

フィルタバンク理論に基づく畳込み辞書学習

～ 構造制約を利用した畳込みネットワーク構築 ～

村松正吾（新潟大学工学部工学科電子情報通信プログラム）

電子情報通学会 基礎・境界ソサイエティ Fundamentals Review (掲載予定)

動作環境 : MATLAB R2023a

準備

```
clear
close all

nsoltDic = ""; % "nsoltdictionary_20230617111056377"

isCodegen = false; % コード生成
setup(isCodegen)
```

```
SaivDr-4.2.2.2 exits.
Skip code generation
```

パラメータ設定

- ブロックサイズ
- 冗長度
- スパース度

```
% Block size
szBlk = [ 8 8 ];

% Redundancy ratio for RICA/K-SVD
redundancyRatio = 5/3;

% Sparsity ratio
sparsityRatio = 3/64;
```

画像の読み込み

- $y \in \mathbb{R}^N$

```
% 原画像の準備
file_yorg = '../data/yorg.png';
if ~exist(file_yorg, 'file')
```

```

unzip('http://www.ess.ic.kanagawa-it.ac.jp/std_img/monoimage2/Mono-
Image2.zip','./results')
yfull = imread('./results/Mono-Image2/512X512/barbara512.bmp');
ycrop = yfull(1:192,end-255:end);
imwrite(ycrop,file_yorg)
end

% 原画像の読み込み
yorg = im2double(imread(file_yorg));
szOrg = size(yorg);

```

画像表示

```

figure
imshow(yorg);
title('Original image y')

```



零平均化

```

%ymean = mean(y,"all");
%y = yorg - ymean;
meansubtract = @(x) x-mean(x,"all");
y = meansubtract(yorg);

```

離散コサイン変換 (DCT)

$$[\mathbf{C}_M]_{k,n} = \sqrt{\frac{2}{M}} \alpha_k \cos \frac{k(n+1/2)\pi}{M}, \quad k, n = 0, 1, \dots, M-1$$

$$\alpha_k = \begin{cases} \frac{1}{\sqrt{2}} & k = 0 \\ 1 & k = 1, 2, \dots, M-1 \end{cases}$$

基底画像

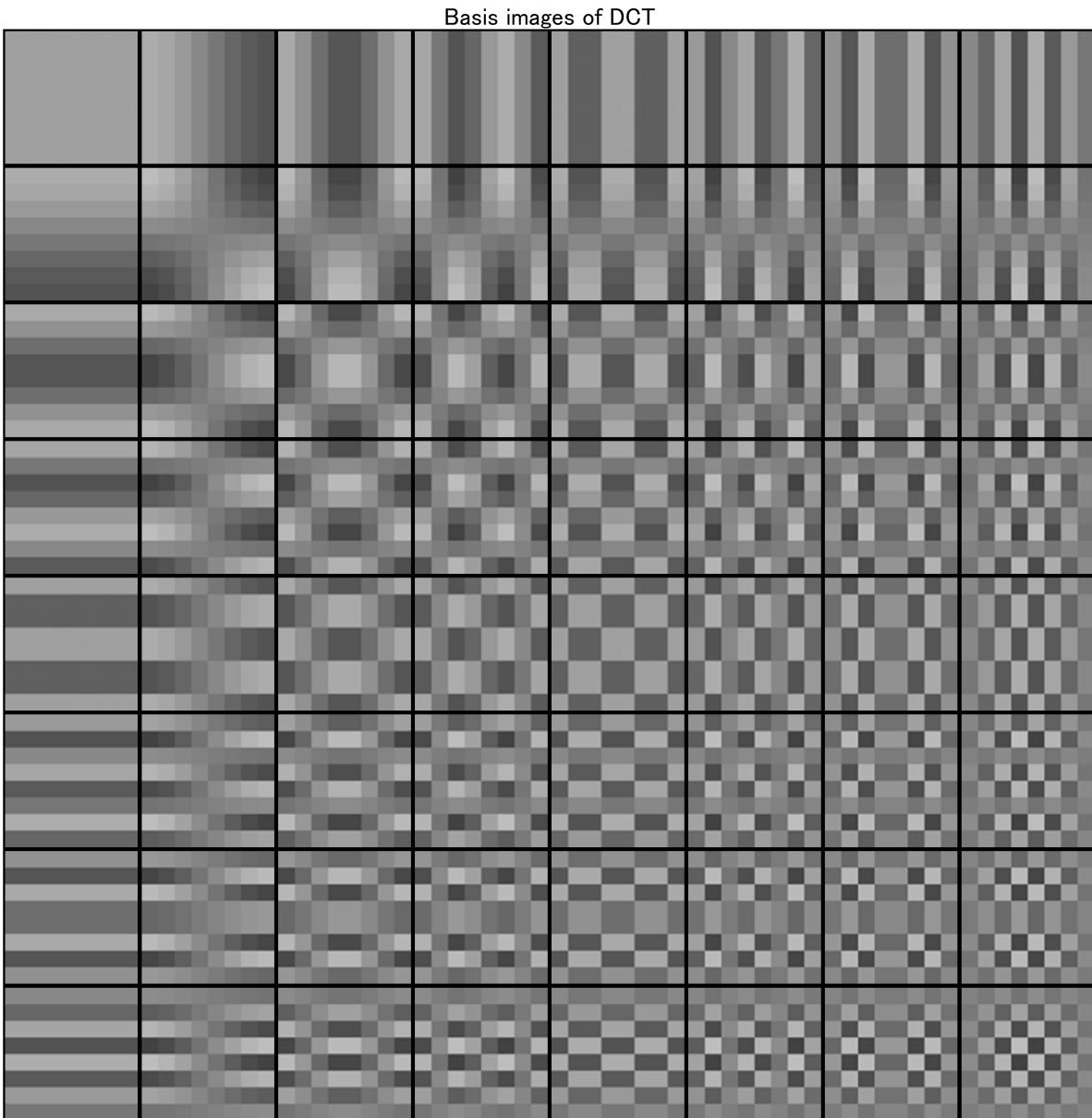
$$\mathbf{B}_{k,\ell} = \mathbf{C}_M^{-1} \mathbf{E}_{k,\ell} \mathbf{C}_M^{-T}, \quad k, \ell = 0, 1, \dots, M-1$$

$$\mathbf{E}_{k,\ell} = \mathbf{e}_k \mathbf{e}_\ell^T$$

```
basisImagesDct = zeros(szBlk(1),szBlk(2),prod(szBlk));
iBasis = 1;
for iRow=1:szBlk(1)
    for iCol=1:szBlk(2)
        E = zeros(szBlk);
        E(iRow,iCol) = 1;
        basisImagesDct(:,:,:,iBasis) = idct2(E,szBlk(1),szBlk(2));
        iBasis = iBasis + 1;
    end
end
```

基底画像の表示

```
figure
montage(imresize(basisImagesDct,8,'nearest')+0.5,'BorderSize',[2 2])
title('Basis images of DCT')
```



ブロック DCT による合成処理とその随伴処理の定義

```
syn_blkdct = @(x) blockproc(x,szBlk,@(block_struct) idct2(block_struct.data));  
adj_blkdct = @(y) blockproc(y,szBlk,@(block_struct) dct2(block_struct.data));
```

随伴関係の確認

```
x = adj_blkdct(y);  
v = randn(size(x));  
u = syn_blkdct(v);  
assert(abs(dot(y(:),u(:))-dot(x(:),v(:)))<1e-9)
```

主成分分析 (PCA)

問題設定:

直交性と次元削減

$$\Phi = \mathbf{I}_M, \forall b, \forall p, \|\mathbf{x}_b\|_0 \leq p < M$$

を制約条件とした最小自乗問題

$$\{\hat{\Phi}, \{\hat{\mathbf{x}}_b\}_b\} = \arg \min_{\{\Phi, \{\mathbf{x}_b\}_b\}} \frac{1}{2S} \sum_{b=1}^S \|\mathbf{y}_b - \Phi \mathbf{x}_b\|_2^2$$

を解く。上式は等価的に

$$\hat{\Phi} = \arg \max_{\Phi} \text{tr}(\Phi_{:,0:p-1}^\top \hat{\Sigma}_y \Phi_{:,0:p-1}) \quad \text{s.t. } \Phi^\top \Phi = \mathbf{I}_M$$

と表現できる。ただし、 $\hat{\Sigma}_y$ は観測ベクトル $\{\mathbf{y}_b\}_b$ （零平均を仮定）の標本分散共分散行列である。

解:

固有値分解

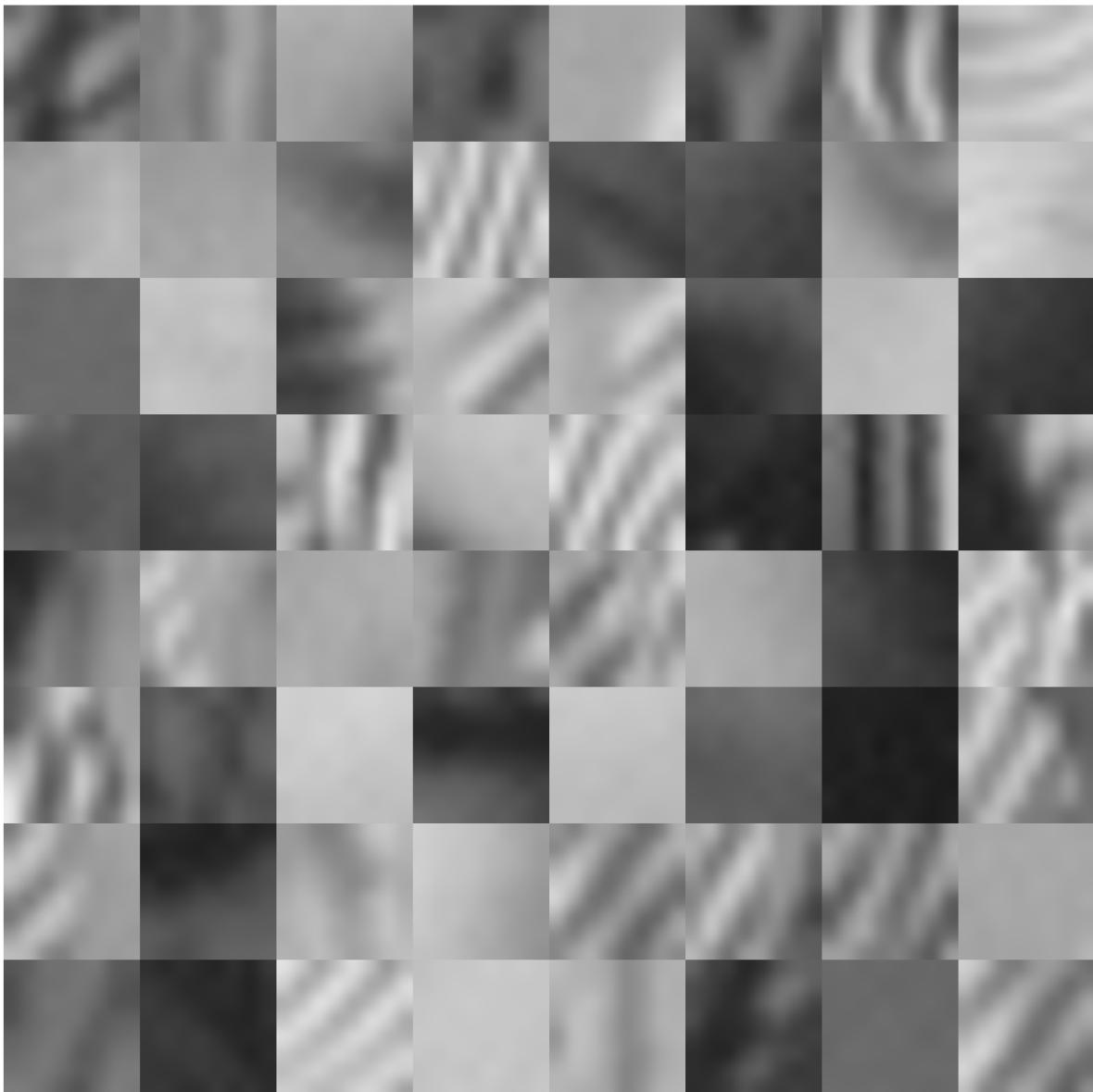
$$\hat{\Phi}^\top \hat{\Sigma}_y \hat{\Phi} = \Lambda$$

ただし、 $\Lambda = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_M)$ 。 $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_M$ は $\hat{\Sigma}_y$ の固有値。

画像 y からのデータ行列 Y の生成

標本平均ブロックを引く代わりに、予め零平均化したデータで学習

```
nPatches = 20*prod(szOrg./szBlk); % PCA/RICA/K-SVD 学習用のパッチをランダム抽出
npos = randsample(prod(szOrg-szBlk),nPatches);
ybs = zeros(szBlk(1),szBlk(2),nPatches, 'like',y);
szSrchy = szOrg(1)-szBlk(1);
for iPatch = 1:nPatches
    ny_ = mod(npos(iPatch)-1,szSrchy)+1;
    nx_ = floor((npos(iPatch)-1)/szSrchy)+1;
    ybs(:,:,:,iPatch) = y(ny_:ny_+szBlk(1)-1,nx_:nx_+szBlk(2)-1);
end
figure
montage(ybs+0.5, 'Size',[8 8]);
```



```
drawnow
```

```
Y = reshape(ybs,prod(szBlk),[]);
```

標本分散共分散行列 $\hat{\Sigma}_y$ の計算

```
SigmaY = cov(Y.'');
```

標本分散共分散行列 $\hat{\Sigma}_y$ の固有値分解

```
[Phi_pca,Lambda] = eig(SigmaY);
```

固有値 λ の大きさの降順に列ベクトルをソート (Sorting column vectors in the descending order of the eigenvalues λ)

```
[~,idx] = sort(diag(Lambda), 'descend');
Phi_pca = Phi_pca(:,idx);
```

