MATH 308 Assignment 16: Exercises 6.4

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We know that $X \sim \mathcal{B}(n,p) \implies f_X(x) = \binom{n}{x} p^x (1-p)^{n-x}$. Thus, the probability of obtaining the value X can be written as a function of the parameter p:

$$f(p) = \binom{n}{X} p^X (1-p)^{n-X}$$

$$\begin{aligned} \implies f' / \binom{n}{X} &= X p^{X-1}(1-p)^{n-X} \\ &\quad - p^X (n-X)(1-p)^{n-X-1} \end{aligned}$$

We can then obtain the MLE $\hat{p}$ by setting $f' = 0$:

$$\begin{aligned} X p^{X-1}(1-p)^{n-X} &= p^X (n-X)(1-p)^{n-X-1} \\ \implies \hat{p}(n-X) &= X(1-\hat{p}) \\ \implies \hat{p}n - \cancel{\hat{p}X} &= X - \cancel{X\hat{p}} \\ \implies \hat{p} &= X/n \quad \square \end{aligned}$$

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$X \sim \mathcal{P}(\lambda) \implies f_X = \frac{\lambda^x e^{-\lambda}}{x!}$. So, the likelihood of obtaining the sample $\{x_1, x_2, \dots x_n\}$ is $L(\lambda) = \prod_{i=1}^n f_X(x_i)$, since the elements of the sample are i.i.d.

$$\begin{aligned} L &= \prod_{i=1}^{n} \frac{\lambda^{x_i} e^{-\lambda}}{x_i!} \\ &= \frac{\lambda^{\Sigma x_i} e^{-n\lambda}}{\prod(x_i!)} \end{aligned}$$

$$\begin{aligned} \implies L' / \prod(x_i!) &= \left(\lambda^{\Sigma x_i} e^{-n\lambda}\right)' \\ &= \Sigma x_i \lambda^{\Sigma x_i - 1} e^{-n\lambda} + \lambda^{\Sigma x_i} e^{-n\lambda}(-n) \\ &= e^{-n\lambda} \lambda^{\Sigma x_i - 1} \left(\Sigma x_i - n\lambda\right) \end{aligned}$$

Setting $L' = 0$, we get $\hat{\lambda}$:

$$\begin{aligned} 0 &= \Sigma x_i - n\hat{\lambda} \\ \implies \hat{\lambda} &= \Sigma x_i / n = \overline{x} \quad \square \end{aligned}$$

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