Robot Control Exp. 2

(Dynamics & Parameter Estimation)

Autumn semester

School of Robotics

BICAR(Biologically-inspired Control and Robot) Lab.

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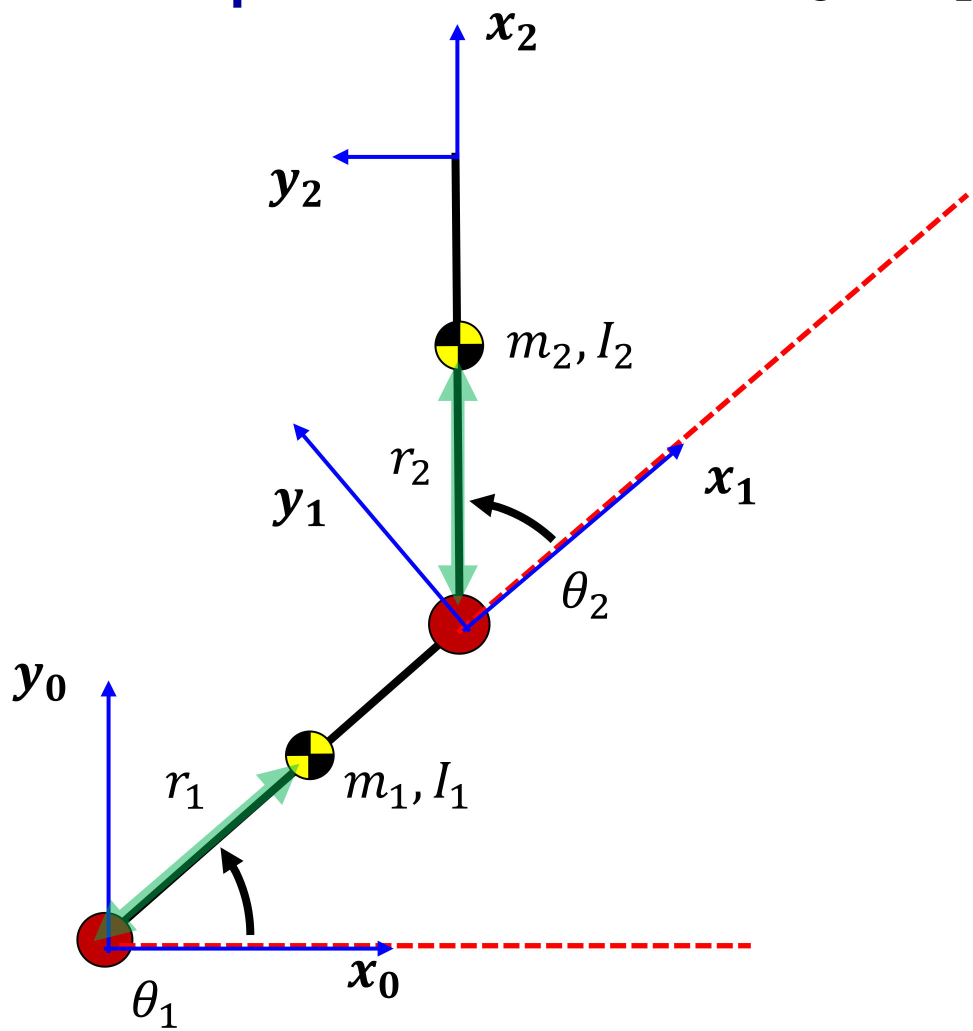
❑ 로봇제어 실습강의 – Dynamics

▪ **Example : 2-DOF**   Link length : $L_1,\ L_2$



D-H Link Parameter Table

Joint	θ_i	d_i	a_i	α_i
1	θ_1	0	L_1	0
2	θ_2	0	L_2	0

$$I_i = \begin{bmatrix} I_{ixx} & 0 & 0 \\ 0 & I_{iyy} & 0 \\ 0 & 0 & I_{izz} \end{bmatrix}$$

$$I_{ixx} = 0, \qquad I_{iyy} = I_{izz} = I_{c1} + I_{m1} = I_{c1} + m_1 r_1^2$$

❑ 로봇제어 실습강의 – Dynamics

- **Velocity of a link**
  - **Rotary joints,** $q_i = \theta_i$

$$T_{i-1}^{i} = \begin{bmatrix} C\theta_i & -C\alpha_i S\theta_i & S\alpha_i S\theta_i & a_i C\theta_i \\ S\theta_i & C\alpha_i C\theta_i & -S\alpha_i C\theta_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\frac{\partial T_{i-1}^{i}}{\partial q_i} = \begin{bmatrix} -S\theta_i & -C\alpha_i C\theta_i & S\alpha_i C\theta_i & -a_i S\theta_i \\ C\theta_i & -C\alpha_i S\theta_i & S\alpha_i S\theta_i & a_i C\theta_i \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Q_i = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\frac{\partial T_{i-1}^{i}}{\partial q_i} = Q_i T_{i-1}^{i} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} C\theta_i & -C\alpha_i S\theta_i & S\alpha_i S\theta_i & a_i C\theta_i \\ S\theta_i & C\alpha_i C\theta_i & -S\alpha_i C\theta_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

❑ 로봇제어 실습강의 – Dynamics

- **Velocity of a link**

The effect of the motion of joint *j* on all the points on link *i*

$$\frac{\partial T_0^i}{\partial q_j} = \begin{cases} T_0^1 T_1^2 \cdots T_{j-2}^{j-1} Q_j T_{j-1}^{j} \cdots T_{i-1}^{i} & for \quad j \le i \\ 0 & for \quad j > i \end{cases}$$

$$U_{ij} \equiv \frac{\partial T_0^i}{\partial q_j} = \begin{cases} T_0^{j-1} Q_j T_{j-1}^{i} & for \quad j \le i \\ 0 & for \quad j > i \end{cases}$$

$$V_i \equiv V_0^i = \frac{d}{dt} r_0^i = \frac{d}{dt}(T_0^1 T_1^2 \cdots T_{i-1}^{i}) r_i^i = \left(\sum_{j=1}^{i} \frac{\partial T_0^i}{\partial q_j} \dot{q}_j\right) r_i^i = \left(\sum_{j=1}^{i} U_{ij} \dot{q}_j\right) r_i^i$$