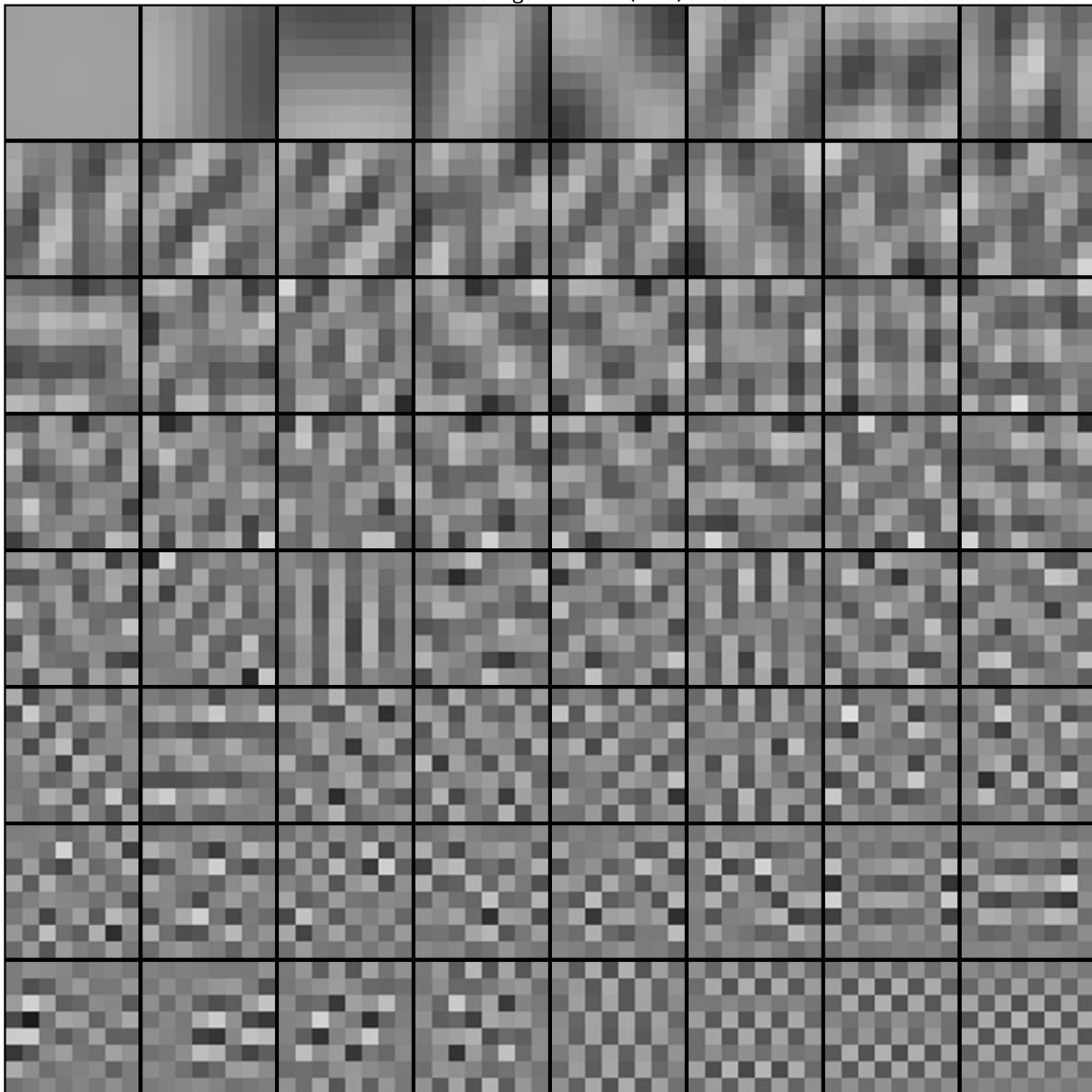
固有ベクトルを基底画像に変換

```
nBases = prod(szBlk);
basisImagesPca = zeros(szBlk(1),szBlk(2),nBases);
for iBasis = 1:nBases
    basisImagesPca(:,:,iBasis) = reshape(Phi_pca(:,iBasis),szBlk(1),szBlk(2));
end
```

基底画像の表示（辞書）

```
figure
montage(imresize(basisImagesPca,8,'nearest')+0.5,'BorderSize',[2 2])
title('Basis images of PCA(KLT)')
```

Basis images of PCA(KLT)



ブロック PCA による合成処理とその随伴処理の定義

```
syn_blkpca = @(x) col2im(Phi_pca*x,szBlk,szOrg,"distinct");
adj_blkpca = @(y) Phi_pca.*im2col(y,szBlk,"distinct");
```

随伴関係の確認

```
x = adj_blkpca(y);
v = randn(size(x));
u = syn_blkpca(v);
assert(abs(dot(y(:),u(:))-dot(x(:),v(:)))<1e-9)
```

再構成独立成分分析 (RICA)

問題設定:

$$\widehat{\Phi} = \arg \min_{\Phi} \frac{1}{2S} \sum_{b=1}^S \|\mathbf{y}_b - \Phi \Phi^\top \mathbf{y}_b\|_2^2 + \frac{\alpha}{S} \sum_{b=1}^S \rho(\Phi^\top \mathbf{y}_b)$$

$$= \arg \min_{\Phi} \frac{(2\alpha)^{-1}}{S} \sum_{b=1}^S \|\mathbf{y}_b - \Phi \Phi^\top \mathbf{y}_b\|_2^2 + \frac{1}{S} \sum_{b=1}^S \rho(\Phi^\top \mathbf{y}_b)$$

ただし, $\{\mathbf{y}_n\}_n \subset \mathbb{R}^M$, $\Phi = (\phi_1, \phi_2, \dots, \phi_P) \in \mathbb{R}^{M \times P}$, $M \geq P$ である.

参考文献:

Le, Quoc V., Alexandre Karpenko, Jiquan Ngiam, and Andrew Y. Ng. "ICA with Reconstruction Cost for Efficient Overcomplete Feature Learning." Advances in Neural Information Processing Systems. Vol. 24, 2011, pp. 1017–1025. <https://papers.nips.cc/paper/4467-ica-with-reconstruction-cost-for-efficient-overcomplete-feature-learning.pdf>.

パラメータ設定

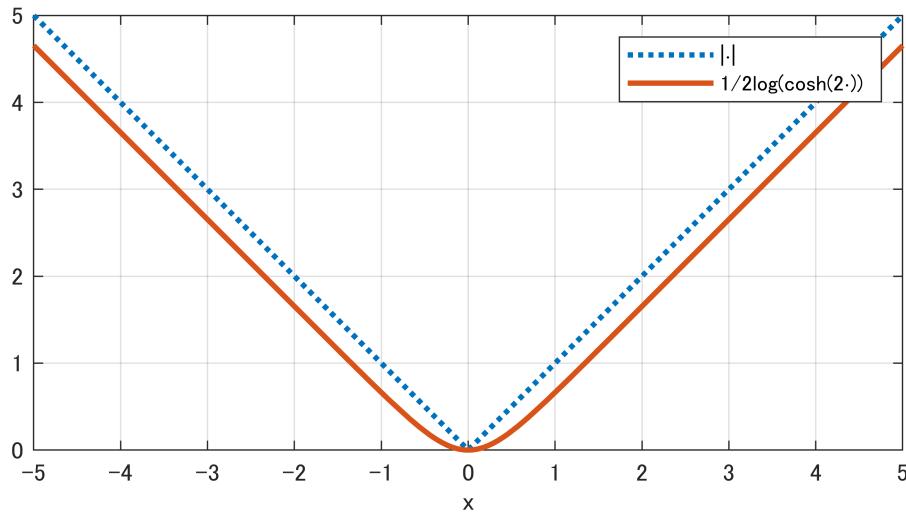
- 繰り返し回数 (Number of iterations)
- 正則化パラメータ (Regularization parameter)

```
% Number of iterations
nItersRica = 1e5;
% Regularization parameter
alpha = 2e-3;
```

コントラスト関数の例

$$\rho(\Phi^\top \mathbf{y}) := \frac{1}{2} \sum_{p=1}^P \log \circ \cosh(2\phi_p^\top \mathbf{y})$$

```
figure
fplot(@(x) abs(x), [-5 5], ':', 'LineWidth', 2, 'DisplayName', '| \cdot |')
hold on
fplot(@(x) log(cosh(2*x))/2, [-5
5], '-', 'LineWidth', 2, 'DisplayName', '1/2log(cosh(2\cdot))')
xlabel('x')
legend
grid on
axis equal
hold off
```



要素画像の数

```
nDims = prod(szBlk);
nAtoms = ceil(redundancyRatio*nDims);
```

辞書 Φ の初期化

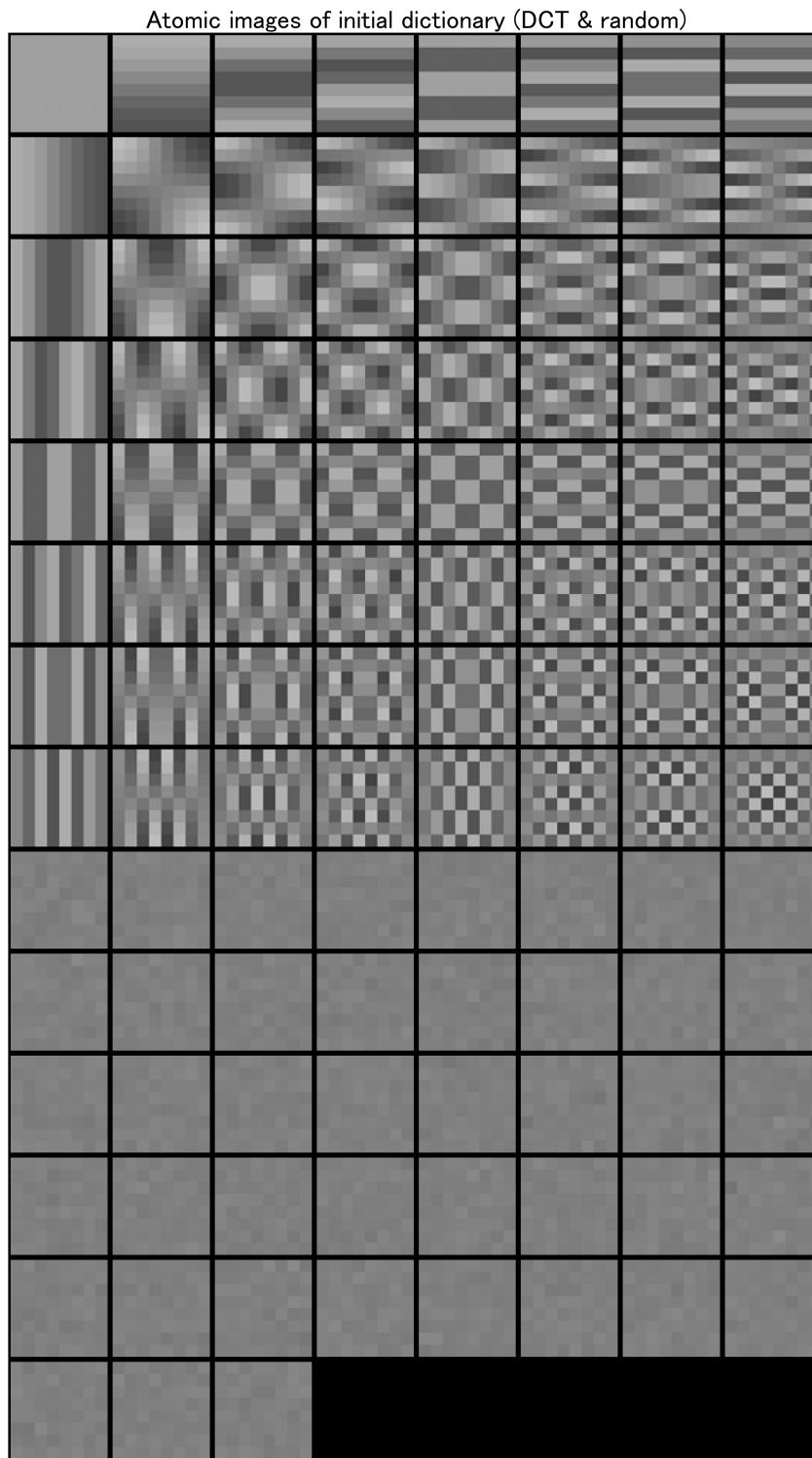
- 二次元離散コサイン変換
- ランダム

```
Phi_rica = randn(nDims,nAtoms);
Phi_rica = Phi_rica/norm(Phi_rica, 'fro');
for iAtom = 1:nDims
    delta = zeros(szBlk);
    delta(iAtom) = 1;
    Phi_rica(:,iAtom) = reshape(idct2(delta),nDims,1);
end
```

要素ベクトルを要素画像に変換

```
atomicImagesRica = zeros(szBlk(1),szBlk(2),nAtoms);
for iAtom = 1:nAtoms
    atomicImagesRica(:,:,:,iAtom) = reshape(Phi_rica(:,iAtom),szBlk(1),szBlk(2));
end
figure
```

```
montage(imresize	atomicImagesRica,8,'nearest')+0.5,'BorderSize',[2 2],'Size',  
[ceil(nAtoms/8) 8])  
title('Atomic images of initial dictionary (DCT & random)')
```



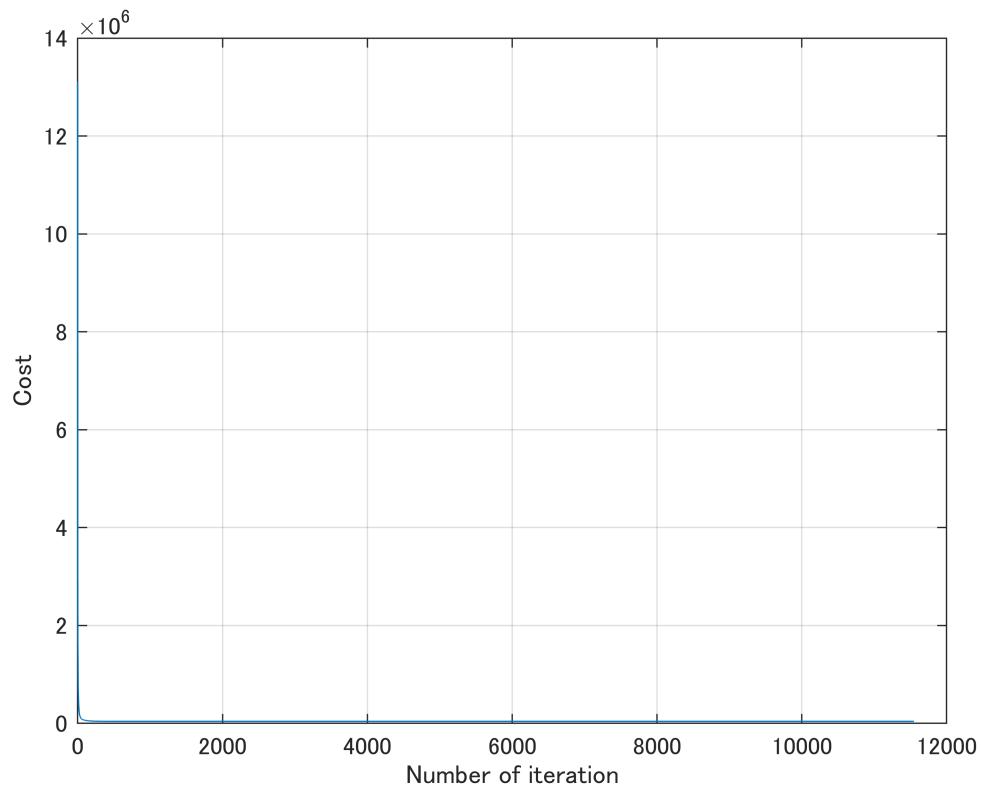
再構成 ICA オブジェクトの作成

PCAに合わせて予め零平均化したデータで学習

```
model = rica(Y.',nAtoms,...  
    'IterationLimit',nItersRica,...  
    'ContrastFcn','logcosh',...  
    'InitialTransformWeight',Phi_rica,...  
    'Lambda',1/(2*alpha));
```

