Lehrstuhl II für Mathematik
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OPTIMIERUNG B
Worksheet 2
Submission until 21.12.2020 at 14:00 o'clock

In this programming task we focus on linear programs. For this, we use the modeling language AMPL in combination with MILP solvers Gurobi and CPLEX. Download the for you suitable package from "Programming Tasks/AMPL" from RWTHmoodle. We also provide there the link to documentation and installation advice for AMPL.
Upload your written solutions as a Pdf and your implementations in a separate archive to Moodle.

Task 1: **1 Points**
The files *coloring.** in RWTHmoodle model an instance of the so-called coloring problem: We color countries on a map so that countries with a common border are colored differently while minimizing the total number of used colors. Download the files *coloring.mod*, *coloring.dat* and *coloring.run* from RWTHmoodle and understand their structure.
Save the *coloring.** files together with the unziped AMPL files in one folder. Solve the coloring problem by starting AMPL from the terminal with the (linux) commando *./ampl coloring.run* and report the found solution. (For how to start AMPL on other operating systems consult the handbook.)

Task 2: **1+3 Points**
Consider the Knapsack problem.

a) Model the Knapsack problem in writing as an integer linear program.

b) Implement your model in AMPL and solve the instances *Knapsack*.dat*. Report an optimal solution for each instance incl. objective value and computation time.

Task 3: **2+2+1 Points**
The file *cities_coordinates.dat* contains coordinates (x, y) of 1000 cities. We want to pair cities together as partner cities. To maximize the cultural exchange, we want to maximize the sum of the distances between the partner cities. Here, we consider the euclidean distance

$$\text{dist}((x_1, y_1), (x_2, y_2)) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

Remark that every city has at most one partner city.

a) Model the problem in writing as an integer linear program.

b) Implement your model in AMPL and solve the instance *cities_coordinates.dat*. Report an optimal solution incl. objective value and computation time.

c) The file *cities_coordinates999.dat* contains one city less. Try to solve this instance using the model from task b). Give an idea why it is more difficult to solve this instance, although it is smaller.

Task 4: **3+5+2 Points**
The mathematical program sudoku is usually defined for a 9×9 matrix. The problem can of course be generalized to any $n^2 \times n^2$ matrix A that is divided into n^2 submatrices of size $n \times n$. The feasible numbers are thereby chosen from the set $\{1, \ldots, n^2\}$.

a) Model the general problem in writing as an integer linear program.

b) Implement your model in AMPL and solve the instances *Sudoku*.dat*. Report an optimal solution for all instances incl. objective value and computation time.

c) For the 16×16 instance there exist multiple solutions. Determine three different solutions and report them.

Give your solutions in human-readable matrix form.