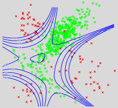


Machine Learning

Lecture *Machine Learning* vom 29-31.3.2023

Felix Becker
(material in collaboration with Lars Gabriel and Mario Stanke)
Institut für Mathematik und Informatik
Universität Greifswald

Image classification problem

Given an image - say 64×64 pixels with 3 color channels - predict a probability distribution over predefined classes e.g. *cat, dog, cow, plant, . . .* . One can then assign the image to the most likely class.

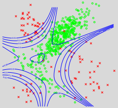


Image classification problem

Given an image - say 64 $\times$ 64 pixels with 3 color channels - predict a probability distribution over predefined classes e.g. *cat, dog, cow, plant, . . .* . One can then assign the image to the most likely class.

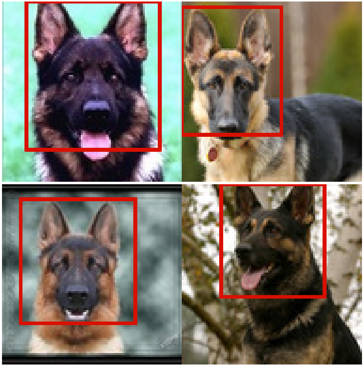
Related problems

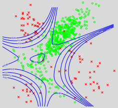
- Object detection
- Image segmentation
- Videos (i.e. timeseries of images) as input

A feature to classify a dog

input images

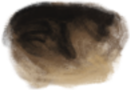
filter



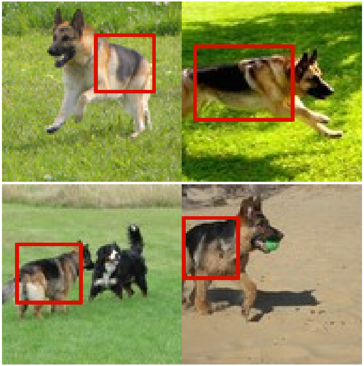


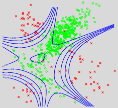
Another feature to classify a dog

filter



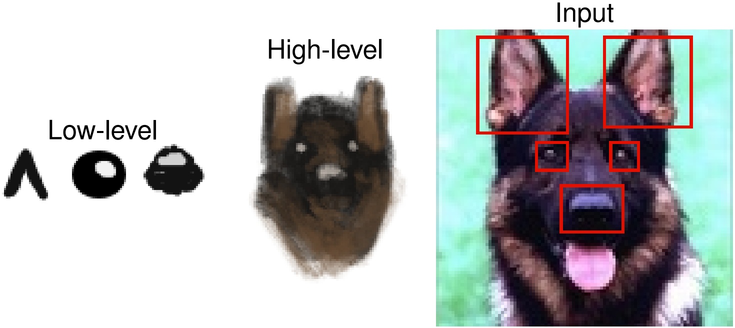
input images

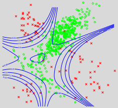




High- and low-level filters

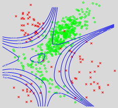
The detection of complex high level filters depends on low-level filters.





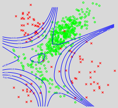
Problems with Fully Connected Artificial Neural Nets (only `Dense` layers)

- high number of parameters



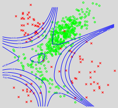
Problems with Fully Connected Artificial Neural Nets (only `Dense` layers)

- high number of parameters
- when images are input:



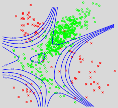
Problems with Fully Connected Artificial Neural Nets (only `Dense` layers)

- high number of parameters
- when images are input:
  - no notion of pixel neighborhoods
  - no translation invariance



Idea of a CNN

- suppose we have K filters like in the previous slides



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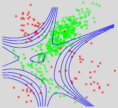

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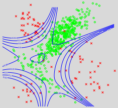

Idea of a CNN

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- each filter can detect some feature *somewhere* in the image
- stacked layers can start with simple features (low-level) that contribute to the detection of more complex features (high-level)
- prediction is based on the filters in the last layer

Cross-correlation (2-dimensional)

Definition 1

Let $A = (a_{ij})_{0 \le i,j < m}$ be a square $m \times m$-dimensional matrix and

$$B = \left(b_{ij}\right)_{\substack{0 \le i < h \\ 0 \le j < w}}$$

be another matrix of shape $h \times w$.
The $h - m + 1 \times w - m + 1$-dimensional matrix C with entries

$$c_{i,j} := \sum_{i'=0}^{m-1} \sum_{j'=0}^{m-1} a_{i',j'} \cdot b_{i+i',j+j'}$$

is the 2-dimensional cross-correlation of A and B. We write $C = A * B$.

