Non-dimensionalization

Why using *adim* variables?

- numerics (avoid ill-conditioned systems)
- analysis of results (less parameters)

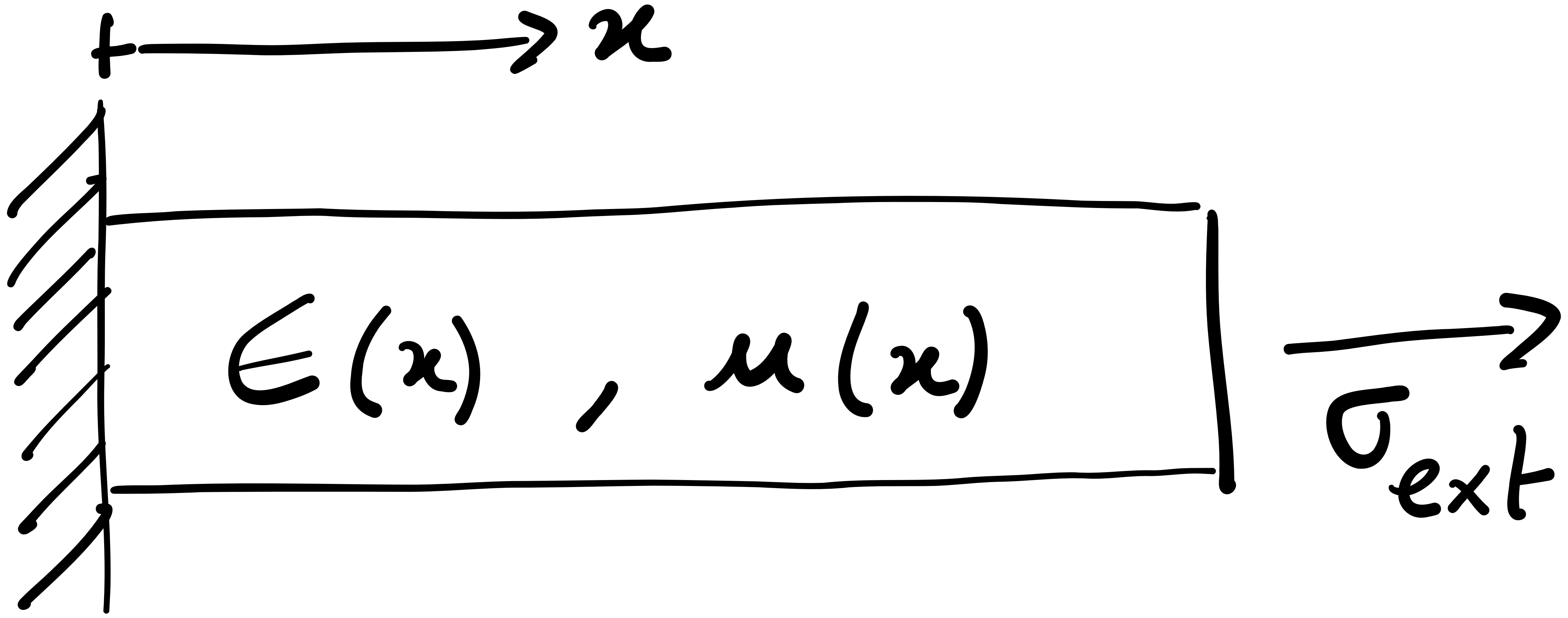
with dim

$$d = 3\mathrm{mm}$$

adim

$$\bar{d} = \frac{d}{L}$$

Illustration on a simple example:



We impose σ_{ext}

What is the extension $\epsilon(x)$?

Non-dimensionalization

$$V = \int_0^L \frac{1}{2} E\, \epsilon^2(x)\, dx - \sigma_{\text{ext}}\, u(L)$$

$$\epsilon(x) = u'(x)$$

$$V = \int_0^L \left(\frac{1}{2} E\, \epsilon^2(x) - \sigma_{\text{ext}}\, \epsilon(x) \right)\, dx$$

$$V = \frac{1}{2} E\, \epsilon^2 - \sigma_{\text{ext}}\, \epsilon$$

We impose σ_{ext} , we look for ϵ

One parameter E (metal 10^{11} Pa, wood 10^9 Pa, PDMS 10^6 Pa)

Stable equilibrium: min V (e.g. with Newton routine)

