외적
Cross Product

MyMusicTaste
나채원

MY MUSIC TASTE

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벡터의 외적이란?

벡터곱(外積, Cross product)

수학에서 3차원 공간의 벡터들간의 이항연산의 일종이다.
연산의 결과가 스칼라인 스칼라곱과는 달리 **연산의 결과가 벡터**이다.

스칼라곱(내적, Dot product)

$$\vec{a}, \vec{b} \in R^n$$

$$\vec{a} \cdot \vec{b} \quad \rightarrow \quad scalar$$

벡터곱(외적, Cross product)

$$\vec{a}, \vec{b} \in R^3$$

$$\vec{a} \times \vec{b} \rightarrow vector$$

$$\vec{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad \vec{a} \times \vec{b} = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$$

좌상 X 우하 – 좌하 X 우상

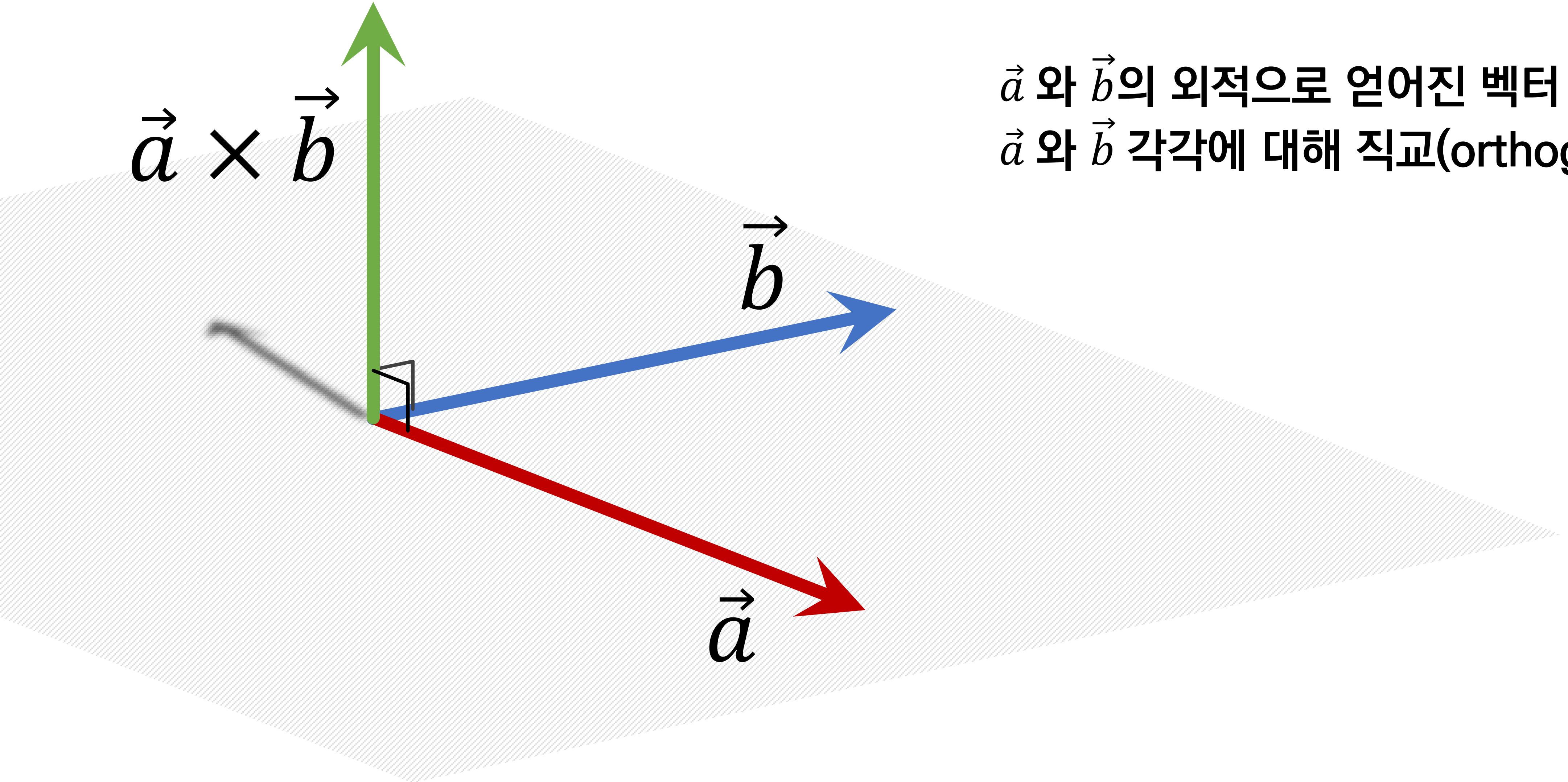
$$\vec{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad \vec{a} \times \vec{b} = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$$

가운데만 반대로 좌하 X 우상 – 좌상 X 우하

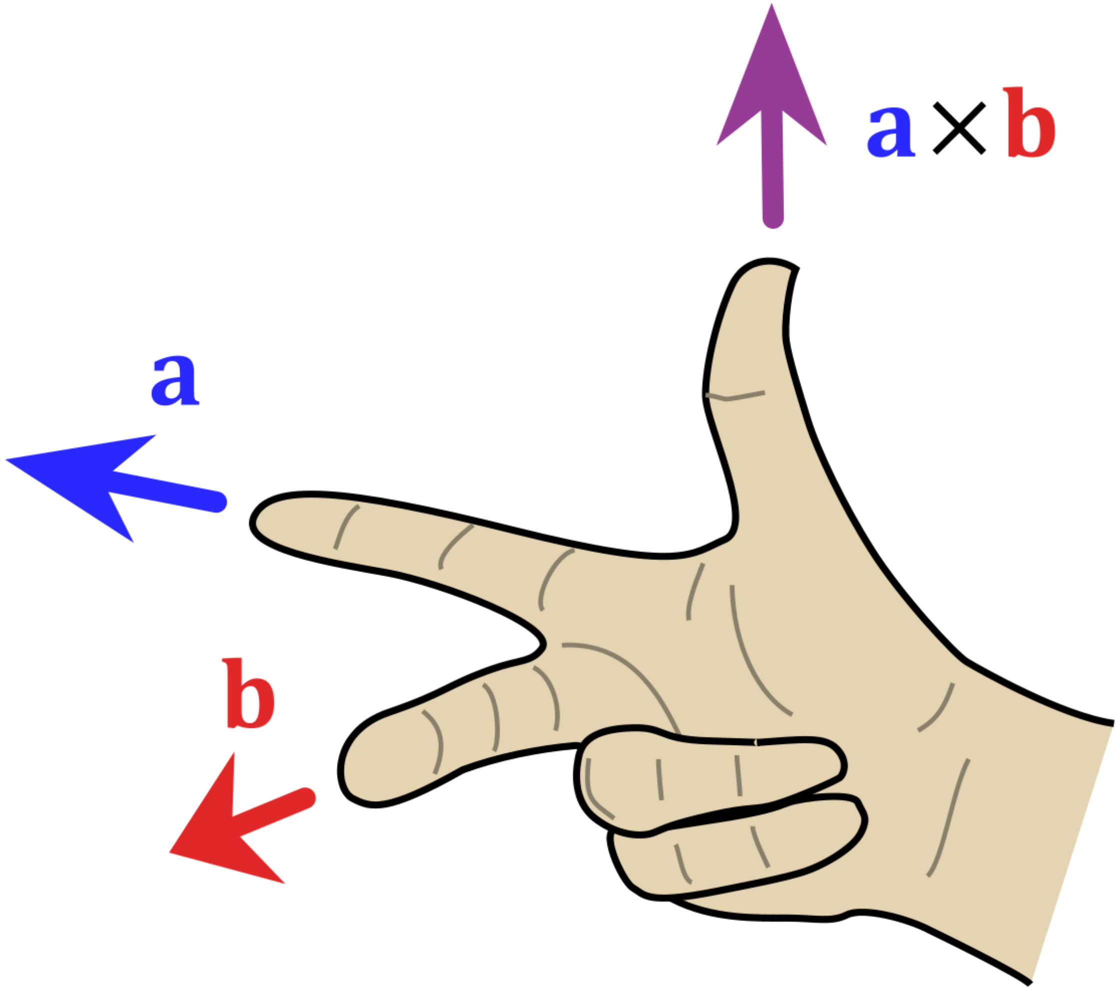
$$\vec{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \qquad \vec{a} \times \vec{b} = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$$

다시 좌상 X 우하 – 좌하 X 우상

$$\vec{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad \vec{a} \times \vec{b} = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$$



$\vec{a}$ 와 $\vec{b}$의 외적으로 얻어진 벡터 $\vec{a} \times \vec{b}$는
$\vec{a}$ 와 $\vec{b}$ 각각에 대해 직교(orthogonal)한다.


a×b
a
b

$\vec{a} \times \vec{b}$가 $\vec{a}$와 직교하는지 확인해보자

$$\vec{a} \times \vec{b} = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix} \qquad \vec{a} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$$

$$(\vec{a} \times \vec{b}) \cdot \vec{a} = 0?$$

$$\begin{bmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{bmatrix} \cdot \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{matrix} a_1(a_2b_3 - a_3b_2) \\ +a_2(a_3b_1 - a_1b_3) \\ +a_3(a_1b_2 - a_2b_1) \end{matrix}$$

$$a_1(a_2b_3 - a_3b_2) + a_2(a_3b_1 - a_1b_3) + a_3(a_1b_2 - a_2b_1)$$

$$\downarrow$$

$$a_1a_2b_3 - a_1a_3b_2 + a_2a_3b_1 - a_2a_1b_3 + a_3a_1b_2 - a_3a_2b_1$$

$$\cancel{a_1 a_2 b_3} - \cancel{a_1 a_3 b_2} + \cancel{a_2 a_3 b_1} - \cancel{a_2 a_1 b_3} + \cancel{a_3 a_1 b_2} - \cancel{a_3 a_2 b_1}$$

$$(\vec{a} \times \vec{b}) \cdot \vec{a} = 0$$

$\vec{a} \times \vec{b}$와 $\vec{b}$에 대해서도 마찬가지이다

$$\begin{bmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{matrix} b_1(a_2b_3 - a_3b_2) \\ +b_2(a_3b_1 - a_1b_3) \\ +b_3(a_1b_2 - a_2b_1) \end{matrix}$$

$$b_1a_2b_3 - b_1a_3b_2 + b_2a_3b_1 - b_2a_1b_3 + b_3a_1b_2 - b_3a_2b_1$$

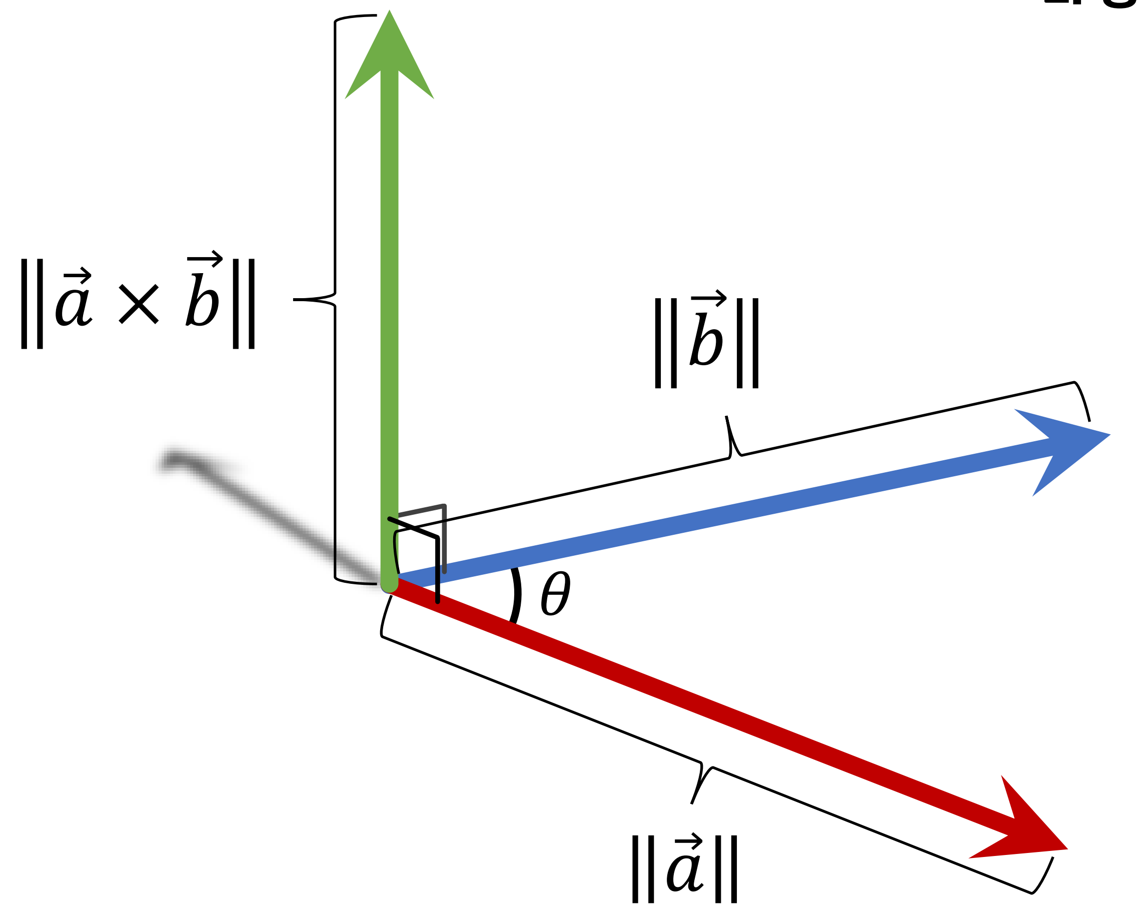
$$\cancel{b_1 a_2 b_3} - \cancel{b_1 a_3 b_2} + \cancel{b_2 a_3 b_1} - \cancel{b_2 a_1 b_3} + \cancel{b_3 a_1 b_2} - \cancel{b_3 a_2 b_1}$$

$$(\vec{a} \times \vec{b}) \cdot \vec{b} = 0$$

2

증명 : 외적과 각의 사인값 사이의 관계

2. 증명: 외적과 각의 사인값 사이의 관계



$$\|\vec{a} \times \vec{b}\| = \|\vec{a}\|\|\vec{b}\|\, sin\, \theta$$

$$\vec{a} \times \vec{b} = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$$

$$\left\| \vec{a} \times \vec{b} \right\|^2 = (a_2 b_3 - a_3 b_2)^2 + (a_3 b_1 - a_1 b_3)^2 + (a_1 b_2 - a_2 b_1)^2$$