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Example 2

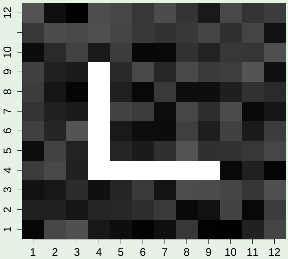
$m = 2$, $h = 4$, $w = 5$.

$$A = \begin{pmatrix} 1 & -1 \\ 2 & -3 \end{pmatrix} \qquad B = \begin{pmatrix} 2 & -3 & 0 & 2 & -1 \\ 0 & 1 & 4 & 0 & 1 \\ 2 & -2 & 7 & 3 & 0 \\ -1 & 0 & 1 & 0 & 4 \end{pmatrix} \qquad C = \begin{pmatrix} 2 & -13 & 6 & 0 \\ 9 & -28 & 9 & 5 \\ 2 & -12 & 6 & -9 \end{pmatrix}$$

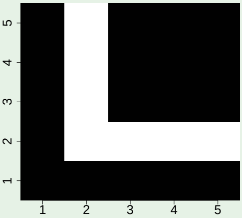
Cross-Correlation of an Image

$$
\begin{pmatrix}
0.56 & 0.54 & 0.59 & 0.41 & 0.56 & 0.54 & 0.45 & 0.50 & 0.44 & 0.58 & 0.44 & 0.54 \\
0.48 & 0.42 & 0.53 & 0.48 & 0.53 & 0.46 & 0.42 & 0.52 & 0.60 & 0.52 & 0.56 & 0.52 \\
0.40 & 0.56 & 0.56 & 0.42 & 0.51 & 0.55 & 0.58 & 0.47 & 0.55 & 0.53 & 0.51 & 0.57 \\
0.40 & 0.44 & 0.58 & 1.00 & 0.51 & 0.47 & 0.51 & 0.43 & 0.54 & 0.48 & 0.56 & 0.46 \\
0.53 & 0.42 & 0.58 & 1.00 & 0.59 & 0.55 & 0.56 & 0.43 & 0.57 & 0.52 & 0.53 & 0.52 \\
0.44 & 0.53 & 0.45 & 1.00 & 0.51 & 0.43 & 0.43 & 0.53 & 0.49 & 0.42 & 0.52 & 0.57 \\
0.41 & 0.51 & 0.53 & 1.00 & 0.46 & 0.43 & 0.54 & 0.42 & 0.57 & 0.42 & 0.53 & 0.53 \\
0.43 & 0.49 & 0.49 & 1.00 & 0.48 & 0.46 & 0.56 & 0.48 & 0.50 & 0.54 & 0.56 & 0.57 \\
0.45 & 0.49 & 0.44 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 0.46 & 0.59 & 0.58 \\
0.59 & 0.45 & 0.50 & 0.48 & 0.48 & 0.60 & 0.46 & 0.41 & 0.46 & 0.55 & 0.57 & 0.40 \\
0.56 & 0.48 & 0.45 & 0.57 & 0.55 & 0.49 & 0.48 & 0.45 & 0.47 & 0.50 & 0.58 & 0.43 \\
0.41 & 0.48 & 0.44 & 0.54 & 0.43 & 0.55 & 0.52 & 0.54 & 0.55 & 0.43 & 0.53 & 0.59
\end{pmatrix}
$$

