DSP Ends - 2021

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Undertaking: I certify that I have not violated the university code of conduct during this exam

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Question 1:

4-Point DFT of sin[(2*Pi/N)*n]

$$DFT = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi kn}{N}}$$

Here $N = 4$

$$x[0] = \sum_{n=0}^{3} \sin\left(\frac{\pi}{2} \times n\right) e^{-j\frac{\pi n(0)}{2}}$$

$$x[0] = 0 + 1 + 0 - 1$$

$$x[0] = 0$$

$$x[1] = \sum_{n=0}^{3} \sin\left(\frac{\pi}{2} \times n\right) e^{-j\frac{\pi}{2}n(1)}$$

$$= 0 + 1e^{-j\pi/2} + 0 - e^{-j3\pi/2}$$

$$= \cos(\pi/2) - j\sin(\pi/2) - \left[\cos 3\pi/2 - j\sin 3\pi/2\right]$$

$$= 0 - j - j = -2j$$

$$x[2] = \sum_{n=0}^{3} \sin\left(\frac{\pi}{2} \times n\right) e^{-j\frac{\pi}{2}n(2)}$$

$$= 0 + 1\left(e^{-j\pi}\right) + 0 - e^{-j3\pi}$$

$$= \left(\cos\pi - j\sin\pi\right) - \left(\cos(3\pi) - \sin(3\pi)\right)$$

$$= -1 - (-1)$$

$$= 0$$

$$x[3] = \sum_{n=0}^{3} \sin\left(\frac{\pi}{2} \times n\right) e^{-j\frac{\pi}{2} \times 3n}$$

$$= 0 + 1e^{-j\frac{3\pi}{2}} + 0 - e^{-j\frac{9\pi}{2}}$$

$$= \left(\cos\frac{3\pi}{2} - j\sin\frac{3\pi}{2}\right) - \left\{\cos\left(\frac{9\pi}{2}\right) - j\sin\left(\frac{9\pi}{2}\right)\right\}$$

$$= (+j) - (0 - j)$$

$$= 2j$$

~~DTFT~~

DFT ~~DTFT~~ of $\sin(2\pi i/N) * N$ is —

$$\boxed{x(k) = \{0, -2j, 0, 2j\}}$$

$\downarrow$

N=4 Point DFT.

Question-2 Properties of DFT

(1) Linearity: $F[a\,x[n] + b\,y[n]]$

$= a\,X(e^{j\omega}) + b\,Y(e^{j\omega})$

(2) Time shifting: $F[x(n-n_0)] = e^{-jn_0\omega}X(e^{j\omega})$

Proof: $F[x(n-n_0)] = \sum_{n=-\infty}^{\infty} x[n-n_0]\,e^{-j\omega n}$

$n' = n - n_0$

$F[x(n-n_0)] = \sum_{n=-\infty}^{\infty} x(n')\,e^{-j\omega(n'+n_0)}$

$= e^{-j\omega n_0}X(e^{j\omega})$

(3) Differencing:

$F[x(n) - x[n-1]] = (1-e^{-j\omega})X(e^{j\omega})$

Proof:
$F[x(n) - x(n-1)] = F[x(n)] - F(x[n-1])$

$X(e^{j\omega}) - X(e^{j\omega})e^{-j\omega} = (1-e^{-j\omega})X(e^{j\omega})$

(4) Frequency shifting :

$$F[x[m] e^{jW_0 m}] = X(e^{j(W-W_0)})$$

(5) Convolution :

$$F[x(n) * y(n)] = X(e^{jW}) Y(e^{jW})$$

Proof =
$$\sum_{n=-\infty}^{\infty} \left[\sum_{n=-\infty}^{\infty} x[m]\, y(n-m) \right] e^{-jnW}$$

$$= \sum_{n=-\infty}^{\infty} x[m] \sum_{n=-\infty}^{\infty} y(n-m)\, e^{-j(n-m)W} e^{-jmW}$$

$$= X(jW) \cdot Y(jW)$$

(6) Time Reversal –

$$F[x[-m]] = X(e^{-jW})$$

(7) ~~Linearity~~ :

