MATH 308 Assignment 15:
Exercises 5.10

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5

a)

Suppose that cookie orders are represented such that, for example, an order of 1 sugar, 2 chocolate chips, 0 oatmeal, 0 peanut butter, and 2 ginger snaps is given by $|c|cc||cc|$. Then, two of the bars are fixed, giving the string $|???????|$, where the nine ? symbols are to be replaced by five c's and two $|$'s. If we choose 5 distinct positions in which to place the c's, the positions of the remaining $|$'s is fixed. Thus, the number of possible cookie orders is $\binom{9}{5}$.

b)

More generally, we can use 'cookie types' to represent occurances of particular elements in the chosen sets. Then, fixing the position of two $|$'s leaves $n+1-2 = n-1$ $|$'s and n c's, for a total of $2n-1$ symbols. Picking element members requires n position choices, giving the total number of set choices as $\binom{2n-1}{n}$.

c)

For a sample S of size n, each bootstrap sample represents the choice of n elements from S with replacement. Thus, the number of bootstrap samples is as above $\binom{2n-1}{n}$.

6

a)

The number of bootstrap samples with k_1 a_1's, k_2 a_2's $\ldots k_n$ a_n's is the same as the number of permutations of the string

$$\underbrace{\overbrace{a_1a_1\ldots a_1}^{k_1\text{times}}\overbrace{a_2a_2\ldots a_2}^{k_2\text{times}}\ldots\overbrace{a_na_n\ldots a_n}^{k_n\text{times}}}_{n\text{ symbols}}$$

Using the formula for string permuations with repeated characters, this yields

$$\frac{n!}{k_1!k_2!\ldots k_n!}$$

which, by definition, equals

$$\binom{n}{k_1, k_2 \ldots k_n} \quad \square$$

b)

A bootstrap sample with k_i a_i's requires choosing k_i of n positions, after which the remaining $n-k_i$ positions can be filled by any of the remaining $n-1$ elements. Thus, the number of such samples is:

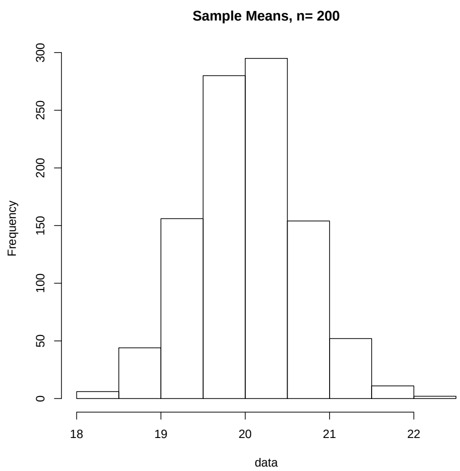
$$(n-1)^{n-k_i}\binom{n}{k_i}$$

The total number of bootstrap samples is simply n^n. Thus, the required probability is

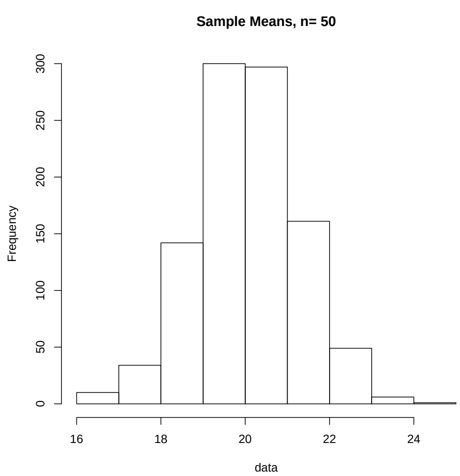
$$\frac{(n-1)^{n-k_i}}{n^n}\binom{n}{k_i}$$

8

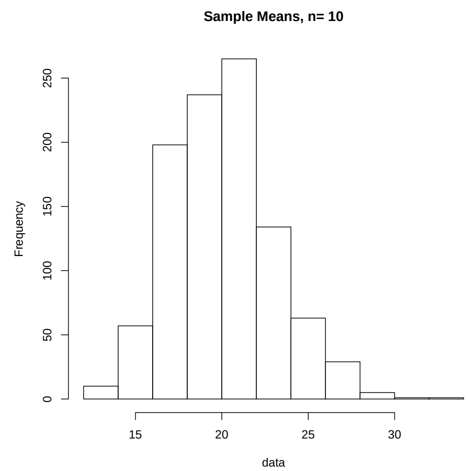
a)



Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
18.21	19.58	20.02	20.02	20.44	22.49



Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
16.19	19.26	20.03	20.03	20.87	24.43