Difficulty: $E = 1$GPa $= 10^9$Pa ==> $V = \frac{1}{2} 10^9\, \epsilon^2 - \sigma_{\text{ext}}\, \epsilon$

ill-conditioned Hessian matrix

Non-dimensionalization

$$V = \frac{1}{2}\, 10^9\, \epsilon^2 - \sigma_{ext}\, \epsilon$$ ill-conditioned Hessian matrix

<u>Mechanical Engineering solution</u> (e.g Abaqus):

=> work with appropriate units: $E = 10^9\, E_{GPa}$ and $\sigma_{ext} = 10^9\, \sigma_{ext,GPa}$

$$V = \frac{1}{2} E\, \epsilon^2 - \sigma_{ext}\, \epsilon$$

$$V = \frac{1}{2}\, 10^9\, E_{GPa}\, \epsilon^2 - 10^9\, \sigma_{ext,GPa}\, \epsilon$$

$$\Rightarrow \frac{V}{10^9} = \frac{1}{2} E_{GPa}\, \epsilon^2 - \sigma_{ext,GPa}\, \epsilon$$

Advantage: no large number in the numerical model

But: still 1 parameter (E_{GPa})

Non-dimensionalization

$$V = \frac{1}{2}\, 10^9\, \epsilon^2 - \sigma_{ext}\, \epsilon$$ ill-conditioned Hessian matrix

<u>Physicist solution</u>: adim variables

$$V = \frac{1}{2} E\, \epsilon^2 - \sigma_{ext}\, \epsilon$$

$$\frac{V}{E} = \frac{1}{2} \epsilon^2 - \frac{\sigma_{ext}}{E}\, \epsilon$$

$$\bar{V} = \frac{1}{2} \epsilon^2 - \bar{\sigma}_{ext}\, \epsilon$$

2 Advantages:

(i) no large number in the numerical model

(ii) no parameter

This is a particular choice of units

Mech. Eng. units:

$$E = 10^9\, E_{GPa} \quad ; \quad \sigma_{ext} = 10^9\, \sigma_{ext,GPa}$$

values *values*

here:

$$E = E \cdot 1 \quad ; \quad \sigma_{ext} = E \cdot \bar{\sigma}_{ext}$$

Non-dimensionalization

Nota bene

(with units)

$$V = \frac{1}{2} E\,\epsilon^2 - \sigma_{ext}\,\epsilon \qquad \text{(real world)}$$

trick: we set $E = 1$ in the numerics

(adim)

$$\bar{V} = \frac{1}{2} \epsilon^2 - \bar{\sigma}_{ext}\,\epsilon \qquad \text{(numerical model)}$$

The Magic behind non-dimensionalization

SI Units

m kg Sec Amp.

