Project - Part 1

Phys7280

Due: Feb 29 2012

Write (or find somewhere) a code to simulate the 2 dimensional Ising model using a simple Metropolis algorithm. So we all talk about the same system use

$$H = -\sum_{n,\mu} s_n s_{n+\mu} \,, \quad Z = \sum_{\{s\}} e^{-\beta H} \,.$$

as your action/Hamiltonian, i.e. use only a single parameter β to characterize the system.

- a) Find the critical coupling β_c. This system can be solved exactly and I will show a specific solution later, but you can just Google β_c for now.
- b) Simulate this system in two small volumes (8^2, 16^2 would be a good choice) and measure the energy density and the magnetization as the function of the coupling β. Show it on a plot! How long is the thermalization time? Does it depend on the volume? Can you confirm, at least approximately, the critical β_c? Can you identify tunneling between the positive and negative magnetization phases? (Add a plot that shows at least one tunneling event.)
- c) Try calculating the specific heat and the magnetic susceptibility - this might be too much for a Metropolis code, i.e. your errors could be huge. Don't worry about that, but make sure you know how you would calculate these quantities with more computational resources or a better code. (Make a plot, no matter what you found.)
- d) Modify your code so that it updates with the single cluster method. If you wrote your original code modular you will not change much beyond the update routine. Since you have good measurements for the energy density, use that to debug the cluster code. Show the agreement with a plot.
- e) Try again c), the susceptibilities.

This might sound a lot of work, but you will get the hang of it. The next phase of the project will build on these codes, so please do them.

Project - Part 2

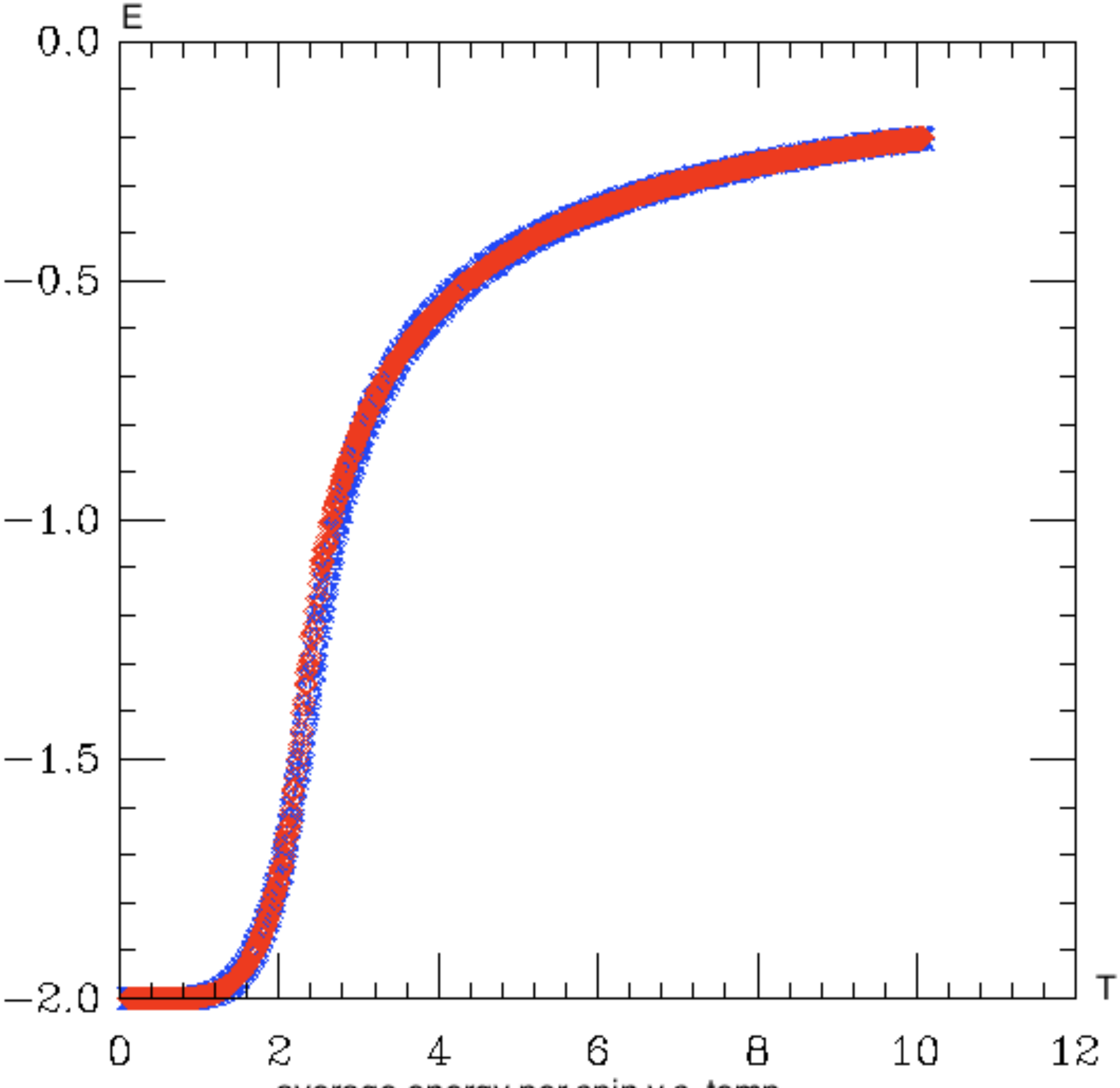
Phys7280

Due: Apr 4 2012

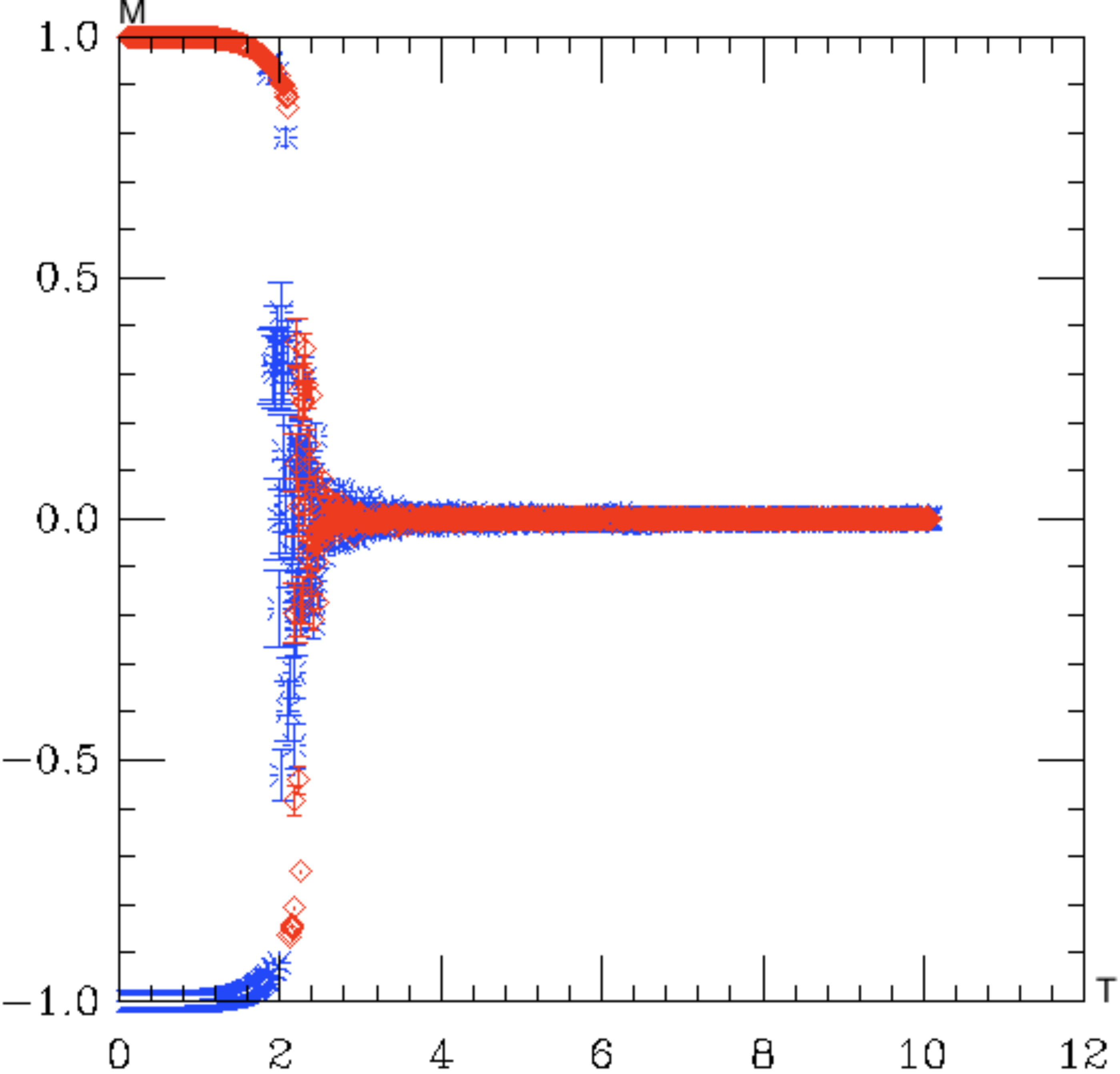
1) Study the finite size scaling of the magnetic susceptibility of the 2 dimensional Ising model. You will need several (4-6) volumes and will have to measure the susceptibility and its errors reliably. (Remember to bin the data when calculating the error.) Estimate the critical exponents. What quantity are you going to fit as the function of what to get that? Recall that a linear fit is usually easy. Fitting exponentials is not.

2) Add a blocking routine to your code. You can use 2×2 blocks with majority rule, or choose a 5-element blocking scheme. Up to you, but please specify. Take your largest volume and block it as many times as possible. Measure the nearest neighbor and next-to-nearest neighbor operators on the blocked lattice. Plot these as the function of the coupling. Where do these curves cross? How well does it estimate the critical point?

3) Based on the measurements of the blocked observables estimate the critical exponent ν by matching the blocked actions. I'll discuss that in class, but the idea is to find (β, β') coupling pairs where where the n times blocked action from β matches the $n+1$ times blocked action from β'. What is the relationship between the correlations lengths at β and β'? How is that related to the exponent ν?

