Homework 2 quantum mechanics applications, scattering, spin

March 7, 2023

- Problem 1: The Hamiltonian of a spin 3/2 particle is given by:

$$\hat{H} = \frac{\alpha}{\hbar^2}(\hat{S_x}^2 + \hat{S_y}^2 - 2\hat{S_z}^2) - \frac{\beta}{\hbar}\hat{S_z} \tag{1}$$

  Find the energy levels for this particle. Consider α and β to be constant.

- Problem 2: consider an electron with angular momentum $l = 1$ and spin 1/2, find:

  a The allowed eigenstates.

  b The eigenstates in terms of the uncoupled basis. (Use a table of the Clebsch-Gordan coefficients to double-check your results).

  d Consider that the electron is in a central potential, thus the eigenfunctions of orbital angular momentum are the spherical harmonics. Write **only one** of the states found in part b in terms of the spherical harmonics. (Do this for a state different than the ones I did in class).

- Problem 3: The Hamiltonian of two particles with angular momentum 1 is given by:

$$\hat{H} = \frac{\epsilon_1}{\hbar^2}(\overrightarrow{\hat{L_1}} + \overrightarrow{\hat{L_2}}) \cdot \overrightarrow{\hat{L_2}} + \frac{\epsilon_2}{\hbar^2}(L_{1z} \hat{+} \hat{L_{2z}})^2 \tag{2}$$

  Find the energy levels and degeneracies for the states whose total angular momentum is the maximum.

- Problem 4: Consider a rotation $U(\beta, \hat{y})$ of an angle β with respect to the $\hat{y}$ axis. Show that the matrix elements $\langle j, m| \, |U(\beta, \hat{y}) \, |j, m\rangle$ are polynomials of degree 2j with respect to $\sin(\beta/2)$ and $\cos(\beta/2)$. Do this for the case j=1/2.

- Problem 5: An electron is in a state of S_z with eigenvalue $+\hbar/2$. Consider the operator $\hat{n} \cdot \overrightarrow{S}$. Find the eigenvalues, eigenstates, and the probability of finding the electron in each of the eigenstates. Consider the unitary vector $\hat{n}$ in spherical coordinates.