❑ 로봇제어 실습강의 – Dynamics

- **Kinetic energy of link *i***
  - **Kinetic energy of a particle with differential mass *dm* in link *i***

$$dK_i = \frac{1}{2}(\dot{x}_i{}^2 + \dot{y}_i{}^2 + \dot{z}_i{}^2)dm = \frac{1}{2} trace(V_i V_i{}^T)dm$$

$$= \frac{1}{2} Tr\left[\sum_{p=1}^{i} U_{ip}\dot{q}_p r_i^i \left(\sum_{r=1}^{i} U_{ir}\dot{q}_r r_i^i\right)^T \right] dm$$

$$= \frac{1}{2} Tr\left[\sum_{p=1}^{i}\sum_{r=1}^{i} U_{ip} r_i^i r_i^{i\,T} U_{ir}^T \dot{q}_p \dot{q}_r \right] dm$$

$$= \frac{1}{2} Tr\left[\sum_{p=1}^{i}\sum_{r=1}^{i} U_{ip} (r_i^i dm\, r_i^{i\,T}) U_{ir}^T \dot{q}_p \dot{q}_r \right]$$

$$Tr(A) \equiv \sum_{i=1}^{n} a_{ii}$$

❑ 로봇제어 실습강의 – Dynamics

- **Kinetic energy of link *i***

$$K_i = \int dK_i = \frac{1}{2} Tr\left[\sum_{p=1}^{i} \sum_{r=1}^{i} U_{ip} \left(\int r_i^i r_i^{iT} dm \right) U_{ir}^T \dot{q}_p \dot{q}_r \right]$$

$$J_i = \int r_i^i r_i^{iT} dm = \begin{bmatrix} \int x_i^2 dm & \int x_i y_i dm & \int x_i z_i dm & \int x_i dm \\ \int x_i y_i dm & \int y_i^2 dm & \int y_i z_i dm & \int y_i dm \\ \int x_i z_i dm & \int y_i z_i dm & \int z_i^2 dm & \int z_i dm \\ \int x_i dm & \int y_i dm & \int z_i dm & \int dm \end{bmatrix}$$

$$\bar{r}_i^i = \begin{bmatrix} \bar{x}_i \\ \bar{y}_i \\ \bar{z}_i \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-I_{xx} + I_{yy} + I_{zz}}{2} & I_{xy} & I_{xz} & m_i \bar{x}_i \\ I_{xy} & \frac{I_{xx} - I_{yy} + I_{zz}}{2} & I_{yz} & m_i \bar{y}_i \\ I_{xz} & I_{yz} & \frac{I_{xx} + I_{yy} - I_{zz}}{2} & m_i \bar{z}_i \\ m_i \bar{x}_i & m_i \bar{y}_i & m_i \bar{z}_i & m_i \end{bmatrix}$$

$$\bar{x}_i = \frac{1}{m_i} \int x_i dm$$

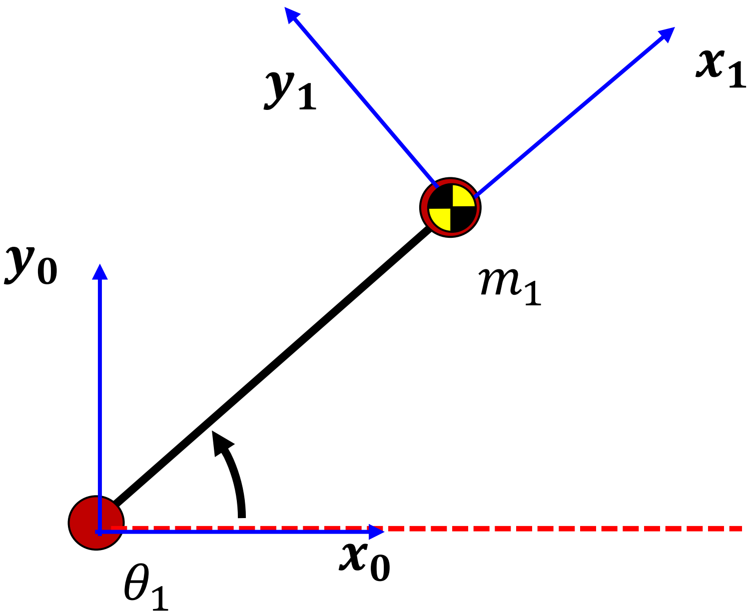
Center of mass

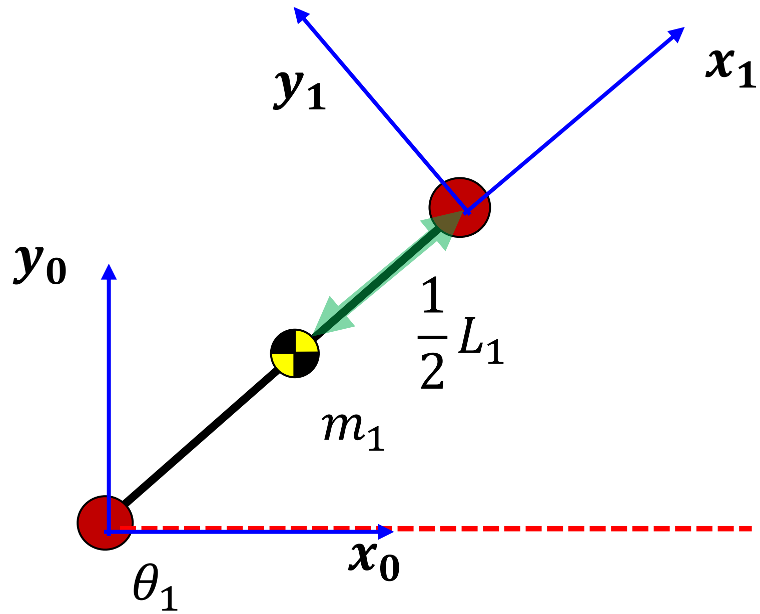
Pseudo-inertia matrix of link i

□ 로봇제어 실습강의 – Dynamics

- **Example**

$$I_{iyy} = I_{izz} = \frac{1}{3} m L_1^2$$





$$\hat{r}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad \mathrm{J}_1 = \begin{bmatrix} \frac{1}{3} m L_1^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_1 \end{bmatrix}$$

$$\hat{r}_1 = \begin{bmatrix} -\frac{1}{2} L_1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad \mathrm{J}_1 = \begin{bmatrix} \frac{1}{3} m L_1^2 & 0 & 0 & -\frac{1}{2} m_1 L_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{1}{2} m_1 L_1 & 0 & 0 & m_1 \end{bmatrix}$$