here: statics, and
no electric charges

m kg

cm N

mm Pa

2 units, which can be chosen *arbitrarily*

Adim Units

L E ← previous example

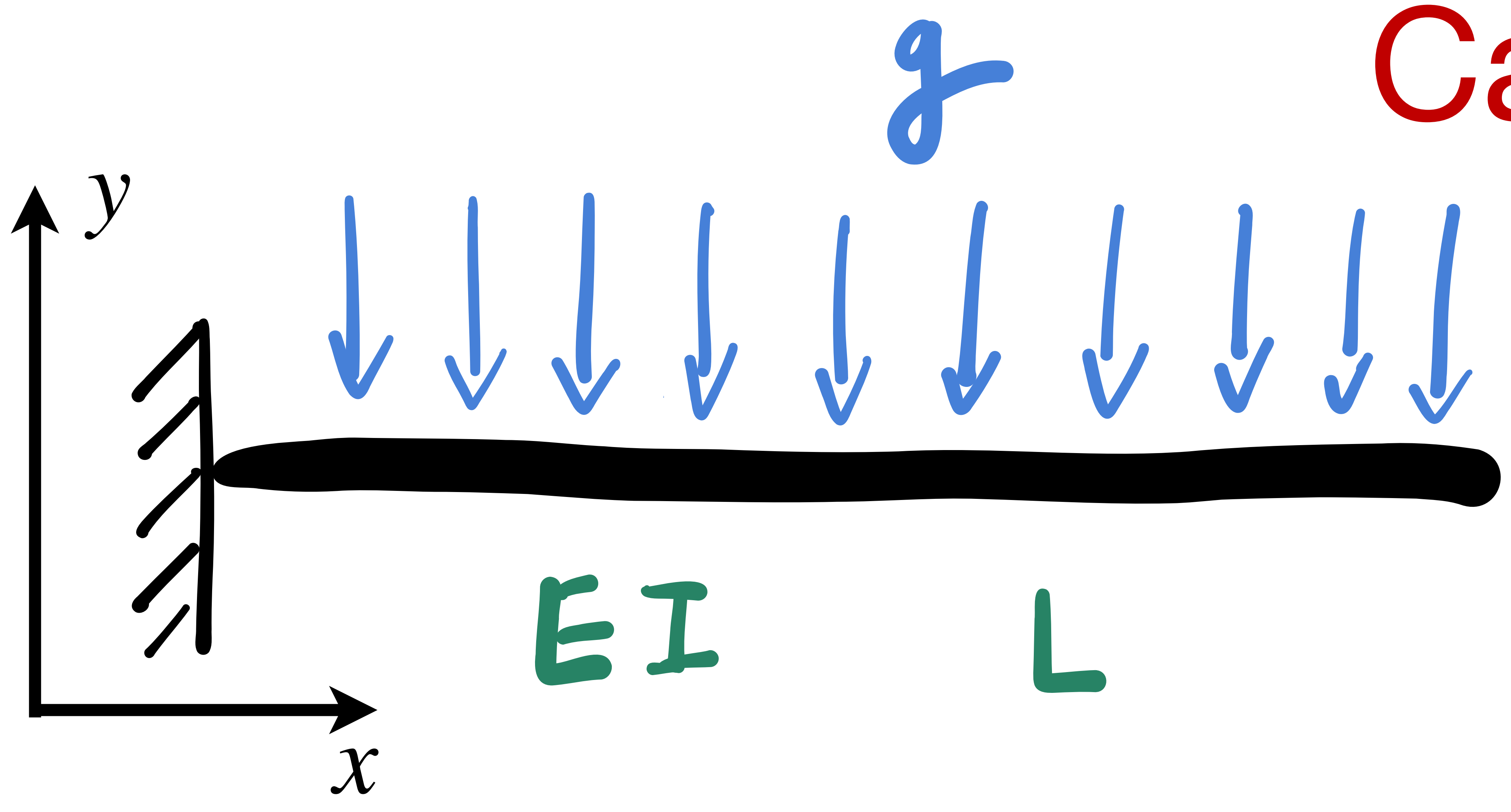
Cantilever test

Cantilever

Well-known in Engineering and Computer Graphics
(here, finite rotations)

Bending rigidity	EI
Length	L
Weight	Mg

Cantilever test



Bending rigidity EI

Total length L

Total weight $M\,g$

Fluid mech. gravito-elastic number

with dim

$$(1)\quad EI\,\frac{d^4 y}{dx^4} + \frac{M\,g}{L} = 0$$

$$(2)\quad \frac{d^4 y}{dx^4} + \frac{M\,g}{EI\,L} = 0$$

introduce adim variables

$\bar{x} = x/L\ ;\ \bar{y} = y/L$

trick: we simply set

$EI = 1$ *and* $L = 1$

adim

$$(3)\quad \frac{d^4 \bar{y}}{d\bar{x}^4} + \frac{M\,g}{EI/L^2} = 0$$

$$(4)\quad \frac{d^4 \bar{y}}{d\bar{x}^4} + \overline{M\,g} = 0$$

The Magic behind non-dimensionalization

SI Units

m kg Sec Amp.

here: statics, and
no electric charges

m kg

2 units, which can be chosen *arbitrarily*

Length Force

Our Units (adim units)

