

# フィルタバンク理論に基づく畳込み辞書学習

## ～ 構造制約を利用した畳込みネットワーク構築 ～

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電子情報通学会 基礎・境界ソサイエティ Fundamentals Review

[https://www.jstage.jst.go.jp/article/essfr/17/2/17\\_116/\\_article/-char/ja](https://www.jstage.jst.go.jp/article/essfr/17/2/17_116/_article/-char/ja)

動作環境 : MATLAB R2024b

### 準備

```
clear
close all

nsoltDic = ""; % "nsoltdictionary_20230621232700260"

isCodegen = true; % コード生成
setup(isCodegen)
```

SaivDr-4.2.2.4 exits.

```
codegen -config cfg -o 'C:\Users\Shogo\Workspace\GitHub\SaivDr\mexcodes\fcn_orthmtxgen_single_on_cpu_mex' ./+saivdr
コードの生成が成功しました: レポートの表示
```

```
codegen -config cfg -o 'C:\Users\Shogo\Workspace\GitHub\SaivDr\mexcodes\fcn_orthmtxgen_diff_single_on_cpu_mex' ./+saivdr
コードの生成が成功しました: レポートの表示
```

```
codegen -config cfg -o 'C:\Users\Shogo\Workspace\GitHub\SaivDr\mexcodes\fcn_orthmtxgen_double_on_cpu_mex' ./+saivdr
コードの生成が成功しました: レポートの表示
```

```
codegen -config cfg -o 'C:\Users\Shogo\Workspace\GitHub\SaivDr\mexcodes\fcn_orthmtxgen_diff_double_on_cpu_mex' ./+saivdr
コードの生成が成功しました: レポートの表示
```

```
codegen -config cfg -o 'C:\Users\Shogo\Workspace\GitHub\SaivDr\mexcodes\fcn_OLpPrFbAtomExtender1dCodeGen_mex' ./+saivdr
コードの生成が成功しました: レポートの表示
```

```
codegen -config cfg -o 'C:\Users\Shogo\Workspace\GitHub\SaivDr\mexcodes\fcn_OLpPrFbAtomConcatenator1dCodeGen_mex' ./+saivdr
コードの生成が成功しました: レポートの表示
```

```
codegen -config cfg -o 'C:\Users\Shogo\Workspace\GitHub\SaivDr\mexcodes\fcn_NsoltAtomExtender2dCodeGen_mex' ./+saivdr
コードの生成が成功しました: レポートの表示
```

```
codegen -config cfg -o 'C:\Users\Shogo\Workspace\GitHub\SaivDr\mexcodes\fcn_NsoltAtomConcatenator2dCodeGen_mex' ./+saivdr
コードの生成が成功しました: レポートの表示
```

```
codegen -config cfg -o 'C:\Users\Shogo\Workspace\GitHub\SaivDr\mexcodes\fcn_NsoltAtomExtender3dCodeGen_mex' ./+saivdr
コードの生成が成功しました: レポートの表示
```

```
codegen -config cfg -o 'C:\Users\Shogo\Workspace\GitHub\SaivDr\mexcodes\fcn_NsoltAtomConcatenator3dCodeGen_mex' ./+saivdr
コードの生成が成功しました: レポートの表示
```

```
codegen -config cfg -o 'C:\Users\Shogo\Workspace\GitHub\SaivDr\mexcodes\fcn_Order1BuildingBlockTypeI_mex' ./+saivdr/
コードの生成が成功しました: レポートの表示
```

```
codegen -config cfg -o 'C:\Users\Shogo\Workspace\GitHub\SaivDr\mexcodes\fcn_Order2BuildingBlockTypeII_mex' ./+saivdr/
コードの生成が成功しました: レポートの表示
```

```
codegen -config cfg -o 'C:\Users\Shogo\Workspace\GitHub\SaivDr\mexcodes\fcn_GradEvalSteps2dCodeGen_mex' ./+saivdr/
コードの生成が成功しました: レポートの表示
```

```
codegen -config cfg -o 'C:\Users\Shogo\Workspace\GitHub\SaivDr\mexcodes\fcn_GradEvalSteps3dCodeGen_mex' ./+saivdr/
コードの生成が成功しました: レポートの表示
```

## パラメータ設定

- ブロックサイズ
- 冗長度
- スパース度

```
% Block size
szBlk = [ 8 8 ];

% Redundancy ratio for RICA/K-SVD
redundancyRatio = 7/3;

% Sparsity ratio
sparsityRatio = 3/64;
```

## 画像の読込

- $y \in \mathbb{R}^N$

```
% 原画像の準備
file_yorg = "../data/yorg.png";
if ~exist(file_yorg, 'file')
    unzip('http://www.ess.ic.kanagawa-it.ac.jp/std_img/monoimage2/Mono-Image2.zip', '../results')
    yfull = imread('../results/Mono-Image2/512X512/barbara512.bmp');
    ycrop = yfull(1:192, end-255:end);
    imwrite(ycrop, file_yorg)
end

% 原画像の読み込み
yorg = im2double(imread(file_yorg));
szOrg = size(yorg);
```

## 画像表示

```
figure
imshow(yorg);
title('Original image y')
```

Original image y



零平均化

```
%ymean = mean(y,"all");
%y = yorg - ymean;
meansubtract = @(x) x-mean(x,"all");
y = meansubtract(yorg);
```

## 離散コサイン変換 (DCT)

$$[C_M]_{k,n} = \sqrt{\frac{2}{M}} \alpha_k \cos \frac{k(n+1/2)\pi}{M}, \quad k, n = 0, 1, \dots, M-1$$

$$\alpha_k = \begin{cases} \frac{1}{\sqrt{2}} & k = 0 \\ 1 & k = 1, 2, \dots, M-1 \end{cases}$$

基底画像

$$B_{k,\ell} = C_M^{-1} E_{k,\ell} C_M^{-T}, \quad k, \ell = 0, 1, \dots, M-1$$

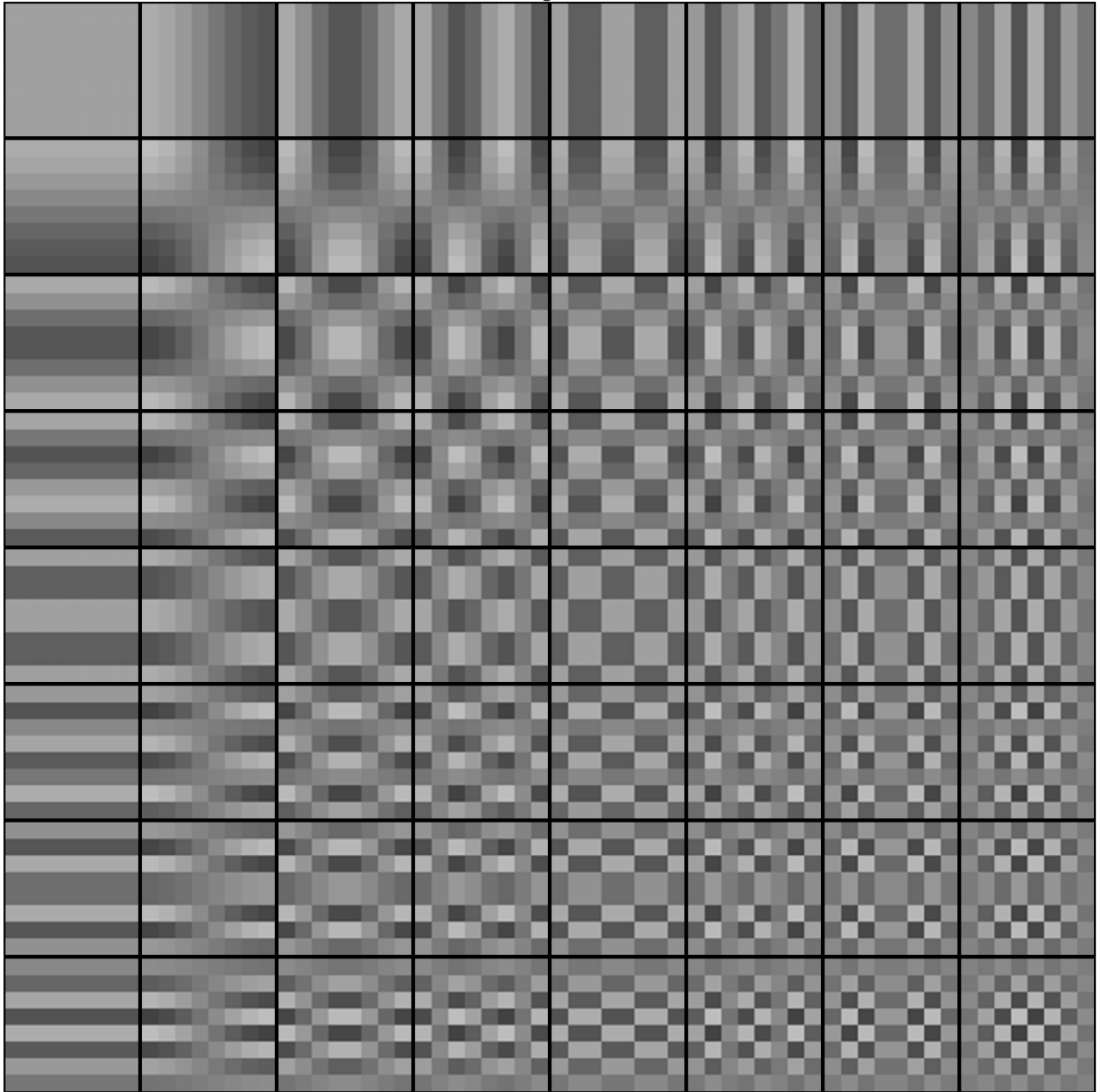
$$E_{k,\ell} = e_k e_\ell^T$$

```
basisImagesDct = zeros(szBlk(1),szBlk(2),prod(szBlk));
iBasis = 1;
for iRow=1:szBlk(1)
    for iCol=1:szBlk(2)
        E = zeros(szBlk);
        E(iRow,iCol) = 1;
        basisImagesDct(:,:,iBasis) = idct2(E,szBlk(1),szBlk(2));
        iBasis = iBasis + 1;
    end
end
```

基底画像の表示

```
figure
montage(imresize(basisImagesDct,8,'nearest')+0.5,'BorderSize',[2 2])
title('Basis images of DCT')
```

Basis images of DCT



### ブロック DCT による合成処理とその随伴処理の定義

```
syn_blkdst = @(x) blockproc(x,szBlk,@(block_struct) idct2(block_struct.data));
adj_blkdst = @(y) blockproc(y,szBlk,@(block_struct) dct2(block_struct.data));
```

### 随伴関係の確認

```
x = adj_blkdst(y);
v = randn(size(x));
u = syn_blkdst(v);
assert(abs(dot(y(:),u(:))-dot(x(:),v(:)))<1e-9)
```

## 主成分分析 (PCA)

### 問題設定:

直交性と次元削減

$$\Phi^T \Phi = \mathbf{I}_M, \forall b, \forall p, \|\mathbf{x}_b\|_0 \leq p < M$$

を制約条件とした最小自乗問題

$$\{\hat{\Phi}, \{\hat{\mathbf{x}}_b\}_b\} = \arg \min_{\{\Phi, \{\mathbf{x}_b\}_b\}} \frac{1}{2S} \sum_{b=1}^S \|\mathbf{y}_b - \Phi \mathbf{x}_b\|_2^2$$

を解く. 上式は等価的に

$$\hat{\Phi} = \arg \max_{\Phi} \text{tr}(\Phi_{:,0:p-1}^T \hat{\Sigma}_y \Phi_{:,0:p-1}) \quad \text{s.t.} \quad \Phi^T \Phi = \mathbf{I}_M$$

と表現できる. ただし,  $\hat{\Sigma}_y$  は観測ベクトル  $\{\mathbf{y}_b\}_b$  (零平均を仮定) の標本分散共分散行列である.

### 解:

固有値分解

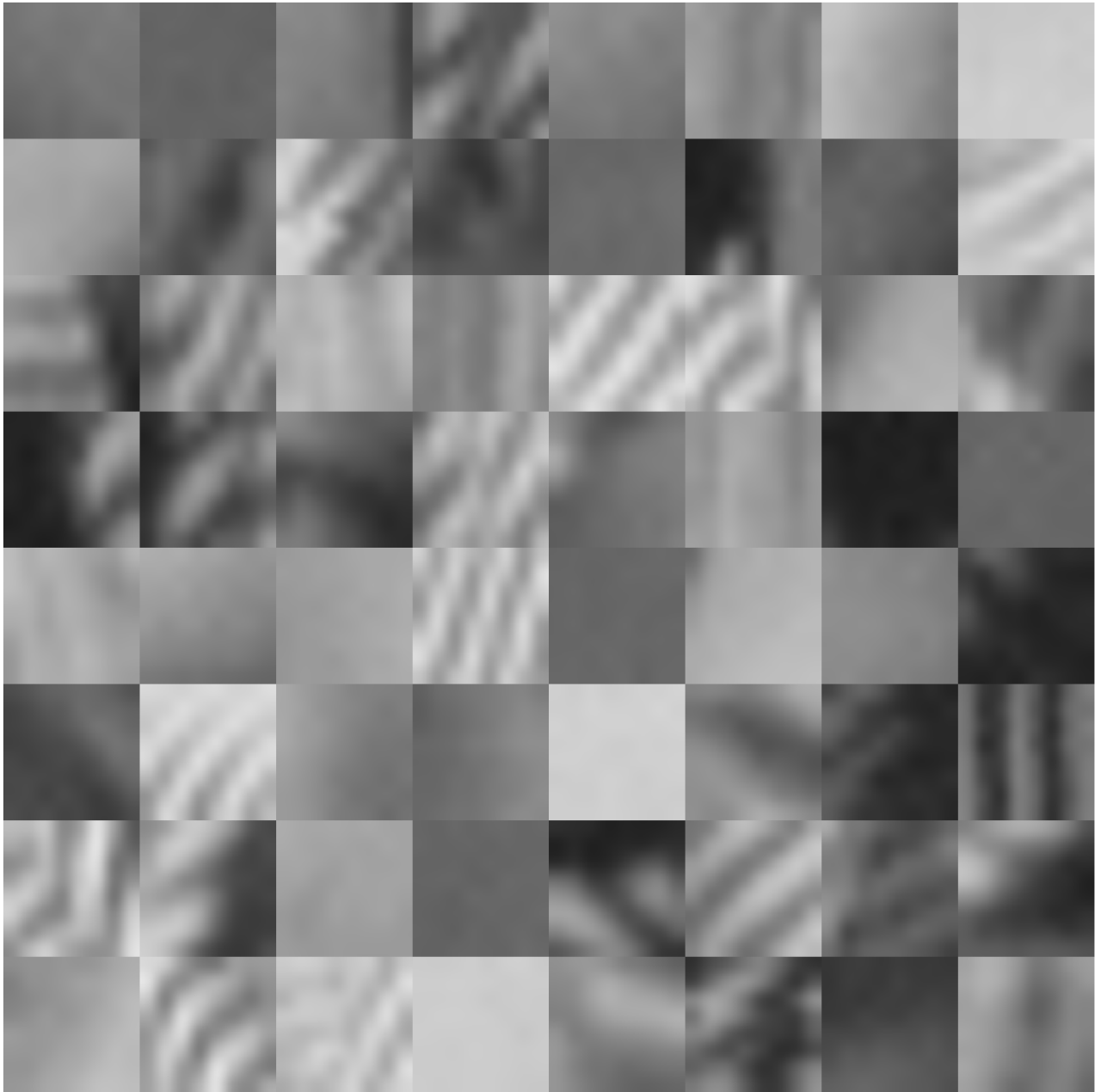
$$\hat{\Phi}^T \hat{\Sigma}_y \hat{\Phi} = \Lambda$$

ただし,  $\Lambda = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_M)$ .  $\lambda_1 \geq \lambda_2 \geq \dots \lambda_M$  は  $\hat{\Sigma}_y$  の固有値.

### 画像 $y$ からのデータ行列 $Y$ の生成

標本平均ブロックを引く代わりに, 予め零平均化したデータで学習

```
nPatches = 20*prod(szOrg./szBlk); % PCA/RICA/K-SVD 学習用のパッチをランダム抽出
npos = randsample(prod(szOrg-szBlk),nPatches);
ybs = zeros(szBlk(1),szBlk(2),nPatches,'like',y);
szSrchy = szOrg(1)-szBlk(1);
for iPatch = 1:nPatches
    ny_ = mod(npos(iPatch)-1,szSrchy)+1;
    nx_ = floor((npos(iPatch)-1)/szSrchy)+1;
    ybs(:, :, iPatch) = y(ny_:ny_+szBlk(1)-1,nx_:nx_+szBlk(2)-1);
end
figure
montage(ybs+0.5, 'Size', [8 8]);
```



```
drawnow
```

```
Y = reshape(ybs,prod(szBlk),[]);
```

標本分散共分散行列  $\hat{\Sigma}_y$  の計算

```
SigmaY = cov(Y.');
```

標本分散共分散行列  $\hat{\Sigma}_y$  の固有値分解

```
[Phi_pca,Lambda] = eig(SigmaY);
```

固有値  $\lambda$  の大きさの降順に列ベクトルをソート (Sorting column vectors in the descending order of the eigenvalues  $\lambda$ )

```
[~,idx] = sort(diag(Lambda),'descend');  
Phi_pca = Phi_pca(:,idx);
```