~~Proof~~

Question - 3

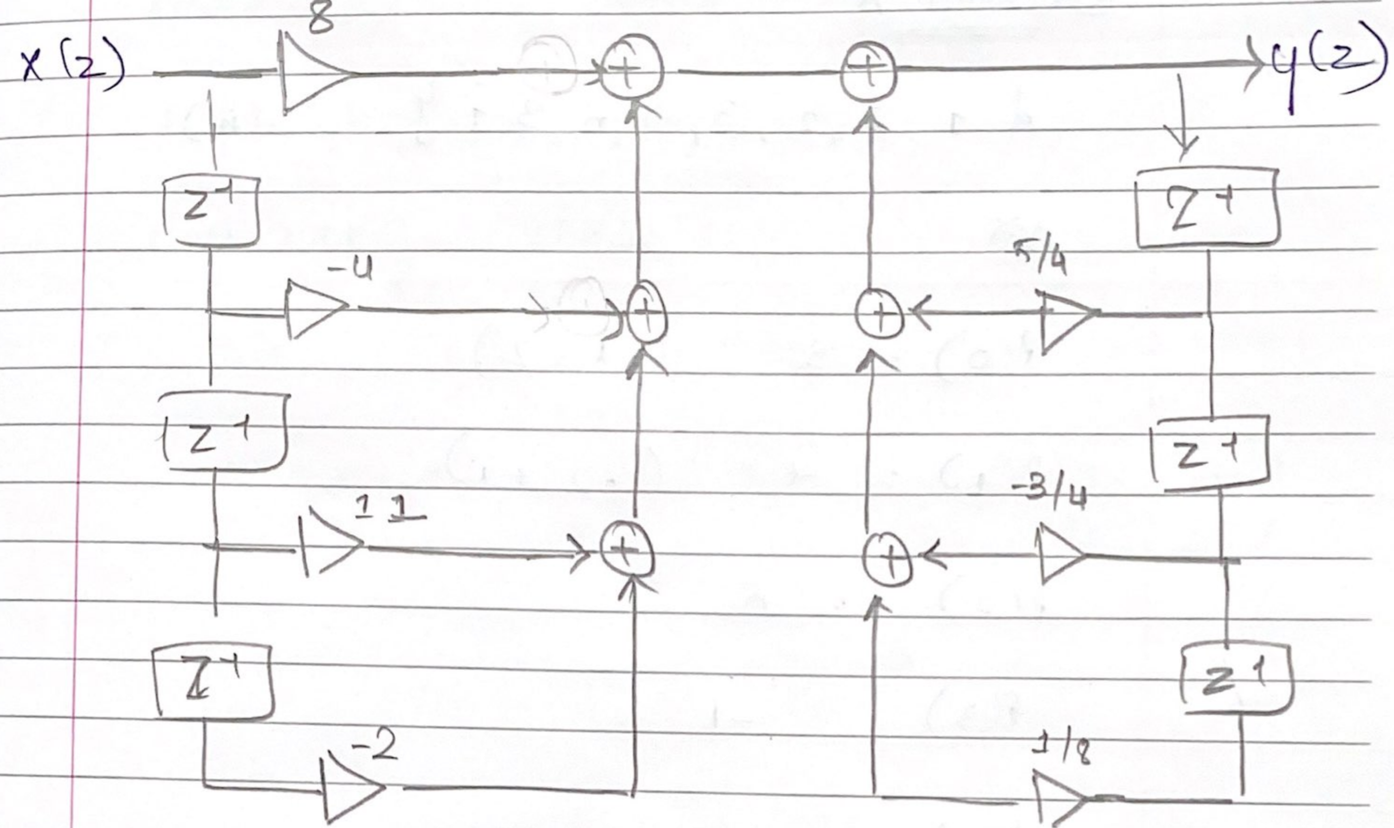
$$H(z) = \frac{\left(8z^3 - 4z^2 + 11z - 2\right)}{\left(z - \frac{1}{4}\right)\left(z^2 - z + \frac{1}{2}\right)}$$

$$= \frac{8z^3 - 4z^2 + 11z - 2}{z^3 - z^2 + \frac{z}{2} - \frac{z^2}{4} + \frac{z}{4} - \frac{1}{8}}$$

$$= \frac{8z^3 - 4z^2 + 11z - 2}{z^3 - \frac{5z^2}{4} + \frac{3z}{4} - \frac{1}{8}}$$

$$\frac{y(z)}{x(z)} = \frac{8 - 4z^{-1} + 11z^{-2} - 2z^{-3}}{1 - \frac{5}{4}z^{-1} + \frac{3}{4}z^{-2} - \frac{1}{8}z^{-3}}$$

$$y(z) = \frac{5}{4}z^{-1}y(z) - \frac{3}{4}z^{-2}y(z) + \frac{1}{8}z^{-3}y(z)$$
$$+ 8x(z) - 4z^{-1}x(z) + 11z^{-2}x(z)$$
$$- 2z^{-3}x(z)$$



