Homework 4 quantum mechanics applications, variational method, time-dependet perturbation theory, Fermi's golden rule.

May 7, 2023

- Problem 1: **You are to work this problem alone, without someone else help. You may use textbooks and the internet** Consider a system with a spinless particle initially in a state s. The system is perturbed, at t=0, with an electric field in the z-direction of the form:

$$E = E_0 \sin \omega t$$

  Consider transitions to the p-state ($m_l = 0, 1, -1$).

  a) Considering only the s and p states, which states are allowed by the perturbation?
  b) Find the probability that the system is at t¿0 in each of the states of part a.

- Problem 2: Conservation of baryon number forbids that a neutron is converted into an anti-neutron. Nevertheless, in the early universe, baryon number must have been violated because we observe matter and very little antimatter today. Consider the transition of a neutron $|n\rangle$ into an anti-neutron $|\overline{n}\rangle$. For this, consider that:

$$|n\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{1}$$

  and

$$|\overline{n}\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2}$$

  (up to a normalization factor). The Hamiltonian for a free particle at rest (such as the neutron or anti-neutron) is:

$$H_0 |n\rangle = m_n c^2 |n\rangle \,, \; H_0 |\overline{n}\rangle = m_n c^2 |\overline{n}\rangle \tag{3}$$

  while the Hamiltonian that shifts $n - \overline{n}$ is:

$$H' |n\rangle = \epsilon |\overline{n}\rangle \,, \; H' |\overline{n}\rangle = \epsilon |n\rangle \tag{4}$$

a. Find the matrix elements of $\langle i| H |j\rangle$ where $H = H_0 + H'$. and then find the eigenvalues and eigenvectors of the Hamiltonian.

b. Consider that at t=0 the state is a neutron. And that at any time, the system evolves as a free particle evolves. Find the probability, as a function of time, that the state is observed to be an anti-neutron.

c. Use the probability found in b. to estimate the half life of the neutron anti-neutron transition. (Half life is the time at which the probability is 1/2).

- Problem 3: **You are to work this problem alone, without someone else help. You may use textbooks and the internet** Consider the infinite square well potential with length L. For the state n=1 use the trial wave function $\psi(x) = x(L-x) + Cx^2(L-x)$. Use the variational method to find the energy of the ground state. Compute the error given by this approximation.

- Problem 4: **You are to work this problem alone, without someone else help. You may use textbooks and the internet** In the electric dipole approximation, one considers the interaction of an atom with radiation such that:

a. The magnetic part of the radiation may be neglected. Show why that is the case.

b. The potential for the electric part may be considered as $V = -q\mathbf{r} \cdot \mathbf{E}$. Where $\mathbf{E}(t) = E_0 \overrightarrow{\epsilon} \cos \omega t$ where $\overrightarrow{\epsilon}$ is a unit polarization vector. If an electron is in the ground state of the Hydrogen atom. Use first-order time-dependent perturbation theory to find the probability that the radiation will ionize the electron in an angle θ and ϕ with respect to the polarization angle.