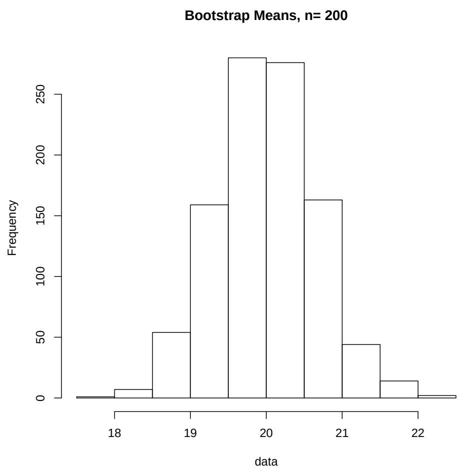
Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
12.56	17.87	19.99	20.05	21.89	33.20

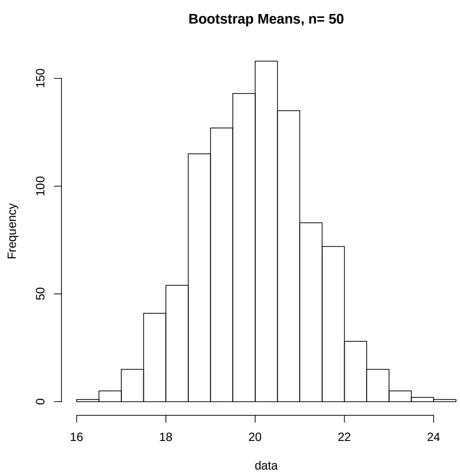
b)

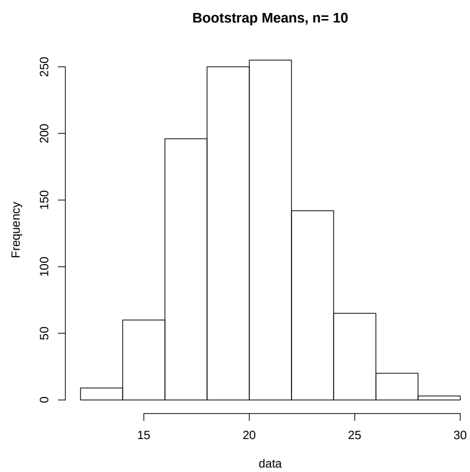
n	Mean	SD
200	20.00	8.94
50	19.98	8.98
10	19.98	9.00

c)

n	Mean	SE
200	20.00	0.65
50	19.98	1.25
10	19.98	2.82



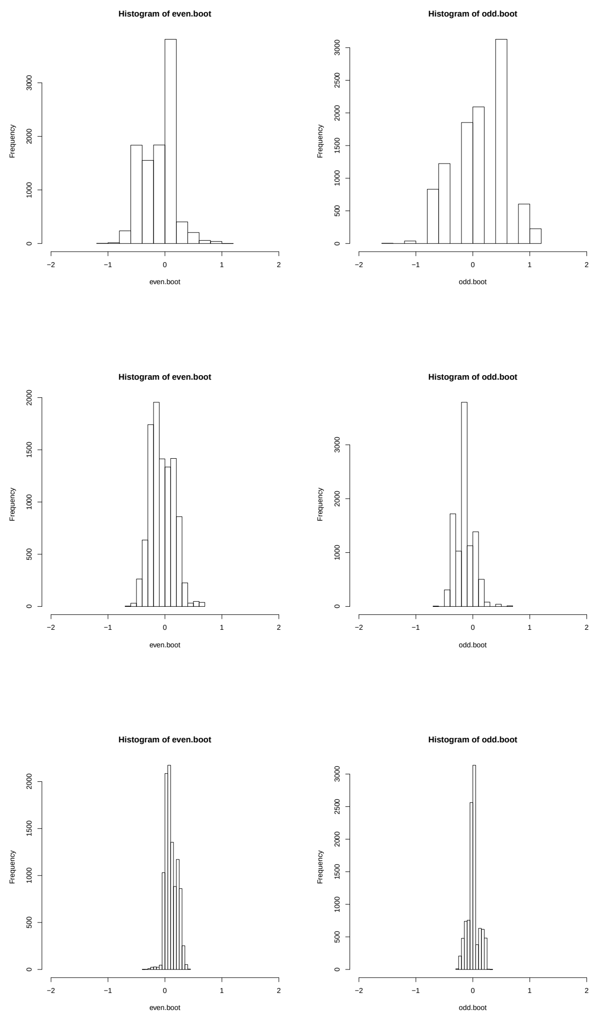




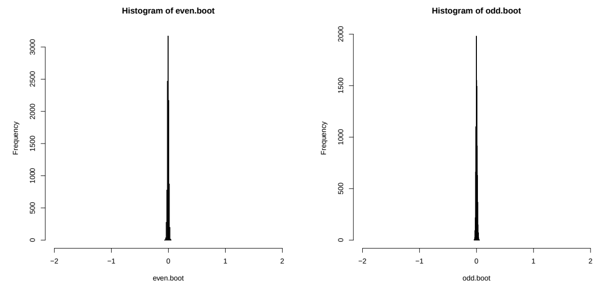
e)

Increasing sample size reduces the bootstrap standard error.