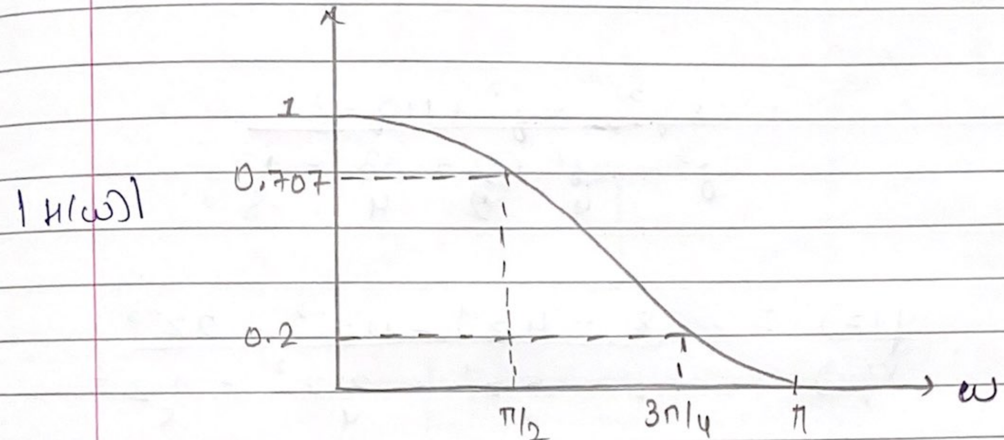
Direct form I

Question : 4

Given that —

$(0.5)^{1/2} \le |H(e^{j\omega})| \le 1 \quad , \quad 0 \le \omega \le \pi/2$

$|H(e^{j\omega})| \le 0.2 \quad , \quad 3\pi/4 \le \omega \le \pi$



$A_1 = 0.707$ and $\omega_P = \pi/2$

$A_2 = 0.2$ and $\omega_s = 3\pi/4$

We design butterworth low pass filter for T = 1 sec

$$\Omega_P = \frac{2}{T} \tan \frac{\omega_P}{2}$$

$$= \frac{2}{1} \tan \pi/4 = \boxed{2 = \Omega_P}$$

lly $\Omega_s = \frac{2}{T} \tan \frac{\omega_s}{2} = \boxed{\Omega_s = 4.82}$

to find order,

$$N \geq \frac{\frac{1}{2}\log\left\{\left(\frac{1}{A_2^2}-1\right)\left(\frac{1}{A_1^2}-1\right)\right\}}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

$$\geq \frac{\frac{1}{2}\log\left\{\left(\frac{1}{(0.2)^2}-1\right)\left(\frac{1}{(0.707)^2}-1\right)\right\}}{\log(1.207)}$$

$$\geq \frac{\frac{1}{2}\log(24(1))}{0.0817}$$

$$\geq \frac{0.6901}{0.0817} \approx 8$$

$$\boxed{N = 9}$$

Determinine cutoff freq :-

$$\Omega_c = \frac{\Omega_p}{\left(\frac{1}{A_1^2}-1\right)^{1/2N}} = \frac{2}{\left(\frac{1}{(0.707)^2}-1\right)^{1/2N}}$$

$$\boxed{\Omega_c = 2 \text{ rad/sec}}$$

Poles

$$H_a(s) = \frac{\Omega_c}{s+\Omega_c} \prod_{k=1}^{(N-1)/2} \frac{\Omega_c^2}{s^2 + b_k \Omega_c s + \Omega_c^2}$$

$$b_k = 2\sin\left(\frac{(2k-1)\pi}{2N}\right)$$

$$b_1 = 2\sin\left(\frac{\pi}{8}\right) =$$

Question - 5

$\{1, 2, 2, 3, 5, 5, 3, 1\} = x(n)$

$P(0) = 3 \quad (1+2)$

$P(1) = 4 \quad (-2+1)$

$P(2) = 5$

$P(3) = -1$

$P(4) = 10$

$P(5) = 0$

$P(6) = 4$

$P(7) = 2$

Now finding G and H

$G(0) = 3 + 5 = 8$

$G(1) = -1 + j = -1+j$

$G(2) = -2$

$G(3) = -1 - j$

$H(0) = 14$, $H(1) = -2j$

$H(2) = 6$, $H(3) = 2j$

Now finding $x(0)$

$x(0) = 8 + 14 = 22$

$x(1) = (-1+j) + -2j\left(\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}\right)$

