

1) Let  $f(s, v)$  be the function solving the problem.

$$f(s, v) = \min(f(s - 1, 270), f(s - 1, 270 - c_s) + v_s)$$

We can see that this problem is just the minimization variant of the knapsack problem if we restate the algorithm as follows:

$$\text{Minimize } \sum_{s=1}^{|S|} c_s x_s$$

$$\text{Subject to } \sum_{s=1}^{|S|} v_s x_s \geq 270 \text{ and } x_s \in (0, 1)$$

We can prove the correctness using proof by induction.

$$\text{Base case: } f(s, 0) = f(0, v) = 0$$

Induction step: When we compute  $f(s', v')$  where  $v' > v$  and  $s' > s$ , we already have  $f(s - 1, v')$  and  $f(s' - 1, v' - c_s)$  computed correctly. The algorithm only has to considering the current item  $s$  either not added to the knapsack, represented by  $f(s - 1, 270)$ , or added to the knapsack, represented by  $f(s - 1, 270 - c_s) + v_s$ . Therefore, the computed value for  $f(s', v')$  would be correct.