이걸 전개하면...

$$\left\|\vec{a} \times \vec{b}\right\|^2 = (a_2b_3 - a_3b_2)^2 + (a_3b_1 - a_1b_3)^2 + (a_1b_2 - a_2b_1)^2$$

$$\downarrow$$

$$\begin{aligned} & a_2^2b_3^2 - 2a_2a_3b_2b_3 + a_3^2b_2^2 \\ + & a_3^2b_1^2 - 2a_1a_3b_1b_3 + a_1^2b_3^2 \\ + & a_1^2b_2^2 - 2a_1a_2b_1b_2 + a_2^2b_1^2 \end{aligned}$$

$$a_2^2 b_3^2 - 2a_2 a_3 b_2 b_3 + a_3^2 b_2^2$$
$$+a_3^2 b_1^2 - 2a_1 a_3 b_1 b_3 + a_1^2 b_3^2$$
$$+a_1^2 b_2^2 - 2a_1 a_2 b_1 b_2 + a_2^2 b_1^2$$

$$\downarrow$$

$$\|\vec{a} \times \vec{b}\|^2 = \begin{array}{c} a_1^2(b_2^2 + b_3^2) + a_2^2(b_1^2 + b_3^2) + a_3^2(b_1^2 + b_2^2) \\ -2(a_2 a_3 b_2 b_3 + a_1 a_3 b_1 b_3 + a_1 a_2 b_1 b_2) \end{array}$$

앞의 식을 잠시 접어두고 이걸 봅시다

$$\vec{a} \cdot \vec{b} = \|\vec{a}\|\|\vec{b}\| \cos\theta = a_1b_1 + a_2b_2 + a_3b_3$$

양변을 제곱한다

$$\|\vec{a}\|^2 \left\|\vec{b}\right\|^2 \cos^2\theta$$
$$= (a_1b_1 + a_2b_2 + a_3b_3)(a_1b_1 + a_2b_2 + a_3b_3)$$

$$(a_1b_1 + a_2b_2 + a_3b_3)(a_1b_1 + a_2b_2 + a_3b_3)$$

$$\downarrow$$

$$a_1^2b_1^2 + a_1a_2b_1b_2 + a_1a_3b_1b_3$$

$$(a_1b_1 + a_2b_2 + a_3b_3)(a_1b_1 + a_2b_2 + a_3b_3)$$

$$\downarrow$$

$$a_1^2b_1^2 + a_1a_2b_1b_2 + a_1a_3b_1b_3$$

$$a_2^2b_2^2 + a_1a_2b_1b_2 \qquad\qquad + a_2a_3b_2b_3$$

$$(a_1b_1 + a_2b_2 + a_3b_3)(a_1b_1 + a_2b_2 + a_3b_3)$$

$$\downarrow$$

$$\begin{aligned}
&a_1^2b_1^2 + a_1a_2b_1b_2 + a_1a_3b_1b_3 \\
&a_2^2b_2^2 + a_1a_2b_1b_2 \qquad\qquad\qquad\quad + a_2a_3b_2b_3 \\
&a_3^2b_3^2 \qquad\qquad\qquad\qquad\quad + a_1a_2b_1b_2 + a_2a_3b_2b_3
\end{aligned}$$

$$
\begin{array}{llll}
a_1^2 b_1^2 & + a_1 a_2 b_1 b_2 & + a_1 a_3 b_1 b_3 & \\
a_2^2 b_2^2 & + a_1 a_2 b_1 b_2 & & + a_2 a_3 b_2 b_3 \\
a_3^2 b_3^2 & & + a_1 a_2 b_1 b_2 & + a_2 a_3 b_2 b_3
\end{array}
$$

$$\downarrow$$

$$
\begin{gathered}
a_1^2 b_1^2 + a_2^2 b_2^2 + a_3^2 b_3^2 \\
+ 2(a_2 a_3 b_2 b_3 + a_1 a_3 b_1 b_3 + a_1 a_2 b_1 b_2)
\end{gathered}
$$

$$a_1^2(b_2^2 + b_3^2) + a_2^2(b_1^2 + b_3^2) + a_3^2(b_1^2 + b_2^2)$$
$$-2(a_2a_3b_2b_3 + a_1a_3b_1b_3 + a_1a_2b_1b_2)$$

$$+$$

$$a_1^2b_1^2 + a_2^2b_2^2 + a_3^2b_3^2$$
$$+2(a_2a_3b_2b_3 + a_1a_3b_1b_3 + a_1a_2b_1b_2)$$

$$a_1^2(b_2^2+b_3^2)+a_2^2(b_1^2+b_3^2)+a_3^2(b_1^2+b_2^2)$$
$$\cancel{-2(a_2a_3b_2b_3+a_1a_3b_1b_3+a_1a_2b_1b_2)}$$

$$+$$

$$a_1^2b_1^2+a_2^2b_2^2+a_3^2b_3^2$$
$$\cancel{+2(a_2a_3b_2b_3+a_1a_3b_1b_3+a_1a_2b_1b_2)}$$

$$
\begin{aligned}
&a_1^2(b_1^2 + b_2^2 + b_3^2)\\
&+a_2^2(b_1^2 + b_2^2 + b_3^2)\\
&+a_3^2(b_1^2 + b_2^2 + b_3^2)
\end{aligned}
$$

$$(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)$$

$$(\underbrace{a_1^2 + a_2^2 + a_3^2}_{\|\vec{a}\|^2})(\underbrace{b_1^2 + b_2^2 + b_3^2}_{\|\vec{b}\|^2})$$

$$\left\|\vec{a} \times \vec{b}\right\|^2 + \|\vec{a}\|^2 \left\|\vec{b}\right\|^2 \cos^2\theta = \|\vec{a}\|^2 \left\|\vec{b}\right\|^2$$

$$\left\|\vec{a}\times\vec{b}\right\|^2 + \|\vec{a}\|^2\left\|\vec{b}\right\|^2\cos^2\theta = \|\vec{a}\|^2\left\|\vec{b}\right\|^2$$

$$\left\|\vec{a}\times\vec{b}\right\|^2 = \|\vec{a}\|^2\left\|\vec{b}\right\|^2 - \|\vec{a}\|^2\left\|\vec{b}\right\|^2\cos^2\theta$$

$$\|\vec{a}\times\vec{b}\|^2 = \|\vec{a}\|^2\|\vec{b}\|^2 - \|\vec{a}\|^2\|\vec{b}\|^2\cos^2\theta$$

$$\left\|\vec{a}\times\vec{b}\right\|^2 = \|\vec{a}\|^2\left\|\vec{b}\right\|^2(1-\cos^2\theta)$$

$$\left\|\vec{a} \times \vec{b}\right\|^2 = \|\vec{a}\|^2 \left\|\vec{b}\right\|^2 (1 - \cos^2 \theta)$$

$$sin^2 \theta + \cos^2 \theta = 1$$

삼각함수 항등식

$$\left\|\vec{a}\times\vec{b}\right\|^2 = \|\vec{a}\|^2\left\|\vec{b}\right\|^2(1-\cos^2\theta)$$

$$sin^2\theta = 1-\cos^2\theta$$

삼각함수 항등식

$$\left\|\vec{a} \times \vec{b}\right\|^2 = \|\vec{a}\|^2 \left\|\vec{b}\right\|^2 sin^2 \theta$$

$$\sqrt{\left\|\vec{a}\times\vec{b}\right\|^2} = \sqrt{\|\vec{a}\|^2\left\|\vec{b}\right\|^2 sin^2\,\theta}$$

$$\|\vec{a} \times \vec{b}\| = \|\vec{a}\|\|\vec{b}\| \sin\theta$$

3

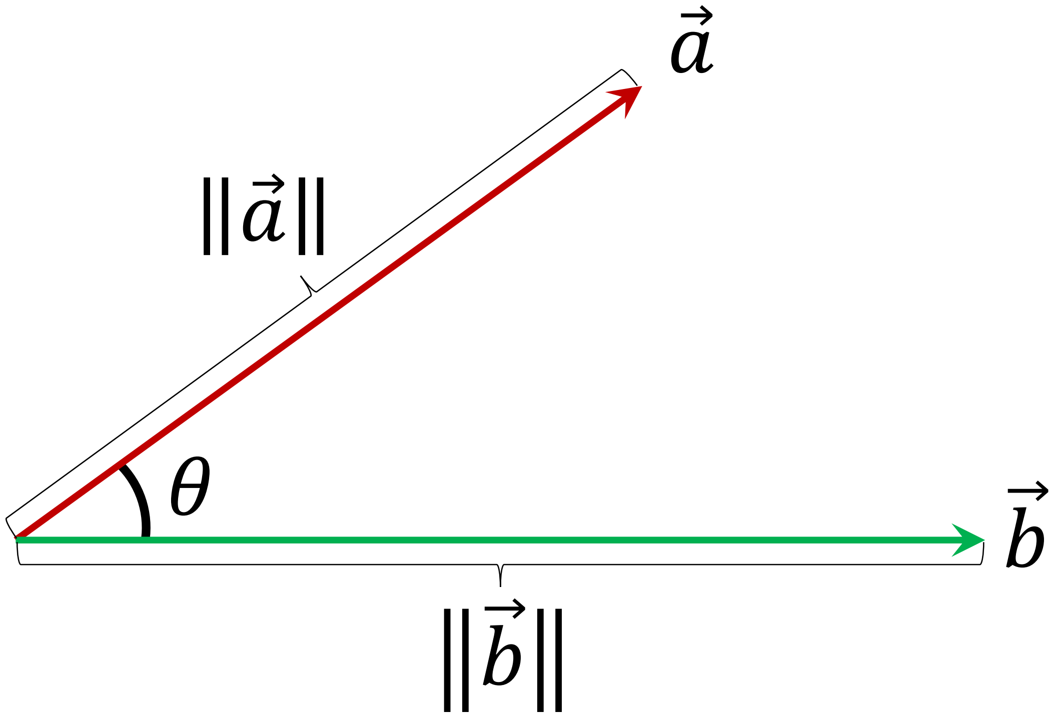
내적과 외적의 비교 / 직관

$$\vec{a} \cdot \vec{b} = \|\vec{a}\| \|\vec{b}\| \cos\theta$$

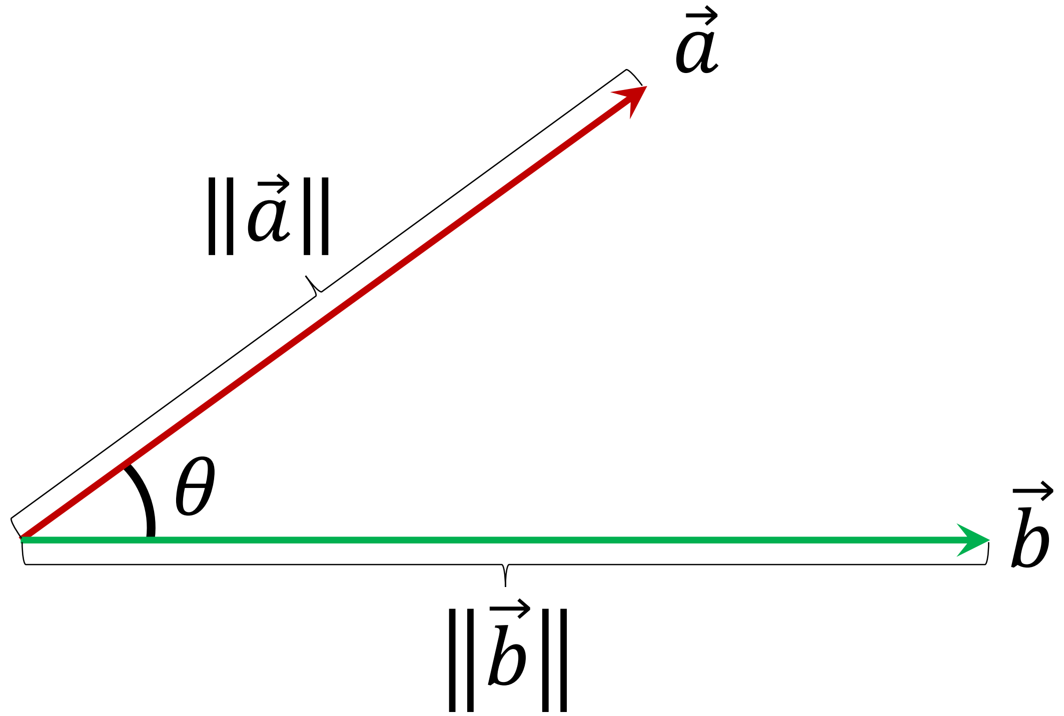
$$\|\vec{a} \times \vec{b}\| = \|\vec{a}\| \|\vec{b}\| \, sin\, \theta$$