“filter” A



input image B



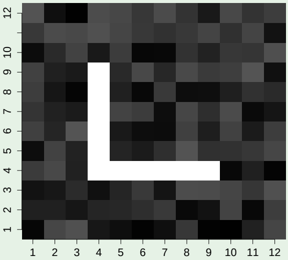
$$
\begin{pmatrix}
-1 & 4 & -1 & -1 & -1 \\
-1 & 4 & -1 & -1 & -1 \\
-1 & 4 & -1 & -1 & -1 \\
-1 & 4 & 4 & 4 & 4 \\
-1 & -1 & -1 & -1 & -1
\end{pmatrix}
$$

output image
$C = A * B$

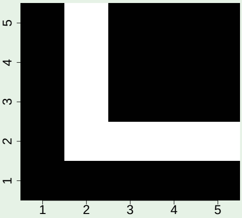
Cross-Correlation of an Image

$$
\begin{pmatrix}
0.56 & 0.54 & 0.59 & 0.41 & 0.56 & 0.54 & 0.45 & 0.50 & 0.44 & 0.58 & 0.44 & 0.54 \\
0.48 & 0.42 & 0.53 & 0.48 & 0.53 & 0.46 & 0.42 & 0.52 & 0.60 & 0.52 & 0.56 & 0.52 \\
0.40 & 0.56 & 0.56 & 0.42 & 0.51 & 0.55 & 0.58 & 0.47 & 0.55 & 0.53 & 0.51 & 0.57 \\
0.40 & 0.44 & 0.58 & 1.00 & 0.51 & 0.47 & 0.51 & 0.43 & 0.54 & 0.48 & 0.56 & 0.46 \\
0.53 & 0.42 & 0.58 & 1.00 & 0.59 & 0.55 & 0.56 & 0.43 & 0.57 & 0.52 & 0.53 & 0.52 \\
0.44 & 0.53 & 0.45 & 1.00 & 0.51 & 0.43 & 0.43 & 0.53 & 0.49 & 0.42 & 0.52 & 0.57 \\
0.41 & 0.51 & 0.53 & 1.00 & 0.46 & 0.43 & 0.54 & 0.42 & 0.57 & 0.42 & 0.53 & 0.53 \\
0.43 & 0.49 & 0.49 & 1.00 & 0.48 & 0.46 & 0.56 & 0.48 & 0.50 & 0.54 & 0.56 & 0.57 \\
0.45 & 0.49 & 0.44 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 1.00 & 0.46 & 0.59 & 0.58 \\
0.59 & 0.45 & 0.50 & 0.48 & 0.48 & 0.60 & 0.46 & 0.41 & 0.46 & 0.55 & 0.57 & 0.40 \\
0.56 & 0.48 & 0.45 & 0.57 & 0.55 & 0.49 & 0.48 & 0.45 & 0.47 & 0.50 & 0.58 & 0.43 \\
0.41 & 0.48 & 0.44 & 0.54 & 0.43 & 0.55 & 0.52 & 0.54 & 0.55 & 0.43 & 0.53 & 0.59
\end{pmatrix}
$$

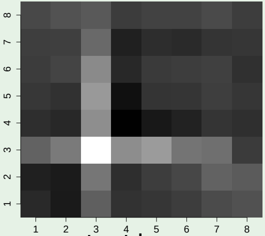
“filter” A



input image B



$$
\begin{pmatrix}
-1 & 4 & -1 & -1 & -1 \\
-1 & 4 & -1 & -1 & -1 \\
-1 & 4 & -1 & -1 & -1 \\
-1 & 4 & 4 & 4 & 4 \\
-1 & -1 & -1 & -1 & -1
\end{pmatrix}
$$