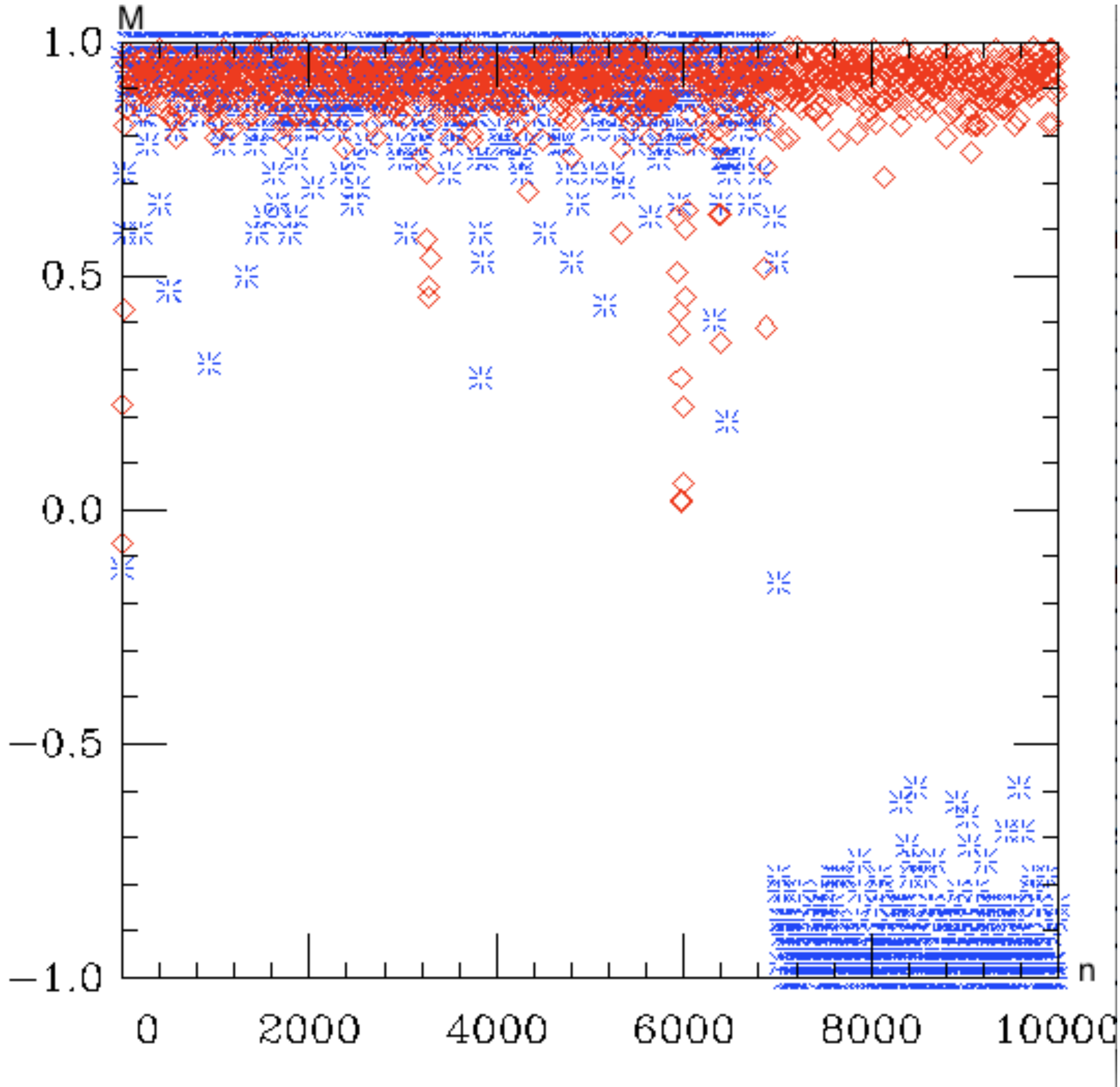


average energy per spin v.s. temp.
made N=L^2 flip attempts per MC step
blocked every 100 MC steps, total 10^4 MC steps.
red is 16^2 and blue is 8^2.
error bars are included but tiny