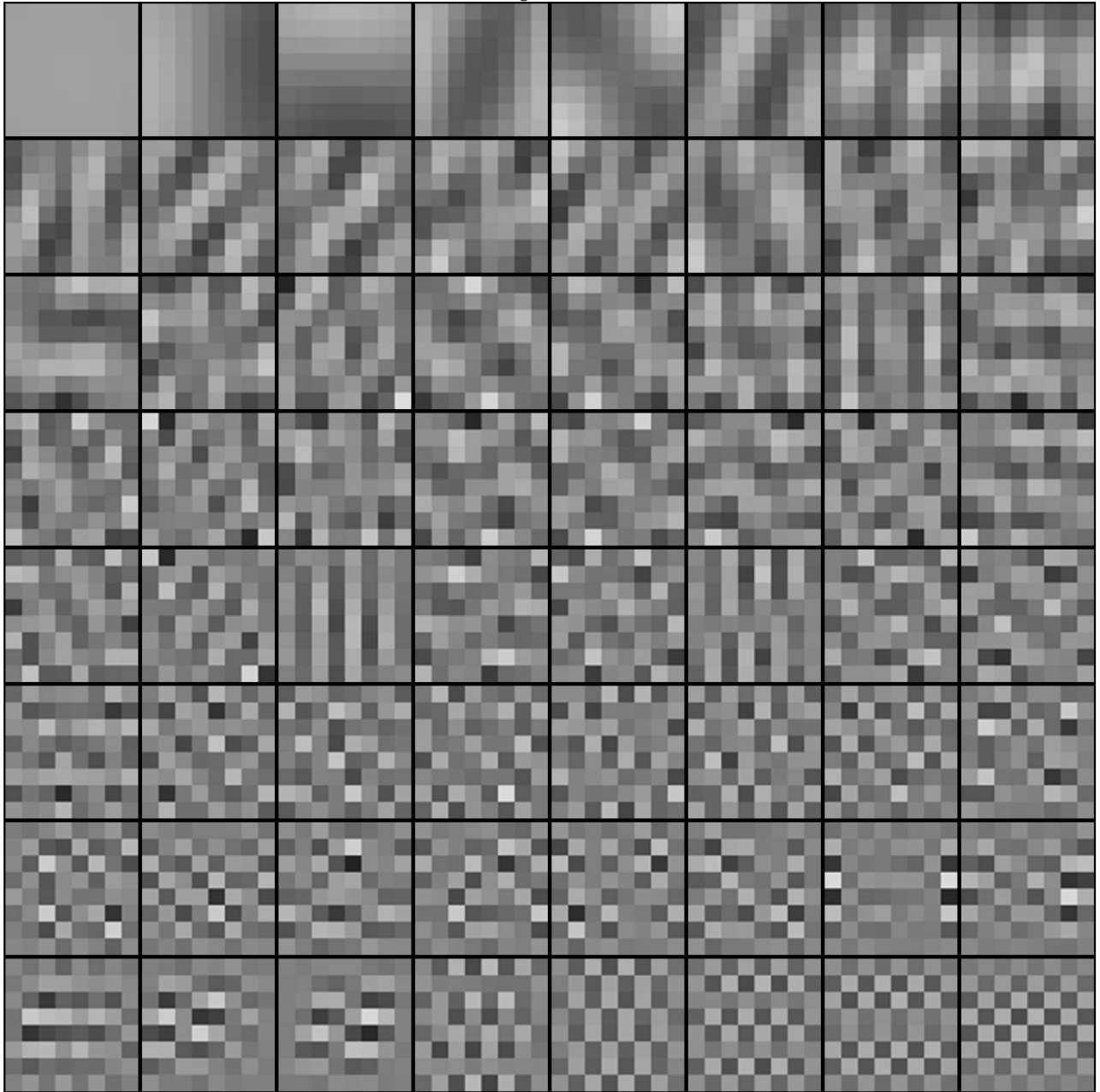
固有ベクトルを基底画像に変換

```
nBases = prod(szBlk);  
basisImagesPca = zeros(szBlk(1),szBlk(2),nBases);  
for iBasis = 1:nBases  
    basisImagesPca(:,:,iBasis) = reshape(Phi_pca(:,iBasis),szBlk(1),szBlk(2));  
end
```

基底画像の表示 (辞書)

```
figure  
montage(imresize(basisImagesPca,8,'nearest')+0.5,'BorderSize',[2 2])  
title('Basis images of PCA(KLT)')
```

Basis images of PCA(KLT)



### ブロック PCA による合成処理とその随伴処理の定義

```
syn_blkpca = @(x) col2im(Phi_pca*x,szBlk,szOrg,"distinct");
adj_blkpca = @(y) Phi_pca.'*im2col(y,szBlk,"distinct");
```

### 随伴関係の確認

```
x = adj_blkpca(y);
v = randn(size(x));
u = syn_blkpca(v);
assert(abs(dot(y(:),u(:))-dot(x(:),v(:)))<1e-9)
```



## 再構成独立成分分析 (RICA)

問題設定:

$$\begin{aligned}\widehat{\Phi} &= \arg \min_{\Phi} \frac{1}{2S} \sum_{b=1}^S \|\mathbf{y}_b - \Phi \Phi^T \mathbf{y}_b\|_2^2 + \frac{\alpha}{S} \sum_{b=1}^S \rho(\Phi^T \mathbf{y}_b) \\ &= \arg \min_{\Phi} \frac{(2\alpha)^{-1}}{S} \sum_{b=1}^S \|\mathbf{y}_b - \Phi \Phi^T \mathbf{y}_b\|_2^2 + \frac{1}{S} \sum_{b=1}^S \rho(\Phi^T \mathbf{y}_b)\end{aligned}$$

ただし,  $\{\mathbf{y}_n\}_n \subset \mathbb{R}^M$ ,  $\Phi = (\phi_1, \phi_2, \dots, \phi_P) \in \mathbb{R}^{M \times P}$ ,  $M \geq P$  である.

参考文献:

Le, Quoc V., Alexandre Karpenko, Jiquan Ngiam, and Andrew Y. Ng. "ICA with Reconstruction Cost for Efficient Overcomplete Feature Learning." Advances in Neural Information Processing Systems. Vol. 24, 2011, pp. 1017–1025. <https://papers.nips.cc/paper/4467-ica-with-reconstruction-cost-for-efficient-overcomplete-feature-learning.pdf>.

パラメータ設定

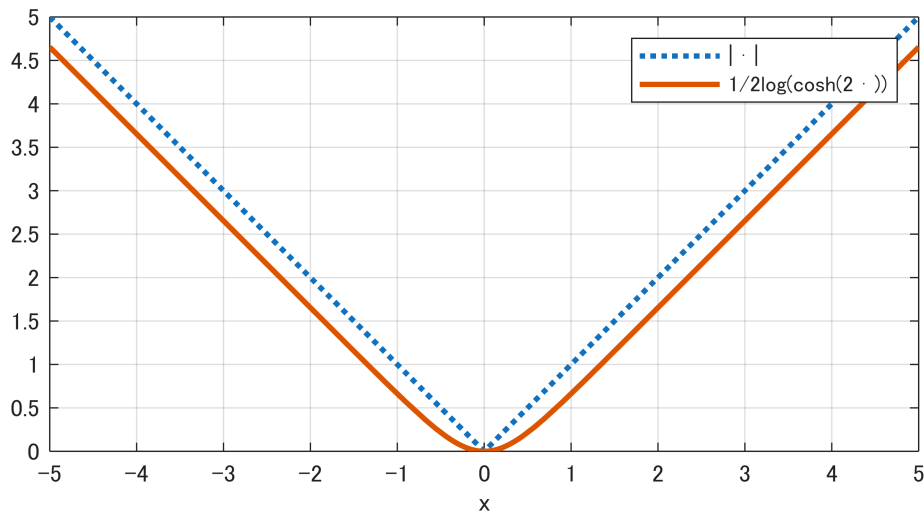
- 繰返し回数 (Number of iterations)
- 正則化パラメータ (Regularization parameter)

```
% Number of iterations
nItersRica = 1e5;
% Regularization parameter
alpha = 2e-3;
```

コントラスト関数の例

$$\rho(\Phi^T \mathbf{y}) := \frac{1}{2} \sum_{p=1}^P \log \circ \cosh(2\phi_p^T \mathbf{y})$$

```
figure
fplot(@(x) abs(x), [-5 5], ':', 'LineWidth', 2, 'DisplayName', '| \cdot |')
hold on
fplot(@(x) log(cosh(2*x))/2, [-5
5], '-', 'LineWidth', 2, 'DisplayName', '1/2 log(cosh(2 \cdot))')
xlabel('x')
legend
grid on
axis equal
hold off
```



### 要素画像の数

```
nDims = prod(szBlk);
nAtoms = ceil(redundancyRatio*nDims);
```

### 辞書 $\Phi$ の初期化

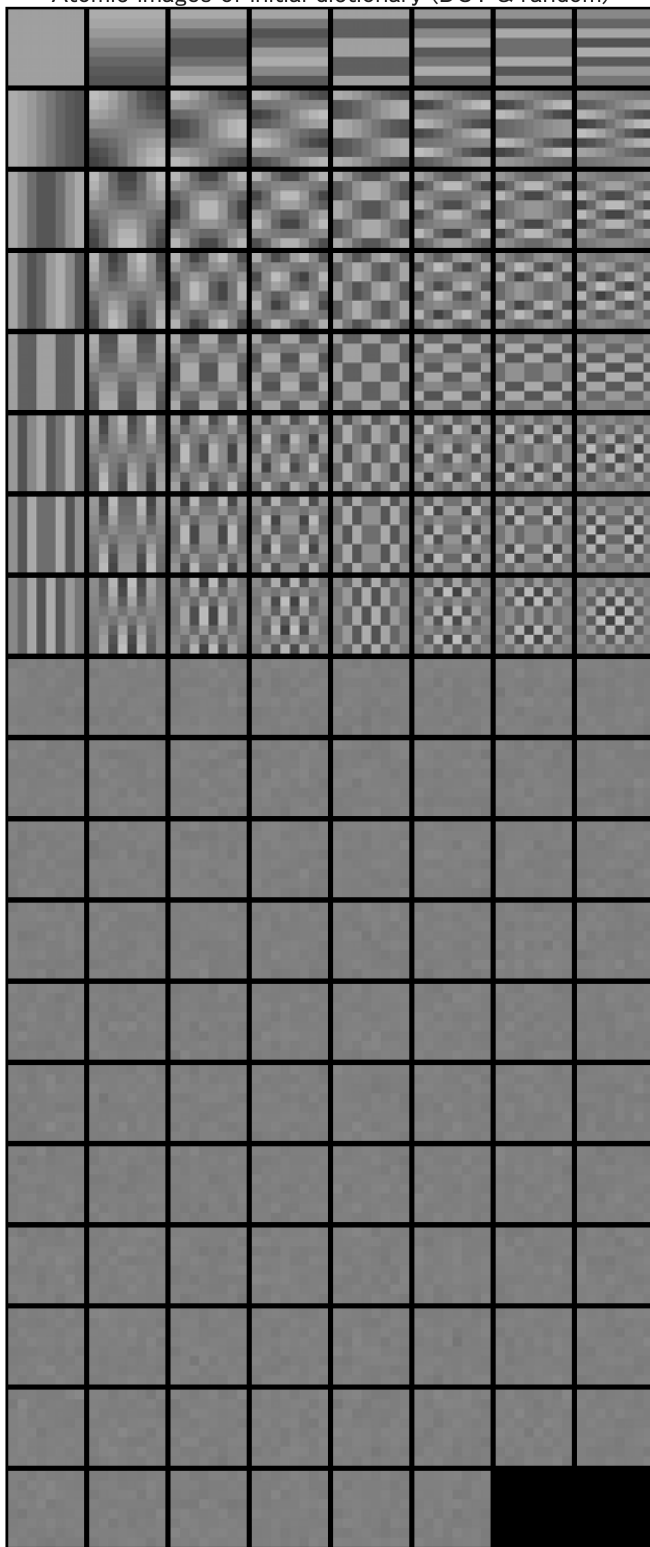
- 二次元離散コサイン変換
- ランダム

```
Phi_rica = randn(nDims,nAtoms);
Phi_rica = Phi_rica/norm(Phi_rica,'fro');
for iAtom = 1:nAtoms
    delta = zeros(szBlk);
    delta(iAtom) = 1;
    Phi_rica(:,iAtom) = reshape(idct2(delta),nDims,1);
end
```

### 要素ベクトルを要素画像に変換

```
atomicImagesRica = zeros(szBlk(1),szBlk(2),nAtoms);
for iAtom = 1:nAtoms
    atomicImagesRica(:,:,iAtom) = reshape(Phi_rica(:,iAtom),szBlk(1),szBlk(2));
end
figure
montage(imresize(atomicImagesRica,8,'nearest')+0.5,'BorderSize',[2 2],'Size',
[ceil(nAtoms/8) 8])
title('Atomic images of initial dictionary (DCT & random)')
```

Atomic images of initial dictionary (DCT & random)



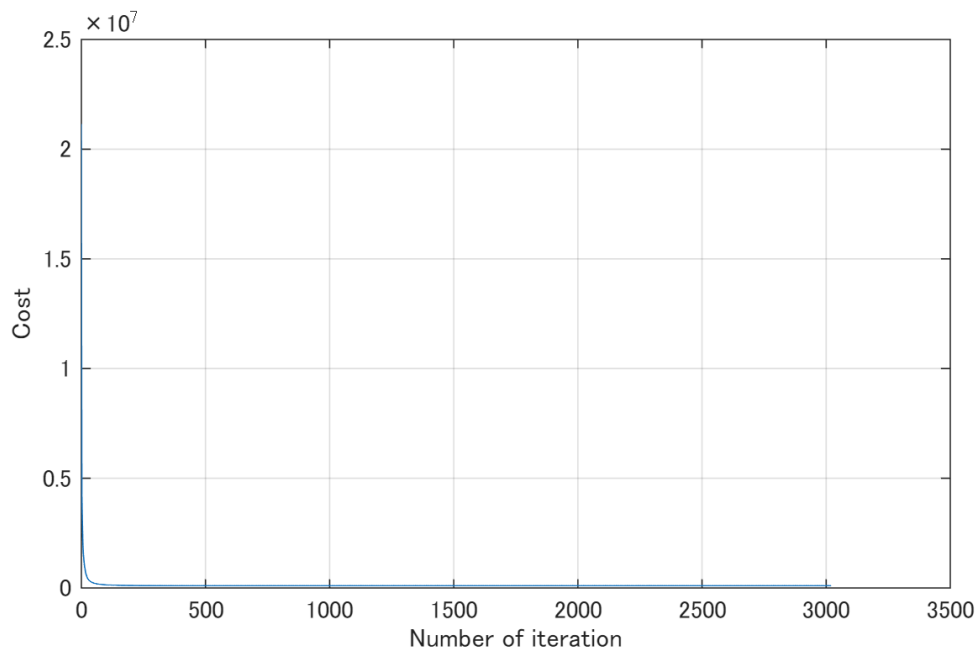
## 再構成 ICA オブジェクトの作成

PCA に合わせて予め零平均化したデータで学習

```
model = rica(Y.',nAtoms,...
    'IterationLimit',nItersRica,...
    'ContrastFcn','logcosh',...
    'InitialTransformWeight',Phi_rica,...
    'Lambda',1/(2*alpha));
```

## コスト評価のグラフ

```
info = model.FitInfo;
figure
plot(info.Iteration,info.Objective)
xlabel('Number of iteration')
ylabel('Cost')
grid on
```



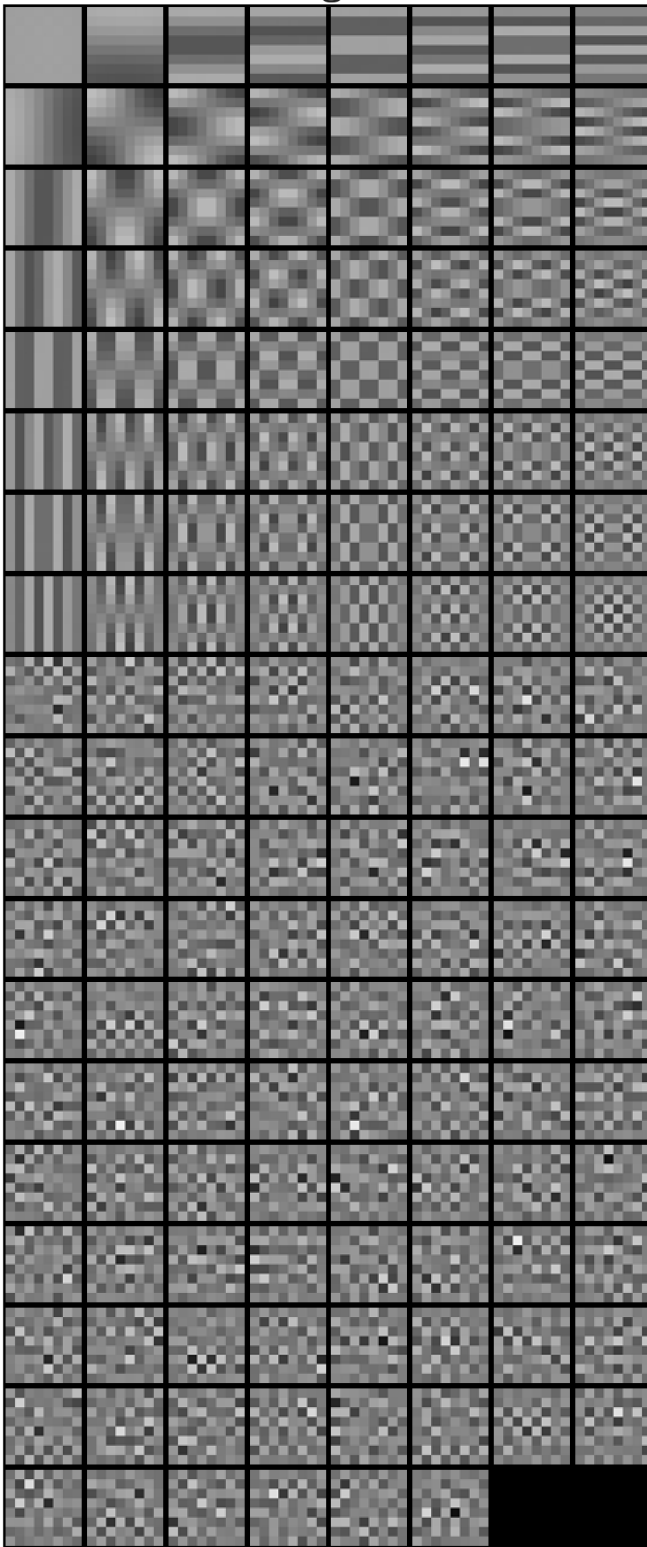
## 要素ベクトルを要素画像に変換

```
Phi_rica = model.TransformWeights;
atomicImagesRica = zeros(szBlk(1),szBlk(2),nAtoms);
for iAtom = 1:nAtoms
    atomicImagesRica(:,:,iAtom) = reshape(Phi_rica(:,iAtom),szBlk(1),szBlk(2));
end
```

## 要素画像の表示（辞書）

```
figure
montage(imresize(atomicImagesRica,8,'nearest')+.5,'BorderSize',[2 2],'Size',
[ceil(nAtoms/8) 8])
title('Atomic images of RICA')
```

Atomic images of RICA



#### ブロック RICA による合成処理とその随伴処理の定義

```
syn_blkrica = @(x) col2im(Phi_rica*x,szBlk,szOrg,"distinct");
adj_blkrica = @(y) Phi_rica.'*im2col(y,szBlk,"distinct");
```

## 随伴関係の確認

```
x = adj_blkrica(y);
v = randn(size(x));
u = syn_blkrica(v);
assert(abs(dot(y(:),u(:))-dot(x(:),v(:)))<1e-9)
```

## K-特異値分解

### パラメータ設定

- 繰り返し回数 (Number of iterations)

```
% Number of iterations
nItersKsvd = 5e3;
```

### 問題設定 (Problem setting):

$$\{\hat{\Phi}, \{\hat{\mathbf{x}}_b\}\} = \arg \min_{\{\Phi, \{\mathbf{x}_b\}\}} \frac{1}{2S} \sum_{b=1}^S \|\mathbf{y}_b - \Phi \hat{\mathbf{x}}_b\|_2^2, \quad \text{s.t. } \forall b, \|\mathbf{x}_b\|_0 \leq K$$

### アルゴリズム:

スパース近似ステップと辞書更新ステップを繰り返す.

