MATH 308 Assignment 15: Exercises 5.10

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a)

Suppose that cookie orders are represented such that, for example, an order of 1 sugar, 2 chocolate chips, 0 oatmeal, 0 peanut butter, and 2 ginger snaps is given by $|c|cc||cc|$. Then, two of the bars are fixed, giving the string $|???????|$, where the nine ? symbols are to be replaced by five c's and two $|$'s. If we choose 5 distinct positions in which to place the c's, the positions of the remaining $|$'s is fixed. Thus, the number of possible cookie orders is $\binom{9}{5}$.

b)

More generally, we can use 'cookie types' to represent occurances of particular elements in the chosen sets. Then, fixing the position of two $|$'s leaves $n+1-2 = n-1$ $|$'s and n c's, for a total of $2n-1$ symbols. Picking element members requires n position choices, giving the total number of set choices as $\binom{2n-1}{n}$.

c)

For a sample S of size n, each bootstrap sample represents the choice of n elements from S with replacement. Thus, the number of bootstrap samples is as above $\binom{2n-1}{n}$.

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a)

The number of bootstrap samples with k_1 a_1's, k_2 a_2's $\ldots k_n$ a_n's is the same as the number of permutations of the string

$$\underbrace{\overbrace{a_1a_1\ldots a_1}^{k_1\text{times}}\overbrace{a_2a_2\ldots a_2}^{k_2\text{times}}\ldots\overbrace{a_na_n\ldots a_n}^{k_n\text{times}}}_{n\text{ symbols}}$$

Using the formula for string permuations with repeated characters, this yields

$$\frac{n!}{k_1!k_2!\ldots k_n!}$$

which, by definition, equals

$$\binom{n}{k_1, k_2 \ldots k_n} \quad \square$$

b)

A bootstrap sample with k_i a_i's requires choosing k_i of n positions, after which the remaining $n-k_i$ positions can be filled by any of the remaining $n-1$ elements. Thus, the number of such samples is:

$$(n-1)^{n-k_i}\binom{n}{k_i}$$

The total number of bootstrap samples is simply n^n. Thus, the required probability is

$$\frac{(n-1)^{n-k_i}}{n^n}\binom{n}{k_i}$$

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