For full credit show all your work.

1. Write the contrapositive of the following statements:

   (a) If $a \leq b$ then $a \leq \frac{a+b}{2} \leq b$.

   (b) If $\forall x \in [a, b]$ $f'(x) > 0$ then $f(x)$ is increasing on $[a, b]$.

2. Prove: If $|a - 3| < 1$ then $\sqrt{a} < 2$.

3. Prove: If $\forall \varepsilon > 0$ we have that $a < \varepsilon$ then $a \leq 0$

4. Prove that $5^{2n} - 1$ is even for all $n \in \mathbb{N}$.

5. Prove that $2^n < n!$ for all $n \geq 4$.

6. Use induction to prove that for all $n \in \mathbb{N}$ that $\displaystyle\sum_{k=1}^{n} \frac{1}{k(k+1)} = \frac{n}{n+1}$.