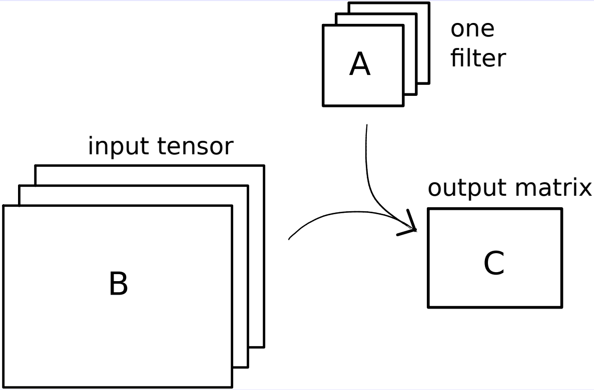
output image

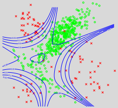
$$C = A * B$$

$$
\begin{pmatrix}
6.6 & 7.4 & 5.0 & 4.19 & 4.7 & 4.3 & 4.7 & 5.2 \\
6.2 & 7.9 & 8.8 & 4.56 & 5.3 & 5.4 & 4.6 & 6.2 \\
5.2 & 5.9 & 9.2 & 3.27 & 4.6 & 5.2 & 3.8 & 5.6 \\
4.7 & 5.2 & 12.1 & 2.57 & 4.6 & 5.0 & 4.0 & 5.5 \\
4.4 & 4.1 & 11.0 & 0.85 & 2.0 & 3.7 & 3.1 & 5.1 \\
7.7 & 9.3 & 19.3 & 11.13 & 11.9 & 10.8 & 8.4 & 7.2 \\
2.7 & 2.7 & 9.6 & 3.69 & 4.4 & 5.7 & 5.4 & 6.7 \\
3.7 & 3.1 & 7.9 & 4.14 & 4.8 & 5.2 & 5.3 & 6.0
\end{pmatrix}
$$

3-dimensional input

- Want to
  1. use multiple filters in parallel and
  2. stack several (convolutional) layers.
- Also, color images are naturally encoded as 3-dimensional (each pixel has a red, green and blue value).
- Solution: Define convolution for 3-dimensional tensor input as well.





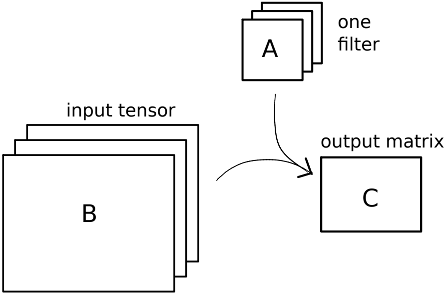
3-dimensional cross-correlation

Let $B = (b_{ijk})_{\substack{0 \leq i < h \\ 0 \leq j < w \\ 0 \leq k < d}}$ be a tensor of shape $h \times w \times d$ and

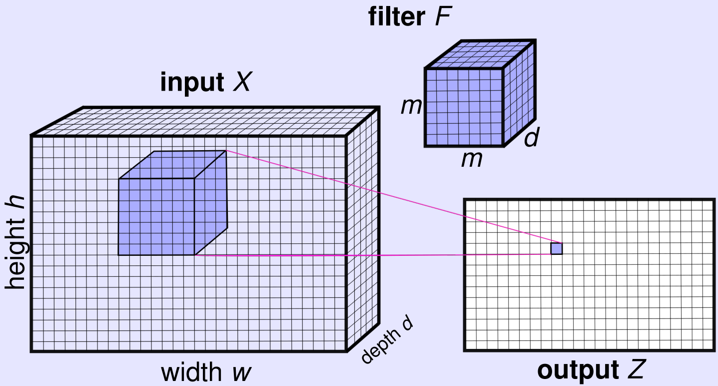
let $A = (a_{ijk})_{\substack{0 \leq i, j < m \\ 0 \leq k < d}}$ be another tensor (“filter”).

The cross-correlation of A and B is then the $h - m + 1 \times w - m + 1$-dimensional matrix $C = A * B$ with entries

$$c_{i,j} := \sum_{i'=0}^{m-1} \sum_{j'=0}^{m-1} \sum_{k=0}^{d-1} a_{i',j',k} \cdot b_{i+i',j+j',k}.$$



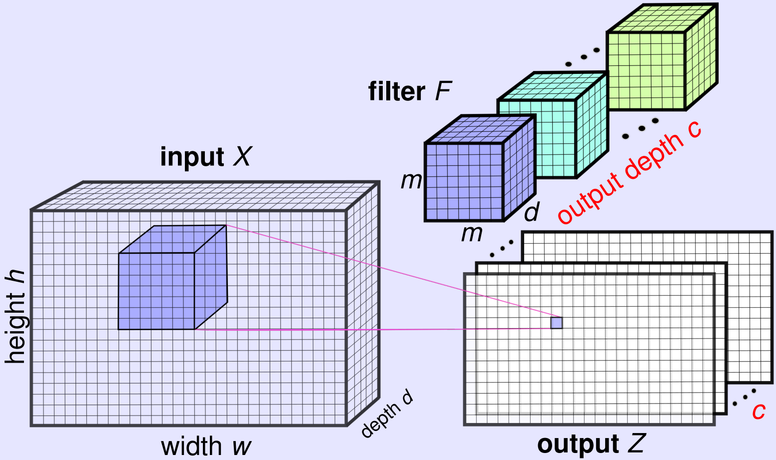
Convolutional Layer



$$z_{i,j} = \sum_{i'=0}^{m-1} \sum_{j'=0}^{m-1} \sum_{k=0}^{d-1} x_{i+i',j+j',k} \cdot f_{i',j',k}$$

