Ahmedabad University
School of Engineering and Applied Science

Winter 2021 Semester

Digital Signal Processing Lab-7
Part -2

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Question 2:

Approach:

Using the given information, we first find sampling frequency and then calculate bandwidth and center frequency .
Using cheby1 function we find transfer function coefficients and then convert it from lowpass to bandpass filter by impinvar function.

We then plot its frequency response and zeros/poles.
For N=2, we simply put coefficients in freqz command that are already found in hand written analysis and plot its frequency response and zeros/poles.

Code:

```
clear all;
close all;
clc;


%%%%%% Calculating for N = 4
N = 4;

Wh = 0.6*pi; % High cutoff frequency
```

```
Wl = 0.3*pi; % Low cutoff frequency

r = 0.11; % Passband Ripple

sampling_time = 0.1; % Sampling Time
sampling_frequency = 1/sampling_time; % Sampling frequency

% Center frequency
Wo = sqrt(Wl*Wh)/sampling_time;
% Bandwidth
Bw = (Wh - Wl)/sampling_time;
% Converting ripple to dB
Rp = -20*log10(1-r);

 % Returns transfer function coefficients
[b,a] = cheby1(N,Rp,1,'s');

% Transform analog filter lowpass filter into bandpass
filter
[bt,at] = lp2bp(b,a,Wo,Bw);

 % Converting from analog to digital
[bz,az] = impinvar(bt,at,sampling_frequency);

% frequency response of lowpass butterworth filter for N=4
figure;
freqz(bz,az);

% poles and zeros of lowpass butterworth filter for N=4
figure;
zplane(bz,az);
figure;
```

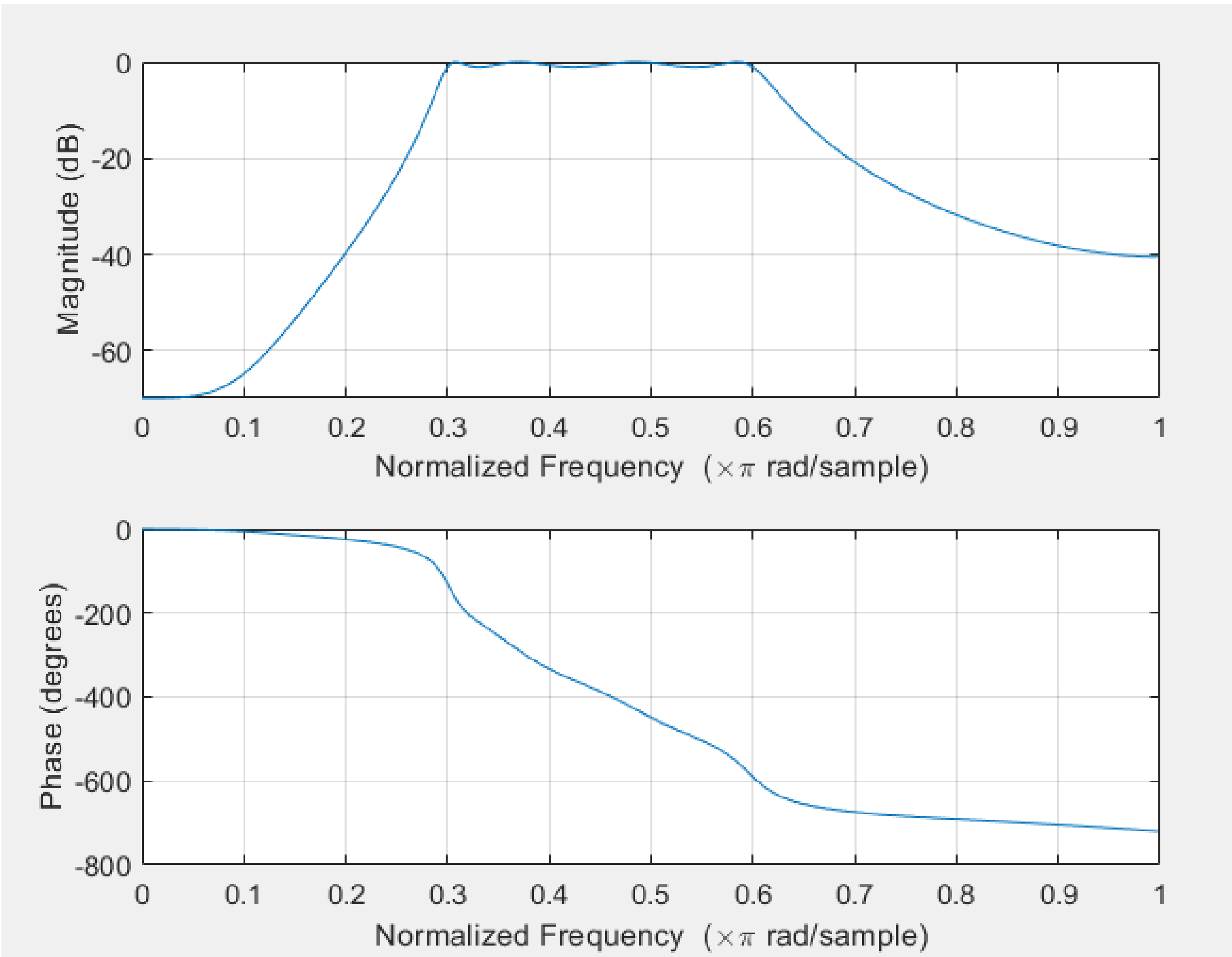
```
% frequency response of lowpass butterworth filter for N=2
b = [0 1.795 -5.29 2.443];
a = [1 -0.6454 0.9 -0.269 0.358];
freqz(b, a);

figure;
% poles and zeros of lowpass butterworth filter for N=2
zplane(b, a);
```

