Growing Cosine Unit: A Novel Oscillatory Activation Function That Can Speedup Training and Reduce Parameters in Convolutional Neural Networks

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Abstract

Convolution neural networks have been successful in solving many socially important and economically significant problems. Their ability to learn complex high-dimensional functions hierarchically can be attributed to the use of nonlinear activation functions. A key discovery that made training deep networks feasible was the adoption of the Rectified Linear Unit (ReLU) activation function to alleviate the vanishing gradient problem caused by using saturating activation functions. Since then many improved variants of the ReLU activation have been proposed. However a majority of activation functions used today are non-oscillatory and monotonically increasing due to their biological plausibility. This paper demonstrates that oscillatory activation functions can improve gradient flow and reduce network size. It is shown that oscillatory activation functions allow neurons to switch classification (sign of output) within the interior of neuronal hyperplane positive and negative half-spaces allowing complex decisions with fewer neurons. A new oscillatory activation function $C(z) = z \cdot \cos z$ that outperforms Sigmoids, Swish, Mish and ReLU on a variety of architectures and benchmarks is presented. This new activation function allows even single neurons to exhibit nonlinear decision boundaries. This paper presents a single neuron solution to the famous XOR problem. Experimental results indicate that replacing the activation function in the convolutional layers with $C(z)$ significantly improves performance on CIFAR-10, CIFAR-100 and Imagenette.

1 Introduction

The quintessential feature of deep convolutional neural networks is their ability to learn arbitrarily complex nonlinear mappings between high-dimensional input to a target output. This universal approximation feature is critically dependent on the nature of the activation function non-linearity used by each layer in the neural network. Training a neural network might be viewed as adjusting a set of parameters to scale, compress, dilate, combine and compose simple nonlinear activation functions to approximate a complex nonlinear target function.

The property of activation functions to be nonlinear is essential, since the composition of any finite number of linear functions is equivalent to a single linear function. Hence, the resultant network composed of purely linear neurons is equivalent to a single linear layer network limited to solving linearly separable problems. Despite the critical importance of the nature of the activation function in determining the performance of neural networks, simple monotonic non decreasing nonlinear

Preprint. Under review.

activation functions are universally used. In this work, we explore the effects of using oscillatory nonlinear activation functions in deep neural networks.

In the past, sigmoidal [10]saturating activation functions were widely used due to their property of approximating the step or signum function while being differentiable. The outputs of s-shaped saturating activations have the important property of being interpretable as a binary yes/no decision and hence are useful. However, deep neural networks composed of purely sigmoidal activation functions are hard to train, due to the vanishing gradient phenomenon which arises when saturating activation functions are used. The adoption of non-saturating and non-sigmoidal Recti-Linear Unit (ReLU) [1] activation function to alleviate the vanishing gradient problem is considered a milestone in the evolution of deep neural networks.

During training, the parameters are continually updated in the direction of the negative gradient. Hence small gradients lead to stagnation in learning and slow parameter updates. The derivative of sigmoidal activation functions with respect to the input input is small outside a small closed interval around zero (usually [-5 , 5]). In particular $\exp(-5) < 0.01$ and hence activation functions composed purely of exponentials, such as logistic-sigmoid and tan-sigmoid will saturate outside this narrow range.

Furthermore, in uni-polar activation functions (functions that take purely non-negative values like logistic-sigmoid), the outputs of a layer can get combined to form large positive values leading to the saturation of neurons in the next layer. Thus, activation functions that do not shift the mean of the input towards positive or negative values (such as tanh(z)) reduce saturation of succeeding layers and hence perform better.

In the past a wide variety of activation functions have been explored [7], which was the inspiration behind the Swish activation function. [11]. Past research indicates that activation functions that have larger derivative values for a wider set of input values perform better. In particular the use of the ReLU like activation functions result in faster training compared to saturating sigmoidal type activation functions because these activation functions do not saturate for a wider range of inputs.

Some drawbacks of ReLU like activation functions:

- The derivative of the loss function J with respect to the weight matrix W^k of layer k is $\frac{\partial J}{\partial W^k} = \delta^k (a^{k-1})^T$. Thus if a^{k-1} is small then this derivative is also small, the weights are not updated and learning stagnates.
- Bias Shift: There is a positive bias in the network for subsequent layers, as the mean activation is always greater than zero. Since the outputs of all ReLU units are non-negative the outputs can combine to produce very large positive inputs to subsequent layers farther away from the input leading to possible saturation and numerical accuracy issues.
- The delta for a particular layer is $\delta^k = \frac{\partial J}{\partial z^k} = ((W^{k+1})^T \delta^{k+1}) \bigodot g'(z^k)$, where $g'(z^k)$ is the derivative of the activation function. So ReLU like activation functions that have zero or small derivative for negative values result is small δ^k values leading to stagnation in learning.

Variants of ReLU: SELU [5], ELU [2] have been successful to an extent in mitigating these shortcomings. Swish [12] and Mish [9] represent a new class of non-monotonic functions that offer promising results across different benchmarks.