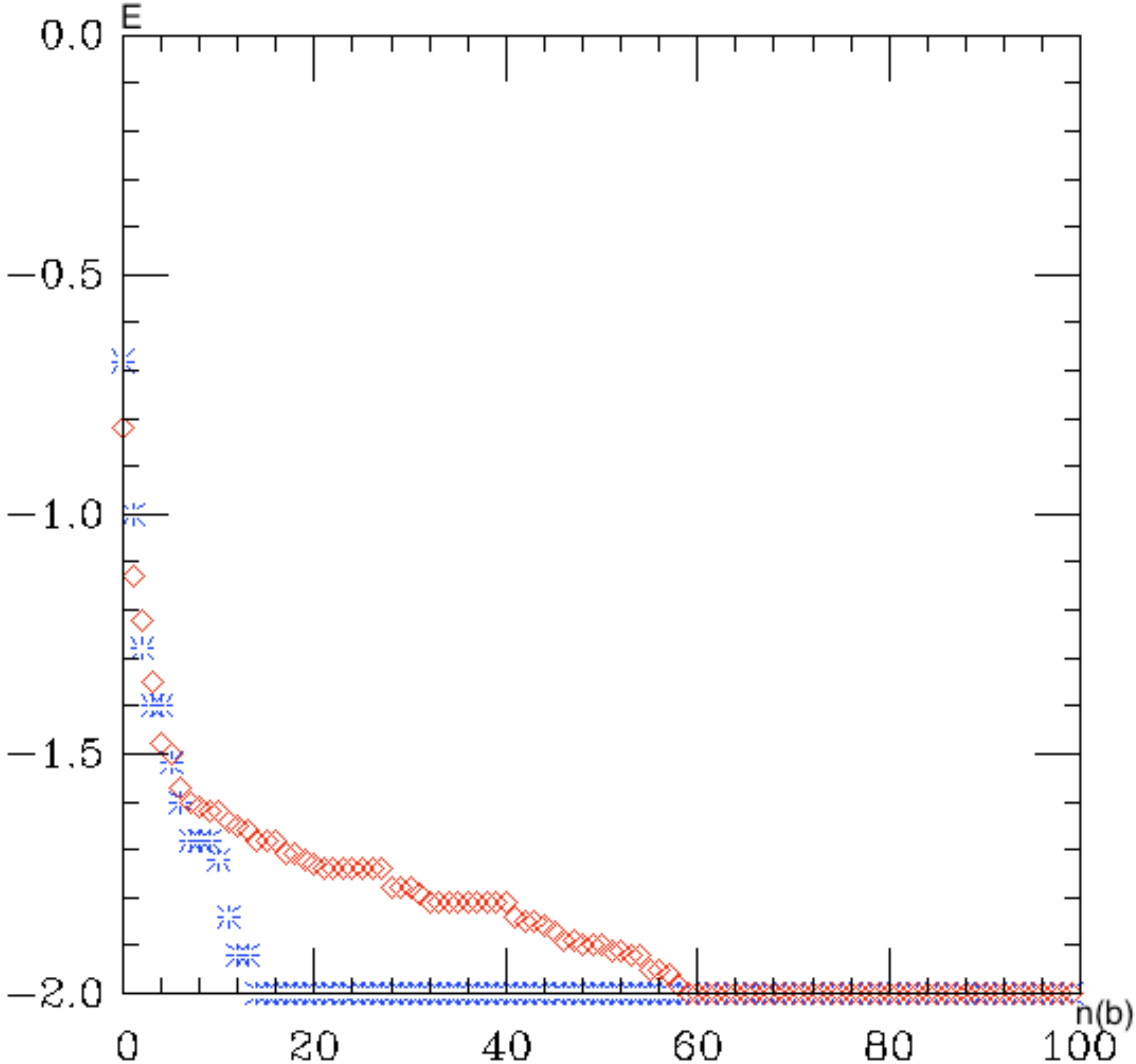


average magnetization per spin v.s temp.
red is 16^2 and blue is 8^2

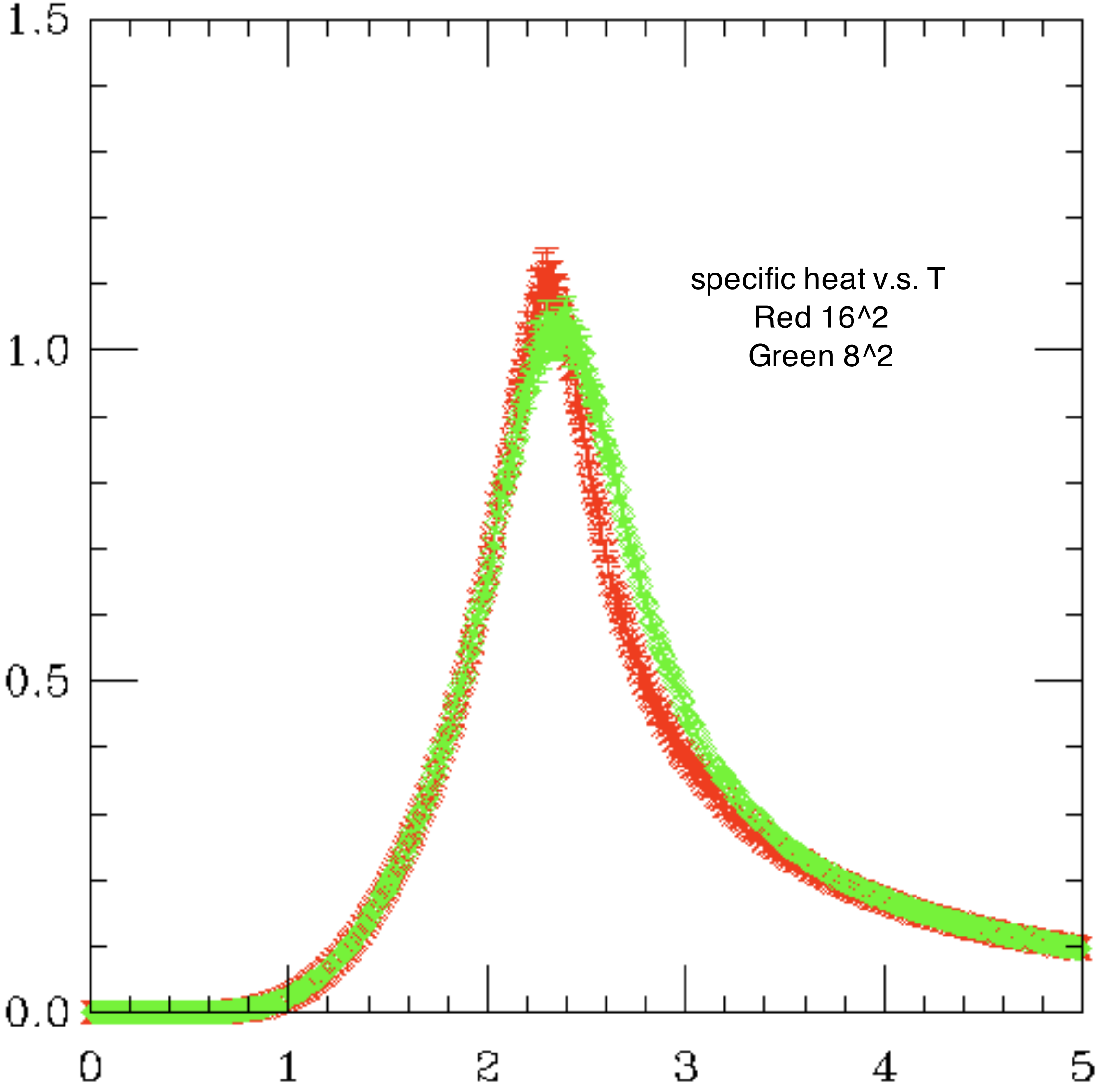
error bars are included but tiny



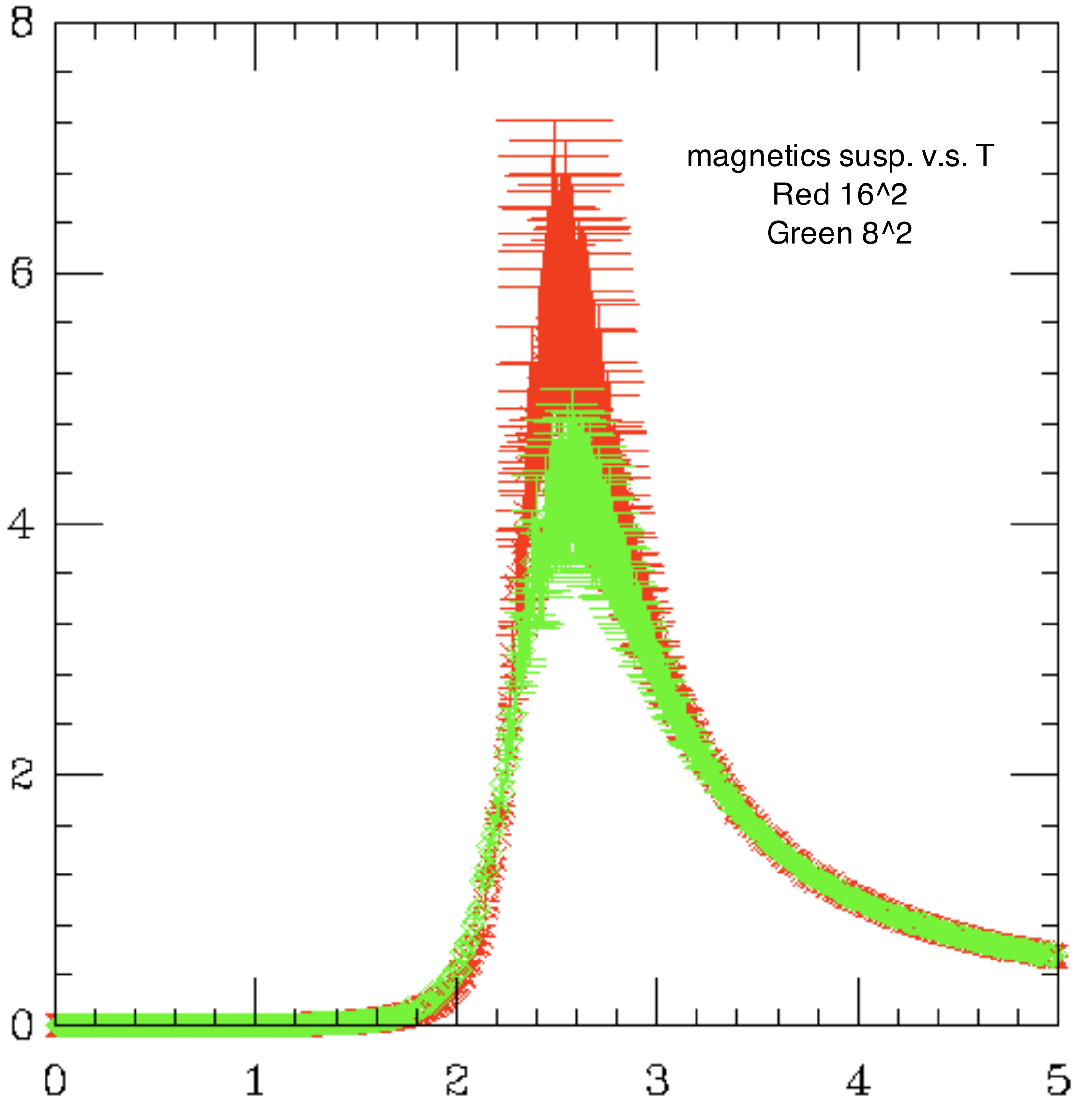
average magnetization per spin v.s. MC steps
red is 16^2 and blue is 8^2
one tunneling event on 8^2

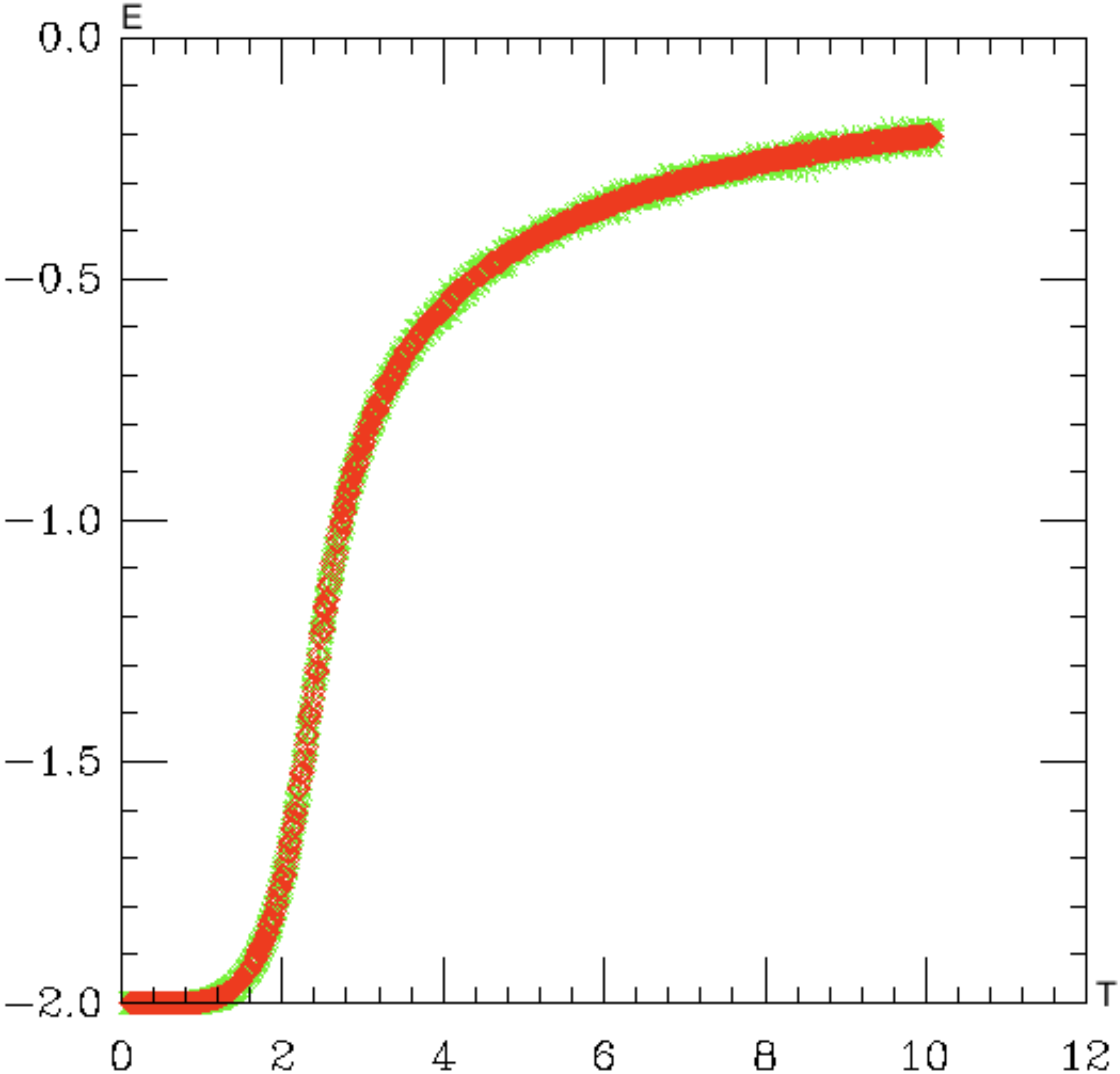


(volume dependence of thermalization)
average energy per spin v.s. blocked MC steps at T=0.5
red is 16^2 and blue is 8^2