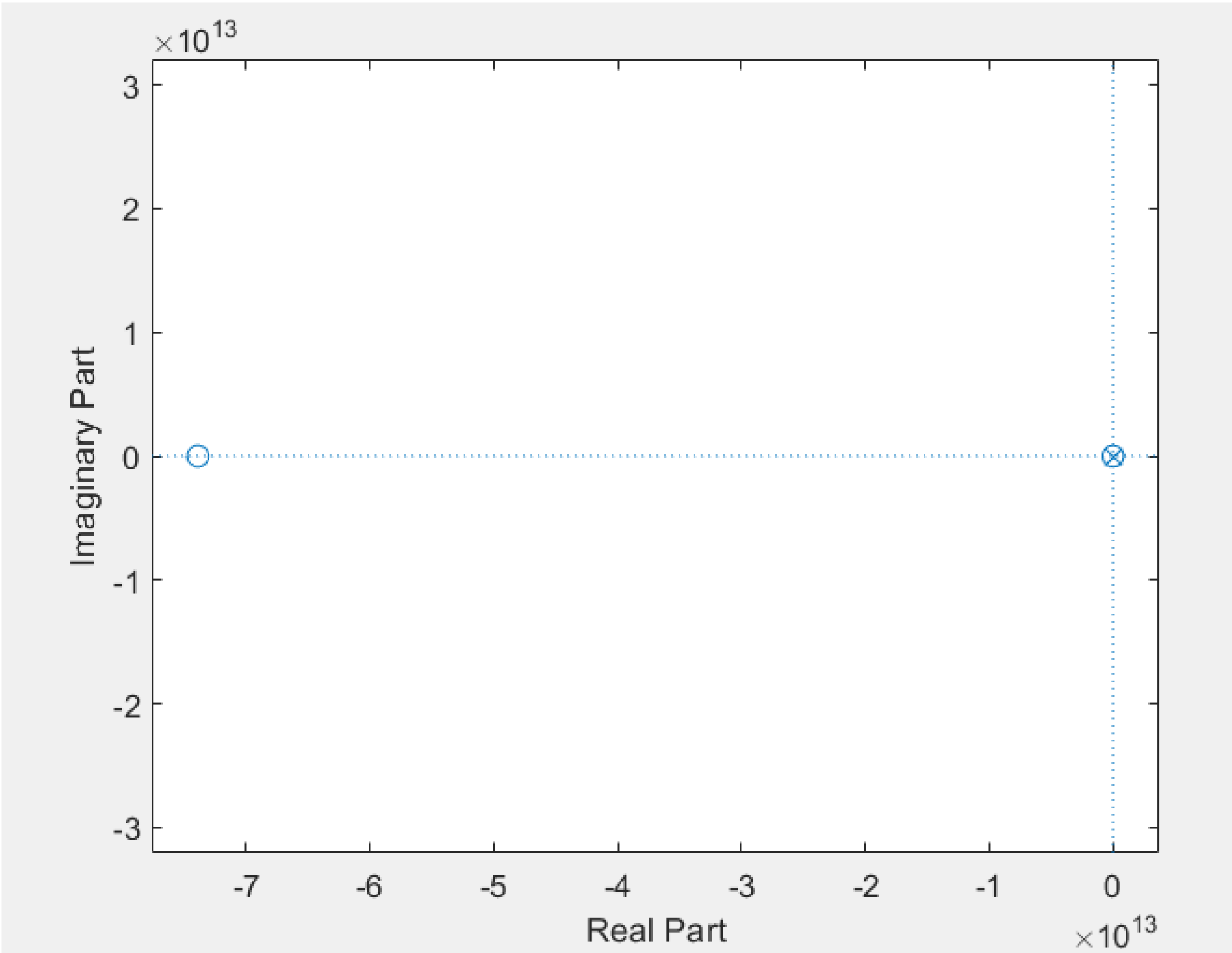
Code for finding partial fractions:

```
clear all;
close all;
clc;

a = [86.6 0 0];
b = [1 10.2921 452.747 1828.36 31559.52];

[r, p, k] = residue(a, b);
disp(r)
disp(p)
disp(k)
```