- スパース近似ステップ

$$\hat{\mathbf{x}}_b = \arg \min_{\mathbf{x}} \frac{1}{2} \|\mathbf{y}_b - \hat{\Phi} \mathbf{x}\|_2^2 \quad \text{s.t. } \|\mathbf{x}\|_0 \leq K$$

- 辞書更新ステップ

$$\hat{\Phi} = \arg \min_{\Phi} \frac{1}{2S} \sum_{b=1}^S \|\mathbf{y}_b - \Phi \hat{\mathbf{x}}_b\|_2^2 = \arg \min_{\Phi} \frac{1}{2S} \left\| \left( \mathbf{Y} - \sum_{p \neq k} \phi_p \hat{\mathbf{X}}_{p,:} \right) - \phi_k \hat{\mathbf{X}}_{k,:} \right\|_F^2$$

### 係数の数

```
nCoefsKsvd = max(floor(sparsityRatio*nDims),1);
```

### 辞書 $\Phi$ の初期化

- 二変量離散コサイン変換
- ランダム

```
Phi_ksvd = randn(nDims,nAtoms);
Phi_ksvd = Phi_ksvd/norm(Phi_ksvd,'fro');
```

```

for iAtom = 1:nDims
    delta = zeros(szBlk);
    delta(iAtom) = 1;
    Phi_ksvd(:,iAtom) = reshape(idct2(delta),nDims,1);
end

```

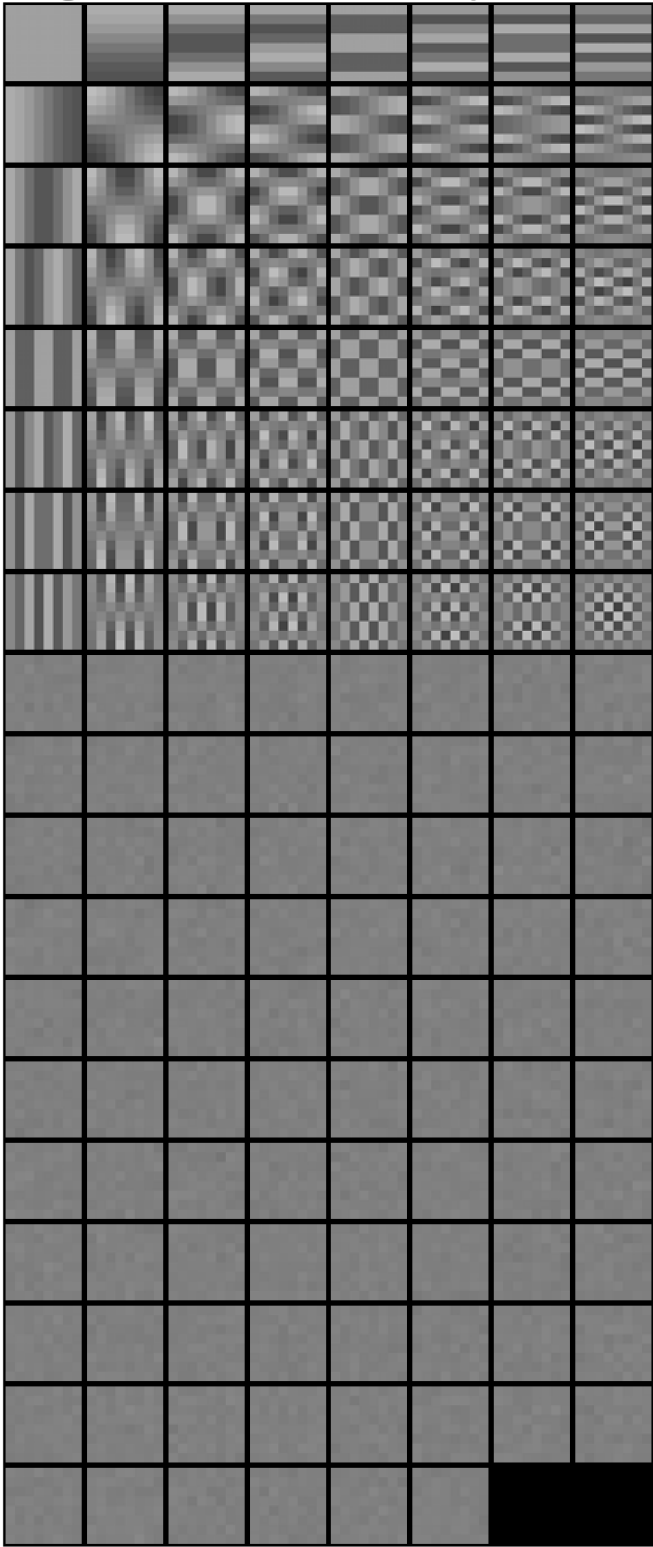
要素ベクトルを要素画像に変換

```

atomicImagesKsvd = zeros(szBlk(1),szBlk(2),nAtoms);
for iAtom = 1:nAtoms
    atomicImagesKsvd(:,:,iAtom) = reshape(Phi_ksvd(:,iAtom),szBlk(1),szBlk(2));
end
figure
montage(imresize(atomicImagesKsvd,8,'nearest')+.5,'BorderSize',[2 2],'Size',
[ceil(nAtoms/8) 8])
title('Atomic images of initial dictionary (DCT & random)')

```

atomic images of initial dictionary (DCI & random



スパース近似ステップと辞書更新ステップの繰り返し

- スパース近似： 直交マッチング追跡 (OMP)



- 辞書更新：特異値分解(SVD)と 1-ランク近似

## 辞書更新の内容

1.  $k \leftarrow 1$
2. 誤差行列  $\mathbf{E}_k$  を定義： $\mathbf{E}_k := \mathbf{Y} - \sum_{p \neq k} \phi_p \hat{\mathbf{X}}_{p,:}$
3. データ行  $\hat{\mathbf{X}}_{k,:}$  の非零値を抽出する行列  $\mathbf{\Omega}_k$  を定義： $\hat{\mathbf{X}}_{k,:}^R = \hat{\mathbf{X}}_{k,:} \mathbf{\Omega}_k \Leftrightarrow \hat{\mathbf{X}}_{k,:}^R \mathbf{\Omega}_k^T = \hat{\mathbf{X}}_{k,:}$
4. 誤差行列  $\mathbf{E}_k$  を行列  $\mathbf{\Omega}_k$  で縮退： $\mathbf{E}_k^R = \mathbf{E}_k \mathbf{\Omega}_k$
5. 縮退した誤差行列  $\mathbf{E}_k^R$  を特異値分解： $\mathbf{E}_k^R = \mathbf{U} \mathbf{S} \mathbf{V}^T = (\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_r) \text{diag}(\sigma_1, \sigma_2, \dots, \sigma_r) (\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_r)^T$
6. 要素ベクトル  $\phi_k$  を更新： $\mathbf{k} \leftarrow \mathbf{u}_1$
7. データ行  $\hat{\mathbf{X}}_{k,:}$  を更新： $\hat{\mathbf{X}}_{k,:} \leftarrow \sigma_1 \mathbf{v}_1^T$
8.  $k \leftarrow k + 1$
9.  $k \leq N$  ならば 2. へ  $k > N$  ならば終了

ただし、 $\sigma_1$  を最大特異値とする。

## 交互ステップの繰返し計算

PCA に合わせて予め零平均化したデータで学習

```
cost = zeros(1,nItersKsvd);
nSamples = size(Y,2);
for iIter = 1:nItersKsvd
    X = zeros(nAtoms,nSamples);
    % Sparse approximation
    for iSample = 1:nSamples
        y_ = Y(:,iSample);
        x = omp(y_,Phi_ksvd,nCoefsKsvd);
        X(:,iSample) = x;
    end
    % Dictionary update
    for iAtom = 1:nAtoms
        idxset = setdiff(1:nAtoms,iAtom);
        xk = X(iAtom,:);
        suppk = find(xk);
        %
        Ekred = Y(:,suppk)-Phi_ksvd(:,idxset)*X(idxset,suppk);
        %
        if ~isempty(suppk)
            [U,S,V] = svd(Ekred,'econ');
            ak = U(:,1);
            xkred = S(1,1)*V(:,1)';
            %
            Phi_ksvd(:,iAtom) = ak;
            X(iAtom,suppk) = xkred;
        end
    end
end
```

```

        end
    end
    cost(iIter) = (norm(Y-Phi_ksvd*X, 'fro')^2)/(2*nSamples);
end

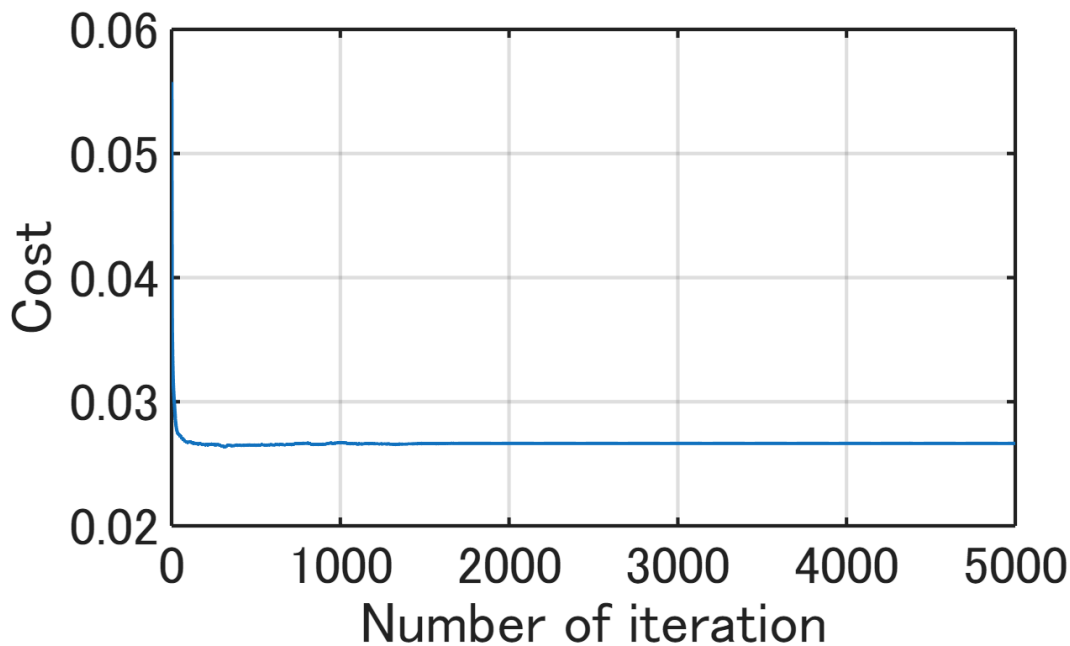
```

コスト評価のグラフ

```

figure
plot(cost)
xlabel('Number of iteration')
ylabel('Cost')
grid on

```



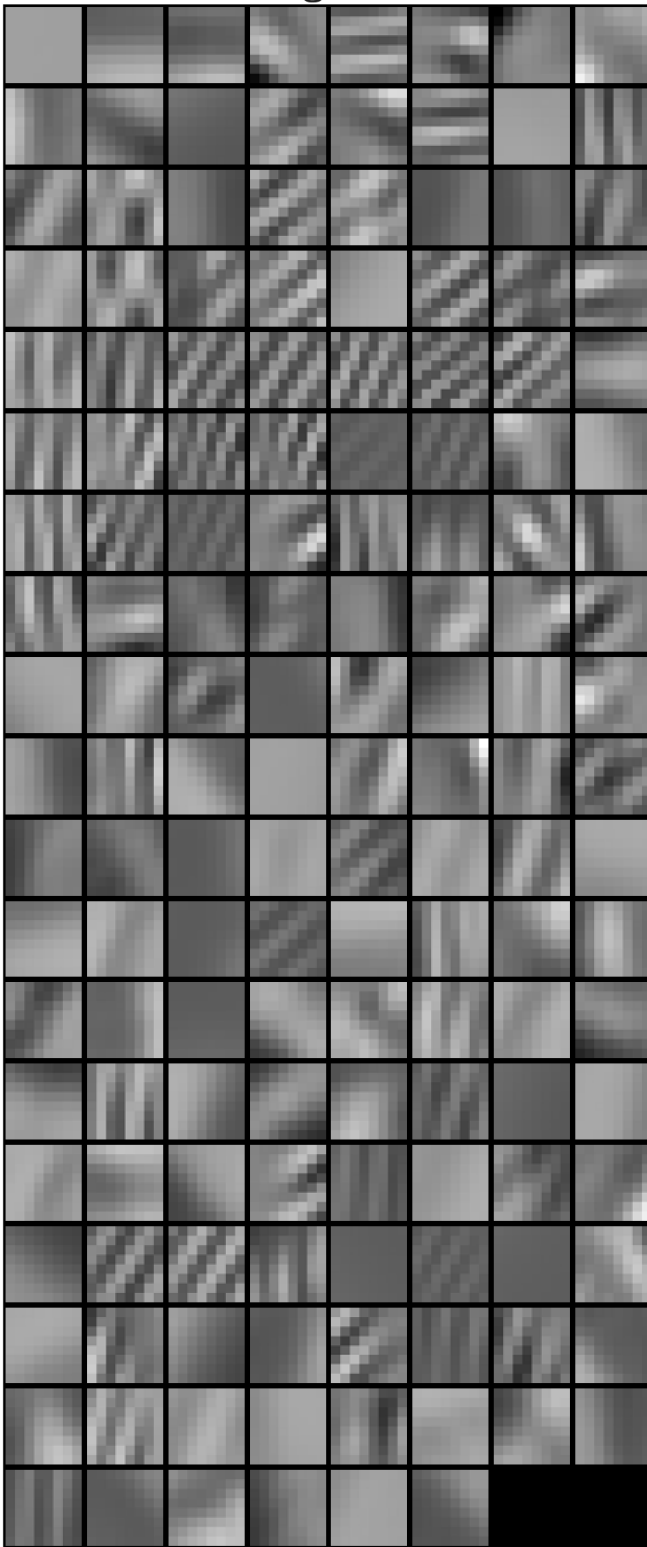
要素ベクトルを要素画像に変換

```

atomicImagesKsvd = zeros(szBlk(1),szBlk(2),nAtoms);
for iAtom = 1:nAtoms
    atomicImagesKsvd(:, :, iAtom) = reshape(Phi_ksvd(:, iAtom), szBlk(1), szBlk(2));
end
figure
montage(imresize(atomicImagesKsvd, 8, 'nearest')+.5, 'BorderSize', [2 2], 'Size',
[ceil(nAtoms/8) 8])
title('Atomic images of K-SVD')

```

Atomic images of K-SVD



#### ブロック K-特異値分解による合成処理とその随伴処理の定義

```
syn_blkksvd = @(x) col2im(Phi_ksvd*x,szBlk,szOrg,"distinct");  
adj_blkksvd = @(y) Phi_ksvd.'*im2col(y,szBlk,"distinct");
```

```
x = adj_blkksvd(y);
v = randn(size(x));
u = syn_blkksvd(v);
assert(abs(dot(y(:),u(:))-dot(x(:),v(:)))<1e-9)
```

## 2 変量ラティス構造冗長フィルタバンク

例として, (偶対称チャネルと奇対称チャネルが等しい) 偶数チャネル、偶数のポリフェーズ次数をもつタイプ I 非分離冗長重複変換(NSOLT)

$$\mathbf{E}(z_v, z_h) = \left( \prod_{n_h=1}^{\nu_h/2} \mathbf{V}_{2n_h}^{\{h\}} \bar{\mathbf{Q}}(z_h) \mathbf{V}_{2n_h-1}^{\{h\}} \mathbf{Q}(z_h) \right) \left( \prod_{n_v=1}^{\nu_v/2} \mathbf{V}_{2n_v}^{\{v\}} \bar{\mathbf{Q}}(z_v) \mathbf{V}_{2n_v-1}^{\{v\}} \mathbf{Q}(z_v) \right) \mathbf{V}_0 \mathbf{E}_0,$$

$$\mathbf{R}(z_v, z_h) = \mathbf{E}^T(z_v^{-1}, z_h^{-1}),$$

を採用する. ただし,

- $\mathbf{E}(z_v, z_h)$ : 分析フィルタバンクの Type-I ポリフェーズ行列
- $\mathbf{R}(z_v, z_h)$ : 合成フィルタバンクの Type-II ポリフェーズ行列
- $z_d \in \mathbb{C}, d \in \{v, h\}$ : Z-変換の変数
- $\nu_d \in \mathbb{N}, d \in \{v, h\}$ : 方向  $d$  のポリフェーズ次数(重複ブロック数)
- $\mathbf{V}_0 = \begin{pmatrix} \mathbf{W}_0 & \mathbf{O} \\ \mathbf{O} & \mathbf{U}_0 \end{pmatrix} \begin{pmatrix} \mathbf{I}_{M/2} \\ \mathbf{O} \\ \mathbf{I}_{M/2} \\ \mathbf{O} \end{pmatrix} \in \mathbb{R}^{P \times M}, \mathbf{V}_n^{\{d\}} = \begin{pmatrix} \mathbf{I}_{P/2} & \mathbf{O} \\ \mathbf{O} & \mathbf{U}_n^{\{d\}} \end{pmatrix} \in \mathbb{R}^{P \times P}, d \in \{v, h\}, \mathbf{W}_0, \mathbf{U}_0, \mathbf{U}_n^{\{d\}} \in \mathbb{R}^{P/2 \times P/2}$  は直交行列
- $\mathbf{Q}(z) = \mathbf{B}_P \begin{pmatrix} \mathbf{I}_{P/2} & \mathbf{O} \\ \mathbf{O} & z^{-1} \mathbf{I}_{P/2} \end{pmatrix} \mathbf{B}_P, \bar{\mathbf{Q}}(z) = \mathbf{B}_P \begin{pmatrix} z \mathbf{I}_{P/2} & \mathbf{O} \\ \mathbf{O} & \mathbf{I}_{P/2} \end{pmatrix} \mathbf{B}_P, \mathbf{B}_P = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathbf{I}_{P/2} & \mathbf{I}_{P/2} \\ \mathbf{I}_{P/2} & -\mathbf{I}_{P/2} \end{pmatrix}$

### 【References】

- [Overview of Filter Banks - MATLAB & Simulink - MathWorks 日本](#)
- MATLAB SaivDr Package: <https://github.com/msiplab/SaivDr>
- S. Muramatsu, K. Furuya and N. Yuki, "Multidimensional Nonseparable Oversampled Lapped Transforms: Theory and Design," in IEEE Transactions on Signal Processing, vol. 65, no. 5, pp. 1251-1264, 1 March 2017, doi: 10.1109/TSP.2016.2633240.
- S. Muramatsu, T. Kobayashi, M. Hiki and H. Kikuchi, "Boundary Operation of 2-D Nonseparable Linear-Phase Paraunitary Filter Banks," in IEEE Transactions on Image Processing, vol. 21, no. 4, pp. 2314-2318, April 2012, doi: 10.1109/TIP.2011.2181527.
- S. Muramatsu, M. Ishii and Z. Chen, "Efficient parameter optimization for example-based design of nonseparable oversampled lapped transform," 2016 IEEE International Conference on Image Processing (ICIP), Phoenix, AZ, 2016, pp. 3618-3622, doi: 10.1109/ICIP.2016.7533034.

- Furuya, K., Hara, S., Seino, K., & Muramatsu, S. (2016). Boundary operation of 2D non-separable oversampled lapped transforms. *APSIPA Transactions on Signal and Information Processing*, 5, E9. doi:10.1017/ATSIP.2016.3.