위의 내적과 외적에 관한 두 공식을 3차원으로 가져와서 비교해보자

$$\vec{a} \cdot \vec{b} = \|\vec{a}\| \|\vec{b}\| \cos\theta$$



$$\vec{a} \cdot \vec{b} = \|\vec{b}\| \|\vec{a}\| \cos\theta$$

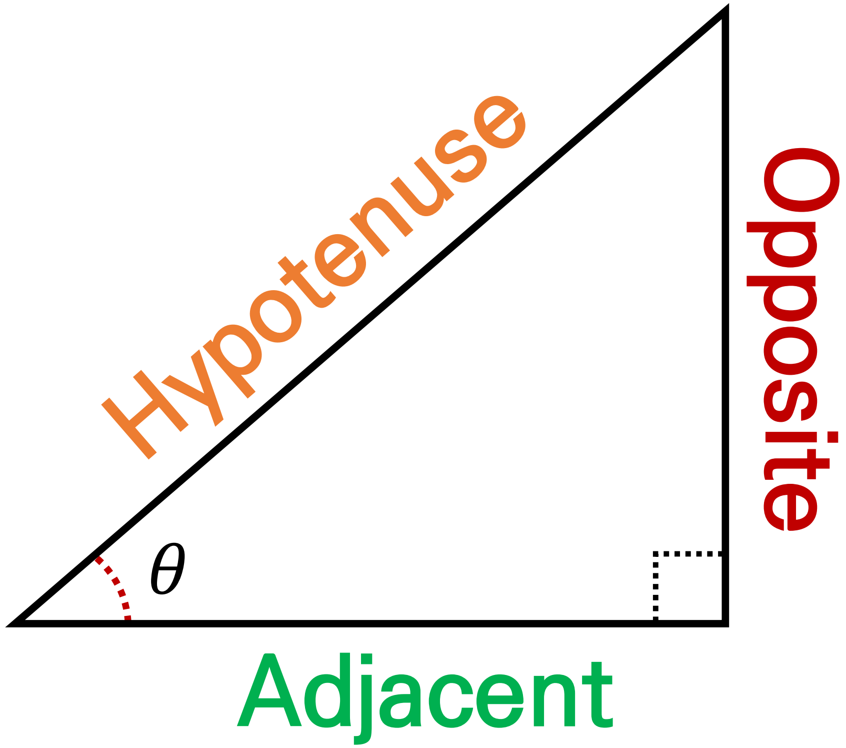


여기서

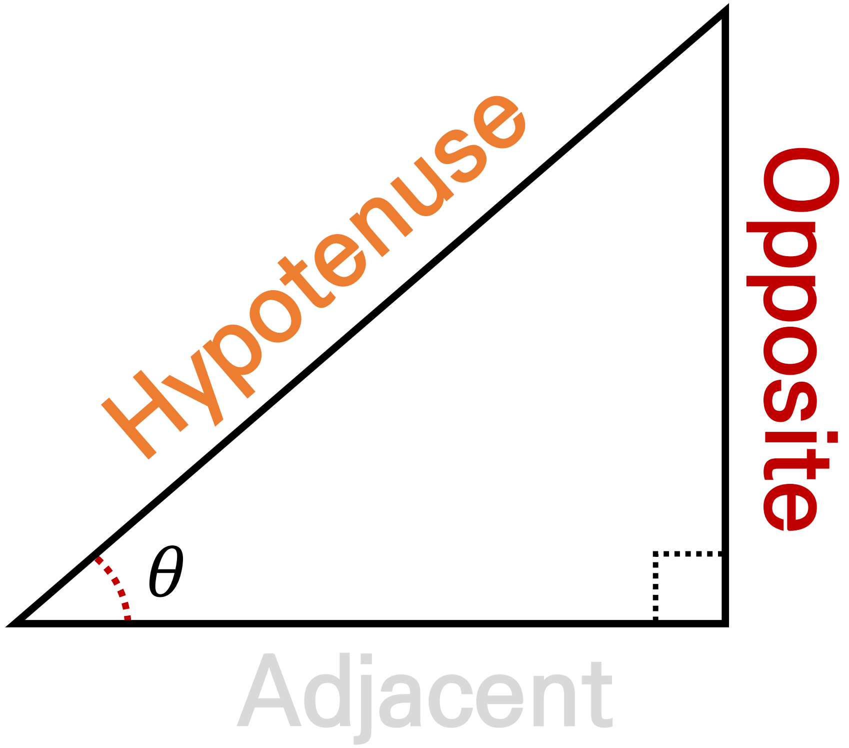
삼각법(Trigonometry)

잠깐 보고 갈게요

SOH CAH TOA

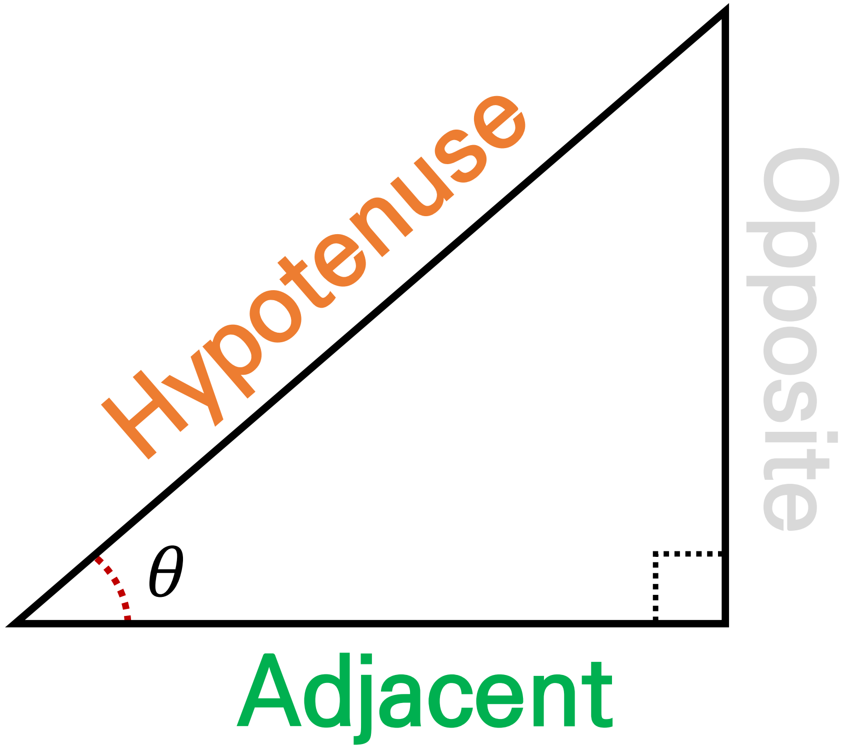


SOH CAH TOA



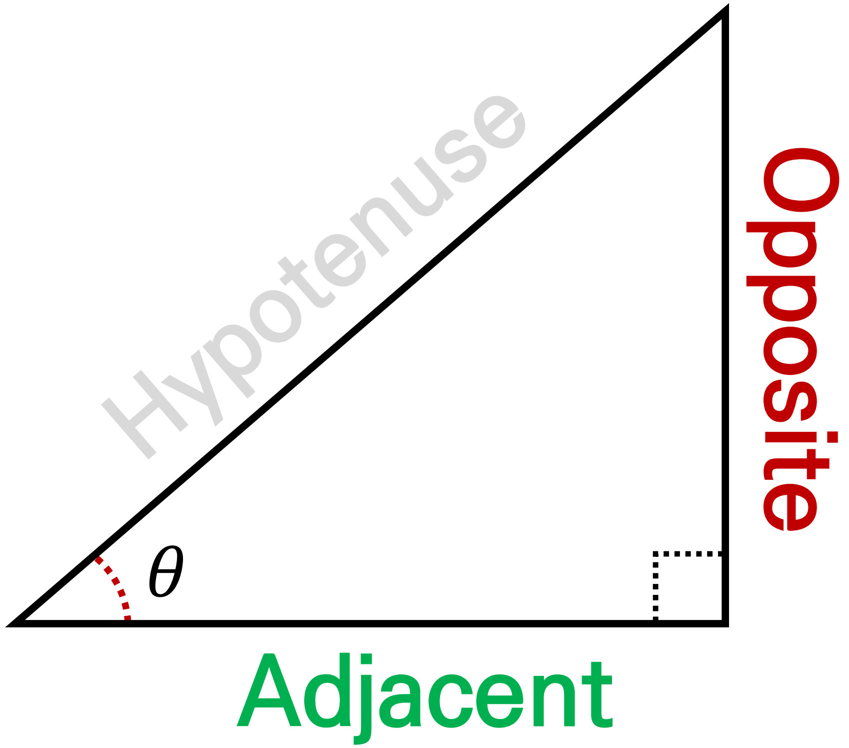
$$\sin\theta = \frac{Opposite}{Hypotenuse}$$

SOH CAH TOA



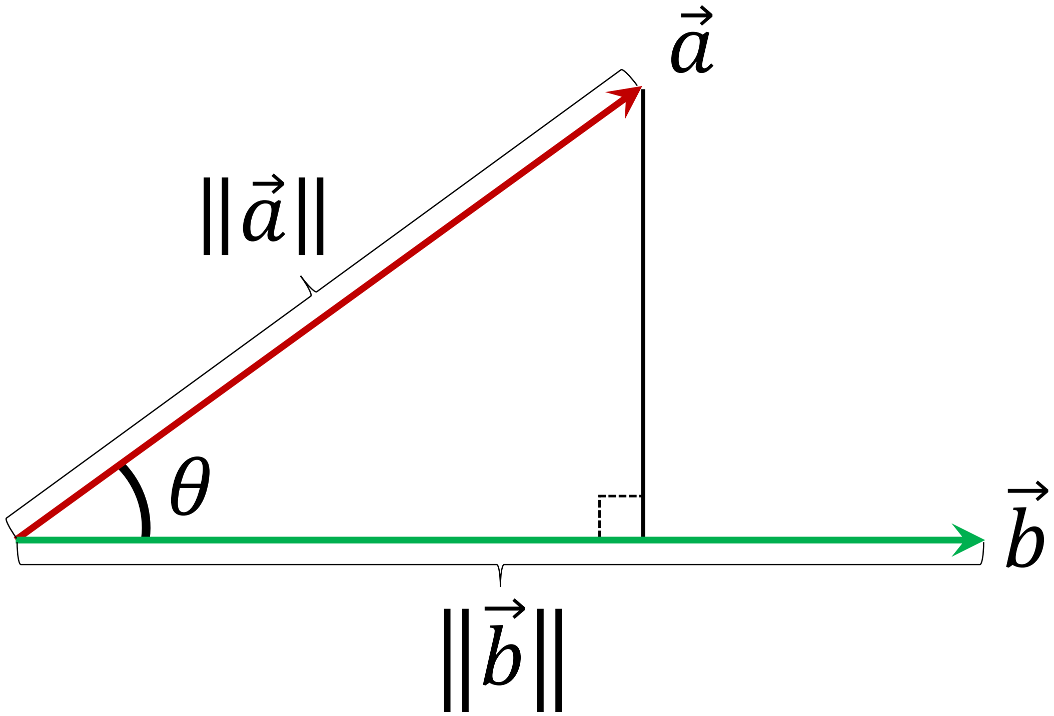
$$\cos\theta = \frac{Adjacent}{Hypotenuse}$$