❑ 로봇제어 실습강의 – Dynamics

▪ **Total kinetic energy of a robot arm**

$$K = \sum_{i=1}^{n} K_i = \frac{1}{2}\sum_{i=1}^{n} Tr\left[\sum_{p=1}^{i}\sum_{r=1}^{i} U_{ip}\left(\int {}^{i}r_i \, {}^{i}r_i^{T} dm\right) U_{ir}^{T} \dot{q}_p \dot{q}_r\right]$$

$$= \frac{1}{2}\sum_{i=1}^{n}\sum_{p=1}^{i}\sum_{r=1}^{i}\left[Tr\left(U_{ip} J_i U_{ir}^{T}\right)\dot{q}_p \dot{q}_r\right]$$

Scalar quantity, function of q_i **and** $\dot{q}_i$ **,** $i = 1, 2, \cdots n$

J_i **: Pseudo-inertia matrix of link *i*, dependent on the mass distribution of link *i* and are expressed w.r.t. the *i*-th frame,**

Need to be computed once for evaluating the kinetic energy

❑ 로봇제어 실습강의 – Dynamics

- **Potential energy of link *i***

$$P_i = -m_i g \bar{r}_0^{\,i} = -m_i g (T_0^i \bar{r}_i^{\,i})$$

$\bar{r}_0^{\,i}$ **: Center of mass w.r.t. base frame**

$\bar{r}_i^{\,i}$**: Center of mass w.r.t. *i*-th frame**

$$g = (g_x, g_y, g_z, 0)$$

$$|g| = 9.8 m/\sec^2$$

g **: gravity row vector expressed in base frame**

- **Potential energy of a robot arm**

$$P = \sum_{i=1}^{n} P_i = \sum_{i=1}^{n} [-m_i g (T_0^i \bar{r}_i^{\,i})]$$

Function of q_i

❑ 로봇제어 실습강의 – Dynamics

- **Lagrangian function**

$$L = K - P = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{i}\sum_{k=1}^{i}\left[Tr(U_{ij}J_iU_{ik}{}^T)\dot{q}_j\dot{q}_k\right] + \sum_{i=1}^{n} m_i g(T_0^i \bar{r}_i{}^i)$$

$$\tau_i = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i}$$

$$= \sum_{j=i}^{n}\sum_{k=1}^{j} Tr(U_{jk}J_jU_{ji}{}^T)\ddot{q}_k + \sum_{j=i}^{n}\sum_{k=1}^{j}\sum_{m=1}^{j} Tr\left(\frac{\partial U_{jk}}{\partial q_m}J_jU_{ji}{}^T\right)\dot{q}_k\dot{q}_m$$

$$-\sum_{j=i}^{n} m_j g U_{ji}\bar{r}_j{}^j$$

❑ 로봇제어 실습강의 – Dynamics

- **Dynamics model**

$$\tau_i = \sum_{k=1}^{n} D_{ik}\ddot{q}_k + \sum_{k=1}^{n}\sum_{m=1}^{n} h_{ikm}\dot{q}_k\dot{q}_m + C_i$$

$$D_{ik} = \sum_{j=\max(i,k)}^{n} Tr(U_{jk} J_j U_{ji}^{\ T})$$

$$h_{ikm} = \sum_{j=i}^{n}\sum_{k=1}^{j}\sum_{m=1}^{j} Tr\left(\frac{\partial U_{jk}}{\partial q_m} J_j U_{ji}^{\ T}\right) = \sum_{j=max(i,k,m)}^{n} Tr(U_{jkm} J_j U_{ji}^{\ T})$$

$$C_i = -\sum_{j=i}^{n} m_j g U_{ji} \bar{r}_j^{\,j}$$

❑ 로봇제어 실습강의 – Dynamics

- **Lagrangian function**

The effect of the motion of joint j on all the points on link i

$$U_{ij} \equiv \frac{\partial T_0^i}{\partial q_j} = \begin{cases} T_0^{j-1} Q_j T_{j-1}^i & for \quad j \le i \\ 0 & for \quad j > i \end{cases}$$

The interaction effects of the motion of joint j and joint k on all the points on link i

$$\frac{\partial U_{ij}}{\partial q_k} \equiv U_{ijk} = \begin{cases} T_0^{j-1} Q_j T_{j-1}^{k-1} Q_k T_{k-1}^i & i \ge k \ge j \\ T_0^{k-1} Q_k T_{k-1}^{j-1} Q_j T_{j-1}^i & i \ge j \ge k \\ 0 \qquad i < j \quad or \quad i < k \end{cases}$$

❑ 로봇제어 실습강의 – Dynamics

```
L2 = L1;

r1 = L1/2;
r2 = L2/2;
Iz1 = 1/3*m1*L1^2;
Iz2 = 1/3*m2*L2^2;
```

- **Example**

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \underbrace{\begin{bmatrix} \tfrac{1}{3}m_1 l^2 + \tfrac{4}{3}m_2 l^2 + m_2 C_2 l^2 & \tfrac{1}{3}m_2 l^2 + \tfrac{1}{2}m_2 l^2 C_2 \\ \tfrac{1}{3}m_2 l^2 + \tfrac{1}{2}m_2 l^2 C_2 & \tfrac{1}{3}m_2 l^2 \end{bmatrix}}_{D} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \underbrace{\begin{bmatrix} -\tfrac{1}{2}m_2 S_2 l^2 \dot{\theta}_2^2 - m_2 S_2 l^2 \dot{\theta}_1 \dot{\theta}_2 \\ \tfrac{1}{2}m_2 S_2 l^2 \dot{\theta}_1^2 \end{bmatrix}}_{H} + \underbrace{\begin{bmatrix} \tfrac{1}{2}m_1 g l C_1 + \tfrac{1}{2}m_2 g l C_{12} + m_2 g l C_1 \\ \tfrac{1}{2}m_2 g l C_{12} \end{bmatrix}}_{C}$$

