Critical Exponent of Species-Size Distribution in Evolution

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Abstract

We analyze the geometry of the species– and genotype-size distribution in evolving and adapting populations of single-stranded self-replicating genomes: here programs in the Avida world. We find that a scale-free distribution (power law) emerges in complex landscapes that achieve a separation of two fundamental time scales: the relaxation time (time for population to return to equilibrium after a perturbation) and the time between mutations that produce fitter genotypes. The latter can be dialed by changing the mutation rate. In the scaling regime, we determine the critical exponent of the distribution of sizes and strengths of avalanches in a system without coevolution, described by first-order phase transitions in single finite niches.

This article presents the work of Wiering and van Hasselt in "Ensemble Algorithms in Reinforcement Learning" [reference]. Ensemble methods merge multiple reinforcement learning (RL) algorithms into a single agent with the objectif of increasing the learning speed and the obtained reward. While ensemble methods have already been used in the context of reinforcement learning for representing and learning a single value function [references 14-16 in paper], Wiering and van Hasselt introduce a novel technique that combines the policy of each RL learning. The individual RL algorithms implemented were: Q-learning, Sarsa, Actor-Critic, QV-learning, and ACLA. The ensemble methods are majority voting, rank voting, Boltzmann multiplication, and Boltzmann addition. They implemented their algorithms for 5 mazes problems with increasing complexity to assess their performance. For all the mazes except the first one, the state space is very large, therefore a neural network was used for value functions approximation. We reimplemented their algorithms and obtained the same results for the first maze. For the other mazes, our neural network did not converge. Possible causes for this non-convergence are discussed in this article.

Introduction

Reinforcement learning is a branch of machine learning in which software agents learn from interacting with their environment. It is a very general framework that can be used to learn tasks of a sequential decision making nature. An environment can exists as different states, and the actions available to the agent depend on the state of this environment. After every action, the agent receives a reward which might be positive or negative. Through iterative experience, the agent seeks a policy, an idea about which action needs to be performed for every possible state of the environment, that maximizes the sum of rewards over time.

For simple tasks, the agent can use a tabular expression to remember values associated with certain states or state-action combinations, also called state(-action) functions. For complex games however, there is an explosion of state-action possibilities. Chess for example, is estimated to have more than 10^{50} chess-board configurations. Not only do we lack the memory to store such a table, we would need more time than the age of the universe to explore all possible states. Therefore function approximators are used to approximate such state(-action) functions. The capture the most essential concepts in order to maximize reward. Neural networks are an example of such function approximator. DeepMind's program AlphaZero is an interesting example that combines neural networks with reinforcement learning algorithms to learn chess. The program was given no domain knowledge except the rules and achieved a superhuman level within 24 hours.

Some basic, well known online model-free value-function based reinforcement algorithms are SARSA and Q-learning. They are both temporal difference (TD) reinforcement learning algorithm that learn by updating a Q-function (action value function). The difference is that Q-learning is off-policy. This means that the optimal action-value function is learned independent of the policy that is being followed, unlike on-policy methods like SARSA, the learning of the action-value does depend on the policy. The Actor-Critic (AC) is an temporal difference, on-policy learning algorithm. But where SARSA and Q-learning only keep track of a single Q-function, AC makes the distinction between a critic value function V that only depends on the state, and an Actor function which will map for each action the states to preference values. wiering(2007) explains a more recent and

advanced on-policy QV-learning method. This algorithm can be seen as mix between Actor-Critic and Q-learning. As Actor-Critc, it learn the state-value function V and an action-value function. But unlike Actor-Critic, it learns the Q-function for that. ACLA is another on-policy RL algorithm derived from Actor-Critic that learns a state value-function V and Actor function P. QV-learning and ACLA have been shown to outperform similar, more basic, RL algorithms (wiering2007) and QV-learning even scores higher in certain problem contexts than more recent RL methods (wiering2009).