Figure:

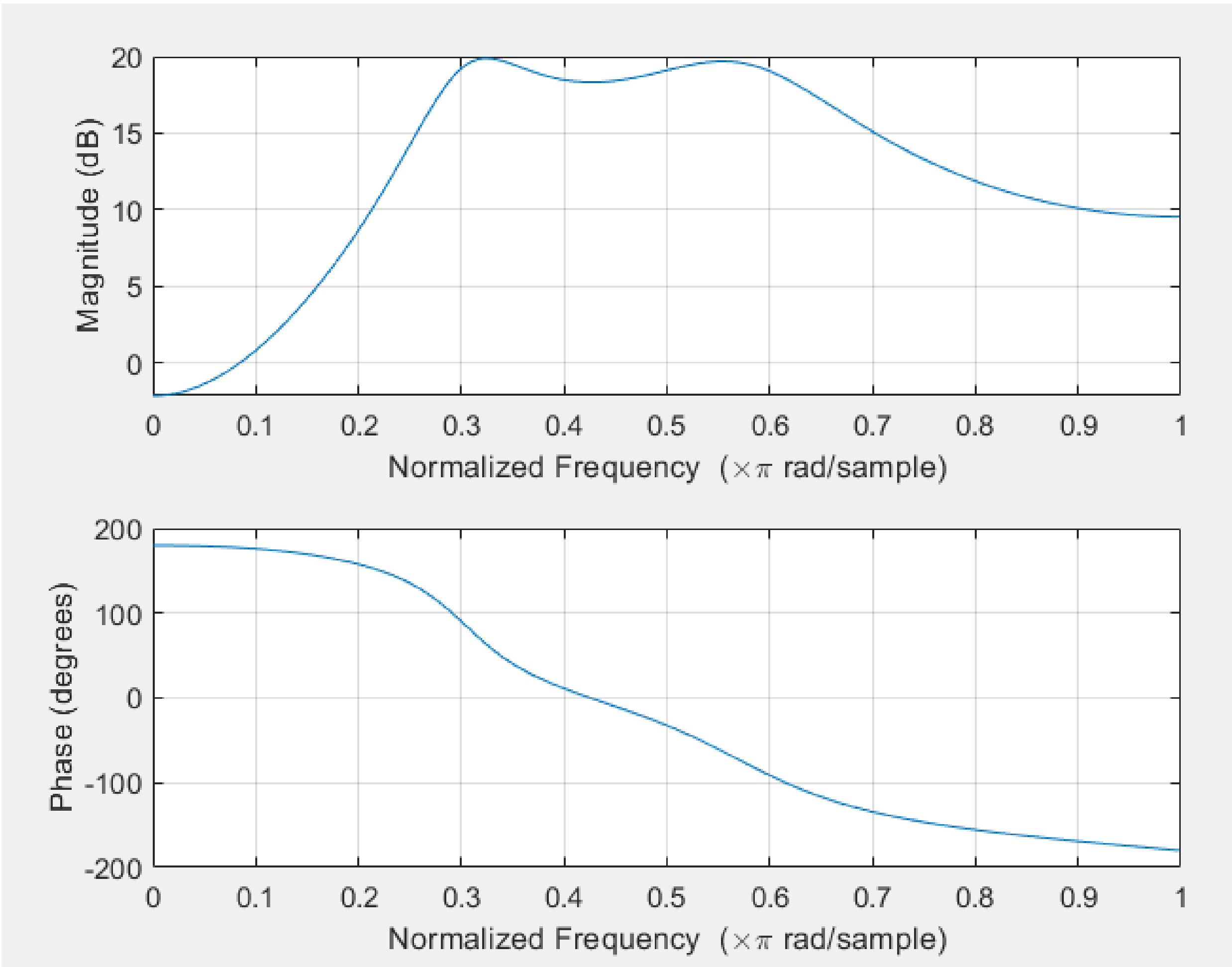
For N = 4

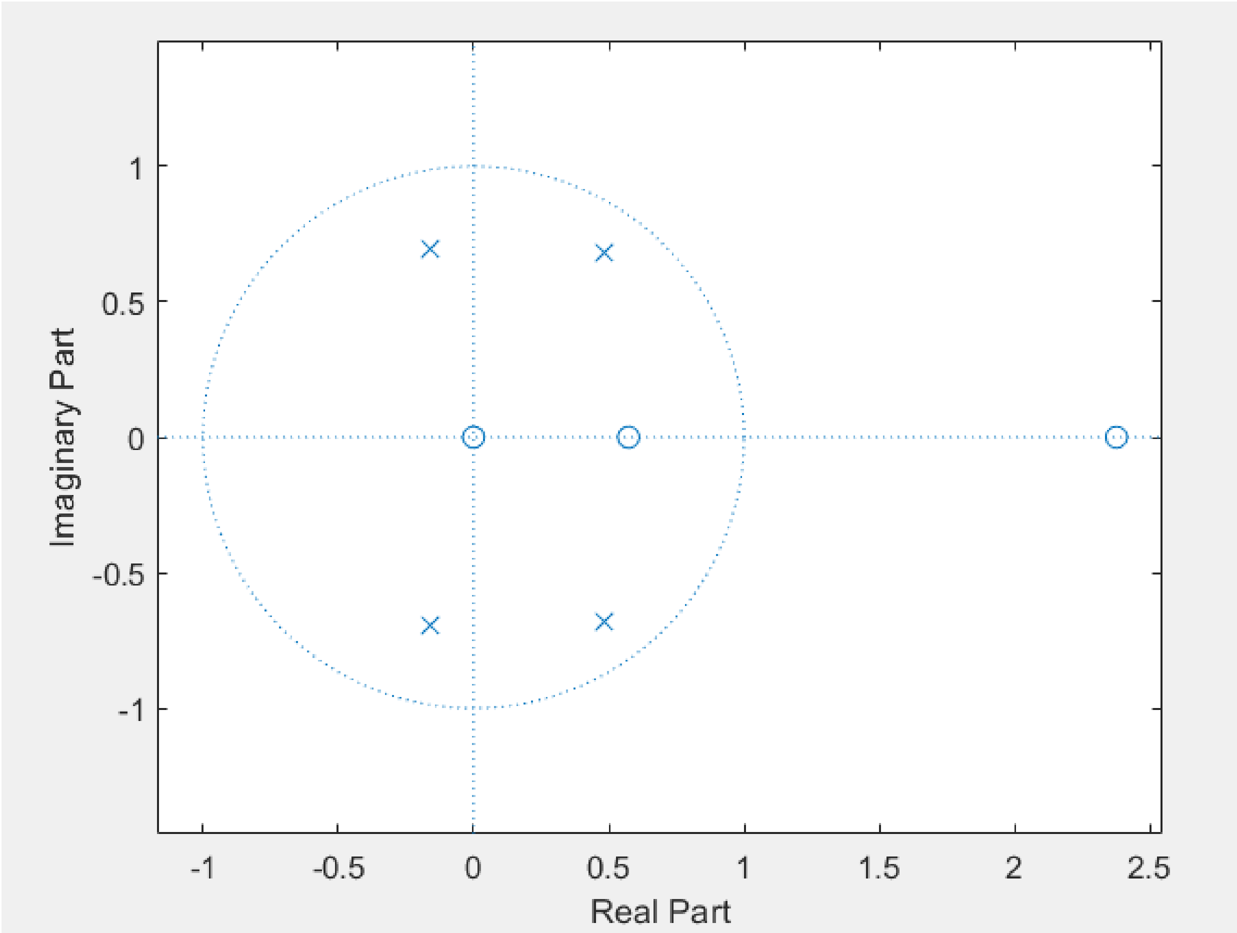




×10^13
3
2
1
0
-1
-2
-3
Imaginary Part
-7
-6
-5
-4
-3
-2
-1
0
Real Part
×10^13

For N = 2;