コスト評価のグラフ

```
info = model.FitInfo;  
figure  
plot(info.Iteration,info.Objective)  
xlabel('Number of iteration')  
ylabel('Cost')  
grid on
```

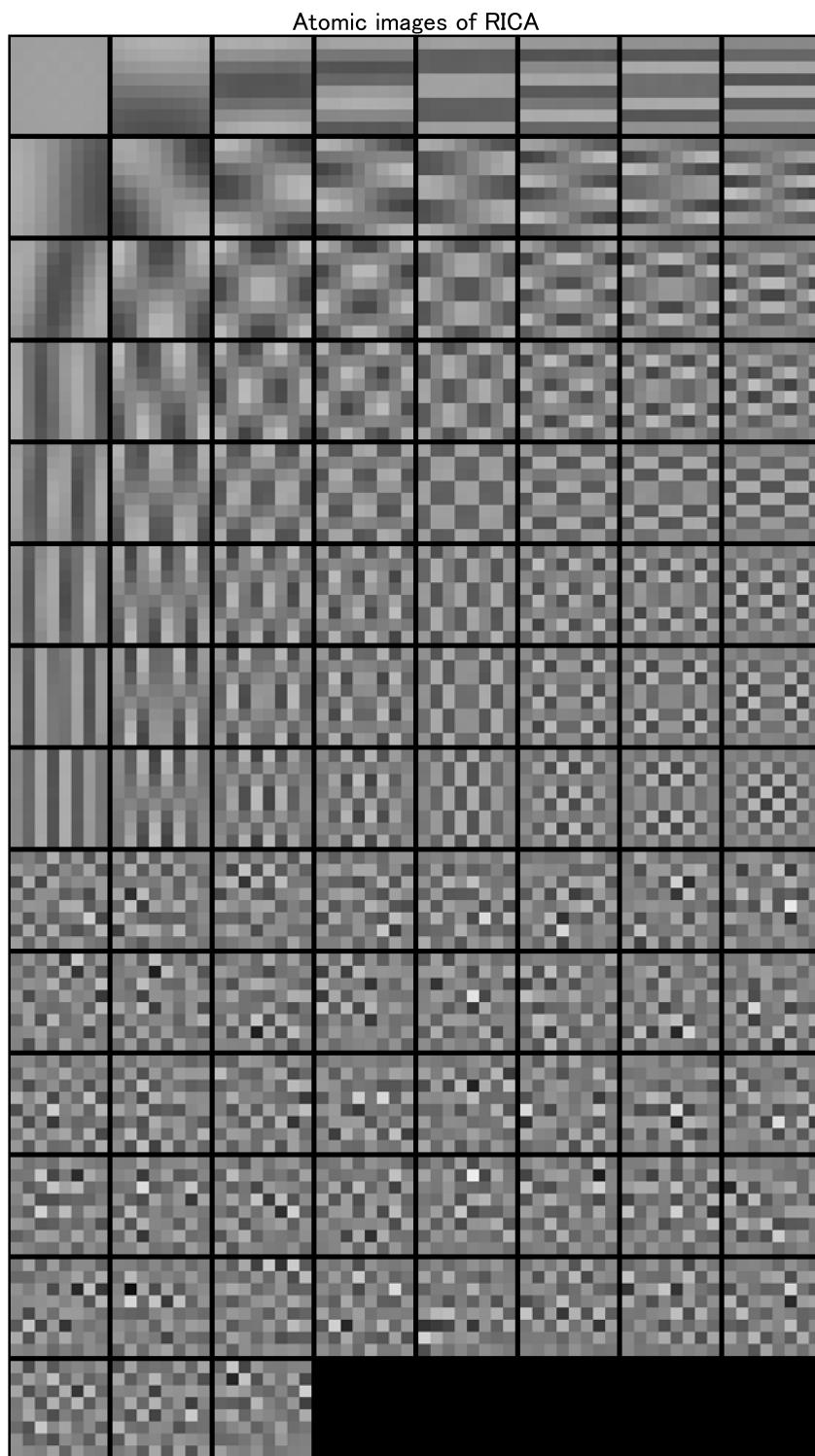


要素ベクトルを要素画像に変換

```
Phi_rica = model.TransformWeights;  
atomicImagesRica = zeros(szBlk(1),szBlk(2),nAtoms);  
for iAtom = 1:nAtoms  
    atomicImagesRica(:,:,:,iAtom) = reshape(Phi_rica(:,iAtom),szBlk(1),szBlk(2));  
end
```

要素画像の表示（辞書）

```
figure  
montage(imresize(atomicImagesRica,8,'nearest')+.5,'BorderSize',[2 2],'Size',  
[ceil(nAtoms/8) 8])  
title('Atomic images of RICA')
```



```

syn_blkrica = @(x) col2im(Phi_rica*x,szBlk,szOrg,"distinct");
adj_blkrica = @(y) Phi_rica.*im2col(y,szBlk,"distinct");

```

随伴関係の確認

```

x = adj_blkrica(y);
v = randn(size(x));
u = syn_blkrica(v);
assert(abs(dot(y(:),u(:))-dot(x(:),v(:)))<1e-9)

```

K-特異値分解

パラメータ設定

- 繰返し回数 (Number of iterations)

```

% Number of iterations
nItersKsvd = 5e3;

```

問題設定 (Problem setting):

$$\{\hat{\Phi}, \{\hat{\mathbf{x}}_b\}\} = \arg \min_{\{\Phi, \{\mathbf{x}_b\}\}} \frac{1}{2S} \sum_{b=1}^S \|\mathbf{y}_b - \Phi \hat{\mathbf{x}}_b\|_2^2, \quad \text{s.t. } \forall b, \|\mathbf{x}_b\|_0 \leq K$$

アルゴリズム :

スパース近似ステップと辞書更新ステップを繰返す.

- スパース近似ステップ

$$\hat{\mathbf{x}}_b = \arg \min_{\mathbf{x}} \frac{1}{2} \|\mathbf{y}_b - \hat{\Phi} \mathbf{x}\|_2^2 \quad \text{s.t. } \|\mathbf{x}\|_0 \leq K$$

- 辞書更新ステップ

$$\hat{\Phi} = \arg \min_{\Phi} \frac{1}{2S} \sum_{b=1}^S \|\mathbf{y}_b - \Phi \hat{\mathbf{x}}_b\|_2^2 = \arg \min_{\Phi} \frac{1}{2S} \left\| \left(\mathbf{Y} - \sum_{p \neq k} \phi_p \hat{\mathbf{X}}_{p,:} \right) - \phi_k \hat{\mathbf{X}}_{k,:} \right\|_F^2$$

係数の数

```

nCoefsKsvd = max(floor(sparsityRatio*nDims),1);

```

辞書 Φ の初期化

- 二変量離散コサイン変換
- ランダム

```

Phi_ksvd = randn(nDims,nAtoms);
Phi_ksvd = Phi_ksvd/norm(Phi_ksvd, 'fro');
for iAtom = 1:nDims
    delta = zeros(szBlk);
    delta(iAtom) = 1;
    Phi_ksvd(:,iAtom) = reshape(idct2(delta),nDims,1);
end

```

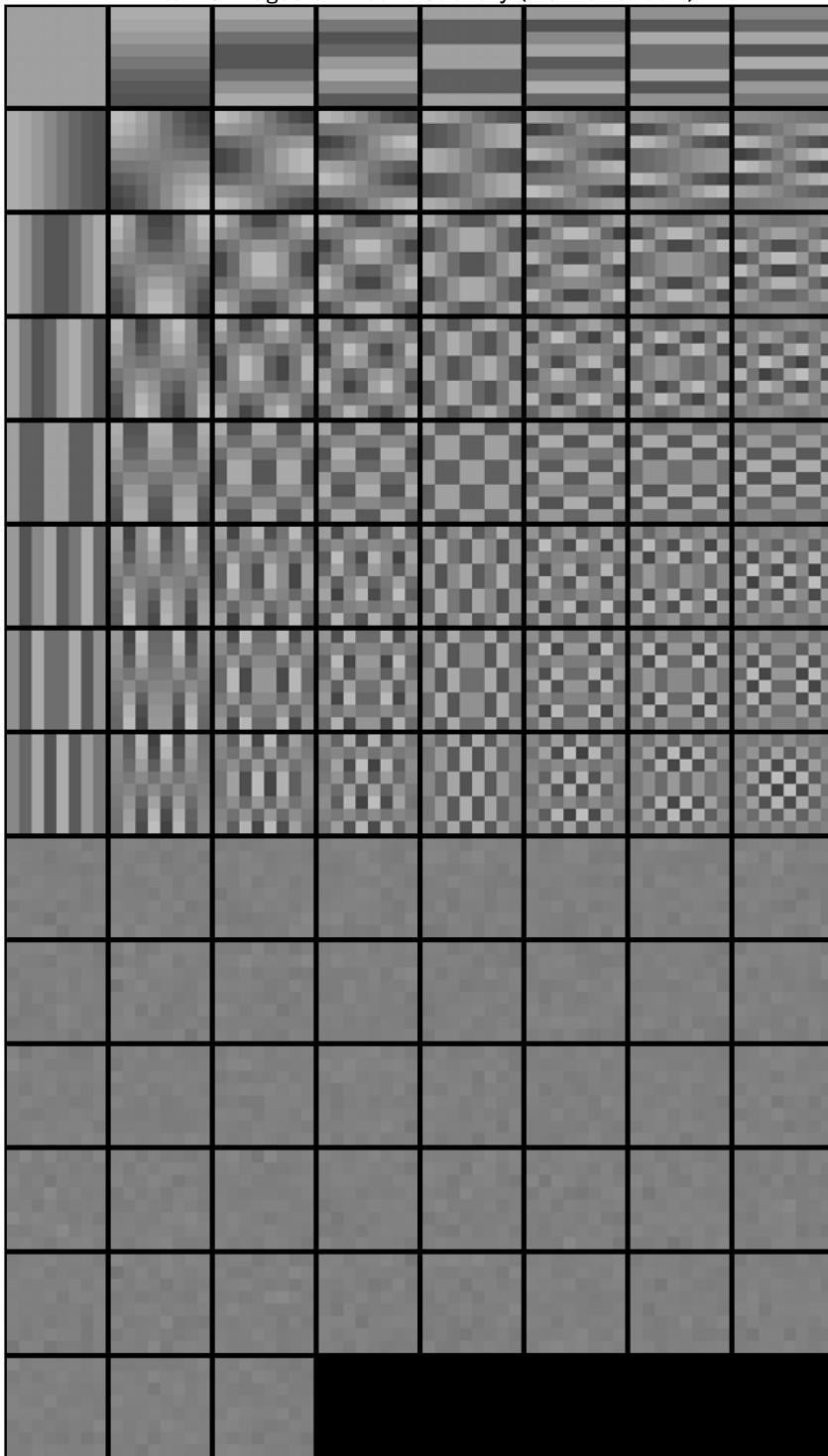
要素ベクトルを要素画像に変換

```

atomicImagesKsvd = zeros(szBlk(1),szBlk(2),nAtoms);
for iAtom = 1:nAtoms
    atomicImagesKsvd(:,:,iAtom) = reshape(Phi_ksvd(:,iAtom),szBlk(1),szBlk(2));
end
figure
montage(imresize(atomicImagesKsvd,8,'nearest')+0.5,'BorderSize',[2 2],'Size',
[ceil(nAtoms/8) 8])
title('Atomic images of initial dictionary (DCT & random)')

```

Atomic images of initial dictionary (DCT & random)



スペース近似ステップと辞書更新ステップの繰り返し

- スペース近似：直交マッチング追跡 (OMP)
- 辞書更新：特異値分解(SVD)と 1-ランク近似

辞書更新の内容

1. $k \leftarrow 1$
2. 誤差行列 \mathbf{E}_k を定義 : $\mathbf{E}_k := \mathbf{Y} - \sum_{p \neq k} \phi_p \hat{\mathbf{X}}_{p,:}$
3. データ行 $\hat{\mathbf{X}}_{k,:}$ の非零値を抽出する行列 Ω_k を定義 : $\hat{\mathbf{X}}_{k,:}^R = \hat{\mathbf{X}}_{k,:}\Omega_k \Leftrightarrow \hat{\mathbf{X}}_{k,:}^R\Omega_k^T = \hat{\mathbf{X}}_{k,:}$
4. 誤差行列 \mathbf{E}_k を行列 Ω_k で縮退 : $\mathbf{E}_k^R = \mathbf{E}_k\Omega_k$
5. 縮退した誤差行列 \mathbf{E}_k^R を特異値分解 : $\mathbf{E}_k^R = \mathbf{U}\mathbf{S}\mathbf{V}^T = (\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_r)\text{diag}(\sigma_1, \sigma_2, \dots, \sigma_r)(\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_r)^T$
6. 要素ベクトル ϕ_k を更新 : $\mathbf{k} \leftarrow \mathbf{u}_1$
7. データ行 $\hat{\mathbf{X}}_{k,:}$ を更新 : $\hat{\mathbf{X}}_{k,:} \leftarrow \sigma_1 \mathbf{v}_1^T$
8. $k \leftarrow k + 1$
9. $k \leq N$ ならば 2. へ $k > N$ ならば終了