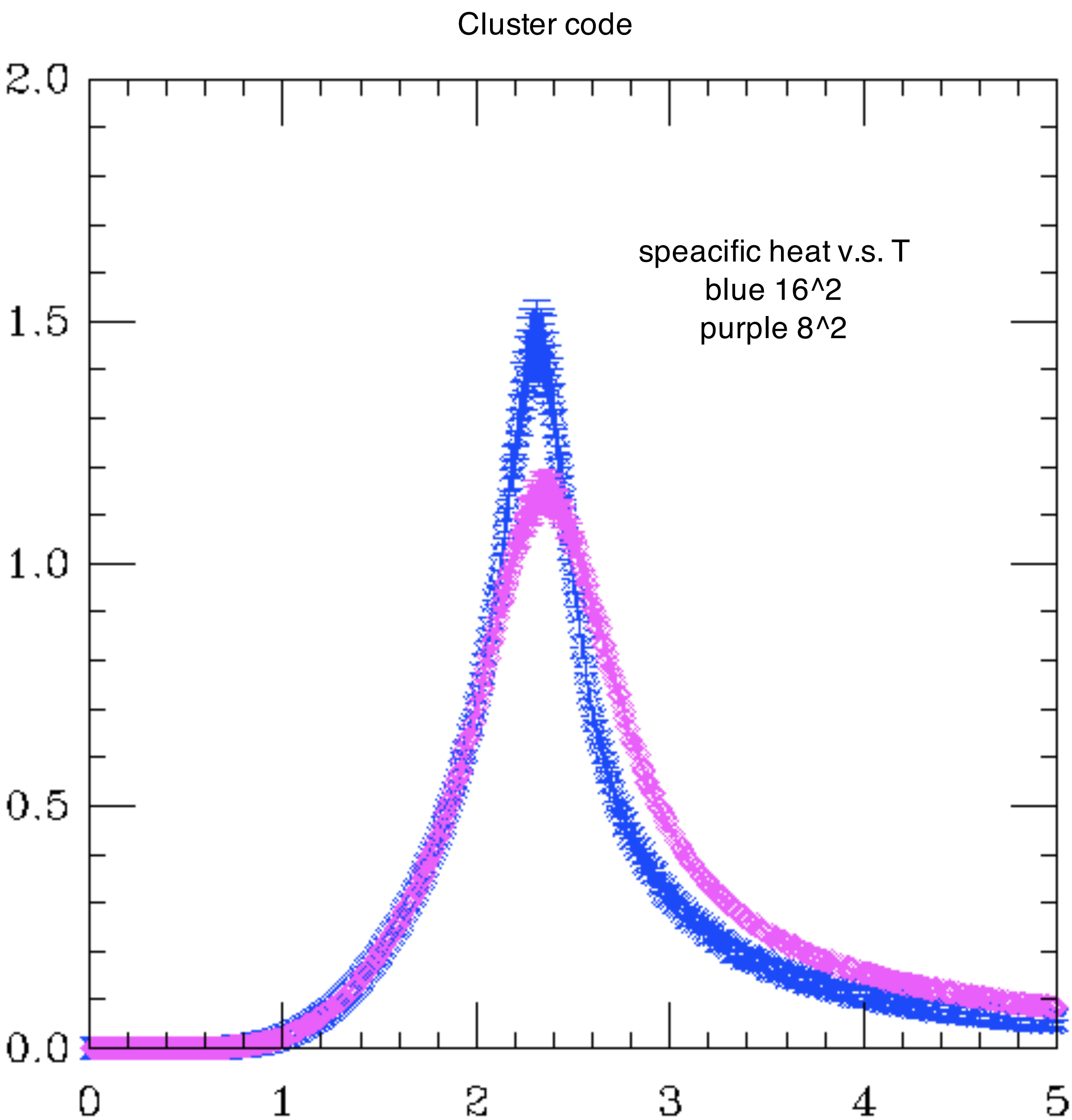
1.5
1.0
0.5
0.0
0
1
2
3
4
5
specific heat v.s. T
Red 16^2
Green 8^2

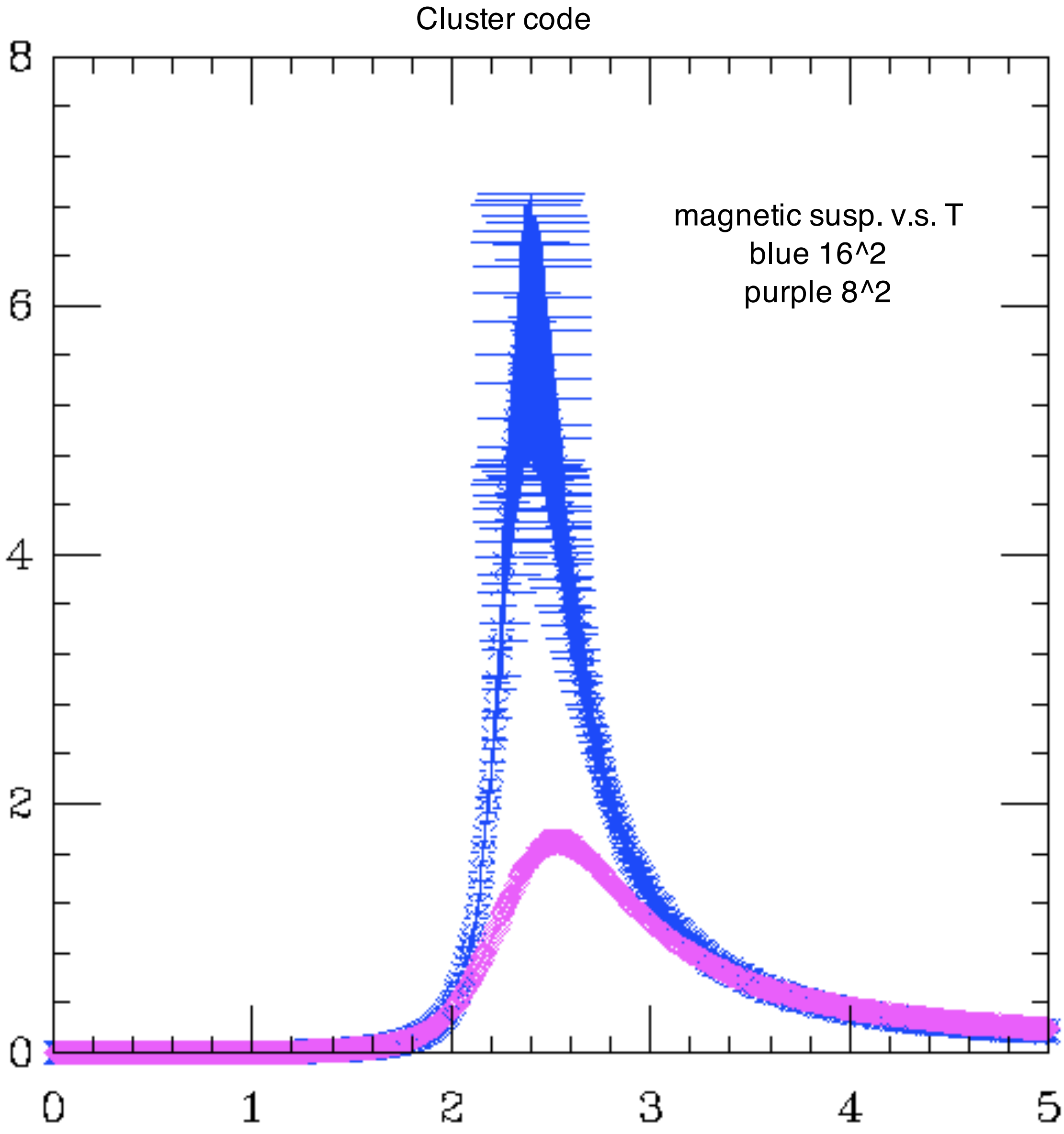

magnetics susp. v.s. T
Red 16^2
Green 8^2
8
6
4
2
0
0
1
2
3
4
5

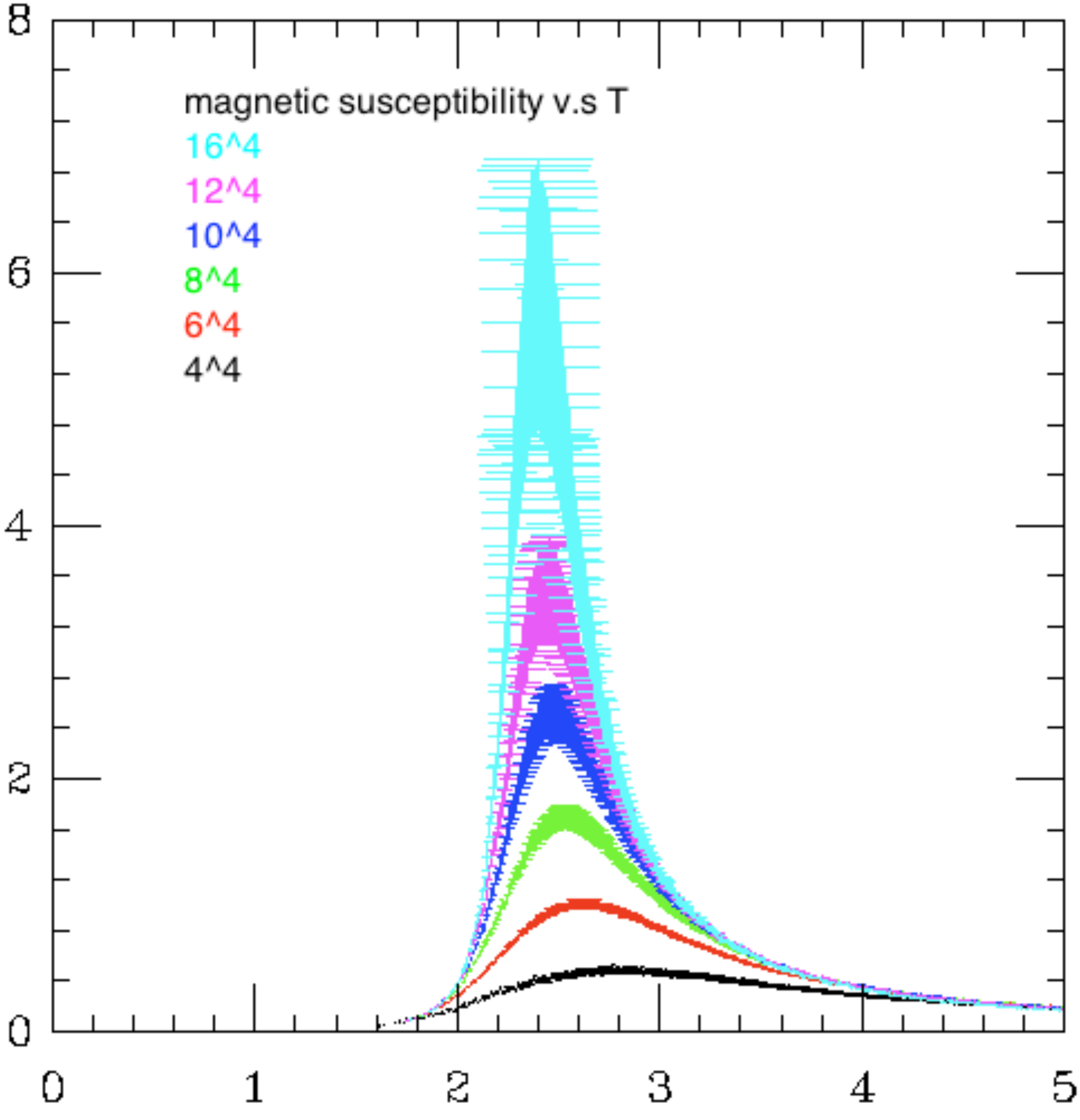


average energy per spin v.s. temp. on 8^2
red is metropolis and green is cluster

error bars are included but tiny


Cluster code
speacific heat v.s. T
blue 16^2
purple 8^2
2.0
1.5
1.0
0.5
0.0
0
1
2
3
4
5