Reinforcement learning methods, however useful, can take many steps to learn. Ensemble methods are a powerful method to combine different Reinforcement Learning (RL) algorithms, which often result in improved learning speed and final performance. [1] used ensemble methods to combine multiple reinforcement learning algorithms for multiple agents for which they used Temporal-Difference(TD) and Residual-Gradient(RG) update methods as well as a policy function. Other ensemble methods have been used in reinforcement learning to combine value functions stored by function approximators (wiering 2008 [14], [15], [16], [17]). However, only Rl algorithms with the same value function can be combined in this way. (Wiering2008) wanted to combine RL methods with different value functions and policies (e.g. Q-learning and ACLA). It is possible however, to combine the different policies that were derived from distinct value functions. Some algorithms that perform this task and take exploration into account at the same time are Majority voting, Rank voting, Boltzmann multiplication, and Boltzmann addition. Majority Voting (VM) combines the best action of the RL algorithms and bases the final decision on the number of times that each action was preferred by the different RL methods. Rank Voting (RV) lets each algorithm rank the different actions and combines these rankings into nal preferences over actions. Boltzmann mulplication(BM) multiplies the Boltzmann probabilities of each action computed by each RL algorithm. Finally, Boltzmann Addition(BA) is very similar to Boltzmann mulplication, but adds instead of multiplies the Boltzmann probabilities of actions.

In this article, we presents the work of Wiering and van Hasselt in "Ensemble Algorithms in Reinforcement Learning". Five RL algorithms (Q-learning, SARSA, Actor-Critic, QV-learning, and ACLA) were compared to 4 ensemble methods (Majority Voting, Rank Voting, Boltzmann multiplication, and Boltzmann Addition). To this end, they had to solve five mazes of varying complexity.

References

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Methods

Reinforcement learning

With reinforcement learning methods, agent are able to learn from interactions with their environment. We assume a Markov Decision Process that is defined by the following parameters. The state-space $ S ={s_1, s_2, ... , s_n}$ with s_t denoting the state at moment t. $A(s)$ is the set of action that are available to the agent if it is in state s, a_t is the action executes in state s_t at time t. The transition function $T(s, a, s')$ maps state-action pairs to a probability over successor states. Finally, there is also the reward function $R(s, a, s')$ that return a reward for every state-action-state transition. This reward function can give stochastic rewards. The goal of an agent is to estimate the optimal policy $\pi^*(s)$. With $\pi^*(s)$, the agent would know for each state what the optimal action is to receive the highest possible cumulative discounted reward in future states.

Q-learning Q-learning is an off-policy, temporal difference (TD) reinforcement learning algorithm that learns by updating a Q-function. This action-value function Q directly approximates the optimal action-value function Q^* and it does this independent of the policy that is being followed. Q-learning updates with new experience (s_t, a_t, r_t, s_{t+1}) in the following way:

$$Q(s_t, a_T) := Q(s_t, a_t)+\alpha(r_t+\gamma max_a Q(s_{t+1}, a)-Q(s_t, a_t))$$

With $0 \leq \alpha \leq 1$ being the learning rate, and $0 \leq \gamma \leq 1$ being the discount factor. Higher γ values will result in the agent taking into account not only the immediate reward of an action, but also future rewards that will be available in future states as a consequence of this action. The advantage of tabular Q-learning is that it will always converge to Q^*. It does not matter what behavioral policy is used, as long as each state-action pair is visited an infinite number of times (wiering 2007 [13]). However, Q-learning combined with function approximators, such as neural networks, have been observed to diverge (wiering 2007).

SARSA Like Q-learning, SARSA is temporal difference (TD) reinforcement learning algorithm that learns the action-value Q-function (wiering 2007). Unlike Q-learning, SARSA is on-policy, meaning that the approximation of optimal Q^* values depend on the policy being followed (wiering 2007 [6]). An experience is defined as the quintuple $(s_t, a_t, r_t, s_{t+1}, a_{t+1})$. This quintuple is what gave rise to the name SARSA (sutton 2018). After every transition, Q-values are updated by the following method:

$$Q(s_t, a_T) := Q(s_t, a_t)+\alpha(r_t+\gamma Q(s_{t+1}, a_{t+1})-Q(s_t, a_t))$$

As with tabular Q-learning, tabular SARSA converges towards Q^* if all state-action pairs are visited infinite number of times.