ただし、 σ_1 を最大特異値とする。

交互ステップの繰返し計算

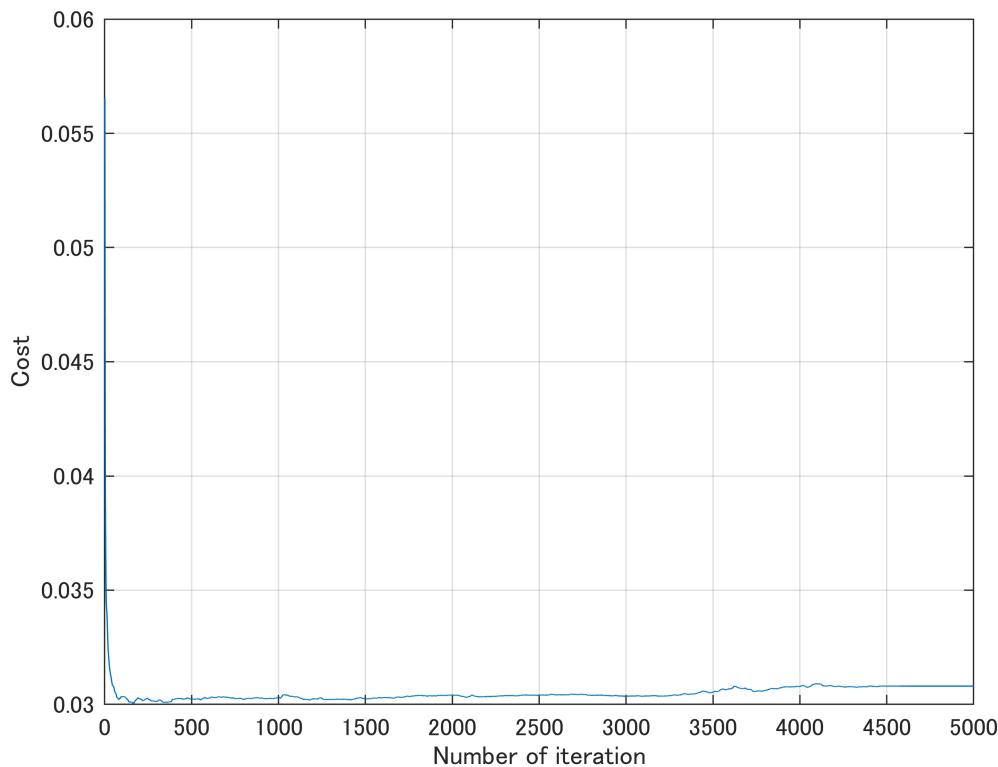
PCA に合わせて予め零平均化したデータで学習

```
cost = zeros(1,nIterKsvd);
nSamples = size(Y,2);
for iIter = 1:nIterKsvd
    X = zeros(nAtoms,nSamples);
    % Sparse approximation
    for iSample = 1:nSamples
        y_ = Y(:,iSample);
        x = omp(y_,Phi_ksvd,nCoefsKsvd);
        X(:,iSample) = x;
    end
    % Dictionary update
    for iAtom = 1:nAtoms
        idxset = setdiff(1:nAtoms,iAtom);
        xk = X(iAtom,:);
        suppK = find(xk);
        %
        Ekred = Y(:,suppK)-Phi_ksvd(:,idxset)*X(idxset,suppK);
        %
        if ~isempty(suppK)
            [U,S,V] = svd(Ekred,'econ');
            ak = U(:,1);
            xkred = S(1,1)*V(:,1)';
            %
            Phi_ksvd(:,iAtom) = ak;
            X(iAtom,suppK) = xkred;
        end
    end
    cost(iIter) = (norm(Y-Phi_ksvd*X,'fro')^2)/(2*nSamples);
```

```
end
```

コスト評価のグラフ

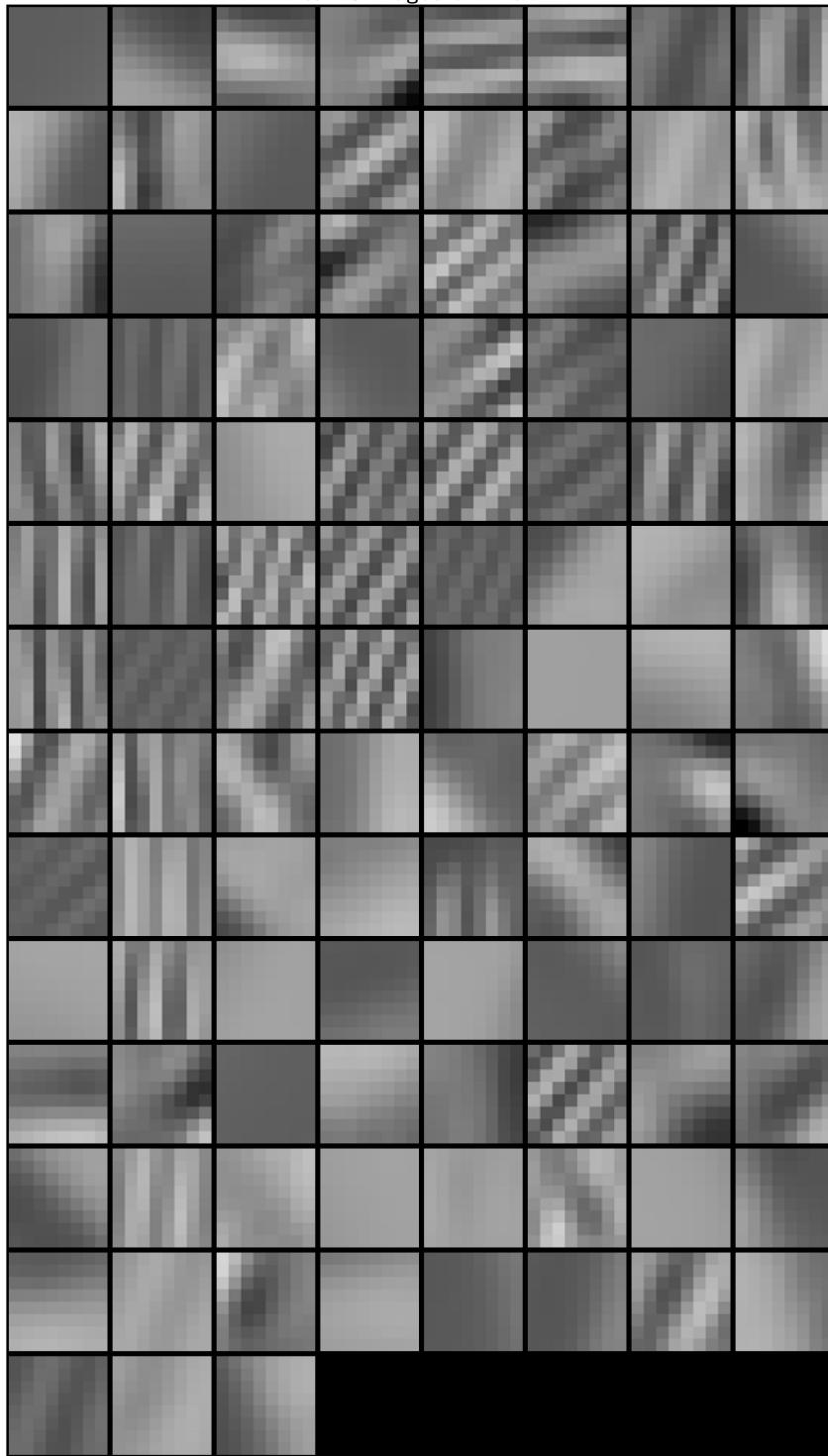
```
figure  
plot(cost)  
xlabel('Number of iteration')  
ylabel('Cost')  
grid on
```



要素ベクトルを要素画像に変換

```
atomicImagesKsvd = zeros(szBlk(1),szBlk(2),nAtoms);  
for iAtom = 1:nAtoms  
    atomicImagesKsvd(:,:,:,iAtom) = reshape(Phi_ksvd(:,iAtom),szBlk(1),szBlk(2));  
end  
figure  
montage(imresize(atomicImagesKsvd,8,'nearest')+0.5,'BorderSize',[2 2],'Size',  
[ceil(nAtoms/8) 8])  
title('Atomic images of K-SVD')
```

Atomic images of K-SVD



ブロック K-特異値分解による合成処理とその随伴処理の定義

```
syn_blkksvd = @(x) col2im(Phi_ksvd*x,szBlk,szOrg,"distinct");
adj_blkksvd = @(y) Phi_ksvd.*im2col(y,szBlk,"distinct");
```

随伴関係の確認

```

x = adj_blkksvd(y);
v = randn(size(x));
u = syn_blkksvd(v);
assert(abs(dot(y(:, ), u(:, ))-dot(x(:, ), v(:, )))<1e-9)

```

2 変量ラティス構造冗長フィルタバンク

例として、（偶対称チャネルと奇対称チャネルが等しい）偶数チャネル、偶数のポリフェーズ次数をもつタイプI非分離冗長重複変換(NSOLT)

$$\mathbf{E}(z_v, z_h) = \left(\prod_{n_h=1}^{\nu_h/2} \mathbf{V}_{2n_h}^{\{h\}} \bar{\mathbf{Q}}(z_h) \mathbf{V}_{2k_h-1}^{\{h\}} \mathbf{Q}(z_h) \right) \left(\prod_{n_v=1}^{\nu_v/2} \mathbf{V}_{2n_v}^{\{v\}} \bar{\mathbf{Q}}(z_v) \mathbf{V}_{2n_v-1}^{\{v\}} \mathbf{Q}(z_v) \right) \mathbf{V}_0 \mathbf{E}_0,$$

$$\mathbf{R}(z_v, z_h) = \mathbf{E}^T(z_v^{-1}, z_h^{-1}),$$

を採用する。ただし、

- $\mathbf{E}(z_v, z_h)$: 分析フィルタバンクの Type-I ポリフェーズ行列
- $\mathbf{R}(z_v, z_h)$: 合成フィルタバンクの Type-II ポリフェーズ行列
- $z_d \in \mathbb{C}, d \in \{v, h\}$: Z-変換の変数
- $\nu_d \in \mathbb{N}, d \in \{v, h\}$: 方向 d のポリフェーズ次数(重複ブロック数)
- $\mathbf{V}_0 = \begin{pmatrix} \mathbf{W}_0 & \mathbf{O} \\ \mathbf{O} & \mathbf{U}_0 \end{pmatrix} \begin{pmatrix} \mathbf{I}_{M/2} \\ \mathbf{O} \\ \mathbf{I}_{M/2} \\ \mathbf{O} \end{pmatrix} \in \mathbb{R}^{P \times M}, \mathbf{V}_n^{\{d\}} = \begin{pmatrix} \mathbf{I}_{P/2} & \mathbf{O} \\ \mathbf{O} & \mathbf{U}_n^{\{d\}} \end{pmatrix} \in \mathbb{R}^{P \times P}, d \in \{v, h\}, \mathbf{W}_0, \mathbf{U}_0, \mathbf{U}_n^{\{d\}} \in \mathbb{R}^{P/2 \times P/2}$ は直交行列
- $\mathbf{Q}(z) = \mathbf{B}_P \begin{pmatrix} \mathbf{I}_{P/2} & \mathbf{O} \\ \mathbf{O} & z^{-1} \mathbf{I}_{P/2} \end{pmatrix} \mathbf{B}_P, \bar{\mathbf{Q}}(z) = \mathbf{B}_P \begin{pmatrix} z \mathbf{I}_{P/2} & \mathbf{O} \\ \mathbf{O} & \mathbf{I}_{P/2} \end{pmatrix} \mathbf{B}_P, \mathbf{B}_P = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathbf{I}_{P/2} & \mathbf{I}_{P/2} \\ \mathbf{I}_{P/2} & -\mathbf{I}_{P/2} \end{pmatrix}$

【References】

- [Overview of Filter Banks - MATLAB & Simulink - MathWorks 日本](#)
- MATLAB SaivDr Package: <https://github.com/msiplab/SaivDr>
- S. Muramatsu, K. Furuya and N. Yuki, "Multidimensional Nonseparable Oversampled Lapped Transforms: Theory and Design," in IEEE Transactions on Signal Processing, vol. 65, no. 5, pp. 1251-1264, 1 March 1, 2017, doi: 10.1109/TSP.2016.2633240.
- S. Muramatsu, T. Kobayashi, M. Hiki and H. Kikuchi, "Boundary Operation of 2-D Nonseparable Linear-Phase Paraunitary Filter Banks," in IEEE Transactions on Image Processing, vol. 21, no. 4, pp. 2314-2318, April 2012, doi: 10.1109/TIP.2011.2181527.
- S. Muramatsu, M. Ishii and Z. Chen, "Efficient parameter optimization for example-based design of nonseparable oversampled lapped transform," 2016 IEEE International Conference on Image Processing (ICIP), Phoenix, AZ, 2016, pp. 3618-3622, doi: 10.1109/ICIP.2016.7533034.

- Furuya, K., Hara, S., Seino, K., & Muramatsu, S. (2016). Boundary operation of 2D non-separable oversampled lapped transforms. *APSIPA Transactions on Signal and Information Processing*, 5, E9. doi:10.1017/AT SIP.2016.3.

2次元画像の階層的分析

$R_M^P(\tau)$ をツリーレベル τ の階層構造フィルタバンクの冗長度とすると、

$$R_M^P(\tau) = \begin{cases} (P-1)\tau + 1, & M = 1, \\ \frac{P-1}{M-1} - \frac{P-M}{(M-1)M^\tau}, & M \geq 2. \end{cases}$$

となる。

パッケージダウンロード

```
% SaivDr パッケージバージョン
SAIVDR_VER = "4.2.2.2";
SAIVDR_DIR = "SaivDr-"+SAIVDR_VER;
if ~exist(SAIVDR_DIR,"dir")
    unzip("https://github.com/msiplab/SaivDr/archive/refs/tags/"+SAIVDR_VER+".zip")
else
    disp(SAIVDR_DIR+" exists.")
end
```

SaivDr-4.2.2.2 exists.

```
ccd = cd(SAIVDR_DIR);
setpath
if isempty(dir("./mexcodes/fcn_*"))
    mybuild
else
    disp("MEX files exist.")
end
```

MEX files exist.

```
cd(ccd)
```

構成パラメータ設定

```
%
% Decimation factor (Strides)
decFactor = [2 2]; % [μv μh]

% Number of channels ( sum(nChannels) >= prod(decFactors) )
nChannels = [3 3]; % [Ps Pa] (Ps=Pa)

% Number of tree levels
nLevels = 3;

% Polyphase Order
```

```

ppOrder = [4 4];
%}

%{
% Decimation factor (Strides)
decFactor = [4 4]; % [μv μh]

% Number of channels ( sum(nChannels) >= prod(decFactors) )
nChannels = [13 13]; % [Ps Pa] (Ps=Pa)

% Number of tree levels
nLevels = 2;

% Polyphase Order
ppOrder = [2 2];
%}

%{
% Decimation factor (Strides)
decFactor = [8 8]; % [μv μh]

% Number of channels ( sum(nChannels) >= prod(decFactors) )
nChannels = [53 53]; % [Ps Pa] (Ps=Pa)

% Number of tree levels
nLevels = 1;

% Polyphase Order
ppOrder = [2 2];
%}

% Redundancy
P = sum(nChannels);
M = prod(decFactor);
redundancyNsolt = ...
    (prod(decFactor)==1)*((P-1)*nLevels+1) + ...
    (prod(decFactor)>1)*((P-1)/(M-1)-(P-M)/((M-1)*M^nLevels))

redundancyNsolt = 1.6641

assert(redundancyNsolt<redundancyRatio)

```

$$L_v \times L_h = \left(\mu_v^\tau + \nu_v \frac{\mu_v^\tau - 1}{\mu_v - 1} \right) \times \left(\mu_h^\tau + \nu_h \frac{\mu_h^\tau - 1}{\mu_h - 1} \right)$$

```

% Filter size [ Ly Lx ]
szFilters = decFactor.^nLevels + ppOrder.*decFactor.*((decFactor.^nLevels-1)./
(decFactor-1)