9

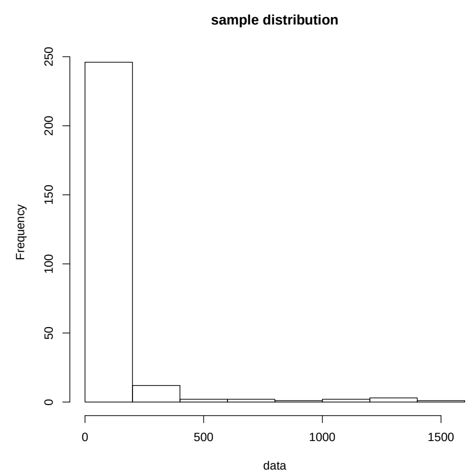


The evenness of N does not seem to affect the histograms, but increasing N reduces the spread.



10

a)

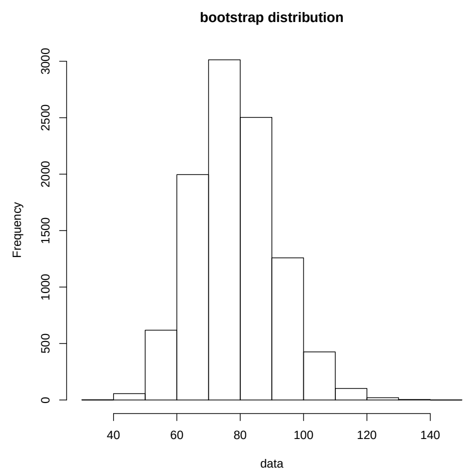


Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
1.00	5.00	14.20	78.08	55.50	1550.00

The data appears to be highly concentrated between 0 and 500, with several outliers that skew the mean.

b)

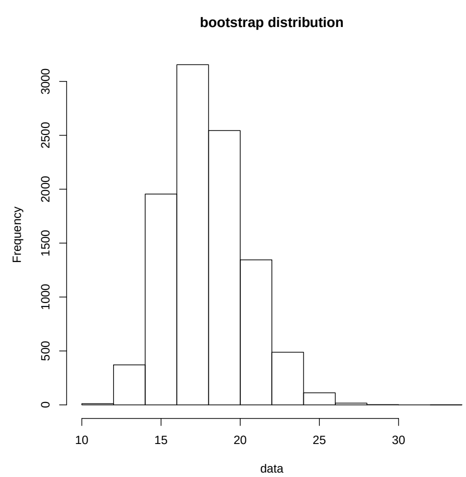
Mean: 78.03
95% confidence interval: 55.0 – 104.61



c)

Bias is 0.37, which is $\approx 3\%$ of the SE.

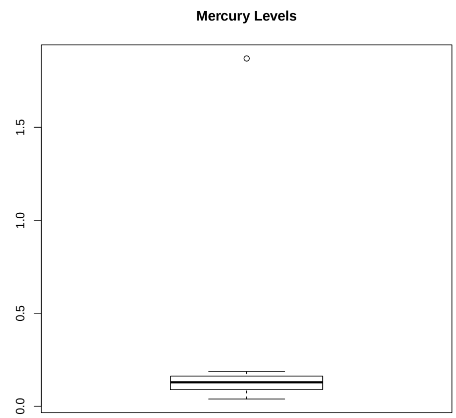
11



Trimmed Mean: 17.90
95% confidence interval: 13.67 – 23.30
Bias is 0.27, which is $\approx 11\%$ of the SE. The new test statistic has a smaller 95% confidence interval at the cost of higher bias.

12

a)



The data is concentrated between 0 and 0.25, with one outlier at 0.9.

b)

Mean: 0.18 ± 0.05
95% Confidence Interval: 0.11–0.31

c)

Mean: 0.19 ± 0.06
95% Confidence Interval: 0.11–0.31

d)

Removing the outlier had little effect on the standard error because it was only one point.

14