Actor-Critic The Actor-Critic (AC) is an temporal difference, on-policy learning algorithm. Where SARSA and Q-learning only keep track on the Q-function, Actor-Critic will update both a Critic and Actor function (wiering 2008 [1]). The Critic function assigns values to the states, irrespective of the action chosen by the agent. The Actor function will for each action map the states to preference values (wiering 2008). An experience is defined as the sequence $((s_t, a_t, r_t, s_{t+1}))$. After each experience, the Critic and Actor get updated as follows [wiering 2008[11]]:

$$\text{Critic: } V(s_t) := V(s_t) + \beta(r_t + \gamma V(s_{t+1}) - V(s_t))$$

$$\text{Actor: } P(s_t, a_t) := P(s_t, a_t) + \alpha(r_t + \gamma V(s_{t+1}) - V(s_t))$$

Where β is the learning rate of the Critic and α the learning rate of the Actor. P-values should not be seen as literal Q-values, but instead as preference values.

QV-learning QV-learning (wiering 2008[6]) is very similar to Actor-Critic. It is also a learning method that learns a state value-function V with TD method, and an Actor function to map the states for each action to preference values. However, QV-learning learns actual Q-values as preference values (wiering 2008). After each experience (s_t, a_t, r_t, s_{t+1}), the V value-function is updates in the same way as in the Actor-Critic method (wiering 2007):

$$V(s_t) := V(s_t) + \beta(r_t + \gamma V(s_{t+1}) - V(s_t))$$

The update for the Q-function is similar to the Actor update, but the $-V(s_t)$ at the end is replaced by $-Q(s_t, a_t)$:

$$\text{Qvalue: } Q(s_t, a_t) := Q(s_t, a_t) + \alpha(r_t + \gamma V(s_{t+1}) - Q(s_t, a_t))$$

ACLA (Actor-Critic Learning Automaton) Actor-Critic Learning Automaton (ACLA)(wiering 2008[6]) is an on-policy learning algorithm that learns a state value-function V and Actor function P. After each experience

wiering 2008 (s_t, a_t, r_t, s_{t+1}), the state value-function is updated in the same way as Actor-Critic or QV-learning (wiering 2007):

$$\delta_t = r_t + \gamma V(s_{t+1}) - V(s_t)$$

$$V(s_t) := V(s_t) + \beta(\delta_t)$$

The actor function maps states to preferences for actions and is updated by an automaton-like updating rule (wiering 2008 [20]) The policy mapping update depends on the sign of δ:

$\delta_t \geq 0$

$$a = a_t : P(s_t, a_t) := P(s_t, a_t) + \alpha(1 - P(s_t, a_t))$$

$$\forall a \neq a_t : P(s_T, a) := P(s_t, a) + \alpha(0 - P(s_t, a))$$

$\delta_t < 0$

$$a = a_t : P(s_t, a_t) := P(s_t, a_t) + \alpha(0 - P(s_t, a_t))$$

$$\forall a \neq a_t : P(s_t, a) := P(s_t, a) + \alpha\left(\frac{P(s_t, a)}{\sum_{b \neq a_t} P(s_t, b)} - P(s_t, a)\right)$$

There are also additional rules to ensure that the target values are between 0 and 1 and existing. If $P(s_t, a)$ is greater than 1, the value gets changed to 1. If $P(s_t, a)$ is smaller than 0, the value gets changed to zero. If the denominator is 0, the new value equals $\frac{1}{|A|-1}$, with $|A|$ being the number of actions (wiering 2008).

