Parallelization of 2-d Ising Model

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Abstract. The Ising model, named after the physicist Ernst Ising, is a mathematical model of ferromagnetism in statistical mechanics. The model allows the identification of phase transitions, as a simplified model of more complicated reality. In this project we will study this model by Monte Carlo simulation implemented in C and MPI, in addition we will compare the performance of serial code and parallel code by speedup.

1 Introduction

Ising model consists of discrete values that represent magnetic dipole moments of atomic spins that can be in one of two states (+1 or 1). The spins are arranged in a lattice allowing each spin to interact with its neighbors. Depending on the interaction, the spin system undergoes a phase transition at different temperature.

2 Description of the Problem

We will consider interacting spins on a 2-d square lattice with Hamiltonian:

$$\mathcal{H} = -\sum_{\langle ij \rangle} s_i s_j \tag{1}$$

where the notation $\langle ij \rangle$ denotes that we are summing over unique pairs of nearest neighbors.

The goal of this project is to parallelize the Metropolis simulation of 2-d Ising model, and compare the performance of the serial code and the parallel code using strong scaling and weak scaling.

The input data of the problem is the temperature with randomly initialized 2-d of spins. The output would be the energy of the thermalized system and the average magnetization. We will also try to implement a graphic window to display the evolution of the system.

2.1 Metropolis simulation of 2-d Ising model

We consider a 2-d Ising model on square lattices of edge length L, using periodic boundary conditions. Each step in Metropolis simulation is flipping a spin in a random location i with probability to be flipped

$$P = \min[1, e^{-\beta \Delta E}], \tag{2}$$

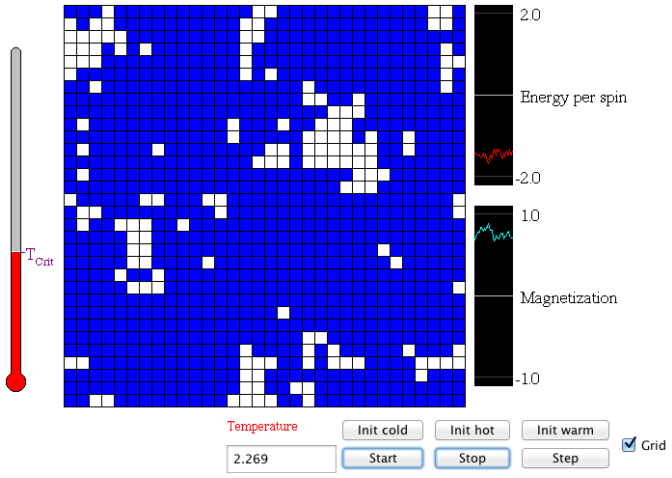

Fig. 1. Illustration of a Monte Carlo Simulation for the Ising model. Taken from [1]

where ΔE corresponds to the energy change induced by the spin flip and $\beta = 1/k_B T$ denotes the inverse temperature. The procedure will be repeated until the average energy of the system converges.

2.2 Parallelize the Metropolis simulation using double checkerboard decomposition

The updating procedure looks as follows [2]:

1. The $L \times L$ lattice is evenly decomposed into N processors, where $\sqrt{N}$ is an integer and L is divisible by $\sqrt{N}$.
2. Each processor initialize the spin configuration of their tile.
3. Each processor performs a Metropolis update of each lattice site in their tile in parallel.
4. The threads of each processor are synchronized.
5. Steps 3 to 4 are repeated until the system is thermalized.

2.3 Data analysis

With fixed temperature, we will study the thermalization time and analyze the performance of the serial code and parallel code with speedup.

3 Analysis

With the lattice size L = 1024, we will simulate the system using both serial and parallel code with N = 4, 16, 64, 256 and 1024. Weak scaling can also be

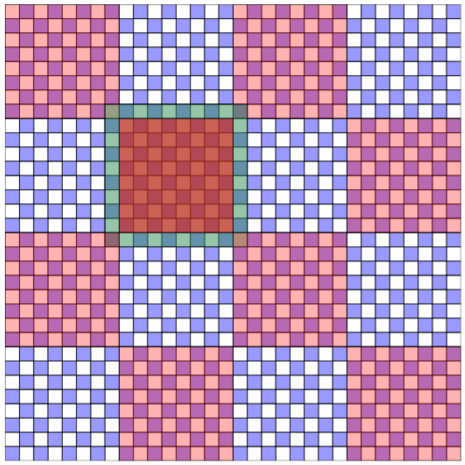

Fig. 2. Double checkerboard decomposition of a square lattice of edge length L=32 onto 16 processors, each processor is assigned a tile of 8×8 sites. Taken from [2]

measured with fixed load on each processors. We will test our code on JANUS and other platforms that available to us.

References

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