$= -1 + j - \frac{2j}{\sqrt{2}} - \frac{2}{\sqrt{2}}$

$= \frac{-\sqrt{2}-2}{\sqrt{2}} + j\left(\frac{\sqrt{2}-2}{\sqrt{2}}\right)$

$x(2) = -2 + 6(-j)$

$x(2) = -2 - 6j$

$x(3) = (-1-j) + (2j)\left(\frac{-1}{\sqrt{2}} \frac{-j}{\sqrt{2}}\right)$

$= (-1-j) - \frac{2j}{\sqrt{2}} + \frac{2}{\sqrt{2}}$

$x(3) = \left(\frac{-\sqrt{2}+2}{\sqrt{2}}\right) + j\left(\frac{-\sqrt{2}-2}{\sqrt{2}}\right)$

$$x(4) = -14 + 8$$

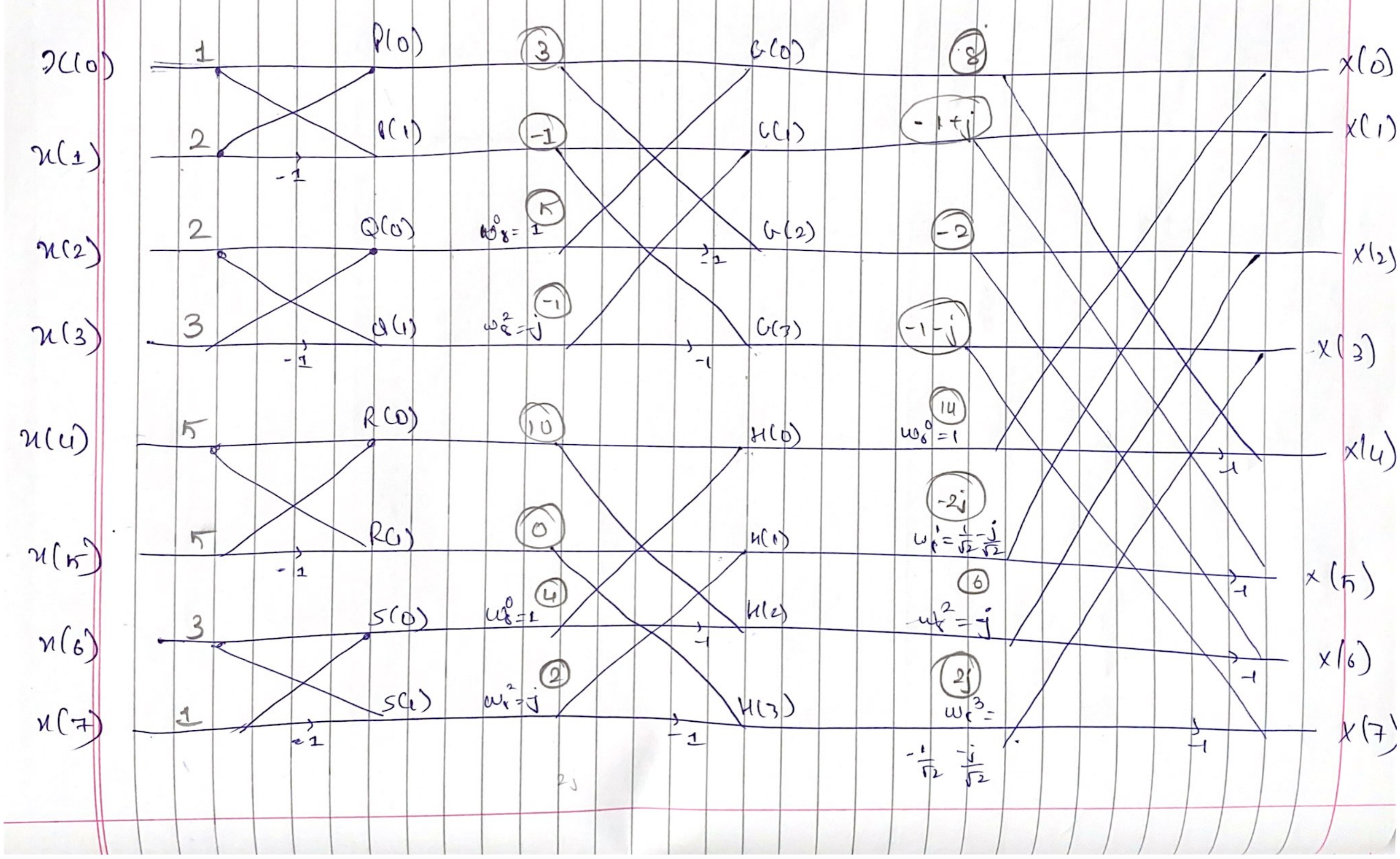
$$x(4) = -6$$

$$x(5) = \left(\frac{+2j}{\sqrt{2}} + \frac{2}{\sqrt{2}} \right) + (-1 + j)$$

$$x(5) = \left(\frac{2 - \sqrt{2}}{\sqrt{2}} \right) + \left(\frac{2 + \sqrt{2}}{\sqrt{2}} \right) j$$