Ensemble algorithms in RL

Combining single classifiers for which the errors are not strongly correlated can lead to a higher accuracy than what is available for a single classifier. With this in mind, ensemble methods have been used in reinforcement learning to combine value functions stored by function approximators (wiering 2008 [14], [15], [16], [17]). However, only Rl algorithms with the same value function can be combined in this way. In our case, want to combine RL methods with different value function and policies (e.g. Q-learning and ACLA). It is possible however, to combine the different policies that were derived from distinct value functions. Next, we will describe four ensemble methods, designed by (Wiering 2008), that combine policies and also take into account exploration.

Majority voting Each of the n RL algorithms defines what it thinks to be the best action. Majority voting will transform these best actions into preferences in the following way:

$$p_t(s_t, a[i]) = \sum_{j=1}^{n} I(a[i], a_t^j)$$

with a_t^j being the best action according to algorithm j at time t and

$$x = y : I(x, y) = 1$$

$$x \neq y : I(x, y) = 0$$

The following Boltzmann distribution based ensemble policy is used for actions selection:

$$\pi_t(s_t, a[i]) = \frac{exp[\frac{p_t(s_t, a[i])}{\tau}]}{\sum_k exp[\frac{p_t(s_t, a[k])}{\tau}]}$$

This policy makes sure that the most probable action is the best action according to most algorithms, but also ensures exploration (wiering 2008).

Rank voting Preference values of the ensemble are give by:

$$p_t(s_t, a[i]) = \sum_{j=1}^{n} r_t^j(a[i])$$

If m actions are possible in state s_t, $r_t^j(a[1])$, $r_t^j(a[2])$, $\ldots$, $r_t^j(a[m])$ denotes the weights for these actions as determined by RL algorithm j. The most probable action is weighted m times, the second best m-1 times, and so on... (wiering 2008). As with majority voting, the rank voting algorithm uses the following Boltzmann distribution based ensemble policy to ensure both exploitation and exploration:

$$\pi_t(s_t, a[i]) = \frac{exp[\frac{p_t(s_t,a[i])}{\tau}]}{\sum_k exp[\frac{p_t(s_t,a[k])}{\tau}]}$$

Boltzmann multiplication Boltzmann multiplication calculates the ensemble preferences by multiplying for each action the action-selection probabilities given by the RL algorithms:

$$p_t(s_t, a[i]) = \prod_j \pi_t^j(s_t, a[i])$$

$\pi_t^j(s_t, a[i])$ is the policy for algorithm j at time t for state s_t and action $a[i]$. Since all the RL algorithms use Boltzmann exploration, preference values are never zero. This is important since if only one RL algorithm return zero, Boltzmann multiplication would result for a zero probability for that action, irrespectively of that the other algorithms return high or low probabilities. The ensemble policy for actions selection is calculated in the following way:

$$\pi_t(s_t, a[i]) = \frac{p_t(s_t, a[i])^{\frac{1}{\tau}}}{\sum_k p_t(s_t, a[k])^{\frac{1}{\tau}}}$$

Boltzmann addition The formula of Boltzmann addition looks very similar to Boltzmann multiplication, with the product being replaced by a sum:

$$p_t(s_t, a[i]) = \sum_j \pi_t^j(s_t, a[i])$$

As concept however, it is a variant of rank voting where $r_t^j = \pi_t^t$. The ensemble policy for actions selection is calculated in the same way as Boltzmann multiplication:

$$\pi_t(s_t, a[i]) = \frac{p_t(s_t, a[i])^{\frac{1}{\tau}}}{\sum_k p_t(s_t, a[k])^{\frac{1}{\tau}}}$$