SOH CAH TOA



$$\tan\theta = \frac{Opposite}{Adjacent}$$

$$\vec{a} \cdot \vec{b} = \|\vec{b}\| \|\vec{a}\| \cos\theta$$

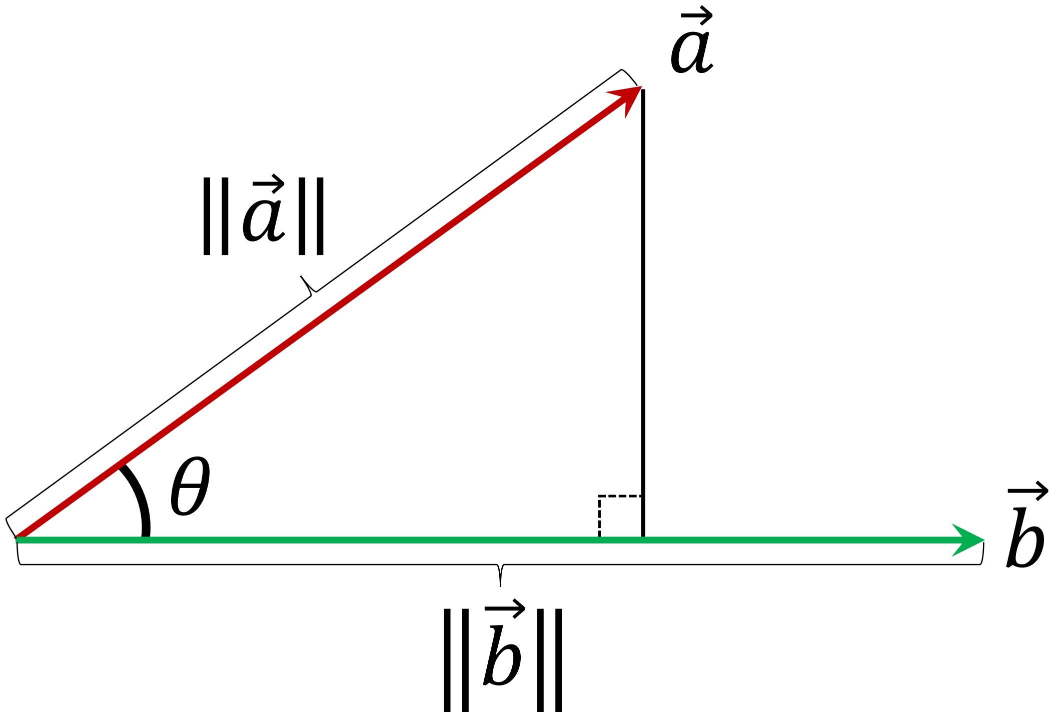


$$cos\theta = \frac{Adjacent}{Hypotenuse}$$

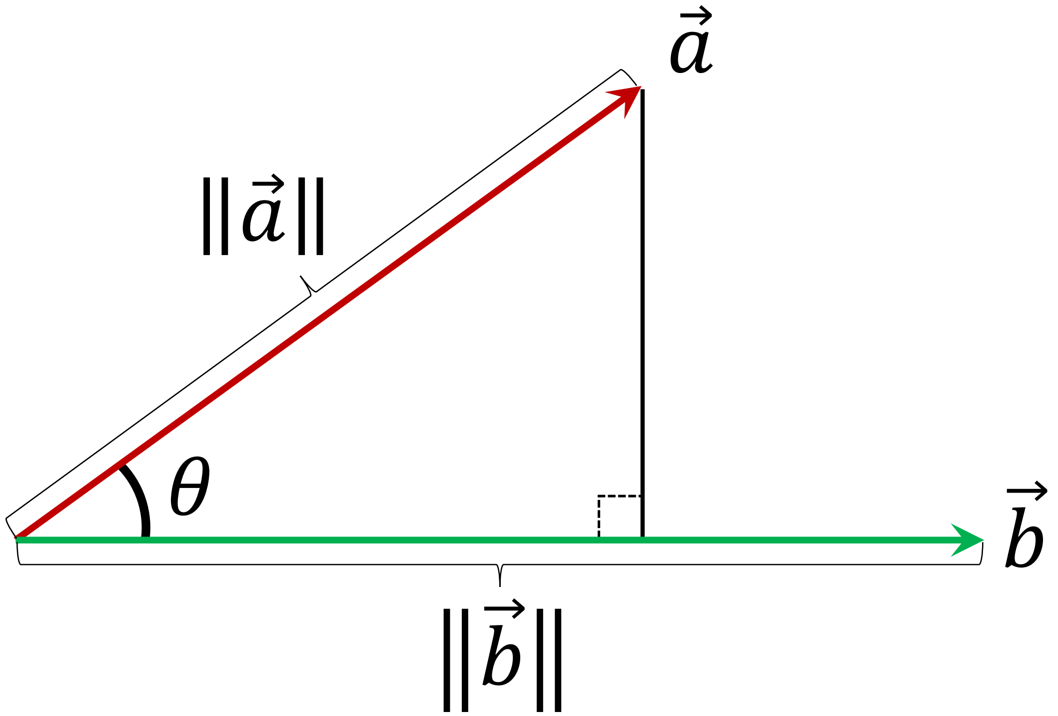
$$\downarrow$$

$$cos\theta = \frac{Adjacent}{\|\vec{a}\|}$$

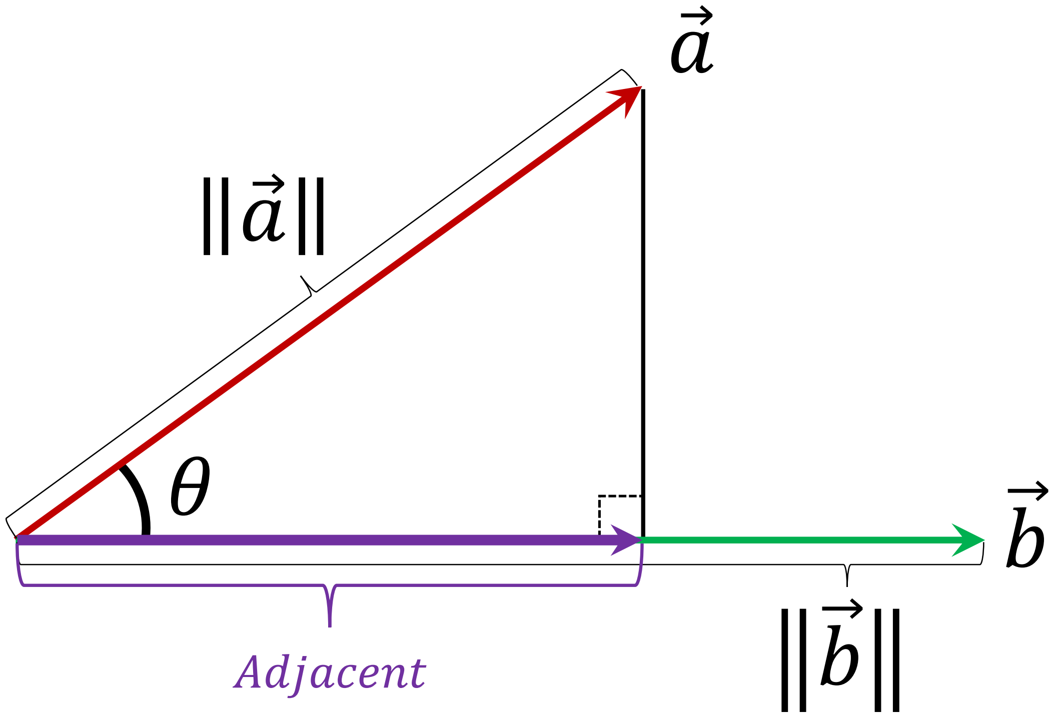
$$\vec{a} \cdot \vec{b} = \|\vec{b}\| \|\vec{a}\| \frac{Adjacent}{\|\vec{a}\|}$$

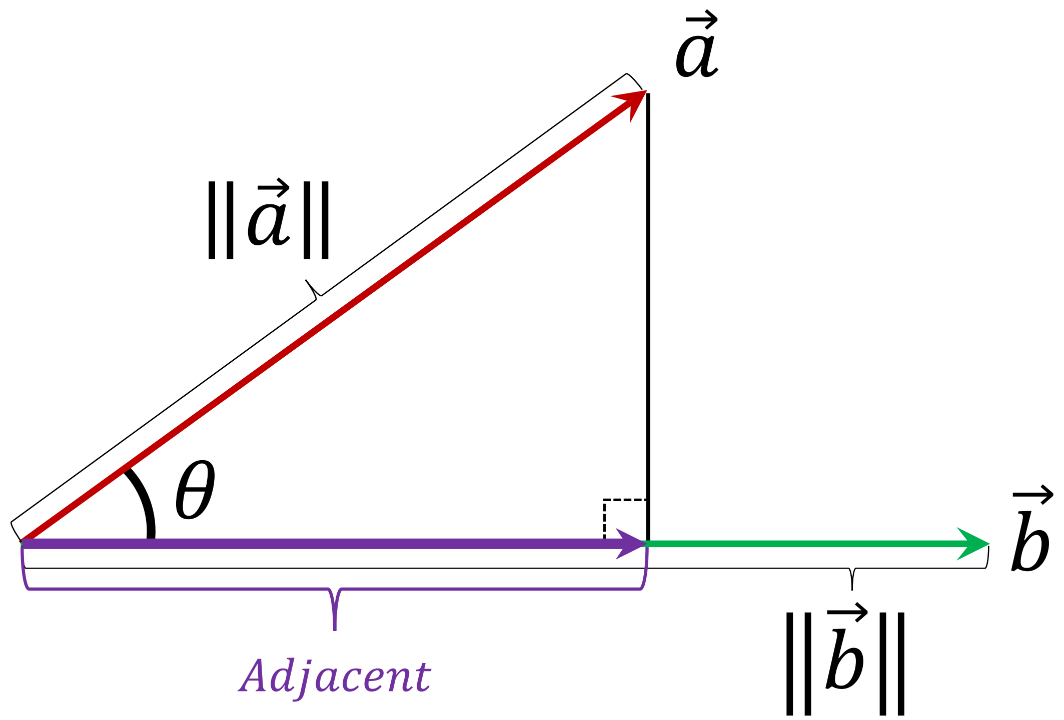


$$\vec{a}\cdot\vec{b} = \|\vec{b}\|\cancel{\|\vec{a}\|}\frac{Adjacent}{\cancel{\|\vec{a}\|}}$$