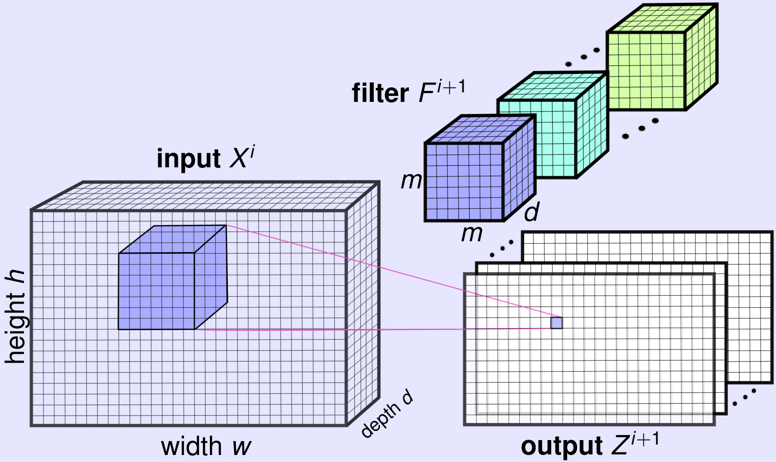
Deep Learning for Computer Vision

Convolutional Layer



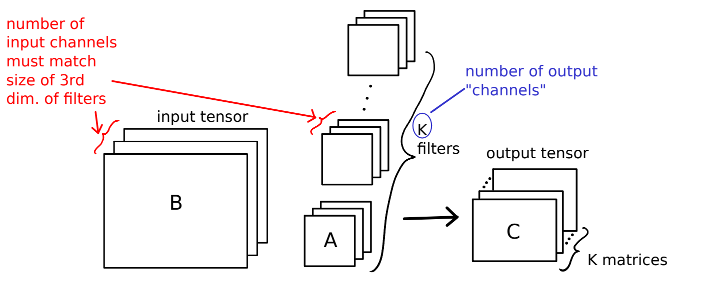
$$z_{i,j,r} = \sum_{i'=0}^{m-1} \sum_{j'=0}^{m-1} \sum_{k=0}^{d-1} x_{i+i',j+j',k} \cdot f_{i',j',k,r} + \underbrace{b_r}_{\text{bias}}$$

Convolutional Layer



$X^{i+1} = \max(Z^{i+1}, 0)$ (**Re**ctified **L**inear **U**nit)

Convolutional Layer



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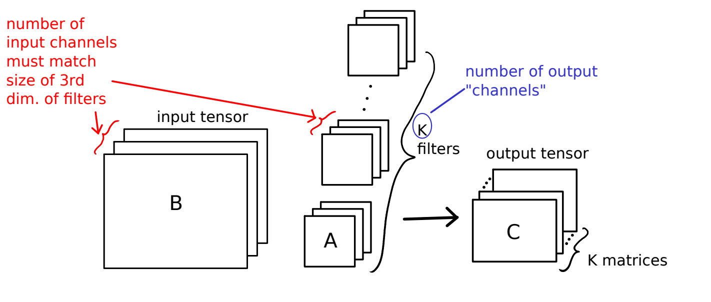

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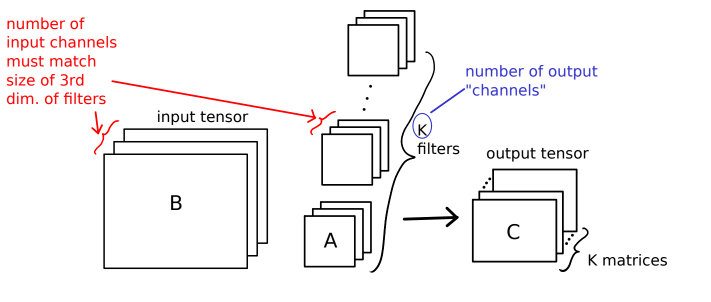

