefs,'UniformOutput',false);
%
nImgs = length(oldtbl.InputImage);
coefarray = cell(nImgs,nInputs);
for iImg = 1:nImgs
    for iInput = 1:nInputs
        coefarray{iImg,iInput} = coefs{iInput}{iImg};
    end
end
% Output as a cell in order to make multiple-input datastore
newdata = [ coefarray table2cell(restbl) ];
end
```

深層学習配列に対する繰返しハード閾値処理(IHT)

```
function [dly,varargout] = iht4dlarray(dlx,analyzer,synthesizer,sparsityRatio)
% IHT Iterative hard thresholding
%
nInputs = length(synthesizer.InputNames);
szBatch = size(dlx,4);

% Iterative hard thresholding w/o normalization
% (A Parseval tight frame is assumed)
gamma = (1.-1e-3);
```

```
nIters = 10;
nCoefs = floor(sparsityRatio*numel(dlx(:,:,:,1)));
[dlcoefs{1:nInputs}] =
analyzer.predict(dlarray(zeros(size(dlx),'like',dlx),'SSCB'));
% IHT
for iter=1:nIters
    % Gradient descent
    dly = synthesizer.predict(dlcoefs{1:nInputs});
    [grad{1:nInputs}] = analyzer.predict(dlx-dly);
    dlcoefs = cellfun(@(x,y) x+gamma*y,dlcoefs,grad,'UniformOutput',false);
    % Hard thresholding
    coefvecs = cellfun(@(x) extractdata(reshape(x,
[],szBatch)),dlcoefs,'UniformOutput',false);
    srtdabscoefs = sort(abs(cell2mat(coefvecs.')),1,'descend');
    thk = reshape(srtdabscoefs(nCoefs,:),1,1,1,szBatch);
    dlcoefs = cellfun(@(x) (abs(x)>thk).*x,dlcoefs,'UniformOutput',false);
    % Monitoring
    %checkSparsity =...
    %nnz(srtdabscoefs>srtdabscoefs(nCoefs,:))/numel(dlx)<=sparsityRatio;
    %assert(checkSparsity)
    %fprintf("IHT(%d) MSE: %6.4f\n",iter,mse(dlx,dly));
end
varargout = dlcoefs;
end
```

NSOLT ネットワークの随伴関係の確認

```
function checkadjointrelation(analysislgraph,synthesislgraph,nLevels,szInput)
import saivdr.dcnn.*
x = rand(szInput,'single');
% Assemble analyzer
analysislgraph4predict = analysislgraph;
for iLayer = 1:length(analysislgraph4predict.Layers)
    layer = analysislgraph4predict.Layers(iLayer);
    if contains(layer.Name,"Lv"+nLevels+"_DcOut") || ...
            ~isempty(regexp(layer.Name,'^Lv\d+_AcOut','once'))
        analysislgraph4predict = analysislgraph4predict.replaceLayer(layer.Name,...
            regressionLayer('Name',layer.Name));
    end
end
analysisnet4predict = assembleNetwork(analysislgraph4predict);

% Assemble synthesizer
synthesislgraph4predict = synthesislgraph;
synthesisnet4predict = assembleNetwork(synthesislgraph4predict);

% Analysis and synthesis process
[s{1:nLevels+1}] = analysisnet4predict.predict(x);
if isvector(s{end-1})
    s{end-1} = permute(s{end-1},[1,3,2]);
```

```
end
y = synthesisnet4predict.predict(s{:});

% Evaluation
display("MSE: " + num2str(mse(x,y)))
end
```

直交マッチング追跡関数の定義

```
function x = omp(y,Phi,nCoefs)
% Initializaton
nDims = size(Phi,1);
nAtoms = size(Phi,2);
e = ones(nAtoms,1);
a = zeros(nAtoms,1);
g = zeros(nAtoms,1);
x = zeros(nAtoms,1);
v = zeros(nDims,1);
r = y - v;
supp = [];
k = 0;
while k < nCoefs
    % Matching process
    rr = r.'*r;
    for m = setdiff(1:nAtoms,supp)
        d = Phi(:,m);
        g(m) = d.'*r; % γm=<dm,r>
        a(m) = g(m)/(d.'*d); % Normalize αm=γm/||dm||^2
        e(m) = rr - g(m)*a(m); % <r-dm/||dm||^2,r>
    end
    % Minimum value search (pursuit)
    [~,mmin]= min(e);
    % Update the support
    supp = union(supp,mmin);
    subPhi = Phi(:,supp);
    x(supp) = pinv(subPhi) * y;
    % Synthesis process
    v = Phi*x;
    % Residual
    r = y - v;
    % Update
    k = k + 1;
end
end
```

NSOLT ネットワークからのツリーレベル情報の抽出

```
function nLevels = extractnumlevels(nsoltnet)
import saivdr.dcnn.*

% Extraction of information
```

```
expidctlayer = '^Lv\d+_E0~?$';
nLevels = 0;
nLayers = height(nsoltnet.Layers);
for iLayer = 1:nLayers
    layer = nsoltnet.Layers(iLayer);
    if ~isempty(regexp(layer.Name,expidctlayer,'once'))
        nLevels = nLevels + 1;
    end
end
end
```

NSOLT ネットワークからのストライド情報の抽出

```
function decFactor = extractdecfactor(nsoltnet)
import saivdr.dcnn.*

% Extraction of information
expfinallayer = '^Lv1_Cmp1+_V0~?$';
nLayers = height(nsoltnet.Layers);
for iLayer = 1:nLayers
    layer = nsoltnet.Layers(iLayer);
    if ~isempty(regexp(layer.Name,expfinallayer,'once'))
        decFactor = layer.DecimationFactor;
    end
end
end
```

NSOLT ネットワークからのチャネル数情報の抽出

```
function nChannels = extractnumchannels(nsoltnet)
import saivdr.dcnn.*

% Extraction of information
expfinallayer = '^Lv1_Cmp1+_V0~?$';
nLayers = height(nsoltnet.Layers);
for iLayer = 1:nLayers
    layer = nsoltnet.Layers(iLayer);
    if ~isempty(regexp(layer.Name,expfinallayer,'once'))
        nChannels = layer.NumberOfChannels;
    end
end
end
```