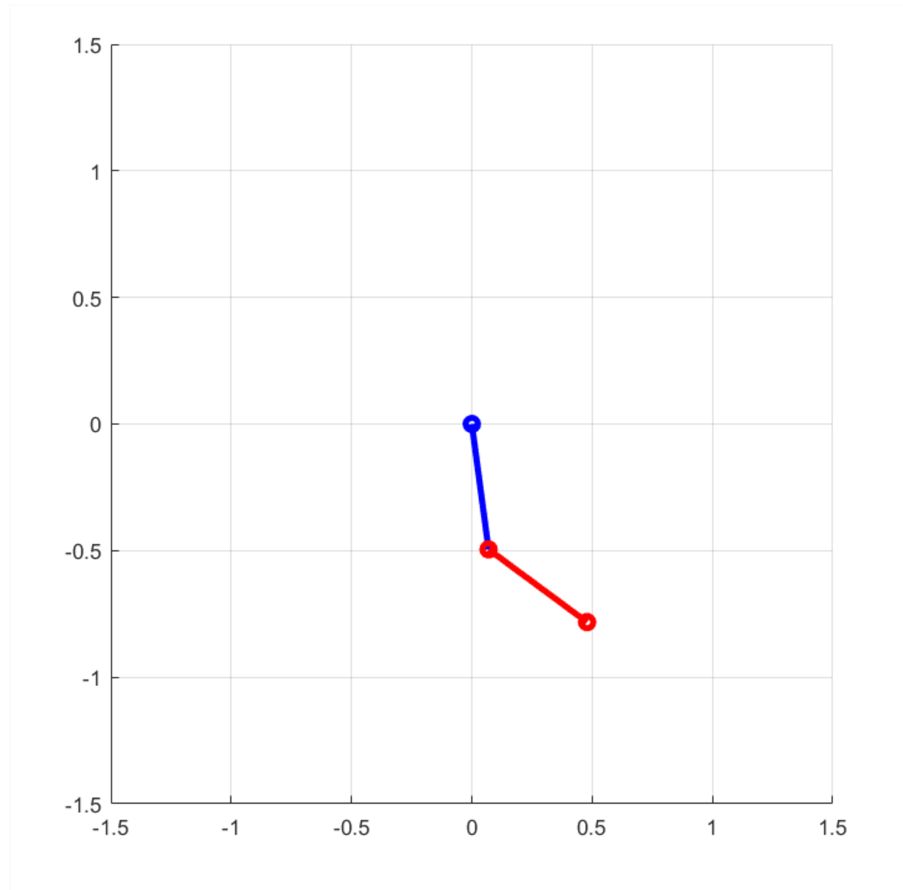
Ref : Robotics: Control, Sensing, Vision, and Intelligence, 1999

❑ 로봇제어 실습강의 – Dynamics simulation

$$M\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{bmatrix} + h + G = \left(\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix}\right)$$

$$\begin{bmatrix}\ddot{\theta}_1\\ \ddot{\theta}_2\end{bmatrix} = M^{-1}\left(\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} - h - G\right)$$

$$Y = \begin{bmatrix}\theta_1\\ \dot{\theta}_1\\ \theta_2\\ \dot{\theta}_2\end{bmatrix} \qquad \frac{dY}{dt} = \begin{bmatrix}\dot{\theta}_1\\ \ddot{\theta}_1\\ \dot{\theta}_2\\ \ddot{\theta}_2\end{bmatrix}$$

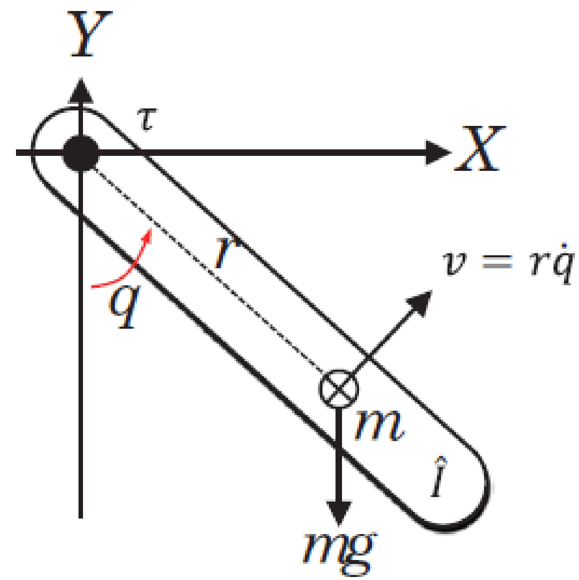


❑ 로봇제어 실습강의 – Dynamics parameter 추정

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+g(q)+d=\tau$$

→ **Unknowns: *m, r, I, I_m, F_s, F_v*, etc.**

If using a real model,



$$\tau = I\ddot{q}+mgr\sin q+I_m\ddot{q}+F_s sign(\dot{q})+F_v\dot{q}$$

$$\Downarrow$$

$$\tau=\begin{bmatrix}\ddot{q} & g\sin q & sign(\dot{q}) & \dot{q}\end{bmatrix}\begin{bmatrix}I+I_m\\ mr\\ F_s\\ F_v\end{bmatrix} \quad\Rightarrow\quad \tau=\boxed{Y(q,\dot{q},\ddot{q})}\theta$$

Regressor

Vector set by kinematic parameters
(position/velocity/acceleration)