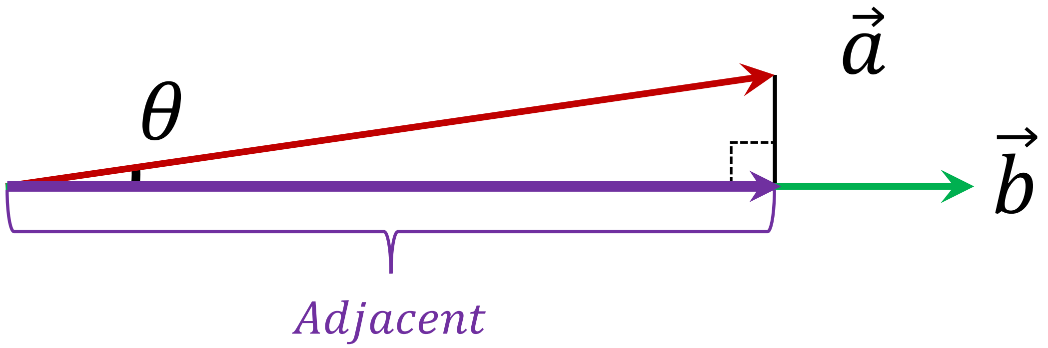


$$\vec{a} \cdot \vec{b} = \|\vec{b}\| Adjacent$$

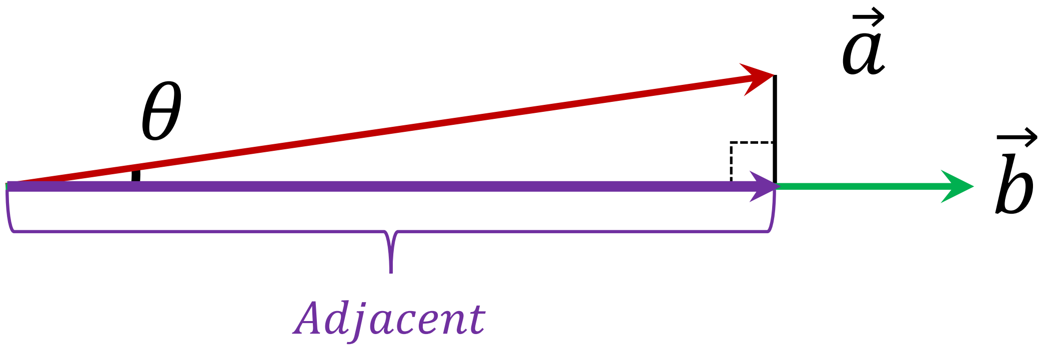




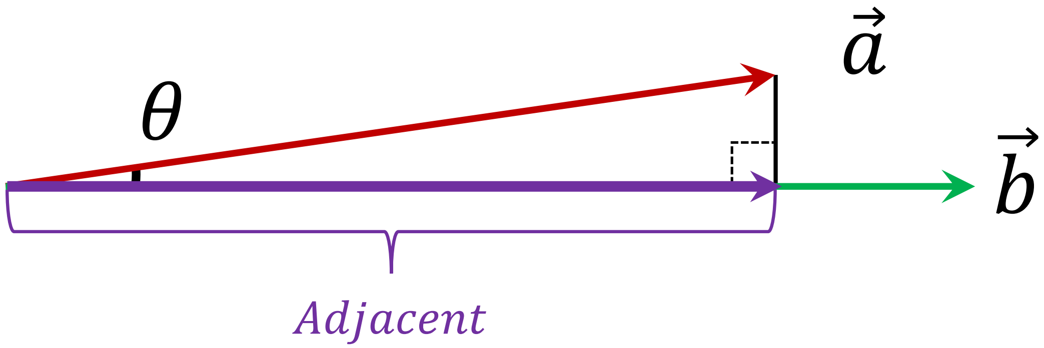
두 벡터의 내적은 두 벡터의 방향이 얼마나 비슷한 지를 나타낸다



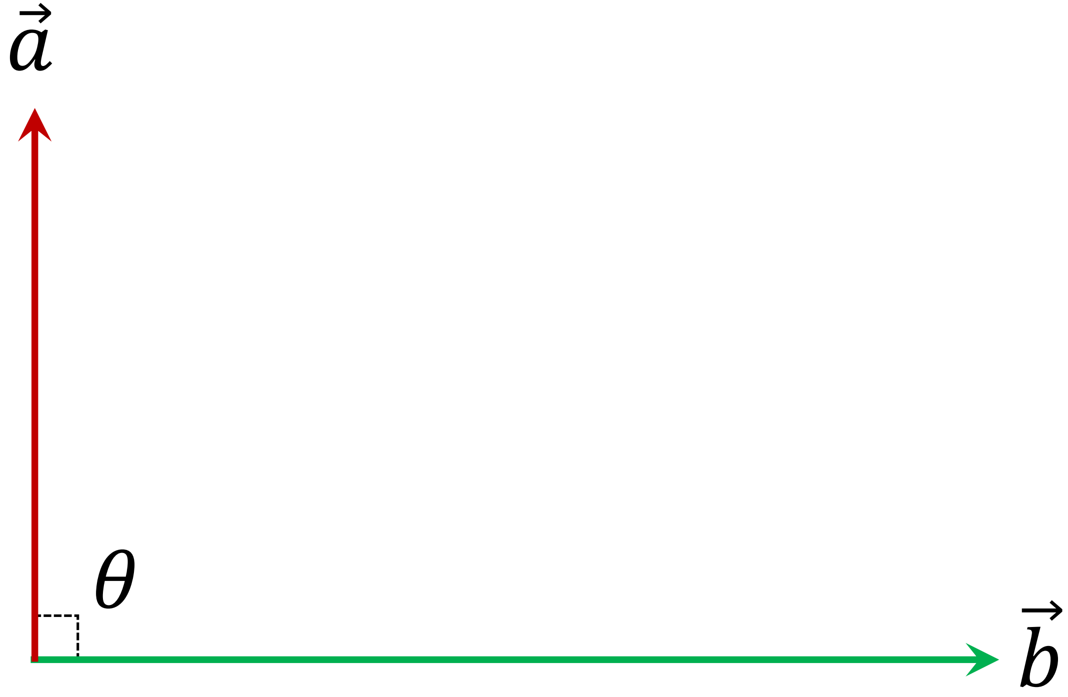
두 벡터의 방향이 비슷할 수록 Adjacent 변의 길이가 길어진다



따라서 두 벡터를 내적한 값 또한 더 커지게 된다

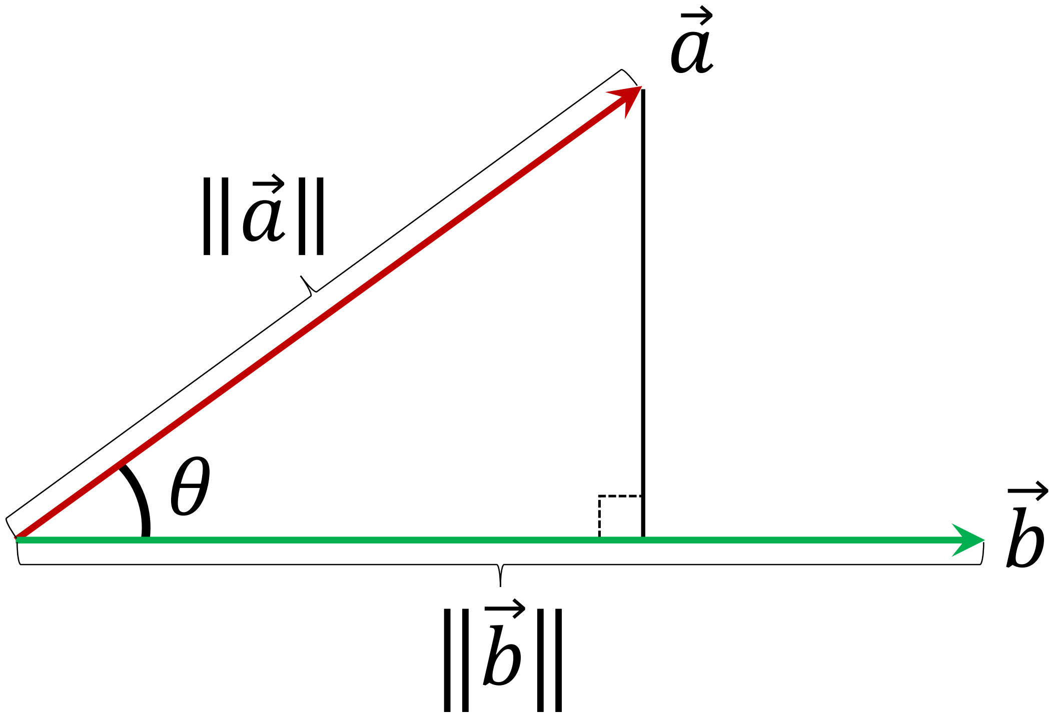


즉, 두 벡터의 방향이 비슷할 수록 내적의 값이 커진다

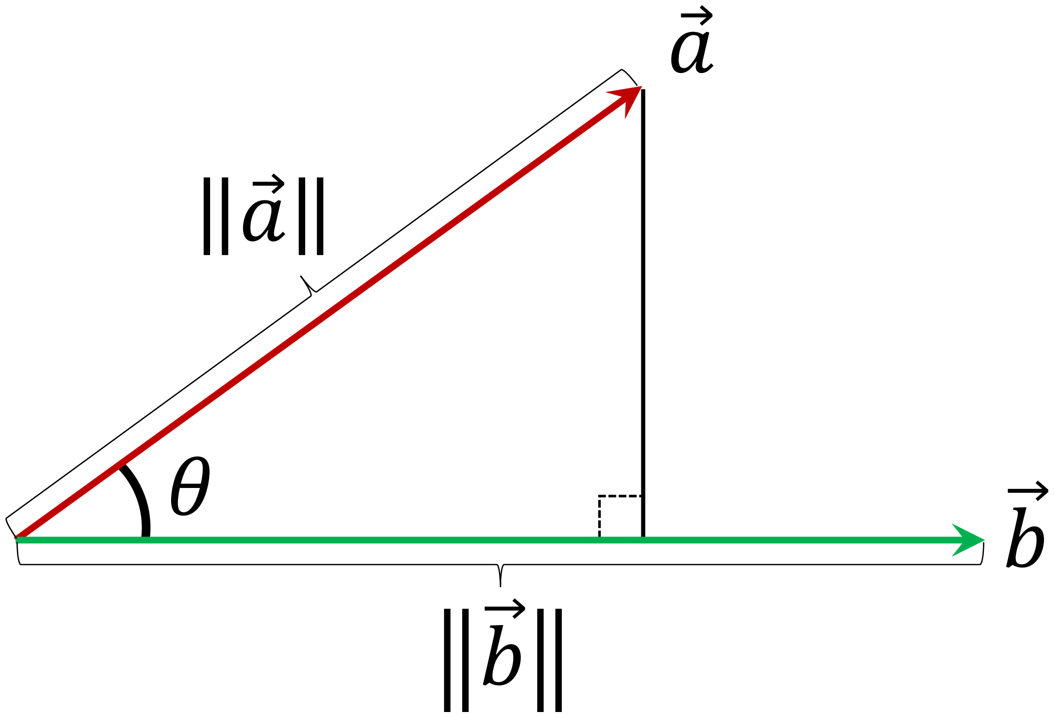


이를 이용해 두 벡터의 내적이 0 이면
두 벡터가 직교한다는 것도 확인할 수 있다

$$\|\vec{a} \times \vec{b}\| = \|\vec{a}\|\|\vec{b}\|\, sin\,\theta$$