Despite the popularity of a wide variety of activation functions and neural network architectures, all networks suffer from a fundamental limitation in that individual neurons can exhibit only linear decision boundaries. Multilayer neural networks with nonlinear activations are needed to achieve nonlinear decision boundaries. This paper explores proposes a new oscillatory activation function that allows individual neurons to exhibit nonlinear decision boundaries thus removing a fundamental limitation of neural networks.

In brief, the contributions of this work are:

- A new activation function, Growing Cosine Unit (GCU) defined by $C(z) = z \cdot \cos(z)$ has been proposed. The advantages of using oscillatory activation functions to improve gradient flow and alleviate the vanish gradient problem has been demonstrated.

- A solution to the classic XOR problem has been demonstrated by successfully training a single neuron with the $C(z) = z \cdot \cos(z)$ activation function to learn this function.

- Two theorems that characterize the limitation of certain class of activation functions are presented.

- A comparison of the proposed GCU activation with popular activation functions on a variety of benchmark datasets is presented in Section 4. These experimental results clearly indicate that the GCU activation is computationally cheaper than Swish and Mish. The GCU activation also reduces training time and allows classification problems to be solved with smaller networks.

2 Oscillatory Activation functions

This paper explores the potential performance benefits and effects of using oscillatory activation functions in neural networks. In the past oscillatory and non-monotonic activation functions have been largely ignored.

In our study, certain Oscillatory activation functions are shown to possess the following advantages:

- Alleviate the vanishing gradient problem. These functions have non-zero derivatives throughout their domain except at isolated points.

- Improved performance for compact network architectures.

- Computationally cheaper than the state-of-the-art Swish and Mish activation functions.

2.1 Learning the XOR function using a single neuron

The famous XOR problem is task of training a neural network to learn the XOR gate function. It was first pointed out by Papert and Minsky [8] that a single neuron cannot learn the XOR function since a single hyperplane (line in this case) cannot separate the output classes for this function definition. This fundamental limitation of single neurons (or single layer networks) lead to pessimistic predictions for the future of neural network research and was responsible for a brief hiatus in the history of AI. In the following section, we show that this issue does not apply to neurons with oscillatory activation function (like GCU).

The XOR problem is the task of learning the following dataset:

$$D = \left\{ (\begin{bmatrix} -1 \\ -1 \end{bmatrix}, -1), (\begin{bmatrix} 1 \\ -1 \end{bmatrix}, 1), (\begin{bmatrix} -1 \\ 1 \end{bmatrix}, 1), (\begin{bmatrix} 1 \\ 1 \end{bmatrix}, -1) \right\} \tag{1}$$

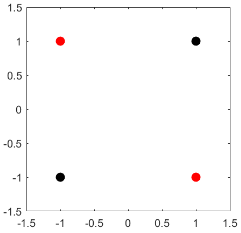


Figure 1: The XOR problem involves the task of learning the XOR function with the smallest network. The red dots must be classified as positive (1) and the black dots must be classified as negative (-1). A single line (hyperplane) cannot separate the two classes.

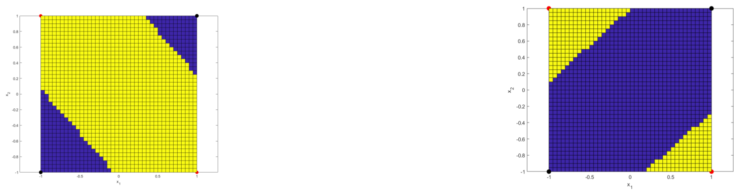


Figure 2: Two solutions to the XOR problem learnt by a single neuron using the GCU activation function. Points in yellow were assigned a class label of +1 and points in blue were assigned a class label of -1 by a single GCU neuron.

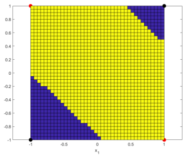


Figure 3: A single neuron solution to the XOR problem learnt by using a simple oscillatory polynomial activation function $P_c(z) = z - \frac{z^3}{2}$. Points in yellow were assigned a class label of +1 and points in blue were assigned a class label of -1 by a single neuron using the polynomial activation.

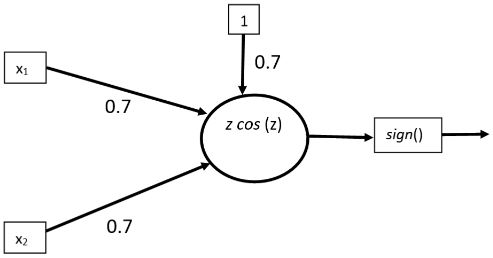


Figure 4: A single neuron solution to the XOR problem. A single neuron with the GCU activation function is capable of learning the XOR dataset shown in (1) exactly. The signum function at the output is used to map output values to ± 1.

The XOR problem was solved by using a single neuron with oscillatory activation functions, mean-square loss and simple Stochastic Gradient Descent (SGD). A learning rate of $\alpha = 0.01$ and the SGD update rule $\Delta \mathbf{w} = \alpha(y - g(z))g'(z)\mathbf{x}, \quad z = \mathbf{w}^T\mathbf{x} + b$ was used. The initial weight vector was initialized with uniform random numbers in the interval [0 , 1].