Experiments

We compared five different RL algorithms (Q-learning, SARSA, Actor-Critic, QV-learning, ACLA) with each other and with four ensemble methods (Majority Voting, Rank Voting, Boltzmann Multiplication, Boltzmann Addition). The goal for the agents was to solve five different maze tasks of varying complexity, In each maze, the agent starts at a certain starting position and needs to reach another goal position. The dominating objective was to move with each step closer to the goal. For the first experiment, the agents learned to solve a small base where start, goal and walls were in static position. To do this, they combined RL algorithms with a tabular expression. For the second to fifth maze, complexity was increased by adding different dynamic elements to the maze. To circumvent the combination explosion that would occur in a tabular expression, neural networks were used as function approximators. In each tile of the maze, the agent can initiate 4 actions: going North, East, South or West. These actions are noisy, meaning that every time an action is taken, there is a 20% chance that the agent performs a random action instead. For each maze, the rewards were given in the following way. If the agent moves into the goal tile, it receives a reward of 100. When it tries to move into a wall or border, it will receive a reward of -2 and remain in the same place (state is not changed). For every other move (moving from one tile to the next), the agent receives a reward of -0.1. Even though it does not collide with a wall or border, it gets a negative score to discourage it from wandering around. An agent learns in different trials. Such trial starts with the agent in the starting position and ends when it reaches the goal positions or has wandered for 1000 consecutive action without reaching the goal.

Small Maze experiment The first experiment was the least complex, so that the agent could use tabular expression for the different state-action pairs. We implemented Sutton's Dyna maze, which consist of 54 tiles (6 rows and 9 columns). The maze also includes 7 walls, a starting and goal, all in fixed position (Fig. 1).

In order to compare our results with those of (wiering 2008), we used the same learning rates, discount factors and greediness (inverse of Boltzmann temperature) parameters. In their paper, they explain how they tested a wide range of parameters to optimize each of the five RL algorithms final performance (by evaluating the average reward). They wanted to compare the best version of each algorithm, which was not possible if the same parameters were used. For the ensemble methods, they used the same parameters as were determined for the single RL algorithms. (wiering 2008) observed the discount factor to have a major impact on the final and cumulative score. For the SARSA method, discount factors 0.90 and 0.95, and for the ACLA method, discount factors 0.99 and 0.90 were compared to each other. We wanted to confirm these findings, so we did simulations with the same discount factors.

Neural networks as universal function approximators Traditionally RL algorithms use tabular expression to rep-

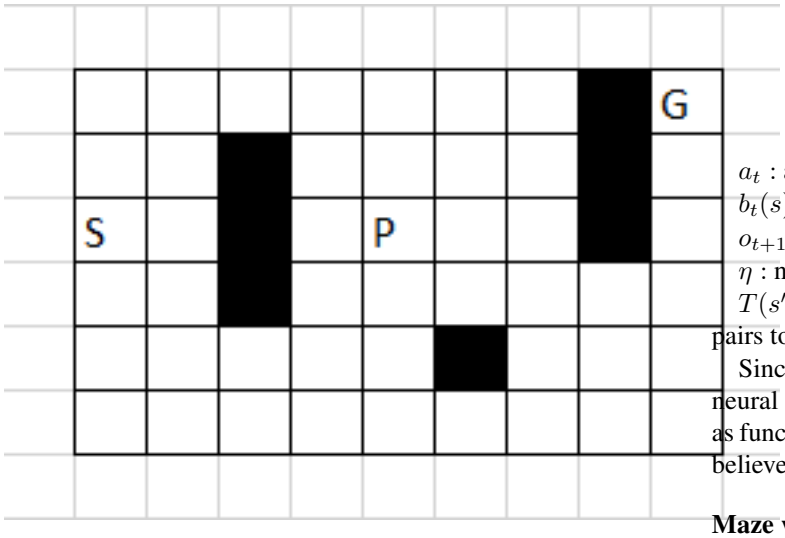


Figure 1: Sutton's Dyna maze

resent a state-function or action-function. In Q-learning for example, a Q-value is remembered for each state-action combination. Each time the agent performs an action, gains experience, the Q-value of that specific state-action pair is updated. For complex games however, there is an explosion of state-action possibilities. Therefore function approximators are used to approximate such state(-action) functions, which try to capture the essence, most essential concepts in order to maximize reward. Neural networks are universal function approximators, this mean that a neural network with a single hidden layer containing a finite number of neurons can approximate any mathematical function to any desired accuracy. For the more complex mazes of experiments 2 to 5, we implemented a neural network that was trained after each move of the agent. Weights were updated after each move using gradient descent for one iteration. This was done by performing one forward and one backward propagation pass.