$$\|\vec{a}\times\vec{b}\| = \|\vec{b}\|\|\vec{a}\|\sin\theta$$

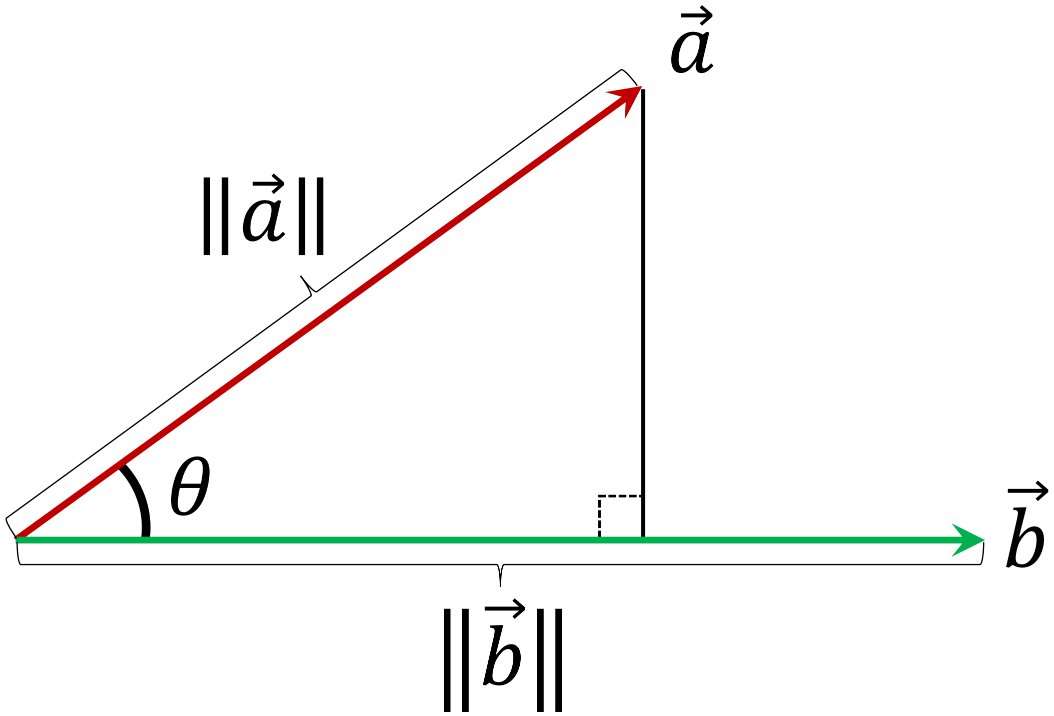


$$\sin\theta = \frac{Opposite}{Hypotenuse}$$

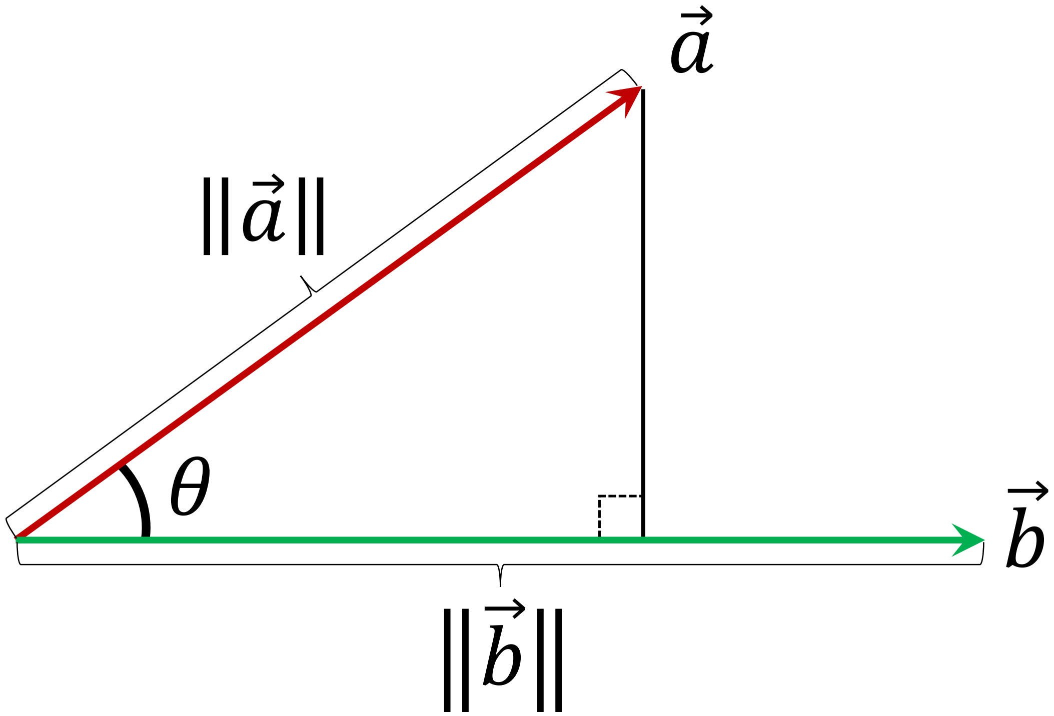
$$\downarrow$$

$$\sin\theta = \frac{Opposite}{\|\vec{a}\|}$$

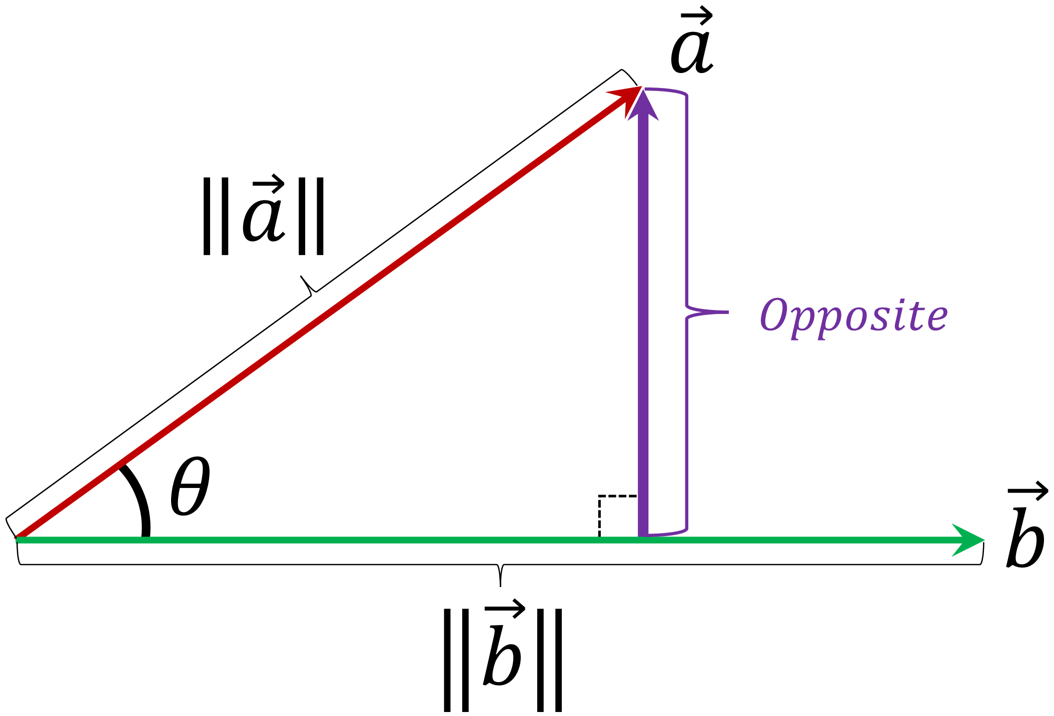
$$\|\vec{a} \times \vec{b}\| = \|\vec{b}\|\|\vec{a}\| \frac{Opposite}{\|\vec{a}\|}$$

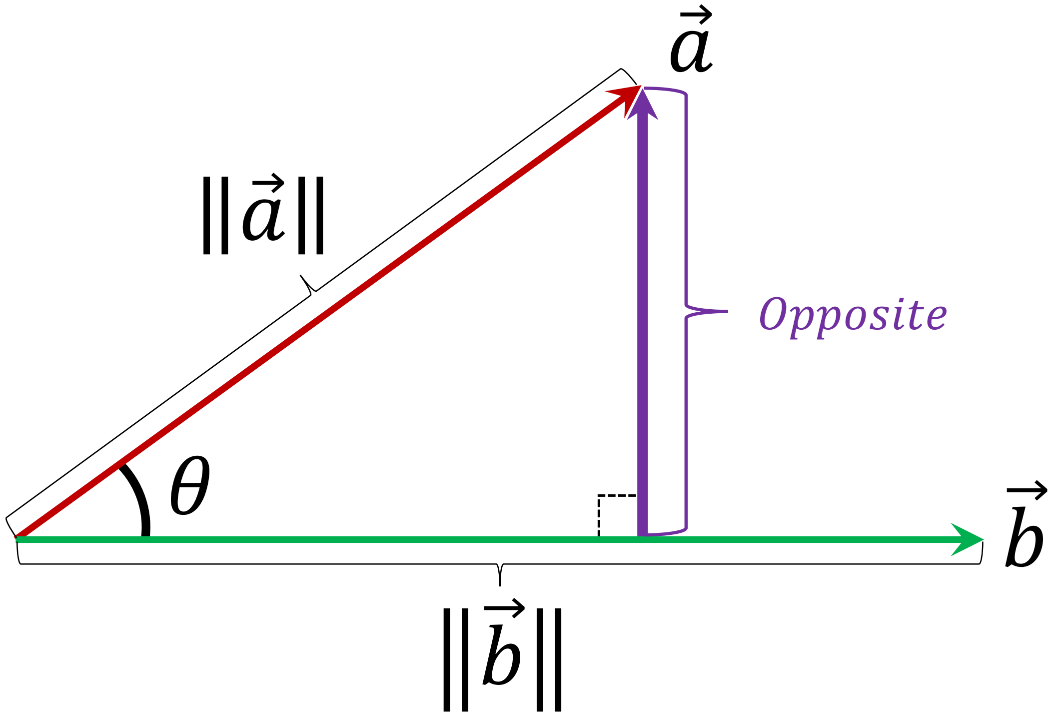


$$\|\vec{a}\times\vec{b}\| = \|\vec{b}\|\cancel{\|\vec{a}\|}\frac{Opposite}{\cancel{\|\vec{a}\|}}$$