The XOR function was successfully learned by a single neuron with the activation functions $g(z)$ chosen to be $g(z) = C(z) = z \cdot \cos(z)$ and $g(z) = P_c(z) = z - \frac{z^3}{2}$ respectively. The target y for each input was taken as the class label namely 1 or -1. After training the output of the neuron is mapped to the class label in the usual manner. That is we assign positive outputs a label of +1 and negative outputs a label of -1. This can be done simply by defining the class label $Class(\mathbf{x})$ for each input **x** to be $Class(\mathbf{x}) = sign(g(\mathbf{w}^T\mathbf{x} + b))$. Where the signum function is defined as

$$sign\,(z) = \begin{cases} 1 & \text{if} \quad z > 0 \\ -1 & \text{if} \quad z < 0 \\ 0 & \text{if} \quad z = 0 \end{cases}$$

Definition 1: The decision boundary of a single neuron is the set $B = \{\mathbf{x} \in \mathbb{R}^n : g(\mathbf{w}^T\mathbf{x} + b) = 0\}$. Where g is the activation function.

That is the boundary is the set of inputs that elicit an output of zero from the neuron. Inputs corresponding to positive outputs are assigned the positive class and inputs corresponding to negative outputs are assigned the negative class (in accordance with $Class(\mathbf{x}) = sign(g(\mathbf{w}^T\mathbf{x} + b)))$ as already discussed.

It is clear from *Definition 1* that the decision boundary for any neuron that uses an activation function satisfying the condition

$$g(z) = 0 \Longleftrightarrow z = 0$$

is

$$z = \mathbf{w}^T\mathbf{x} + b = 0$$

In other words the decision boundary is a single hyperplane ($B = H$).

However if g is an oscillatory function like $C(z) = z\cos z$, the decision boundary is the set $\{\mathbf{x} \in R^n : \text{GCU}(\mathbf{w}^T\mathbf{x} + b) = 0\}$ and consists of infinitely many hyperplanes in the input space, since $C(z)$ has infinitely many roots. In particular the decision boundary with the GCU activation is a set of uniformly spaced parallel hyperplanes $\mathbf{w}^T\mathbf{x} + b = \frac{\pi}{2} + n\pi$. Thus the input space is divided into parallel strips separated by the hyperplanes and point in adjacent strips are assigned different classes alternately. These parallel strips can be seen in the solution to the XOR problem (Fig. 4).

2.2 Characterization of Activation Functions

In the following, we prove that no single neuron with a strictly monotone activation function can learn the XOR function.

We adopt the following notation: The output (activation) of a single neuron is given by $a = g(\mathbf{w}^T\mathbf{x}+b)$, where g is the activation function. The hyperplane boundary associated with a neuron is the set of points:

$$H = \{\mathbf{x} \in \mathbb{R}^n : \mathbf{w}^T\mathbf{x} + b = 0\}$$

The positive and negative half spaces are similarly defined to be:

$$H_- = \{\mathbf{x} \in \mathbb{R}^n : \mathbf{w}^T\mathbf{x} + b < 0\}$$

$$H_+ = \{\mathbf{x} \in \mathbb{R}^n : \mathbf{w}^T\mathbf{x} + b > 0\}$$

Any hyperplane divides the input space $\mathbb{R}^n$ into 3 connected regions: the positive half-space H_+, the negative half-space H_- and an affine-space H. The weight vector **w** points into the positive half-space H_+.

The distance between a point **x** and the hyperplane decision boundary H is given by:

$$d(\mathbf{x}, H) = \frac{|\mathbf{w}^T\mathbf{x} + b|}{\|w\|}$$

.

Proposition 1: Consider a single neuron with weight vector **w** and bias b using an activation function that is monotonically strictly increasing with $g(0) = 0$. The class label assigned to an input **x** by this neuron is defined to be $C(\mathbf{x}) = sign(\mathbf{w}^T\mathbf{x} + b)$. If a point $\mathbf{x}^1$ assigned to a particular class is at a distance d_1 from the hyperplane $H = \{\mathbf{x} \in \mathbb{R}^n : \mathbf{w}^T\mathbf{x} + b = 0\}$, then any other point $\mathbf{x}^2$ at a distance $d_2 > d_1$ in the same halfspace as $\mathbf{x}^1$ will be assigned to the same class by this neuron.

Proof:

Case 1: Consider the case where $\mathbf{x}^1 \in H_+$ AND $\mathbf{x}^2 \in H_+$

By assumption, $z_1 = \mathbf{w}^T\mathrm{x}^1 + b > 0$ AND $z_2 = \mathbf{w}^T\mathrm{x}^2 + b > 0$. Also $d_1 < d_2$

Using the formula for $d(\mathbf{x,H})$:

$$\frac{|\mathbf{w}^T\mathbf{x}^1 + b|}{\|w\|} < \frac{|\mathbf{w}^T\mathbf{x}^2 + b|}{\|w\|}$$

$$\frac{\mathbf{w}^T\mathbf{x}^1 + b}{\|w\|} < \frac{\mathbf{w}^T\mathbf{x}^2 + b}{\|w\|}$$

$$\mathbf{w}^T\mathbf{x}^1 + b < \mathbf{w}^T\mathbf{x}^2 + b$$

$$z_1 < z_2$$

$0 < z_1$ by assumption, thus $0 < z_1 < z_2$.