```

```

szFilters = 1x2
    56      56

% Patch size for training
szPatchTrn = 2.^nextpow2(szFilters+1) % > [ Ly Lx ]

szPatchTrn = 1x2
    64      64

assert(all(szPatchTrn>szFilters))

% Number of patches per image
nSubImgs = floor(nPatches*prod(szBlk./szPatchTrn))

nSubImgs = 240

assert(nSubImgs > 0)

% No DC-leakage
noDcLeakage = true

noDcLeakage = logical
    1

```

辞書の設定

```

if exist("../data/"+nsoltDic+".mat","file")
    S = load("../data/"+nsoltDic);
    analysisnet = S.analysisnet;
    synthesisnet = S.synthesisnet;
    nLevels_ = extractnumlevels(analysisnet);
    decFactor_ = extractdecfactor(analysisnet);
    nChannels_ = extractnumchannels(analysisnet);

    assert(nLevels==nLevels_)
    assert(all(decFactor==decFactor_))
    assert(all(nChannels==nChannels_))
else
    % Number of iterations
    nItersNsolt = 8;

    % Standard deviation of initial angles
    stdInitAng = 1e-1; %pi/6;

    % Mini batch size
    miniBatchSize = 15;

    % Number of Epochs (1 Epoch = nSubImgs/miniBatchSize iterations)
    maxEpochs = 25;

    % Number of iterations

```

```

maxIters = nSubImgs/miniBatchSize * maxEpochs

% Training options
opts = trainingOptions('sgdm', ... % Stochastic gradient descent w/ momentum
    ... 'Momentum', 0.9000, ...
    'InitialLearnRate', 1.0e-03, ...
    ... 'LearnRateScheduleSettings', 'none', ...
    'L2Regularization', 0.0, ... 1.0e-04, ...
    ... 'GradientThresholdMethod', 'l2norm', ...
    ... 'GradientThreshold', Inf, ...
    'MaxEpochs', maxEpochs, ... 30, ...
    'MiniBatchSize', miniBatchSize, ... 128, ...
    'Verbose', 1, ...
    ... 'VerboseFrequency', 50, ...
    ... 'ValidationData', [], ...
    ... 'ValidationFrequency', 50, ...
    ... 'ValidationPatience', Inf, ...
    ... 'Shuffle', 'once', ...
    ... 'CheckpointPath', '', ...
    ... 'ExecutionEnvironment', 'auto', ...
    ... 'WorkerLoad', [], ...
    ... 'OutputFcn', [], ...
    'Plots', 'none', ... 'training-progress', ...
    ... 'SequenceLength', 'longest', ...
    ... 'SequencePaddingValue', 0, ...
    ... 'SequencePaddingDirection', 'right', ...
    ... 'DispatchInBackground', 0, ...
    'ResetInputNormalization', 0); ... 1

```

層構造の構築

```

import saivdr.dcnn.*
analysislgraph = fcn_creatensoltlgraph2d([], ...
    'InputSize', szPatchTrn, ...
    'NumberOfChannels', nChannels, ...
    'DecimationFactor', decFactor, ...
    'PolyPhaseOrder', ppOrder, ...
    'NumberOfLevels', nLevels, ...
    'NumberOfVanishingMoments', noDcLeakage, ...
    'Mode', 'Analyzer');

synthesislgraph = fcn_creatensoltlgraph2d([], ...
    'InputSize', szPatchTrn, ...
    'NumberOfChannels', nChannels, ...
    'DecimationFactor', decFactor, ...
    'PolyPhaseOrder', ppOrder, ...
    'NumberOfLevels', nLevels, ...
    'NumberOfVanishingMoments', noDcLeakage, ...
    'Mode', 'Synthesizer');

figure
subplot(1,2,1)

```

```

plot(analysislgraph)
title('Analysis NSOLT')
subplot(1,2,2)
plot(synthesislgraph)
title('Synthesis NSOLT')

% Construction of deep learning network.
synthesisnet = dlnetwork(synthesislgraph);

% Initialize
nLearnables = height(synthesisnet.Learnables);
for iLearnable = 1:nLearnables
    if synthesisnet.Learnables.Parameter(iLearnable)=="Angles"
        layerName = synthesisnet.Learnables.Layer(iLearnable);
        synthesisnet.Learnables.Value(iLearnable) = ...
            cellfun(@(x) x+stdInitAng*randn(size(x)), ...
            synthesisnet.Learnables.Value(iLearnable), 'UniformOutput',false);
    end
end

% Copy the synthesizer's parameters to the analyzer
synthesislgraph = layerGraph(synthesisnet);
analysislgraph = fcn_cpparamssyn2ana(analysislgraph,synthesislgraph);
analysisnet = dlnetwork(analysislgraph);

```

随伴関係（完全再構成）の確認

NSOLT はパーセバルタイト性を満たす。

```

nOutputs = nLevels+1;
x = rand(szPatchTrn, 'single');
s = cell(1,nOutputs);
dlx = dlarray(x, 'SSCB'); % Deep learning array (SSCB:
Spatial,Spatial,Channel,Batch)
[s{1:nOutputs}] = analysisnet.predict(dlx);
dly = synthesisnet.predict(s{:});
display("MSE: " + num2str(mse(dlx,dly)))

```

要素画像の初期状態

```

import saivdr.dcnn.*
figure
atomicimshow(synthesisnet,[],2^(nLevels-1))
title('Atomic images of initial NSOLT')

```

訓練画像の準備

画像データストアからランダムにパッチを抽出

PCA に合わせて予め零平均化したデータで学習

```

imds = imageDatastore(file_yorg,"ReadFcn",@(x)
meansubtract(im2single(imread(x))));
patchds =
randomPatchExtractionDatastore(imds,imds,szPatchTrn,'PatchesPerImage',nSubImgs);
figure
minibatch = preview(patchds);
responses = minibatch.ResponseImage;
responses = cellfun(@(x) x + 0.5,responses,'UniformOutput',false);
figure
montage(responses,'Size',[2 4]);
drawnow

```

畳み込み辞書学習

問題設定:

$$\{\hat{\theta}, \{\hat{\mathbf{x}}_n\}\} = \arg \min_{\{\theta, \{\mathbf{x}_n\}\}} \frac{1}{2S} \sum_{n=1}^S \|\mathbf{y}_n - \mathbf{D}_{\theta} \hat{\mathbf{x}}_n\|_2^2, \quad \text{s.t. } \forall n, \|\mathbf{x}_n\|_0 \leq K,$$

ただし、 \mathbf{D}_{θ} は設計パラメータベクトル θ をもつ畳み込み辞書.

アルゴリズム:

スパース近似ステップと辞書更新ステップを繰返す.

- スパース近似ステップ

$$\hat{\mathbf{x}}_n = \arg \min_{\mathbf{x}_n} \frac{1}{2} \|\mathbf{y}_n - \hat{\mathbf{D}} \mathbf{x}_n\|_2^2 \quad \text{s.t. } \|\mathbf{x}_n\|_0 \leq K$$

- 辞書更新ステップ

$$\hat{\theta} = \arg \min_{\theta} \frac{1}{2S} \sum_{n=1}^S \|\mathbf{y}_n - \mathbf{D}_{\theta} \hat{\mathbf{x}}_n\|_2^2$$

$$\hat{\mathbf{D}} = \mathbf{D}_{\hat{\theta}}$$

採用するスパース近似と辞書更新の手法:

- スパース近似 : (正規化なし) 繰返しハード閾値処理(IHT)
- 辞書更新 : モーメンタム付き確率的勾配降下法(SGD)

```

% Check if IHT works for dlarray
%x = dlarray(randn(szPatchTrn,'single'), 'SSCB');
%[y,coefs{1:nOutputs}] = iht(x,analysisnet,synthesisnet,sparsityRatio);

```

辞書学習の繰返し計算

```

import saivdr.dcnn.*
%profile on
for iIter = 1:nIterNsolt

    % Sparse approximation (Applied to produce an object of
    TransformedDatastore)
    coefimgds = transform(patchds, @(x)
iht4patchds(x,analysisnet,synthesisnet,sparsityRatio));

    % Synthesis dictionary update
    trainlgraph = synthesislgraph.replaceLayer('Lv1_Out',...
        regressionLayer('Name','Lv1_Out'));
    trainednet = trainNetwork(coefimgds,trainlgraph,opts);

    % Analysis dictionary update (Copy parameters from synthesizer to analyzer)
    trainedlgraph = layerGraph(trainednet);
    analysislgraph = fcn_cpparamssyn2ana(analysislgraph,trainedlgraph);
    analysisnet = dlnetwork(analysislgraph);

    % Check the adjoint relation (perfect reconstruction)
    checkadjointrelation(analysislgraph,trainedlgraph,nLevels,szPatchTrn);

    % Replace layer
    synthesislgraph = trainedlgraph.replaceLayer('Lv1_Out',...
        nsoltIdentityLayer('Name','Lv1_Out'));
    synthesisnet = dlnetwork(synthesislgraph);

end
%profile off
%profile viewer

```

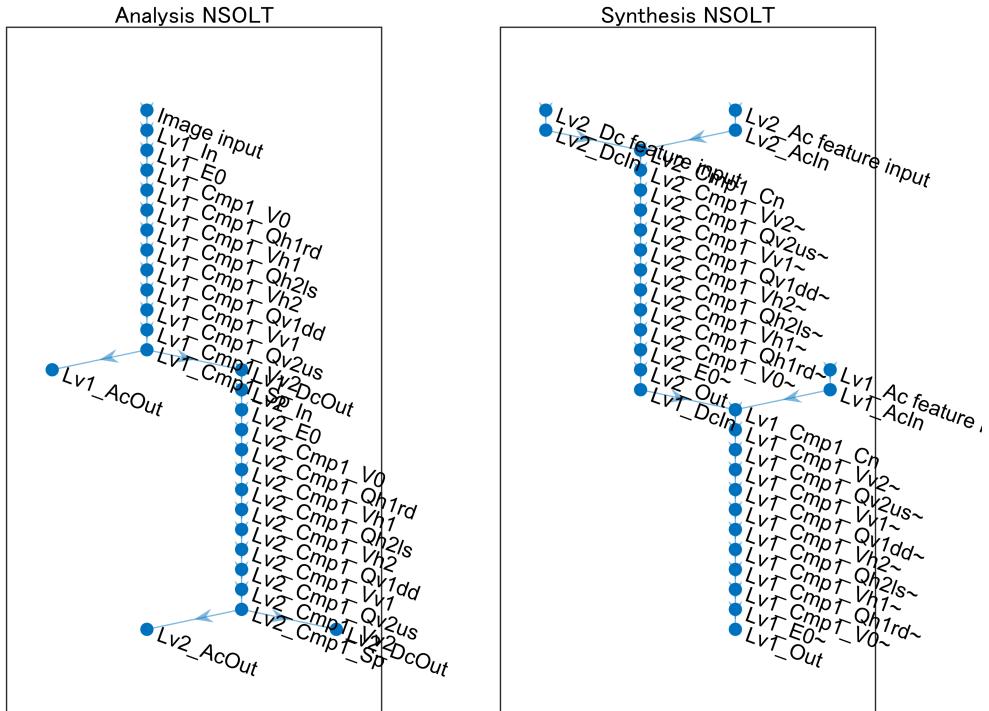
訓練ネットワークの保存

```

import saivdr.dcnn.*
synthesislgraph = layerGraph(synthesisnet);
analysislgraph = fcn_cpparamssyn2ana(analysislgraph,synthesislgraph);
analysisnet = dlnetwork(analysislgraph);
save(sprintf('../results/
nsoltdictionary_%s',datetime('now','Format','yyyyMMddHHmmssSSS')), 'analysisnet', 'syn
thesisnet', 'nLevels')
end

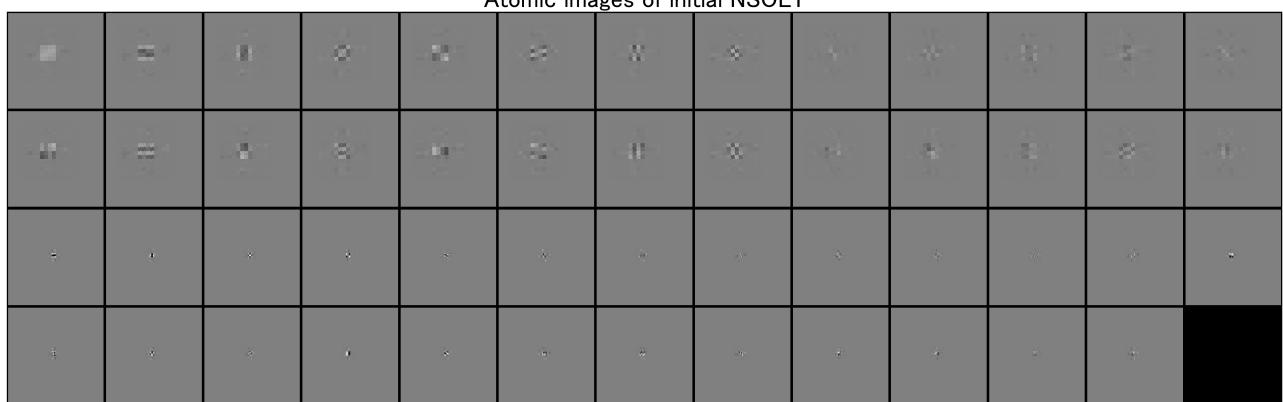
maxIters = 400

```



Copy angles from Lv1_Cmp1_V0~ to Lv1_Cmp1_V0
 Copy angles from Lv1_Cmp1_Vh1~ to Lv1_Cmp1_Vh1
 Copy angles from Lv1_Cmp1_Vh2~ to Lv1_Cmp1_Vh2
 Copy angles from Lv1_Cmp1_Vv1~ to Lv1_Cmp1_Vv1
 Copy angles from Lv1_Cmp1_Vv2~ to Lv1_Cmp1_Vv2
 Copy angles from Lv2_Cmp1_V0~ to Lv2_Cmp1_V0
 Copy angles from Lv2_Cmp1_Vh1~ to Lv2_Cmp1_Vh1
 Copy angles from Lv2_Cmp1_Vh2~ to Lv2_Cmp1_Vh2
 Copy angles from Lv2_Cmp1_Vv1~ to Lv2_Cmp1_Vv1
 Copy angles from Lv2_Cmp1_Vv2~ to Lv2_Cmp1_Vv2
 "MSE: 6.2082e-11"

Atomic images of initial NSOLT





単一の GPU で学習中。

エポック	反復	経過時間 (h h : mm : ss)	ミニバッチ RMSE	ミニバッチ損失	基本学習率
1	1	00 : 00 : 13	3. 54	6. 3	0. 000000e+00
4	50	00 : 05 : 41	3. 07	4. 7	0. 000000e+00
7	100	00 : 11 : 15	3. 13	4. 9	0. 000000e+00
10	150	00 : 16 : 43	3. 30	5. 4	0. 000000e+00
13	200	00 : 22 : 42	3. 14	4. 9	0. 000000e+00
16	250	00 : 28 : 33	2. 93	4. 3	0. 000000e+00
19	300	00 : 34 : 49	3. 10	4. 8	0. 000000e+00
22	350	00 : 41 : 13	2. 86	4. 1	0. 000000e+00
25	400	00 : 47 : 20	2. 94	4. 3	0. 000000e+00