❑ 로봇제어 실습강의 – Dynamics parameter 추정

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+g(q)+d=\tau$$

→ **Unknowns:** ***m, r, I, I_m, F_s, F_v,*** **etc.**

If get 4 data set, we can calculate the dynamics parameters

$$\begin{bmatrix}\tau_{(1)}\\ \tau_{(2)}\\ \tau_{(3)}\\ \tau_{(4)}\end{bmatrix}=\begin{bmatrix}Y\left(q_{(1)},\dot{q}_{(1)},\ddot{q}_{(1)}\right)\\ Y\left(q_{(2)},\dot{q}_{(2)},\ddot{q}_{(2)}\right)\\ Y\left(q_{(3)},\dot{q}_{(3)},\ddot{q}_{(3)}\right)\\ Y\left(q_{(4)},\dot{q}_{(4)},\ddot{q}_{(4)}\right)\end{bmatrix}\theta$$

$$\Downarrow$$

$$\begin{bmatrix}I+I_m\\ mr\\ F_s\\ F_v\end{bmatrix}=\begin{bmatrix}\ddot{q}_{(1)} & g\sin q_{(1)} & sign(\dot{q}_{(1)}) & \dot{q}_{(1)}\\ \ddot{q}_{(2)} & g\sin q_{(2)} & sign(\dot{q}_{(2)}) & \dot{q}_{(2)}\\ \ddot{q}_{(3)} & g\sin q_{(3)} & sign(\dot{q}_{(3)}) & \dot{q}_{(3)}\\ \ddot{q}_{(4)} & g\sin q_{(4)} & sign(\dot{q}_{(4)}) & \dot{q}_{(4)}\end{bmatrix}^{-1}\begin{bmatrix}\tau_{(1)}\\ \tau_{(2)}\\ \tau_{(3)}\\ \tau_{(4)}\end{bmatrix}$$

But, two conditions should be required so that unique solution exists
→ 1) inverse matrix, 2) measurable acceleration data

❑ 로봇제어 실습강의 – Dynamics parameter 추정

- **Problems: how to measure all joint accel. and sustain vector dimension**

$$p = M(q)\dot{q}$$ **P: generalized momentum**

$$\dot{p} = \dot{M}(q)\dot{q} + M(q)\ddot{q} \qquad \dot{p} = \dot{M}(q)\dot{q} - C(q,\dot{q})\dot{q} - g(q) - d + \tau$$

$$\dot{p} = C^T(q,\dot{q})\dot{q} - g(q) - d + \tau \qquad \because \dot{M} = C + C^T$$

$$\rightarrow \dot{p} - W_1(q,\dot{q})\theta = \tau, \quad \text{where } \boxed{W_1(q,\dot{q})\theta = C^T(q,\dot{q})\dot{q} - g(q) - d,} \text{ regressor}$$

→ **Integrating above eqn.**

$$p - \left[\int W_1(q,\dot{q})dt\right]\theta = \int \tau dt \quad \text{Here, } \boxed{p = M(q)\dot{q} = W_2(q,\dot{q})\theta}$$

$$\Rightarrow \boxed{u = Y(q,\dot{q},t)\theta}$$

$$\boxed{\begin{aligned} Y(q,\dot{q},t) &= W_2(q,\dot{q}) - \int W_1(q,\dot{q})dt \\ u &= \int \tau dt \end{aligned}}$$

❑ 로봇제어 실습강의 – Dynamics parameter 추정

→ **For estimation error minimization**

$$\text{minimize} \ \sum_{i=1}^{n} \frac{1}{2} \epsilon_{(i)}^2 \qquad \text{where} \quad \epsilon_{(i)} = u_{(i)} - Y_{(i)} \hat{\theta}_{(n)}$$

$$u_{(n)} = \sum_{i=1}^{n} \tau_{(i)} \Delta T$$

$$Y_{(n)} = W_{2,(n)} - \sum_{i=1}^{n} W_{1,(i)} \Delta T$$

$$\frac{\partial \left(\sum_{i=1}^{n} \frac{1}{2} \epsilon_{(i)}^T \epsilon_{(i)} \right)}{\partial \hat{\theta}_{(n)}} = \sum_{i=1}^{n} \frac{\partial \epsilon_{(i)}^T}{\partial \hat{\theta}_{(n)}} \epsilon_{(i)} \qquad \text{since} \quad \frac{\partial \epsilon_{(i)}^T}{\partial \hat{\theta}_{(n)}} = -Y_{(i)}^T$$

$$= -Y_{(1)}^T \left[u_{(1)} - Y_{(1)} \hat{\theta}_{(n)} \right] - Y_{(2)}^T \left[u_{(2)} - Y_{(2)} \hat{\theta}_{(n)} \right]$$

$$\cdots - Y_{(n)}^T \left[u_{(n)} - Y_{(n)} \hat{\theta}_{(n)} \right]$$

$$= \left[\sum_{i=1}^{n} Y_{(i)}^T Y_{(i)} \right] \theta_{(n)} - \left[\sum_{i=1}^{n} Y_{(i)}^T u_{(i)} \right] = 0$$

$$\theta_{(n)} = \left[\sum_{i=1}^{n} Y_{(i)}^T Y_{(i)} \right]^{-1} \left[\sum_{i=1}^{n} Y_{(i)}^T u_{(i)} \right]$$

□ 로봇제어 실습강의 – Dynamics parameter 추정

- **Example** $\tau - I\ddot{q} + mgr\sin q + I_m\ddot{q} + F_s sign(\dot{q}) + F_v\dot{q}$

$$p = (I + I_m)\dot{q} = W_2(\dot{q})\theta = \begin{bmatrix}\dot{q} & 0 & 0 & 0\end{bmatrix}\begin{bmatrix} I + I_m \\ mr \\ F_s \\ F_v \end{bmatrix}$$

$$\dot{p} - W_1(q,\dot{q})\theta = \tau \qquad W_1(q,\dot{q})\theta = -mgr\sin q - F_s sign(\dot{q}) - F_v\dot{q}$$

$$= \begin{bmatrix} 0 & -g\sin q & -sign(\dot{q}) & -\dot{q} \end{bmatrix}\begin{bmatrix} I + I_m \\ mr \\ F_s \\ F_v \end{bmatrix}$$

$$\boxed{Y(q,\dot{q},t)\theta = u}$$

$$Y(q,\dot{q},t) = W_2(\dot{q}) - \int W_1(q,\dot{q})\,dt$$

$$u = \int \tau\,dt$$

□ 로봇제어 실습강의 – Dynamics parameter 추정

- **Example** $\tau - I\ddot{q} + mgr\sin q + I_m\ddot{q} + F_s sign(\dot{q}) + F_v\dot{q}$

State model

$$x = [x_1, x_2]^T = [q, \dot{q}]^T \qquad \frac{dx}{dt} = \begin{bmatrix} x_2 \\ (\tau - mgr\sin x_1 - F_s sign(x_2) - F_v x_2)/(I + I_m) \end{bmatrix}$$