## 2次元画像の階層的分析

$R_M^P(\tau)$  をツリーレベル  $\tau$  の階層構造フィルタバンクの冗長度とすると、

$$R_M^P(\tau) = \begin{cases} (P-1)\tau + 1, & M = 1, \\ \frac{P-1}{M-1} - \frac{P-M}{(M-1)M^\tau}, & M \geq 2. \end{cases}$$

となる.

## 構成パラメータ設定

```
%%{
% Decimation factor (Strides)
decFactor = [2 2]; % [μv μh]

% Number of channels ( sum(nChannels) >= prod(decFactors) )
nChannels = [4 4]; % [Ps Pa] (Ps=Pa)

% Number of tree levels
nLevels = 4;

% Polyphase Order
ppOrder = [4 4];
%%}

%{
% Decimation factor (Strides)
decFactor = [4 4]; % [μv μh]

% Number of channels ( sum(nChannels) >= prod(decFactors) )
nChannels = [13 13]; % [Ps Pa] (Ps=Pa)

% Number of tree levels
nLevels = 2;

% Polyphase Order
ppOrder = [2 2];
%}

%{
% Decimation factor (Strides)
decFactor = [8 8]; % [μv μh]

% Number of channels ( sum(nChannels) >= prod(decFactors) )
```

```

nChannels = [53 53]; % [Ps Pa] (Ps=Pa)

% Number of tree levels
nLevels = 1;

% Polyphase Order
ppOrder = [2 2];
%}

% Redundancy
P = sum(nChannels);
M = prod(decFactor);
redundancyNsolt = ...
    (prod(decFactor)==1)*((P-1)*nLevels+1) + ...
    (prod(decFactor)>1)*((P-1)/(M-1)-(P-M)/((M-1)*M^nLevels))

redundancyNsolt =
2.3281

```

```

assert(redundancyNsolt<redundancyRatio)

```

$$L_v \times L_h = \left( \mu_v^\tau + \nu_v \frac{\mu_v(\mu_v^\tau - 1)}{\mu_v - 1} \right) \times \left( \mu_h^\tau + \nu_h \frac{\mu_h(\mu_h^\tau - 1)}{\mu_h - 1} \right)$$

```

% Filter size [ Ly Lx ]
maxDecFactor = decFactor.^nLevels;
szFilters = maxDecFactor + ppOrder.*decFactor.*(maxDecFactor-1)./(decFactor-1)

szFilters = 1x2
    136    136

```

```

% Patch size for training
szPatchTrn = maxDecFactor.*ceil(szFilters./maxDecFactor) % > [ Ly Lx ]

szPatchTrn = 1x2
    144    144

```

```

%szPatchTrn = 2.^nextpow2(szFilters) % > [ Ly Lx ]
assert(all(szPatchTrn>szFilters))

```

```

% Number of patches per image
nSubImgs = floor(nPatches*prod(szBlk./szPatchTrn))

```

```

nSubImgs =
47

```

```

assert(nSubImgs > 0)

```

```

% No DC-leakage
noDcLeakage = true

```

```
noDcLeakage = logical
```

```
1
```

## 辞書の設定

```
if exist("../data/"+nsoltDic+".mat","file")
    S = load("../data/"+nsoltDic);
    analysisnet = S.analysisnet;
    synthesisnet = S.synthesisnet;
    nLevels_ = extractnumlevels(analysisnet);
    decFactor_ = extractdecfactor(analysisnet);
    nChannels_ = extractnumchannels(analysisnet);

    assert(nLevels==nLevels_)
    assert(all(decFactor==decFactor_))
    assert(all(nChannels==nChannels_))
else
    % Number of iterations
    nItersNsolt = 10;

    % Standard deviation of initial angles
    stdInitAng = 1e-1; %pi/6;

    % Mini batch size
    miniBatchSize = 10;

    % Number of Epochs (1 Epoch = nSubImgs/miniBatchSize iterations)
    maxEpochs = 30;

    % Number of iterations
    maxIters = nSubImgs/miniBatchSize * maxEpochs

    % Training options
    opts = trainingOptions('sgdm', ... % Stochastic gradient descent w/ momentum
        ... 'Momentum', 0.9000,...
        ... 'InitialLearnRate', 5.0e-03,...
        ... 'LearnRateScheduleSettings', 'none',...
        ... 'L2Regularization', 0.0, ... 1.0e-04,...
        ... 'GradientThresholdMethod', 'l2norm',...
        ... 'GradientThreshold', Inf,...
        ... 'MaxEpochs', maxEpochs, ... 30,...
        ... 'MiniBatchSize', miniBatchSize, ... 128,...
        ... 'Verbose', 1,...
        ... 'VerboseFrequency', 50,...
        ... 'ValidationData', [],...
        ... 'ValidationFrequency', 50,...
        ... 'ValidationPatience', Inf,...
        ... 'Shuffle', 'once',...
        ... 'CheckpointPath', '',...
        ... 'ExecutionEnvironment', 'auto',...
    );
```

```

... 'WorkerLoad', [], ...
... 'OutputFcn', [], ...
'Plots', 'none', ... 'training-progress', ...
... 'SequenceLength', 'longest', ...
... 'SequencePaddingValue', 0, ...
... 'SequencePaddingDirection', 'right', ...
... 'DispatchInBackground', 0, ...
'ResetInputNormalization', 0); ... 1

```

## 層構造の構築

```

import saivdr.dcn.*
analysislgraph = fcn_creatensoltlgraph2d([], ...
    'InputSize', szPatchTrn, ...
    'NumberOfChannels', nChannels, ...
    'DecimationFactor', decFactor, ...
    'PolyPhaseOrder', ppOrder, ...
    'NumberOfLevels', nLevels, ...
    'NumberOfVanishingMoments', noDcLeakage, ...
    'Mode', 'Analyzer');
synthesislgraph = fcn_creatensoltlgraph2d([], ...
    'InputSize', szPatchTrn, ...
    'NumberOfChannels', nChannels, ...
    'DecimationFactor', decFactor, ...
    'PolyPhaseOrder', ppOrder, ...
    'NumberOfLevels', nLevels, ...
    'NumberOfVanishingMoments', noDcLeakage, ...
    'Mode', 'Synthesizer');

figure
subplot(1,2,1)
plot(analysislgraph)
title('Analysis NSOLT')
subplot(1,2,2)
plot(synthesislgraph)
title('Synthesis NSOLT')

% Construction of deep learning network.
synthesisnet = dlnetwork(synthesislgraph);

% Initialize
nLearnables = height(synthesisnet.Learnables);
for iLearnable = 1:nLearnables
    if synthesisnet.Learnables.Parameter(iLearnable)=="Angles"
        layerName = synthesisnet.Learnables.Layer(iLearnable);
        synthesisnet.Learnables.Value(iLearnable) = ...
            cellfun(@(x) x+stdInitAng*randn(size(x)), ...
                synthesisnet.Learnables.Value(iLearnable), 'UniformOutput', false);
    end
end
end

```



```
% Copy the synthesizer's parameters to the analyzer
synthesislgraph = layerGraph(synthesisnet);
analysislgraph = fcn_cpparamssyn2ana(analysislgraph,synthesislgraph);
analysisnet = dlnetwork(analysislgraph);
```

## 随伴関係（完全再構成）の確認

NSOLT はパーセバルタイト性を満たす.

```
nOutputs = nLevels+1;
x = rand(szPatchTrn, 'single');
s = cell(1,nOutputs);
dlx = dlarray(x, 'SSCB'); % Deep learning array (SSCB:
Spatial, Spatial, Channel, Batch)
[s{1:nOutputs}] = analysisnet.predict(dlx);
dly = synthesisnet.predict(s{:});
display("MSE: " + num2str(mse(dlx,dly)))
```

## 要素画像の初期状態

```
import saivdr.dcn.*
figure
atomicimshow(synthesisnet,[],2^(nLevels-1))
title('Atomic images of initial NSOLT')
```

## 訓練画像の準備

画像データストアからランダムにパッチを抽出

PCA に合わせて予め零平均化したデータで学習

```
imds = imageDatastore(file_yorg, "ReadFcn", @(x)
meansubtract(im2single(imread(x))));
patchds =
randomPatchExtractionDatastore(imds,imds,szPatchTrn, 'PatchesPerImage', nSubImgs);
figure
minibatch = preview(patchds);
responses = minibatch.ResponseImage;
responses = cellfun(@(x) x + 0.5, responses, 'UniformOutput', false);
figure
montage(responses, 'Size', [2 4]);
drawnow
```

## 畳み込み辞書学習

問題設定:

$$\{\hat{\theta}, \{\hat{\mathbf{x}}_n\}\} = \arg \min_{\{\theta, \{\mathbf{x}_n\}\}} \frac{1}{2S} \sum_{n=1}^S \|\mathbf{y}_n - \mathbf{D}_{\theta} \hat{\mathbf{x}}_n\|_2^2, \quad \text{s.t. } \forall n, \|\mathbf{x}_n\|_0 \leq K,$$

ただし、 $\mathbf{D}_{\theta}$  は設計パラメータベクトル  $\theta$  をもつ畳み込み辞書.

## アルゴリズム:

スパース近似ステップと辞書更新ステップを繰り返す.

- スパース近似ステップ

$$\hat{\mathbf{x}}_n = \arg \min_{\mathbf{x}_n} \frac{1}{2} \|\mathbf{y}_n - \hat{\mathbf{D}}\mathbf{x}_n\|_2^2 \quad \text{s.t. } \|\mathbf{x}_n\|_0 \leq K$$

- 辞書更新ステップ

$$\hat{\boldsymbol{\theta}} = \arg \min_{\boldsymbol{\theta}} \frac{1}{2S} \sum_{n=1}^S \|\mathbf{y}_n - \mathbf{D}_{\boldsymbol{\theta}}\hat{\mathbf{x}}_n\|_2^2$$

$$\hat{\mathbf{D}} = \mathbf{D}_{\hat{\boldsymbol{\theta}}}$$

採用するスパース近似と辞書更新の手法:

- スパース近似: (正規化なし) 繰り返しハード閾値処理(IHT)
- 辞書更新: モーメントム付き確率的勾配降下法(SGD)

```
% Check if IHT works for dlarray
%x = dlarray(randn(szPatchTrn,'single'),'SSCB');
%[y,coefs{1:nOutputs}] = iht(x,analysisnet,synthesisnet,sparsityRatio);
```

## 辞書学習の繰り返し計算

```
import saivdr.dcn.*
%profile on
for iIter = 1:nItersNsolt

    % Sparse approximation (Applied to produce an object of
    TransformedDatastore)
    coefimgds = transform(patchds, @(x)
    iht4patchds(x,analysisnet,synthesisnet,sparsityRatio));

    % Synthesis dictionary update
    trainlgraph = synthesislgraph.replaceLayer('Lv1_Out',...
        regressionLayer('Name','Lv1_Out'));
    trainednet = trainNetwork(coefimgds,trainlgraph,opts);

    % Analysis dictionary update (Copy parameters from synthesizer to analyzer)
    trainedlgraph = layerGraph(trainednet);
    analysislgraph = fcn_cpparamssyn2ana(analysislgraph,trainedlgraph);
    analysisnet = dlnetwork(analysislgraph);

    % Check the adjoint relation (perfect reconstruction)
    checkadjointrelation(analysislgraph,trainedlgraph,nLevels,szPatchTrn);
```

```

% Replace layer
synthesislgraph = trainedlgraph.replaceLayer('Lv1_Out',...
    nsoltIdentityLayer('Name','Lv1_Out'));
synthesisnet = dlnetwork(synthesislgraph);

end
%profile off
%profile viewer

```

## 訓練ネットワークの保存

```

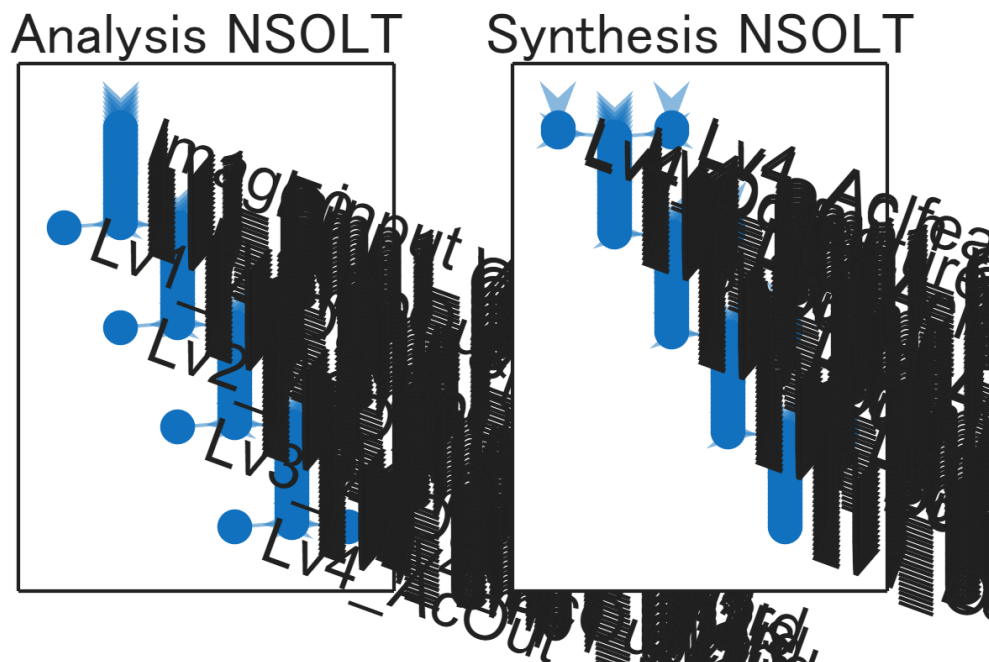
import saivdr.dcn.*
synthesislgraph = layerGraph(synthesisnet);
analysislgraph = fcn_cppparamssyn2ana(analysislgraph,synthesislgraph);
analysisnet = dlnetwork(analysislgraph);
save(sprintf(' ../results/
nsoltdictionary_%s',datetime('now','Format','yyyyMMddHHmmssSSS')), 'analysisnet', 'syn
thesisnet', 'nLevels')
end

```

```

maxIters =
141

```



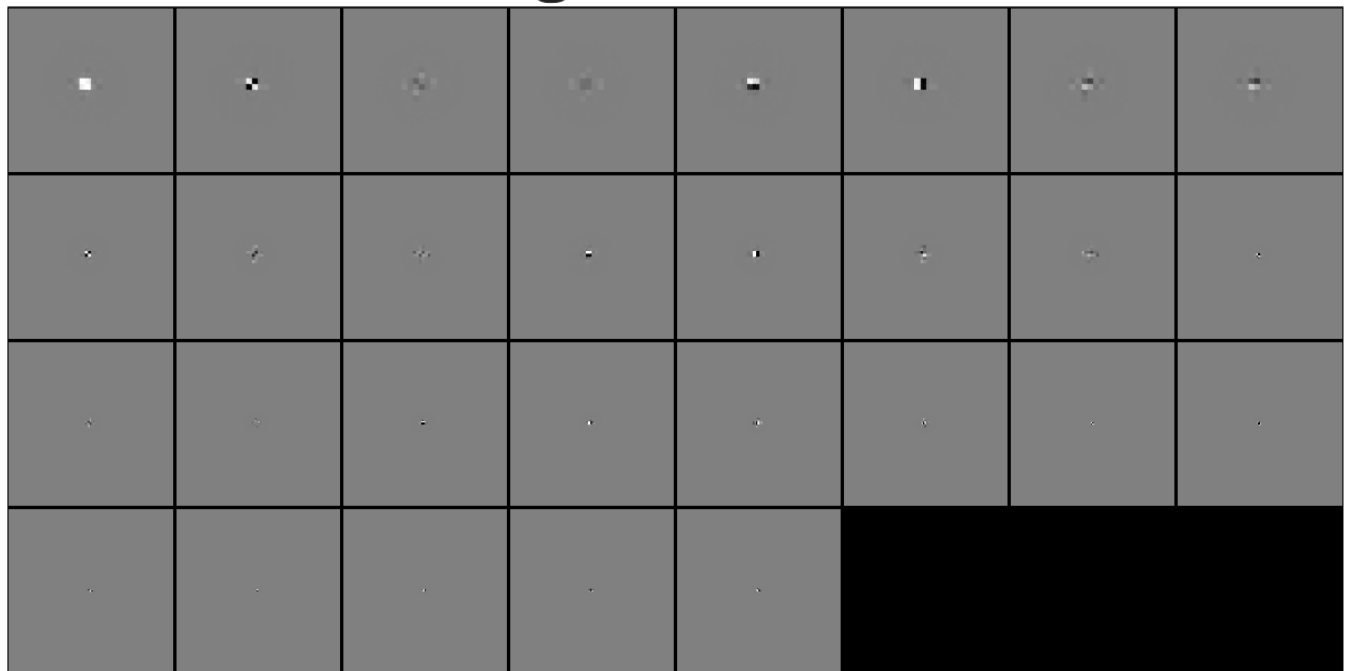
```

Copy angles from Lv1_Cmp1_V0~ to Lv1_Cmp1_V0
Copy angles from Lv1_Cmp1_Vh1~ to Lv1_Cmp1_Vh1
Copy angles from Lv1_Cmp1_Vh2~ to Lv1_Cmp1_Vh2
Copy angles from Lv1_Cmp1_Vh3~ to Lv1_Cmp1_Vh3
Copy angles from Lv1_Cmp1_Vh4~ to Lv1_Cmp1_Vh4
Copy angles from Lv1_Cmp1_Vv1~ to Lv1_Cmp1_Vv1
Copy angles from Lv1_Cmp1_Vv2~ to Lv1_Cmp1_Vv2
Copy angles from Lv1_Cmp1_Vv3~ to Lv1_Cmp1_Vv3
Copy angles from Lv1_Cmp1_Vv4~ to Lv1_Cmp1_Vv4
Copy angles from Lv2_Cmp1_V0~ to Lv2_Cmp1_V0
Copy angles from Lv2_Cmp1_Vh1~ to Lv2_Cmp1_Vh1
Copy angles from Lv2_Cmp1_Vh2~ to Lv2_Cmp1_Vh2