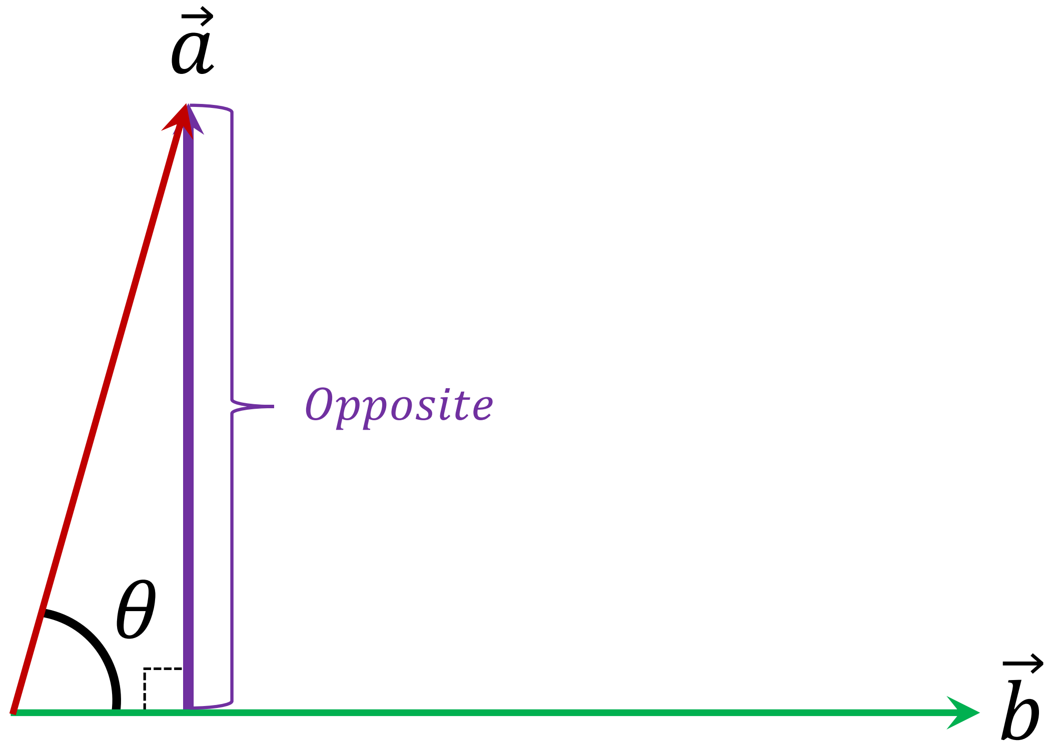


$$\|\vec{a} \times \vec{b}\| = \|\vec{b}\| Opposite$$

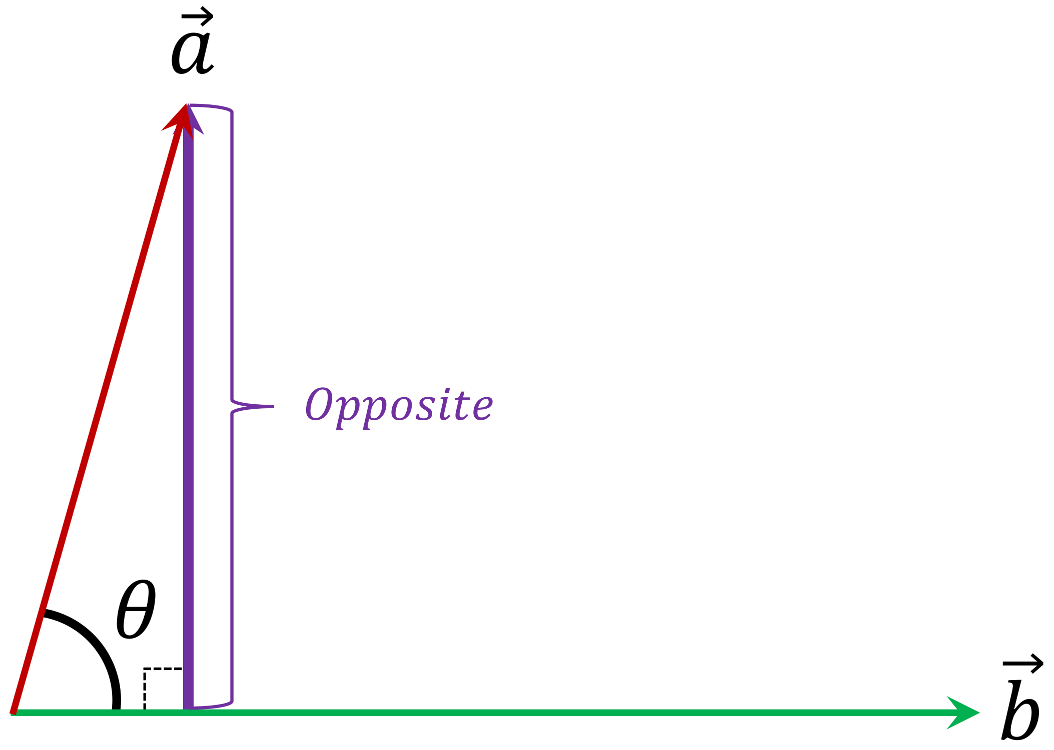




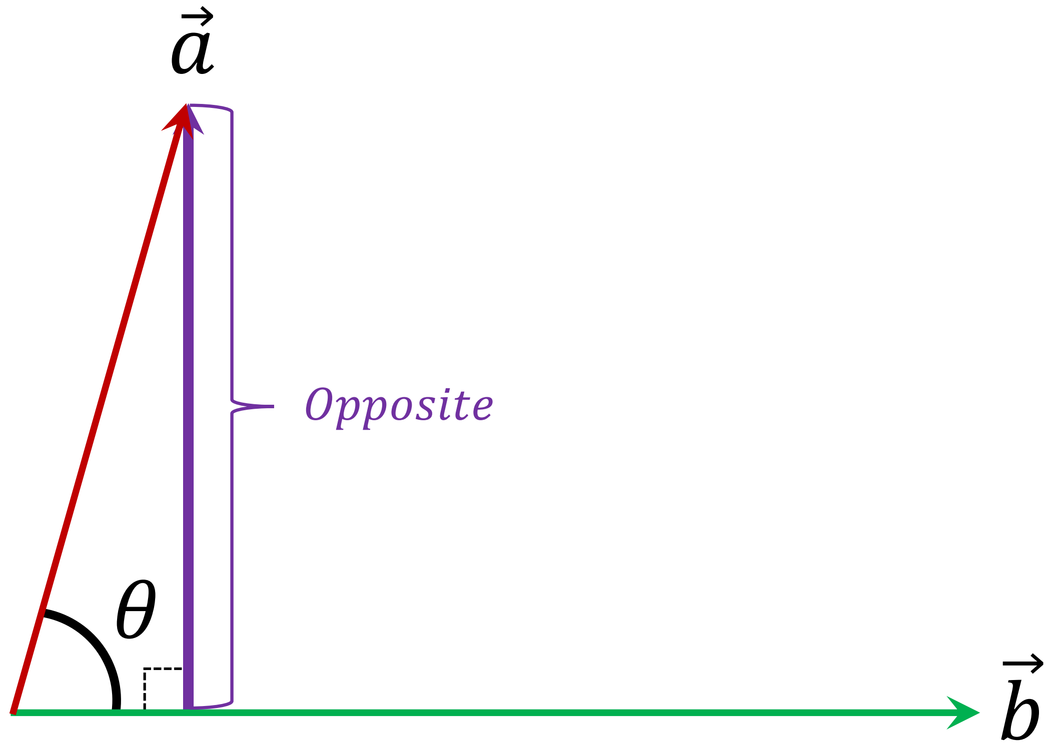
두 벡터의 외적은 두 벡터의 방향이 얼마나 다른 지를 나타낸다



두 벡터의 방향이 다를 수록 Opposite 변의 길이가 길어진다

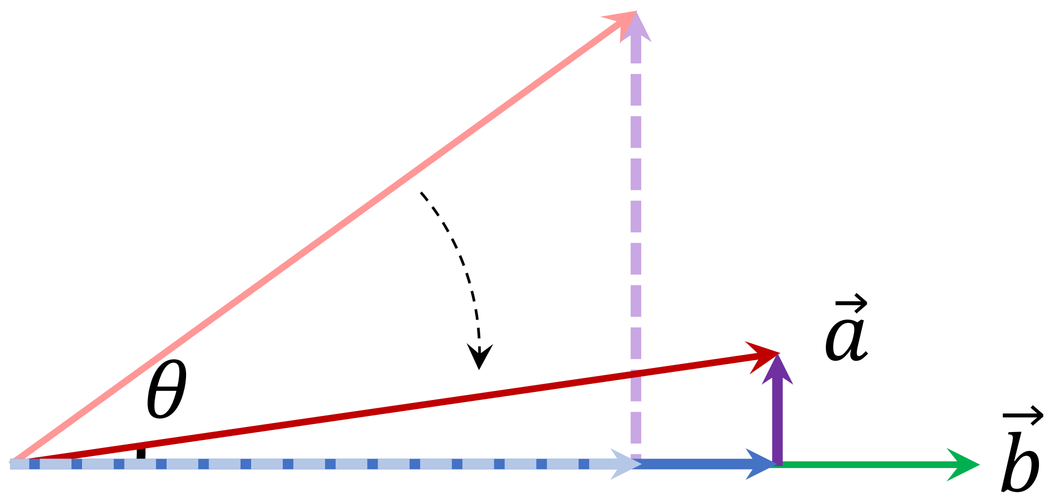


따라서 두 벡터를 외적한 값 또한 더 커지게 된다



즉, 두 벡터의 방향이 다를 수록 외적의 값이 커진다

두 벡터의 방향이 같으면 외적의 값이 0이 된다



내적 ⬆

외적 ⬇

내적

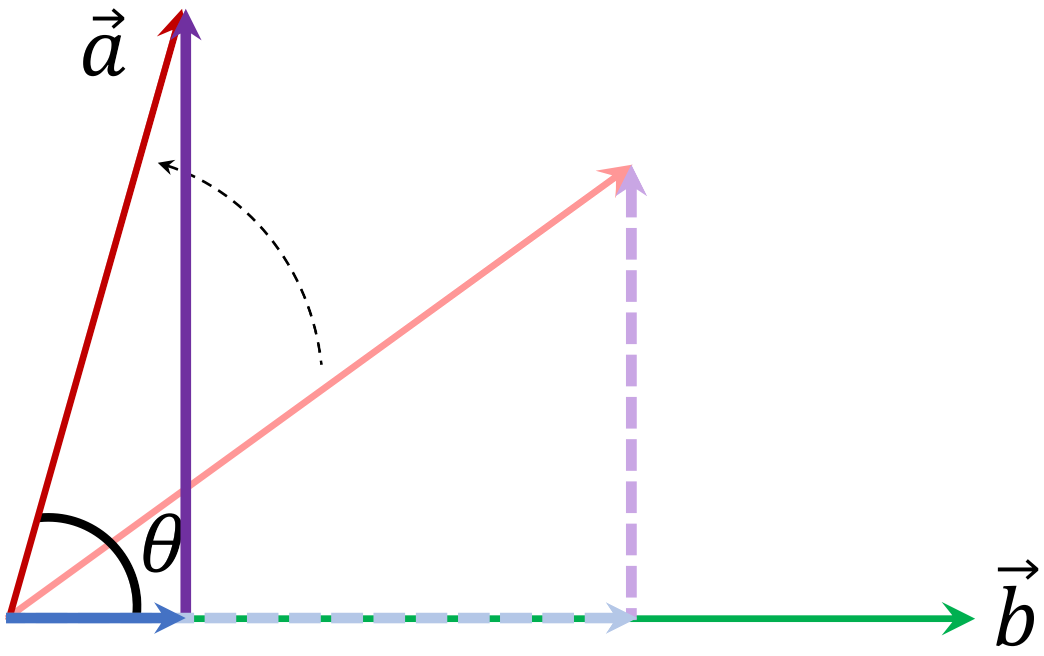
MAX

$$\vec{a} \cdot \vec{b} = \|\vec{a}\|\|\vec{b}\| \times 1$$

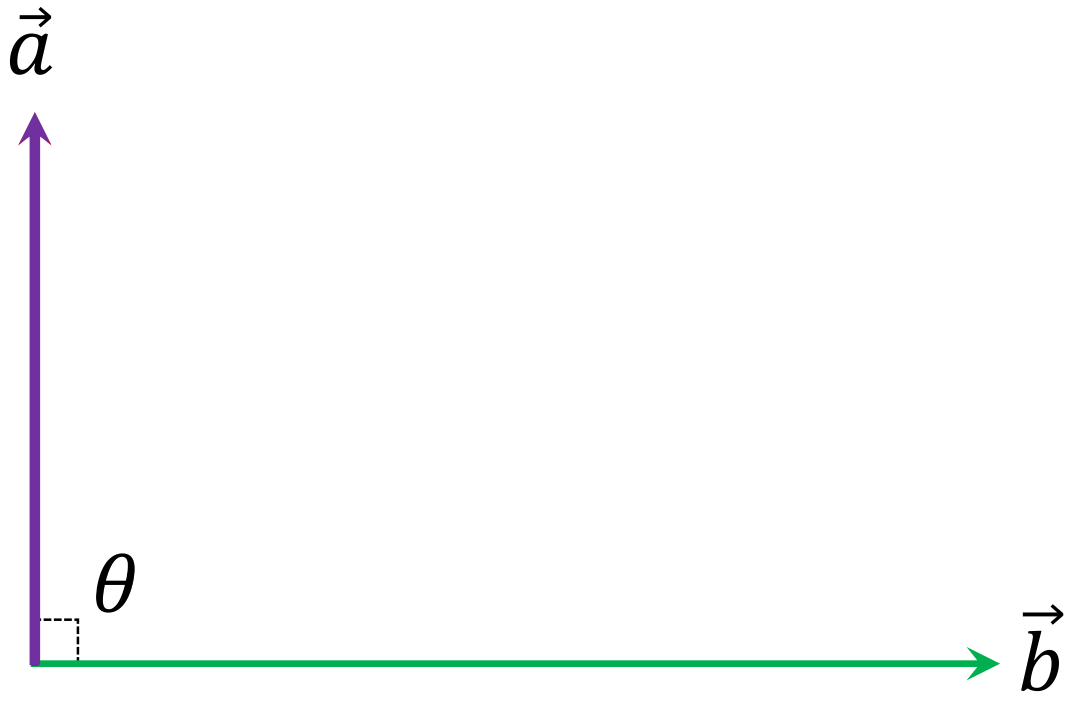
외적

MIN

$$\|\vec{a} \times \vec{b}\| = \|\vec{a}\|\|\vec{b}\| \times 0$$



내적 ⬇ 외적 ⬆



내적

MIN

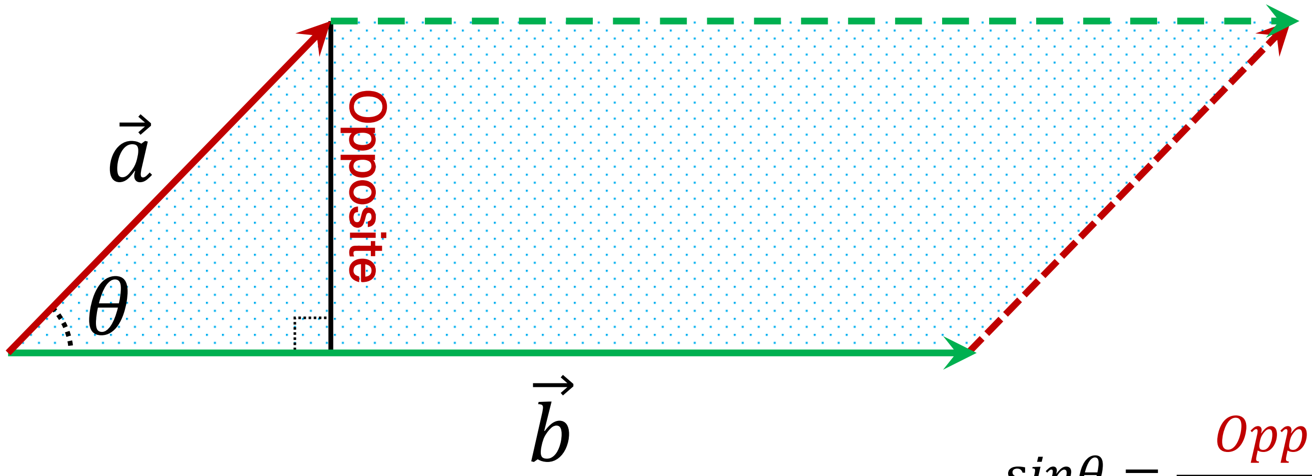
$$\vec{a} \cdot \vec{b} = \|\vec{a}\|\|\vec{b}\| \times 0$$

외적

MAX

$$\|\vec{a} \times \vec{b}\| = \|\vec{a}\|\|\vec{b}\| \times 1$$

외적으로 알 수 있는 것
한가지 더!



$$sin\theta = \frac{Opposite}{Hypotenuse}$$

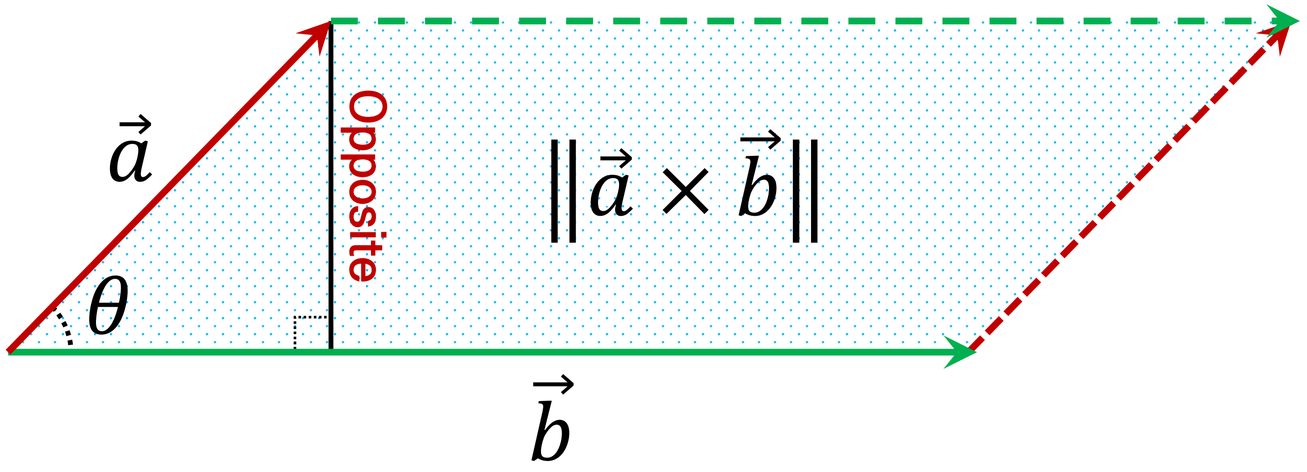
$\downarrow$

$$sin\theta = \frac{Opposite}{\|\vec{a}\|}$$

$\downarrow$

$$Opposite = \|\vec{a}\|sin\theta$$

넓이 = 밑변 길이 X 높이 = $\|\vec{b}\|Opposite$



$$\|\vec{a}\|\|\vec{b}\| sin\theta = \|\vec{a} \times \vec{b}\|$$

4

벡터의 삼중적의 확장

벡터의 삼중적

두 벡터의 벡터곱에 또 다른 벡터를 벡터곱한 것

$$\vec{a} \times (\vec{b} \times \vec{c})$$

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$$

$$\vec{b} = b_x\hat{\imath} + b_y\hat{\jmath} + b_z\hat{k}$$

$$\vec{c} = c_x\hat{\imath} + c_y\hat{\jmath} + c_z\hat{k}$$

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ b_x & b_y \\ c_x & c_y \end{matrix}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ b_x & b_y \\ c_x & c_y \end{matrix}$$

$$\hat{\imath}\left(b_y c_z - b_z c_y\right)$$

$$\left|\begin{array}{ccc} \hat{\imath} & \hat{\jmath} & \hat{k} \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{array}\right| \begin{array}{cc} \hat{\imath} & \hat{\jmath} \\ b_x & b_y \\ c_x & c_y \end{array}$$

$$\hat{\imath}\left(b_y c_z - b_z c_y\right) + \hat{\jmath}\left(b_z c_x - b_x c_z\right)$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} \begin{matrix} \hat{i} & \hat{j} \\ b_x & b_y \\ c_x & c_y \end{matrix}$$

$$\hat{i}\left(b_y c_z - b_z c_y\right) + \hat{j}\left(b_z c_x - b_x c_z\right) + \hat{k}\left(b_x c_y - b_y c_x\right)$$

$$\vec{b} \times \vec{c}$$

$$=$$

$$\hat{\imath}\left(b_y c_z - b_z c_y\right) + \hat{\jmath}(b_z c_x - b_x c_z) + \hat{k}\left(b_x c_y - b_y c_x\right)$$

$$\vec{b} \times \vec{c}$$

$$=$$

$$\hat{\imath}\left(b_y c_z - b_z c_y\right) + \hat{\jmath}\left(b_z c_x - b_x c_z\right) + \hat{k}\left(b_x c_y - b_y c_x\right)$$

$$\vec{a} \times (\vec{b} \times \vec{c})$$
$$=$$
$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_x & a_y & a_z \\ b_y c_z - b_z c_y & b_z c_x - b_x c_z & b_x c_y - b_y c_x \end{vmatrix}$$