Partially observable maze For the partially observable maze experiment, we started from the same Sutton's Dyna maze. However in this case, the starting position varies and is unknown to the agent. Additionally, the agent can only observe part of the maze. After each action, it receives an observation, a tuple of four values (representing the tiles North, South, East and West of the agent). for example, if there is a wall to the North, a border to the East and empty tiles in the other directions, the observation tuple looks like (1,0,1,0). In addition to the 20 % noise of action selection, there is also a 10 % observational noise for each independent tile. Because the position is unknown to the agent, Markov localization is used to keep track of a believe state. Initially this believe state has a uniform distribution over all the tiles without obstacle. Updates are performed as following:

$$b_{t+1}(s) = \eta P(o_{t+1}|s) \sum_{s'} T(s', a_t, s) b_t(s')$$

a_t : action at time t
$b_t(s)$: believe state
o_{t+1} : observation
η : normalization factor
$T(s', a_t, s)$: transition function that maps the state-action pairs to a probability function over successor states.

Since there are to many states for tabular expression, a neural network with 20 sigmoidal hidden neurons was used as function approximator. As input, the network received the believe state as input, a tuple of length 54.

Maze with dynamic obstacles For third experiment, 4 to 8 obstacles (walls) were generated at random locations at the start of each trial. Start and goal positions were fixed in the same place as Sutton's Dyna maze. If no possible path exists from start to goal, the agent would not have to solve it and an new maze was generated instead. The agent needed to learn the knowledge of a path planner. For this a neural network was implemented with 60 sigmoidal hidden units. As input, the network received a tuple of length 108 (since there are 54 tiles in the maze and we want to know from each tile whether an agent or wall is on it). 54 of them depict the agent position (0 or 1 for each tile), and 54 depict the wall positions (0 or 1 for each tile).

Maze dynamic goal positions The fourth maze is very similar to the third maze. Instead of dynamic obstacles, there is a dynamic goal position. Obstacles and starting position are placed as in Sutton's Dyna maze. This time, a neural network with 20 sigmoidal hidden units and a tuple of length 108 as input was used as function approximator. This time 54 values represent the agent position, and the other 54 values the goal position.

Generalized maze The last experiment combined the problem of a maze with dynamic obstacles and a maze with dynamic goals positions into a generalized maze experiment (wierling 2008 [23]). In this maze, the goal and obstacles are positioned at random positions at the start each trial. A neural network with 100 sigmoidal hidden units receives 162 (54x3) input values, 54 to describe the agent position, 54 to describe the obstacle positions, and 54 to describe the goal position.