学習終了：最大数のエポックが完了しました。

```

Copy angles from Lv1_Cmp1_V0~ to Lv1_Cmp1_V0
Copy angles from Lv1_Cmp1_Vh1~ to Lv1_Cmp1_Vh1
Copy angles from Lv1_Cmp1_Vh2~ to Lv1_Cmp1_Vh2
Copy angles from Lv1_Cmp1_Vv1~ to Lv1_Cmp1_Vv1
Copy angles from Lv1_Cmp1_Vv2~ to Lv1_Cmp1_Vv2
Copy angles from Lv2_Cmp1_V0~ to Lv2_Cmp1_V0
Copy angles from Lv2_Cmp1_Vh1~ to Lv2_Cmp1_Vh1
Copy angles from Lv2_Cmp1_Vh2~ to Lv2_Cmp1_Vh2
Copy angles from Lv2_Cmp1_Vv1~ to Lv2_Cmp1_Vv1
Copy angles from Lv2_Cmp1_Vv2~ to Lv2_Cmp1_Vv2
    "MSE: 3.324e-14"

```

単一の GPU で学習中。

エポック	反復	経過時間 (h h : mm : ss)	ミニバッチ RMSE	ミニバッチ損失	基本学習率
1	1	00 : 00 : 14	2. 97	4. 4	0. 000000e+00
4	50	00 : 06 : 22	2. 81	4. 0	0. 000000e+00
7	100	00 : 11 : 52	2. 63	3. 5	0. 000000e+00
10	150	00 : 17 : 25	2. 70	3. 7	0. 000000e+00
13	200	00 : 23 : 01	2. 95	4. 4	0. 000000e+00
16	250	00 : 28 : 33	2. 55	3. 3	0. 000000e+00
19	300	00 : 34 : 09	2. 81	3. 9	0. 000000e+00

2 2	3 5 0	0 0 : 3 9 : 4 4	2. 7 0	3. 6	0. 0
2 5	4 0 0	0 0 : 4 5 : 1 6	2. 8 5	4. 1	0. 0

学習終了: 最大数のエポックが完了しました。

```
Copy angles from Lv1_Cmp1_V0~ to Lv1_Cmp1_V0
Copy angles from Lv1_Cmp1_Vh1~ to Lv1_Cmp1_Vh1
Copy angles from Lv1_Cmp1_Vh2~ to Lv1_Cmp1_Vh2
Copy angles from Lv1_Cmp1_Vv1~ to Lv1_Cmp1_Vv1
Copy angles from Lv1_Cmp1_Vv2~ to Lv1_Cmp1_Vv2
Copy angles from Lv2_Cmp1_V0~ to Lv2_Cmp1_V0
Copy angles from Lv2_Cmp1_Vh1~ to Lv2_Cmp1_Vh1
Copy angles from Lv2_Cmp1_Vh2~ to Lv2_Cmp1_Vh2
Copy angles from Lv2_Cmp1_Vv1~ to Lv2_Cmp1_Vv1
Copy angles from Lv2_Cmp1_Vv2~ to Lv2_Cmp1_Vv2
    "MSE: 3.8698e-14"
```

単一の GPU で学習中。

エポック	反復	経過時間 (h h : mm : ss)	ミニバッチ RMSE	ミニバッチ損失	基本学
1	1	0 0 : 0 0 : 1 3	2. 7 4	3. 8	0. 0
4	5 0	0 0 : 0 5 : 4 1	2. 4 6	3. 0	0. 0
7	1 0 0	0 0 : 1 1 : 5 1	2. 5 9	3. 3	0. 0
10	1 5 0	0 0 : 1 7 : 5 3	2. 4 4	3. 0	0. 0
13	2 0 0	0 0 : 2 3 : 5 5	2. 8 0	3. 9	0. 0
16	2 5 0	0 0 : 3 0 : 0 3	2. 4 5	3. 0	0. 0
19	3 0 0	0 0 : 3 6 : 0 5	2. 6 6	3. 5	0. 0
22	3 5 0	0 0 : 4 2 : 0 9	2. 5 3	3. 2	0. 0
25	4 0 0	0 0 : 4 8 : 1 3	2. 6 7	3. 6	0. 0

学習終了: 最大数のエポックが完了しました。

```
Copy angles from Lv1_Cmp1_V0~ to Lv1_Cmp1_V0
Copy angles from Lv1_Cmp1_Vh1~ to Lv1_Cmp1_Vh1
Copy angles from Lv1_Cmp1_Vh2~ to Lv1_Cmp1_Vh2
Copy angles from Lv1_Cmp1_Vv1~ to Lv1_Cmp1_Vv1
Copy angles from Lv1_Cmp1_Vv2~ to Lv1_Cmp1_Vv2
Copy angles from Lv2_Cmp1_V0~ to Lv2_Cmp1_V0
Copy angles from Lv2_Cmp1_Vh1~ to Lv2_Cmp1_Vh1
Copy angles from Lv2_Cmp1_Vh2~ to Lv2_Cmp1_Vh2
Copy angles from Lv2_Cmp1_Vv1~ to Lv2_Cmp1_Vv1
Copy angles from Lv2_Cmp1_Vv2~ to Lv2_Cmp1_Vv2
    "MSE: 2.7927e-14"
```

単一の GPU で学習中。

エポック	反復	経過時間 (h h : mm : ss)	ミニバッチ RMSE	ミニバッチ損失	基本学
1	1	0 0 : 0 0 : 1 1	2. 5 7	3. 3	0. 0
4	5 0	0 0 : 0 5 : 5 4	2. 8 5	4. 1	0. 0
7	1 0 0	0 0 : 1 1 : 5 9	2. 5 5	3. 3	0. 0
10	1 5 0	0 0 : 1 8 : 0 4	2. 7 6	3. 8	0. 0
13	2 0 0	0 0 : 2 4 : 1 7	2. 4 5	3. 0	0. 0
16	2 5 0	0 0 : 3 0 : 2 0	2. 7 0	3. 6	0. 0
19	3 0 0	0 0 : 3 6 : 2 3	2. 2 0	2. 4	0. 0
22	3 5 0	0 0 : 4 2 : 2 5	2. 9 4	4. 3	0. 0
25	4 0 0	0 0 : 4 8 : 2 7	2. 7 5	3. 8	0. 0

学習終了: 最大数のエポックが完了しました。

```
Copy angles from Lv1_Cmp1_V0~ to Lv1_Cmp1_V0
Copy angles from Lv1_Cmp1_Vh1~ to Lv1_Cmp1_Vh1
Copy angles from Lv1_Cmp1_Vh2~ to Lv1_Cmp1_Vh2
Copy angles from Lv1_Cmp1_Vv1~ to Lv1_Cmp1_Vv1
Copy angles from Lv1_Cmp1_Vv2~ to Lv1_Cmp1_Vv2
Copy angles from Lv2_Cmp1_V0~ to Lv2_Cmp1_V0
```

Copy angles from Lv2_Cmp1_Vh1~ to Lv2_Cmp1_Vh1
 Copy angles from Lv2_Cmp1_Vh2~ to Lv2_Cmp1_Vh2
 Copy angles from Lv2_Cmp1_Vv1~ to Lv2_Cmp1_Vv1
 Copy angles from Lv2_Cmp1_Vv2~ to Lv2_Cmp1_Vv2
 "MSE: 2.549e-14"

単一の GPU で学習中。

エポック	反復	経過時間 (h h : mm : s s)	ミニバッチ	R M S E	ミニバッチ損失	基本学
1	1	00 : 00 : 13		2. 4 4	3. 0	0. 0
4	5 0	00 : 06 : 19		2. 6 1	3. 4	0. 0
7	1 0 0	00 : 12 : 13		2. 4 3	3. 0	0. 0
10	1 5 0	00 : 18 : 00		2. 5 3	3. 2	0. 0
13	2 0 0	00 : 24 : 13		2. 8 2	4. 0	0. 0
16	2 5 0	00 : 30 : 16		2. 4 5	3. 0	0. 0
19	3 0 0	00 : 36 : 07		2. 4 8	3. 1	0. 0
22	3 5 0	00 : 42 : 14		2. 5 9	3. 4	0. 0
25	4 0 0	00 : 47 : 48		2. 6 1	3. 4	0. 0

学習終了: 最大数のエポックが完了しました。

Copy angles from Lv1_Cmp1_V0~ to Lv1_Cmp1_V0
 Copy angles from Lv1_Cmp1_Vh1~ to Lv1_Cmp1_Vh1
 Copy angles from Lv1_Cmp1_Vh2~ to Lv1_Cmp1_Vh2
 Copy angles from Lv1_Cmp1_Vv1~ to Lv1_Cmp1_Vv1
 Copy angles from Lv1_Cmp1_Vv2~ to Lv1_Cmp1_Vv2
 Copy angles from Lv2_Cmp1_V0~ to Lv2_Cmp1_V0
 Copy angles from Lv2_Cmp1_Vh1~ to Lv2_Cmp1_Vh1
 Copy angles from Lv2_Cmp1_Vh2~ to Lv2_Cmp1_Vh2
 Copy angles from Lv2_Cmp1_Vv1~ to Lv2_Cmp1_Vv1
 Copy angles from Lv2_Cmp1_Vv2~ to Lv2_Cmp1_Vv2
 "MSE: 2.4454e-14"

単一の GPU で学習中。

エポック	反復	経過時間 (h h : mm : s s)	ミニバッチ	R M S E	ミニバッチ損失	基本学
1	1	00 : 00 : 11		2. 4 5	3. 0	0. 0
4	5 0	00 : 05 : 53		2. 6 3	3. 4	0. 0
7	1 0 0	00 : 11 : 44		2. 3 8	2. 8	0. 0
10	1 5 0	00 : 17 : 28		2. 2 8	2. 6	0. 0
13	2 0 0	00 : 23 : 05		2. 6 7	3. 6	0. 0
16	2 5 0	00 : 28 : 48		2. 3 8	2. 8	0. 0
19	3 0 0	00 : 34 : 52		2. 4 1	2. 9	0. 0
22	3 5 0	00 : 40 : 30		2. 5 9	3. 4	0. 0
25	4 0 0	00 : 46 : 25		2. 5 5	3. 3	0. 0

学習終了: 最大数のエポックが完了しました。

Copy angles from Lv1_Cmp1_V0~ to Lv1_Cmp1_V0
 Copy angles from Lv1_Cmp1_Vh1~ to Lv1_Cmp1_Vh1
 Copy angles from Lv1_Cmp1_Vh2~ to Lv1_Cmp1_Vh2
 Copy angles from Lv1_Cmp1_Vv1~ to Lv1_Cmp1_Vv1
 Copy angles from Lv1_Cmp1_Vv2~ to Lv1_Cmp1_Vv2
 Copy angles from Lv2_Cmp1_V0~ to Lv2_Cmp1_V0
 Copy angles from Lv2_Cmp1_Vh1~ to Lv2_Cmp1_Vh1
 Copy angles from Lv2_Cmp1_Vh2~ to Lv2_Cmp1_Vh2
 Copy angles from Lv2_Cmp1_Vv1~ to Lv2_Cmp1_Vv1
 Copy angles from Lv2_Cmp1_Vv2~ to Lv2_Cmp1_Vv2
 "MSE: 2.8741e-14"

単一の GPU で学習中。

エポック	反復	経過時間 (h h : mm : s s)	ミニバッチ	R M S E	ミニバッチ損失	基本学
1	1	00 : 00 : 11		2. 4 5	3. 0	0. 0

1	1	00 : 00 : 11	2. 40	2. 9	0. 0
4	50	00 : 06 : 02	2. 33	2. 7	0. 0
7	100	00 : 12 : 15	2. 50	3. 1	0. 0
10	150	00 : 18 : 20	2. 65	3. 5	0. 0
13	200	00 : 24 : 30	2. 45	3. 0	0. 0
16	250	00 : 30 : 39	2. 57	3. 3	0. 0
19	300	00 : 36 : 39	2. 30	2. 6	0. 0
22	350	00 : 42 : 48	2. 43	3. 0	0. 0
25	400	00 : 48 : 43	2. 39	2. 9	0. 0

学習終了: 最大数のエポックが完了しました。

Copy angles from Lv1_Cmp1_V0~ to Lv1_Cmp1_V0
 Copy angles from Lv1_Cmp1_Vh1~ to Lv1_Cmp1_Vh1
 Copy angles from Lv1_Cmp1_Vh2~ to Lv1_Cmp1_Vh2
 Copy angles from Lv1_Cmp1_Vv1~ to Lv1_Cmp1_Vv1
 Copy angles from Lv1_Cmp1_Vv2~ to Lv1_Cmp1_Vv2
 Copy angles from Lv2_Cmp1_V0~ to Lv2_Cmp1_V0
 Copy angles from Lv2_Cmp1_Vh1~ to Lv2_Cmp1_Vh1
 Copy angles from Lv2_Cmp1_Vh2~ to Lv2_Cmp1_Vh2
 Copy angles from Lv2_Cmp1_Vv1~ to Lv2_Cmp1_Vv1
 Copy angles from Lv2_Cmp1_Vv2~ to Lv2_Cmp1_Vv2

"MSE: 2.5514e-14"

単一の GPU で学習中。

エポック	反復	経過時間 (h h : mm : ss)	ミニバッチ RMSE	ミニバッチ損失	基本学
1	1	00 : 00 : 13	2. 37	2. 8	0. 0
4	50	00 : 06 : 13	2. 59	3. 4	0. 0
7	100	00 : 12 : 21	2. 35	2. 8	0. 0
10	150	00 : 18 : 26	2. 48	3. 1	0. 0
13	200	00 : 24 : 33	2. 42	2. 9	0. 0
16	250	00 : 30 : 38	2. 20	2. 4	0. 0
19	300	00 : 36 : 41	2. 39	2. 8	0. 0
22	350	00 : 42 : 52	2. 08	2. 2	0. 0
25	400	00 : 49 : 03	2. 62	3. 4	0. 0

学習終了: 最大数のエポックが完了しました。

Copy angles from Lv1_Cmp1_V0~ to Lv1_Cmp1_V0
 Copy angles from Lv1_Cmp1_Vh1~ to Lv1_Cmp1_Vh1
 Copy angles from Lv1_Cmp1_Vh2~ to Lv1_Cmp1_Vh2
 Copy angles from Lv1_Cmp1_Vv1~ to Lv1_Cmp1_Vv1
 Copy angles from Lv1_Cmp1_Vv2~ to Lv1_Cmp1_Vv2
 Copy angles from Lv2_Cmp1_V0~ to Lv2_Cmp1_V0
 Copy angles from Lv2_Cmp1_Vh1~ to Lv2_Cmp1_Vh1
 Copy angles from Lv2_Cmp1_Vh2~ to Lv2_Cmp1_Vh2
 Copy angles from Lv2_Cmp1_Vv1~ to Lv2_Cmp1_Vv1
 Copy angles from Lv2_Cmp1_Vv2~ to Lv2_Cmp1_Vv2

