MATH 308 Assignment 20: Power Function

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The null is rejected if $\overline{X} > \mu_0 + \frac{z_{1-\alpha}}{\sqrt{n}} \equiv C$. But, $\overline{X} \sim \mathcal{N}\left(\mu, 1^2/n\right) \implies \overline{X} = \mu + \frac{Z}{\sqrt{n}}$

$$\begin{aligned}
\therefore \beta(\mu) &= P(\overline{X} > C) \\
&= 1 - P(\overline{X} \leq C) \\
&= 1 - P\left(\mu + \frac{Z}{\sqrt{n}} \leq C\right) \\
&= 1 - P(Z \leq (C-\mu)\sqrt{n}) \\
&= 1 - \Phi((C-\mu)\sqrt{n}) \\
&= 1 - \Phi(z_{1-\alpha} + (\mu_0 - \mu)\sqrt{n})
\end{aligned}$$

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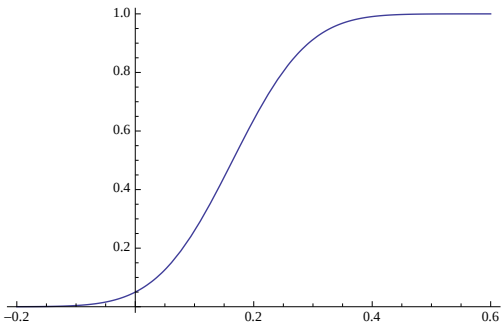
Setting $\alpha = 0.05, \mu_0 = 0$ and $n = 100$, we get

$$\beta(\mu) = 1 - \Phi(z_{0.95} - 10\mu)$$



3

$$\begin{aligned}
0.8 &= 1 - \Phi(z_{0.95} + (0-1)\sqrt{n}) \\
&\implies z_{0.2} = z_{0.95} - \sqrt{n} \\
&\implies n = (z_{0.95} - z_{0.2})^2 \approx 7
\end{aligned}$$