Since g is strictly increasing and $g(0) = 0 : 0 < g(z_1) < g(z_2)$

$$g(z_2) > 0$$

$$Class(\mathbf{x}^2) = sign(\mathbf{w}^T\mathbf{x}^2 + b) = sign(g(z_2)) = 1$$

Thus $x_2 \in H_+$ and hence x_2 belongs to the same class as x_1.

Case 2: Consider the case where $\mathbf{x}^1 \in H_-$ AND $\mathbf{x}^2 \in H_-$

By assumption, $z_1 = \mathbf{w}^T\mathrm{x}^1 + b < 0$ AND $z_2 = \mathbf{w}^T\mathrm{x}^2 + b < 0$. Also $d_1 < d_2$

Using the formula for $d(\mathbf{x},\mathbf{H})$:

$$\frac{|\mathbf{w}^T\mathbf{x}^1 + b|}{\|w\|} < \frac{|\mathbf{w}^T\mathbf{x}^2 + b|}{\|w\|}$$

$$\frac{-(\mathbf{w}^T\mathbf{x}^1 + b)}{\|w\|} < \frac{-(\mathbf{w}^T\mathbf{x}^2 + b)}{\|w\|}$$

$$\mathbf{w}^T\mathbf{x}^1 + b > \mathbf{w}^T\mathbf{x}^2 + b$$

$$z_1 > z_2$$

$0 > z_1$ by assumption, thus $0 > z_1 > z_2$. Since g is strictly increasing and $g(0) = 0 : 0 > g(z_1) > g(z_2)$

$$0 > g(z_2)$$

$$Class(\mathbf{x}^2) = sign(\mathbf{w}^T\mathbf{x}^2 + b) = sign(g(z_2)) = -1$$

Thus $x_2 \in H_-$ and hence x_2 belongs to the same class as x_1.

Thus it is clear from Proposition 1, that if a point is assigned a particular class, other points further away from the boundary are automatically assigned the same class by strictly monotonic activation functions. However oscillatory activation functions are not subject to this limitation,hence can learn the XOR classification with a single neuron.

Definition 2: A function $f : X \to \mathbb{R}$ is said to be sign equivalent to a function $g : X \to \mathbb{R}$ iff $sign(f(x)) = sign(g(x))$ for all $x \in X$.

It is clear that sign equivalence is actually an Equivalence relation on the set of all real-valued functions on a set. Further we note that the set of functions $G = \{f : \mathbb{R} \to \mathbb{R} : \quad f(0) = 0\}$ is a vector space. Also the subset of functions of G that are sign equivalent to $I(z) = z$ form a convex cone in G.

Proposition 2: Consider a single neuron that uses an activation function that is sign equivalent to the identity function $I(z) = z$, that is $sign(g(z)) = sign(z)$. If $\mathbf{x}^1, \mathbf{x}^2 \in H_+$, then $Class(\mathbf{x}^1) = Class(\mathbf{x}^2) = 1$ and if $\mathbf{x}^1, \mathbf{x}^2 \in H_-$, then $Class(\mathbf{x}^1) = Class(\mathbf{x}^2) = -1$.

Proof:

Case 1: Let $sign(g(z)) = sign(z)$ AND $\mathbf{x}^1, \mathbf{x}^2 \in H_+$

$\Rightarrow z_1 = \mathbf{w}^T \mathrm{x}^1 + b > 0$ AND $z_2 = \mathbf{w}^T \mathbf{x}^2 + b > 0$

$\Rightarrow Class(\mathbf{x}^1) = sign(g(z_1)) = sign(z_1) = 1$ AND $Class(\mathbf{x}^2) = sign(g(z_2)) = sign(z_2) = 1$

Thus $\mathbf{x}^1$ and $\mathbf{x}^2$ belong to the same class.

Case 2: Let $sign(g(z)) = sign(z)$ AND $\mathbf{x}^1, \mathbf{x}^2 \in H_-$

$\Rightarrow z_1 = \mathbf{w}^T \mathrm{x}^1 + b < 0$ AND $z_2 = \mathbf{w}^T \mathbf{x}^2 + b < 0$

$\Rightarrow Class(\mathbf{x}^1) = sign(g(z_1)) = sign(z_1) = -1$ AND $Class(\mathbf{x}^2) = sign(g(z_2)) = sign(z_2) = -1$

Thus $\mathbf{x}^1$ and $\mathbf{x}^2$ belong to the same class.

From Proposition 2 it is clear that a single neuron using the Swish activation function cannot solve the XOR problem.