```

Copy angles from Lv2\_Cmp1\_Vh3~ to Lv2\_Cmp1\_Vh3  
 Copy angles from Lv2\_Cmp1\_Vh4~ to Lv2\_Cmp1\_Vh4  
 Copy angles from Lv2\_Cmp1\_Vv1~ to Lv2\_Cmp1\_Vv1  
 Copy angles from Lv2\_Cmp1\_Vv2~ to Lv2\_Cmp1\_Vv2  
 Copy angles from Lv2\_Cmp1\_Vv3~ to Lv2\_Cmp1\_Vv3  
 Copy angles from Lv2\_Cmp1\_Vv4~ to Lv2\_Cmp1\_Vv4  
 Copy angles from Lv3\_Cmp1\_V0~ to Lv3\_Cmp1\_V0  
 Copy angles from Lv3\_Cmp1\_Vh1~ to Lv3\_Cmp1\_Vh1  
 Copy angles from Lv3\_Cmp1\_Vh2~ to Lv3\_Cmp1\_Vh2  
 Copy angles from Lv3\_Cmp1\_Vh3~ to Lv3\_Cmp1\_Vh3  
 Copy angles from Lv3\_Cmp1\_Vh4~ to Lv3\_Cmp1\_Vh4  
 Copy angles from Lv3\_Cmp1\_Vv1~ to Lv3\_Cmp1\_Vv1  
 Copy angles from Lv3\_Cmp1\_Vv2~ to Lv3\_Cmp1\_Vv2  
 Copy angles from Lv3\_Cmp1\_Vv3~ to Lv3\_Cmp1\_Vv3  
 Copy angles from Lv3\_Cmp1\_Vv4~ to Lv3\_Cmp1\_Vv4  
 Copy angles from Lv4\_Cmp1\_V0~ to Lv4\_Cmp1\_V0  
 Copy angles from Lv4\_Cmp1\_Vh1~ to Lv4\_Cmp1\_Vh1  
 Copy angles from Lv4\_Cmp1\_Vh2~ to Lv4\_Cmp1\_Vh2  
 Copy angles from Lv4\_Cmp1\_Vh3~ to Lv4\_Cmp1\_Vh3  
 Copy angles from Lv4\_Cmp1\_Vh4~ to Lv4\_Cmp1\_Vh4  
 Copy angles from Lv4\_Cmp1\_Vv1~ to Lv4\_Cmp1\_Vv1  
 Copy angles from Lv4\_Cmp1\_Vv2~ to Lv4\_Cmp1\_Vv2  
 Copy angles from Lv4\_Cmp1\_Vv3~ to Lv4\_Cmp1\_Vv3  
 Copy angles from Lv4\_Cmp1\_Vv4~ to Lv4\_Cmp1\_Vv4  
 "MSE: 3.9846e-10"

## Acquire images of initial results





単一の GPU で学習中。

エポック	反復	経過時間 (h h : mm : s s)	ミニバッチ RMSE	ミニバッチ損失	基本学
1	1	00 : 00 : 10	6. 79	23. 1	0. 0
13	50	00 : 02 : 18	5. 74	16. 5	0. 0
25	100	00 : 04 : 25	6. 02	18. 1	0. 0
30	120	00 : 05 : 16	5. 90	17. 4	0. 0

学習終了：最大エポック数が完了しました。

```
Copy angles from Lv1_Cmp1_V0~ to Lv1_Cmp1_V0
Copy angles from Lv1_Cmp1_Vh1~ to Lv1_Cmp1_Vh1
Copy angles from Lv1_Cmp1_Vh2~ to Lv1_Cmp1_Vh2
Copy angles from Lv1_Cmp1_Vh3~ to Lv1_Cmp1_Vh3
Copy angles from Lv1_Cmp1_Vh4~ to Lv1_Cmp1_Vh4
Copy angles from Lv1_Cmp1_Vv1~ to Lv1_Cmp1_Vv1
Copy angles from Lv1_Cmp1_Vv2~ to Lv1_Cmp1_Vv2
Copy angles from Lv1_Cmp1_Vv3~ to Lv1_Cmp1_Vv3
Copy angles from Lv1_Cmp1_Vv4~ to Lv1_Cmp1_Vv4
Copy angles from Lv2_Cmp1_V0~ to Lv2_Cmp1_V0
Copy angles from Lv2_Cmp1_Vh1~ to Lv2_Cmp1_Vh1
Copy angles from Lv2_Cmp1_Vh2~ to Lv2_Cmp1_Vh2
Copy angles from Lv2_Cmp1_Vh3~ to Lv2_Cmp1_Vh3
Copy angles from Lv2_Cmp1_Vh4~ to Lv2_Cmp1_Vh4
Copy angles from Lv2_Cmp1_Vv1~ to Lv2_Cmp1_Vv1
Copy angles from Lv2_Cmp1_Vv2~ to Lv2_Cmp1_Vv2
Copy angles from Lv2_Cmp1_Vv3~ to Lv2_Cmp1_Vv3
Copy angles from Lv2_Cmp1_Vv4~ to Lv2_Cmp1_Vv4
Copy angles from Lv3_Cmp1_V0~ to Lv3_Cmp1_V0
Copy angles from Lv3_Cmp1_Vh1~ to Lv3_Cmp1_Vh1
Copy angles from Lv3_Cmp1_Vh2~ to Lv3_Cmp1_Vh2
Copy angles from Lv3_Cmp1_Vh3~ to Lv3_Cmp1_Vh3
Copy angles from Lv3_Cmp1_Vh4~ to Lv3_Cmp1_Vh4
Copy angles from Lv3_Cmp1_Vv1~ to Lv3_Cmp1_Vv1
Copy angles from Lv3_Cmp1_Vv2~ to Lv3_Cmp1_Vv2
Copy angles from Lv3_Cmp1_Vv3~ to Lv3_Cmp1_Vv3
```

Copy angles from Lv3\_Cmp1\_Vv4~ to Lv3\_Cmp1\_Vv4  
 Copy angles from Lv4\_Cmp1\_V0~ to Lv4\_Cmp1\_V0  
 Copy angles from Lv4\_Cmp1\_Vh1~ to Lv4\_Cmp1\_Vh1  
 Copy angles from Lv4\_Cmp1\_Vh2~ to Lv4\_Cmp1\_Vh2  
 Copy angles from Lv4\_Cmp1\_Vh3~ to Lv4\_Cmp1\_Vh3  
 Copy angles from Lv4\_Cmp1\_Vh4~ to Lv4\_Cmp1\_Vh4  
 Copy angles from Lv4\_Cmp1\_Vv1~ to Lv4\_Cmp1\_Vv1  
 Copy angles from Lv4\_Cmp1\_Vv2~ to Lv4\_Cmp1\_Vv2  
 Copy angles from Lv4\_Cmp1\_Vv3~ to Lv4\_Cmp1\_Vv3  
 Copy angles from Lv4\_Cmp1\_Vv4~ to Lv4\_Cmp1\_Vv4  
 "MSE: 3.4618e-14"

単一の GPU で学習中。

エポック	反復	経過時間 (h h : mm : s s)	ミニバッチ RMSE	ミニバッチ損失	基本学
1	1	00 : 00 : 08	5. 56	15. 4	0. 0
13	50	00 : 02 : 15	5. 16	13. 3	0. 0
25	100	00 : 04 : 22	5. 24	13. 7	0. 0
30	120	00 : 05 : 15	5. 19	13. 5	0. 0

学習終了: 最大エポック数が完了しました。

Copy angles from Lv1\_Cmp1\_V0~ to Lv1\_Cmp1\_V0  
 Copy angles from Lv1\_Cmp1\_Vh1~ to Lv1\_Cmp1\_Vh1  
 Copy angles from Lv1\_Cmp1\_Vh2~ to Lv1\_Cmp1\_Vh2  
 Copy angles from Lv1\_Cmp1\_Vh3~ to Lv1\_Cmp1\_Vh3  
 Copy angles from Lv1\_Cmp1\_Vh4~ to Lv1\_Cmp1\_Vh4  
 Copy angles from Lv1\_Cmp1\_Vv1~ to Lv1\_Cmp1\_Vv1  
 Copy angles from Lv1\_Cmp1\_Vv2~ to Lv1\_Cmp1\_Vv2  
 Copy angles from Lv1\_Cmp1\_Vv3~ to Lv1\_Cmp1\_Vv3  
 Copy angles from Lv1\_Cmp1\_Vv4~ to Lv1\_Cmp1\_Vv4  
 Copy angles from Lv2\_Cmp1\_V0~ to Lv2\_Cmp1\_V0  
 Copy angles from Lv2\_Cmp1\_Vh1~ to Lv2\_Cmp1\_Vh1  
 Copy angles from Lv2\_Cmp1\_Vh2~ to Lv2\_Cmp1\_Vh2  
 Copy angles from Lv2\_Cmp1\_Vh3~ to Lv2\_Cmp1\_Vh3  
 Copy angles from Lv2\_Cmp1\_Vh4~ to Lv2\_Cmp1\_Vh4  
 Copy angles from Lv2\_Cmp1\_Vv1~ to Lv2\_Cmp1\_Vv1  
 Copy angles from Lv2\_Cmp1\_Vv2~ to Lv2\_Cmp1\_Vv2  
 Copy angles from Lv2\_Cmp1\_Vv3~ to Lv2\_Cmp1\_Vv3  
 Copy angles from Lv2\_Cmp1\_Vv4~ to Lv2\_Cmp1\_Vv4  
 Copy angles from Lv3\_Cmp1\_V0~ to Lv3\_Cmp1\_V0  
 Copy angles from Lv3\_Cmp1\_Vh1~ to Lv3\_Cmp1\_Vh1  
 Copy angles from Lv3\_Cmp1\_Vh2~ to Lv3\_Cmp1\_Vh2  
 Copy angles from Lv3\_Cmp1\_Vh3~ to Lv3\_Cmp1\_Vh3  
 Copy angles from Lv3\_Cmp1\_Vh4~ to Lv3\_Cmp1\_Vh4  
 Copy angles from Lv3\_Cmp1\_Vv1~ to Lv3\_Cmp1\_Vv1  
 Copy angles from Lv3\_Cmp1\_Vv2~ to Lv3\_Cmp1\_Vv2  
 Copy angles from Lv3\_Cmp1\_Vv3~ to Lv3\_Cmp1\_Vv3  
 Copy angles from Lv3\_Cmp1\_Vv4~ to Lv3\_Cmp1\_Vv4  
 Copy angles from Lv4\_Cmp1\_V0~ to Lv4\_Cmp1\_V0  
 Copy angles from Lv4\_Cmp1\_Vh1~ to Lv4\_Cmp1\_Vh1  
 Copy angles from Lv4\_Cmp1\_Vh2~ to Lv4\_Cmp1\_Vh2  
 Copy angles from Lv4\_Cmp1\_Vh3~ to Lv4\_Cmp1\_Vh3  
 Copy angles from Lv4\_Cmp1\_Vh4~ to Lv4\_Cmp1\_Vh4  
 Copy angles from Lv4\_Cmp1\_Vv1~ to Lv4\_Cmp1\_Vv1  
 Copy angles from Lv4\_Cmp1\_Vv2~ to Lv4\_Cmp1\_Vv2  
 Copy angles from Lv4\_Cmp1\_Vv3~ to Lv4\_Cmp1\_Vv3  
 Copy angles from Lv4\_Cmp1\_Vv4~ to Lv4\_Cmp1\_Vv4  
 "MSE: 3.2571e-14"

単一の GPU で学習中。

エポック	反復	経過時間 (h h : mm : s s)	ミニバッチ RMSE	ミニバッチ損失	基本学
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1	1	00 : 00 : 08	5. 08	12. 9	0. 0
13	50	00 : 02 : 15	5. 24	13. 8	0. 0
25	100	00 : 04 : 21	5. 13	13. 1	0. 0
30	120	00 : 05 : 13	5. 13	13. 1	0. 0

学習終了：最大エポック数が完了しました。

```
Copy angles from Lv1_Cmp1_V0~ to Lv1_Cmp1_V0
Copy angles from Lv1_Cmp1_Vh1~ to Lv1_Cmp1_Vh1
Copy angles from Lv1_Cmp1_Vh2~ to Lv1_Cmp1_Vh2
Copy angles from Lv1_Cmp1_Vh3~ to Lv1_Cmp1_Vh3
Copy angles from Lv1_Cmp1_Vh4~ to Lv1_Cmp1_Vh4
Copy angles from Lv1_Cmp1_Vv1~ to Lv1_Cmp1_Vv1
Copy angles from Lv1_Cmp1_Vv2~ to Lv1_Cmp1_Vv2
Copy angles from Lv1_Cmp1_Vv3~ to Lv1_Cmp1_Vv3
Copy angles from Lv1_Cmp1_Vv4~ to Lv1_Cmp1_Vv4
Copy angles from Lv2_Cmp1_V0~ to Lv2_Cmp1_V0
Copy angles from Lv2_Cmp1_Vh1~ to Lv2_Cmp1_Vh1
Copy angles from Lv2_Cmp1_Vh2~ to Lv2_Cmp1_Vh2
Copy angles from Lv2_Cmp1_Vh3~ to Lv2_Cmp1_Vh3
Copy angles from Lv2_Cmp1_Vh4~ to Lv2_Cmp1_Vh4
Copy angles from Lv2_Cmp1_Vv1~ to Lv2_Cmp1_Vv1
Copy angles from Lv2_Cmp1_Vv2~ to Lv2_Cmp1_Vv2
Copy angles from Lv2_Cmp1_Vv3~ to Lv2_Cmp1_Vv3
Copy angles from Lv2_Cmp1_Vv4~ to Lv2_Cmp1_Vv4
Copy angles from Lv3_Cmp1_V0~ to Lv3_Cmp1_V0
Copy angles from Lv3_Cmp1_Vh1~ to Lv3_Cmp1_Vh1
Copy angles from Lv3_Cmp1_Vh2~ to Lv3_Cmp1_Vh2
Copy angles from Lv3_Cmp1_Vh3~ to Lv3_Cmp1_Vh3
Copy angles from Lv3_Cmp1_Vh4~ to Lv3_Cmp1_Vh4
Copy angles from Lv3_Cmp1_Vv1~ to Lv3_Cmp1_Vv1
Copy angles from Lv3_Cmp1_Vv2~ to Lv3_Cmp1_Vv2
Copy angles from Lv3_Cmp1_Vv3~ to Lv3_Cmp1_Vv3
Copy angles from Lv3_Cmp1_Vv4~ to Lv3_Cmp1_Vv4
Copy angles from Lv4_Cmp1_V0~ to Lv4_Cmp1_V0
Copy angles from Lv4_Cmp1_Vh1~ to Lv4_Cmp1_Vh1
Copy angles from Lv4_Cmp1_Vh2~ to Lv4_Cmp1_Vh2
Copy angles from Lv4_Cmp1_Vh3~ to Lv4_Cmp1_Vh3
Copy angles from Lv4_Cmp1_Vh4~ to Lv4_Cmp1_Vh4
Copy angles from Lv4_Cmp1_Vv1~ to Lv4_Cmp1_Vv1
Copy angles from Lv4_Cmp1_Vv2~ to Lv4_Cmp1_Vv2
Copy angles from Lv4_Cmp1_Vv3~ to Lv4_Cmp1_Vv3
Copy angles from Lv4_Cmp1_Vv4~ to Lv4_Cmp1_Vv4
```

"MSE: 3.3624e-14"

単一の GPU で学習中。

エポック	反復	経過時間 (h h : mm : s s)	ミニバッチ RMSE	ミニバッチ損失	基本学習
1	1	00 : 00 : 08	4. 82	11. 6	0. 0
13	50	00 : 02 : 14	5. 14	13. 2	0. 0
25	100	00 : 04 : 21	4. 97	12. 4	0. 0
30	120	00 : 05 : 14	4. 98	12. 4	0. 0

学習終了：最大エポック数が完了しました。

```
Copy angles from Lv1_Cmp1_V0~ to Lv1_Cmp1_V0
Copy angles from Lv1_Cmp1_Vh1~ to Lv1_Cmp1_Vh1
Copy angles from Lv1_Cmp1_Vh2~ to Lv1_Cmp1_Vh2
Copy angles from Lv1_Cmp1_Vh3~ to Lv1_Cmp1_Vh3
Copy angles from Lv1_Cmp1_Vh4~ to Lv1_Cmp1_Vh4
Copy angles from Lv1_Cmp1_Vv1~ to Lv1_Cmp1_Vv1
Copy angles from Lv1_Cmp1_Vv2~ to Lv1_Cmp1_Vv2
Copy angles from Lv1_Cmp1_Vv3~ to Lv1_Cmp1_Vv3
Copy angles from Lv1_Cmp1_Vv4~ to Lv1_Cmp1_Vv4
Copy angles from Lv2_Cmp1_V0~ to Lv2_Cmp1_V0
```

```

Copy angles from Lv2_Cmp1_Vh1~ to Lv2_Cmp1_Vh1
Copy angles from Lv2_Cmp1_Vh2~ to Lv2_Cmp1_Vh2
Copy angles from Lv2_Cmp1_Vh3~ to Lv2_Cmp1_Vh3
Copy angles from Lv2_Cmp1_Vh4~ to Lv2_Cmp1_Vh4
Copy angles from Lv2_Cmp1_Vv1~ to Lv2_Cmp1_Vv1
Copy angles from Lv2_Cmp1_Vv2~ to Lv2_Cmp1_Vv2
Copy angles from Lv2_Cmp1_Vv3~ to Lv2_Cmp1_Vv3
Copy angles from Lv2_Cmp1_Vv4~ to Lv2_Cmp1_Vv4
Copy angles from Lv3_Cmp1_V0~ to Lv3_Cmp1_V0
Copy angles from Lv3_Cmp1_Vh1~ to Lv3_Cmp1_Vh1
Copy angles from Lv3_Cmp1_Vh2~ to Lv3_Cmp1_Vh2
Copy angles from Lv3_Cmp1_Vh3~ to Lv3_Cmp1_Vh3
Copy angles from Lv3_Cmp1_Vh4~ to Lv3_Cmp1_Vh4
Copy angles from Lv3_Cmp1_Vv1~ to Lv3_Cmp1_Vv1
Copy angles from Lv3_Cmp1_Vv2~ to Lv3_Cmp1_Vv2
Copy angles from Lv3_Cmp1_Vv3~ to Lv3_Cmp1_Vv3
Copy angles from Lv3_Cmp1_Vv4~ to Lv3_Cmp1_Vv4
Copy angles from Lv4_Cmp1_V0~ to Lv4_Cmp1_V0
Copy angles from Lv4_Cmp1_Vh1~ to Lv4_Cmp1_Vh1
Copy angles from Lv4_Cmp1_Vh2~ to Lv4_Cmp1_Vh2
Copy angles from Lv4_Cmp1_Vh3~ to Lv4_Cmp1_Vh3
Copy angles from Lv4_Cmp1_Vh4~ to Lv4_Cmp1_Vh4
Copy angles from Lv4_Cmp1_Vv1~ to Lv4_Cmp1_Vv1
Copy angles from Lv4_Cmp1_Vv2~ to Lv4_Cmp1_Vv2
Copy angles from Lv4_Cmp1_Vv3~ to Lv4_Cmp1_Vv3
Copy angles from Lv4_Cmp1_Vv4~ to Lv4_Cmp1_Vv4

```

"MSE: 2.8204e-14"

単一の GPU で学習中。

エポック	反復	経過時間 (h h : mm : s s)	ミニバッチ RMSE	ミニバッチ損失	基本学
1	1	00 : 00 : 08	4. 95	12. 3	0. 0
13	50	00 : 02 : 14	5. 02	12. 6	0. 0
25	100	00 : 04 : 19	4. 43	9. 8	0. 0
30	120	00 : 05 : 11	4. 88	11. 9	0. 0

