Midterm exam Quantum Mechanics Applications

Universidad EIA

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Name: ______________________________

Problem 1 The total Hamiltonian of a system is given by:

$$\hat{H} = E_0 \begin{pmatrix} 1+\lambda & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 \\ 0 & 0 & 3 & -2\lambda \\ 0 & 0 & -2\lambda & 7 \end{pmatrix} \tag{1}$$

where $\lambda << 1$

a) (12%) Consider $\hat{H} = \hat{H}_0 + \lambda \hat{W}$. Find the four eigenvalues and four eigenstates of the unperturbed Hamiltonian $\hat{H}_0$.

b (8%)Use non-degenerate perturbation theory to calculate the first order correction to each of the energies found in a).

c (10%)Use non-degenerate perturbation theory to calculate the second order correction to one of the energies found in a) (**Note: not all energies are corrected at second order. You must calculate one that is non-zero**).

c (10%)Use non-degenerate perturbation theory to calculate the first order correction to one of the eigenstates of $\hat{H}_0$.(**Note:There are four eigenstates, but you only have to pick one. If you find that the correction to the chosen eigenstate is zero, pick another eigenstate, do this until you find an eigenstate with non-zero correction**).

Problem 2 A particle with mass m is subjected to a central potential which depends on r, such that

$$V(r) = \begin{cases} -V_0 \ for \ 0 < r < a \\ 0 \ for \ r > a \end{cases} \tag{2}$$

where $V_0 > 0$. Find:

a) (10%) If the wave number of the wave function in the potential is k and the one of the outside the potential is k', find the allowed wave functions for the case of very low energy.

b) (10%) Find the phase shift δ resulting from the scattering.

Problem 3: The Hamiltonian related to the interaction of two spin 1/2 particles has the form : $\hat{F} = A + B\overrightarrow{\sigma_1} \cdot \overrightarrow{\sigma_2}$ where A and B are positive constants and $\overrightarrow{\sigma_i}$ acts on particle i.

a) (10%) Show that for this system it is possible to measure simultaneously $\hat{F}$ and the operators related to the total angular momentum $\hat{J}^2$ and $\hat{J}_z$.

b) (10%) Find the allowed eigenstates in the J and M basis (the coupled basis).

c) (12%) Find the coupled basis in terms of the un-coupled basis.

Question: (8%) In perturbation theory we learned that it is not possible to treat the degenerate case the same was as the non-degenerate case. Why do we have to make that distinction?