Cluster code
magnetic susp. v.s. T
blue 16^2
purple 8^2
0
2
4
6
8
0
1
2
3
4
5



magnetic susceptibility v.s T
16^4
12^4
10^4
8^4
6^4
4^4
0
2
4
6
8
0
1
2
3
4
5

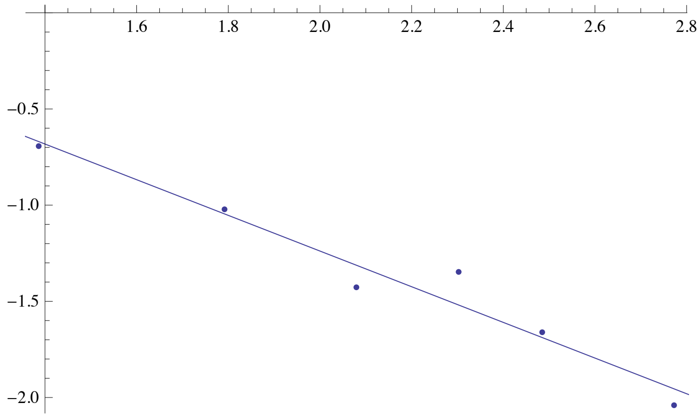
```
Vol   peak   loc.peak    mag.error
16^2  2.4    5.82718e+00  -1.07540e+00
12^2  2.46   3.51359e+00  -3.89768e-01
10^2  2.48   2.53473e+00  -2.02205e-01
8^2   2.51   1.70386e+00  -9.08360e-02
6^2   2.63   1.01564e+00  -3.18710e-02
4^2   2.77   5.03336e-01  -2.51457e-02

(*fit critical exponent for correlation length *)
```

Out[1464]=

	Estimate	Standard Error	t-Statistic	P-Value
a	0.61589	0.22367	2.75356	0.0511868
v	1.07844	0.119108	9.05432	0.000824539

Out[1440]=

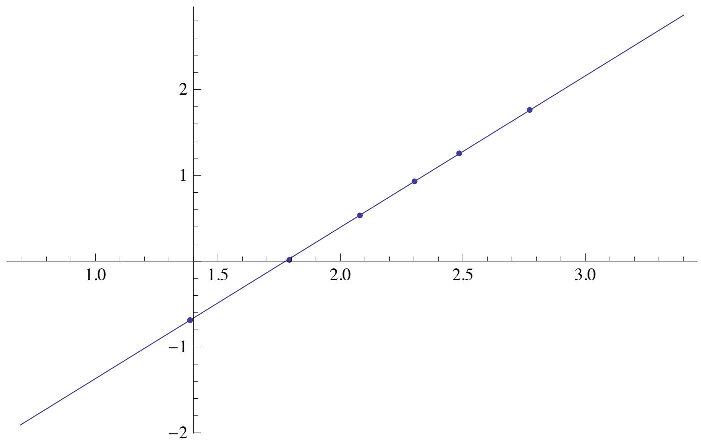


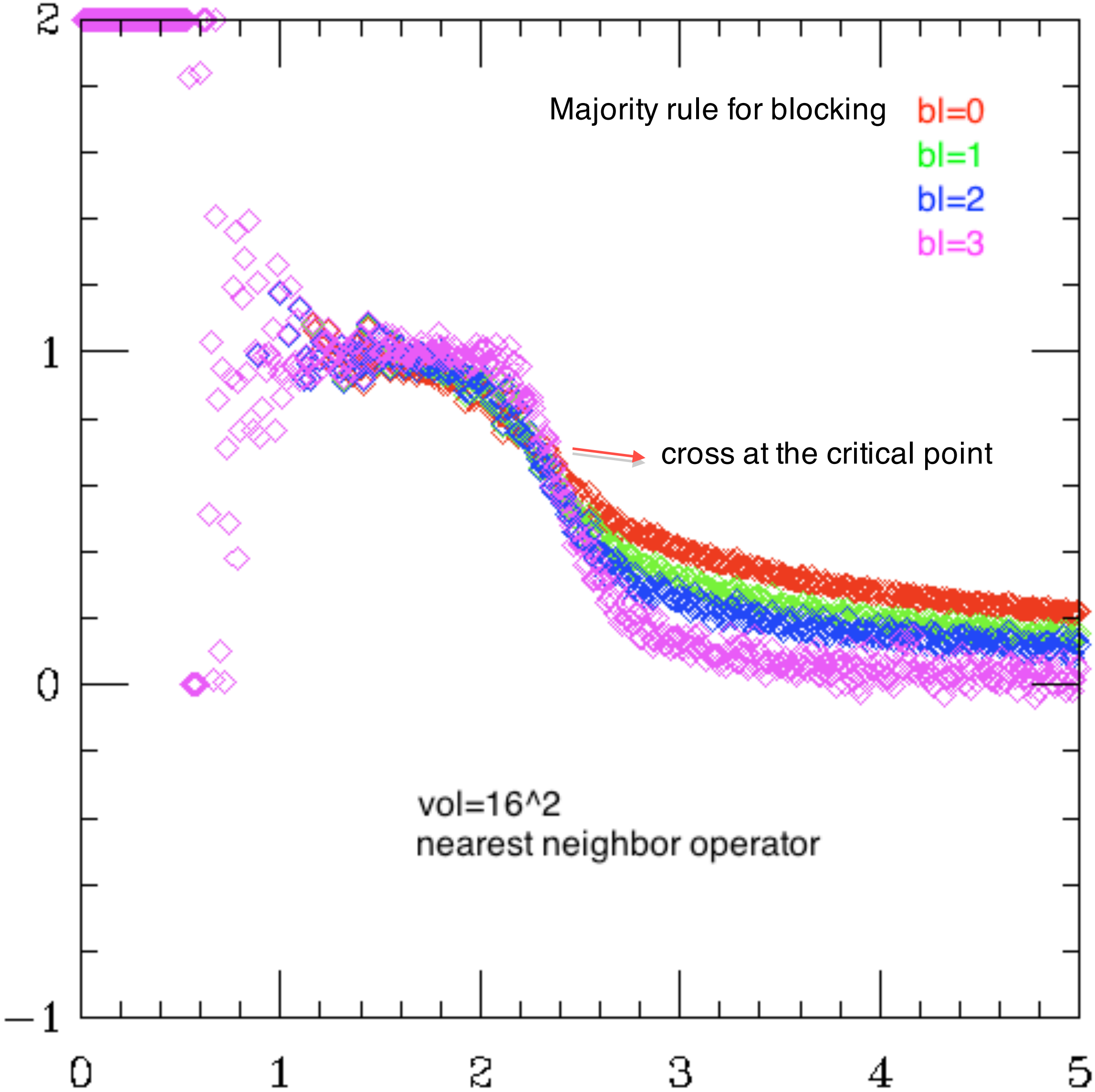

```
(*fit critical exponent for magnetics susceptibility *)
```

Out[1507]=

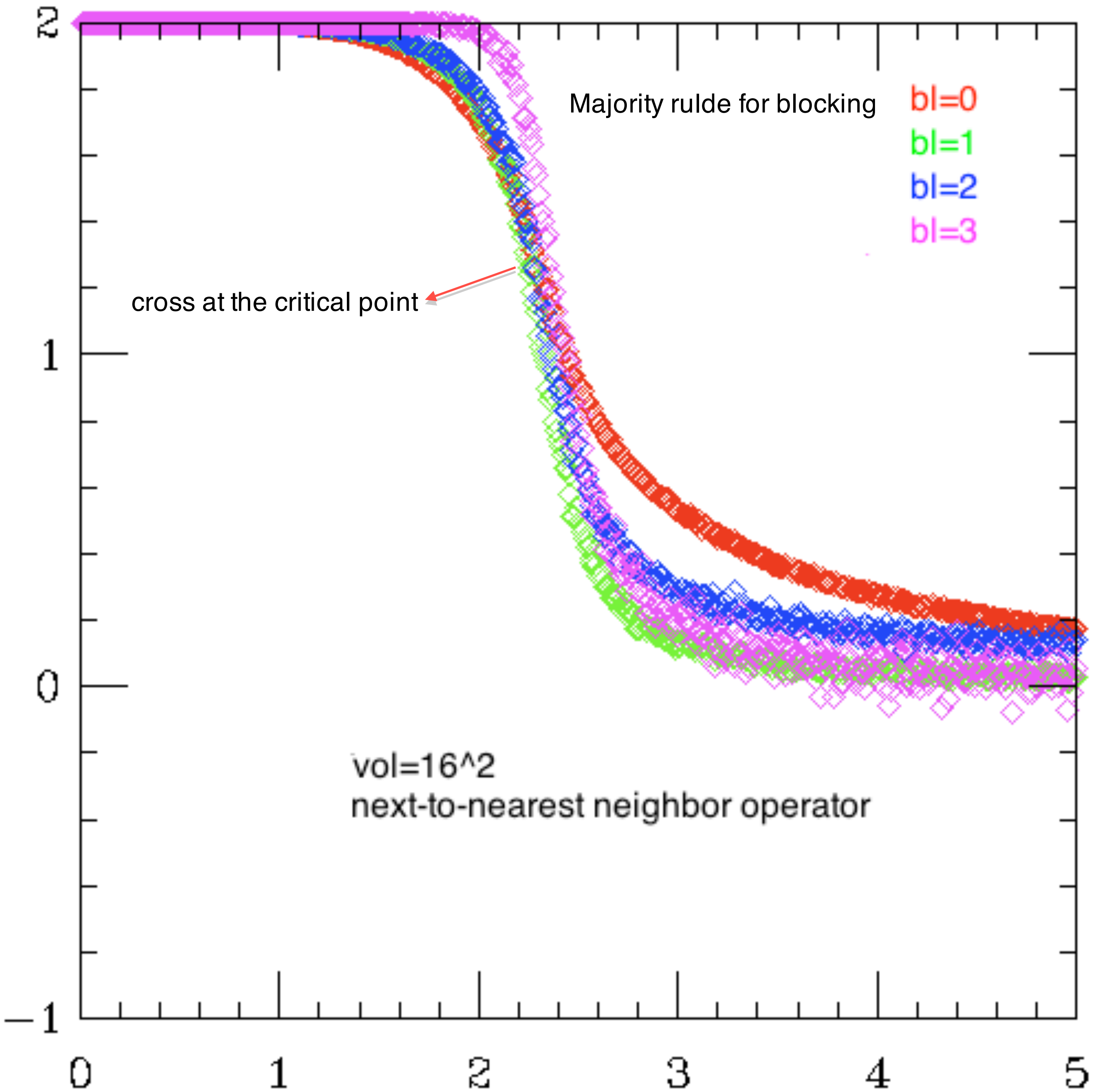
	Estimate	Standard Error	t-Statistic	P-Value
b	−3.1317	0.00551656	−567.691	5.77691×10^{-11}
gamma	1.7638	0.00389826	452.459	1.43159×10^{-10}