学習終了: 最大エポック数が完了しました。

```

Copy angles from Lv1_Cmp1_V0~ to Lv1_Cmp1_V0
Copy angles from Lv1_Cmp1_Vh1~ to Lv1_Cmp1_Vh1
Copy angles from Lv1_Cmp1_Vh2~ to Lv1_Cmp1_Vh2
Copy angles from Lv1_Cmp1_Vh3~ to Lv1_Cmp1_Vh3
Copy angles from Lv1_Cmp1_Vh4~ to Lv1_Cmp1_Vh4
Copy angles from Lv1_Cmp1_Vv1~ to Lv1_Cmp1_Vv1
Copy angles from Lv1_Cmp1_Vv2~ to Lv1_Cmp1_Vv2
Copy angles from Lv1_Cmp1_Vv3~ to Lv1_Cmp1_Vv3
Copy angles from Lv1_Cmp1_Vv4~ to Lv1_Cmp1_Vv4
Copy angles from Lv2_Cmp1_V0~ to Lv2_Cmp1_V0
Copy angles from Lv2_Cmp1_Vh1~ to Lv2_Cmp1_Vh1
Copy angles from Lv2_Cmp1_Vh2~ to Lv2_Cmp1_Vh2
Copy angles from Lv2_Cmp1_Vh3~ to Lv2_Cmp1_Vh3
Copy angles from Lv2_Cmp1_Vh4~ to Lv2_Cmp1_Vh4
Copy angles from Lv2_Cmp1_Vv1~ to Lv2_Cmp1_Vv1
Copy angles from Lv2_Cmp1_Vv2~ to Lv2_Cmp1_Vv2
Copy angles from Lv2_Cmp1_Vv3~ to Lv2_Cmp1_Vv3
Copy angles from Lv2_Cmp1_Vv4~ to Lv2_Cmp1_Vv4
Copy angles from Lv3_Cmp1_V0~ to Lv3_Cmp1_V0
Copy angles from Lv3_Cmp1_Vh1~ to Lv3_Cmp1_Vh1
Copy angles from Lv3_Cmp1_Vh2~ to Lv3_Cmp1_Vh2
Copy angles from Lv3_Cmp1_Vh3~ to Lv3_Cmp1_Vh3
Copy angles from Lv3_Cmp1_Vh4~ to Lv3_Cmp1_Vh4
Copy angles from Lv3_Cmp1_Vv1~ to Lv3_Cmp1_Vv1
Copy angles from Lv3_Cmp1_Vv2~ to Lv3_Cmp1_Vv2
Copy angles from Lv3_Cmp1_Vv3~ to Lv3_Cmp1_Vv3

```



Copy angles from Lv3\_Cmp1\_Vv4~ to Lv3\_Cmp1\_Vv4  
 Copy angles from Lv4\_Cmp1\_V0~ to Lv4\_Cmp1\_V0  
 Copy angles from Lv4\_Cmp1\_Vh1~ to Lv4\_Cmp1\_Vh1  
 Copy angles from Lv4\_Cmp1\_Vh2~ to Lv4\_Cmp1\_Vh2  
 Copy angles from Lv4\_Cmp1\_Vh3~ to Lv4\_Cmp1\_Vh3  
 Copy angles from Lv4\_Cmp1\_Vh4~ to Lv4\_Cmp1\_Vh4  
 Copy angles from Lv4\_Cmp1\_Vv1~ to Lv4\_Cmp1\_Vv1  
 Copy angles from Lv4\_Cmp1\_Vv2~ to Lv4\_Cmp1\_Vv2  
 Copy angles from Lv4\_Cmp1\_Vv3~ to Lv4\_Cmp1\_Vv3  
 Copy angles from Lv4\_Cmp1\_Vv4~ to Lv4\_Cmp1\_Vv4

"MSE: 2.9844e-14"

単一の GPU で学習中。

エポック	反復	経過時間 (h h : mm : s s)	ミニバッチ RMSE	ミニバッチ損失	基本学
1	1	00 : 00 : 08	4.90	12.0	0.0
13	50	00 : 02 : 11	4.83	11.7	0.0
25	100	00 : 04 : 18	4.89	11.9	0.0
30	120	00 : 05 : 10	4.82	11.6	0.0

学習終了: 最大エポック数が完了しました。

Copy angles from Lv1\_Cmp1\_V0~ to Lv1\_Cmp1\_V0  
 Copy angles from Lv1\_Cmp1\_Vh1~ to Lv1\_Cmp1\_Vh1  
 Copy angles from Lv1\_Cmp1\_Vh2~ to Lv1\_Cmp1\_Vh2  
 Copy angles from Lv1\_Cmp1\_Vh3~ to Lv1\_Cmp1\_Vh3  
 Copy angles from Lv1\_Cmp1\_Vh4~ to Lv1\_Cmp1\_Vh4  
 Copy angles from Lv1\_Cmp1\_Vv1~ to Lv1\_Cmp1\_Vv1  
 Copy angles from Lv1\_Cmp1\_Vv2~ to Lv1\_Cmp1\_Vv2  
 Copy angles from Lv1\_Cmp1\_Vv3~ to Lv1\_Cmp1\_Vv3  
 Copy angles from Lv1\_Cmp1\_Vv4~ to Lv1\_Cmp1\_Vv4  
 Copy angles from Lv2\_Cmp1\_V0~ to Lv2\_Cmp1\_V0  
 Copy angles from Lv2\_Cmp1\_Vh1~ to Lv2\_Cmp1\_Vh1  
 Copy angles from Lv2\_Cmp1\_Vh2~ to Lv2\_Cmp1\_Vh2  
 Copy angles from Lv2\_Cmp1\_Vh3~ to Lv2\_Cmp1\_Vh3  
 Copy angles from Lv2\_Cmp1\_Vh4~ to Lv2\_Cmp1\_Vh4  
 Copy angles from Lv2\_Cmp1\_Vv1~ to Lv2\_Cmp1\_Vv1  
 Copy angles from Lv2\_Cmp1\_Vv2~ to Lv2\_Cmp1\_Vv2  
 Copy angles from Lv2\_Cmp1\_Vv3~ to Lv2\_Cmp1\_Vv3  
 Copy angles from Lv2\_Cmp1\_Vv4~ to Lv2\_Cmp1\_Vv4  
 Copy angles from Lv3\_Cmp1\_V0~ to Lv3\_Cmp1\_V0  
 Copy angles from Lv3\_Cmp1\_Vh1~ to Lv3\_Cmp1\_Vh1  
 Copy angles from Lv3\_Cmp1\_Vh2~ to Lv3\_Cmp1\_Vh2  
 Copy angles from Lv3\_Cmp1\_Vh3~ to Lv3\_Cmp1\_Vh3  
 Copy angles from Lv3\_Cmp1\_Vh4~ to Lv3\_Cmp1\_Vh4  
 Copy angles from Lv3\_Cmp1\_Vv1~ to Lv3\_Cmp1\_Vv1  
 Copy angles from Lv3\_Cmp1\_Vv2~ to Lv3\_Cmp1\_Vv2  
 Copy angles from Lv3\_Cmp1\_Vv3~ to Lv3\_Cmp1\_Vv3  
 Copy angles from Lv3\_Cmp1\_Vv4~ to Lv3\_Cmp1\_Vv4  
 Copy angles from Lv4\_Cmp1\_V0~ to Lv4\_Cmp1\_V0  
 Copy angles from Lv4\_Cmp1\_Vh1~ to Lv4\_Cmp1\_Vh1  
 Copy angles from Lv4\_Cmp1\_Vh2~ to Lv4\_Cmp1\_Vh2  
 Copy angles from Lv4\_Cmp1\_Vh3~ to Lv4\_Cmp1\_Vh3  
 Copy angles from Lv4\_Cmp1\_Vh4~ to Lv4\_Cmp1\_Vh4  
 Copy angles from Lv4\_Cmp1\_Vv1~ to Lv4\_Cmp1\_Vv1  
 Copy angles from Lv4\_Cmp1\_Vv2~ to Lv4\_Cmp1\_Vv2  
 Copy angles from Lv4\_Cmp1\_Vv3~ to Lv4\_Cmp1\_Vv3  
 Copy angles from Lv4\_Cmp1\_Vv4~ to Lv4\_Cmp1\_Vv4

"MSE: 2.649e-14"

単一の GPU で学習中。

エポック	反復	経過時間 (h h : mm : s s)	ミニバッチ RMSE	ミニバッチ損失	基本学
------	----	--------------------------	------------	---------	-----

1	1	00 : 00 : 08	4. 90	12. 0	0. 0
13	50	00 : 02 : 15	4. 95	12. 2	0. 0
25	100	00 : 04 : 22	4. 82	11. 6	0. 0
30	120	00 : 05 : 13	4. 83	11. 7	0. 0

学習終了：最大エポック数が完了しました。

```
Copy angles from Lv1_Cmp1_V0~ to Lv1_Cmp1_V0
Copy angles from Lv1_Cmp1_Vh1~ to Lv1_Cmp1_Vh1
Copy angles from Lv1_Cmp1_Vh2~ to Lv1_Cmp1_Vh2
Copy angles from Lv1_Cmp1_Vh3~ to Lv1_Cmp1_Vh3
Copy angles from Lv1_Cmp1_Vh4~ to Lv1_Cmp1_Vh4
Copy angles from Lv1_Cmp1_Vv1~ to Lv1_Cmp1_Vv1
Copy angles from Lv1_Cmp1_Vv2~ to Lv1_Cmp1_Vv2
Copy angles from Lv1_Cmp1_Vv3~ to Lv1_Cmp1_Vv3
Copy angles from Lv1_Cmp1_Vv4~ to Lv1_Cmp1_Vv4
Copy angles from Lv2_Cmp1_V0~ to Lv2_Cmp1_V0
Copy angles from Lv2_Cmp1_Vh1~ to Lv2_Cmp1_Vh1
Copy angles from Lv2_Cmp1_Vh2~ to Lv2_Cmp1_Vh2
Copy angles from Lv2_Cmp1_Vh3~ to Lv2_Cmp1_Vh3
Copy angles from Lv2_Cmp1_Vh4~ to Lv2_Cmp1_Vh4
Copy angles from Lv2_Cmp1_Vv1~ to Lv2_Cmp1_Vv1
Copy angles from Lv2_Cmp1_Vv2~ to Lv2_Cmp1_Vv2
Copy angles from Lv2_Cmp1_Vv3~ to Lv2_Cmp1_Vv3
Copy angles from Lv2_Cmp1_Vv4~ to Lv2_Cmp1_Vv4
Copy angles from Lv3_Cmp1_V0~ to Lv3_Cmp1_V0
Copy angles from Lv3_Cmp1_Vh1~ to Lv3_Cmp1_Vh1
Copy angles from Lv3_Cmp1_Vh2~ to Lv3_Cmp1_Vh2
Copy angles from Lv3_Cmp1_Vh3~ to Lv3_Cmp1_Vh3
Copy angles from Lv3_Cmp1_Vh4~ to Lv3_Cmp1_Vh4
Copy angles from Lv3_Cmp1_Vv1~ to Lv3_Cmp1_Vv1
Copy angles from Lv3_Cmp1_Vv2~ to Lv3_Cmp1_Vv2
Copy angles from Lv3_Cmp1_Vv3~ to Lv3_Cmp1_Vv3
Copy angles from Lv3_Cmp1_Vv4~ to Lv3_Cmp1_Vv4
Copy angles from Lv4_Cmp1_V0~ to Lv4_Cmp1_V0
Copy angles from Lv4_Cmp1_Vh1~ to Lv4_Cmp1_Vh1
Copy angles from Lv4_Cmp1_Vh2~ to Lv4_Cmp1_Vh2
Copy angles from Lv4_Cmp1_Vh3~ to Lv4_Cmp1_Vh3
Copy angles from Lv4_Cmp1_Vh4~ to Lv4_Cmp1_Vh4
Copy angles from Lv4_Cmp1_Vv1~ to Lv4_Cmp1_Vv1
Copy angles from Lv4_Cmp1_Vv2~ to Lv4_Cmp1_Vv2
Copy angles from Lv4_Cmp1_Vv3~ to Lv4_Cmp1_Vv3
Copy angles from Lv4_Cmp1_Vv4~ to Lv4_Cmp1_Vv4
```

"MSE: 3.6517e-14"

単一の GPU で学習中。

エポック	反復	経過時間 (h h : mm : s s)	ミニバッチ RMSE	ミニバッチ損失	基本学習
1	1	00 : 00 : 08	5. 00	12. 5	0. 0
13	50	00 : 02 : 16	5. 16	13. 3	0. 0
25	100	00 : 04 : 23	4. 78	11. 4	0. 0
30	120	00 : 05 : 15	4. 90	12. 0	0. 0

学習終了：最大エポック数が完了しました。

```
Copy angles from Lv1_Cmp1_V0~ to Lv1_Cmp1_V0
Copy angles from Lv1_Cmp1_Vh1~ to Lv1_Cmp1_Vh1
Copy angles from Lv1_Cmp1_Vh2~ to Lv1_Cmp1_Vh2
Copy angles from Lv1_Cmp1_Vh3~ to Lv1_Cmp1_Vh3
Copy angles from Lv1_Cmp1_Vh4~ to Lv1_Cmp1_Vh4
Copy angles from Lv1_Cmp1_Vv1~ to Lv1_Cmp1_Vv1
Copy angles from Lv1_Cmp1_Vv2~ to Lv1_Cmp1_Vv2
Copy angles from Lv1_Cmp1_Vv3~ to Lv1_Cmp1_Vv3
Copy angles from Lv1_Cmp1_Vv4~ to Lv1_Cmp1_Vv4
Copy angles from Lv2_Cmp1_V0~ to Lv2_Cmp1_V0
```

```

Copy angles from Lv2_Cmp1_Vh1~ to Lv2_Cmp1_Vh1
Copy angles from Lv2_Cmp1_Vh2~ to Lv2_Cmp1_Vh2
Copy angles from Lv2_Cmp1_Vh3~ to Lv2_Cmp1_Vh3
Copy angles from Lv2_Cmp1_Vh4~ to Lv2_Cmp1_Vh4
Copy angles from Lv2_Cmp1_Vv1~ to Lv2_Cmp1_Vv1
Copy angles from Lv2_Cmp1_Vv2~ to Lv2_Cmp1_Vv2
Copy angles from Lv2_Cmp1_Vv3~ to Lv2_Cmp1_Vv3
Copy angles from Lv2_Cmp1_Vv4~ to Lv2_Cmp1_Vv4
Copy angles from Lv3_Cmp1_V0~ to Lv3_Cmp1_V0
Copy angles from Lv3_Cmp1_Vh1~ to Lv3_Cmp1_Vh1
Copy angles from Lv3_Cmp1_Vh2~ to Lv3_Cmp1_Vh2
Copy angles from Lv3_Cmp1_Vh3~ to Lv3_Cmp1_Vh3
Copy angles from Lv3_Cmp1_Vh4~ to Lv3_Cmp1_Vh4
Copy angles from Lv3_Cmp1_Vv1~ to Lv3_Cmp1_Vv1
Copy angles from Lv3_Cmp1_Vv2~ to Lv3_Cmp1_Vv2
Copy angles from Lv3_Cmp1_Vv3~ to Lv3_Cmp1_Vv3
Copy angles from Lv3_Cmp1_Vv4~ to Lv3_Cmp1_Vv4
Copy angles from Lv4_Cmp1_V0~ to Lv4_Cmp1_V0
Copy angles from Lv4_Cmp1_Vh1~ to Lv4_Cmp1_Vh1
Copy angles from Lv4_Cmp1_Vh2~ to Lv4_Cmp1_Vh2
Copy angles from Lv4_Cmp1_Vh3~ to Lv4_Cmp1_Vh3
Copy angles from Lv4_Cmp1_Vh4~ to Lv4_Cmp1_Vh4
Copy angles from Lv4_Cmp1_Vv1~ to Lv4_Cmp1_Vv1
Copy angles from Lv4_Cmp1_Vv2~ to Lv4_Cmp1_Vv2
Copy angles from Lv4_Cmp1_Vv3~ to Lv4_Cmp1_Vv3
Copy angles from Lv4_Cmp1_Vv4~ to Lv4_Cmp1_Vv4

```

"MSE: 2.4553e-14"

単一の GPU で学習中。

エポック	反復	経過時間 (h h : mm : s s)	ミニバッチ RMSE	ミニバッチ損失	基本学
1	1	00 : 00 : 08	4. 85	11. 8	0. 0
13	50	00 : 02 : 15	5. 18	13. 4	0. 0
25	100	00 : 04 : 22	4. 84	11. 7	0. 0
30	120	00 : 05 : 13	4. 95	12. 2	0. 0

学習終了: 最大エポック数が完了しました。

```

Copy angles from Lv1_Cmp1_V0~ to Lv1_Cmp1_V0
Copy angles from Lv1_Cmp1_Vh1~ to Lv1_Cmp1_Vh1
Copy angles from Lv1_Cmp1_Vh2~ to Lv1_Cmp1_Vh2
Copy angles from Lv1_Cmp1_Vh3~ to Lv1_Cmp1_Vh3
Copy angles from Lv1_Cmp1_Vh4~ to Lv1_Cmp1_Vh4
Copy angles from Lv1_Cmp1_Vv1~ to Lv1_Cmp1_Vv1
Copy angles from Lv1_Cmp1_Vv2~ to Lv1_Cmp1_Vv2
Copy angles from Lv1_Cmp1_Vv3~ to Lv1_Cmp1_Vv3
Copy angles from Lv1_Cmp1_Vv4~ to Lv1_Cmp1_Vv4
Copy angles from Lv2_Cmp1_V0~ to Lv2_Cmp1_V0
Copy angles from Lv2_Cmp1_Vh1~ to Lv2_Cmp1_Vh1
Copy angles from Lv2_Cmp1_Vh2~ to Lv2_Cmp1_Vh2
Copy angles from Lv2_Cmp1_Vh3~ to Lv2_Cmp1_Vh3
Copy angles from Lv2_Cmp1_Vh4~ to Lv2_Cmp1_Vh4
Copy angles from Lv2_Cmp1_Vv1~ to Lv2_Cmp1_Vv1
Copy angles from Lv2_Cmp1_Vv2~ to Lv2_Cmp1_Vv2
Copy angles from Lv2_Cmp1_Vv3~ to Lv2_Cmp1_Vv3
Copy angles from Lv2_Cmp1_Vv4~ to Lv2_Cmp1_Vv4
Copy angles from Lv3_Cmp1_V0~ to Lv3_Cmp1_V0
Copy angles from Lv3_Cmp1_Vh1~ to Lv3_Cmp1_Vh1
Copy angles from Lv3_Cmp1_Vh2~ to Lv3_Cmp1_Vh2
Copy angles from Lv3_Cmp1_Vh3~ to Lv3_Cmp1_Vh3
Copy angles from Lv3_Cmp1_Vh4~ to Lv3_Cmp1_Vh4
Copy angles from Lv3_Cmp1_Vv1~ to Lv3_Cmp1_Vv1
Copy angles from Lv3_Cmp1_Vv2~ to Lv3_Cmp1_Vv2
Copy angles from Lv3_Cmp1_Vv3~ to Lv3_Cmp1_Vv3

