

8-point Decimation in time fast Fourier Algorithm

In DFT :-
$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j \frac{2\pi}{N} kn}$$

in order to obtain FFT,
we divide this into odd and even numbers

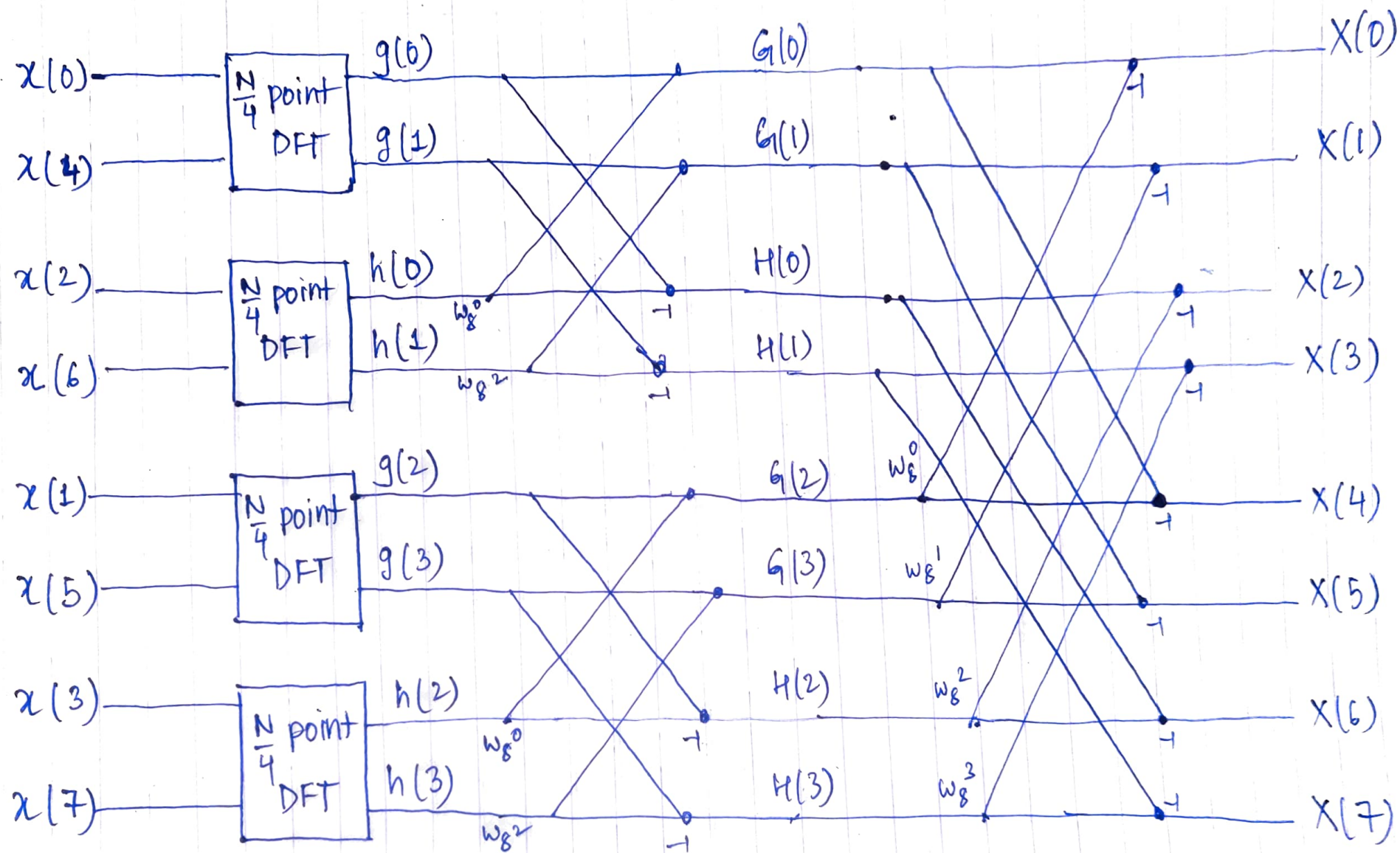
$$\begin{aligned} X(k) &= \underbrace{\sum_{m=0}^{\frac{N}{2}-1} x(2m) e^{-j \frac{2\pi}{N} k(2m)}}_{\text{Even Samples}} + \underbrace{\sum_{m=0}^{\frac{N}{2}-1} x(2m+1) e^{-j \frac{2\pi}{N} k(2m+1)}}_{\text{odd-Samples}} \\ &= \sum_{m=0}^{\frac{N}{2}-1} x(2m) e^{-j \frac{2\pi km}{N/2}} + \sum_{m=0}^{\frac{N}{2}-1} x(2m+1) e^{-j \frac{2\pi mk}{N/2}} \cdot \underbrace{e^{-j \frac{2\pi k}{N}}}_{\text{Constant}} \\ &= \underbrace{\sum_{m=0}^{\frac{N}{2}-1} x(2m) e^{-j \frac{2\pi km}{N/2}}}_{\frac{N}{2} \text{ point DFT}} + e^{-j \frac{2\pi k}{N}} \underbrace{\sum_{m=0}^{\frac{N}{2}-1} x(2m+1) e^{-j \frac{2\pi mk}{N/2}}}_{\frac{N}{2} \text{ point DFT}} \end{aligned}$$

↑
twiddle factor (W_N^k)

but Here we want a 8-point Decimation
So we will consider $N/4$ point DFT

$$\begin{aligned} X(k) &= \left\{ \frac{N}{4} \text{ point DFT} + W_{\frac{N}{2}}^k \left[\frac{N}{4} \text{ point DFT} \right] \right\} \\ &\quad + \\ &\quad W_{\frac{N}{2}}^k \left\{ \frac{N}{4} \text{ point DFT} + W_{\frac{N}{2}}^k \left[\frac{N}{4} \text{ point DFT} \right] \right\} \end{aligned}$$

final butterfly structure



Now the Computational Complexity for DFT will be

- a.) N^2 for Complex Multiplication
- b.) $N(N-1)$ for Complex addition

And the Computational Complexity for FFT will be

- a.) $\frac{N}{2} \log_2 N$ for complex multiplication

- b.) $N \log N$ for Complex addition

So we can say FFT is much faster than DFT when it comes to computational complexity

Now I have used a table to justify it with values for DFT & FFT multiplication. (and also have indicated the ratio).

	N	DFT (N^2)	FFT ($\frac{N}{2} \log_2 N$)	Ratio
1.]	2	4	1	4:1
2.]	4	16	4	4:1
3.]	8	64	12	
4.]	64	4096	192	
5.]	1024	1048576	5120	