"MSE: 2.5752e-14"

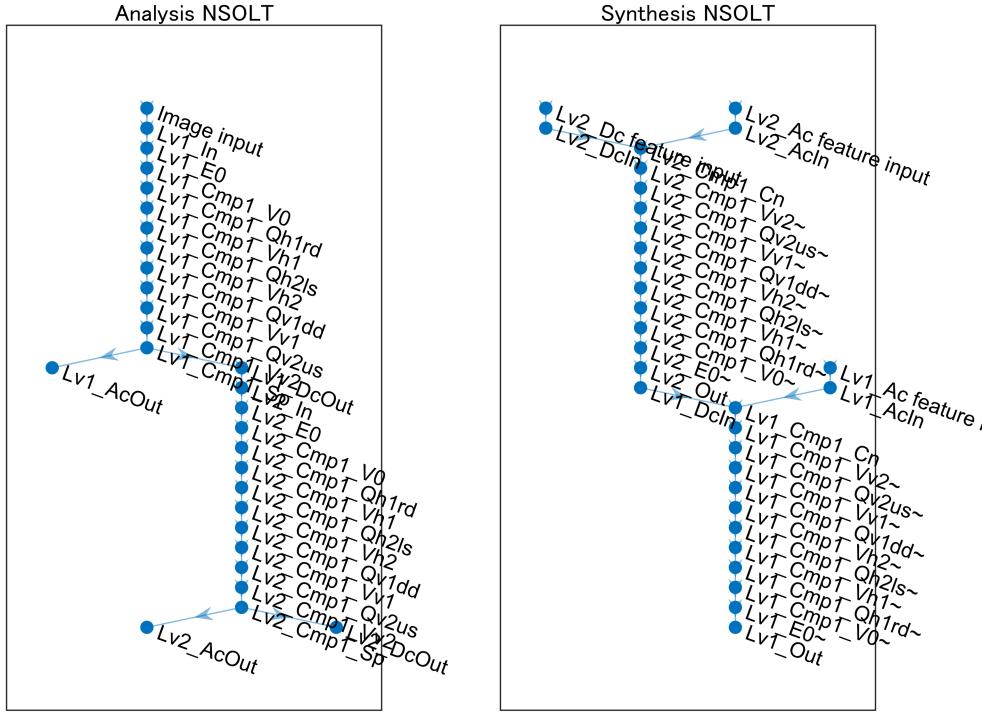
Copy angles from Lv1_Cmp1_V0~ to Lv1_Cmp1_V0
 Copy angles from Lv1_Cmp1_Vh1~ to Lv1_Cmp1_Vh1
 Copy angles from Lv1_Cmp1_Vh2~ to Lv1_Cmp1_Vh2
 Copy angles from Lv1_Cmp1_Vv1~ to Lv1_Cmp1_Vv1
 Copy angles from Lv1_Cmp1_Vv2~ to Lv1_Cmp1_Vv2
 Copy angles from Lv2_Cmp1_V0~ to Lv2_Cmp1_V0
 Copy angles from Lv2_Cmp1_Vh1~ to Lv2_Cmp1_Vh1
 Copy angles from Lv2_Cmp1_Vh2~ to Lv2_Cmp1_Vh2
 Copy angles from Lv2_Cmp1_Vv1~ to Lv2_Cmp1_Vv1
 Copy angles from Lv2_Cmp1_Vv2~ to Lv2_Cmp1_Vv2

```
analysislgraph = layerGraph(analysisnet);
synthesislgraph = layerGraph(synthesisnet);
```

```

figure
subplot(1,2,1)
plot(analysislgraph)
title('Analysis NSOLT')
subplot(1,2,2)
plot(synthesislgraph)
title('Synthesis NSOLT')

```



要素画像の表示

```

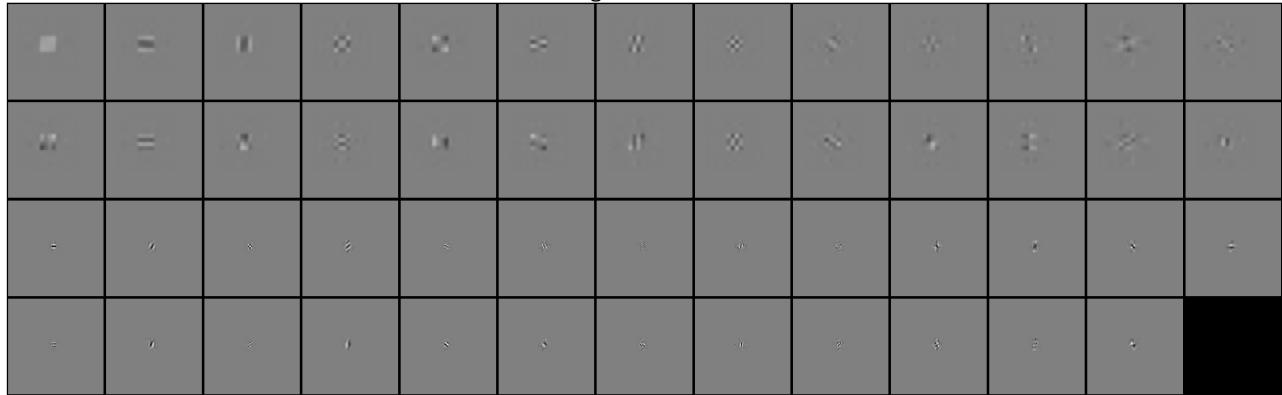
import saivdr.dcnn.*

figure

atomicimshow(synthesisnet,[],2^(nLevels-1))
title('Atomic images of trained NSOLT')

```

Atomic images of trained NSOLT



推論用 NSOLT ネットワークの構築

```
% Assemble analyzer
analysisgraph4predict = analysisgraph;
analysisgraph4predict = analysisgraph4predict.replaceLayer('Image input',...
    imageInputLayer(szOrg,'Name','Image input','Normalization','none'));
for iLayer = 1:height(analysisgraph4predict.Layers)
    layer = analysisgraph4predict.Layers(iLayer);
    if contains(layer.Name,"Lv"+nLevels+"_DcOut") || ...
        ~isempty(regexp(layer.Name,'^Lv\d+_AcOut','once'))
        analysisgraph4predict = analysisgraph4predict.replaceLayer(layer.Name,...
            regressionLayer('Name',layer.Name));
    end
end
analysisnet4predict = assembleNetwork(analysisgraph4predict);

% Assemble synthesizer
synthesisgraph4predict = synthesisgraph;
synthesisgraph4predict = synthesisgraph4predict.replaceLayer('Lv1_Out',...
    regressionLayer('Name','Lv1_Out'));
for iLayer = 1:height(synthesisgraph4predict.Layers)
    layer = synthesisgraph4predict.Layers(iLayer);
    if contains(layer.Name,'Ac feature input')
        iLv = str2double(layer.Name(3));
        sbSize = szOrg.* (decFactor.^(-iLv));
        newlayer = ...
            imageInputLayer([sbSize
        (sum(nChannels)-1)],'Name',layer.Name,'Normalization','none');
        synthesisgraph4predict = synthesisgraph4predict.replaceLayer(... 
            layer.Name,newlayer);
    elseif contains(layer.Name,sprintf('Lv%0d_Dc feature input',nLevels))
        iLv = str2double(layer.Name(3));
        sbSize = szOrg.* (decFactor.^(-iLv));
        newlayer = ...
            imageInputLayer([sbSize 1],'Name',layer.Name,'Normalization','none');
    end
end
```

```

    synthesislgraph4predict = synthesislgraph4predict.replaceLayer(...  

        layer.Name,newlayer);  

    end  

end  

synthesisnet4predict = assembleNetwork(synthesislgraph4predict);

```

随伴関係（完全再構成）の確認

NSOLT はパーセバルタイト性を満たす。

```

u = rand(szOrg,'single');  

[s{1:nLevels+1}] = analysisnet4predict.predict(u);  

v = synthesisnet4predict.predict(s{1:nLevels+1});  

assert(mse(u,v)<1e-9)

```

NSOLT による合成処理とその随伴処理の定義

```

nsoltconfig.nLevels = nLevels;  

szCoefs = zeros(nLevels+1,3);  

for iLevel = 1:nLevels+1  

    s_iLevel = s{iLevel};  

    szCoefs(iLevel,1) = size(s_iLevel,1);  

    szCoefs(iLevel,2) = size(s_iLevel,2);  

    szCoefs(iLevel,3) = size(s_iLevel,3);  

end  

nsoltconfig.szCoefs = szCoefs;  

syn_nsolt = @(x) synthesisnsolt(x,synthesisnet4predict,nsoltconfig);  

adj_nsolt = @(y) analysisnsolt(y,analysisnet4predict,nsoltconfig);

```

随伴関係の確認

```

x = adj_nsolt(y);  

v = randn(size(x));  

u = syn_nsolt(v);  

assert(abs(dot(y(:),u(:))-dot(x(:),v(:)))<1e-3)

```

繰返しハード閾値処理(IHT)によるスペース近似の比較

辞書の準備

```

blkdctwon = { syn_blkdct, adj_blkdct, "Block DCTwoN", false };  

blkdct = { syn_blkdct, adj_blkdct, "Block DCT", true };  

blkpcawon = { syn_blkpca, adj_blkpca, "Block PCAwoN", false };  

blkpca = { syn_blkpca, adj_blkpca, "Block PCA", true };  

blkrica = { syn_blkrica, adj_blkrica, "Block RICA", true };  

blkksvd = { syn_blkksvd, adj_blkksvd, "Block K-SVD", true };  

nsoltwon = { syn_nsolt, adj_nsolt, "NSOLTwon", false };  

nsolt = { syn_nsolt, adj_nsolt, "NSOLT", true };  

dicset = { blkdctwon, blkdct, blkpcawon, blkpca, blkrica, blkksvd, nsoltwon,  

nsolt };  

nDics = length(dicset);

```

IHT

$$\mathbf{x}^{(t+1)} \leftarrow \mathcal{H}_{BK}\left(\mathbf{x}^{(t)} + \mu^{(t)} \widehat{\mathbf{D}}^\top (\mathbf{y} - \widehat{\mathbf{D}}\mathbf{x}^{(t)})\right)$$

$$t \leftarrow t + 1$$

- T. Blumensath and M. E. Davies, "Normalized Iterative Hard Thresholding: Guaranteed Stability and Performance," in IEEE Journal of Selected Topics in Signal Processing, vol. 4, no. 2, pp. 298-309, April 2010, doi: 10.1109/JSTSP.2010.2042411.

```
nItersIht = 2000;

% 平均値を引いた画像を用意（近似後に平均値を加算）
ymean = mean(yorg,"all");
y = yorg - ymean;
% 準備
c = 1e-3;
kappa = 1.1/(1-c);
nCoefs = floor(sparsityRatio*prod(szOrg));
psnrs = zeros(nItersIht,nDics);
ssims = zeros(nItersIht,nDics);
yaprxs = cell(1,nDics);
% 繰り返し処理
for iDic = 1:nDics
    dic_ = dicset{iDic};
    synproc = dic_{1};
    adjproc = dic_{2};
    dicname = dic_{3};
    isStepSizeNormalized = dic_{4};
    % IHT
    display(dicname)
    s = adjproc(y); % D^Ty
    xt = zeros(size(s),'like',s); % x1 = 0;
    if isStepSizeNormalized % 正規化あり
        suppt = find(hardthresh(s,nCoefs)); % Γ1 = supp(H_K(D^Ty))
        maskt = (abs(s)~=0);
    end
    for iIter=1:nItersIht
        % Gradient descent
        gt = adjproc(y-synproc(xt)); % g = D^T(y-Dx)
        if ~isStepSizeNormalized % 正規化なし
            mu = (1-c);
            xtp1 = hardthresh(xt+mu*gt,nCoefs);
        else % 正規化あり
            ggt = gt(suppt); % g_Γn
            ugt = synproc(maskt.*gt); % D_Γn^T g_Γn
            mu = (ggt.*ugt)/(ugt(:).*ugt(:));
            ttp1 = hardthresh(xt+mu*gt,nCoefs); % ~xn+1 = H_K(xn + μn gn)
            supptp1 = find(tp1); % Γn+1 = supp(~xn+1)
```

```

        if length(supptp1)==length(suppt) && all(supptp1==suppt)
            xtp1 = ttp1; % xn+1 = ~xn+1
        else
            dxt = ttp1-xt; % ~xn+1 - xn
            omega = (1-c)*(norm(dxt,'fro'))/norm(synproc(dxt),'fro'))^2;
            if mu <= omega
                xtp1 = ttp1; % xn+1 = ~xn+1
            else
                while mu > omega
                    mu = mu/(kappa*(1-c));
                    ttp1 = hardthresh(xt+mu*gt,nCoefs); % ~xn+1 = H_K(xn + μn
                gn)
                    dxt = ttp1-xt; % ~xn+1 - xn
                    omega = (1-c)*(norm(dxt,'fro'))/norm(synproc(dxt),'fro'))^2;
                end
                supptp1 = find(ttp1); % Γn+1 = supp(~xn+1)
                xtp1 = ttp1; % xn+1 = ~xn+1
            end
        end
        % Update
        suppt = supptp1;
        maskt = zeros(size(maskt), 'like', maskt);
        maskt(suppt) = 1;
    end
    xt = xtp1;
    % Monitoring
    checkSparsity = nnz(xt)/prod(szOrg)<=sparsityRatio;
    assert(checkSparsity)
    yaprx_ = synproc(xt);
    psnr_ = psnr(cast(yaprx_, 'like', y), y);
    ssim_ = ssim(cast(yaprx_, 'like', y), y);
    psnrs(iIter, iDic) = psnr_;
    ssims(iIter, iDic) = ssim_;
    %fprintf("IHT(%d) PSNR: %6.4f\n", iIter, psnr_);
end
yaprxs{iDic} = yaprx_ + ymean;
end