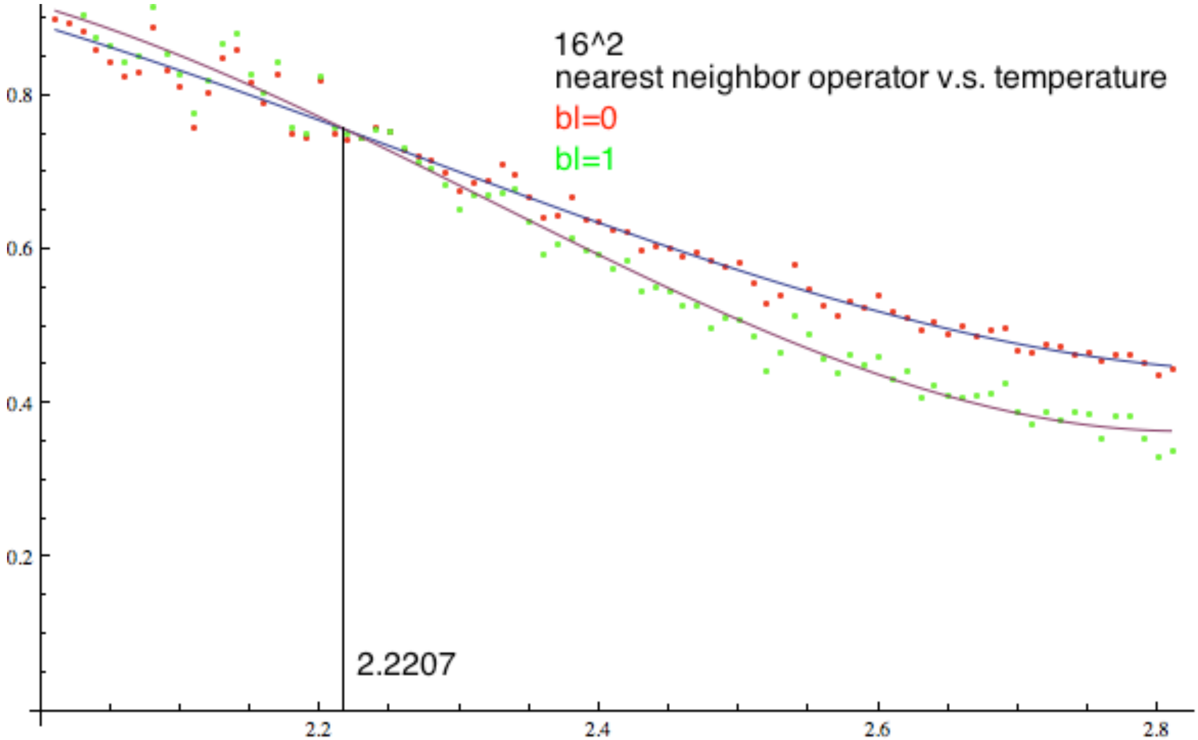
Out[1511]=

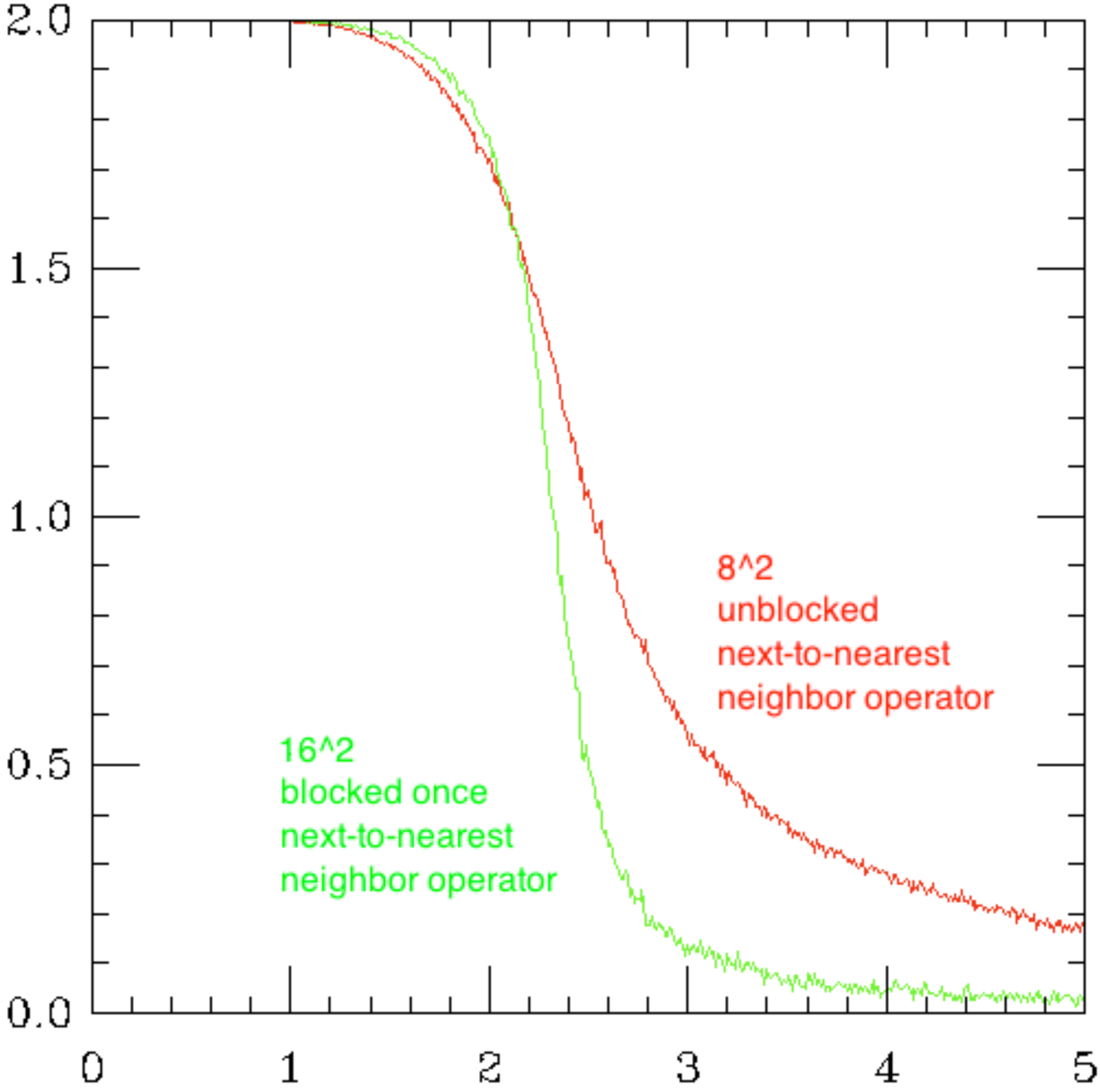




Majority rule for blocking
bl=0
bl=1
bl=2
bl=3
cross at the critical point
vol=16^2
nearest neighbor operator
2
1
0
−1
0
1
2
3
4
5

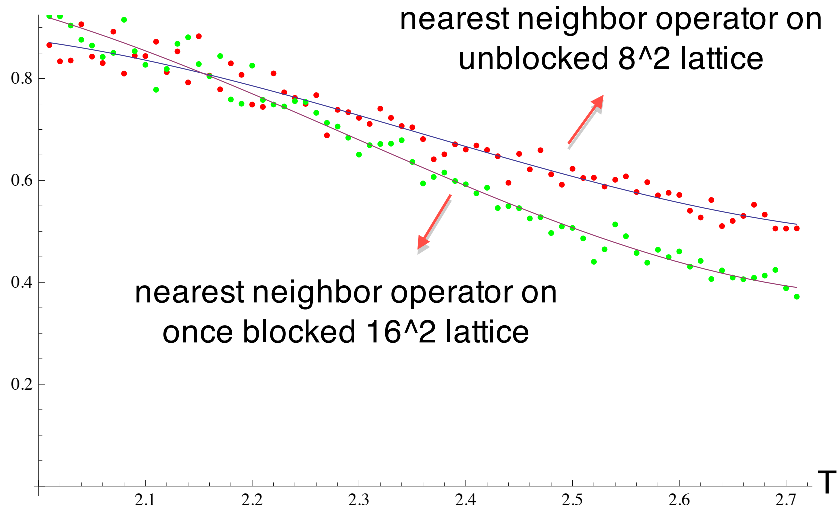

Majority rulde for blocking
bl=0
bl=1
bl=2
bl=3
cross at the critical point
vol=16^2
next-to-nearest neighbor operator
2
1
0
−1
0
1
2
3
4
5


16^2
nearest neighbor operator v.s. temperature
bl=0
bl=1
2.2207
0.2
0.4
0.6
0.8
2.2
2.4
2.6
2.8



2.0
1.5
1.0
0.5
0.0
0
1
2
3
4
5
8^2
unblocked
next-to-nearest
neighbor operator
16^2
blocked once
next-to-nearest
neighbor operator