Magnitude (dB)
20
15
10
5
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Normalized Frequency (×π rad/sample)
Phase (degrees)
200
100
0
-100
-200
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Normalized Frequency (×π rad/sample)



1
0.5
0
-0.5
-1
Imaginary Part
-1
-0.5
0
0.5
1
1.5
2
2.5
Real Part

DSP - Lab - 8

AU1841070
Nancy Rodadia

(2) Given that,

Cutoff freq (ω_L) = 0.3π
Passband ripple (δ_P) = 0.11
higher cutoff freq (ω_H) = 0.6π
Sampling Time (T) = 0.1

For, N = 2

$$\varepsilon^2 = \frac{1}{(1-\delta_P)^2} - 1$$

$$= \frac{1}{(1-0.11)^2} - 1$$

$$= \frac{1}{0.7921} - 1$$

$$\varepsilon = \sqrt{0.26246}$$

$$\boxed{\varepsilon = 0.5123}$$

$$\beta = \left[\frac{\sqrt{1+\varepsilon^2}+1}{\varepsilon}\right]^{1/N}$$

$$= \left[\frac{\sqrt{1+0.26246}+1}{0.5123}\right]^{1/2}$$

$$= \sqrt{\frac{2.12359}{0.5123}} = \sqrt{4.14521}$$

$$\boxed{\beta = 2.036}$$

Calculating Major axis (r_1)

$$r_1 = \left(\frac{\beta^2+1}{2\beta}\right)$$

$$= \frac{4.145296+1}{4.072}$$

$$\boxed{r_1 = 1.2636}$$

Calculating Minor axis (r_2)

$$r_2 = \left(\frac{\beta^2-1}{2\beta}\right)$$

$$= \frac{4.145296-1}{4.072}$$

$$\boxed{r_2 = 0.7724}$$

For angular positions of poles,

$$\phi_K = \frac{(2K + N + 1)\pi}{2N}$$

For $K=0$, $\phi_0 = \frac{3\pi}{4}$

For $K=1$, $\phi_1 = \frac{5\pi}{4}$

Calculating x_K and y_K

For $K=0$,

$$x_0 = r_2 \cos \phi_0$$

$$= (0.7724) \cos \frac{3\pi}{4}$$

$$= (0.7724)(-0.7071)$$

$$\boxed{x_0 = -0.54616}$$

$$y_0 = r_1 \sin \phi_0$$

$$= (1.2636)(\sin 3\pi/4)$$

$$= (1.2636)(0.7071)$$

$$\boxed{y_0 = 0.89349}$$

For $K=1$,

$$x_1 = r_2 \cos\phi_1$$

$$= 0.7724 \times \cos \frac{5\pi}{4}$$

$$= (0.7724)(-0.7071)$$

$$\boxed{\therefore x_1 = -0.54616}$$

$$y_1 = r_1 \cancel{\cos}\ \sin\phi_1$$

$$= (1.2636) \times \sin 5\pi/4$$

$$= (1.2636)(-0.7071)$$

$$\boxed{y_1 = -0.89349}$$

Equation of Poles

$$S_K = x_K + j y_K$$

$$S_0 = x_0 + j y_0$$

$$\boxed{S_0 = -0.54616 + j0.89349}$$

and

$$\boxed{S_1 = -0.54616 - j0.89349}$$

System function: $H(s) = \dfrac{b_0}{(s-s_1)(s-s_2)}$

$$b_0 = \frac{s_0 \cdot s_1}{\sqrt{1-\varepsilon^2}}$$

$$= \frac{[-0.54616 + j(0.89349)][-0.54616 - j0.89349]}{1.12359}$$

$$b_0 = \frac{1.0966}{1.12359} = 0.9760$$

$$\Omega_L = \frac{\omega_L}{T} = \frac{0.3\pi}{0.1} = 3\pi$$

$$\Omega_H = \frac{\omega_H}{T} = \frac{0.6\pi}{0.1} = 6\pi$$

Lowpass to Bandpass

$$s = \left(\frac{s^2 + (\Omega_L)(\Omega_H)}{s(\Omega_H - \Omega_L)} \right)$$

$$= 1.202731 + 18(\pi)(\pi)$$

System function (Hs) = $\frac{b_0}{(s-s_0)(s-s_1)}$

Substituting values of b_0, s_0 & s_1.