We choose

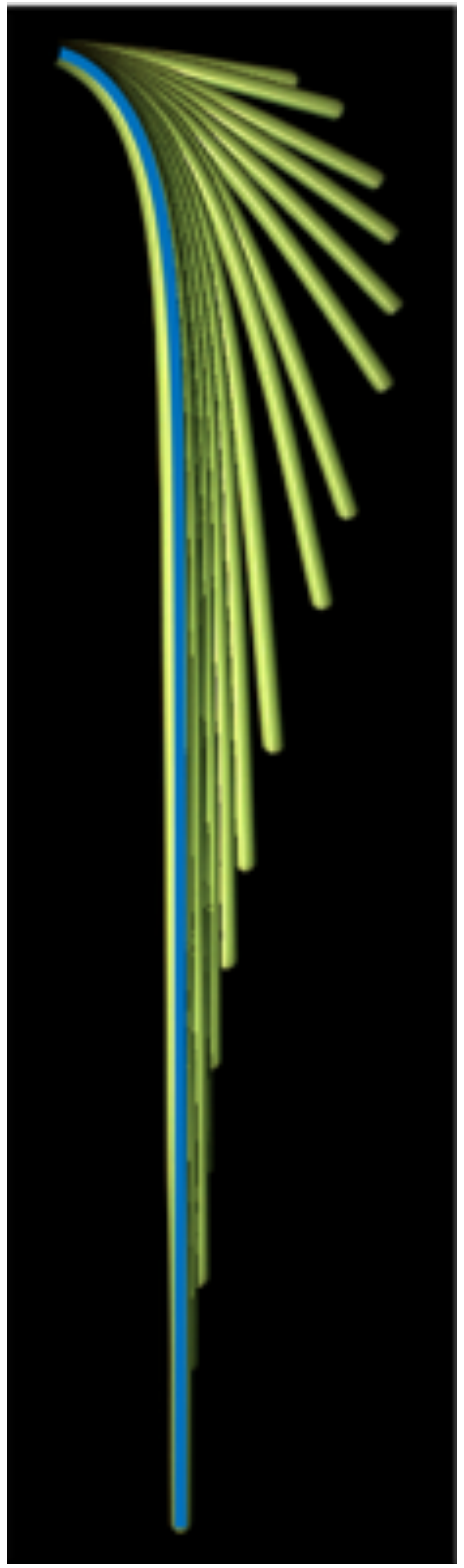
$L \quad EI/L^2$

Equiv. to
formally write

$L=1 \quad EI=1$

no loss of generality!

Cantilever test



Well-known in Engineering and Computer Graphics
(here, finite rotations)

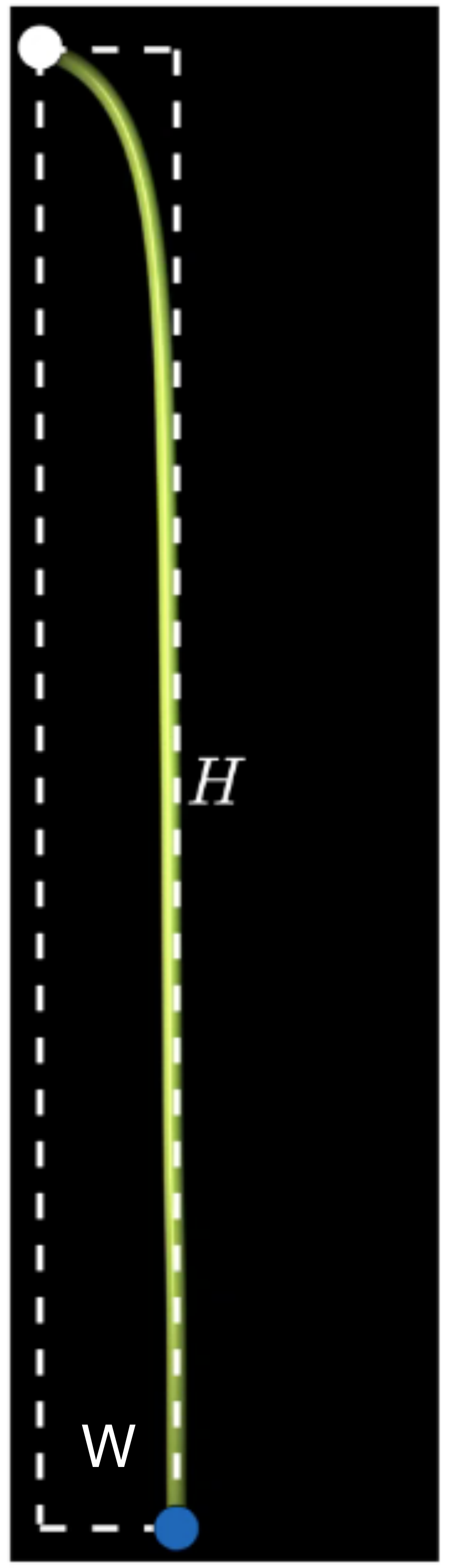
$$3 \left\{ \begin{array}{ll} \text{Bending rigidity} & EI \\ \text{Length} & L \\ \text{Weight} & Mg \end{array} \right.$$