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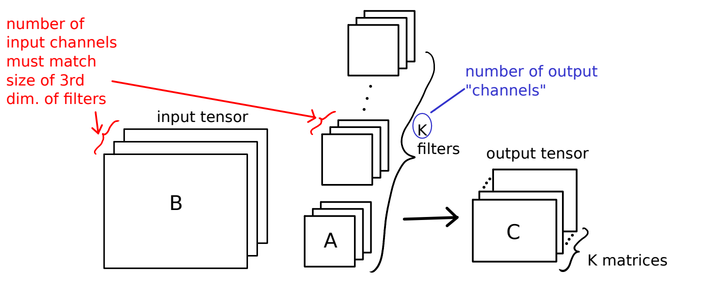
- Stride (Schrittweite) s: Skip $s - 1$ positions in each direction when 'sliding' A over $B \Rightarrow$ decreases output layer size up to a factor of s^2.

Convolutional Layer



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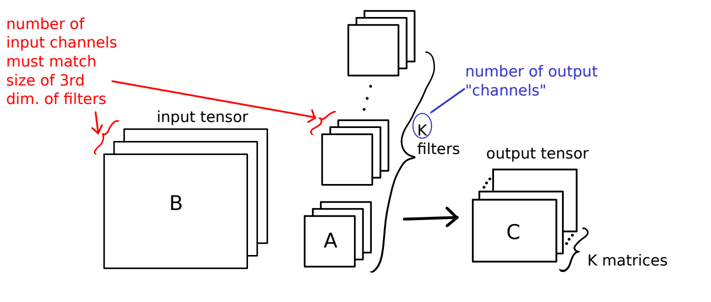

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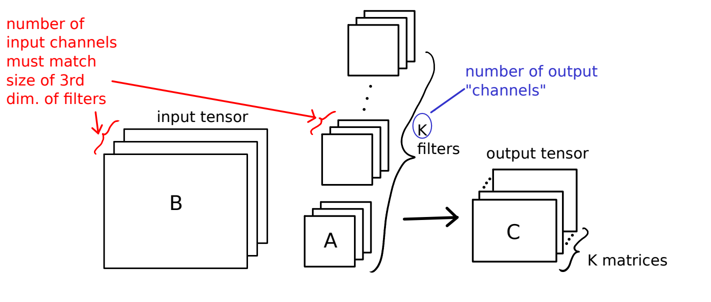

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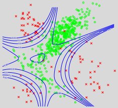
- The matrices A are learned, not set manually. The derivative wrt. to the filter matrix parameters is computed during BackProp.
- Convolution is a special case of a fully-connected layer, in which certain parameters are shared (*parameter sharing*).
- Output neurons of convolution can detect lower-level features like ("lower left corner", "pupil") and be combined in deeper layers.



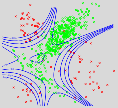
Pooling-Layers

Max-Pooling (`tf.keras.layers.MaxPool2D`)

- similar to a convolutional layer
- requires a `pool_size` m like the filter size
- does not have any parameters
- computes output

$$z_{i,j,r} = \max_{\substack{i' \in [0,\, m) \\ j' \in [0,\, m)}} x_{s \cdot i + i', s \cdot j + j', r}$$

- is usually applied with a stride $s \geq 2$ and therefore reduces height and width
- often $s = m$, can have different strides for each dimension
- intuition:



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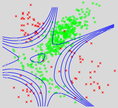
- intuition: checks local presence of *at least one* feature (e.g. a green stroke) rather than summing up if there are multiple



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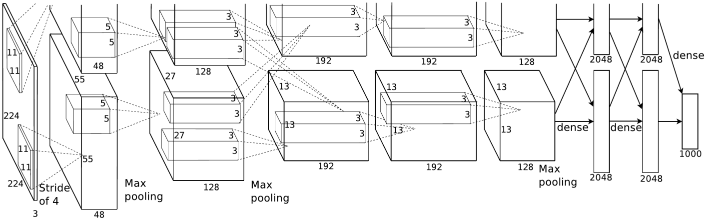
With an analogous definition, average pooling *averages* over regions of size $m \times m$, but is used less often.

Multi-Layered CNN example

Photo classification

- CNN from 2012 (“AlexNet”)
- classification into 1000 categories



Alex Krizhevsky, Ilya Sutskever and Geoffrey Hinton, “ImageNet Classification with Deep Convolutional Neural Networks”, *NIPS*, 2012