

## Ay190 Worksheet 2 – Problem 3

Michael Eastwood

mweastwood@astro.caltech.edu

### 1. Second Derivative

We begin by Taylor expanding:

$$f(x+h) = f(x) + f'(x)h + \frac{1}{2}f''(x)h^2 + \frac{1}{6}f'''(x)h^3 + \mathcal{O}(h^4) \quad (1a)$$

$$f(x-h) = f(x) - f'(x)h + \frac{1}{2}f''(x)h^2 - \frac{1}{6}f'''(x)h^3 + \mathcal{O}(h^4) \quad (1b)$$

Summing these equations, we find:

$$f(x+h) + f(x-h) = 2f(x) + f''(x)h^2 + \mathcal{O}(h^4) \quad (2)$$

So we have the second-order central finite difference approximation for  $f''(x)$ :

$$\boxed{f''(x) = \frac{1}{h^2} \left( f(x+h) - 2f(x) + f(x-h) \right) + \mathcal{O}(h^2)} \quad (3)$$