$$H(s) = \frac{0.9760}{s^2 - ss_0 - ss_1 + s_0 s_1}$$

$$H(s) = \frac{0.9760}{\left[\begin{array}{l} s^2 - s(-0.546 + j(0.893)) + 1.0966 \\ - s(-0.546 - j(0.893)) \end{array}\right]}$$

$$H(s) = \frac{0.9760}{s^2 + 1.0923s + 1.0966}$$

Lowpass to Bandpass.

$$s = \left[\frac{s^2 + \Omega_L \Omega_H}{s(\Omega_H - \Omega_L)}\right]$$

$$= \frac{s^2 + 18(3.14)^2}{s \times 3(3.14)}$$

$$= \frac{s^2 + 177.4728}{9.42\,s}$$

Substituting value of s in H(s)

$$H(s) = \frac{0.9760}{\left\{ \left(\frac{s^2 + 177.4728}{(9.42)s} \right)^2 + 1.0923 \left(\frac{s^2 + 177.4728}{(9.42)s} \right) + 1.0966 \right\}}$$

$$= \frac{0.9760}{\left(\frac{s^4 + 31495.6 + 354.94s}{(9.42\,s)^2} \right) + (9.42s)\left(\frac{1.0923 s^2 + 193.853\, s}{(9.42\, s)^2} \right) + \frac{916.5582\; \cancel{1002.3209}\, s^2}{(9.42s)^2}}$$

$$= \frac{(0.9760)(9.42s)^2}{s^4 + 31495.6 + (354.946)s + 10.2894s^3 + 1826\, s^2 + 916.5582 s^2}$$

$$= \frac{(86.606)s^2}{s^4 + 10.2894 s^3 + 2742.5 s^2 + 354.946 s + 31495.6}$$

$$= \frac{86.6 s^2}{s^4 + 10.2921 s^3 + 452.747 s^2 + 1828.36 s + 31559.52}$$

Solving the above partial fractions in matlab we get following –

$$H(s) = \frac{0.4340 - 3.3814j}{s - (-3.3606 + 17.9740j)}$$

$$+ \frac{0.4340 + 3.3814j}{s - (-3.3606 - 17.9740j)}$$

$$+ \frac{(-0.4340) + 1.7586j}{s - (-1.7854 + 9.5499j)}$$

$$+ \frac{(-0.4340) - 1.7586j}{s - (-1.7854 - 9.5499j)}$$

Applying Impulse Invariance method

$$H(z) = \frac{0.4340 - 3.3814j}{1 - [\exp(-(-3.3606 + 17.9740j) \times T)]\, z^{-1}}$$

$$+ \cancel{\left(\frac{0.4340 + 3.3814j}{s - (-3.3606 - 17.9740j)}\right)}$$

$$+ \frac{0.4340 + 3.3814j}{1 - [\exp(-(-3.3606 - 17.974j) \times T]\, z^{-1}}$$

$$+ \frac{(-0.4340) + 1.7586j}{1 - [\exp(-(-1.7854 + 9.54j) \times T]\, z^{-1}} + \frac{(-0.4340) - 1.75j}{1 - [\exp(-(-1.78 - 9.54j)T]\, z^{-1}}$$

$$H(z) = \frac{0.4340 - 3.3814j}{1-(-0.1503 + j\,0.60)z^{-1}}$$

$$+ \frac{0.4340 - 3.3814j}{1-(-0.15 - j0.600)z^{-1}}$$

$$+ \frac{(-0.4340) + 1.7586j}{1-(-0.41 + j\,0.60)z^{-1}}$$

$$+ \frac{(-0.4340) - 1.7586j}{1-(0.41 - j\,0.60)z^{-1}}$$

After simplification we get —

$$H(z) = \frac{1.795z^{-1} - 5.29z^{-2} + 2.443z^{-3}}{1 - 0.6454z^{-1} + 0.9z^{-2} - 0.269z^{-3} + 0.358z^{-4}}$$