MATH 308 Assignment 12: Sequences in Coin Flips

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March 26, 2014

1 Expected Number of Occurences

For each coin flip i, let X_i be the event that the sequence 01111111 occurs starting at position i. Then:

$$\mathbb{E}\left(\mathbb{1}(X_i)\right) = \begin{cases} 2^{-8}, & \text{if } 1 \leq i \leq 93 \\ 0, & \text{if } i > 93 \end{cases}$$

So, the expected number of occurences is:

$$\begin{aligned} N &= \mathbb{E}\left(\sum_{i=1}^{100} \mathbb{1}(X_i)\right) \\ &= \sum_{i=1}^{100} \mathbb{E}\left(\mathbb{1}(X_i)\right) \\ &= \sum_{i=1}^{93} \mathbb{E}\left(\mathbb{1}(X_i)\right) \\ &= 93/2^8 \approx 0.363 \end{aligned}$$

2 Probability of Occurence

Using the result above, we can approximate the coin-flipping experiment as a Poisson process with rate parameter $\lambda = 93/2^8$.

The requisite probability can then be calculated as `1-ppois(0,93/2^8)`, which comes out to ≈ 0.30, which is slightly under the result from Assignment 2.