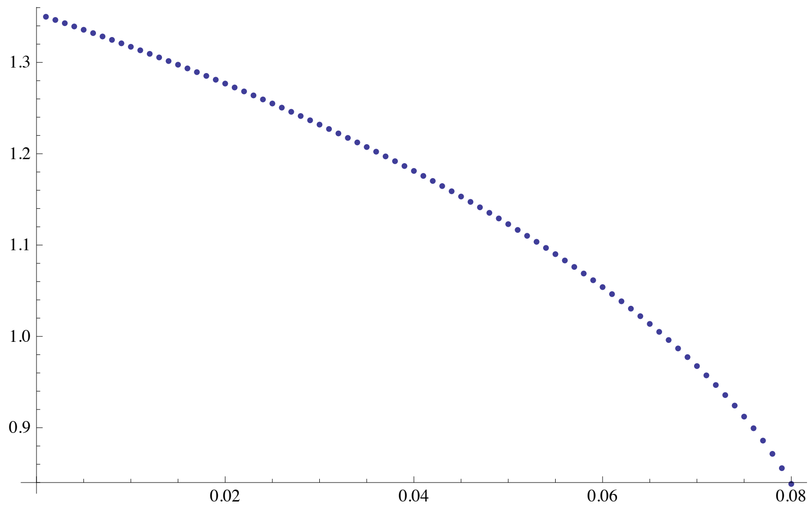
Out[1658]=

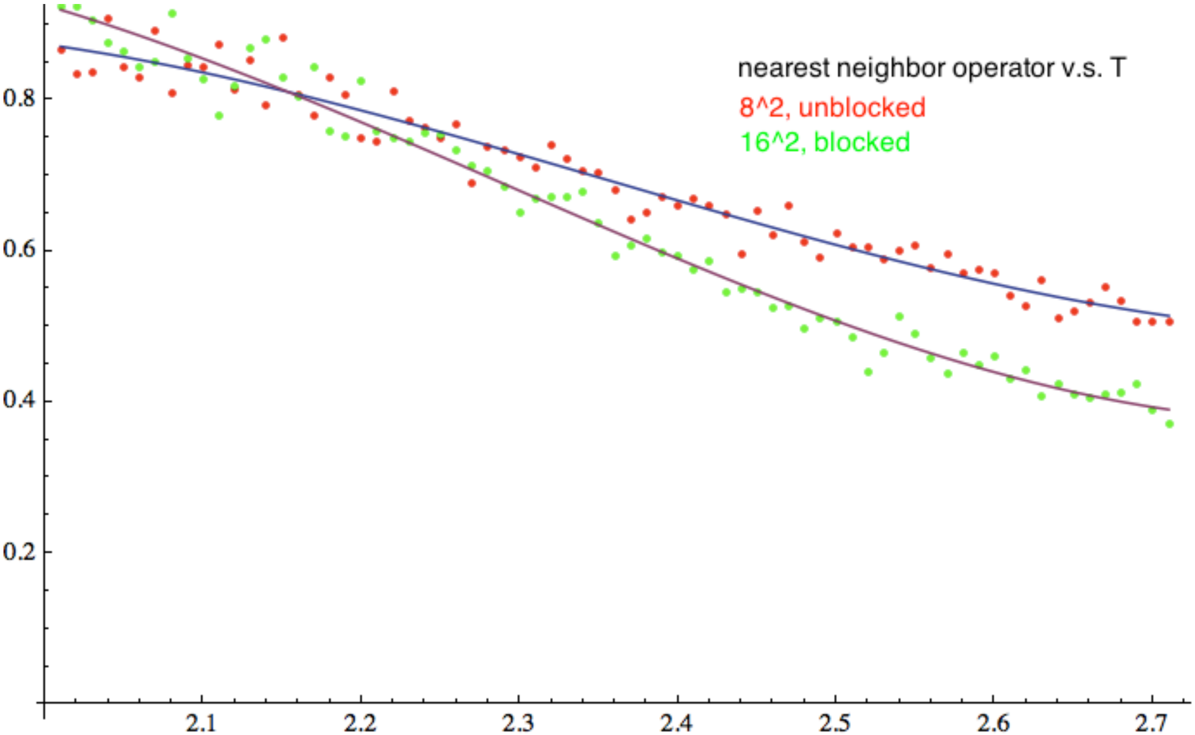


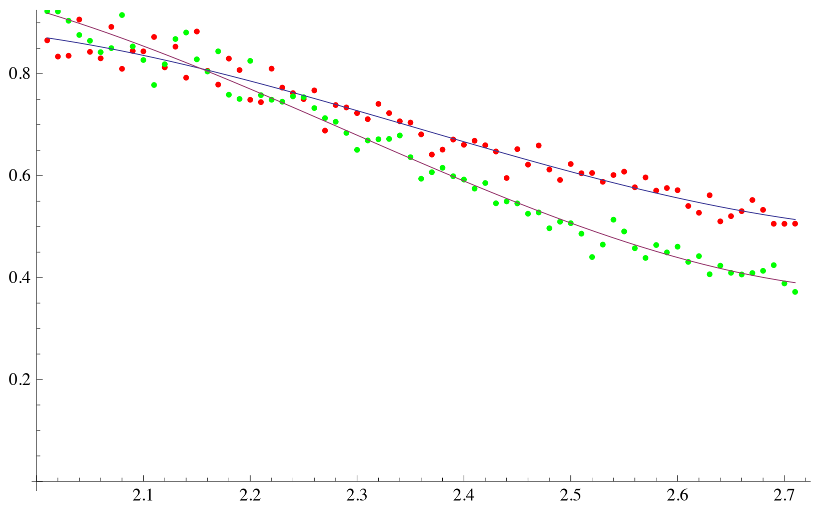
fit the critical exponent from MCRG;
The error is hard to estimate because it
depends on how far the data are taken from the critical point;
This is an esimation matching once blocked 16 ^ 2 and unblocked 8 ^ 2 in the form :
v = Log2 / Log[(T ' - Tc) / (T - Tc)];
where T and T ' are matching temperatures
on oncec blocked 16 ^ 2 and unblocked 8 ^ 2 lattices;

The fitted value for critical exponent of correlation length is

{v → 1.15414}




nearest neighbor operator v.s. T
8^2, unblocked
16^2, blocked
0.8
0.6
0.4
0.2
2.1
2.2
2.3
2.4
2.5
2.6
2.7



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depends on how far the data are taken from the critical point;
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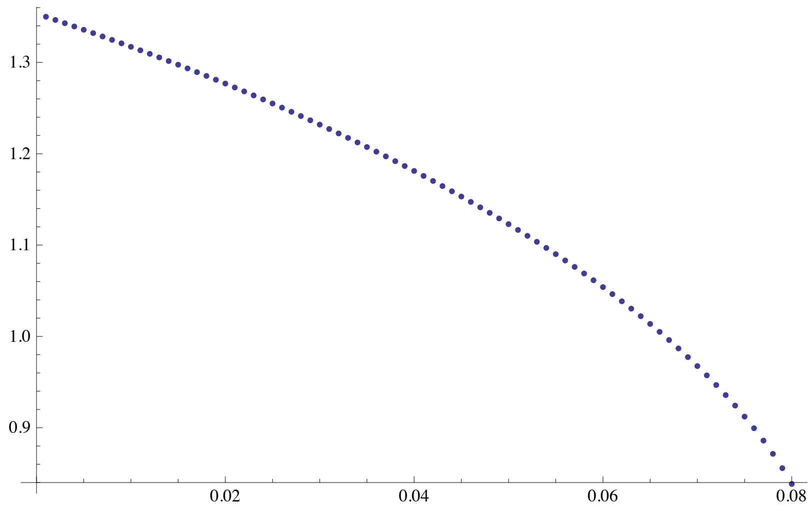


fit the critical exponent from MCRG

The error is hard to estimate because it depends on how far the data are taken from the critical point;

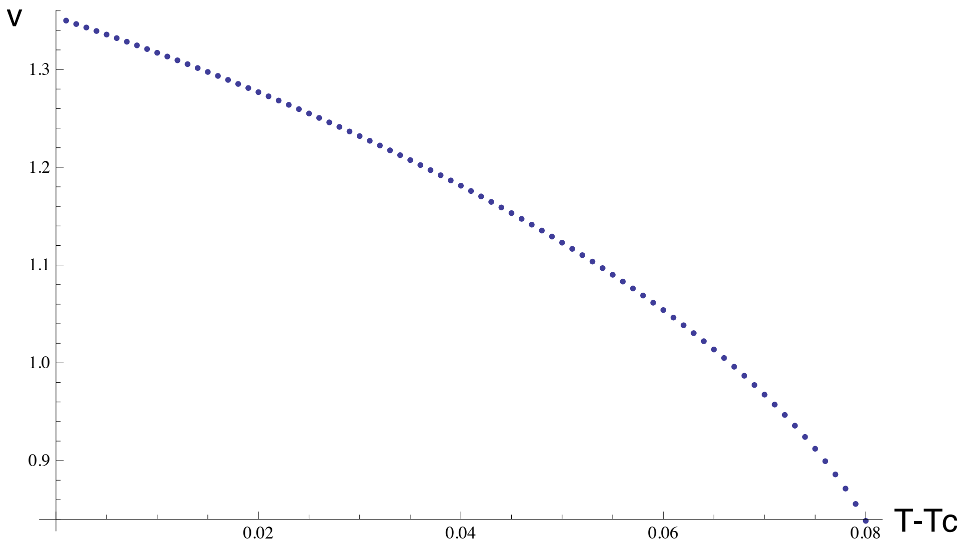
This is an esimation matching once blocked 16 ^ 2 and unblocked 8 ^ 2 in the form :