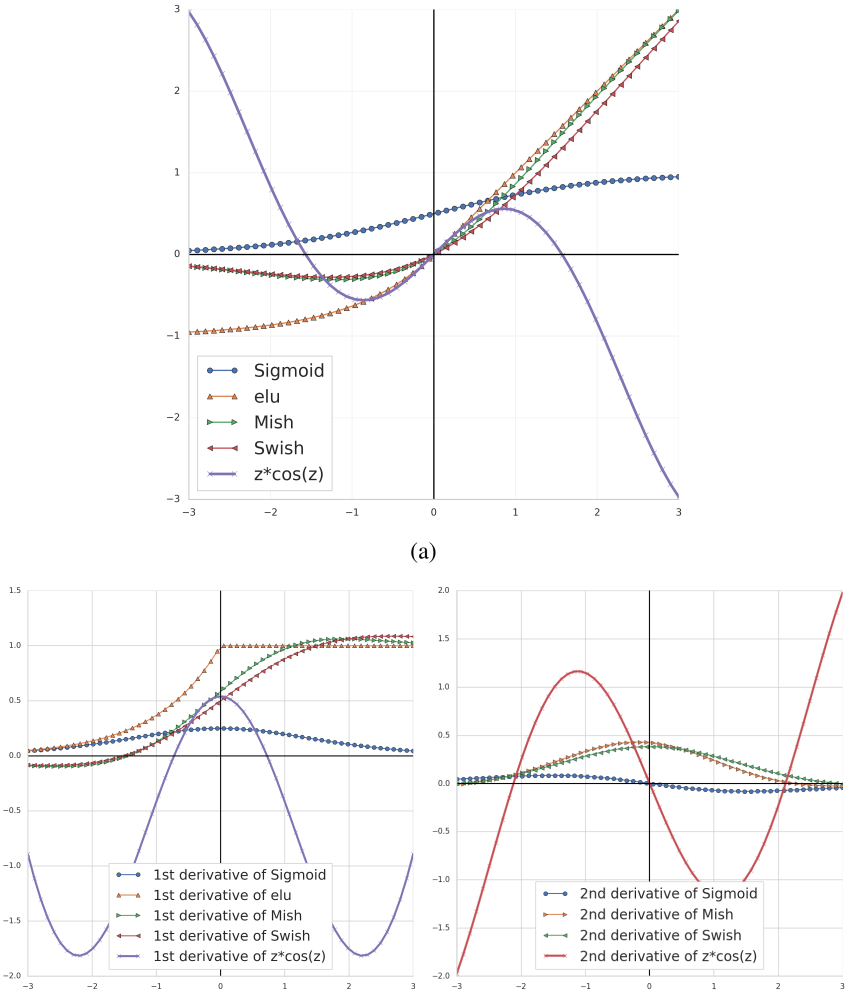
The Swish activation $S(z) = \frac{z}{1+exp(-z)}$

It is clear that $sign(S(z)) = sign(z)$ (since $\frac{1}{1+exp(-z)} > 0$).

Similarly a single neuron using the Mish activation function cannot solve the XOR problem. The Mish activation $M(z) = z \tanh(\log(1 + \exp(z)))$, It is clear that $sign(M(z)) = sign(z)$ (since $\tanh(\log(1 + \exp(z))) > 0$).

Based on propositions 1 and 2, single neurons that use monotonic activation functions and activation functions that are are sign equivalent to $I(z) = z$ cannot solve the XOR problem. To solve the XOR problem with a single neuron we must search for an activation that violates both the above conditions. In our work, the oscillatory function $C(z) = z \cos z$ that violates both the above conditions is proposed and used to solve the XOR problem with a single neuron.

3 Comparison of Computational complexity for activation functions



(a)

(b)

(c)

Figure 5: (a) Plot of GCU ($z \cos z$) and other activation functions. The corresponding first (b) and second (c) derivatives.

Fig. 5 compares the features of different activation functions. It is clear that $C(z)$ and the other activation functions are very close to $I(z) = z$ for small values of z. This is desirable and has a regularizing effect since the network behaves like a linear classifier when initialized with small weights. During training the weights get updated and the nonlinear range of GCU is utilized as needed. In particular a GCU network can serve as a linear classifier if necessary avoiding overfitting effects. Also the GCU activation temporarily saturates close to its first maximum and minimum values and mimics the behaviour of sigmoids. For larger inputs GCU oscillates and is an unbounded function.

Name	Function
Logistic-sigmoid	$\sigma(z) = \frac{1}{1+\exp{(-z)}}$
tan-sigmoid	$T(z) = \tanh z = \frac{\exp(z)-\exp(-z)}{\exp(z)+\exp(-z)}$
Rectified Linear Unit (ReLU)	$R(z) = \begin{cases} z & \text{if} \quad z > 0 \\ 0 & \text{if} \quad z < 0 \end{cases}$
Leaky ReLU	$L(z) = \begin{cases} z & \text{if} \quad z > 0 \\ 0.01z & \text{if} \quad z < 0 \end{cases}$
Swish	$S(z) = \frac{z}{1+\exp{(-z)}}$
Mish	$M(z) = z\tanh{(\log(1+\exp{(z)}))}$
Growing Cosine Unit (GCU)	$C(z) = z\cos z$

Table 1: A list of activation functions considered in this paper and their definitions.