State

$$\frac{dx}{dt} = \begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = \begin{bmatrix} x_2 \\ f(x_1, x_{2,\dots}) \end{bmatrix}$$

```
function dxdt = OneLinkRobot (t, x)

global I;
global Im;
global m;
global g;
global r;
global Fs;
global Fv;
global tau;
```

dxdt $= [x(2)\ ;\ (tau - m*g*r*\sin(x(1)) - Fs*sign(x(2)) - Fv*x(2))/(I + \mathrm{Im})];$

❑ 로봇제어 실습강의 – Dynamics parameter 추정

- **Example** $\tau - I\ddot{q} + mgr\sin q + I_m\ddot{q} + F_s sign(\dot{q}) + F_v\dot{q}$

$$I = 0.05; I_m = 0.05; m = 0.5; r = 0.2; F_s = 0.1; F_v = 0.1$$

→ **Answer:** $\left[I + I_m, mr, F_s, F_v\right]^T = \left[0.1, 0.1, 0.1, 0.1\right]^T$

```
global I;
global Im;
global m;
global g;
global r;
global Fs;
global Fv;
global tau;
I=0.05;  Im=0.05;  m=0.5;  g=9.806;  r=0.2;  Fs=0.1;  Fv=0.1;
```

❑ 로봇제어 실습강의 – Dynamics parameter 추정

- **Example** $\tau - I\ddot{q} + mgr\sin q + I_m\ddot{q} + F_s sign\left(\dot{q}\right) + F_v\dot{q}$

```
dT = 0.002;
q = 0;  qdot = 0;
l = 0.5;
n = 1;
W1_int = [ 0 , 0 , 0 , 0 ];
u = 0;
P = eye(4,4);
theta = [ 0 ; 0 ; 0 ; 0 ];
for t = 0 : dT : 5.0
   % Arbitrary set of control input, if possible, should include various frequency
   tau = sin(t) + cos(10*t);
[st,x] = ode45('OneLinkRobot', [0, dT] , [q; qdot] );
index = size(x);  q = x(index(1), 1);  qdot = x(index(1), 2);
```

❑ 로봇제어 실습강의 – Dynamics parameter 추정

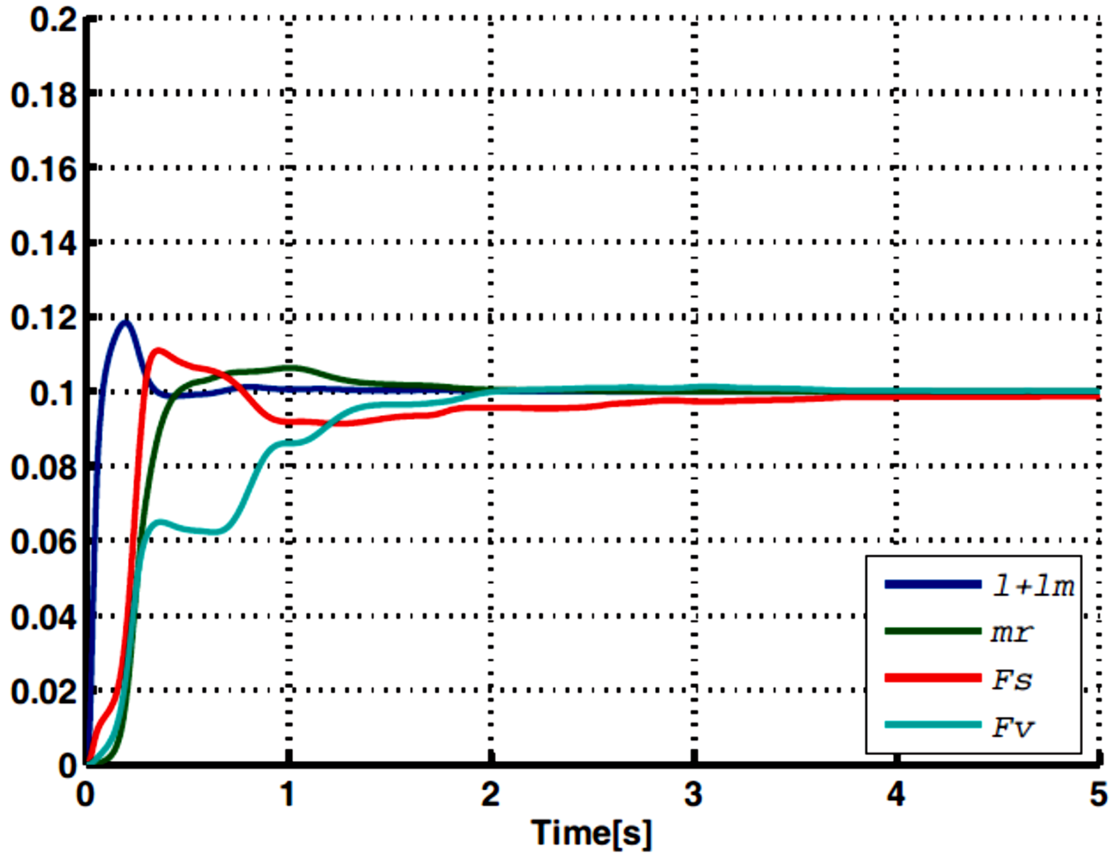
- **Example** $\tau - I\ddot{q} + mgr\sin q + I_m\ddot{q} + F_s sign\left(\dot{q}\right) + F_v\dot{q}$

```
        % Kalman Filter based parameter estimation algorithm
        W1_int = W1_int + [ 0, -g*sin(q) , -sign(qdot) , -qdot ]*dT;
        W2 = [ qdot , 0 , 0 , 0 ];
        Y = W2 - W1_int;
        u = u + tau * dT;
        P = P - P*Y'*inv(1+Y*P*Y')*Y*P;
        K = P*Y';
        theta = theta + K * (u - Y*theta);

        % Data save
        save_time(n,:) = t;
        save_theta(n,:) = theta;
        n=n+1;
end
```