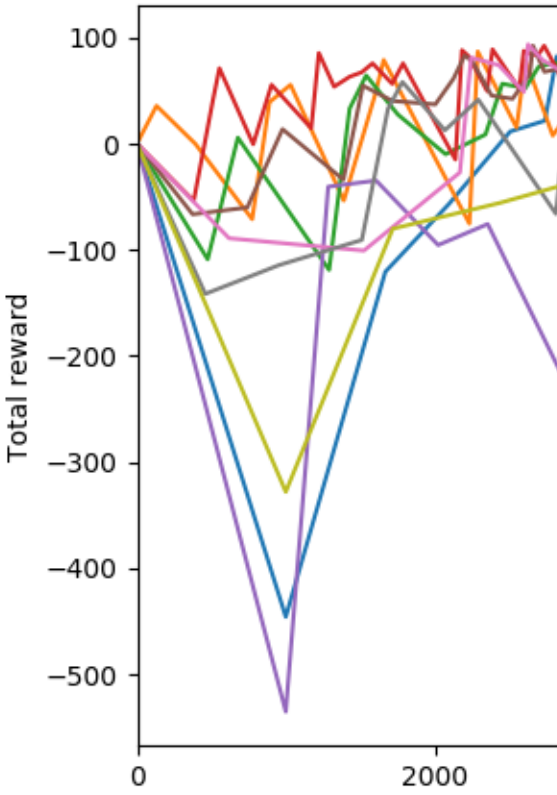
Results

Simple Maze

The evaluation was done by noting the Final Reward intake during the last 2500 learning-steps of each simulation. Also the cumulative average reward was calculated after every

2500 learning-steps. Each simulation has 50,000 learning-steps. Table 1 shows the average results and the standard deviation of 500 such simulations.

Method	α	β	γ	G	Final	C
Q	0.2	-	0.9	1	5.20 +/- 0.19	8
SARSA	0.2	-	0.9	1	5.20 +/- 0.18	9
AC	0.1	0.2	0.95	1	5.21 +/- 0.20	9
QV	0.2	0.2	0.9	1	5.21 +/- 0.18	9
ACLA	0.005	0.1	0.99	9	5.18 +/- 0.14	8
Majority Voting	-	-	-	1.6	5.19 +/- 0.19	9
Rank Voting	-	-	-	0.6	4.89 +/- 0.29	8
Boltzmann Mult.	-	-	-	0.2	5.23 +/- 0.15	9
Boltzmann Add.	-	-	-	1	5.04 +/- 0.32	9



It was observed that Majority voting and Boltzmann Multiplication have the best Final as well as the Cumulative reward intakes. In the current experiment, Boltzmann addition has the worst performance.

In order to examine the effect of the discount factor (γ) , the SARSA and the ACLA algorithms are rerun with different discount factors. The results are shown in table 2.

Method	α	β	γ	G	Final	Cumulative
SARSA	0.2	-	0.95	1	4.96 +/- 0.90	87.04 +/- 17.08
ACLA	0.005	0.1	0.9	9	4.27 +/- 1.8	60.87 +/- 29.48

There is a significant drop in the performance of both algorithms as compared to the previous simulations.

Figure 1 and 2 plots the total reward obtained by the algorithms vs the timestep. For the sake of clarity results are plotted till 15,000 timesteps. It can be observed that all algorithms converge to a stable performance well within 15,000.

