Task 4.

a) Model the general problem in writing as an integer linear program.

$$x_{ijk} = \begin{cases} 1 & \text{if element } (i,j) \text{ in } A \text{ contains } k. \\ 0 & \text{otherwise} \end{cases}$$

① constraint on columns

$$\sum_{i=1}^{n^2} x_{ijk} = 1, \quad j = 1 \dots n^2, \quad k = 1 \dots n^2$$

② constraint on rows.

$$\sum_{j=1}^{n^2} x_{ijk} = 1, \quad i = 1 \dots n^2, \quad k = 1 \dots n^2$$

③ constraint on submatrices of size $n \times n$.

$$\sum_{j=n \cdot p-(n-1)}^{n \cdot p} \sum_{i=n \cdot q-(n-1)}^{n \cdot q} x_{ijk} = 1, \quad k = 1 \dots n^2, \quad p = 1 \dots n, \quad q = 1 \dots n$$

④ make sure each entry in A is filled.

$$\sum_{k=1}^{n^2} x_{ijk} = 1, \quad i = 1 \dots n^2, \quad j = 1 \dots n^2$$

⑤ constraint on known entries.

$x_{ijk} = 1 \quad \forall (i,j) \in A$, which has been assigned a number at the beginning time.

Task 4

(b) size = 9

computation time: Solution determined by presolve

```
3 9 . . . . . 8 2
8 . 4 . . . 5 . 3
. . 6 . 2 . 7 . .
. . . 3 . 2 . . .
. 3 5 6 . 1 2 4 .
. . . 4 . 7 . . .
. . 2 . 1 . 4 . .
9 . 3 . . . 1 . 8
5 1 . . . . . 7 .
```

```
3 9 7 1 4 5 6 8 2
8 2 4 7 6 9 5 1 3
1 5 6 8 2 3 7 9 4
4 6 1 3 9 2 8 5 7
7 3 5 6 8 1 2 4 9
2 8 9 4 5 7 3 6 1
6 7 2 9 1 8 4 3 5
9 4 3 5 7 6 1 2 8
5 1 8 2 3 4 9 7 6
```

size=16

computation time: 0.20sec

```
Root node processing (before b&c):
  Real time             =    0.20 sec. (234.49 ticks)
Parallel b&c, 12 threads:
  Real time             =    0.00 sec. (0.00 ticks)
  Sync time (average)   =    0.00 sec.
  Wait time (average)   =    0.00 sec.
                          -------------
Total (root+branch&cut) =    0.20 sec. (234.49 ticks)
```

```
  .    2    .    .    .    .    .    .    .    .    .    .    .
  .    .    .    .    9    8    .    .    .    .    .    .    .
  .    .    .    .    .    .    .    .    5    .    .    .    .
  .    5    3    .    .    .    .    9    .    .    .    .    .
  4    .    .    .    .    .    .    .    .    .    .    .    .
  .    .    .   10    .    .    .    .    2    .    .    .    .
  .    .    .    .    .    .    .    5    .    .    .    .    .
  .    .    .    .    4    .    .    6    .    .    .    .    .
  .    .    .    .    .    .    .    .    .   16   15   13    .
  .    .    .    .    .    .    .    .    .    .    3    .    9
  .    4   14    .    .    .    9    .    .    .    .    .    .
  .    .    .    .    7    .    .    .    .    .    .    .    .
  .    .    .    .    6    .    .   11    .    .    .    .    .
```

```
  1    2   16   12   10    5    6    3   14    8   15    4    7   13    9   11
 13    8   10    5   12   15   11    7    9   16    2    6    4   14    3    1
  7    6   11    4    9    8   14   13   10    1   12    3    5   16    2   15
  3    9   15   14   16    2    1    4    5   11    7   13   10    8    6   12
 14    5    3   13   15   10    2    9    1    6    4    7    8   12   11   16
  4   12    2    8   14    7   16    1   11    5   10    9    3    6   15   13
  6   16    1   10   11   12    3    8    2   14   13   15    9    5    4    7
 11   15    7    9   13    6    4    5   12    3   16    8    1    2   14   10
  2    1    8    3    4   16   15    6   13   10    9   11   12    7    5   14
 10    7    9   11    1   14    5   12    8    2    3   16    6   15   13    4
  5   13   12   15    8   11    7    2    6    4   14    1   16    3   10    9
 16    4   14    6    3   13    9   10    7   15    5   12    2   11    1    8
  9    3    6    7    2    4    8   16   15   13   11   10   14    1   12    5
  8   11    4    2    7    9   13   14    3   12    1    5   15   10   16    6
 15   10    5    1    6    3   12   11   16    9    8   14   13    4    7    2
 12   14   13   16    5    1   10   15    4    7    6    2   11    9    8    3
```

size=20

no results

```
presolve: constraint Submatrices[25,4,3] cannot hold:
        body <= -1 cannot be >= 0; difference = -1
presolve: constraint Submatrices[9,2,1] cannot hold:
        body <= -1 cannot be >= 0; difference = -1
presolve: constraint Submatrices[5,2,1] cannot hold:
        body <= -1 cannot be >= 0; difference = -1
presolve: constraint Submatrices[1,1,2] cannot hold:
        body <= -1 cannot be >= 0; difference = -1
presolve: constraint Rows[3,24] cannot hold:
        body <= -1 cannot be >= 0; difference = -1
3 presolve messages suppressed.
```