

# MATH 308 Assignment 16:

## Exercises 6.4

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**1**

We know that  $X \sim \mathcal{B}(n, p) \implies f_X(x) = \binom{n}{x} p^x (1-p)^{n-x}$ . Thus, the probability of obtaining the value  $X$  can be written as a function of the parameter  $p$ :

$$f(p) = \binom{n}{X} p^X (1-p)^{n-X}$$

$$\implies f' / \binom{n}{X} = X p^{X-1} (1-p)^{n-X} - p^X (n-X) (1-p)^{n-X-1}$$

We can then obtain the MLE  $\hat{p}$  by setting  $f' = 0$ :

$$\begin{aligned} X p^{X-1} (1-p)^{n-X} &= p^X (n-X) (1-p)^{n-X-1} \\ \implies \hat{p} (n-X) &= X (1-\hat{p}) \\ \implies \hat{p} n - \hat{p} X &= X - X \hat{p} \\ \implies \hat{p} &= X/n \quad \square \end{aligned}$$

**2**

$X \sim \mathcal{P}(\lambda) \implies f_X = \frac{\lambda^x e^{-\lambda}}{x!}$ . So, the likelihood of obtaining the sample  $\{x_1, x_2, \dots, x_n\}$  is  $L(\lambda) = \prod_{i=1}^n f_X(x_i)$ , since the elements of the sample are i.i.d.

$$\begin{aligned} L &= \prod_{i=1}^n \frac{\lambda^{x_i} e^{-\lambda}}{x_i!} \\ &= \frac{\lambda^{\sum x_i} e^{-n\lambda}}{\prod (x_i!)} \end{aligned}$$

$$\begin{aligned} \implies L' / \prod (x_i!) &= (\lambda^{\sum x_i} e^{-n\lambda})' \\ &= \sum x_i \lambda^{\sum x_i - 1} e^{-n\lambda} + \lambda^{\sum x_i} e^{-n\lambda} (-n) \\ &= e^{-n\lambda} \lambda^{\sum x_i - 1} (\sum x_i - n\lambda) \end{aligned}$$

Setting  $L' = 0$ , we get  $\hat{\lambda}$ :

$$\begin{aligned} 0 &= \sum x_i - n\hat{\lambda} \\ \implies \hat{\lambda} &= \sum x_i / n = \bar{x} \quad \square \end{aligned}$$

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