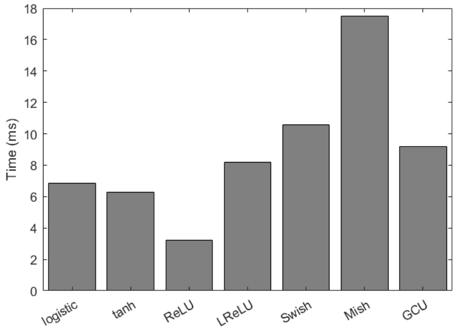


Figure 6: Average time over 1000 independent runs. Where each run consisted of applying the activation to a vector of length 10^6 with elements uniformly distributed in the interval [-5 , 5].

Table 1 shows that the proposed GCU activation function is computationally cheaper than Swish and Mish activation functions. For example GCU uses one transcendental function call and one multiplication whereas the Mish activation function uses 3 transcendental function calls and one multiplication. It is also evident from Fig.6 that the proposed GCU activation is computationally cheaper than the Swish and Mish activation functions.

4 Comparison of performance on benchmark datasets

4.1 Experimental set up

In the following a comparison of the CNN models with the proposed GCU activation on the CIFAR-10 [6], CIFAR-100, and Imagenette [4] datasets is presented. Imagenette is a subset of ImageNet [3], which consists of ten classes of easily recognized objects. The RMSprop optimizer [14] is used with the categorical cross entropy loss function(softmax classification head). For computational reasons, we were unable to test this on the full ImageNet, which remains a future tasks. Experiments on CIFAR-10, CIFAR-100 were carried out with an initial learning, decay rate of $10^{-4}, 10^{-6}$ respectively. For Imagenette, this was 10^{-6}, with no decay. The Xavier Uniform initializer instantiates the weights of the kernel layers. The GCU activation is used only for the convolutional layers and not for the dense layers, being computationally costlier than the ReLU activation.

CONV. Layer	Dense Layer	Top - 1 Acc. %	SD Acc.	Loss	SD Loss
ReLU	ReLU	74.13	0.56	0.74	0.016
GCU	**ReLU**	**75.64**	**0.47**	**0.73**	**0.004**
Swish	Swish	71.74	0.48	0.82	0.014
Swish	ReLU	71.70	1.05	0.84	0.016
Mish	Mish	74.22	0.62	0.77	0.004
Mish	ReLU	73.20	0.74	0.79	0.011

Table 2: Validation set accuracy of various activation functions on the CIFAR-10 dataset

CONV. Layer	Dense Layer	Top- 1 Acc. %	SD Acc.	Loss	SD Loss
ReLU	ReLU	41.29	0.43	2.31	0.016
GCU	**ReLU**	**43.42**	**0.36**	**2.23**	**0.004**
Swish	Swish	39.37	0.40	2.43	0.014
Swish	ReLU	38.46	0.42	2.45	0.016
Mish	Mish	41.13	0.36	2.33	0.004
Mish	ReLU	39.83	0.37	2.39	0.011

Table 3: Validation set accuracy of various activation functions on the CIFAR-100 dataset

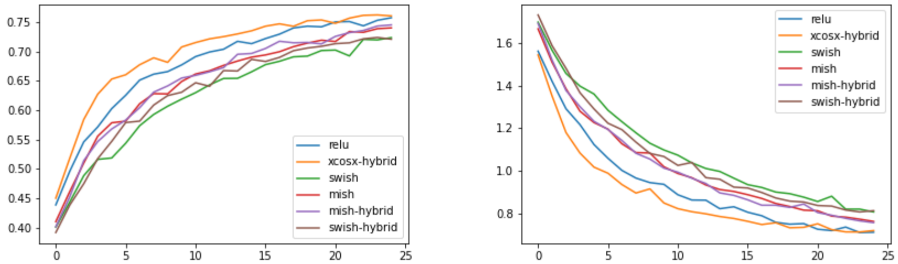
The results for each of these are reported over 5 runs, each of 25 epochs. We leverage a compact network architecture for CIFAR-10,100, as detailed in the Appendix section. The architecture consists of only 4 convolutional layers, followed by dense layers. For ImageNette, we utilize the VGG-16 backbone. [13].

For both architectures tested, we experiment with the choice of activation functions at two locations: one for all of the convolutional layer and the other for the dense layers.

Convolution Layer	Activation Dense Layer	Top- 1 Acc. %	SD Acc.	Loss	SD Loss
ReLU	ReLU	60.28	0.60	1.21	0.02
GCU	**ReLU**	**68.27**	**1.01**	**1.00**	**0.03**
GCU	GCU	67.87	0.37	1.07	0.02
Swish	Swish	43.02	0.65	1.69	0.01
Swish	ReLU	42.96	0.27	1.71	0.03
Mish	Mish	48.72	1.79	1.56	0.06
Mish	ReLU	44.32	2.16	1.84	0.13

