BDA - Chapter 3 Soln

March 1, 2021 12:37 PM

2. Comparison of two multinomial observations: on September 25, 1988, the evening of a presidential campaign debate, ABC News conducted a survey of registered voters in the United States; 639 persons were polled before the debate, and 639 different persons were polled after. The results are displayed in Table 3.2. Assume the surveys are independent simple random samples from the population of registered voters. Model the data with two different multinomial distributions. For $j = 1, 2$, let α_j be the proportion of voters who preferred Bush, out of those who had a preference for either Bush or Dukakis at the time of survey j. Plot a histogram of the posterior density for $\alpha_2 - \alpha_1$. What is the posterior probability that there was a shift toward Bush?

Survey	Bush	Dukakis	No opinion/other	Total
pre-debate	294	307	38	639
post-debate	288	332	19	639

Recall: multinomial = binomial w/ >2 outcomes

↳ $p(y|\theta) \propto \prod_{i=1}^{k} \theta_i^{y_i}$ $\quad \sum \theta_i = 1$

conjugate prior = dirichlet = generalized Beta

↳ $p(\theta|\alpha) \propto \prod_{i=1}^{k} \theta_i^{\alpha_i - 1}$

↳ $\alpha_i = 1$ common uninformative prior choice

↳ α_i intuitively is like adding $\alpha_i - 1$ more observations of that category to the likelihood

Pre Debate vote $\quad y \sim \text{Multinomial}\left(\left(\underset{\theta_1}{\frac{294}{639}}, \underset{\theta_2}{\frac{307}{639}}, \underset{\theta_3}{\frac{38}{639}}\right), \underset{n}{639}\right)$

Uninformative Prior: $(\theta_1, \theta_2, \theta_3) \sim \text{Dirichlet}(1, 1, 1)$

$\Rightarrow \quad (\theta_1, \theta_2, \theta_3) \mid y = \text{Dirichlet}(295, 308, 39)$

Post Debate: $\quad (\theta_1', \theta_2', \theta_3') \mid y = \text{Dirichlet}(289, 333, 20)$

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$$\alpha_1 = \frac{\theta_1}{\theta_1 + \theta_2} \qquad \alpha_2 = \frac{\theta_1'}{\theta_1' + \theta_2'}$$. What is $\alpha_1 | y$ and $\alpha_2 | y$?

Intuitively, we are marginalizing out the third option.

From properties of Dirichlet, this leaves a dirichlet with 2 outcomes instead of 3.

$\hookrightarrow$ this is just a Beta distribution!

See Ch. 3 q' and Appendix A for details.

$$\therefore \quad \alpha_1 | y \sim \text{Beta}(295, 308)$$
$$\alpha_2 | y \sim \text{Beta}(289, 333)$$

Prob of shift $\sim 19\%$.

9. Conjugate normal model: suppose y is an independent and identically distributed sample of size n from the distribution $\mathrm{N}(\mu, \sigma^2)$, where the prior distribution for (μ, σ^2) is $\text{N-Inv-}\chi^2(\mu, \sigma^2 | \mu_0, \sigma_0^2/\kappa_0; \nu_0, \sigma_0^2)$; that is, $\sigma^2 \sim \text{Inv-}\chi^2(\nu_0, \sigma_0^2)$ and $\mu|\sigma^2 \sim \mathrm{N}(\mu_0, \sigma^2/\kappa_0)$. The posterior distribution, $p(\mu, \sigma^2|y)$, is also normal-inverse-χ^2; derive explicitly its parameters in terms of the prior parameters and the sufficient statistics of the data.

algebra bash :)