❑ 로봇제어 실습강의 – Dynamics parameter 추정

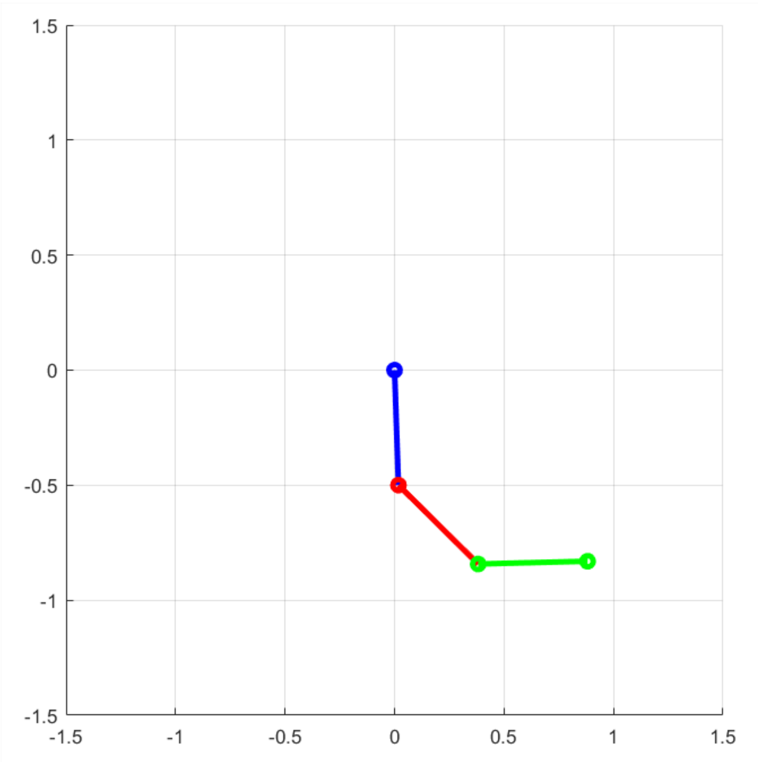
- **Example** $\tau - I\ddot{q} + mgr\sin q + I_m\ddot{q} + F_s sign(\dot{q}) + F_v\dot{q}$



❑ 로봇제어 실습강의 – HW

- **HW1.**

  → Using the Lagrangian function, derive 3-DOF robot and perform a free-fall simulation. Discuss the results.




```
L1 = 0.5;    L2 = 0.5;    L3 = 0.5;
r1 = 0.1;    r2 = 0.1;    r3 = 0.1;

m1 = 0.2;    m2 = 0.2;    m3 = 0.2;

Iz1= 0.05;   Iz2= 0.05;   Iz3 = 0.05;
```

❑ 로봇제어 실습강의 – HW2

- **HW2.**

→ Using the following Dynamics model, estimate the 2-DOF dynamics parameter.

$$M(q)=\begin{bmatrix} I_1+I_2+m_2l_1^2+2m_2r_2l_1c_2+I_{m1} & I_2+m_2r_2l_1c_2 \\ I_2+m_2r_2l_1c_2 & I_2+I_{m2} \end{bmatrix}$$

$$C(q,\dot{q})=\begin{bmatrix} -m_2r_2l_1s_2\dot{q}_2 & -m_2r_2l_1s_2(\dot{q}_1+\dot{q}_2) \\ m_2r_2l_1s_2\dot{q}_1 & 0 \end{bmatrix}$$

$$g(q)=\begin{bmatrix} -m_1r_1gc_1-m_2l_1gc_1-m_2r_2gc_{12} \\ -m_2r_2gc_{12} \end{bmatrix} \quad d=\begin{bmatrix} F_{s1}sign(\dot{q}_1)+F_{v1}\dot{q}_1 \\ F_{s2}sign(\dot{q}_2)+F_{v2}\dot{q}_2 \end{bmatrix}$$

$$I_1=I_2=0.05, I_{m1}=I_{m2}=0.05, m_1=m_2=0.2, g=9.806$$

$$F_{s1}=F_{s2}=0.1, F_{v1}=F_{v2}=0.1, l_1=l_2=0.5$$

$$r_1=r_2=0.1$$

❑ 로봇제어 실습강의 – HW2

- **HINT**

$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} = M\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix} + C\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix} + g + d$$

$$\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix} = M^{-1}\left(\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} - C\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix} - g - d\right)$$

$$p = M\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix} = W_2(\dot{q}_1, \dot{q}_2)\theta \qquad \theta = \begin{bmatrix} I_1 + m_2 l_1^2 + I_{m1} \\ I_2 \\ m_2 r_2 l_1 \\ m_1 r_1 + m_2 l_1 \\ m_2 r_2 \\ I_{m2} \\ F_{s1} \\ F_{s2} \\ F_{v1} \\ F_{v2} \end{bmatrix}$$

$$x_1 = q_1$$
$$x_2 = q_2$$
$$x_3 = \dot{q}_1$$
$$x_4 = \dot{q}_2$$

$$\frac{dx}{dt} = \begin{matrix}\dot{q}_1\\ \dot{q}_2\\ \ddot{q}_1\\ \ddot{q}_2\end{matrix} = \begin{bmatrix}x_3\\ x_4\\ \ddot{q}_1\\ \ddot{q}_2\end{bmatrix}$$

Modify dynamic model

Real model (unknown)

$I_1 = I_2 = 0.05, I_{m1} = I_{m2} = 0.05, m_1 = m_2 = 0.2, g = 9.806$
$F_{s1} = F_{s2} = 0.1, F_{v1} = F_{v2} = 0.1, l_1 = l_2 = 0.5$
$r_1 = r_2 = 0.1$

dynamics simulation

$$\theta = \begin{bmatrix}0\\0\\0\\0\\0\\0\\0\\0\\0\\0\end{bmatrix} \Rightarrow \boxed{\textbf{Estimation algorithm}} \Rightarrow \theta = \begin{bmatrix} I_1 + m_2 l_1^2 + I_{m1} \\ I_2 \\ m_2 r_2 l_1 \\ m_1 r_1 + m_2 l_1 \\ m_2 r_2 \\ I_{m2} \\ F_{s1} \\ F_{s2} \\ F_{v1} \\ F_{v2} \end{bmatrix} = \begin{bmatrix}0.15\\0.05\\0.01\\0.12\\0.02\\0.05\\0.1\\0.1\\0.1\\0.1\end{bmatrix}$$