```

Copy angles from Lv3\_Cmp1\_Vv4~ to Lv3\_Cmp1\_Vv4  
 Copy angles from Lv4\_Cmp1\_V0~ to Lv4\_Cmp1\_V0  
 Copy angles from Lv4\_Cmp1\_Vh1~ to Lv4\_Cmp1\_Vh1  
 Copy angles from Lv4\_Cmp1\_Vh2~ to Lv4\_Cmp1\_Vh2  
 Copy angles from Lv4\_Cmp1\_Vh3~ to Lv4\_Cmp1\_Vh3  
 Copy angles from Lv4\_Cmp1\_Vh4~ to Lv4\_Cmp1\_Vh4  
 Copy angles from Lv4\_Cmp1\_Vv1~ to Lv4\_Cmp1\_Vv1  
 Copy angles from Lv4\_Cmp1\_Vv2~ to Lv4\_Cmp1\_Vv2  
 Copy angles from Lv4\_Cmp1\_Vv3~ to Lv4\_Cmp1\_Vv3  
 Copy angles from Lv4\_Cmp1\_Vv4~ to Lv4\_Cmp1\_Vv4  
 "MSE: 2.2377e-14"

単一の GPU で学習中。

エポック	反復	経過時間 (h h : mm : s s)	ミニバッチ RMSE	ミニバッチ損失	基本
1	1	00 : 00 : 08	5.09	13.0	0.0
13	50	00 : 02 : 15	4.85	11.8	0.0
25	100	00 : 04 : 24	5.05	12.7	0.0
30	120	00 : 05 : 15	4.85	11.8	0.0

学習終了: 最大エポック数が完了しました。

Copy angles from Lv1\_Cmp1\_V0~ to Lv1\_Cmp1\_V0  
 Copy angles from Lv1\_Cmp1\_Vh1~ to Lv1\_Cmp1\_Vh1  
 Copy angles from Lv1\_Cmp1\_Vh2~ to Lv1\_Cmp1\_Vh2  
 Copy angles from Lv1\_Cmp1\_Vh3~ to Lv1\_Cmp1\_Vh3  
 Copy angles from Lv1\_Cmp1\_Vh4~ to Lv1\_Cmp1\_Vh4  
 Copy angles from Lv1\_Cmp1\_Vv1~ to Lv1\_Cmp1\_Vv1  
 Copy angles from Lv1\_Cmp1\_Vv2~ to Lv1\_Cmp1\_Vv2  
 Copy angles from Lv1\_Cmp1\_Vv3~ to Lv1\_Cmp1\_Vv3  
 Copy angles from Lv1\_Cmp1\_Vv4~ to Lv1\_Cmp1\_Vv4  
 Copy angles from Lv2\_Cmp1\_V0~ to Lv2\_Cmp1\_V0  
 Copy angles from Lv2\_Cmp1\_Vh1~ to Lv2\_Cmp1\_Vh1  
 Copy angles from Lv2\_Cmp1\_Vh2~ to Lv2\_Cmp1\_Vh2  
 Copy angles from Lv2\_Cmp1\_Vh3~ to Lv2\_Cmp1\_Vh3  
 Copy angles from Lv2\_Cmp1\_Vh4~ to Lv2\_Cmp1\_Vh4  
 Copy angles from Lv2\_Cmp1\_Vv1~ to Lv2\_Cmp1\_Vv1  
 Copy angles from Lv2\_Cmp1\_Vv2~ to Lv2\_Cmp1\_Vv2  
 Copy angles from Lv2\_Cmp1\_Vv3~ to Lv2\_Cmp1\_Vv3  
 Copy angles from Lv2\_Cmp1\_Vv4~ to Lv2\_Cmp1\_Vv4  
 Copy angles from Lv3\_Cmp1\_V0~ to Lv3\_Cmp1\_V0  
 Copy angles from Lv3\_Cmp1\_Vh1~ to Lv3\_Cmp1\_Vh1  
 Copy angles from Lv3\_Cmp1\_Vh2~ to Lv3\_Cmp1\_Vh2  
 Copy angles from Lv3\_Cmp1\_Vh3~ to Lv3\_Cmp1\_Vh3  
 Copy angles from Lv3\_Cmp1\_Vh4~ to Lv3\_Cmp1\_Vh4  
 Copy angles from Lv3\_Cmp1\_Vv1~ to Lv3\_Cmp1\_Vv1  
 Copy angles from Lv3\_Cmp1\_Vv2~ to Lv3\_Cmp1\_Vv2  
 Copy angles from Lv3\_Cmp1\_Vv3~ to Lv3\_Cmp1\_Vv3  
 Copy angles from Lv3\_Cmp1\_Vv4~ to Lv3\_Cmp1\_Vv4  
 Copy angles from Lv4\_Cmp1\_V0~ to Lv4\_Cmp1\_V0  
 Copy angles from Lv4\_Cmp1\_Vh1~ to Lv4\_Cmp1\_Vh1  
 Copy angles from Lv4\_Cmp1\_Vh2~ to Lv4\_Cmp1\_Vh2  
 Copy angles from Lv4\_Cmp1\_Vh3~ to Lv4\_Cmp1\_Vh3  
 Copy angles from Lv4\_Cmp1\_Vh4~ to Lv4\_Cmp1\_Vh4  
 Copy angles from Lv4\_Cmp1\_Vv1~ to Lv4\_Cmp1\_Vv1  
 Copy angles from Lv4\_Cmp1\_Vv2~ to Lv4\_Cmp1\_Vv2  
 Copy angles from Lv4\_Cmp1\_Vv3~ to Lv4\_Cmp1\_Vv3  
 Copy angles from Lv4\_Cmp1\_Vv4~ to Lv4\_Cmp1\_Vv4  
 "MSE: 2.5301e-14"

Copy angles from Lv1\_Cmp1\_V0~ to Lv1\_Cmp1\_V0  
 Copy angles from Lv1\_Cmp1\_Vh1~ to Lv1\_Cmp1\_Vh1  
 Copy angles from Lv1\_Cmp1\_Vh2~ to Lv1\_Cmp1\_Vh2  
 Copy angles from Lv1\_Cmp1\_Vh3~ to Lv1\_Cmp1\_Vh3  
 Copy angles from Lv1\_Cmp1\_Vh4~ to Lv1\_Cmp1\_Vh4

```

Copy angles from Lv1_Cmp1_Vv1~ to Lv1_Cmp1_Vv1
Copy angles from Lv1_Cmp1_Vv2~ to Lv1_Cmp1_Vv2
Copy angles from Lv1_Cmp1_Vv3~ to Lv1_Cmp1_Vv3
Copy angles from Lv1_Cmp1_Vv4~ to Lv1_Cmp1_Vv4
Copy angles from Lv2_Cmp1_V0~ to Lv2_Cmp1_V0
Copy angles from Lv2_Cmp1_Vh1~ to Lv2_Cmp1_Vh1
Copy angles from Lv2_Cmp1_Vh2~ to Lv2_Cmp1_Vh2
Copy angles from Lv2_Cmp1_Vh3~ to Lv2_Cmp1_Vh3
Copy angles from Lv2_Cmp1_Vh4~ to Lv2_Cmp1_Vh4
Copy angles from Lv2_Cmp1_Vv1~ to Lv2_Cmp1_Vv1
Copy angles from Lv2_Cmp1_Vv2~ to Lv2_Cmp1_Vv2
Copy angles from Lv2_Cmp1_Vv3~ to Lv2_Cmp1_Vv3
Copy angles from Lv2_Cmp1_Vv4~ to Lv2_Cmp1_Vv4
Copy angles from Lv3_Cmp1_V0~ to Lv3_Cmp1_V0
Copy angles from Lv3_Cmp1_Vh1~ to Lv3_Cmp1_Vh1
Copy angles from Lv3_Cmp1_Vh2~ to Lv3_Cmp1_Vh2
Copy angles from Lv3_Cmp1_Vh3~ to Lv3_Cmp1_Vh3
Copy angles from Lv3_Cmp1_Vh4~ to Lv3_Cmp1_Vh4
Copy angles from Lv3_Cmp1_Vv1~ to Lv3_Cmp1_Vv1
Copy angles from Lv3_Cmp1_Vv2~ to Lv3_Cmp1_Vv2
Copy angles from Lv3_Cmp1_Vv3~ to Lv3_Cmp1_Vv3
Copy angles from Lv3_Cmp1_Vv4~ to Lv3_Cmp1_Vv4
Copy angles from Lv4_Cmp1_V0~ to Lv4_Cmp1_V0
Copy angles from Lv4_Cmp1_Vh1~ to Lv4_Cmp1_Vh1
Copy angles from Lv4_Cmp1_Vh2~ to Lv4_Cmp1_Vh2
Copy angles from Lv4_Cmp1_Vh3~ to Lv4_Cmp1_Vh3
Copy angles from Lv4_Cmp1_Vh4~ to Lv4_Cmp1_Vh4
Copy angles from Lv4_Cmp1_Vv1~ to Lv4_Cmp1_Vv1
Copy angles from Lv4_Cmp1_Vv2~ to Lv4_Cmp1_Vv2
Copy angles from Lv4_Cmp1_Vv3~ to Lv4_Cmp1_Vv3
Copy angles from Lv4_Cmp1_Vv4~ to Lv4_Cmp1_Vv4

```

```

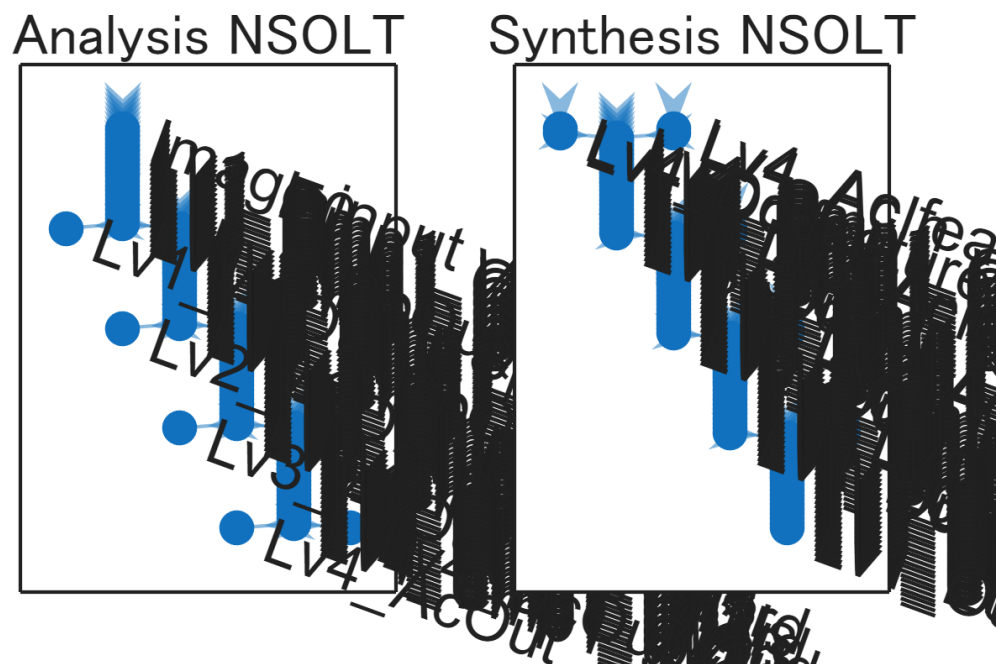
analysislgraph = layerGraph(analysisnet);
synthesislgraph = layerGraph(synthesisnet);

```

```

figure
subplot(1,2,1)
plot(analysislgraph)
title('Analysis NSOLT')
subplot(1,2,2)
plot(synthesislgraph)
title('Synthesis NSOLT')

```

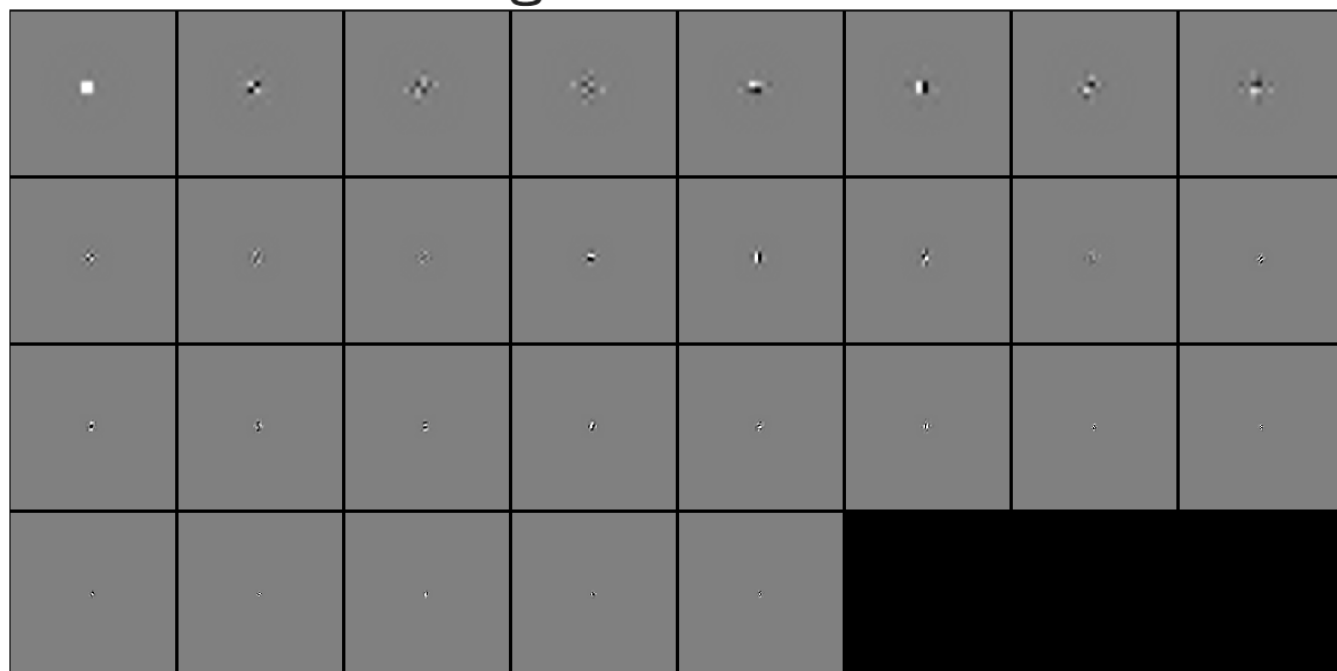


### 要素画像の表示

```
import saivdr.dcn.*

figure

atomicimshow(synthesisnet,[],2^(nLevels-1))
title('Atomic images of trained NSOLT')
```



## 推論用 NSOLT ネットワークの構築

```
% Assemble analyzer
analysislgraph4predict = analysislgraph;
analysislgraph4predict = analysislgraph4predict.replaceLayer('Image input',...
    imageInputLayer(szOrg, 'Name', 'Image input', 'Normalization', 'none'));
for iLayer = 1:height(analysislgraph4predict.Layers)
    layer = analysislgraph4predict.Layers(iLayer);
    if contains(layer.Name, "Lv"+nLevels+"_DcOut") || ...
        ~isempty(regexp(layer.Name, '^Lv\d+_AcOut', 'once'))
        analysislgraph4predict = analysislgraph4predict.replaceLayer(layer.Name, ...
            regressionLayer('Name', layer.Name));
    end
end
analysisnet4predict = assembleNetwork(analysislgraph4predict);

% Assemble synthesizer
synthesislgraph4predict = synthesislgraph;
synthesislgraph4predict = synthesislgraph4predict.replaceLayer('Lv1_Out', ...
    regressionLayer('Name', 'Lv1_Out'));
for iLayer = 1:height(synthesislgraph4predict.Layers)
    layer = synthesislgraph4predict.Layers(iLayer);
    if contains(layer.Name, 'Ac feature input')
        iLv = str2double(layer.Name(3));
        sbSize = szOrg.*(decFactor.^(-iLv));
        newlayer = ...
            imageInputLayer([sbSize
(sum(nChannels)-1)], 'Name', layer.Name, 'Normalization', 'none');
        synthesislgraph4predict = synthesislgraph4predict.replaceLayer(...
            layer.Name, newlayer);
    elseif contains(layer.Name, sprintf('Lv%0d_Dc feature input', nLevels))
        iLv = str2double(layer.Name(3));
        sbSize = szOrg.*(decFactor.^(-iLv));
        newlayer = ...
            imageInputLayer([sbSize 1], 'Name', layer.Name, 'Normalization', 'none');
        synthesislgraph4predict = synthesislgraph4predict.replaceLayer(...
            layer.Name, newlayer);
    end
end
synthesisnet4predict = assembleNetwork(synthesislgraph4predict);
```

### 随伴関係（完全再構成）の確認

NSOLT はパーセバルタイト性を満たす.

```
u = rand(szOrg, 'single');
[s{1:nLevels+1}] = analysisnet4predict.predict(u);
v = synthesisnet4predict.predict(s{1:nLevels+1});
assert(mse(u,v)<1e-9)
```

## NSOLT による合成処理とその随伴処理の定義

```
nsoltconfig.nLevels = nLevels;
szCoefs = zeros(nLevels+1,3);
for iLevel = 1:nLevels+1
    s_iLevel = s{iLevel};
    szCoefs(iLevel,1) = size(s_iLevel,1);
    szCoefs(iLevel,2) = size(s_iLevel,2);
    szCoefs(iLevel,3) = size(s_iLevel,3);
end
nsoltconfig.szCoefs = szCoefs;
syn_nsolt = @(x) synthesisnsolt(x,synthesisnet4predict,nsoltconfig);
adj_nsolt = @(y) analysisnsolt(y,analysisnet4predict,nsoltconfig);
```

### 随伴関係の確認

```
x = adj_nsolt(y);
v = randn(size(x));
u = syn_nsolt(v);
assert(abs(dot(y(:),u(:))-dot(x(:),v(:)))<1e-3)
```

## 繰返しハード閾値処理(IHT)によるスパース近似の比較

### 辞書の準備

```
blkdctwon = { syn_blkdst, adj_blkdst, "Block DCTwoN", false };
blkdst = { syn_blkdst, adj_blkdst, "Block DCT", true };
blkpcawon = { syn_blkpca, adj_blkpca, "Block PCAwon", false };
blkpca = { syn_blkpca, adj_blkpca, "Block PCA", true };
blkrica = { syn_blkrica, adj_blkrica, "Block RICA", true };
blkksvd = { syn_blkksvd, adj_blkksvd, "Block K-SVD", true };
nsoltwon = { syn_nsolt, adj_nsolt, "NSOLTwon", false };
nsolt = { syn_nsolt, adj_nsolt, "NSOLT", true };
dicset = { blkdctwon, blkdst, blkpcawon, blkpca, blkrica, blkksvd, nsoltwon,
nsolt };
nDics = length(dicset);
```

### IHT

$$\mathbf{x}^{(t+1)} \leftarrow \mathcal{H}_{BK}(\mathbf{x}^{(t)} + \mu^{(t)} \hat{\mathbf{D}}^T(\mathbf{y} - \hat{\mathbf{D}}\mathbf{x}^{(t)}))$$

$$t \leftarrow t + 1$$

- T. Blumensath and M. E. Davies, "Normalized Iterative Hard Thresholding: Guaranteed Stability and Performance," in IEEE Journal of Selected Topics in Signal Processing, vol. 4, no. 2, pp. 298-309, April 2010, doi: 10.1109/JSTSP.2010.2042411.

```
nItersIht = 2000;
```