Table 4: Performance comparison on the Imagenette dataset

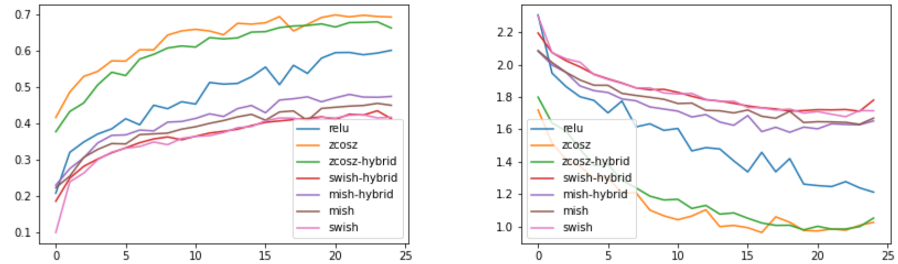
4.2 Results



(a) Accuracy

(b) Loss

Figure 7: Validation Accuracy(a) and loss(b) on the CIFAR-10 dataset



(a) Accuracy

(b) Loss

Figure 8: Validation Accuracy(a) and loss(b) on the Imagenette dataset

The experiments (Table 1-3) show that use of GCU activation for convolutional layers and ReLU for the dense layers provides the best performance among all architectures considered. This is particularly evident on the VGG-16 network trained on the Imagenette dataset, where the GCU models outperform all ReLU architectures by 7+%. The models with GCU in the convolutional layers also converge faster during training as highlighted by Fig 7-8.

4.3 Visualizing learnt filters

For the ImageNette dataset in the previous section, we visualize the output of the filters in successive layers from the input to the output. It is clear from Figs. 9, 10 and 11 that both ReLU and GCU convolutional layers hierarchically detect the features of a bird in the input image. However it is quite clear that the feature detectors with GCU activation function are more confident and correspond to larger outputs (red pixels correspond to larger values). In particular the 5 rightmost columns in Fig. 11 clearly show the detection of the bird image in red. Thus it appears that convolutional filters with GCU activation are able to segment and detect the bird image significantly more clearly and accurately than with the ReLU activation function. These filter output visualizations qualitatively confirm the quantitative higher accuracy results with the GCU activation shown in Tables 2 and 3.

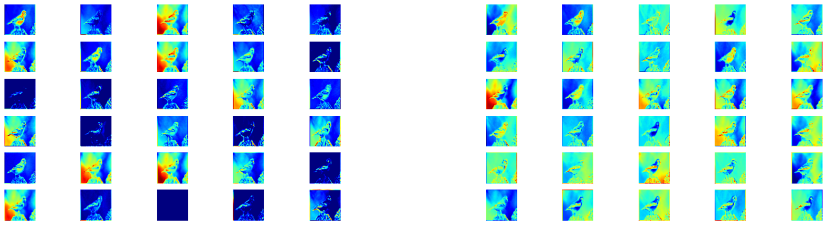


Figure 9: Comparisons of filter output in Layer 3 with ReLU and GCU activation functions. Outputs from ReLU filters are shown in the 5 leftmost columns while the 5 rightmost columns show the outputs of filters with the GCU activation.

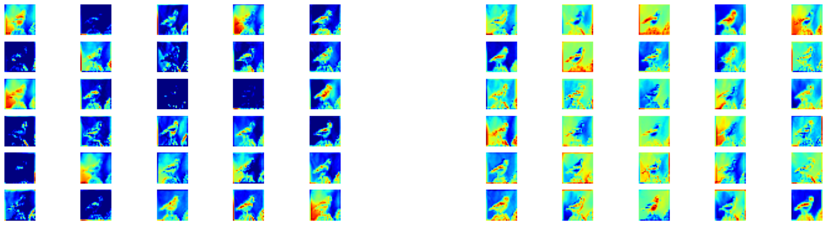


Figure 10: Comparisons of filter output in Layer 6 with ReLU and GCU activation functions. Outputs from ReLU filters are shown in the 5 leftmost columns while the 5 rightmost columns show the outputs of filters with the GCU activation.

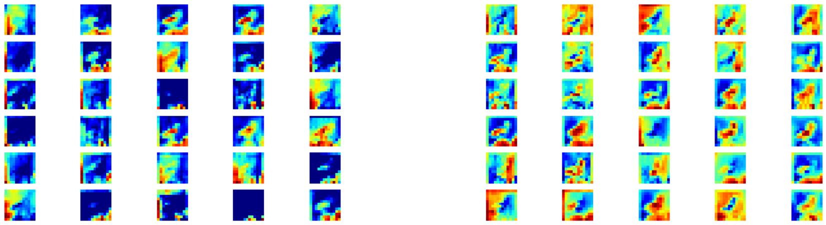


Figure 11: Comparisons of filter output in Layer 10 with ReLU and GCU activation functions. Outputs from ReLU filters are shown in the 5 leftmost columns while the 5 rightmost columns show the outputs of filters with the GCU activation.

5 Future Work

The findings in this research indicate that a wider class of functions that drastically differ from the popular ReLU like functions can serve as useful activation functions in CNNs. Future work will explore more functions to attempt to identify even better activation functions. The question of whether even better activation functions exists remains open. Also the question of whether oscillatory activation functions exist in biological neural networks remains unanswered.The question of whether more complex activation functions allow neural network function approximators to learn the target function with fewer neurons is worth pursuing further.