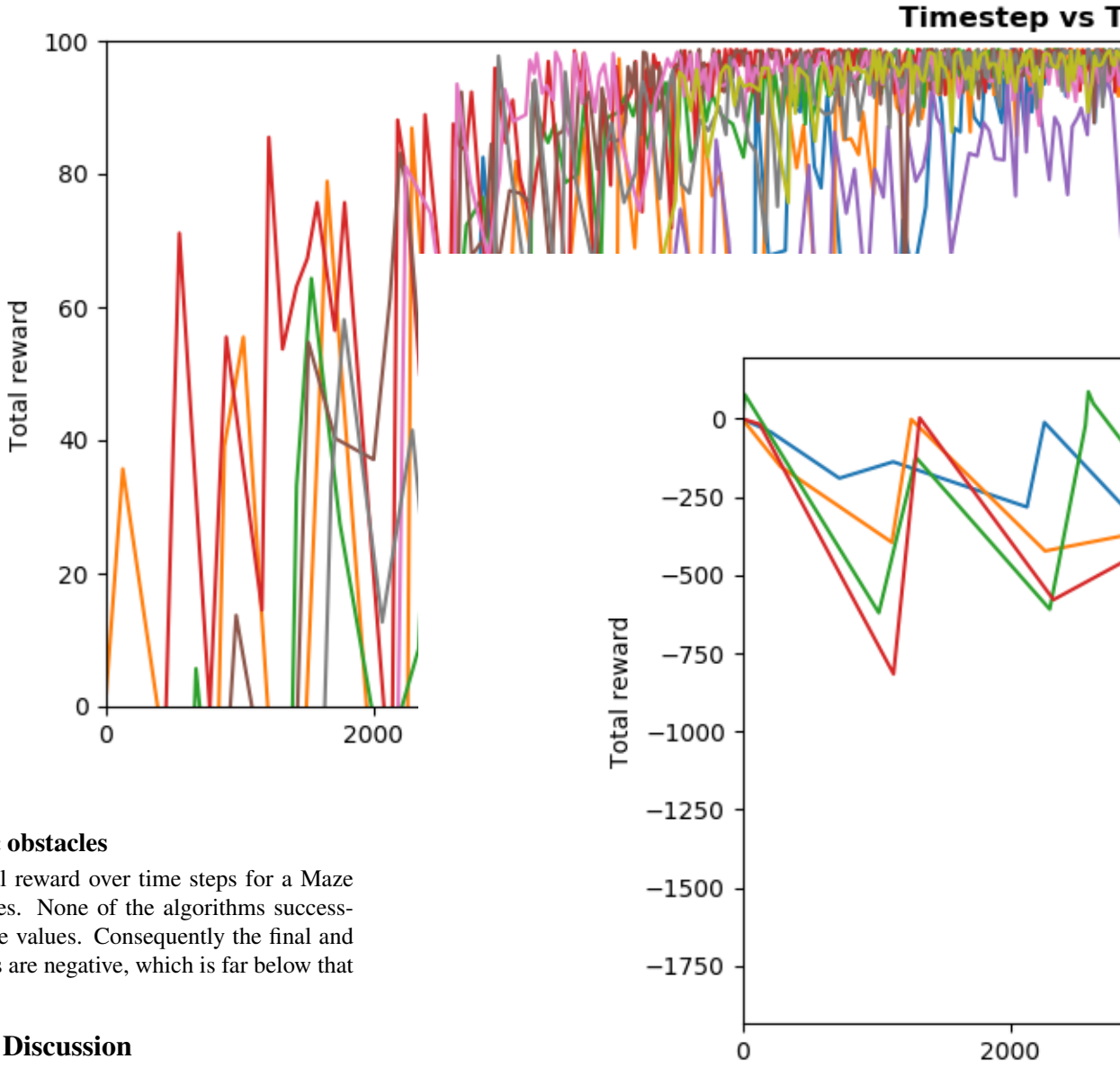


Figure 2: Figure_3

Maze with dynamic obstacles

Figure 3 plots the total reward over time steps for a Maze with Dynamic obstacles. None of the algorithms successfully converge to stable values. Consequently the final and the cumulative rewards are negative, which is far below that of simple maze.

Discussion

Although the article of (wiering 2008) is very interesting and shows new insights, we are of the opinion that they could have taken extra steps to assure reproducibility. Some methods were ambiguously explained, so that it took some effort to figure out what was meant exactly. The source was also not available, which would have solved previous remark. For example, while implementing the neural network, it is not clear how the weights are updates. We decided to update the weights after each move gradient descent for one iteration. However, it is possible that in the original paper, they used multiple gradient descent iterations after each move. Or maybe, action replay as in, “Human-level control through deep reinforcement learning”, from DeepMind where used. Here, actions, states and rewards are stored in memory and every so often, the weights are updated by learning from

the experiences stored in memory. This could be one of the reason why our neural networks were not able to converge for the dynamic maze and other more complex maze experiments, while in the original paper, they did. Other possible explanation are numerical errors.

For the small maze experiment, we observed very similar results to those in the paper. All final and cumulative values were within the standard deviation.

(Wiering 2008) determined all learning parameters by performing various trials with different parameter values. They noted in their paper that the discount factor had a major effect on the performance. To assess this effect, the SARSA and the ACLA algorithms were trained on the simple maze with changed discount. We observe a significant change in the final and the cumulative reward, which highlights the importance of this parameter in the overall performance of the algorithms.

References