```

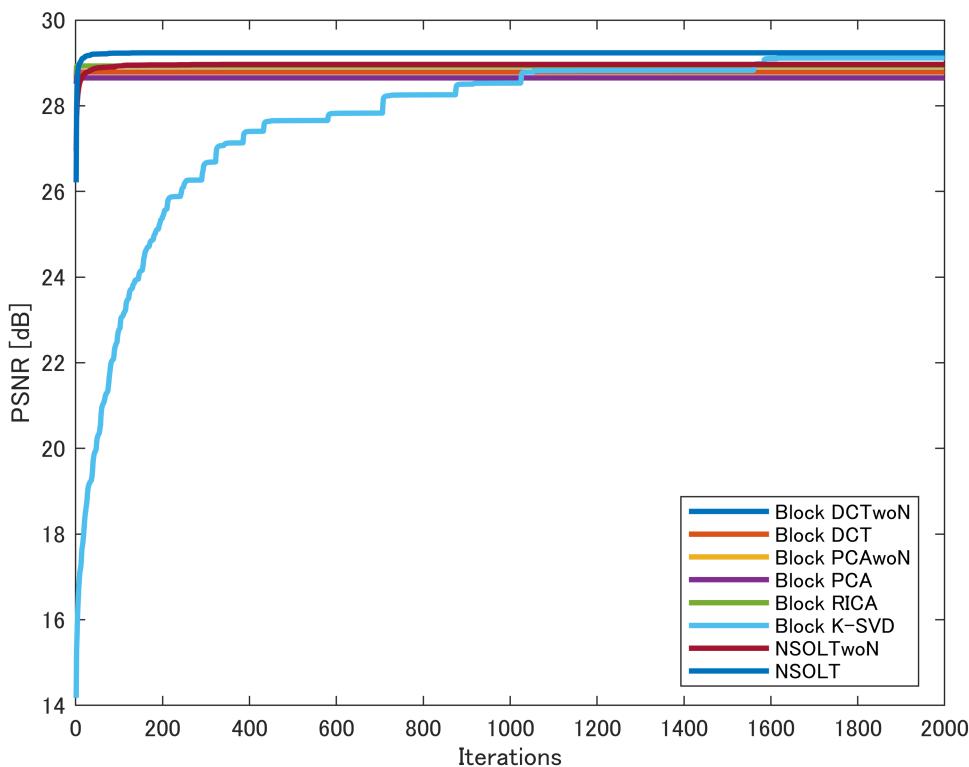
```

dicname =
"Block DCTwoN"
dicname =
"Block DCT"
dicname =
"Block PCAwoN"
dicname =
"Block PCA"
dicname =
"Block RICA"
dicname =
"Block K-SVD"
dicname =
"NSOLTtwoN"
dicname =
"NSOLT"

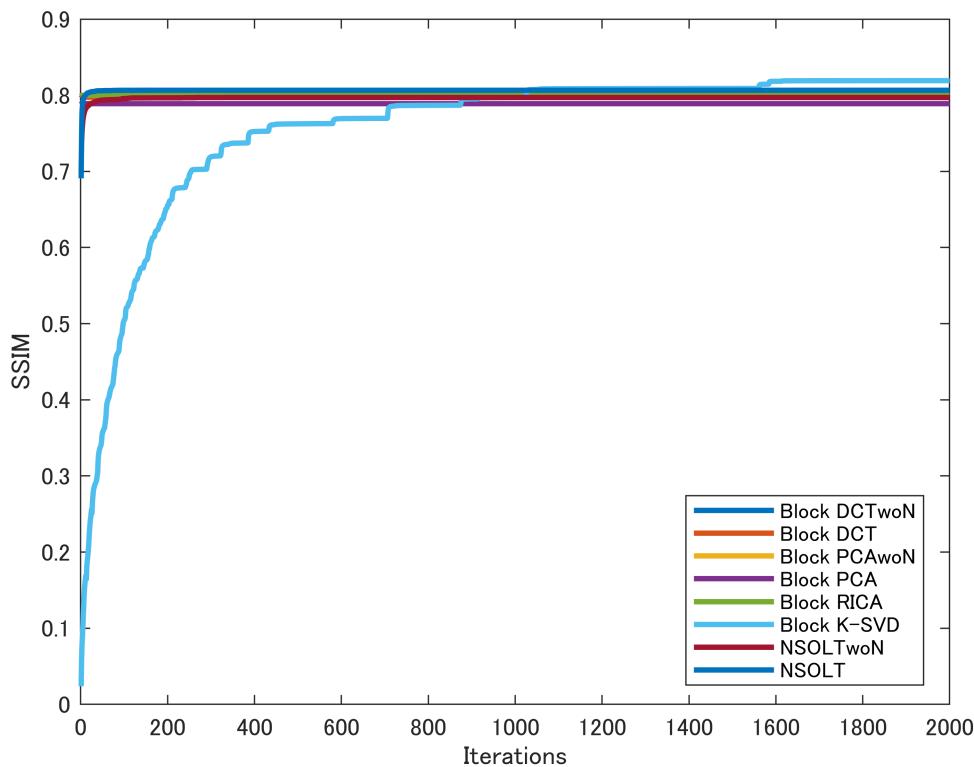
```

近似結果の表示

```
dicnames =  
[blkdctwon{3},blkdct{3},blkpcawon{3},blkpca{3},blkrica{3},blkksvd{3},nsoltwon{3},nsol  
lt{3}];  
psnrtbl = array2table(psnrs,'VariableNames',dicnames);  
psnrtbl = horzcat(table((1:nIterSIht).','VariableNames','Iterations'),psnrtbl);  
ssimtbl = array2table(ssims,'VariableNames',dicnames);  
ssimtbl = horzcat(table((1:nIterSIht).','VariableNames','Iterations'),ssimtbl);  
  
% PSNR のグラフ  
figure  
plot(psnrtbl,"Iterations",dicnames,'LineWidth',2)  
ylabel('PSNR [dB]')  
legend('Location','best')
```



```
% SSIM のグラフ  
figure  
plot(ssimtbl,"Iterations",dicnames,'LineWidth',2)  
ylabel('SSIM')  
legend('Location','best')
```



```
% 原画像の表示
figure
tiledlayout(2,ceil((nDics+1)/2))
nexttile
imshow(yorg)
title("Original image")
% 近似画像の表示
for idx = 1:nDics
    yaprx = yaprxs{idx};
    dicname = dicnames(idx)
    file_yaprx = "./results/yaprx_" + replace(lower(dicname), ' ', '_') + ".png";
    imwrite(yaprx,file_yaprx)
    %
    nexttile
    imshow(yaprxs{idx})
    title(dicname+" "+num2str(psnrs(end,idx))+" dB")
end

dicname =
"Block DCTwoN"
dicname =
"Block DCT"
dicname =
"Block PCAwoN"
dicname =
"Block PCA"
dicname =
"Block RICA"
dicname =
"Block K-SVD"
dicname =
"NSOLTwoN"
dicname =
"NSOLT"
```

```

"Block PCAwoN"
dicname =
"Block PCA"
dicname =
"Block RICA"
dicname =
"Block K-SVD"
dicname =
"NSOLTtwoN"
dicname =
"NSOLT"

```

Original image Block DCTwoN 28.787 dB Block DCT 28.787 dB Block PCAwoN 28.650 dB Block PCA 28.6508 dB



Block RICA 28.928 dB Block K-SVD 29.117 dB NSOLTtwoN 28.9648 dB NSOLT 29.2361 dB



【関数定義】

NSOLT 合成処理関数

```

function y = synthesisnsolt(x,synthesisnet4predict,config)
nLevels = config.nLevels;
szCoefs = config.szCoefs;
s = cell(1,nLevels+1);
sidx = 1;
for iLevel = 1:nLevels+1
    sz_iLevel = szCoefs(iLevel,:);
    eidx = sidx+prod(sz_iLevel)-1;
    x_iLevel = x(sidx:eidx);
    s{iLevel} = reshape(x_iLevel,sz_iLevel);
    sidx = eidx+1;
end
y = synthesisnet4predict.predict(s{1:nLevels+1});

```

```
end
```

NSOLT 分析処理関数

```
function x = analysisnsolt(y,analysisnet4predict,config)
nLevels = config.nLevels;
szCoefs = config.szCoefs;
[s{1:nLevels+1}] = analysisnet4predict.predict(y);
nCoefs = sum(prod(szCoefs,2),1);
%  
x = [];
x = zeros(nCoefs,1);
sidx = 1;
for iLevel = 1:nLevels+1
    %  
x = [x; s{iLevel}(:)];
    eidx = sidx - 1 + prod(szCoefs(iLevel,:));
    x(sidx:eidx) = s{iLevel}(:);
    sidx = eidx + 1;
end
end
```

ハード閾値処理

```
function y = hardthresh(x,K)
v = sort(abs(x(:)), 'descend');
thk = v(K);
y = (abs(x)>thk).*x;
end
```

深層学習配列に対する繰返しハード閾値処理(IHT)のバッチ処理

```
function newdata = iht4patchds(olddb,analyzer,synthesizer,sparsityRatio)
% IHT for InputImage in randomPatchExtractionDatastore
%
nInputs = length(synthesizer.InputNames);

% Apply IHT process for every input patch
restbl = removevars(olddb,'InputImage');
dlv = dlarray(cat(4,olddb.InputImage{:}), 'SSCB');
[~,dlcoefs{1:nInputs}] = iht4dlarray(dlv,analyzer,synthesizer,sparsityRatio);
coefs = cellfun(@(x) permute(num2cell(extractdata(x),1:3),[4 1 2
3]),dlcoefs,'UniformOutput',false);
%
nImgs = length(olddb.InputImage);
coefarray = cell(nImgs,nInputs);
for iImg = 1:nImgs
    for iInput = 1:nInputs
        coefarray{iImg,iInput} = coefs{iInput}{iImg};
    end
end
% Output as a cell in order to make multiple-input datastore
```

```

newdata = [ coefarray table2cell(restbl) ];
end

```

深層学習配列に対する繰返しハード閾値処理(IHT)

```

function [dly,varargout] = iht4dlarray(dlX,analyzer,synthesizer,sparsityRatio)
% IHT Iterative hard thresholding
%
nInputs = length(synthesizer.InputNames);
szBatch = size(dlX,4);

% Iterative hard thresholding w/o normalization
% (A Parseval tight frame is assumed)
gamma = (1.-1e-3);
nIters = 10;
nCoefs = floor(sparsityRatio*numel(dlX(:,:,:,:)));
[dlcoefs{1:nInputs}] =
analyzer.predict(dlarray(zeros(size(dlX),'like',dlX),'SSCB'));
% IHT
for iter=1:nIters
    % Gradient descent
    dly = synthesizer.predict(dlcoefs{1:nInputs});
    [grad{1:nInputs}] = analyzer.predict(dlX-dly);
    dlcoefs = cellfun(@(x,y) x+gamma*y,dlcoefs,grad,'UniformOutput',false);
    % Hard thresholding
    coefvecs = cellfun(@(x) extractdata(reshape(x,
[],szBatch)),dlcoefs,'UniformOutput',false);
    srtdabscoefs = sort(abs(cell2mat(coefvecs.'))),1,'descend');
    thk = reshape(srtdabscoefs(nCoefs,:),1,1,1,szBatch);
    dlcoefs = cellfun(@(x) (abs(x)>thk).*x,dlcoefs,'UniformOutput',false);
    % Monitoring
    %checkSparsity =...
    %nnz(srtdabscoefs>srtdabscoefs(nCoefs,:))/numel(dlX)<=sparsityRatio;
    %assert(checkSparsity)
    %fprintf("IHT(%d) MSE: %6.4f\n",iter,mse(dlX,dly));
end
varargout = dlcoefs;
end

```

NSOLT ネットワークの随伴関係の確認

```

function checkadjointrelation(analysislgraph,synthesislgraph,nLevels,szInput)
import saivdr.dcnn.*
x = rand(szInput,'single');
% Assemble analyzer
analysislgraph4predict = analysislgraph;
for iLayer = 1:length(analysislgraph4predict.Layers)
    layer = analysislgraph4predict.Layers(iLayer);
    if contains(layer.Name,"Lv"+nLevels+"_DcOut") || ...
        ~isempty(regexp(layer.Name,'^Lv\d+_AcOut','once'))
        analysislgraph4predict = analysislgraph4predict.replaceLayer(layer.Name,...

```

```

        regressionLayer('Name',layer.Name));
    end
end
analysisnet4predict = assembleNetwork(analysislgraph4predict);

% Assemble synthesizer
synthesislgraph4predict = synthesislgraph;
synthesisnet4predict = assembleNetwork(synthesislgraph4predict);

% Analysis and synthesis process
[s{1:nLevels+1}] = analysisnet4predict.predict(x);
if isvector(s{end-1})
    s{end-1} = permute(s{end-1},[1,3,2]);
end
y = synthesisnet4predict.predict(s{:});

% Evaluation
display("MSE: " + num2str(mse(x,y)))
end

```

直交マッチング追跡関数の定義

```

function x = omp(y,Phi,nCoefs)
% Initializaton
nDims = size(Phi,1);
nAtoms = size(Phi,2);
e = ones(nAtoms,1);
a = zeros(nAtoms,1);
g = zeros(nAtoms,1);
x = zeros(nAtoms,1);
v = zeros(nDims,1);
r = y - v;
supp = [];
k = 0;
while k < nCoefs
    % Matching process
    rr = r.*r;
    for m = setdiff(1:nAtoms,supp)
        d = Phi(:,m);
        g(m) = d.*r; %  $g_m = \langle d_m, r \rangle$ 
        a(m) = g(m)/(d.*d); % Normalize  $a_m = g_m / \|d_m\|^2$ 
        e(m) = rr - g(m)*a(m); %  $e_m = r - d_m / \|d_m\|^2, r$ 
    end
    % Minimum value search (pursuit)
    [~,mmin]= min(e);
    % Update the support
    supp = union(supp,mmin);
    subPhi = Phi(:,supp);
    x(supp) = pinv(subPhi) * y;
    % Synthesis process
    v = Phi*x;

```

```
% Residual
r = y - v;
% Update
k = k + 1;
end
end
```

NSOLT ネットワークからのツリーレベル情報の抽出

```
function nLevels = extractnumlevels(nsoltNet)
import saivdr.dcnn.*

% Extraction of information
expidctlayer = '^Lv\d+_E0~?';
nLevels = 0;
nLayers = height(nsoltNet.Layers);
for iLayer = 1:nLayers
    layer = nsoltNet.Layers(iLayer);
    if ~isempty(regexp(layer.Name, expidctlayer, 'once'))
        nLevels = nLevels + 1;
    end
end
end
```

NSOLT ネットワークからのストライド情報の抽出

```
function decFactor = extractdecfactor(nsoltNet)
import saivdr.dcnn.*

% Extraction of information
expfinallayer = '^Lv1_Cmp1+_V0~?';
nLayers = height(nsoltNet.Layers);
for iLayer = 1:nLayers
    layer = nsoltNet.Layers(iLayer);
    if ~isempty(regexp(layer.Name, expfinallayer, 'once'))
        decFactor = layer.DecimationFactor;
    end
end
end
```

NSOLT ネットワークからのチャネル数情報の抽出

```
function nChannels = extractnumchannels(nsoltNet)
import saivdr.dcnn.*

% Extraction of information
expfinallayer = '^Lv1_Cmp1+_V0~?';
nLayers = height(nsoltNet.Layers);
for iLayer = 1:nLayers
    layer = nsoltNet.Layers(iLayer);
    if ~isempty(regexp(layer.Name, expfinallayer, 'once'))
```

```
nChannels = layer.NumberOfChannels;  
end  
end  
end
```

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