6 Conclusion

This paper explored the possible advantage of using oscillatory activation functions that differ drastically from ReLU like activation functions in the convolutional layers of deep CNNs. Extensive comparisons of performance on CIFAR-10, CIFAR-100 and Imagenette indicate that a new activation function $C(z) = z\cos(z)$ significantly outperforms all popular activation functions on testing-set accuracy. The new activation function GCU allows certain classification tasks to be solved with significantly fewer neurons. In particular the famous XOR problem which hitherto required a network with a minimum of 3 neurons for its solution was solved with a single neuron using the proposed oscillatory GCU activation function. Intriguingly the decision boundary of a single GCU neuron is observed to consist of infinitely many parallel hyperplanes instead of a single hyperplane. Experimental results indicate that the use of oscillatory activation functions improve gradient flow and alleviate the vanishing gradient problem. Improved gradient flow can be attributed to GCU activation having small derivative values only close to isolated points in the domain instead of on entire infinite intervals. However a more detailed theoretical analysis is necessary to validate the advantages of having oscillatory activation functions.

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7 Appendix

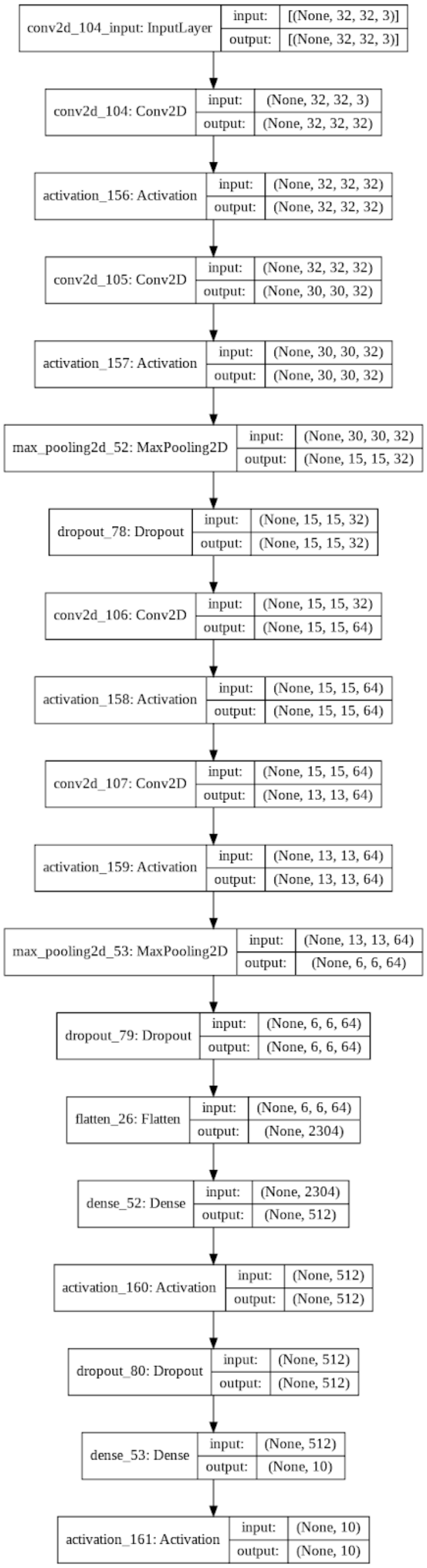
conv2d_104_input: InputLayer | input: [(None, 32, 32, 3)] | output: [(None, 32, 32, 3)]
conv2d_104: Conv2D | input: (None, 32, 32, 3) | output: (None, 32, 32, 32)
activation_156: Activation | input: (None, 32, 32, 32) | output: (None, 32, 32, 32)
conv2d_105: Conv2D | input: (None, 32, 32, 32) | output: (None, 30, 30, 32)
activation_157: Activation | input: (None, 30, 30, 32) | output: (None, 30, 30, 32)
max_pooling2d_52: MaxPooling2D | input: (None, 30, 30, 32) | output: (None, 15, 15, 32)
dropout_78: Dropout | input: (None, 15, 15, 32) | output: (None, 15, 15, 32)
conv2d_106: Conv2D | input: (None, 15, 15, 32) | output: (None, 15, 15, 64)
activation_158: Activation | input: (None, 15, 15, 64) | output: (None, 15, 15, 64)
conv2d_107: Conv2D | input: (None, 15, 15, 64) | output: (None, 13, 13, 64)
activation_159: Activation | input: (None, 13, 13, 64) | output: (None, 13, 13, 64)
max_pooling2d_53: MaxPooling2D | input: (None, 13, 13, 64) | output: (None, 6, 6, 64)
dropout_79: Dropout | input: (None, 6, 6, 64) | output: (None, 6, 6, 64)
flatten_26: Flatten | input: (None, 6, 6, 64) | output: (None, 2304)
dense_52: Dense | input: (None, 2304) | output: (None, 512)
activation_160: Activation | input: (None, 512) | output: (None, 512)
dropout_80: Dropout | input: (None, 512) | output: (None, 512)
dense_53: Dense | input: (None, 512) | output: (None, 10)
activation_161: Activation | input: (None, 10) | output: (None, 10)

Figure 12: Architecture used for the CIFAR-10 dataset