% 平均値を引いた画像を用意（近似後に平均値を加算）



```

ymean = mean(yorg,"all");
y = yorg - ymean;
% 準備
c = 1e-3;
kappa = 1.1/(1-c);
nCoefs = floor(sparsityRatio*prod(szOrg));
psnrs = zeros(nItersIht,nDics);
ssims = zeros(nItersIht,nDics);
yaprxs = cell(1,nDics);
% 繰り返し処理
for iDic = 1:nDics
    dic_ = dicset{iDic};
    synproc = dic_{1};
    adjproc = dic_{2};
    dicname = dic_{3};
    isStepSizeNormalized = dic_{4};
    % IHT
    display(dicname)
    s = adjproc(y); %  $D^T y$ 
    xt = zeros(size(s),'like',s); %  $x_1 = 0$ ;
    if isStepSizeNormalized % 正規化あり
        suppt = find(hardthresh(s,nCoefs)); %  $\Gamma_1 = \text{supp}(H_K(D^T y))$ 
        maskt = (abs(s)~=0);
    end
    for iIter=1:nItersIht
        % Gradient descent
        gt = adjproc(y-synproc(xt)); %  $g = D^T(y - D x_n)$ 
        if ~isStepSizeNormalized % 正規化なし
            mu = (1-c);
            xtp1 = hardthresh(xt+mu*gt,nCoefs);
        else % 正規化あり
            ggt = gt(suppt); %  $g_{\Gamma_n}$ 
            ugt = synproc(maskt.*gt); %  $D_{\Gamma_n}^T g_{\Gamma_n}$ 
            mu = (ggt.'*ggt)/(ugt(:).'*ugt(:));
            ttp1 = hardthresh(xt+mu*gt,nCoefs); %  $\tilde{x}_{n+1} = H_K(x_n + \mu_n g_n)$ 
            supptp1 = find(ttp1); %  $\Gamma_{n+1} = \text{supp}(\tilde{x}_{n+1})$ 
            if length(supptp1)==length(suppt) && all(supptp1==suppt)
                xtp1 = ttp1; %  $x_{n+1} = \tilde{x}_{n+1}$ 
            else
                dxt = ttp1-xt; %  $\tilde{x}_{n+1} - x_n$ 
                omega = (1-c)*(norm(dxt,'fro')/norm(synproc(dxt),'fro'))^2;
                if mu <= omega
                    xtp1 = ttp1; %  $x_{n+1} = \tilde{x}_{n+1}$ 
                else
                    while mu > omega
                        mu = mu/(kappa*(1-c));
                        ttp1 = hardthresh(xt+mu*gt,nCoefs); %  $\tilde{x}_{n+1} = H_K(x_n + \mu_n$ 
                    gn)

                    dxt = ttp1-xt; %  $\tilde{x}_{n+1} - x_n$ 
                    omega = (1-c)*(norm(dxt,'fro')/norm(synproc(dxt),'fro'))^2;

```

```

        end
        supptp1 = find(ttp1); %  $\Gamma_{n+1} = \text{supp}(\sim x_{n+1})$ 
        xtp1 = ttp1; %  $x_{n+1} = \sim x_{n+1}$ 
    end
    end
    % Update
    suppt = supptp1;
    maskt = zeros(size(maskt), 'like', maskt);
    maskt(suppt) = 1;
end
xt = xtp1;
% Monitoring
checkSparsity = nnz(xt)/prod(szOrg)<=sparsityRatio;
assert(checkSparsity)
yaprx_ = synproc(xt);
psnr_ = psnr(cast(yaprx_, 'like', y), y);
ssim_ = ssim(cast(yaprx_, 'like', y), y);
psnrs(iIter, iDic) = psnr_;
ssims(iIter, iDic) = ssim_;
%fprintf("IHT(%d) PSNR: %6.4f\n", iIter, psnr_);
end
yaprxs{iDic} = yaprx_ + ymean;
end

```

```

dicname =
"Block DCTwoN"
dicname =
"Block DCT"
dicname =
"Block PCAwoN"
dicname =
"Block PCA"
dicname =
"Block RICA"
dicname =
"Block K-SVD"
dicname =
"NSOLTwon"
dicname =
"NSOLT"

```

## 近似結果の表示

```

dicnames =
[blkdctwon{3}, blkdct{3}, blkpcawon{3}, blkpca{3}, blkrica{3}, blkksvd{3}, nsoltwon{3}, nsolt{3}];
psnrtbl = array2table(psnrs, 'VariableNames', dicnames);
psnrtbl = horzcat(table((1:nItrsIht).', 'VariableNames', "Iterations"), psnrtbl);
ssimtbl = array2table(ssims, 'VariableNames', dicnames);
ssimtbl = horzcat(table((1:nItrsIht).', 'VariableNames', "Iterations"), ssimtbl);

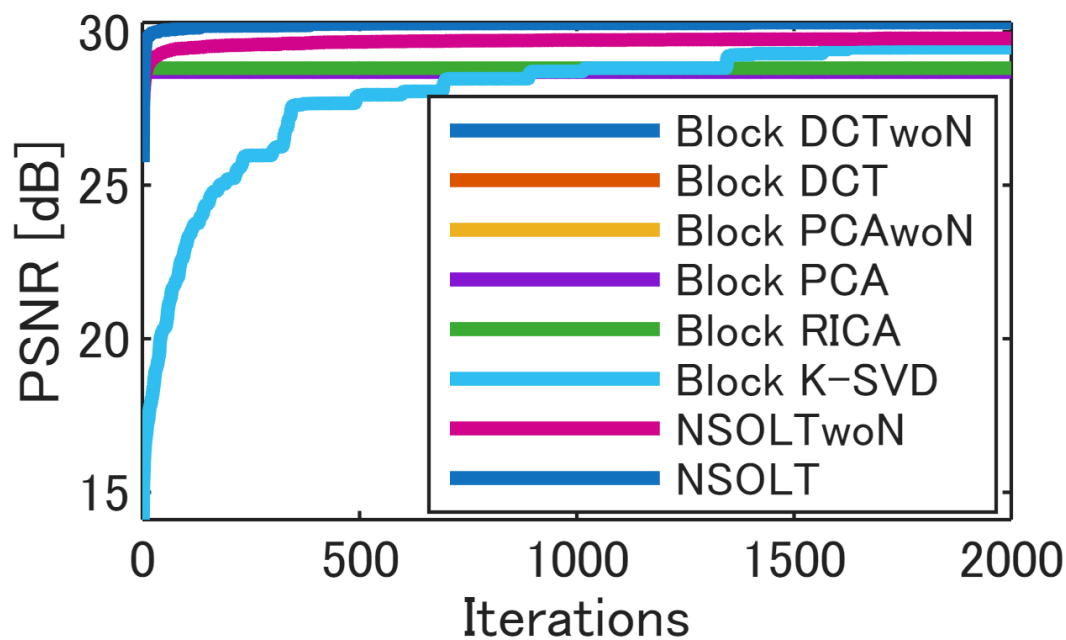
% PSNR のグラフ
figure

```

```

plot(psnrtbl,"Iterations",dicnames,'LineWidth',2)
ylabel('PSNR [dB]')
legend('Location','best')

```

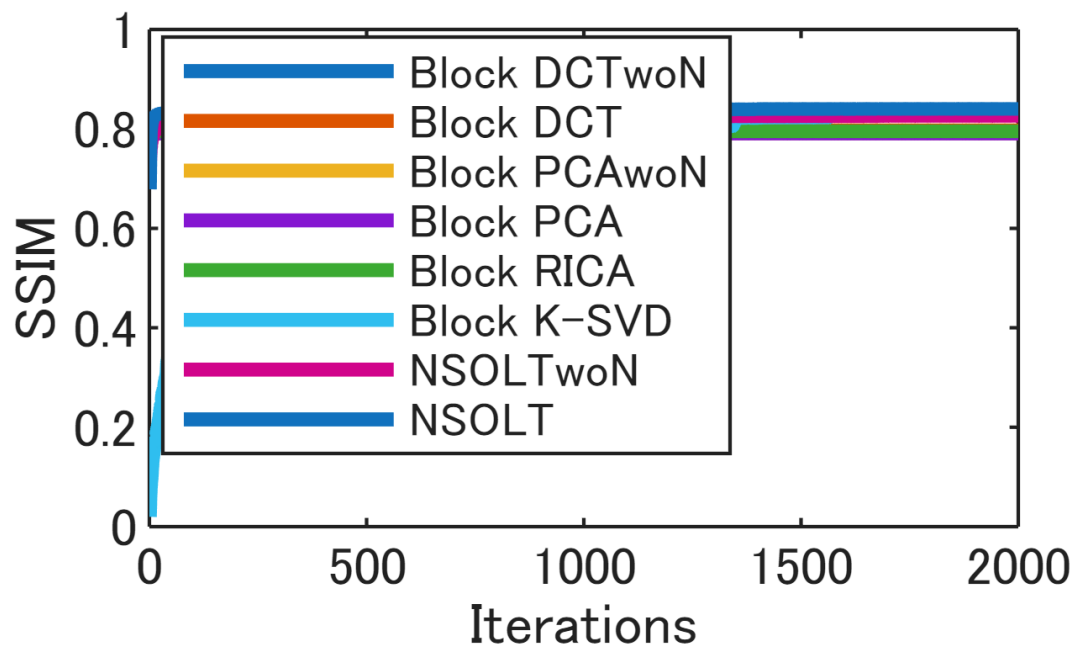


% SSIM のグラフ

```

figure
plot(ssimtbl,"Iterations",dicnames,'LineWidth',2)
ylabel('SSIM')
legend('Location','best')

```



```

% 原画像の表示
figure
tiledlayout(2,ceil((nDics+1)/2))
nexttile
imshow(yorg)
title("Original image")
% 近似画像の表示
for idx = 1:nDics
    yaprx = yaprxs{idx};
    dicname = dicnames(idx)
    file_yaprx = "../results/yaprx_" + replace(lower(dicname),' ','_') + ".png";
    imwrite(yaprx,file_yaprx)
    %
    nexttile
    imshow(yaprxs{idx})
    title(dicname+" "+num2str(psnrs(end,idx))+" dB")
end

```

```

dicname =
"Block DCTwoN"
dicname =
"Block DCT"
dicname =
"Block PCAwoN"
dicname =
"Block PCA"
dicname =
"Block RICA"
dicname =
"Block K-SVD"
dicname =
"NSOLTwoN"
dicname =
"NSOLT"

```

Original image Block DCTwoN Block DCT Block PCAwoN Block PCA Block RICA Block K-SVD NSOLTwoN NSOLT



< RECC 28.8066 29.4761 29.8030 29.62 dB



## 【関数定義】

### NSOLT 合成処理関数

```
function y = synthesisnsolt(x,synthesisnet4predict,config)
nLevels = config.nLevels;
szCoefs = config.szCoefs;
s = cell(1,nLevels+1);
sidx = 1;
for iLevel = 1:nLevels+1
    sz_iLevel = szCoefs(iLevel,:);
    eidx = sidx+prod(sz_iLevel)-1;
    x_iLevel = x(sidx:eidx);
    s{iLevel} = reshape(x_iLevel,sz_iLevel);
    sidx = eidx+1;
end
y = synthesisnet4predict.predict(s{1:nLevels+1});
end
```

### NSOLT 分析処理関数

```
function x = analysisnsolt(y,analysisnet4predict,config)
nLevels = config.nLevels;
szCoefs = config.szCoefs;
[s{1:nLevels+1}] = analysisnet4predict.predict(y);
nCoefs = sum(prod(szCoefs,2),1);
%x = [];
x = zeros(nCoefs,1);
sidx = 1;
for iLevel = 1:nLevels+1
    %x = [x; s{iLevel}(:)];
    eidx = sidx - 1 + prod(szCoefs(iLevel,:));
    x(sidx:eidx) = s{iLevel}(:);
    sidx = eidx + 1;
end
end
```

### ハード閾値処理

```
function y = hardthresh(x,K)
v = sort(abs(x(:)), 'descend');
thk = v(K);
y = (abs(x)>thk).*x;
end
```

### 深層学習配列に対する繰返しハード閾値処理(IHT)のバッチ処理

```
function newdata = iht4patchds(oldtbl,analyzer,synthesizer,sparsityRatio)
% IHT for InputImage in randomPatchExtractionDatastore
%
```

```

nInputs = length(synthesizer.InputNames);

% Apply IHT process for every input patch
restbl = removevars(oldtbl,'InputImage');
dlv = dlmarray(cat(4,oldtbl.InputImage{:}),'SSCB');
[~,dlcoefs{1:nInputs}] = iht4dlarray(dlv,analyzer,synthesizer,sparsityRatio);
coefs = cellfun(@(x) permute(num2cell(extractdata(x),1:3),[4 1 2
3]),dlcoefs,'UniformOutput',false);
%
nImgs = length(oldtbl.InputImage);
coefarray = cell(nImgs,nInputs);
for iImg = 1:nImgs
    for iInput = 1:nInputs
        coefarray{iImg,iInput} = coefs{iInput}{iImg};
    end
end
% Output as a cell in order to make multiple-input datastore
newdata = [ coefarray table2cell(restbl) ];
end

```

### 深層学習配列に対する繰返しハード閾値処理(IHT)

```

function [dly,varargout] = iht4dlarray(dlx,analyzer,synthesizer,sparsityRatio)
% IHT Iterative hard thresholding
%
nInputs = length(synthesizer.InputNames);
szBatch = size(dlx,4);

% Iterative hard thresholding w/o normalization
% (A Parseval tight frame is assumed)
gamma = (1.-1e-3);
nIters = 10;
nCoefs = floor(sparsityRatio*numel(dlx(:, :, :, 1)));
[dlcoefs{1:nInputs}] =
analyzer.predict(dlmarray(zeros(size(dlx),'like',dlx),'SSCB'));
% IHT
for iter=1:nIters
    % Gradient descent
    dly = synthesizer.predict(dlcoefs{1:nInputs});
    [grad{1:nInputs}] = analyzer.predict(dlx-dly);
    dlcoefs = cellfun(@(x,y) x+gamma*y,dlcoefs,grad,'UniformOutput',false);
    % Hard thresholding
    coefvecs = cellfun(@(x) extractdata(reshape(x,
[,szBatch])),dlcoefs,'UniformOutput',false);
    srtdabscoefs = sort(abs(cell2mat(coefvecs.')),1,'descend');
    thk = reshape(srtdabscoefs(nCoefs,:),1,1,1,szBatch);
    dlcoefs = cellfun(@(x) (abs(x)>thk).*x,dlcoefs,'UniformOutput',false);
    % Monitoring
    %checkSparsity =...
    %nnz(srtdabscoefs>srtdabscoefs(nCoefs,:))/numel(dlx)<=sparsityRatio;
    %assert(checkSparsity)

```

```

    fprintf("IHT(%d) MSE: %6.4f\n",iter,mse(dlx,dly));
end
varargout = dlcoefs;
end

```

## NSOLT ネットワークの随伴関係の確認

```

function checkadjointrelation(analysislgraph,synthesislgraph,nLevels,szInput)
import saivdr.dcn.*
x = rand(szInput,'single');
% Assemble analyzer
analysislgraph4predict = analysislgraph;
for iLayer = 1:length(analysislgraph4predict.Layers)
    layer = analysislgraph4predict.Layers(iLayer);
    if contains(layer.Name,"Lv"+nLevels+"_DcOut") || ...
        ~isempty(regexpi(layer.Name,'^Lv\d+_AcOut','once'))
        analysislgraph4predict = analysislgraph4predict.replaceLayer(layer.Name,...
            regressionLayer('Name',layer.Name));
    end
end
analysisnet4predict = assembleNetwork(analysislgraph4predict);

% Assemble synthesizer
synthesislgraph4predict = synthesislgraph;
synthesisnet4predict = assembleNetwork(synthesislgraph4predict);

% Analysis and synthesis process
[s{1:nLevels+1}] = analysisnet4predict.predict(x);
if isvector(s{end-1})
    s{end-1} = permute(s{end-1},[1,3,2]);
end
y = synthesisnet4predict.predict(s{:});

% Evaluation
display("MSE: " + num2str(mse(x,y)))
end

```

## 直交マッチング追跡関数の定義

```

function x = omp(y,Phi,nCoefs)
% Initializaton
nDims = size(Phi,1);
nAtoms = size(Phi,2);
e = ones(nAtoms,1);
a = zeros(nAtoms,1);
g = zeros(nAtoms,1);
x = zeros(nAtoms,1);
v = zeros(nDims,1);
r = y - v;
supp = [];
k = 0;

```

```

while k < nCoefs
    % Matching process
    rr = r.'*r;
    for m = setdiff(1:nAtoms,supp)
        d = Phi(:,m);
        g(m) = d.'*r; %  $\gamma_m = \langle dm, r \rangle$ 
        a(m) = g(m)/(d.'*d); % Normalize  $\alpha_m = \gamma_m / ||dm||^2$ 
        e(m) = rr - g(m)*a(m); %  $\langle r - dm / ||dm||^2, r \rangle$ 
    end
    % Minimum value search (pursuit)
    [~,mmin]= min(e);
    % Update the support
    supp = union(supp,mmin);
    subPhi = Phi(:,supp);
    x(supp) = pinv(subPhi) * y;
    % Synthesis process
    v = Phi*x;
    % Residual
    r = y - v;
    % Update
    k = k + 1;
end
end

```

## NSOLT ネットワークからのツリーレベル情報の抽出

```

function nLevels = extractnumlevels(nsoltnet)
import saivdr.dcn.*

% Extraction of information
expidctlayer = '^Lv\d+_E0~?$';
nLevels = 0;
nLayers = height(nsoltnet.Layers);
for iLayer = 1:nLayers
    layer = nsoltnet.Layers(iLayer);
    if ~isempty(regexpi(layer.Name,expidctlayer,'once'))
        nLevels = nLevels + 1;
    end
end
end
end

```

## NSOLT ネットワークからのストライド情報の抽出

```

function decFactor = extractdecfactor(nsoltnet)
import saivdr.dcn.*

% Extraction of information
expfinalayer = '^Lv1_Cmp1+_V0~?$';
nLayers = height(nsoltnet.Layers);
for iLayer = 1:nLayers
    layer = nsoltnet.Layers(iLayer);

```



```

        if ~isempty(regexpi(layer.Name, expfinallayer, 'once'))
            decFactor = layer.DecimationFactor;
        end
    end
end
end

```

## NSOLT ネットワークからのチャンネル数情報の抽出

```

function nChannels = extractnumchannels(nsoltnet)
import saivdr.dcn.*

% Extraction of information
expfinallayer = '^Lv1_Cmp1+_V0~?$',
nLayers = height(nsoltnet.Layers);
for iLayer = 1:nLayers
    layer = nsoltnet.Layers(iLayer);
    if ~isempty(regexpi(layer.Name, expfinallayer, 'once'))
        nChannels = layer.NumberOfChannels;
    end
end
end
end

```

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