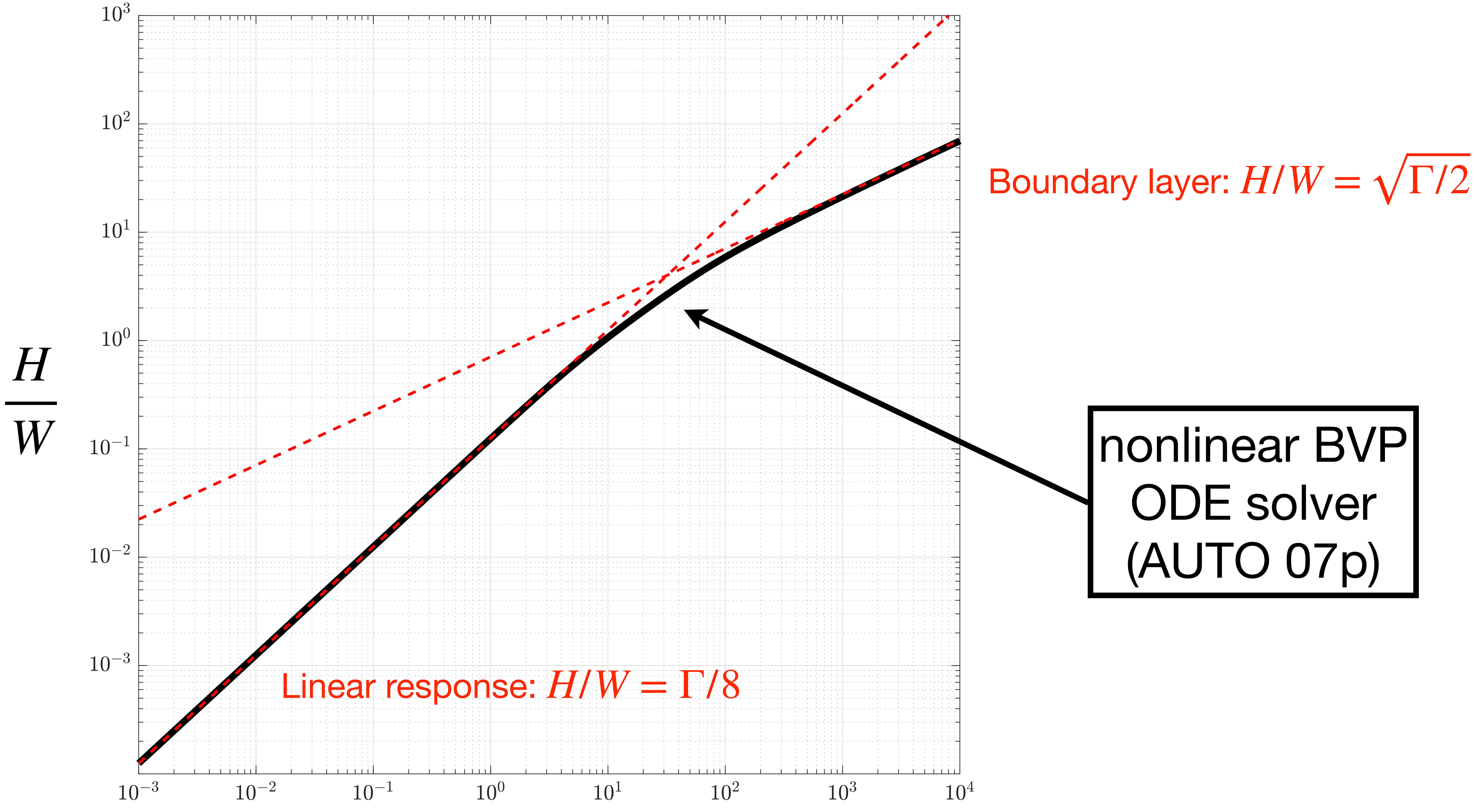
-2

1

Adim weight: $\Gamma = \dfrac{MgL^2}{EI}$

Cantilever test - master curve





$$\Gamma = \frac{MgL^2}{EI}$$ (adim weight)