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_x & a_y & a_z \\ b_y c_z - b_z c_y & b_z c_x - b_x c_z & b_x c_y - b_y c_x \end{vmatrix} \begin{matrix} \hat{\imath} & \hat{\jmath} \\ a_x & a_y \\ b_y c_z - b_z c_y & b_z c_x - b_x c_z \end{matrix}$$

$$
\left|\begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_y c_z - b_z c_y & b_z c_x - b_x c_z & b_x c_y - b_y c_x \end{array}\right| \begin{array}{cc} \hat{i} & \hat{j} \\ a_x & a_y \\ b_y c_z - b_z c_y & b_z c_x - b_x c_z \end{array}
$$

$$
\hat{i}(a_y b_x c_y - a_y b_y c_x - a_z b_z c_x + a_z b_x c_z)
$$

$$\left|\begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_yc_z - b_zc_y & b_zc_x - b_xc_z & b_xc_y - b_yc_x \end{array}\right| \begin{array}{cc} \hat{i} & \hat{j} \\ a_x & a_y \\ b_yc_z - b_zc_y & b_zc_x - b_xc_z \end{array}$$

$$\hat{j}(a_zb_yc_z - a_zb_zc_y - a_xb_xc_y + a_xb_yc_x)$$

$$\left|\begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_y c_z - b_z c_y & b_z c_x - b_x c_z & b_x c_y - b_y c_x \end{array}\right| \begin{array}{cc} \hat{i} & \hat{j} \\ a_x & a_y \\ b_y c_z - b_z c_y & b_z c_x - b_x c_z \end{array}$$

$$\hat{k}(a_x b_z c_x - a_x b_x c_z - a_y b_y c_z + a_y b_z c_y)$$

다시 처음 계산으로 돌아와서...

$$\left|\begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_y c_z - b_z c_y & b_z c_x - b_x c_z & b_x c_y - b_y c_x \end{array}\right| \begin{array}{cc} \hat{i} & \hat{j} \\ a_x & a_y \\ b_y c_z - b_z c_y & b_z c_x - b_x c_z \end{array}$$

$$\hat{\imath}(a_y b_x c_y - a_y b_y c_x - a_z b_z c_x + a_z b_x c_z)$$

더미변수 추가

$$\left|\begin{array}{ccc} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_x & a_y & a_z \\ b_y c_z - b_z c_y & b_z c_x - b_x c_z & b_x c_y - b_y c_x \end{array}\right| \begin{array}{cc} \hat{\imath} & \hat{\jmath} \\ a_x & a_y \\ b_y c_z - b_z c_y & b_z c_x - b_x c_z \end{array}$$

$$\hat{\imath}(a_y b_x c_y - a_y b_y c_x - a_z b_z c_x + a_z b_x c_z + a_x b_x c_x - a_x b_x c_x)$$

$$\hat{\imath}(a_y b_x c_y - a_y b_y c_x - a_z b_z c_x + a_z b_x c_z + a_x b_x c_x - a_x b_x c_x)$$

$$\downarrow$$

$$b_x(a_x c_x + a_y c_y + a_z c_z) - a_y b_y c_x - a_z b_z c_x - a_x b_x c_x$$

$$b_x(a_xc_x + a_yc_y + a_zc_z) - a_yb_yc_x - a_zb_zc_x - a_xb_xc_x$$

$$\downarrow$$

$$b_x(a_xc_x + a_yc_y + a_zc_z) - c_x(a_xb_x + a_yb_y + a_zb_z)$$

$$\underbrace{b_x(a_xc_x + a_yc_y + a_zc_z)}_{\vec{a}\cdot\vec{c}} - \underbrace{c_x(a_xb_x + a_yb_y + a_zb_z)}_{\vec{a}\cdot\vec{b}}$$

$$\downarrow$$

$$\hat{\imath}(b_x(\vec{a}\cdot\vec{c}) - c_x(\vec{a}\cdot\vec{b}))$$

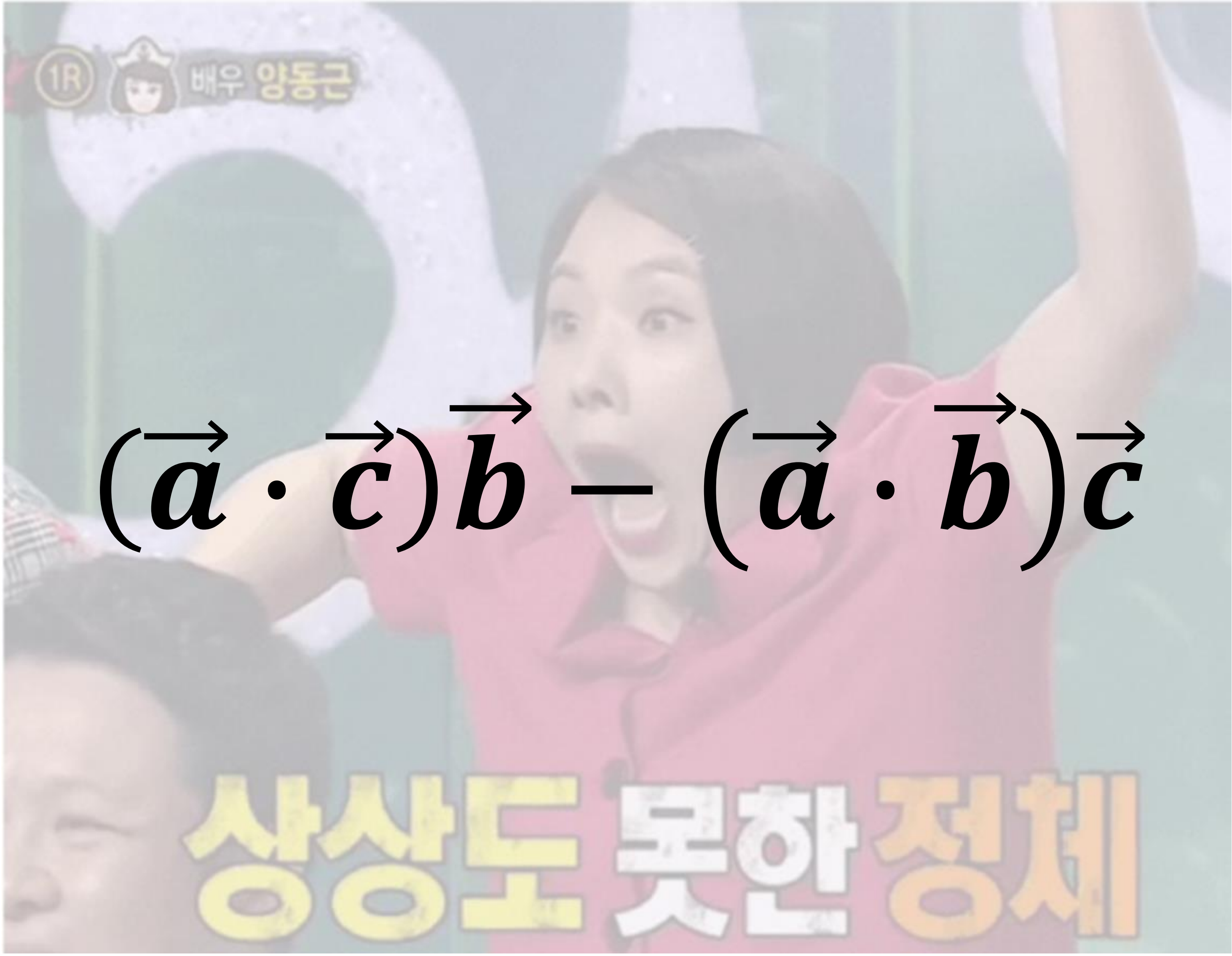
나머지도 똑같은 방법으로 계산해주면...

$$\hat{\imath}(b_x(\vec{a}\cdot\vec{c}) - c_x(\vec{a}\cdot\vec{b}))$$
$$+\,\hat{\jmath}(b_y(\vec{a}\cdot\vec{c}) - c_y(\vec{a}\cdot\vec{b}))$$
$$+\,\hat{k}(b_z(\vec{a}\cdot\vec{c}) - c_z(\vec{a}\cdot\vec{b}))$$

$$
\begin{aligned}
&b_x\hat{\imath}(\vec{a}\cdot\vec{c}) - c_x\hat{\imath}(\vec{a}\cdot\vec{b}) \\
&+ b_y\hat{\jmath}(\vec{a}\cdot\vec{c}) - c_y\hat{\jmath}(\vec{a}\cdot\vec{b}) \\
&+ b_z\hat{k}(\vec{a}\cdot\vec{c}) - c_z\hat{k}(\vec{a}\cdot\vec{b})
\end{aligned}
$$

$$(\vec{a}\cdot\vec{c})\left(b_x\hat{\imath}+\ b_y\hat{\jmath}+\ b_z\hat{k}\right)-\left(\vec{a}\cdot\vec{b}\right)(c_x\hat{\imath}+c_y\hat{\jmath}+c_z\hat{k})$$

$$(\vec{a}\cdot\vec{c})\underbrace{\left(b_x\hat{\imath} + \ b_y\hat{\jmath} + \ b_z\hat{k}\right)}_{\vec{b}} - \left(\vec{a}\cdot\vec{b}\right)\underbrace{(c_x\hat{\imath} + c_y\hat{\jmath} + c_z\hat{k})}_{\vec{c}}$$



$$(\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$$

벡터의 삼중적 공식
쉽게 외우기

$$\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}\left(\vec{a} \cdot \vec{b}\right)$$

$$\overset{3}{\vec{a}} \times \left(\overset{1}{\vec{b}} \times \overset{2}{\vec{c}}\right) =$$

$$\overset{3}{\vec{a}} \times (\overset{1}{\vec{b}} \times \overset{2}{\vec{c}}) = \overset{1}{\vec{b}}$$

1번 곱하기

$$\overset{3}{\vec{a}} \times (\overset{1}{\vec{b}} \times \overset{2}{\vec{c}}) = \overset{1}{\vec{b}}(\overset{3}{\vec{a}} \cdot \overset{2}{\vec{c}})$$

1번 곱하기 나머지 두개의 내적

$$\overset{3}{\vec{a}} \times \left(\overset{1}{\vec{b}} \times \overset{2}{\vec{c}}\right) = \overset{1}{\vec{b}}\left(\overset{3}{\vec{a}} \cdot \overset{2}{\vec{c}}\right) -$$

빼기

$$\overset{3}{\vec{a}} \times \left(\overset{1}{\vec{b}} \times \overset{2}{\vec{c}}\right) = \overset{1}{\vec{b}}\left(\overset{3}{\vec{a}} \cdot \overset{2}{\vec{c}}\right) - \overset{2}{\vec{c}}$$

2번 곱하기

$$\overset{3}{\vec{a}} \times (\overset{1}{\vec{b}} \times \overset{2}{\vec{c}}) = \overset{1}{\vec{b}}(\overset{3}{\vec{a}} \cdot \overset{2}{\vec{c}}) - \overset{2}{\vec{c}}(\overset{3}{\vec{a}} \cdot \overset{1}{\vec{b}})$$

2번 곱하기 나머지 두개의 내적

To Be Continued...