start **end**

□ 로봇제어 실습강의 – HW2

- **HINT**

$$M(q)=\begin{bmatrix} I_1+I_2+m_2l_1^2+2m_2r_2l_1c_2+I_{m1} & I_2+m_2r_2l_1c_2 \\ I_2+m_2r_2l_1c_2 & I_2+I_{m2} \end{bmatrix}$$

$$C(q,\dot{q})=\begin{bmatrix} -m_2r_2l_1s_2\dot{q}_2 & -m_2r_2l_1s_2(\dot{q}_1+\dot{q}_2) \\ m_2r_2l_1s_2\dot{q}_1 & 0 \end{bmatrix}$$

$$g(q)=\begin{bmatrix} -m_1r_1gc_1-m_2l_1gc_1-m_2r_2gc_{12} \\ -m_2r_2gc_{12} \end{bmatrix}$$

$$d=\begin{bmatrix} F_{s1}sign(\dot{q}_1)+F_{v1}\dot{q}_1 \\ F_{s2}sign(\dot{q}_2)+F_{v2}\dot{q}_2 \end{bmatrix}$$

$$M=\begin{bmatrix} (I_1+m_2l_1^2+I_{m1})+(I_2)+2(m_2r_2l_1)\cos(q_2) & (I_2)+(m_2r_2l_1)\cos(q_2) \\ (I_2)+(m_2r_2l_1)\cos(q_2) & (I_2)+(I_{m2}) \end{bmatrix}$$

$$\Downarrow$$

$$M=\begin{bmatrix} \theta_1+\theta_2+\theta_3 2\cos(q_2) & \theta_2+\theta_3\cos(q_2) \\ \theta_2+\theta_3\cos(q_2) & \theta_2+\theta_6 \end{bmatrix}$$

$$\theta=\begin{bmatrix} I_1+m_2l_1^2+I_{m1} \\ I_2 \\ m_2r_2l_1 \\ m_1r_1+m_2l_1 \\ m_2r_2 \\ I_{m2} \\ F_{s1} \\ F_{s2} \\ F_{v1} \\ F_{v2} \end{bmatrix}$$

$$p=M\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}=\begin{bmatrix} \theta_1\dot{q}_1+\theta_2\dot{q}_1+\theta_3 2\cos(q_2)\dot{q}_1+\theta_2\dot{q}_2+\theta_3\cos(q_2)\dot{q}_2 \\ \theta_2\dot{q}_1+\theta_3\cos(q_2)\dot{q}_1+\theta_2\dot{q}_2+\theta_6\dot{q}_2 \end{bmatrix}=W_2\theta$$

❑ 로봇제어 실습강의 – HW2

- **HINT**

$$M(q)=\begin{bmatrix} I_1+I_2+m_2l_1^2+2m_2r_2l_1c_2+I_{m1} & I_2+m_2r_2l_1c_2 \\ I_2+m_2r_2l_1c_2 & I_2+I_{m2} \end{bmatrix}$$

$$C(q,\dot q)=\begin{bmatrix} -m_2r_2l_1s_2\dot q_2 & -m_2r_2l_1s_2(\dot q_1+\dot q_2) \\ m_2r_2l_1s_2\dot q_1 & 0 \end{bmatrix}$$

$$g(q)=\begin{bmatrix} -m_1r_1gc_1-m_2l_1gc_1-m_2r_2gc_{12} \\ -m_2r_2gc_{12} \end{bmatrix}$$

$$d=\begin{bmatrix} F_{s1}sign(\dot q_1)+F_{v1}\dot q_1 \\ F_{s2}sign(\dot q_2)+F_{v2}\dot q_2 \end{bmatrix}$$

$$C^T=\begin{bmatrix} -m_2r_2l_1\sin(q_2)\dot q_2 & m_2r_2l_1\sin(q_2)\dot q_1 \\ -m_2r_2l_1\sin(q_2)(\dot q_1+\dot q_2) & 0 \end{bmatrix}=\begin{bmatrix} -\theta_3\sin(q_2)\dot q_2 & \theta_3\sin(q_2)\dot q_1 \\ -\theta_3\sin(q_2)(\dot q_1+\dot q_2) & 0 \end{bmatrix}$$

$$\theta=\begin{bmatrix} I_1+m_2l_1^2+I_{m1} \\ I_2 \\ m_2r_2l_1 \\ m_1r_1+m_2l_1 \\ m_2r_2 \\ I_{m2} \\ F_{s1} \\ F_{s2} \\ F_{v1} \\ F_{v2} \end{bmatrix}$$

$$C^T\begin{bmatrix} \dot q_1 \\ \dot q_2 \end{bmatrix}=\begin{bmatrix} -\theta_3\sin(q_2)\dot q_2\dot q_1+\theta_3\sin(q_2)\dot q_1\dot q_2 \\ -\theta_3\sin(q_2)(\dot q_1+\dot q_2)\dot q_1 \end{bmatrix}=\begin{bmatrix} 0 \\ -\theta_3\sin(q_2)(\dot q_1+\dot q_2)\dot q_1 \end{bmatrix}$$

$$-g(q)=\begin{bmatrix} (m_1r_1+m_2l_1)g\cos(q_1)+m_2r_2g\cos(q_1+q_2) \\ m_2r_2g\cos(q_1+q_2) \end{bmatrix}=\begin{bmatrix} \theta_4g\cos(q_1)+\theta_5g\cos(q_1+q_2) \\ \theta_5g\cos(q_1+q_2) \end{bmatrix}$$

$$-d=\begin{bmatrix} -\theta_7sign(\dot q_1)-\theta_9\dot q_1 \\ -\theta_8sign(\dot q_2)-\theta_{10}\dot q_2 \end{bmatrix}$$

$$W_1\theta=C^T\begin{bmatrix} \dot q_1 \\ \dot q_2 \end{bmatrix}-g-d=\begin{bmatrix} \theta_4g\cos(q_1)+\theta_5g\cos(q_1+q_2)-\theta_7sign(\dot q_1)-\theta_9\dot q_1 \\ -\theta_3\sin(q_2)(\dot q_1+\dot q_2)\dot q_1+\theta_5g\cos(q_1+q_2)-\theta_8sign(\dot q_2)-\theta_{10}\dot q_2 \end{bmatrix}$$