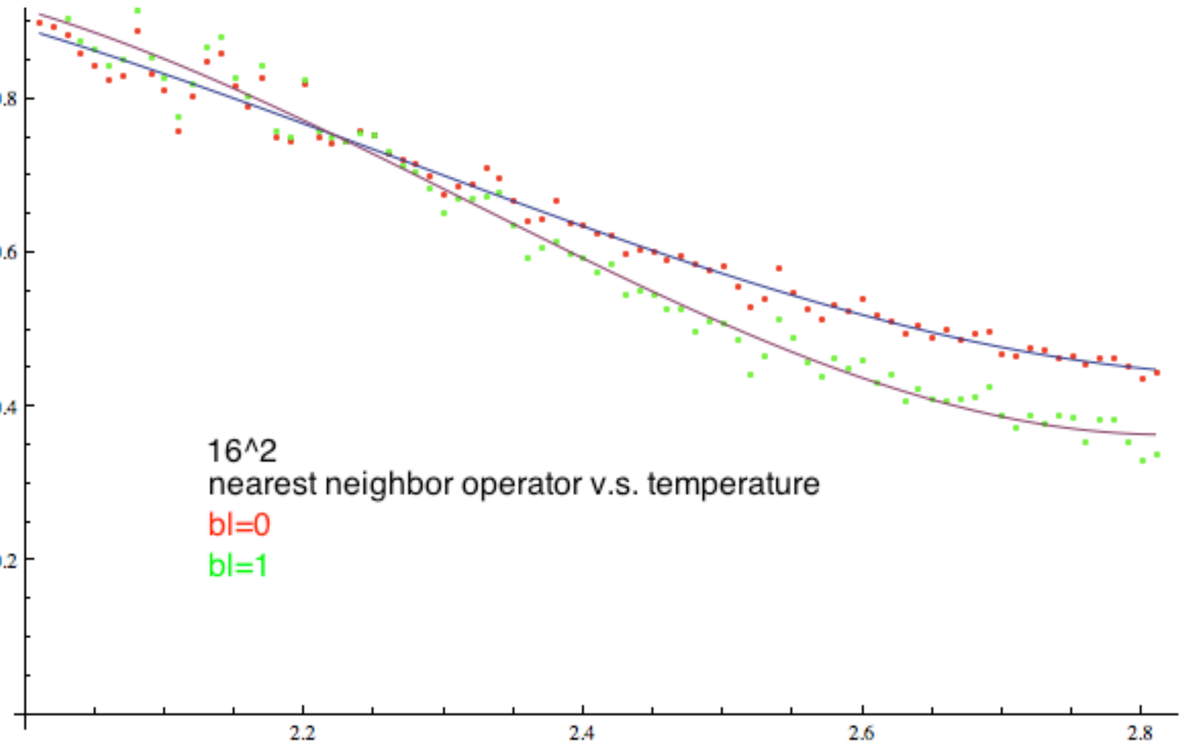
$$v = \log 2 / \log[(T - Tc) / (T' - Tc)]$$

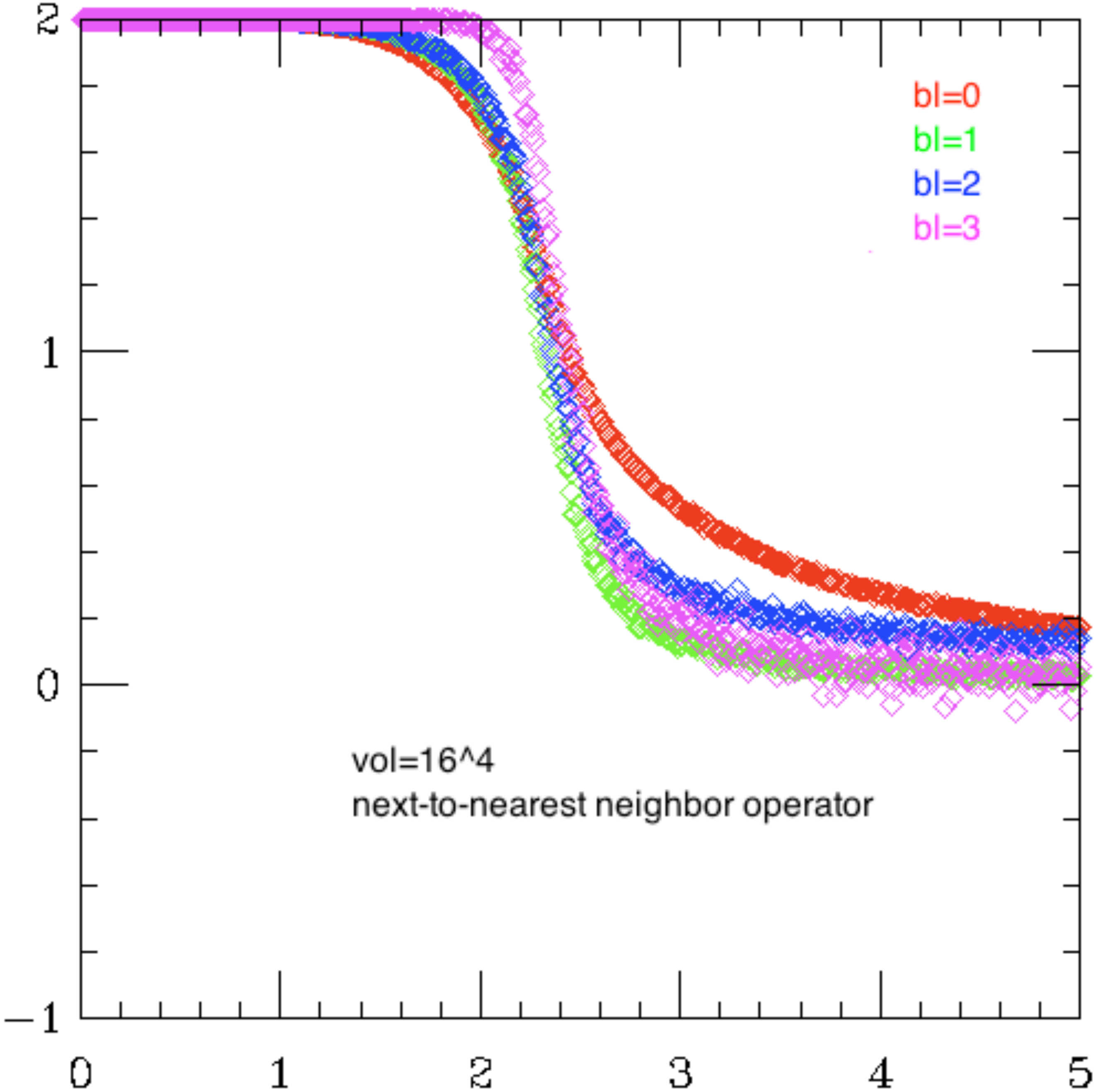
taking T and T ' from Tc to Tc + 0.08;

The fitted value for critical exponent of correlation length is :

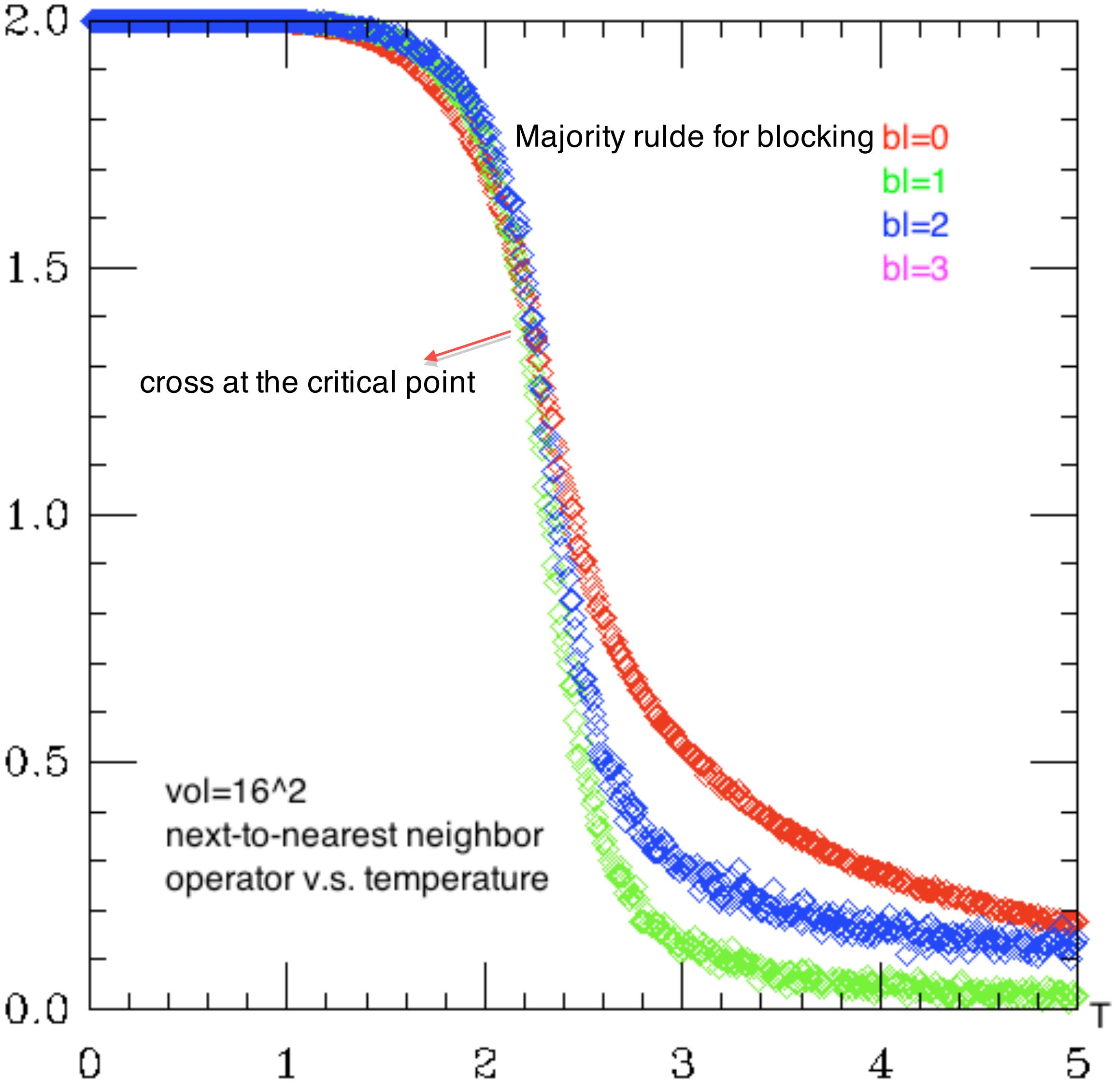
$\{v \to 1.15414\}$




0.8
0.6
0.4
0.2
2.2
2.4
2.6
2.8
16^2
nearest neighbor operator v.s. temperature
bl=0
bl=1



2
1
0
−1
0
1
2
3
4
5
bl=0
bl=1
bl=2
bl=3
vol=16^4
next-to-nearest neighbor operator


2.0
1.5
1.0
0.5
0.0
0
1
2
3
4
5
T
Majority rulde for blocking bl=0
bl=1
bl=2
bl=3
cross at the critical point
vol=16^2
next-to-nearest neighbor
operator v.s. temperature