$$x(6) = 6j - 2$$

$$x(7) = \frac{2j}{\sqrt{2}} + \frac{2}{\sqrt{2}} - 1 - j$$



x(0)
x(1)
x(2)
x(3)
x(4)
x(5)
x(6)
x(7)
1
2
2
3
5
5
3
1
-1
-1
-1
-1
P(0)
P(1)
Q(0)
Q(1)
R(0)
R(1)
S(0)
S(1)
3
-1
5
-1
10
0
4
2
$W_8^0 = 1$
$W_8^2 = -j$
$W_8^0 = 1$
$W_8^2 = -j$
G(0)
G(1)
G(2)
G(3)
H(0)
H(1)
H(2)
H(3)
-1
-1
-1
-1
8
-1+j
-2
-1-j
14
-2j
6
2j
$W_8^0 = 1$
$W_8^1 = \frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$
$W_8^2 = -j$
$W_8^3 = -\frac{1}{\sqrt{2}} - \frac{j}{\sqrt{2}}$
-1
-1
-1
-1
X(0)
X(1)
X(2)
X(3)
X(4)
X(5)
X(6)
X(7)

Question : 6

* Mapping of digital filter with analog filters in bilinear transformation —

Mapping in Impulse invarient was given by —

$$z = e^{sT}$$

In bilinear,

$$s = \frac{2}{T}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right)$$

$$\Rightarrow \quad s = \frac{2}{T}\left(\frac{z-1}{z+1}\right)$$

Now, $z = re^{j\omega}$

$$s = \frac{2}{T}\left(\frac{re^{j\omega} - 1}{re^{j\omega} + 1}\right)$$

Now multipling by $re^{j\omega} + 1$ on both sides —

$$s = \frac{2}{T}\left(\frac{re^{j\omega} - 1}{re^{j\omega} + 1}\right)\left(\frac{re^{j\omega} + 1}{re^{j\omega} + 1}\right)$$

$$= \frac{2}{T}\left[\frac{r^2 - 1}{1 + r^2 + 2r\cos\omega} + \frac{j\,2r\sin\omega}{1 + r^2 + 2r\cos\omega}\right]$$

NOW $S = \cancel{\sigma + j\omega}\ \sigma + j\Omega$

σ - real part

Ω - img part

$$\sigma = \frac{2}{T}\left[\frac{r^2-1}{1+r^2+2r\cos\omega}\right] \quad -① $$

$$\Omega = \frac{2}{T}\left[\frac{2r\sin\omega}{1+r^2+2r\cos\omega}\right] \quad -② $$

From eqn ①,

if $r < 1$, then $\sigma < 0$

if $r > 1$ then $\sigma > 0$

and $r = 1$ then $\sigma = 0$

From eqn ②

$\sigma = 0$ then $r = 1$

$$\Omega = \frac{2}{T}\left(\frac{2\sin\omega}{1+1+2\cos\omega}\right)$$

$$= \frac{2}{T}\left(\frac{\sin\omega}{1+\cos\omega}\right)$$

$$= \frac{2}{T}\left(\frac{2\sin\omega/2\ \cos\omega/2}{1+2\cos^2\omega/2-1}\right)$$

$$\Omega = \frac{2}{T}\left(\frac{2\sin \omega/2}{\cos \omega/2}\right) = \frac{2}{T}\tan \omega/2$$ (3)

Now if $\omega = \pi$ then

$$\Omega = \frac{2}{T} \times \tan \pi/2$$

$$= \frac{2}{T} \times \infty = \infty$$

Similarly if $\omega = -\pi$ then

$$\Omega = \frac{2}{T} \times \tan(-\pi/2) = -\infty$$

Hence mapping from $-\pi$ to π in digital domain is transformed to $-\infty$ to ∞ in analog

$\therefore \quad -\pi \le \omega \le \pi \Rightarrow -\infty \le \Omega \le \infty$

Hence we overcome ~~this method~~ mapping drawback of impulse invariant method using bilinear transformation.